How to share it out: The value of information in teams*

Alex Gershkov[♮] Jianpei Li[♯] Paul Schweinzer[§]

27-Nov-2014

Abstract

We study the role of information exchange, leadership and coordination in team or partnership structures. For this purpose, we view individuals jointly engaging in productive processes—a 'team'—as endowed with individual and privately held information on the joint production process. Once individual information is shared, team members decide individually on the effort they exert in the joint production process. This effort, however, is not contractible; only the joint output (or profit) of the team can be observed. Our central question is whether or not incentives can be provided to a team in this environment such that team members communicate their private information and exert efficient productive efforts on the basis of this communication. Our main result shows that there exists a simple ranking-based contract which implements both desiderata in a wide set of situations.

JEL: *C7, D7, D8, L2.*
Keywords: *Moral hazard, Adverse selection, Leadership, Teams.*

"Forming a business partnership is the next best thing to getting married."
The Manufacturing Jeweler, March 1897

*We are grateful to Peter Spencer and Ruqu Wang for helpful comments and discussions. Gershkov wishes to thank the Israel Science Foundation for financial support. Schweinzer is grateful for the hospitality of the University of International Business and Economics. All authors thank for financial support through the University of York Research and Impact Support Fund. [♮]The Hebrew University of Jerusalem, Jerusalem, 91905 Israel and School of Economics, University of Surrey, Guildford, UK, alexg@huji.ac.il. [♯]School of International Trade and Economics, University of International Business and Economics, East Huixin Street No. 10, Beijing 100029, China, lijianpei@uibe.edu.cn. [§]Department of Economics, University of York, Heslington, York YO10 5DD, United Kingdom, paul.schweinzer@york.ac.uk.

1 Introduction

This paper analyzes combined moral hazard and adverse selection problems in teams. Team members exert unobservable effort and the generated output is entirely allocated among the members of the team. Created output depends not only on the chosen efforts but also on an underlying productivity parameter. The source of adverse selection is that the information about this parameter may be privately known to some team members. Therefore, an efficient mechanism must generate the right incentives for both information revelation and the appropriate effort exertion. We ask the following question: is there a mechanism that can accomplish both goals if some additional information on relative performance (e.g., a noisy ranking of the team member's efforts) is available?[1]

The most direct examples of the applicability of the model's formal structure can be found in the professional services industries, especially in law, accounting and, until recently, investment banking, where partnerships are the dominant form of organization (Greenwood & Empson, 2003). Individual performance evaluation is used by many partnerships for promotion decisions or the allocation of bonus payments. The sharing rule we propose can be used to ascertain both efficient information exchange and subsequent effort exertion under these reward schemes.

The global hedge fund industry manages assets worth in excess of \$2.63 trillion (Hedge Fund Research, 2014). Despite this staggering amount, the economics literature has paid little attention to the incentives which motivate the individuals who operate these funds.[2] Our framework represents the strategic environment in which fund managers or, more generally, teams of privately informed partners, do their work. The organizational structure of partnerships (or, synonymously throughout the paper, teams) seems to fit the needs of the asset management industry well. Most hedge funds and investment firms in this industry are privately run as general/limited partnerships. 'Investment clubs' are partnerships in which a small number of members pool their resources to make joint investments. Other examples of investment partnerships, often without limited liability, include 'single family offices,' i.e., private companies that manage investments and trusts for a single, usually very wealthy, family. The main reason why the partnership structure is attractive in the mentioned cases is that the partners have 'skin in the game.' Hence, regulatory oversight is usually minimal because the few heavy-weight partners involved are generally trusted to exert due care in their investment decisions.[3]

The principal elements of our model are private information, unobservable efforts and team

[1] It is well known that the answer to our research question in the absence of additional information on team member efforts is negative. In particular Holmström (1982) shows in a complete information framework that moral hazard is incompatible with efficiency if efforts cannot be observed.

[2] The incentives given to these individuals are not trivial: according to Vardi (2013), the top 40 hedge fund managers and traders earned a combined \$16.7 billion in 2012.

[3] We provide a discussion of further applications and examples in our concluding section.

structure.[4] As organizations can be seen to exist precisely in order to resolve or process informational problems (Coase, 1937; Marschak & Radner, 1972), the introduction of asymmetric information into what is otherwise a classical team production problem seems to be natural. To fix ideas, consider a situation in which one of the team members receives a private signal that affects the outcome of team production.[5] This informed team member may not find it in her best interest to reveal this information truthfully to others. Our main result shows that a team remuneration scheme based on a ranking of partners' efforts exists which can overcome this 'communications dilemma' and implement both efficient information sharing and subsequent efficient efforts. Hence, our mechanism indicates how partner incentives can be structured in order to avoid perverse incentive effects.

The paper's findings can be summarized as follows. Our main result shows that a profit sharing rule exists which subdivides realized team output unevenly among all team members in symmetric equilibrium. This rule ensures the communication of relevant private information by one team member—whom we call the 'team leader'—and subsequent efficient effort exertion by all team members (including the leader) although efforts are not assumed to be contractible. Moreover, the proposed profit sharing rule allocates the entire realized output among the team members and thus balances its budget in and out of equilibrium. The derived sharing rule depends on some statistic of exerted efforts on which team remuneration can be based, for instance, the precision of a contractible ranking of partners' efforts interpreted as a contest among team members. This result is derived for a general environment only restricted by concave output (as a function of the sum of efforts) and convex effort costs.

The main element that our analysis adds to the literature and which allows for a positive solution to the combined problems of Holmström (1982) and Hermalin (1998) is the noisy ranking of team members' efforts. This relative performance information seems to be regularly collected and naturally available as part of incentive schemes in many organizations (Lazear & Shaw, 2007). Moreover, since the required effort information is ordinal rather than cardinal, collecting these statistics represents a weaker informational requirement than what is usually embodied in standard piece-rate based contracts.

Given the classic results of Holmström (1982) for moral hazard in teams and Hermalin (1998) for the adverse selection leadership case, our positive result may be surprising because, in the combined problem, the profit sharing rule needs to address complex, twofold incentives. First, the privately informed leader is able to misrepresent her private information about joint productivity in order to deceive the other team members into providing inefficiently high (or

[4] In team or partnership structures, partners share the profit among themselves. Thus, any incentive mechanism is subject to the constraint to balance the team's budget. Note that many other bilateral or multilateral contractual situations are also subject to the same (implicit) budget restriction (Spulber, 2009, p57, p97).

[5] In our basic model, we only consider a single team member with private information. We generalize this environment later, as part of our extensions, to an arbitrary number of privately informed team members.

low) efforts while planning to capitalize on this response through a low (or high) effort herself. The second incentive problem that the sharing rule must address is that, because of uncontractible efforts, all team members may be tempted to 'free ride' by exerting inefficiently low efforts even if the leader provided correct information. Intuitively, the presented sharing rule can dissuade the leader from this behavior by making sure that, even for misrepresented private information, efficient effort provision given that report remains a best response for both the leader and the other team members. Since our profit sharing rule is explicitly constructed to guarantee this, a pair of misleading report and subsequent inefficient effort is not profitable. Consequently, as the leader has incentives to report her information truthfully, the other team members may base their response on this report which allows for a jointly efficient set of efforts. Our sharing rule is able to overcome the second moral hazard problem through incorporating an appropriately structured contest among all team members which ensures that free riding incentives are counterbalanced with individual winning probabilities based on some statistic of players' efforts.

We subsequently are able to generalize this main result in four directions: *i*) the case of selection among rival projects in which each team member proposes a project on the quality of which she is privately informed, *ii*) for the case where the leader receives only a 'noisy signal' of the true productivity parameter, *iii*) the case of 'information pooling' in which any number of team members need to contribute their private information in order to make efficient production possible, and *iv*) the case of 'leading by example' in which the leader can exert either contractible or non-contractible upfront efforts. In all four extensions the precise formulation of the required sharing rule changes but our principal result, that full efficiency is implementable, is robust to these model variations.

In summary we present a general solution to the communication and coordination problem couched in a classic joint production problem among symmetric team members. We interpret our result as underpinning the emergence of a profit sharing rule as a function of private information which is, in our model, a required factor of the joint production process. If information or knowledge is dispersed, our model implements efficient cooperation between team members who voluntarily share their private information. This captures the process in which a team or partnership can integrate the specialist knowledge of its members as a precondition for subsequently overcoming the free rider problem.

The plan for the remainder of the paper is to first provide a short overview of the two main literatures unified by this paper followed by the model definition and the characterization of efficient efforts in section 2. Section 3 then presents our main result: the derivation of a general profit sharing rule which ensures the communication of private information through a single team leader and subsequent efficient effort provision by all team members. Section 4 proceeds to illustrate several extensions of the main model, i.e., full characterizations of the efficiency inducing sharing rule *i*) in the case of rival project selection, *ii*) in the presence of stochastic signals, *iii*) when information is dispersed among multiple team members (not just the leader), and *iv*) for the case of 'leading by example' in which the leader exerts upfront observable efforts. Finally, section 4 discusses the conditions under which our sharing rule satisfies limited liability. In the concluding section, we offer a discussion of a further set of applications and examples centering on the ideas of leadership and coordination. The proofs of the main results can be found in appendix A. A proof ensuring the existence of the equilibrium that we derive under a broad class of specifications can be found in appendix B, together with an example illustrating equilibrium existence in further cases.

Related Literature

The present paper combines two distinct literatures on team production and information-based leadership into a unified contracting framework. The classical contributions to the literature on moral hazard in teams are Alchian & Demsetz (1972) and Holmström (1982) who establish the impossibility of efficiency in team production under a budget balancing constraint.[6] When efforts are unobservable, players have an incentive to free ride because they share their marginal contributions with other players but bear the marginal costs on their own. Following these, Legros & Matthews (1993) analyze when approximate efficiency in team production can be achieved if one player chooses inefficient effort with a small probability in order to 'monitor' other players. Strausz (1999) shows that efficiency can be achieved if sequential instead of simultaneous effort exertion is considered and players can observe their predecessors' input. Battaglini (2006) analyzes multi-dimensional team production problems and shows that efficiency can be achieved if the dimensionality of team output is sufficiently large. Gershkov et al. (2009) show that a contract based on a partial but verifiable ranking of agents' efforts can implement efficiency.[7] These contributions typically consider pure moral hazard problems and focus on how to mitigate players' free-riding incentives. We contribute to this literature by extending the classical team production setup through the addition of asymmetric information on team productivity. We show that a ranking-based sharing rule can be used to achieve efficiency in this nested problem of moral hazard and adverse selection. Although based on a the idea of a verifiable noisy ranking of players' efforts similar to Gershkov et al. (2009), the new sharing rule has to be carefully designed to take care of both free riding and information transmission incentives.

Interest in information-based leadership problems was initiated by Hermalin (1998); he stud-

[6] This team production literature is distinct from the Principal-Agent framework because of the absence of a principal and the implied requirement for the budget to balance.

[7] Kvaløy & Olsen (2006), and Bonatti & Hörner (2011) analyze team production problems in a repeated setting. Blanes i Vidal & Möller (2013) study the tradeoff between players' incentives to adopt an alternative project and their incentives to invest into the current project.

ies a privately informed player who communicates her information to others and participates in team production. Hermalin (1998) defines a leader as a team member who induces voluntary following by credibly transmitting private information. He shows that an observable sacrifice or tangible upfront investment by the leader (leading-by-example) can mitigate the adverse selection problem. Following this, Komai et al. (2007) analyze whether it is better to concentrate information with a single player or making it transparent among all players. Komai & Stegeman (2010) broaden the study of leading-by-example games to binary participation choice and non-linear utility functions. Zhou (2011) extends this information-based leadership framework to the study of organizational hierarchies. Recent and comprehensive surveys are Ahlquist & Levi (2011) and Hermalin (2012). The contributions in this literature typically focus on information transmission between the players and how to mitigate the impact of adverse selection, while generally leaving the moral hazard problem unresolved. Contrasting with this, we develop a ranking-based compensation scheme in which the prize structure depends on the level of team output and the leader's announcement of the state of the world. This mechanism encourages the leader to truthfully reveal her private information while at the same time eliminating the free-riding incentive of all team members.

There has been intense interest in combined adverse selection and moral hazard problems in principal-agent settings. See Guesnerie et al. (1989) for a comprehensive review of the early literature. In a repeated setting, Rahman (2012) characterizes an optimal contract if the monitor's observations are private and costly. Gershkov & Perry (2012) characterize optimal contracts in a dynamic principal-agent setting with moral hazard and adverse selection (persistent as well as repeated). In a more general dynamic environment, Garrett & Pavan (2012) allow for the possibility of agent turnover and characterize the optimal retention policy. Garrett & Pavan (2014) analyze how the dynamic power of incentives optimally varies with the relationship tenure in a dynamic environment with moral hazard and adverse selection.

2 The model

There is a set $\mathcal{N}$ of $n \geq 2$ symmetric, risk-neutral players. Each player $i \in \mathcal{N}$ exerts effort $e_i \in [0, \infty)$ which needs not, in principle, be verifiable. Effort cost $c(e_i)$ is assumed to be strictly convex with $c(0) = 0$ and $c'(0) = 0$. Efforts generate increasing and concave team output of $y(\alpha, e_1+\cdots+e_n)$ which depends on the sum of players' efforts and the realization of some random variable α, interpreted as productivity parameter, which is distributed according to function F on interval $[a, b]$, with $0 < a < b \leq \infty$. Output $y(\cdot, \cdot)$ is twice continuously differentiable with $y_2(\alpha, 0) > 0$ for any $\alpha \in [a, b]$.[8] We assume that the team output or production function $y(\alpha, e_1 + \cdots + e_n)$ is supermodular, that is, exhibits positive cross derivatives between e_i and α. The signal α is privately observed by player 1, the team leader, while all other team members only know the distribution of α. Throughout, we denote the observed signal by α^* and the reported signal by α'. We call the actual realization of output y^*.

We assume that, in addition to the information about the generated output, there is a tournament that specifies a ranking of the agents according to their exerted efforts. We assume that the ranking is noisy and depends only on the agents' exerted efforts. The outcome of the tournament is observable and verifiable. We employ the following notation: $f^J(e_i, \mathbf{e}_{-i})$ is the probability that player i is ranked jth. We assume that these functions f are symmetric with respect to the identity of the players. Since output is shared entirely among partners and probabilities are additive we have, for any $i \in \mathcal{N}$, e_i and $\mathbf{e}_{-i}$,

$$\sum_{J=1}^{n} f^J(e_i, \mathbf{e}_{-i}) = 1. \tag{1}$$

In addition to differentiability of $f(e_i, \mathbf{e}_{-i})$ with respect to all arguments we assume that, for any $\mathbf{e}_{-i}$, $f^1(e_i, \mathbf{e}_{-i})$ increases with e_i, that is, the probability to be ranked first increases with own effort. A team contract specifies the shares of team output for each player. Budget balancing requires that these shares sum to one across players.

2.1 Efficiency benchmark

We start by defining the socially efficient level of team efforts. Efficient efforts are defined as the set of efforts which maximize social welfare as chosen by a benevolent planner (who knows α^* and can dictate agents' efforts)

$$\max_{e} \; y(\alpha^*, e_1 + \cdots + e_n) - \sum_{i=1}^{n} c(e_i). \tag{2}$$

Hence, symmetric first-best efforts $e^*(\alpha^*) = e_1^*(\alpha^*) = \cdots = e_n^*(\alpha^*)$ are defined through

$$y_2(\alpha^*, ne(\alpha^*)) = c'(e(\alpha^*)). \tag{3}$$

Note that supermodularity of the output function implies that $e^*(\alpha^*)$ is increasing. Therefore, the tournament does not play any role in the efficient outcome but can be used as an information device for implementing the efficient effort choice.

[8] Throughout the paper, h_i, $h \in \{y, f, s\}$, denotes the partial derivative of h with respect to the i^{th} argument. The second derivative with respect to the same i^{th} argument is denoted $h_{i,i}$ and the second order mixed partial derivative with respect to the i^{th} and j^{th} arguments is written $h_{i,j}$. As usual, h' denotes the first derivative of a function with a single argument.

2.2 Dual incentive problem

In our basic setup, only the leader has private information on the value of the group's productivity parameter α^*. Although this information is valuable to everyone, a problem arises if the players share team output in some fixed way because the leader may then have an incentive to lie: intuitively, the leader may find it individually beneficial to claim that the group is in a 'high-productivity' state through some report $\alpha' > \alpha^*$ to induce all other team members to exert high efforts, even if she plans to put in less. The other team members, anticipating this, may then disregard the leader's report. Thus, in this framework, an efficient team contract, while keeping the budget balanced, has to solve a dual incentive problem: *i*) eliciting true information from the leader and *ii*) encouraging efficient effort from both the leader and the other team members.

3 Results

In this section we present the incentive mechanism and our results for the case where information is isolated in the sense that only the team leader, called player 1, has private information. This setup is later generalized to dispersed information where each player receives a private signal on team productivity.

The designer suggests the following mechanism consisting of a ranking-based sharing rule which divides the total generated output y^* and a dynamic structure. At the first stage, after the leader learns her private information and all players observe the proposed sharing rule, players either accept or disagree to participate in the mechanism. If the contract is rejected by at least one agent, the game ends. Conditional on acceptance of all agents, the privately informed player reports her information publicly. At the second stage, all players exert efforts and, after the realization of both output and the ranking of the tournament, the generated team output is shared according to the proposed sharing rule.[9]

This sharing rule depends on the report of the team leader α' and the realized output y^*. We denote by $s^\ell(y^*, \alpha')$ the share of the agent who was ranked ℓ^{th} according to the tournament, when the realized output is y^* and the report of the leader is α'. Budget balancedness implies that, for any y^* and α',

$$\sum_{\ell=1}^{n} s^\ell(y^*, \alpha') = 1. \tag{4}$$

We now show that a leader report $\alpha' = \alpha^*$ is part of a (Perfect) Bayesian Nash equilibrium strategy of the game defined by the above mechanism and that, subsequently, exerting the

[9] This is not the only mechanism that implements efficient efforts. In particular, the direct mechanism, in which the team leader reports her signal privately to the designer and the designer sends effort recommendations to all agents using a similar sharing rule, implements efficiency as well.

efficient effort choices $e^*(\alpha^*)$ constitute the counterpart strategies for all players. The expected utility of player $i \in \mathcal{N}$ from choosing effort level e_i after observing report α' in the true state of the world α^* while the other players choose their equilibrium effort given the reported state, $e^*(\alpha')$, is $u_i(e_i, \mathbf{e}^*_{-i}(\alpha'), \alpha^*) =$

$$\mathbb{E}_{\alpha^*}\left[y(\alpha^*, \sigma(e_i, \mathbf{e}^*_{-i}(\alpha'))) \left(\sum_{\ell=1}^{n} f^\ell(e_i, \mathbf{e}^*_{-i}(\alpha')) s^\ell(y^*, \alpha') \right) \Bigg| \alpha' \right] - c(e_i) \tag{5}$$

in which $(s^1(\cdot,\cdot), \ldots, s^n(\cdot,\cdot))$ is the output- and report-dependent sharing rule. Competitors' report-dependent efforts are

$$\mathbf{e}^*_{-i}(\alpha') = (\underbrace{e^*(\alpha'), ..., e^*(\alpha')}_{n-1 \text{ times}}),$$

output depends on $\sigma(e_i, \mathbf{e}^*_{-i}(\alpha')) = e_i + (n-1)e^*(\alpha')$ and expectations in (5) are over α^* conditional on the reported α'.

Our first result states that ex post efficient efforts by all players, $e^*(\alpha^*)$, can always be obtained as an equilibrium of our game.

Proposition 1. *Efficient, symmetric efforts for all players defined in* (3) *can be implemented through the winner's share* $s^1(y^*, \alpha') =$

$$\frac{1}{n} + \frac{n-1}{ny^* f^1_1(e^*(\alpha'), \mathbf{e}^*_{-i}(\alpha'))}\left[c'(e^*(\alpha')) - \frac{y_2(\alpha^*(y^*, \alpha'), ne^*(\alpha')))}{n} \right] \tag{6}$$

in which $\alpha^*(y^*, \alpha')$ *is the solution to* $y^* = y(\alpha^*, ne^*(\alpha'))$ *and the losers' share* $s^j(y^*, \alpha') = \frac{1-s^1(y^*,\alpha')}{n-1}$ *for all* $j \neq 1$.[10]

All main proofs can be found in appendix A. We postpone the discussion of equilibrium existence to appendix B.

The idea of the proof of proposition 1 is to construct a sharing rule which encourages the team leader to exert the efficient effort level $e^*(\alpha')$ given her own report α' even if the report does not correspond to the true state of the world $\alpha' \neq \alpha^*$. This sharing rule, in addition to solving the moral hazard problem between all agents, provides appropriate incentives for the team leader to report the correct state of the world at the first stage. Given report-contingent equilibrium effort choices by all players, the function $\alpha^*(y^*, \alpha')$ can be interpreted as the productivity parameter that an outsider can deduce from observing realized output together with the leader's report.

The interpretation of how the mechanism works is as follows. Focus first on the second stage of the game, i.e., after some signal α' has been reported by player 1 at the first stage. Individual efforts have two effects: first, they enlarge the total output available for all players to share.

[10] All losers are treated equally under this sharing rule. Alternatively, a sharing rule with (6) as the winning share and appropriately designed, multiple losing shares could also implement full efficiency.

Since the costs of these efforts are born individually, there is the usual free-riding incentive in teams. The contest designed around the appropriately chosen reward system (6) introduces a second effect in which increasing the own effort increases also the chance of winning while simultaneously decreasing the other players' chances. By trading off the first against the second effect, the mechanism can provide players with incentives to exert report-contingent efficient efforts $e^*(\alpha')$.[11] However, under the presence of asymmetric information, the sharing rule needs to balance these effects both in equilibrium and following any possible leader misreport.

Given equilibrium behavior at stage two, the designed mechanism ensures that the privately informed player 1 finds it disadvantageous to choose a pair consisting of a misreport α' at stage one and an inefficient, signal-contingent effort choice at the second stage of the game. A pair consisting of a low misreport (enticing low efforts $e^*(\alpha')$ of the uninformed players in equilibrium), together with higher than efficient effort is undesirable under the reward structure (6) because individual, convex effort cost is too high relative to the appropriately chosen winner's share of (lower) total output. Similarly, a high misreport (enticing high efforts $e^*(\alpha')$ of the uninformed players in equilibrium) together with low own efforts (and costs) to win a larger prize is discouraged because a well designed losing prize decreases in realized output. Since, therefore, the informed player 1 has appropriate incentives to truthfully report her signal, the uninformed team members can rely on a truthful report in equilibrium and exert efficient efforts.

Remark 1. *We can separate the effects of moral hazard and adverse selection on our sharing rule. In Gershkov et al. (2009), for the case of commonly known state of the world α^*, the report-independent winner's share that implements efficiency is*

$$s^1(y^*, \alpha^*) = \frac{1}{n} + \frac{(n-1)^2 y_2(\alpha^*, ne^*(\alpha^*))}{n^2 f_1^1(e^*(\alpha^*), \mathbf{e}_{-i}^*(\alpha^*)) y(\alpha^*, ne^*(\alpha^*))} \tag{7}$$

with each of the losers receiving $s^{j\neq 1}(y^*, \alpha^*) = \frac{1-s^1(y^*,\alpha^*)}{n-1}$.

Remember that our new sharing rule (6) *provides efficient incentives to the uninformed agents when they believe that the informed agent reported the right state of the world. Inserting $\alpha' = \alpha^*$ into sharing rule* (6) *and recalling that efficiency implies both $y_2(\alpha^*, ne^*(\alpha^*)) = c'(e^*(\alpha^*))$ and $\alpha^*(y^*, \alpha^*) = \alpha^*$, we get $s^1(y^*, \alpha^*) =$*

$$\begin{aligned} &\frac{1}{n} + \frac{n-1}{n}\left[\frac{c'(e^*(\alpha^*))}{f_1^1(e^*(\alpha^*), \mathbf{e}_{-i}^*(\alpha^*)) y(\alpha^*, ne^*(\alpha^*))} - \frac{y_2(\alpha^*(y^*, \alpha^*), ne^*(\alpha^*))}{f_1^1(e^*(\alpha^*), \mathbf{e}_{-i}^*(\alpha^*)) y(\alpha^*, ne^*(\alpha^*)) n}\right] \\ &= \frac{1}{n} + \frac{n-1}{ny(\alpha^*, ne^*(\alpha^*))}\left[\frac{y_2(\alpha^*, ne^*(\alpha^*))}{f_1^1(e^*(\alpha^*), \mathbf{e}_{-i}^*(\alpha^*))} - \frac{y_2(\alpha^*, ne^*(\alpha^*))}{f_1^1(e^*(\alpha^*), \mathbf{e}_{-i}^*(\alpha^*)) n}\right] \end{aligned} \tag{8}$$

which immediately delivers (7). *Therefore, the agents' shares along the equilibrium path are the same as in Gershkov et al. (2009). Hence, the 'correction' of the sharing rule to take care of*

[11] This tradeoff has been previously reported (in a slightly less general setting) by Gershkov et al. (2009) in a game of complete information.

adverse selection can be expressed as

$$\frac{n-1}{n}\left[\frac{c'(e^*(\alpha'))}{y^* f_1^1(e^*(\alpha'),\mathbf{e}_{-i}^*(\alpha'))}-\frac{y_2(\alpha^*(y^*,\alpha'),ne^*(\alpha'))}{ny^* f_1^1(e^*(\alpha'),\mathbf{e}_{-i}^*(\alpha'))}-\frac{(n-1)y_2(\alpha^*,ne^*(\alpha^*))}{nf_1^1(e^*(\alpha^*),\mathbf{e}_{-i}^*(\alpha^*))y(\alpha^*,ne^*(\alpha^*))}\right] \tag{9}$$

which we interpret as the (off-equilibrium) value of information to the winner.[12] *This correction makes the shares report-dependent and is not present in Gershkov et al. (2009).*

Notice that the loser's share $\frac{1-s^1(y^*,\alpha')}{n-1}$ is given by

$$\frac{1}{n}-\frac{1}{ny^* f_1^1(e^*(\alpha'),\mathbf{e}_{-i}^*(\alpha'))}\left[c'(e^*(\alpha'))-\frac{y_2(\alpha^*(y^*,\alpha'),ne^*(\alpha')))}{n}\right]. \tag{10}$$

Therefore, the difference between the winner's and the loser's compensations is given by

$$\left[c'(e^*(\alpha'))-\frac{y_2(\alpha^*(y^*,\alpha'),ne^*(\alpha')))}{n}\right]\frac{1}{f_1^1(e^*(\alpha'),\mathbf{e}_{-i}^*(\alpha'))} \tag{11}$$

where the expression in the square brackets is the private marginal disutility from effort exertion. This element aligns agent incentives with the socially efficient objective which, in turn, generates the correct incentives for the agents to report information and to exert socially efficient efforts.

Note that the derivative of the success function $f^1(e^*(\alpha'),\mathbf{e}_{-i}^*(\alpha'))$ in equilibrium with respect to e_i can be interpreted as the responsiveness (or precision) of the ranking to a deviation from equilibrium by player i. In other words, this derivative expresses the extent to which winning probabilities change if player i changes efforts. From (6), we get an immediate comparative statics result with respect to the precision of the success function $f_1^1(e^*(\alpha'),\mathbf{e}_{-i}^*(\alpha'))$.

Corollary 1. *The share of the winner $s^1(y^*,\alpha')$ decreases on the equilibrium path with the precision of the ranking $f_1^1(e^*(\alpha'),\mathbf{e}_{-i}^*(\alpha'))$.*

This is intuitive (and proved formally in the appendix), since high ranking precision increases the incentives for the agents. Therefore, if $f_1^1(e^*(\alpha'),\mathbf{e}_{-i}^*(\alpha'))$ increases, agents want to exert higher efforts. To restore their incentives, the share of the winner should be adjusted/decreased.

Example 1: We illustrate our efficiency result from proposition 1 in a simple example with n players, Tullock ranking technology $f^1(e_i,\mathbf{e}_{-i})=e_i^r/\sum_j e_j^r$ with $r>0$,[13] linear production $y(\alpha^*,\sum_i e_i)=\alpha^*\sum_i e_i$, and quadratic effort cost $e_i^2/2$. Note that in this example the efficient

[12] This is different from the notion of value of information in Hermalin (1998, footnote 12) who shows that second best team welfare under the true signal exceeds team welfare under the expected signal. (The same would be true in our model.)

[13] For completeness, we define $f^1(0,\ldots,0)=1/n$; the implied discontinuity at point $(0,\ldots,0)$ plays no role in this example.

effort level is $e^*(\alpha^*) = \alpha^*$. This is implemented through the following ranking-based sharing rule:

$$\begin{aligned} s^1(y^*, \alpha') &= \frac{1}{n} - \frac{n-1}{n^3 f_1^1(\alpha', \alpha')\alpha'} + \frac{(n-1)\alpha'}{ny^* f_1^1(\alpha', \alpha')} = \frac{1}{n} - \frac{1}{nr} + \frac{n\alpha'^2}{y^* r}, \\ s^{j\neq 1}(y^*, \alpha') &= \frac{1}{n} + \frac{1}{n^3 f_1^1(\alpha', \alpha')\alpha'} - \frac{\alpha'}{ny^* f_1^1(\alpha', \alpha')} = \frac{1}{n} + \frac{1}{n(n-1)r} - \frac{n\alpha'^2}{(n-1)y^* r} \end{aligned} \tag{12}$$

in which $s^1(\cdot)$ of the final team output is awarded to the first-ranked player while $s^{j\neq 1}(\cdot)$ is awarded to all other players. In this example, limited liability—defined as non-negative shares of output for all players—is satisfied on the equilibrium path if $r \geq 1$.$\triangleleft$

Remark 2. *An advantage of the contest approach is that it requires only ordinal, noisy information on agents' efforts which is arguably easier to collect than information on the precise effort realizations. Nevertheless, the fact that we are able to implement efficient efforts implies that a noisy ordinal ranking of efforts is a sufficient statistic in the sense of Holmström (1982, Section 3) for the cardinal effort information employed in standard contracts.*

Remark 3. *Assume for the moment that it is commonly known that the leader reports truthfully. Then we know from Holmström (1982) that there exists no sharing rule that simultaneously ensures efficient efforts and balances its budget. Hence, without the additional information on the noisy ranking incorporated in our contest we cannot obtain efficient effort exertion. In other words, if a sharing rule cannot condition on effort information (including some noisy ranking of efforts), it cannot induce both truthful reporting and efficient effort exertion. In this sense, Gershkov et al. (2009) show that a sharing rule which takes into account ranking information but does not vary with output cannot generally implement efficiency.*

4 Extensions and robustness

4.1 Project selection

In many cases, more than one team member may have a project in mind and information on the quality (productivity) of a project is player i's private information. However, due to limited resources, only one project may be implementable by the team (at any given time). In such situations, it may be desirable to select the best project for the team before encouraging each player to exert efficient effort. An efficient team contract then needs to ensure that *i*) the best project is selected and *ii*) every player exerts efficient effort at the production stage.

For presentational simplicity we assume that each team member proposes a single potential project. Use α_i^* and α_i' to represent the observed team productivity of player i's project and player i's report, respectively. We replace the first stage of our base game by a *project selection* *stage* at which each player submits a verifiable report α'_i. The team member who makes the highest among these upfront reports is selected as team leader and her project is chosen for the team to implement. After the project is selected, team members provide efforts as before.

We investigate now whether there exists a ranking-based sharing rule $(s^1, \frac{1-s^1}{n-1}, \ldots, \frac{1-s^1}{n-1})$ which ensures the selection of the best project and, at the same time, efficient effort exertion in production (implying that first stage reports must also be truthful). Note that again the sharing rule keeps the team budget balanced by construction.

Proposition 2. *The sharing rule from proposition 1 with α' replaced by $\max\{\alpha'_1, \ldots, \alpha'_n\}$ ensures simultaneously efficient project selection and efficient effort provision.*

Proposition 2 illustrates that it is possible to achieve both efficient project selection and, simultaneously, implement the efficient provision of efforts in the subsequent team production using a simple, symmetric ranking-based contract. Intuitively—since the winner of the first stage is going to exert the same efforts as all other players, even in case of deviating from truth telling—every player expects to get $1/n$ of the generated social surplus at the beginning of the first stage. This aligns the players' preferences with the social preferences and generates efficient incentives to report individually private project productivity information truthfully.

4.2 Noisy signals

In this section we illustrate that precise information of the team leader is not crucial for efficient incentive provision. That is, one may fear that the positive result of proposition 1 follows from the fact that given the observed output y^* and knowing the equilibrium effort $e^*(\alpha')$, the designer may learn the exact private information of the team leader and 'punish' her in case of misreporting. Here we show that it is not the case. We consider an output or production function of the form $y(\alpha^*; \varepsilon; \mathbf{e}) = y(\alpha^* + \varepsilon, \sum_{i=1}^n e_i)$ where $\varepsilon \sim G[\underline{\varepsilon}, \bar{\varepsilon}]$ with $\mathbb{E}(\varepsilon) = 0$ and density g. That is, we assume that the team leader observes the production parameter with some noise. Moreover, we assume that this uncontractible uncertainty only realizes after the effort exertion. The ex post efficient efforts then solve

$$\max_{e_1,\ldots,e_n} \mathbb{E}_\varepsilon \left[y\left(\alpha^* + \varepsilon, \sum_{i=1}^n e_i\right)\right] - \left(\sum_{i=1}^n c(e_i)\right). \tag{13}$$

Given our assumptions on the production function, the ex post efficient effort level, $e^*(\alpha^*)$ is given by

$$\mathbb{E}_\varepsilon \left[y_2\left(\alpha^* + \varepsilon, ne^*(\alpha^*)\right)\right] = c'\left(e^*(\alpha^*)\right). \tag{14}$$

The dynamic game structure in the case of a noisy signal is similar to the deterministic case of the base model: Player 1 reports α' and, at the second stage and given this report, all players simultaneously choose efforts. We would like to find a sharing rule assigning output shares to the first-, second-, third-ranked players etc.

$$\left(s^1\left(y^*,\alpha'\right),\frac{1-s^1\left(y^*,\alpha'\right)}{n-1},\ldots,\frac{1-s^1\left(y^*,\alpha'\right)}{n-1}\right) \tag{15}$$

such that, for any observed α^* and reported α', player 1 will choose the report-contingent efficient effort $e_1 = e^*(\alpha')$ and it is a best response for every other player to also choose the report-contingent efficient efforts $e^*(\alpha')$.

The expected utility of player 1 if she observes α^*, reports α' and exerts effort e_1, when the other players choose the report-contingent efficient effort $e^*(\alpha')$ is given by $u_1(e_1,\mathbf{e}^*_{-1}(\alpha'),\alpha^*) =$

$$\mathbb{E}_{\varepsilon}\left[y\left(\alpha^*+\varepsilon,\sigma\left(e_1,\mathbf{e}^*_{-1}(\alpha')\right)\right)\left(\sum_{h=1}^{n} f^h(e_1,\mathbf{e}^*_{-1}(\alpha'))s^h(y^*,\alpha')\right)\right]-c(e_1). \tag{16}$$

The next result characterizes the sharing rule that induces truth telling by the team leader and efficient effort exertion by all team members in the presence of a noisy productivity signal.

Proposition 3. *The sharing rule given in (6) implements the first best outcome.*

In summary, this result illustrates that our main efficiency result in proposition 1 does not depend critically on the leader's quality of information. On the contrary, we show that any expectation-zero noise term can be accommodated by our efficient sharing rule (6) without affecting the intuition of our positive result.

4.3 Information aggregation

In this section we assume that each of the n players obtains an individual signal α_i^* in the otherwise unchanged environment from section 3. The aggregation of these signals determines the productivity of the team project, so that output now takes the form $y^*(\alpha^*,e) = y\left(\sum_i \alpha_i^*, e_1+\cdots+e_n\right)$. These signals $(\alpha_1^*,\ldots,\alpha_n^*)$ are drawn from the commonly known distribution Z with support $[a,b]^n$. The main difference to the base model is that now not only player 1 but also each other player must have the appropriate incentives to report their complementary signals truthfully.

We apply the revelation principle and restrict attention to direct mechanisms in which the agents report their private information to the mechanism and the mechanism provides all agents with effort recommendations. The revelation principle implies that, in looking for an efficient mechanism, we can focus on those sharing rules under which the agents find it optimal to report their signals truthfully and to follow their recommendation.

For simplicity, we denote by $\mathcal{A}^* = \alpha_1^*+\cdots+\alpha_n^*$ the true productivity and by $\mathcal{A}_i' = \alpha_1^*+\cdots+\alpha_{i-1}^*+\alpha_i'+\alpha_{i+1}^*+\cdots+\alpha_n^*$ the aggregated productivity in the case where all agents but i report truthfully while agent i reports α'_i. The vector of ex post efficient efforts corresponding to (3) is now defined as the set of efforts which maximize social welfare as chosen by a benevolent planner (who knows $\mathcal{A}^* = \alpha_1^* + \cdots + \alpha_n^*$), i.e.,

$$\max_{e} \; y\left(\mathcal{A}^*, e_1 + \cdots + e_n\right) - \sum_i c(e_i). \tag{17}$$

Hence, symmetric ex post efficient efforts $e^*(\mathcal{A}^*) = e_1^*(\mathcal{A}^*) = \cdots = e_n^*(\mathcal{A}^*)$ are defined through

$$y_2(\mathcal{A}^*, e^*(\mathcal{A}^*)) = c'(e^*(\mathcal{A}^*)). \tag{18}$$

To implement efficiency, the designer recommends player efforts $e^*(\mathcal{A}')$, in which $\mathcal{A}' = \sum_i \alpha'$. The next proposition with its proof in appendix A shows that under sharing rule (6), adjusted for the present information structure, it is indeed in the players' interest to follow this recommendation and report their signals truthfully, $\alpha'_i = \alpha_i^*$.

Proposition 4. *The efficient outcome is implementable in the setup with information aggregation, i.e., in the case in which all team members receive individual signals α_i^* on joint team productivity $\mathcal{A}^*$.*

Example 2: We continue our example by replacing the production function with $y(\mathcal{A}^*, \mathbf{e}) = \mathcal{A}^* \sum_i e_i$, in which α_i^* is player i's private information about team productivity and the true team productivity is measured by $\mathcal{A}^* = \sum_i \alpha_i^*$. We replace the first stage of the base game with a stage in which each player privately reports their α'_i to the designer who subsequently sends the effort recommendation $e^*(\mathcal{A}') = \mathcal{A}' = \sum_i \alpha'$ to the team members. Compared to the sharing rule of example 1, the similar rule consisting of winning share

$$s^1(y^*, \mathcal{A}') = \frac{1}{n} - \frac{1}{nr} + \frac{n\mathcal{A}'^2}{y^* r} \tag{19}$$

and losing shares $s^{j\neq 1}(y^*, \mathcal{A}') = \dfrac{1 - s^1(y^*, \mathcal{A}')}{n-1}$ implements the first best outcome.◁

4.4 Leading by Example

In this section we change the structure of the interaction and allow the leader to choose her effort before the other players. This effort is assumed to be observable by her team partners. We show that in such a case, there exists a simpler ranking-based sharing rule which implements the efficient outcome. This sharing rule will only depend on observed output.

Here, we consider the following sequential game: at the first stage, the leader chooses effort e_1. Then, at the second stage, all other players $j \neq 1$ observe e_1 and choose their own efforts $e_j(e_1(\cdot))$. Following this, a noisy ranking of all players' efforts realizes. The winner receives fraction s^1 of final team output and each of the losers receives share $\frac{1-s^1}{n-1}$.

In this environment, the sharing rule may be conditioned on the observed output y^* alone because the leader's effort e_1 is observed by all other players before they choose their own efforts. Thus, the leader's effort serves as a signal of the team's productivity parameter α^*. Moreover, and this is crucial, this time structure limits the strategic possibilities of the leader. While in the original game—in which everyone chooses efforts simultaneously—the leader was able to deviate in both her report α' and the chosen effort (so multidimensional deviations had to be taken into account), one of these channels is shut here. In the current structure, a misreport is more costly to the leader, as she cannot report α' and subsequently choose an effort which is inconsistent with this report. Therefore, the report of productivity α is unnecessary to implement the efficient allocation as it can be deduced from the leader's effort choice.

Proposition 5. *Assume the sequential game described above. The sharing rule consisting of*

$$s^1(y^*) = \frac{1}{n} + \frac{n-1}{n} \frac{c'(e^*(\breve{\alpha}(y^*))) - \frac{y_2(\breve{\alpha}(y^*), ne^*(\breve{\alpha}(y^*)))}{n}}{y^* f_1^1(e^*(\breve{\alpha}(y^*)), \mathbf{e}^*_{-i}(\breve{\alpha}(y^*)))} \tag{20}$$

and $s^j(y^*) = \frac{1-s^1(y^*)}{n-1}$ *for all* $j \neq 1$ *in which* $\breve{\alpha}(y^*)$ *is the solution to* $y^* = y(\alpha, ne_1(\alpha))$, *implements efficient efforts.*

Example 3: We continue our example with $y = \alpha^* \sum_i e_i$, $c(e_i) = \frac{1}{2}e_i^2$, and $f^1(e_i, \mathbf{e}_{-i}) = e_i^r / \sum e_j^r$ by replacing the simultaneous game with the sequential structure described above. In this sequential game, a tournament with shares

$$s^1 = \frac{(n-1)+r}{nr}, \qquad s^j = \frac{r-1}{nr} \text{ for } j \neq 1 \tag{21}$$

implements efficiency. To see this, note that on observing the effort choice of player 1, e_1, players $j \neq 1$ believe that the productivity parameter of the team is $\alpha^* = e_1$. Given sharing rule (21), it is a best response for the uninformed players to follow their leader by choosing exactly $e_j = e_1$. At the first stage, anticipating that the uninformed players are going to follow suit by choosing $e_j = e_1$, player 1's best strategy is to choose effort $e_1 = \alpha^*$, thus communicating the true state of world and implementing efficiency. Note that sharing rule (21) is independent of output. This is the case only due to the fact that this example uses a linear production function; output independence may not be obtained in general.◁

In a setup with linear production and quadratic effort costs, Hermalin (1998, p1192) finds that leading by example is superior to a range of other mechanisms. Nevertheless, leading by example fails to achieve full efficiency because the usual moral hazard problem remains. The reason for this failure is the fixed sharing rule that Hermalin (1998) uses throughout the paper. We show that a well-designed tournament, by orchestrating competition among the players, removes the free-riding incentives while ensuring truthful information revelation.

Remark 3 explains that fixed shares can generally not provide incentives for the efficient provision of efforts. This remains true with upfront exertion of observable efforts by a privately informed leader.

We assumed so far that only final output is contractible. If the leader's upfront effort is also contractible, however, then the next proposition states that there exists a sharing rule that provides the correct incentives for all agents and conditions only on the leader's observed effort.

Proposition 6. *Assume the sequential game described above with contractible leader's effort. In this game, there exists a sharing rule which depends solely upon the leader's effort that implements efficiency.*

4.5 Limited liability along the equilibrium path

In this section we illustrate that, for a widely used class of contest success functions, we can achieve limited liability (i.e., all players' shares $s^i(\cdot) \in [0,1]$ for any $i \in \mathcal{N}$) along the equilibrium path. For this section only, we restrict attention to symmetric, ratio-based contest success functions. We define a success function $\hat{f}^1(x_i)$ as *ratio-based* if it only depends on the vector of ratios of a player's effort over each of her opponents' efforts $x_i = \left(\frac{e_i}{e_1}, \ldots, \frac{e_i}{e_{i-1}}, \frac{e_i}{e_{i+1}}, \ldots, \frac{e_i}{e_n}\right)$.[14] We take symmetry to imply that, for any two players $\ell \neq m$ and for any two vectors of efforts, $(e_1, \ldots, e_n)$ and $(\tilde{e}_1, \ldots, \tilde{e}_n)$ with $e_k = \tilde{e}_k$ for $k \notin \{\ell, m\}$ and $e_\ell = \tilde{e}_m$ and $e_m = \tilde{e}_\ell$, we have

$$\hat{f}^1(x_\ell) = \hat{f}^1(\tilde{x}_m). \tag{22}$$

The Tullock success function is an example of such a symmetric, ratio-based function. When agents exert identical equilibrium efforts $e^*(\alpha^*)$, then the above implies that for any $i \in \{1, \ldots, n\}$, $\ell, m \neq i$, we have that $\frac{\partial \hat{f}^1(\mathbf{1})}{\partial x_{i\ell}} = \frac{\partial \hat{f}^1(\mathbf{1})}{\partial x_{im}}$ where $\mathbf{1}$ is the $n-1$ dimensional vector with 1 at every position. The relationship between the ratio-based success function and the original effort-based success function is such that

$$\hat{f}^1\left(\frac{e_i}{e_1}, \frac{e_i}{e_2}, \ldots, \frac{e_i}{e_{i-1}}, \frac{e_i}{e_{i+1}}, \ldots, \frac{e_i}{e_n}\right) = f^1\left(e_i, \mathbf{e}_{-i}\right) \tag{23}$$

with derivative

$$\begin{aligned} \frac{d}{de_i}\hat{f}^1\left(\frac{e_i}{e_1}, \frac{e_i}{e_2}, \ldots, \frac{e_i}{e_{i-1}}, \frac{e_i}{e_{i+1}}, \ldots, \frac{e_i}{e_n}\right) &= \sum_{j \neq i} \frac{1}{e_j} \frac{\partial}{\partial x_{ij}} \hat{f}^1\left(x_{i1}, \ldots, x_{ii-1}, x_{ii+1}, \ldots, x_{in}\right) \\ &= f_1^1\left(e_i, \mathbf{e}_{-i}\right). \end{aligned} \tag{24}$$

[14] In order to avoid technical complications with unbounded ratios, we require efforts to be positive for the purposes of this limited liability discussion. In other words, $e_i \in [\delta, \infty)$ in which $\delta > 0$ can be arbitrarily close to zero.

In symmetric equilibrium we therefore have

$$\frac{d}{de_i}\hat{f}^1(\mathbf{1}) \; = \sum_{j\neq i} \frac{1}{e^*(\alpha')}\frac{\partial}{\partial x_{ij}}\hat{f}^1(\mathbf{1}) = f_1^1\left(e^*(\alpha'), \mathbf{e}^*_{-\mathbf{i}}(\alpha')\right). \tag{25}$$

In our base model, efficient efforts are defined by $y_2(\alpha^*, ne(\alpha^*)) = c'(e^*(\alpha^*))$. Substituting the latter with the prior in sharing rule (6) gives us

$$s^1(y^*, \alpha^*) \; = \frac{1}{n} + \frac{(n-1)\, y_2(\alpha^*(y^*, \alpha^*), ne^*(\alpha^*))}{ny(\alpha^*, ne^*(\alpha^*)) f_1^1(e^*(\alpha^*), \mathbf{e}^*_{-i}(\alpha^*))}\left[\frac{n-1}{n}\right] > \frac{1}{n} \tag{26}$$

implying that the equilibrium winner's share is always greater than a loser's share.

Winning share (26) satisfies limited liability along the equilibrium path if, in addition to the inequality already shown in (26), $s^1(y^*, \alpha^*) \leq 1$. This is the case if

$$f_1^1(e^*(\alpha^*), \mathbf{e}^*_{-i}(\alpha^*)) \geq \frac{n-1}{n}\frac{y_2(\alpha^*(y^*, \alpha^*), ne^*(\alpha^*))}{y(\alpha^*, ne^*)}. \tag{27}$$

The last inequality implies that

$$f_1^1(e^*(\alpha^*), \mathbf{e}^*_{-i}(\alpha^*)) \geq \frac{1}{e^*}\frac{n-1}{n^2}\frac{ne^* y_2(\alpha^*(y^*, \alpha^*), ne^*(\alpha^*))}{y(\alpha^*, ne^*)} \tag{28}$$

in which the last fraction on the right-hand side is smaller than 1 because of concavity of y. Thus, (25) together with concavity of production implies that, for any symmetric, ratio-based success function, we obtain as sufficient condition for limited liability to be satisfied that

$$\hat{f}_1^1(\mathbf{1}) \geq \frac{1}{n^2}. \tag{29}$$

Hence, losers receive positive shares in equilibrium if the ranking technology is sufficiently accurate, which is intuitive. In case of a very high ranking precision, the winner-take-all contest generates incentives for higher than the efficient effort level. In such cases, the winner's prize should be reduced in order to adjust the agents' incentives.

5 Concluding remarks

In addition to the motivation offered in the introduction, our model can be interpreted as providing a solution to the coordination problem in which the interests of the involved individuals coincide. It would be therefore jointly optimal if they would individually all select this jointly most beneficial option. Nevertheless, in many such situations, it is not in the individual's self-interest to choose the action which implements the greater good.

Leadership is but one means by which social groups attempt to solve the coordination problem and can take many forms in general public life.[15] The present paper analyzes the

[15] The information-based foundation that we offer for leadership is, of course, not the only possible explanation. Alternatives include delegation, sharing of responsibility, inclusion of stakeholders and others.

question of what constitutes the coordinative essence of leadership in team structures where (some of) the team members are privately informed on some aspect of the profitability of a joint project. Such proprietary information arises naturally if, for example, some team member occupies a role in a predefined organizational structure by virtue of which she acquires and disseminates information.

An effective coordinating scheme then needs to implement *i)* the communication of private information and *ii)* the efficient effort provision by all team members although these efforts may not be directly observed. In the environment we consider, this gives rise to the nested and triple problem of *adverse selection* because of the leader's private information, *moral hazard* because individual efforts are unobservable, and *balanced budget* because of the team structure which renders the classic principal-agent and budget breaker solutions inapplicable.

Examples which emphasize the coordination aspect and feature the properties outlined above can be found in military history. During the First World War, for instance, officers in most armies used a 'trench whistle' to communicate isolated timing information to a team. Its high pitched sound was used to coordinate large scale attacks. At the officers' blow of their whistles, the soldiers would go 'over the top' of the trenches and attack the enemy. Note that signalling the attack at the wrong time may result under this interpretation in over or under exertion of team member efforts relative to the efficient level which may benefit or harm the standing of the whistling leader.

A final example of carefully designed incentive structures in partnerships is the 19th century American whaling industry beautifully described in Hilt (2006). The author describes how managing partners provided appropriate incentives to the whaler's captains and crews on their entirely unobservable multi-year expeditions. During the 1830s, *part of the industry* changed its structure from the previously unincorporated partnerships to corporative ownership. "This represented a significant departure from the traditional reliance on concentrated ownership to resolve incentive conflicts in the industry, and it failed: none of the whaling corporations survived beyond the 1840s, and few experienced much financial success, at a time the American whaling industry as a whole continued to expand." (Hilt, 2006, p198)

Appendix A: Omitted proofs

Proof of proposition 1. We prove the proposition in two steps. In step 1 we show that, given the leader's report α', it is a mutually best response for the leader and the other players to choose effort $e^*(\alpha')$ under sharing rule (6). In step 2 we show that, anticipating the equilibrium at the second stage, the leader's optimal strategy is to report truthfully.

Step 1. For a given report α', given that every other player chooses $e^*(\alpha')$, the team-leader's first-order condition with respect to effort choice is

$$\begin{aligned}
&\frac{\partial u_1(e_1, \mathbf{e}^*_{-1}(\alpha'), \alpha^*)}{\partial e_1} \\
&= y_2(\alpha^*, \sigma(e_1, \mathbf{e}^*_{-1}(\alpha'))) \left(\sum_{\ell=1}^{n} f^\ell(e_1, \mathbf{e}^*_{-1}(\alpha')) s^\ell(y^*, \alpha')\right) \\
&\quad + y(\alpha^*, \sigma(e_1, \mathbf{e}^*_{-1}(\alpha'))) \left(\sum_{\ell=1}^{n} f^\ell_1(e_1, \mathbf{e}^*_{-1}(\alpha')) s^\ell(y^*, \alpha')\right) \\
&\quad + y(\alpha^*, \sigma(e_1, \mathbf{e}^*_{-1}(\alpha'))) y_2(\alpha^*, \sigma(e_1, \mathbf{e}^*_{-1}(\alpha'))) \left(\sum_{\ell=1}^{n} f^\ell(e_1, \mathbf{e}^*_{-1}(\alpha')) s^\ell_1(y^*, \alpha')\right) \\
&\quad - c'(e_1)
\end{aligned} \tag{30}$$

which equals

$$\begin{aligned}
&y_2(\alpha^*, \sigma(e_1, \mathbf{e}^*_{-1}(\alpha'))) \left(\sum_{\ell=1}^{n} f^\ell(e_1, \mathbf{e}^*_{-1}(\alpha')) s^\ell(y^*, \alpha')\right) - c'(e_1) \\
&\quad + y(\alpha^*, \sigma(e_1, \mathbf{e}^*_{-1}(\alpha'))) \left(\sum_{\ell=1}^{n-1} f^\ell_1(e_1, \mathbf{e}^*_{-1}(\alpha')) \left(s^\ell(y^*, \alpha') - s^n(y^*, \alpha')\right)\right) \\
&\quad + y(\alpha^*, \sigma(e_1, \mathbf{e}^*_{-1}(\alpha'))) y_2(\alpha^*, \sigma(e_1, \mathbf{e}^*_{-1}(\alpha'))) \left(\sum_{\ell=1}^{n} f^\ell(e_1, \mathbf{e}^*_{-1}(\alpha')) s^\ell_1(y^*, \alpha')\right)
\end{aligned} \tag{31}$$

where $f^\ell_1(e_1, \mathbf{e}_{-1}) = \frac{\partial}{\partial e_1} f^\ell(e_1, \mathbf{e}_{-1})$. The equality holds because of equation (1), balanced budget, and $\sum_\ell f^\ell_1(e_1, \mathbf{e}^*_{-1}(\alpha')) = \sum_\ell s^\ell_1(y^*, \alpha') = 0$. In case of equal effort levels of all agents, $e_1 = e^*(\alpha')$, $f^\ell(e^*(\alpha'), \mathbf{e}^*_{-1}(\alpha')) = 1/n$ for all $\ell = 1, \ldots, n$. Then, setting the first-order condition with respect to effort choice to zero gives

$$\begin{aligned}
0 &= \frac{\partial u_1(e^*(\alpha'), \mathbf{e}^*_{-1}(\alpha'), \alpha^*)}{\partial e_1} = \frac{y_2(\alpha^*, ne^*(\alpha'))}{n} \\
&\quad + y^* \left(\sum_{\ell=1}^{n-1} f^\ell_1(e^*(\alpha'), \mathbf{e}^*_{-1}(\alpha')) \left(s^\ell(y^*, \alpha') - s^n(y^*, \alpha')\right)\right) - c'(e^*(\alpha'))
\end{aligned} \tag{32}$$

with output in equilibrium $y^* = y(\alpha^*, ne^*(\alpha'))$. Therefore, we get that

$$\sum_{\ell=1}^{n-1} f^\ell_1(e^*(\alpha'), \mathbf{e}^*_{-1}(\alpha')) \left(s^\ell(y^*, \alpha') - s^n(y^*, \alpha')\right) = \frac{c'(e^*(\alpha')) - y_2(\alpha^*, ne^*(\alpha'))/n}{y^*}. \tag{33}$$

Under the simple prize structure with $s^2(y^*, \alpha') = \cdots = s^n(y^*, \alpha')$, this gives

$$f_1^1(e^*(\alpha'), \mathbf{e}_{-1}^*(\alpha')) \left(s^1(y^*, \alpha') - s^n(y^*, \alpha')\right) = \frac{c'(e^*(\alpha')) - y_2(\alpha^*, ne^*(\alpha'))/n}{y^*} \tag{34}$$

This equation is satisfied under sharing rule (6), proving that it is indeed a best response for the leader to choose $e^*(\alpha')$ given every other player choosing $e^*(\alpha')$. A similar argument implies that also all the uninformed agents prefer to exert the report-contingent efficient effort level $e^*(\alpha')$ for any report of the team leader α'.

Step 2. We now have to show that, at the first stage, the team leader announces the true signal. At the reporting stage, the leader who exerts efforts $e^*(\alpha')$ reports α' such as to maximize expected utility (5), i.e.,

$$\underset{\alpha'}{max}\ u_1(e^*(\alpha'), \mathbf{e}_{-1}^*(\alpha'), \alpha^*) \ = \underset{\alpha'}{max}\ \left(\frac{y\left(\alpha^*, ne^*(\alpha')\right)}{n} - c(e^*(\alpha'))\right) \tag{35}$$

which, from (3) is maximized at the report $\alpha' = \alpha^*$ implying that player 1 reports truthfully. All team members will find it optimal to accept the contract because each player will get $1/n$ of the generated efficient social surplus which must be positive because $y_2(\alpha^*, 0) > 0$ and $c'(0) = 0$. Therefore, the players' ex post efficient efforts $e^*(\alpha^*)$ are implementable. □

Proof of corollary 1. Recall that the winner's share is

$$s^1(y^*, \alpha') = \frac{1}{n} + \frac{n-1}{ny^*}\left[\frac{c'(e^*(\alpha'))}{f_1^1(e^*(\alpha'), \mathbf{e}_{-i}^*(\alpha'))} - \frac{y_2(\alpha^*(y^*, \alpha'), ne^*(\alpha'))}{f_1^1(e^*(\alpha'), \mathbf{e}_{-i}^*(\alpha'))n}\right]. \tag{36}$$

Given the truth telling behavior of the team leader, we can rewrite the equilibrium winner's share as follows

$$\begin{aligned}
s^1(y^*, \alpha^*) &= \frac{1}{n} + \frac{n-1}{n}\left[\frac{c'(e^*(\alpha^*))}{f_1^1(e^*(\alpha^*), \mathbf{e}_{-i}^*(\alpha^*))y^*} - \frac{y_2(\alpha^*(y^*, \alpha^*), ne^*(\alpha^*))}{f_1^1(e^*(\alpha^*), \mathbf{e}_{-i}^*(\alpha^*))y^*n}\right] \\
&= \frac{1}{n} + \frac{n-1}{n}\left[\frac{c'(e^*(\alpha^*))}{f_1^1(e^*(\alpha^*), \mathbf{e}_{-i}^*(\alpha^*))y^*} - \frac{y_2(\alpha^*, ne^*(\alpha^*))}{f_1^1(e^*(\alpha^*), \mathbf{e}_{-i}^*(\alpha^*))y^*n}\right] \\
&= \frac{1}{n} + \frac{n-1}{n}\left[\frac{y_2(\alpha^*, ne^*(\alpha^*))}{f_1^1(e^*(\alpha^*), \mathbf{e}_{-i}^*(\alpha^*))y^*} - \frac{y_2(\alpha^*, ne^*(\alpha^*))}{f_1^1(e^*(\alpha^*), \mathbf{e}_{-i}^*(\alpha^*))y^*n}\right] \\
&= \frac{1}{n} + \left(\frac{n-1}{n}\right)^2 \frac{1}{f_1^1\left(e^*(\alpha^*), \mathbf{e}_{-\mathbf{i}}^*(\alpha^*)\right)} \frac{y_2(\alpha^*, ne^*(\alpha^*))}{y(\alpha^*, ne^*(\alpha^*))}
\end{aligned} \tag{37}$$

where the second line follows since $\alpha^* = \alpha^*(y^*, \alpha^*)$ and the third line follows since in the efficient allocation we have $c'(e^*(\alpha^*)) = y_2(\alpha^*, ne^*(\alpha^*))$. □

Proof of proposition 2. The sharing rule stipulates that the first-ranked player in effort ranking at the production stage gets share s^1 of team output while the remainder is shared equally among the other partners. We show that the same sharing rule (6) with α' replaced by $\max\{\alpha'_1, \cdots, \alpha'_n\}$, together with losers' shares $\left(\frac{1-s^1}{n-1}, \ldots, \frac{1-s^1}{n-1}\right)$, implements full efficiency.

Assume that player 1 won the first stage by reporting some α'_1 which is not necessarily equal to α^*_1. At the production stage, given that player 1's project is already selected and sharing rule (6) is in place, the game is the same as in the base model and every player chooses report-contingent efficient effort $e^*(\alpha'_1)$.

Now look at the project selection stage. Anticipating that at the production stage all players exert $e^*(\max\{\alpha'_1, \cdots, \alpha'_n\})$, each player now chooses her report α'_i in order to maximize her own expected payoff, assuming other players report truthfully. Player i's payoff if her project is selected is

$$y\left(\alpha^*_i, e^*_i(\alpha'_i), \mathbf{e}^*_{-i}(\alpha'_i)\right) \left(\underbrace{f^1(e^*(\alpha'_i), \mathbf{e}^*_{-i}(\alpha'_i))}_{=1/n} s^1 + \underbrace{(1 - f^1(e^*(\alpha'_i), \mathbf{e}^*_{-i}(\alpha'_i)))}_{=(n-1)\frac{1}{n}} \frac{1-s^1}{n-1} \right) - c(e^*(\alpha'_i)).$$

If some other player j's project is selected, player i's payoff is

$$y(\alpha_j, e^*_i(\alpha_j), \mathbf{e}^*_{-i}(\alpha_j)) \left(\underbrace{f^1(e^*(\alpha_j), \mathbf{e}^*_{-i}(\alpha_j))}_{=1/n} s^1 + \underbrace{(1 - f^1(e^*(\alpha_j), \mathbf{e}^*_{-i}(\alpha_j)))}_{=(n-1)\frac{1}{n}} \frac{1-s^1}{n-1} \right) - c(e^*(\alpha_j)).$$

In the first case, truthful reporting maximizes own utility for the same reason as in the base model. In the second case, truthful reporting is still the best strategy because it does not affect player i's expected payoff. Thus, it is player i's optimal strategy to report her signal truthfully. Since every player reports truthfully, the best project is selected. □

Proof of proposition 3. The derivative of the team leader's expected utility with respect to her effort is given by

$$\begin{aligned}
&\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \Bigg[y_2 \left(\alpha^* \right. \\
&\left. + \varepsilon ; \sigma \left(e_1, \mathbf{e}_{-1}^* \left(\alpha'\right)\right)\right) \Bigg\{ f^1 \left(e_1, \mathbf{e}_{-1}^* \left(\alpha'\right)\right) s^1 \left(y \left(\alpha^* + \varepsilon ; \sigma \left(e_1, \mathbf{e}_{-1}^* \left(\alpha'\right)\right)\right), \alpha'\right) \\
&+ \left(1 - f^1 \left(e_1, \mathbf{e}_{-1}^* \left(\alpha'\right)\right)\right) \frac{1 - s^1 \left(y \left(\alpha^* + \varepsilon ; \sigma \left(e_1, \mathbf{e}_{-1}^* \left(\alpha'\right)\right)\right), \alpha'\right)}{n-1} \Bigg\} \\
&+ y \left(\alpha^* + \varepsilon ; \sigma \left(e_1, \mathbf{e}_{-1}^* \left(\alpha'\right)\right)\right) f_1^1 \left(e_1, \mathbf{e}_{-1}^* \left(\alpha'\right)\right) \{ s^1 \left(y \left(\alpha^* + \varepsilon ; \sigma \left(e_1, \mathbf{e}_{-1}^* \left(\alpha'\right)\right)\right), \alpha'\right) \\
&- \frac{1 - s^1 \left(y \left(\alpha^* + \varepsilon ; \sigma \left(e_1, \mathbf{e}_{-1}^* \left(\alpha'\right)\right)\right), \alpha'\right)}{n-1} \} \\
&+ y \left(\alpha^* + \varepsilon ; \sigma \left(e_1, \mathbf{e}_{-1}^* \left(\alpha'\right)\right)\right) y_2 \left(\alpha^* + \varepsilon ; \sigma \left(e_1, \mathbf{e}_{-1}^* \left(\alpha'\right)\right)\right) \\
&\times s_1^1 \left(y \left(\alpha^* + \varepsilon ; \sigma \left(e_1, \mathbf{e}_{-1}^* \left(\alpha'\right)\right)\right), \alpha'\right) \Bigg\{ f^1 \left(e_1, \mathbf{e}_{-1}^* \left(\alpha'\right)\right) \\
&- \frac{1 - f^1 \left(e_1, \mathbf{e}_{-1}^* \left(\alpha'\right)\right)}{n-1} \Bigg\} \Bigg] g \left(\varepsilon\right) d\varepsilon - c'(e_1).
\end{aligned} \tag{38}$$

We will show that for the stated sharing rule, the first order condition is satisfied. Recall that $f^1 \left(e^* \left(\alpha'\right), \mathbf{e}_{-1}^* \left(\alpha'\right)\right) = 1/n$. Therefore, the first-order condition boils down to

$$\begin{aligned}
&\int_{\underline{\varepsilon}}^{\bar{\varepsilon}} \left[\frac{y_2 \left(\alpha^* + \varepsilon ; ne^* \left(\alpha'\right)\right)}{n} + y \left(\alpha^* + \varepsilon ; ne^* \left(\alpha'\right)\right) f_1^1 \left(e_1, \mathbf{e}_{-1}^* \left(\alpha'\right)\right) \times \right. \\
&\left. \times \left(n \frac{s^1 \left(y \left(\alpha^* + \varepsilon ; ne^* \left(\alpha'\right)\right), \alpha'\right)}{n-1} - \frac{1}{n-1} \right) \right] g \left(\varepsilon\right) d\varepsilon - c'(e^* \left(\alpha'\right)) = 0.
\end{aligned} \tag{39}$$

Inserting the rule (6) completes the proof. We still have to show that it is optimal for the leader to report truthfully her signal. For the leader,

$$\max_{\alpha'} \; u_1(e_1, e^*(\alpha'), \alpha^*) \; = \max_{\alpha'} \; \left(\frac{\mathbb{E} y \left(\alpha^* + \varepsilon, ne^* \left(\alpha'\right)\right)}{n} - c \left(e^* \left(\alpha'\right)\right) \right) \tag{40}$$

is solved at $\alpha' = \alpha^*$ by efficiency condition (14). □

Proof of proposition 4. To implement efficiency, the designer recommends player efforts $e^*(\mathcal{A}')$, in which $\mathcal{A}' = \sum_i \alpha'$. We show that under sharing rule (6), adjusted for the present information structure, i.e., the sharing rule

$$\frac{1}{n} + \frac{n-1}{ny^* f_1^1(e^*(\mathcal{A}'), \mathbf{e}_{-i}^*(\mathcal{A}'))} \left[c'(e^*(\mathcal{A}')) - \frac{y_2(\mathcal{A}^*(y^*, \mathcal{A}'), ne^*(\mathcal{A}')))}{n} \right] \tag{41}$$

in which $\mathcal{A}^*(y^*, \mathcal{A}')$ is the solution to $y^* = y(\mathcal{A}^*, ne^*(\mathcal{A}'))$, it is indeed in the players' interest to follow this recommendation and report their signals truthfully, $\alpha_i' = \alpha_i^*$. At the second stage, we define player i's expected utility from action pair (α_i', e_i) given equilibrium behavior $\left(\alpha_j^*, e^*(\mathcal{A}')\right)$, for $j \neq i$, of everyone else as $u_i((\alpha_i', e_i), \mathbf{e}^*_{-i}(\mathcal{A}'), \mathcal{A}^*) =$

$$\mathbb{E}_{\mathcal{A}^*}\left[y\left(\mathcal{A}^*, \sigma\left(e_i, \mathbf{e}^*_{-i}(\mathcal{A}')\right)\right)\left(\sum_{h=1}^{n} f^h(e_i, \mathbf{e}^*_{-i}(\mathcal{A}'))s^h(y^*, \mathcal{A}')\right)\middle|\mathcal{A}'\right] - c(e_i) \tag{42}$$

where observed output is $y^* = y\left(\mathcal{A}^*, \sigma\left(e_i, \mathbf{e}^*_{-i}(\mathcal{A}')\right)\right)$. For a given report α_i' we obtain player i's first-order condition with respect to efforts e_i as

$$\begin{aligned}
&\frac{\partial u_i((\alpha_i', e_i), \mathbf{e}^*_{-i}(\mathcal{A}'), \mathcal{A}^*)}{\partial e_i} \\
&= y_2\left(\mathcal{A}^*, \sigma\left(e_i, \mathbf{e}^*_{-i}(\mathcal{A}')\right)\right)\left(\sum_{h=1}^{n} f^h(e_i, \mathbf{e}^*_{-i}(\mathcal{A}'))s^h(y^*, \mathcal{A}')\right) \\
&\quad + y\left(\mathcal{A}^*, \sigma\left(e_i, \mathbf{e}^*_{-i}(\mathcal{A}')\right)\right)\left(\sum_{h=1}^{n} f_1^h(e_i, \mathbf{e}^*_{-i}(\mathcal{A}'))s^h(y^*, \mathcal{A}')\right) \\
&\quad + y\left(\mathcal{A}^*, \sigma\left(e_i, \mathbf{e}^*_{-i}(\mathcal{A}')\right)\right) y_2\left(\mathcal{A}^*, \sigma\left(e_i, \mathbf{e}^*_{-i}(\mathcal{A}')\right)\right)\left(\sum_{h=1}^{n} f^h(e_i, \mathbf{e}^*_{-i}(\mathcal{A}'))s_1^h(y^*, \mathcal{A}')\right) - c'(e_i)
\end{aligned} \tag{43}$$

which is the same as condition (30) derived for player 1 in the proof of proposition 1, with the only difference that $\mathcal{A}^*$ and $\mathcal{A}'$ are now sums. Therefore, exactly as in the base model, even after misreporting, all agents will follow the designer's recommendation and choose efforts $e^*(\mathcal{A}')$.

At the first stage, therefore, each player i reports α_i' such as to maximize expected utility (42) which, in case of symmetric second stage and truth telling of all players other than i, implies that she chooses

$$\underset{\alpha_i'}{max}\ u_i((\alpha_i', e^*\left(\mathcal{A}_i'\right)), \mathbf{e}^*_{-i}(\mathcal{A}_i'), \mathcal{A}^*) \ = \underset{\alpha_i'}{max}\ \left(\frac{y\left(\mathcal{A}^*, ne^*\left(\mathcal{A}_i'\right)\right)}{n} - c(e^*(\mathcal{A}_i'))\right) \tag{44}$$

which, from (18) is maximized at the report $\alpha_i' = \alpha_i^*$ implying that player i reports truthfully. Thus, the sharing rule (6) derived in proposition 1, with α' replaced by $\mathcal{A}'$, implements ex post efficient efforts as defined in (18) also in the setup with information aggregation. □

Proof of proposition 5. We show that this sharing rule *i)* induces the leader to choose $e_1 = e^*(\alpha^*)$, and *ii)* all the other agents to follow the leader and to choose also e_1. We start with analyzing the incentives of agent $j \neq 1$, given that all the other agents follow the described strategy. The expected utility of agent $j \neq 1$ if he chooses effort e is given by

$$y\left(\alpha^*, e + (n-1)e^*\left(\alpha^*\right)\right)\left[f^1\left(e, \mathbf{e}_{-\mathbf{j}}{}^*\left(\alpha^*\right)\right)s^1\left(y\right) + \left(1 - f^1\left(e, \mathbf{e}_{-\mathbf{j}}{}^*\left(\alpha^*\right)\right)\right)\frac{1 - s^1\left(y\right)}{n-1}\right] - c(e). \tag{45}$$

The derivative of the last expression with respect to e is given by

$$\begin{aligned}
&y_2\left(\alpha^*, e+(n-1)e^*\left(\alpha^*\right)\right)\left[f^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)s^1\left(y\right)+\left(1-f^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)\right)\frac{1-s^1\left(y\right)}{n-1}\right] \\
&-c'(e)+y\left(\alpha^*, e+(n-1)e^*\left(\alpha^*\right)\right)\left[f_1^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)\left(\frac{ns^1\left(y\right)}{n-1}-\frac{1}{n-1}\right)\right. \\
&\left.+s^{1'}\left(y\right)\left(f^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)-\frac{\left(1-f^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)\right)}{n-1}\right)y_2\left(\alpha^*, e+(n-1)e^*\left(\alpha^*\right)\right)\right].
\end{aligned} \tag{46}$$

For $e = e_1$ to be an equilibrium, it must be that the last derivative at point $e = e_1$ is 0. Since $f^1\left(e^*\left(\alpha^*\right), \mathbf{e_{-j}}^*\left(\alpha^*\right)\right) = 1/n$, we get

$$c'(e^*(\alpha^*))-\frac{y_2(\alpha^*, ne^*(\alpha^*))}{n}=y(\alpha^*, ne^*(\alpha^*))\left[f_1^1\left(e^*\left(\alpha^*\right), \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)\left(\frac{ns^1(y)}{n-1}-\frac{1}{n-1}\right)\right]. \tag{47}$$

Inserting

$$s^1\left(y\right)=\frac{1}{n}+\frac{n-1}{n}\frac{c'\left(e^*\left(\breve{\alpha}\left(y^*\right)\right)\right)-\frac{y_2\left(\breve{\alpha}(y^*), ne^*\left(\breve{\alpha}(y^*)\right)\right)}{n}}{y^*f_1^1\left(e^*\left(\breve{\alpha}\left(y^*\right)\right), \mathbf{e_{-j}}^*\left(\breve{\alpha}\left(y^*\right)\right)\right)} \tag{48}$$

and noticing that in the equilibrium $\breve{\alpha}\left(y^*\right) = \alpha^*$ gives the required condition.

For the leader, her expected utility if she chooses effort e_1 when the state of the world is α^* is given by

$$y\left(\alpha^*, ne_1\right)\frac{1}{n}-c(e_1). \tag{49}$$

This is maximized at

$$c'\left(e_1\right)=y_2\left(\alpha^*, ne_1\right) \tag{50}$$

which is the efficient effort level, given the state of the world α^*. □

Proof of proposition 6. We show that the following winner's share

$$s^1\left(e_1\right)=\frac{1}{n}+\frac{n-1}{n}\frac{c'\left(e_1\right)-\frac{y_2\left(\alpha^{**}(e_1), ne_1\right)}{n}}{y\left(\alpha^{**}\left(e_1\right), ne_1\right)f_1^1(e_1, e_1)} \tag{51}$$

in which $\alpha^{**}\left(e_1\right)$ is the solution to $e^*\left(\alpha^{**}\right) = e_1$, together with $s^j(y^*) = \frac{1-s^1(y^*)}{n-1}$ for all $j \neq 1$, induces *i*) the leader to choose $e_1 = e^*\left(\alpha^*\right)$, and *ii*) all the other agents to follow the leader and to choose also e_1. We start with analyzing the incentives of agent $j \neq 1$, given that all the other agents choose $e_1 = e^*\left(\alpha^*\right)$. The expected utility of agent $j \neq 1$ if he chooses effort e is given by

$$y\left(\alpha^*, e+(n-1)e^*\left(\alpha^*\right)\right)\left[f^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)s^1+\left(1-f^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)\right)\frac{1-s^1}{n-1}\right]-c(e). \tag{52}$$

Since $s^1(e_1)$ is independent of the realized output, agent j cannot affect it. The derivative of the last expression with respect to e is given by

$$
\begin{aligned}
&y_2\left(\alpha^*, e+(n-1)e^*\left(\alpha^*\right)\right)\left[f^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)s^1+\left(1-f^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)\right)\frac{1-s^1}{n-1}\right] \\
&\quad + y\left(\alpha^*, e+(n-1)e^*\left(\alpha^*\right)\right)\left[f_1^1\left(e, \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)\left(\frac{ns^1}{n-1}-\frac{1}{n-1}\right)\right]-c'(e).
\end{aligned} \tag{53}
$$

For $e = e_1$ to be an equilibrium, it must be that the derivative is 0 at this point. Since $f^1\left(e^*\left(\alpha^*\right), \mathbf{e_{-j}}^*\left(\alpha^*\right)\right) = 1/n$ and $c'(e^*\left(\alpha^*\right)) = y_2(\alpha^*, ne(\alpha^*))$, we obtain

$$
c'\left(e^*\left(\alpha^*\right)\right) - \frac{y_2\left(\alpha^*, ne^*\left(\alpha^*\right)\right)}{n} = y\left(\alpha^*, ne^*\left(\alpha^*\right)\right)\left[f_1^1\left(e^*\left(\alpha^*\right), \mathbf{e_{-j}}^*\left(\alpha^*\right)\right)\left(\frac{ns^1}{n-1}-\frac{1}{n-1}\right)\right] \tag{54}
$$

which implies

$$
s^1\left(e_1\right) = \frac{1}{n} + \frac{n-1}{n}\frac{c'\left(e_1\right) - \frac{y_2\left(\alpha^*, ne_1\right)}{n}}{y\left(\alpha^{**}, ne_1\right)f_1^1(e_1, \mathbf{e_1})}. \tag{55}
$$

Since in equilibrium $\alpha^{**}\left(e_1\right) = \alpha^*$, player j indeed chooses e_1 under sharing rule (51).

For the leader, her expected utility if she chooses effort e_1 when the state of the world is α^* is given by

$$
y\left(\alpha^*, ne_1\right)\frac{1}{n} - c(e_1) \tag{56}
$$

which is maximized at

$$
c'\left(e_1\right) = y_2\left(\alpha^*, ne_1\right) \tag{57}
$$

which is the efficient effort level, given the state of the world α^*. □

Appendix B: Equilibrium existence

We now examine in which cases the efficient efforts implemented through sharing rule (6) constitute an equilibrium. Since equilibrium existence depends on the detailed specification of the curvature of the ranking technology, the production function and costs, we switch into a particular class in which we demonstrate that the exertion of efficient efforts constitutes a global utility maximum under our proposed sharing rule (6).

This is not the only case in which equilibria exist in our model. In order to illustrate this, we add an example of a commonly used model setup in which our candidate equilibrium exists. This example falls outside the class investigated in the following proposition.

Proposition 7. *We restrict attention to the class of problems consisting of output* $y(\alpha^*, e_1 + (n-1)e^*(\alpha')) = \alpha^*\bar{w}(e_1 + (n-1)e^*(\alpha'))$*, cost* $c(e_1) = (e_1^x)/x$*, for* $x > 1$*,* $\bar{w} > 0$ *and generalized Tullock contest success technology with precision parameter* r*. Moreover, we restrict permissible*

α to the compact range $[a, na]$ for $a > 0$. A sufficient condition for efficient effort provision by every player and truthful type reporting by player 1 to be an equilibrium in this class is that $x = r$.

Proof of proposition 7. We start with the second stage effort choice problem given any report α'. Consider the objective

$$u_1\left(e_1, \mathbf{e}^*_{-1}(\alpha'), \alpha^*\right) = y(\alpha^*, e_1)\left(\frac{(1-f^1(e_1))(1-s^1(e_1))}{n-1} + f^1(e_1)s^1(e_1)\right) - c(e_1) \quad (58)$$

where

$$s^1(e_1) = \frac{(n-1)\left(\frac{c'^*(\alpha')}{f_1^1(e^*(\alpha'))} - \frac{y_2(\tilde{\alpha}(e_1), e^*(\alpha'))}{nf_1^1(e^*(\alpha'))}\right)}{ny(\alpha^*, e_1)} + \frac{1}{n}. \quad (59)$$

We use shorthand notation $y(\alpha^*, \hat{e}) = y(\alpha^*, \hat{e} + (n-1)e^*(\alpha'))$, $f^1(\hat{e}) = f^1(\hat{e}, \mathbf{e_{-i}}^*(\alpha'))$ with $\hat{e} \in \{e_1, e^*(\alpha')\}$ and similarly for all other expressions. Then $\dfrac{\partial u_1}{\partial e_1} =$

$$\begin{aligned}
&y(\alpha^*, e_1)\Bigg[\frac{(1-f^1(e_1))\left(\frac{(n-1)y_2(\alpha^*, e_1)\mu}{ny(\alpha^*, e_1)^2} + \frac{(n-1)\tilde{\alpha}'(e_1)y_{1,2}(\tilde{\alpha}(e_1), e^*(\alpha'))}{n^2 f_1^1(e^*(\alpha'))y(\alpha^*, e_1)}\right)}{n-1} \\
&+ f^1(e_1)\left(-\frac{(n-1)y_2(\alpha^*, e_1)\mu}{ny(\alpha^*, e_1)^2} - \frac{(n-1)\tilde{\alpha}'(e_1)y_{1,2}(\tilde{\alpha}(e_1), e^*(\alpha'))}{n^2 f_1^1(e^*(\alpha'))y(\alpha^*, e_1)}\right) \\
&+ f_1^1(e_1)\left(\frac{(n-1)\mu}{ny(\alpha^*, e_1)} + \frac{1}{n}\right) - \frac{f_1^1(e_1)\left(-\frac{(n-1)\mu}{ny(\alpha^*, e_1)} - \frac{1}{n} + 1\right)}{n-1}\Bigg] \\
&+ y_2(\alpha^*, e_1)\left\{f^1(e_1)\left(\frac{(n-1)\mu}{ny(\alpha^*, e_1)} + \frac{1}{n}\right) + \frac{(1-f^1(e_1))\left(-\frac{(n-1)\mu}{ny(\alpha^*, e_1)} - \frac{1}{n} + 1\right)}{n-1}\right\} \\
&- c'(e_1)
\end{aligned} \quad (60)$$

in which $\mu = \dfrac{c'^*(\alpha')}{f_1^1(e^*(\alpha'))} - \dfrac{y_2(\tilde{\alpha}(e_1), e^*(\alpha'))}{nf_1^1(e^*(\alpha'))}$. (60) simplifies to

$$\begin{aligned}
\frac{\partial u_1}{\partial e_1} = &\frac{y_2(\alpha^*, e_1)}{n} - c'(e_1) + \frac{f_1^1(e_1)}{f_1^1(e^*(\alpha'))}\left(c'^*(\alpha')) - \frac{y_2(\tilde{\alpha}(e_1), e^*(\alpha'))}{n}\right) \\
&- \tilde{\alpha}'(e_1)\frac{y_{1,2}(\tilde{\alpha}(e_1), e^*(\alpha'))}{n^2 f_1^1(e^*(\alpha'))}\left\{nf^1(e_1) - 1\right\}.
\end{aligned} \quad (61)$$

Inserting

$$\tilde{\alpha}'(e_1) = \frac{y_2(\alpha^*, e_1)}{y_1(\tilde{\alpha}(e_1), e^*(\alpha'))} \quad (62)$$

we obtain

$$\begin{aligned}
\frac{\partial u_1}{\partial e_1} = &\frac{y_2(\alpha^*, e_1)}{n} - c'(e_1) + \frac{f_1^1(e_1)}{f_1^1(e^*(\alpha'))}\left(c'^*(\alpha')) - \frac{y_2(\tilde{\alpha}(e_1), e^*(\alpha'))}{n}\right) \\
&- \frac{y_2(\alpha^*, e_1)}{y_1(\tilde{\alpha}(e_1), e^*(\alpha'))}\frac{y_{1,2}(\tilde{\alpha}(e_1), e^*(\alpha'))}{n^2 f_1^1(e^*(\alpha'))}\left\{nf^1(e_1) - 1\right\}.
\end{aligned} \quad (63)$$

For the linear case $y(\alpha^*, \hat{e}) = \alpha^* w(\hat{e} + (n-1)e^*(\alpha')) = \alpha^* w(\hat{e})$ using again the shortened notation for the function $w(\cdot)$ in the last step we get

$$\begin{aligned}\frac{\partial u_1}{\partial e_1} = \frac{\alpha^* w'(e_1)}{n} - c'(e_1) + \frac{f_1^1(e_1)}{f_1^1(e^*(\alpha'))}\left(c'^*(\alpha')) - \frac{\alpha^* w(e_1) w'^*(\alpha'))}{n w(e^*(\alpha'))}\right) \\ - \frac{\alpha^* w'(e_1)}{w(e^*(\alpha')} \frac{w'^*(\alpha'))}{n^2 f_1^1(e^*(\alpha'))} \{n f^1(e_1) - 1\}\end{aligned} \tag{64}$$

in which we substituted the linear adjustment

$$\tilde{\alpha}'(e_1) = \frac{\alpha^* w'(e_1)}{w(e^*(\alpha'))}. \tag{65}$$

Using linear $w(\hat{e}) = \bar{w}(\hat{e} + (n-1)e^*(\alpha'))$ and monomial cost $c(\hat{e}) = \hat{e}^x/x$, the foc equals

$$\begin{aligned}\frac{\partial u_1}{\partial e_1} = \frac{\alpha^* \bar{w}}{n} - e_1^{x-1} + \frac{f_1^1(e_1)}{f_1^1(e^*(\alpha'))}\left(e^*(\alpha')^{x-1} - \frac{\alpha^* \bar{w}(e_1 + e^*(\alpha')(n-1))}{n^2 e^*(\alpha')}\right) \\ - \frac{\alpha^*}{n e^*(\alpha')} \frac{\bar{w}}{n^2 f_1^1(e^*(\alpha'))} \left\{n f^1(e_1) - 1\right\}.\end{aligned} \tag{66}$$

We use $ke^*(\alpha')$ in order to allow for any possible effort deviation. Then substituting $e_1 = ke^*(\alpha')$, the report-contingent efficient $e^*(\alpha') = (\bar{w}\alpha')^{\frac{1}{x-1}}$ and Tullock technology into the ratio of success function slopes gives

$$\begin{aligned}\frac{f_1^1(e_1)}{f_1^1(e^*(\alpha'))} &= \left(\frac{(n-1) r e_1^{r-1} e^*(\alpha')^r}{(e_1^r + (n-1)e^*(\alpha')^r)^2}\right) \Big/ \left(\frac{(n-1)r}{e^*(\alpha') n^2}\right) \\ &= \frac{e^*(\alpha') n^2 (e_1 e^*(\alpha'))^r}{e_1 \left(e_1^r + (n-1)e^*(\alpha')^r\right)^2} \\ &= \frac{n^2 \left(k(\alpha'\bar{w})^{\frac{2}{x-1}}\right)^r}{k\left(\left(k(\alpha'\bar{w})^{\frac{1}{x-1}}\right)^r + (n-1)\left((\alpha'\bar{w})^{\frac{1}{x-1}}\right)^r\right)^2} \\ &= \frac{n^2 k^{r-1}}{(k^r + n - 1)^2}.\end{aligned} \tag{67}$$

Making the same substitutions in the remainder of (66) step by step gives, for $e_1 = ke^*(\alpha')$

$$\begin{aligned}\frac{\partial u_1}{\partial e_1} = \frac{\alpha^* \bar{w}}{n} - (e^*(\alpha')k)^{x-1} + \frac{f_1^1(ke^*(\alpha'))}{f_1^1(e^*(\alpha'))}\left(e^*(\alpha')^{x-1} - \frac{\alpha^* \bar{w}(e^*(\alpha')k + e^*(\alpha')(n-1))}{e^*(\alpha') n^2}\right) \\ - \frac{\alpha^* \bar{w}(n f^1(e^*(\alpha')k) - 1)}{e^*(\alpha') n^3 f_1^1(e^*(\alpha'))},\end{aligned} \tag{68}$$

inserting Tullock technology gives

$$\begin{aligned}\frac{\partial u_1}{\partial e_1} = \frac{\alpha^* \bar{w}}{n} - (e^*(\alpha')k)^{x-1} + \frac{f_1^1(ke^*(\alpha'))}{f_1^1(e^*(\alpha'))}\left(e^*(\alpha')^{x-1} - \frac{\alpha^* \bar{w}(e^*(\alpha')k + e^*(\alpha')(n-1))}{e^*(\alpha') n^2}\right) \\ - \frac{\alpha^* \bar{w}\left(\frac{n}{(n-1)e^*(\alpha')^r (e^*(\alpha')k)^{-r} + 1} - 1\right)}{(n-1) n r},\end{aligned} \tag{69}$$

and finally inserting $e^*(\alpha') = (\bar{w}\alpha')^{\frac{1}{x-1}}$ gives

$$\begin{aligned}\frac{\partial u_1}{\partial e_1} &= \frac{\alpha^* \bar{w}}{n} - \left(k(\alpha'\bar{w})^{\frac{1}{x-1}}\right)^{x-1} \\ &+ \frac{n^2 k^{r-1}}{\left(k^r+n-1\right)^2}\left(\left(\left(\alpha'\bar{w}\right)^{\frac{1}{x-1}}\right)^{x-1} - \frac{\alpha^*\bar{w}(\alpha'\bar{w})^{-\frac{1}{x-1}}\left(k(\alpha'\bar{w})^{\frac{1}{x-1}} + (n-1)(\alpha'\bar{w})^{\frac{1}{x-1}}\right)}{n^2}\right) \\ &- \frac{\alpha^*\bar{w}\left(\frac{n}{(n-1)\left((\alpha'\bar{w})^{\frac{1}{x-1}}\right)^r\left(k(\alpha'\bar{w})^{\frac{1}{x-1}}\right)^{-r}+1} - 1\right)}{(n-1)nr}\end{aligned} \tag{70}$$

which simplifies into

$$\frac{\partial u_1}{\partial e_1} = \bar{w}\left(\frac{\alpha^*}{n} - \alpha' k^{x-1} - \frac{\alpha^*\left(k^r-1\right)}{nr\left(k^r+n-1\right)} + \frac{k^{r-1}\left(\alpha' n^2 - \alpha^*(k+n-1)\right)}{\left(k^r+n-1\right)^2}\right). \tag{71}$$

As a special case, we substitute $x = r$ and get

$$\frac{\partial u_1}{\partial e_1} = \frac{n}{\alpha^*}k^{r-1}\left(\frac{\alpha' n^2 - \alpha^*(k+n-1)}{\left(k^r+n-1\right)^2} - \alpha'\right) + 1 - \frac{k^r-1}{r\left(k^r+n-1\right)}. \tag{72}$$

We need to find a condition which ensures that this is positive for $k < 1$ and negative for $k > 1$.

1. $k < 1$: We need to ensure that

$$\frac{n}{\alpha^*}k^{r-1}\left(\frac{\alpha' n^2 - \alpha^*(k+n-1)}{\left(k^r+n-1\right)^2} - \alpha'\right) + 1 > \frac{k^r-1}{r\left(k^r+n-1\right)} \tag{73}$$

the right-hand side of which is negative whenever $k < 1$. In order for the left-hand side to be positive, we need

$$\frac{\alpha^* k^{1-r}}{n} + \frac{\alpha' n^2 - \alpha^*(k+n-1)}{\left(k^r+n-1\right)^2} > \alpha' \tag{74}$$

which is implied by

$$\alpha' \underbrace{\left(\frac{n^2}{\left(k^r+n-1\right)^2} - 1\right)}_{=A} + \alpha^* \underbrace{\left(\frac{k^{1-r}}{n} - \frac{1}{k^r+n-1}\right)}_{=B} > 0. \tag{75}$$

$A > 0$ for all $k < 1$ and $r > 0$ and $B > 0$ if $k < 1$ and

$$r > \frac{\log\left(\frac{k(n-1)}{n-k}\right)}{\log(k)} \geq \frac{n}{n-1} \tag{76}$$

where the final right-hand side term is the limit of the increasing log-ratio as $k \to 1$. We showed that for $x = r$, it is true that

$$\frac{\alpha^*}{n} - \alpha' k^{x-1} - \frac{\alpha^*\left(k^r - 1\right)}{nr\left(k^r + n - 1\right)} + \frac{k^{r-1}\left(\alpha' n^2 - \alpha^*(k+n-1)\right)}{\left(k^r + n - 1\right)^2} > 0. \tag{77}$$

However, since the derivative of the left-hand side of the last inequality with respect to x is $-\alpha' k^{x-1} \ln k$ which is positive for any $k < 1$, the last inequality holds for any $x \geq r$.

2. $k > 1$: We start from (71) and want to show that

$$\frac{k^{r-1}\left(\alpha' n^2 - \alpha^*(k+n-1)\right)}{\left(k^r + n - 1\right)^2} - \alpha' k^{x-1} < \frac{\alpha^*\left(k^r - 1\right)}{nr\left(k^r + n - 1\right)} - \frac{\alpha^*}{n}$$

implied by

$$\frac{n}{\alpha^* k}\left(\frac{k^r\left(\alpha' n^2 - \alpha^*(k+n-1)\right)}{\left(k^r + n - 1\right)^2} - \alpha' k^x\right) + 1 < \frac{1}{\frac{nr}{k^r - 1} + r} \tag{78}$$

the right-hand side of which is positive. Thus, we need to show that

$$\frac{n}{\alpha^* k}\left(\frac{k^r\left(\alpha' n^2 - \alpha^*(k+n-1)\right)}{\left(k^r + n - 1\right)^2} - \alpha' k^x\right) + 1 < 0 \tag{79}$$

or, equivalently, that

$$\frac{\alpha^* k^{1-r}}{\alpha' n} + \frac{\alpha' n^2 - \alpha^*(k+n-1)}{\alpha'\left(k^r + n - 1\right)^2} < k^{x-r} \tag{80}$$

which is implied by

$$\frac{n^2}{\left(k^r + n - 1\right)^2} - \frac{\alpha^*(k+n-1)}{\alpha'\left(k^r + n - 1\right)^2} < \frac{k^{-r}\left(\alpha' n k^x - \alpha^* k\right)}{\alpha' n} \tag{81}$$

which gives

$$\frac{\alpha^* k - \alpha' n k^x}{n k^r} < \frac{\alpha^*(k+n-1) - \alpha' n^2}{\left(k^r + n - 1\right)^2}. \tag{82}$$

We restrict possible $\alpha \in [a, b = sa]$, with $s > n$, and—since (82) is linear in α on both sides—obtain two subcases:

(a) Highest misreport $\alpha^* = a$, $\alpha' = b$: resulting in

$$\frac{ak^{-r}\left(k - snk^{x}\right)}{n} < \frac{a\left(k - sn^2 + n - 1\right)}{\left(k^r + n - 1\right)^2} \tag{83}$$

which holds for x sufficiently higher than r. For instance, for $x = r$, we obtain

$$\frac{k}{k^r} - 1 < sn - 1 + \frac{n(k - 1 + n - sn^2)}{(k^r - 1 + n)^2} \tag{84}$$

in which the left-hand side is negative and the right-hand side is positive for $k > 1$ because

$$\begin{aligned} \frac{sn\left(k^r + n - 1\right)^2 + n\left(k - 1 + n - sn^2\right) - \left(k^r - 1 + n\right)^2}{\left(k^r - 1 + n\right)^2} &> 0 \iff \\ ns\left(k^r + n - 1\right)^2 - \left(k^r + n - 1\right)^2 + n\left(k + n^2(-s) + n - 1\right) &> 0 \end{aligned} \tag{85}$$

which equals

$$(ns - 1)\left(k^r + n - 1\right)^2 > n^3 s - n(k + n - 1). \tag{86}$$

Recall that the left-hand side equals the right-hand side at $k = 1$ by construction. The left-hand side derivative is $2rk^{r-1}(ns-1)\left(k^r + n - 1\right) > 0$ and the rhs derivative is $-n < 0$. Hence, for $k > 1$, (84) holds.

(b) Lowest misreport $\alpha^* = b$, $\alpha' = a$: resulting in

$$\frac{ak^{-r}\left(sk - nk^{x}\right)}{n} < \frac{a(k - 1 + n)s - an^2}{\left(k^r + n - 1\right)^2} \tag{87}$$

which also holds for x sufficiently higher than r. For instance, for $x = r$, we obtain

$$\frac{k}{k^r} - 1 < \frac{n}{s} - \frac{n\left(n^2 - s(k + n - 1)\right)}{s\left(k^r + n - 1\right)^2} - 1 \tag{88}$$

in which the left-hand side is negative for $k > 1$ and the right-hand side is positive if

$$\frac{n}{s} - 1 > \frac{n\left(n^2 - s(k + n - 1)\right)}{s\left(k^r + n - 1\right)^2}. \tag{89}$$

We can rewrite the last inequality as follows

$$\begin{aligned} \frac{n - s}{s} &> \frac{n\left(n^2 - s(k + n - 1)\right)}{s\left(k^r + n - 1\right)^2} \iff \\ (n - s)\left(k^r + n - 1\right)^2 &> n^3 - sn(k + n - 1). \end{aligned} \tag{90}$$

We have equality for $k = 1$. The derivative of the right-hand side of the last inequality is $-sn$ which is negative for $s > 0$, while the derivative of the left-hand side is $(n - s)\left(k^r + n - 1\right)2rk^{r-1}$ which is positive for $n > s$. Hence, for $k > 1$, (88) holds.

Given player 1's choice of $e_1 = e^*(\alpha')$ at the second stage,[16] we now move on to the reporting stage where she chooses α' in order to maximize utility

$$\begin{aligned} &\underset{\alpha'}{max}\ y(\alpha^*, ne(\alpha'))\left(\sum_{\ell=1}^{n} f^\ell(\mathbf{e}^*(\alpha'))s^\ell(y^*,\alpha')\right) - c(e(\alpha')) \\ &= \underset{\alpha'}{max}\ \ u_1(\mathbf{e}^*(\alpha'), \alpha^*) \\ &= y(\alpha^*, ne(\alpha'))\frac{1}{n} - c(e(\alpha')) \end{aligned} \tag{91}$$

because $f^\ell(\mathbf{e}^*(\alpha')) = 1/n$ for every ℓ and $\sum_\ell s^\ell(y^*, \alpha') = 1$. This yields the first-order condition

$$y_2(\alpha^*, ne(\alpha')) = c'(e(\alpha')) \tag{92}$$

which equals the social planner's efficiency condition. Therefore, if the solution to the planner's problem is unique, then player 1 shares the same objective and will choose to truthfully report $\alpha' = \alpha^*$. □

The following example shows that it is easy to find instances violating the sufficient conditions of proposition 7 while still exhibiting the equilibrium identified in proposition 1.

<u>Example 4</u>: Consider the following two-players example outside of the class for which we show existence in proposition 7: (i) square-root team production $y(\alpha^*, e_1, e_2) = \alpha^*\bar{w}\sqrt{e_1 + e_2}$ and (ii) 'exponential difference' contest success function defined for two players as

$$f^1(e_1, e_2) \ = \frac{1}{1 + \exp\left(r(e_2 - e_1)\right)}, \text{ for } r > 0. \tag{93}$$

All other specifications are as in proposition 7. Assume that player two behaves according to our equilibrium prescription, i.e., $e_2 = e^*(\alpha') = 8^{\frac{1}{1-2x}}(\alpha'\bar{w})^{\frac{2}{2x-1}}$ (from the solution to the planner's problem). In this example setup, we obtain player one's objective as $u_1(e_1, e^*_{-1}(\alpha'), \alpha^*) =$

$$\frac{\alpha^*\bar{w}(e_1 + e^*(\alpha'))\left(s^1(y^*, \alpha')\left(\exp(r(e_1 - e^*(\alpha'))) - 1\right) + 1\right)}{\exp(r(e_1 - e^*(\alpha'))) + 1} \tag{94}$$

in which the equivalent of the sharing rule (6) is

$$s^1(y^*, \alpha') = \frac{1}{2}\left(1 - \frac{\alpha^*\bar{w}e_1 + \alpha^*\bar{w}e^*(\alpha') - 4e^*(\alpha')^x}{\alpha^*\bar{w}e_1e^*(\alpha')r + \alpha^*\bar{w}e^*(\alpha')^2r}\right). \tag{95}$$

Consider parameter values $x = 2$, $r = 2.5$, $\alpha \in [1, 50]$. A plot of player one's objective against $e^*(\alpha')$ by player two in figure 1 shows no profitable deviations.

Hence, this example illustrates that situations can be found which exhibit the equilibrium behavior derived in proposition 1 and lie outside of the class defined by the sufficient conditions presented in proposition 7.$\triangleleft$

[16] Since the effort choice problem of the uninformed players is identical to that of the leader, this argument directly implies that also $e_j = e^*(\alpha')$ for every $j > 1$.

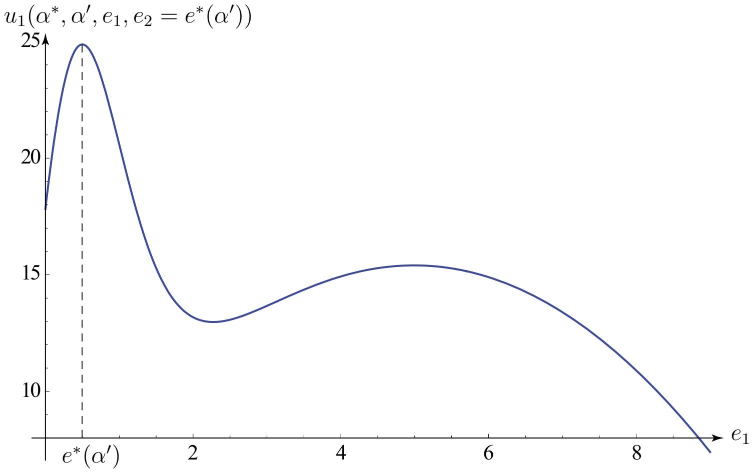

Figure 1: Possible deviations from $e^*(\alpha')$ for $\alpha \in [1, 50]$; the objective possess no other maxima.

References

Ahlquist, J. S., & Levi, M. (2011). Leadership: What it means, what it does, and what we want to know about it. Annual Review of Political Science, 14, 1–24.

Alchian, A., & Demsetz, H. (1972). Production, information costs, and economic organisation. American Economic Review, 62, 777–95.

Battaglini, M. (2006). Joint production in teams. Journal of Economic Theory, 130(1), 138–67.

Blanes i Vidal, J., & Möller, M. (2013). Decision-making and implementation in teams. CEPR, Discussion Paper #1208.

Bonatti, A., & Hörner, J. (2011). Collaborating. American Economic Review, 101(2), 632–63.

Coase, R. H. (1937). The nature of the firm. Economica, 4(16), 386–405.

Garrett, D., & Pavan, A. (2012). Managerial turnover in a changing world. Journal of Political Economy, 120(5), 879–925.

Garrett, D., & Pavan, A. (2014). Dynamic managerial compensation: On the optimality of seniority-based schemes. Northwestern University, Center for Mathematical Studies in Economics and Management Science, Discussion Paper.

Gershkov, A., Li, J., & Schweinzer, P. (2009). Efficient tournaments within teams. Rand Journal of Economics, 40(1), 103–19.

Gershkov, A., & Perry, M. (2012). Dynamic contracts with moral hazard and adverse selection. Review of Economic Studies, 79, 268–306.

Greenwood, R., & Empson, L. (2003). The professional partnership: Relic or exemplary form of governance? Organization Studies, 24(6), 909–33.

Guesnerie, R., Picard, P., & Rey, P. (1989). Adverse selection and moral hazard with risk neutral agents. European Economic Review, 33(4), 807–823.

Hedge Fund Research. (2014). Hedge fund asserts surge to new record to begin 2014. Hedge Fund Research, Inc., Januar 21, https://www.hedgefundresearch.com/pdf/pr_20140121.pdf.

Hermalin, B. E. (1998). Toward an economic theory of leadership: Leading by example. American Economic Review, 88(5), 1188–206.

Hermalin, B. E. (2012). Leadership and corporate culture. In R. Gibbons & J. Roberts (Eds.), Handbook of organizational economics (pp. 432–78). Princeton University Press.

Hilt, E. (2006). Incentives in corporations: Evidence from the american whaling industry. Journal of Law and Economics, 49(1), 197–227.

Holmström, B. (1982). Moral hazard in teams. Bell Journal of Economics, 13, 324–40.

Komai, M., & Stegeman, M. (2010). Leadership based on asymmetric information. Rand Journal of Economics, 41(1), 35–63.

Komai, M., Stegeman, M., & Hermalin, B. E. (2007). Leadership and information. American Economic Review, 97(3), 944–47.

Kvaløy, O., & Olsen, T. E. (2006). Team incentives in relational employment contracts. Journal of Labour Economics, 24, 139–69.

Lazear, E. P., & Shaw, K. L. (2007). Personnel economics: The economist's view of human resources. Journal of Economic Perspectives, 21(4), 91–114.

Legros, P., & Matthews, S. (1993). Efficient and nearly-efficient partnerships. Review of Economic Studies, 60(3), 599–611.

Marschak, J., & Radner, R. (1972). Economic theory of teams. New Haven, CT.

Rahman, D. (2012). But who will monitor the monitor? American Economic Review, 102(6), 2767–97.

Spulber, D. F. (2009). The theory of the firm. Cambridge: Cambridge University Press.

Strausz, R. (1999). Efficiency in sequential partnerships. Journal of Economicy Theory, 85, 140–56.

Vardi, N. (2013). The 40 highest-earning hedge fund managers and traders. Forbes, 26-February.

Zhou, J. (2011). Economics of leadership and hierarchy. University of California, Berkeley, Working paper.