Strategic Advertising and Directed Search*

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Abstract

We characterize equilibrium in a frictional environment where prices are not perfectly observed. Sellers send costly advertisements, which endogenously create buyers heterogeneity, to direct buyers' uncoordinated search, as in Butters (1977). The model helps understand advertising patterns in markets where sellers are capacity constrained (e.g., labor markets). We analyze how advertising intensity depends on the buyer-seller ratio and trade mechanisms. Equilibrium advertising intensity increases with the number of buyers only with a relatively small buyer-seller ratio. In large markets, price-posting dominates auctions. Unlike most directed-search models, the equilibrium is not constrained efficient.

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1 Introduction

In many markets, agents have limited amounts of goods/services to exchange. For instance, in the labor market, firms wanting to fill vacancies have limited numbers of available jobs for workers.[1] Given the prevalence of capacity constraints, how goods are allocated through the frictional matching process is central to economic analyses of labor, housing, marriage, and online markets. Many factors affect the matching process e.g., trading protocols, frictions, market tightness, and search intensities. This article focuses on one factor—informative advertising, which informs buyers about trading opportunities. Advertising is increasingly important in organizing trades in the aforementioned markets. For example, Monster.com, a leading online jobmatching website, had 63 million monthly jobseekers, posted over 1 million job advertisements, and raised $US996 million in revenue in 2008, mainly from advertising, yet the literature has paid scant attention to advertising's strategic use and its welfare consequences when capacity constraints are present.

Our primary interest is to determine how advertising affects trading. Unlike most advertising literature, where sellers can sell as many goods as they want, our framework explicitly considers capacity constraints and search frictions.[2] We make three contributions. Firstly, we determine equilibrium advertising patterns likely to emerge in large frictional markets (e.g., labor markets), and examine their efficiency properties. Secondly, we derive testable implications about relationships between advertising intensities and market fundamentals such as market tightness, trading protocols, and entry costs. Finally, we contribute to the directed-search literature by incorporating the imperfect observability of seller-initiated terms of trade and stochastic communication. We show that predictions of canonical directed-search models are robust to these extensions.[3]

More precisely, we build on a directed-search framework in which capacity-constrained sellers sell goods using price-posting or auctions, as in Burdett, Shi, and Wright (2001) and Julien, Kennes, and King (2000), respectively. Sellers post (reserve) prices and their locations to attract buyers, but this information reaches buyers only through costly advertisements. *Ex-ante* homogeneous buyers endogenously become heterogeneous as the advertisement arrival is stochastic. Therefore, different buyers may receive different advertisements from distinct sellers. Thus, buyers must choose wisely which seller to visit, as rationing or purchase delay will cause disutility.

[1]Other examples include landlords/real estate agents supplying accommodation in housing markets, participants seeking partners on dating sites, and auctioneers selling goods in online marketplaces (e.g., eBay and Yahoo!).

[2]For literature on the economics of advertising, see the comprehensive survey of Bagwell (2007).

[3]In canonical models, posting prices is costless and every buyer observes all posted prices. These assumptions appear to be *ad hoc* as the analysis usually focuses on large markets comprising infinite sellers.

The industrial organization literature examines extensively how firms facilitate trade with informative advertising.[4] One might think the advertising patterns in labor and commodity goods markets are similar. Two features of labor markets, however, create important differences. Firstly, each seller (firms) in labor markets have a limited number of vacancies (goods) and therefore there is competition among workers (buyers); such a capacity constraint is usually not binding in commodity goods markets. Secondly, and in relation to the first point, besides physical search frictions also present in commodity goods markets, a coordination friction is commonly observed in labor markets because workers compete with each other uncoordinatedly. Therefore, while commodity goods markets buyers may not care how intensively sellers advertise, labor-market workers must be concerned about advertising intensity, as high advertising intensity tends to inform more workers about jobs. This, in turn, increases the probability of being rationed.

We characterize a unique symmetric equilibrium with positive advertising intensity. Equilibrium advertising intensity is an inverted U-shape with respect to market tightness. This result contrasts sharply with previous findings in the commodity-goods advertising literature, where advertising intensity is strictly decreasing in market concentration (Butters, 1977 and Grossman and Shapiro, 1984).[5] Since each seller only needs a few buyers in order to be matched, she has weaker incentives to advertise when there are too few/ too many buyers. In particular, with a small buyer-seller ratio, fierce competition among sellers weakens their incentives to advertise. Likewise, with many buyers, a low level of advertising intensity almost guarantees sales and therefore sellers do not advertise much either. Equilibrium advertising intensity is maximal when buyer and seller numbers are relatively balanced. A direct implication for labor markets is that firms' search incentives will be small if vacancies' skill requirements are either very high or very low.

Other than market tightness, trading protocols are important determinants of equilibrium advertising intensity.[6] To highlight this aspect, we compare two trading protocols with extreme commitment powers: auctions and price-posting. We find that for any market tightness, equilibrium advertising intensity with auctions is higher than with price-posting. Lack of commitment under auctions gives sellers higher profits when there are more buyers

[4]In his seminal work, Butters (1977) formalizes the idea that advertising reach is both costly and stochastic in the context of homogenous goods markets. As in our model, such advertising informs buyers about prices and sellers' locations. Other aspects of advertising studied in this literature include informing match value (Grossman and Shapiro, 1984), reducing search cost (Robert and Stahl, 1993), and alleviating hold-up problems (Anderson and Renault, 2006).

[5]Menzio (2007) studies cheap-talk advertising in the directed-search framework. Information posted by sellers is soft' as the third party cannot verify it. Menzio shows that advertising informativeness has an inverted U-shape in market tightness.

[6]For a trading protocols comparison, see Kultti (1999, 2003) and Julien, Kennes, and King (2001).

visiting them. Thus, sellers have stronger incentives to invest in advertising in markets where auctions are popular trading protocols. The matching rate and sellers' expected revenues are, however, equal with both mechanisms, suggesting sellers are worse off with auctions as they incur higher advertising costs.

Our model also generates novel results about co-movements of market transparency (information) and prices. Common wisdom suggests buyers' better information intensifies sellers' competition and thereby drives prices down. This argument is, however, not necessarily true for capacity-constrained sellers. For small markets, Lester (2011) shows that prices can increase with better-informed buyers. In contrast, we endogenize market transparency by letting sellers choose advertising intensities. We extend Lester's result to large markets and show that equilibrium advertising intensity and price are positively correlated when market tightness is relatively small.

Finally, the efficiency analysis suggests that, unlike most directed-search models, the decentralized market equilibrium is not constrained efficient. There is too much advertising and too little entry. Informative advertising determines matching efficiency in two ways. Firstly, it informs more buyers about product availability and allows them to participate, thereby enhancing efficiency. Secondly, however, high advertising intensity causes more buyers to overlap their choice sets, which amplifies market frictions. In the decentralized market, sellers who have entered the market ignore the latter effect and overinvest in advertising. High advertising intensity, and therefore high advertising costs, reduces expected profits and generates insufficient entry.

This paper is organized as follows. We first review the related literature. Section 2 describes the basic environment. Section 3 characterizes the equilibrium outcome under auctions in the finite economy and explores the limit properties of this equilibrium as an exact-equilibrium in the large economy. Section 4 characterizes equilibrium under alternative price-posting mechanisms. Section 5 compares equilibrium advertising intensities under two mechanisms, the relationship between equilibrium prices and market transparency, and the efficiency analysis. Section 6 concludes.

1.1 Related literature

The two main ingredients in our analysis are directed search and informative advertising. The literature has analyzed each feature separately but not jointly. Putting these elements within the same framework not only captures important features of realistic markets, but also yields non-trivial comparisons between advertising patterns in commodity goods and labor markets. Butters (1977), Grossman and Shapiro (1984), and Robert and Stahl (1993) studied advertising in environments where sellers have unlimited capacity, as in commodity goods markets. An important application of the current framework is labor markets, where sellers (firms) have only limited capacities (vacancies). This constraint drastically changes firms' incentives to advertise and resulting advertising patterns.

Some recent papers incorporate search intensities in directed-search models. Albrecht, Gautier, and Vroman (2006), and Galenianos and Kircher (2009) extend the standard directed-search model so buyers can endogenously choose how many firms they visit. Lester (2010) permits each seller to decide how many units of goods to offer. We model advertisements, which can be considered sellers' search intensities, as the probability of reaching buyers. Unlike previous work, sellers' search intensities directly affect how many advertisements each buyer receives, and thus the market's information structure. Unlike search intensities that do not affect information structure, here, more advertisements do not necessarily lead to higher probabilities of trading.

One closely related paper is Menzio (2007), which focuses on how heterogeneous firms can direct workers' search through cheap-talk advertising. He finds that, for intermediate market-tightness values, there exists an equilibrium where the advertisements' content is positively correlated with actual productivity i.e., the advertising informativeness has an inverted U-shape in market tightness. Surprisingly, in this paper with homogeneous goods, we obtain a similar result—advertising intensity, measured by how many advertisements each seller sends, has an inverted U-shape in market tightness.

Finally, several papers investigate relationships between market transparency and equilibrium price. Anderson and Renault (2000) analyze a duopoly sequential-search model and show that entry of informed buyers may lead to prices increasing. Most closely related to our paper is Lester (2011), which focuses on a directed-search model with homogeneous products. In his environment, uninformed buyers can participate through random search while posted prices direct informed buyers.[7] Lester shows that, in a small market, an increased proportion of informed buyers can increase prices. As in Varian (1980), these papers exogenously assume the composition of informed and uninformed buyers. Moreover, informed buyers observe all posted prices. Our approach, however, is motivated by Butters (1977), which induces the endogenous fraction of the population of informed buyers. We show that the positive correlation between market transparency and price result in Anderson and Renault (2000) and Lester (2011) can be extended to large markets.

[7]In our model, uninform buyers cannot trade. Allowing random search will dilute the value of costly advertising, which leads to the non-existence of pure-strategy equilibrium.

2 The model

The market considered has M sellers and N buyers with $M, N \geq 2$. The buyer-to-seller ratio is denoted by $\phi = N/M$, which reflects the corresponding market tightness. We denote by $\mathcal{M}$ and $\mathcal{N}$, the sets of sellers and buyers, respectively. Each seller is endowed with only one unit of a homogeneous perishable good that has no value to sellers. Buyers are ex-ante identical and each buyer demands one unit of the good. Both sellers and buyers are risk-neutral so that if a buyer purchases the good at price p, his net payoff is $1 - p$, while the seller's payoff is simply p.

Before the market's opening, buyers do not know sellers' locations nor corresponding terms of trade. Sellers must inform buyers about their locations and prices via costly advertising. As in Butters (1977), advertisement reach is stochastic. Buyers can only trade with sellers from which they have received advertisements. From now on, buyers who receive at least one advertisement are called informed buyers'.

We now describe in detail the game buyers and sellers play, the trading protocols, and the advertising technology sellers can use to inform buyers and the induced information sets arising from this advertising technology. Since buyers cannot coordinate about which seller to visit (endogenous friction), certain restrictions are imposed on their selecting strategies.

Game structure. The game has three stages. In the first stage, sellers simultaneously and independently post a mechanism and choose advertising intensities to attract buyers. In the second stage, all informed buyers simultaneously and independently choose which seller to visit. This selection decision is irreversible, even if buyers eventually do not obtain goods. Uninformed buyers cannot trade. In the final stage, sellers transfer goods to buyers according to terms of trade announced in the first stage.

Trading Protocols. We analyze the two most commonly studied direct and anonymous mechanisms in the directed-search literature: first-price auctions with a reserve price and price-posting.[8] With price-posting, sellers fully commit to the posted prices. If more than one buyer selects the same seller, this seller randomly picks one of them. While price-posting is widely observed in many markets, allowing buyers to bid ex-post through an auction allows the seller to exploit ex-post surplus when several buyers compete for her goods. In this case, goods are sold at the posted (reserve) price only when one buyer visits a seller. When more than one buyer selects the same seller, however, the price is bid up to 1 and sellers obtain the entire match's surplus.

[8]Sellers' mechanisms are direct if transfer and allocation do not depend on the mechanisms posted by other sellers. Mechanisms are anonymous if they treat all buyers participating in them symmetrically.

Advertising. Informative advertising is modeled in the spirit of Butters (1977) and Grossman and Shapiro (1984) so that advertisements reach buyers stochastically.[9] Seller $i \in \mathcal{M}$ chooses an advertising intensity $q_i \in [0,1]$ so each buyer has probability q_i of receiving advertisements from seller i. The advertising is informative in the sense that advertisements include the seller's posted price and location. Without this information, buyers cannot trade.

The advertising cost function is denoted by $C(q)$ and has these properties: $C'(q) > 0$, $C''(q) \geq 0$, $C(0) = 0$, and $C'(0) = 0$. Note that advertising cost does not depend on the number of buyers.[10] This assumption is motivated by labor market advertising and searching behavior. Today, most, if not all, jobs are posted on mediums such as jobmatching sites, job agencies, or newspapers, and most workers find jobs through those channels. Thus, if the probability of reaching any buyer depends solely on how many platforms upon which a seller posts advertisements, the number of buyers is irrelevant to advertising expenditure.

A micro-founded example for our advertising technology can be constructed easily. Consider a constant-reach independent readership (CRIR) advertising technology. A seller chooses how many websites upon which to place advertisements. Assume each website has a readership of x in the population. Then, a fraction x of the population is exposed to an advertisement published on any given website. If a seller places advertisements in s websites, the probability a given buyer sees none of these advertisements is $(1-x)^s$. The reach of such an advertising campaign is $q = 1-(1-x)^s$. Equivalently, we can write the number of websites upon which advertisements must be placed to achieve a reach of q as $s = \ln(1-q)/\ln(1-x)$. Suppose each website charges y for posting advertisements. Then, in order to reach each buyer with probability q, sellers must incur a cost equal to $C(q) = y \ln(1-q)/\ln(1-x)$. This cost function satisfies our advertising cost specification.

Buyers' Information Sets. Although all buyers are ex-ante identical, the stochastic advertising technology endogenously induces some buyer heterogeneity because buyers can receive advertisements from a different number of sellers. To illustrate this, define buyer j's information set $\mathcal{I}_j \in \mathbb{P}(\{p^1, ..., p^M\})$ which contains the vector of all prices of sellers that buyer j sees after the advertising stage, where $\mathbb{P}(\mathcal{Z})$ is the power set of set $\mathcal{Z}$. It is important to highlight that buyer j does not know seller i's location unless $p^i \in I_j$. Additionally, for any $\mathcal{I}$, $|\mathcal{I}|$ denotes the size of $\mathcal{I}$.

Belief System. When buyers receives advertisements from sellers, they know the price and

[9] Ireland (1993) and Eaton, McDonald, and Meriluoto (2010) allow for target advertising so sellers can choose the fraction of buyers receiving advertisements.

[10] In Butters (1977), urn-ball matching advertising requires that the probability of reaching a buyer decreases in the total number of buyers, holding the resources devoted to advertising fixed. This type of advertising technology is more applicable to conventional marketing models (e.g., mailbox advertising and phone-calls), where firms contact consumers directly.

location but are assumed to never observe a seller's advertising intensity. This assumption is natural since we are considering a market with imperfectly observed prices. Moreover, it is even harder for buyers to know advertising intensities than prices.[11] Given sellers' capacity constraint, buyers care about each seller's advertising intensity because high advertising intensity tends to attract more buyers and therefore creates a higher probability of rationing. Since advertising intensities are unobservable, buyers must form beliefs about them. A buyer's belief function, $\mu : \mathbb{P}(\{p^1, ..., p^M\}) \to [0,1]^M$, maps from his information set to the profile of advertising intensities. In particular, $\mu^i(\mathcal{I})$ denotes the conjectured level of seller i's advertising intensity. For example, a belief system is said to be passive if $\mu^i(\mathcal{I}) = \hat{q}^i, \forall i \in \mathcal{M}$ and $\mathcal{I} \in \mathbb{P}(\{p^1, ..., p^M\})$ for some fixed $\hat{q}^i \in [0,1]$.

Selecting Strategy. Buyers select sellers based on their information sets and the beliefs they form. A buyer's selecting strategy that specifies the probability with which he selects a seller is given by σ: $\mathbb{P}(\{p^1, ..., p^M\}) \to \triangle_{M+1}$, where $\triangle_{M+1}$ is the M dimensional simplex (uninformed buyers do not participate with probability 1). The probability that buyer j selects seller i is denoted by $\sigma^i(\mathcal{I}_j)$. These restrictions are imposed on the buyer's selecting strategies:

(i) $\sigma^i = 0$ if $p^i \notin \mathcal{I}$;

(ii) $\sum_{i=1}^{M} \sigma^i(\mathcal{I}) \leq 1$ for all $\mathcal{I}$ and it holds as an equality if $|\mathcal{I}| \neq \emptyset$;

(iii) $\sigma^i(\mathcal{I}_j) = \sigma^l(\mathcal{I}_j)$ if $p^i, p^l \in \mathcal{I}_i$ and $p^i = p^l$;

(iv) if $|\mathcal{I}_j| = |\mathcal{I}_k|$ and there exists a one-to-one mapping: $\eta : \mathcal{I}_j \to \mathcal{I}_k$ so that $p^i = \eta(p^i)$ for every $p^i \in \mathcal{I}_j$, then $\sigma^i(\mathcal{I}_j) = \sigma^l(\mathcal{I}_k)$ where $p^l \in \mathcal{I}_k$ and $p^l = \eta(p^i)$.

Interpretation of the first two conditions is straightforward. Buyers cannot visit any seller from whom he has received no advertisements. Conditions (iii) and (iv) are modified versions of the standard assumptions in directed-search models .[12] In particular, condition (iii) says that if a buyer observes two sellers charging the same price, he should select each seller with the same probability. This condition ensures *anonymity* over observed sellers.

[11] Note that in the U.S. large, stock-market-listed companies must provide some public information about expenditures and typically do not report advertising expenditures. In contrast, small, unlisted firms need not provide any sort of public information, making it difficult for buyers to ascertain their advertising budgets. Moreover, this assumption also allows us to bypass the difficulty in establishing the existence of symmetric pure-strategy equilibrium in large markets. The small-market results we derived with observable advertising intensity are available upon request.

[12] In canonical directed-search models, *seller anonymity* requires buyers to select two sellers with the same probability if they charge the same price, and *seller symmetry* requires all buyers to use the same selecting strategy since they observe the same set of prices.

Condition (iv) is stronger than the requirement that buyers possessing the same information set should use the same selecting strategy. Suppose there is a one-to-one mapping between two information sets and each price in the mapping domain equals its image; condition (iv) requires seeing the two information sets as being similar' and buyers possessing these two information sets should select sellers in a similar' fashion.[13]

Equilibrium. We use the Symmetric Perfect Bayesian Equilibrium (SPBE) concept and focus on those equilibria with strictly positive advertising intensities.

3 Auctions with reserve prices

Suppose the prevailing sales mechanism is a first-price auction with reserve-price-posting.[14] Here, sellers are committed to reserve prices only in a bilateral match and will run first-price auctions when multiple buyers select them.

Given this trading protocol, we solve the game backwards. Consider the final stage, when the auction takes place. When attending the auction held by seller i, a buyer observes the total number of auction participants. His bidding strategy depends on the reserve price, r^i, and the number of participants, n^i, because sellers do not ex-ante commit to the posted price when $n^i > 1$. Bertrand-type reasoning yields the optimal bidding strategy:

$$b(r^i, n^i) = \begin{cases} r^i & \text{if } \; n^i = 1; \\ 1 & \text{if } \; n^i > 1. \end{cases} \tag{1}$$

Seller i's realized profit is:

$$\pi^i(r^i, n^i) = \begin{cases} 0 & \text{if } \; n^i = 0; \\ r^i & \text{if } \; n^i = 1; \\ 1 & \text{if } \; n^i > 1. \end{cases} \tag{2}$$

We now turn to the game's second stage and study a micro-founded matching process by analyzing buyers' choices in selecting which seller to visit.

[13]This example illustrates the intuition of Assumption iv: if buyer 1's information set is $\{p^1, p^2\}$, buyer 2's information set is $\{p^3, p^4\}$ and $p^1 = p^3$, $p^2 = p^4$. Assumption iv requires $\sigma_1^1 = \sigma_2^3$ and $\sigma_1^2 = \sigma_2^4$.

[14]Note that here first- and second-price auctions yield the same outcomes.

3.1 Buyers' choices

Unlike in standard directed-search models, a buyer's selection problem over sellers is more complicated with imperfectly observed prices, as advertising intensities matter. A buyer receiving an advertisement from seller i needs to assess the probability of being alone at seller i's store. This is because it is the only way he can obtain a positive surplus if he selects seller i. This probability, however, depends on: (i) seller i's advertising intensity and posted reserve price, which directly affect other buyers' probabilities of selecting seller i; (ii) other sellers' advertising intensities and posted reserve prices, which indirectly affect other buyers' probabilities of selecting seller i; and (iii) other buyers' choices for any given information set.

To better establish a buyer's selection problem, we first consider M sellers competing for two buyers, denoted by 1 and 2. Since both buyers are *ex-ante* identical, without loss of generality, we use buyer 1 as the representative buyer. Further, since we are solving for a symmetric equilibrium, we treat seller i as the only potential deviator.

If buyer 1 receives advertisements only from seller i, he must select seller i. Now suppose buyer 1 receives advertisements from at least two sellers, including seller i. His expected payoff from selecting seller i is $U_1^i = (1 - r^i)\Pr[\text{being alone at seller } i]$. One possibility is that buyer 2 does not select seller i as he has not received seller i's advertisement. From buyer 1's perspective, the probability of this event is $P_0^i \equiv 1 - \mu^i(\mathcal{I}_1)$. Another possibility is that, although buyer 2 receives seller i's advertisement, he chooses not to visit seller i. We denote the probability of this event by P_+^i. Note that the probability of buyer 2 not selecting seller i conditional on receiving i's advertisement depends on $\mathcal{I}_2$. From buyer 1's perspective, the probability that buyer 2 has the information set $\mathcal{I}_2$ conditional $r^i \in \mathcal{I}_2$ is given by

$$\prod_{\substack{r^j \in \mathcal{I}_2 \\ j \neq i}} \mu^j(\mathcal{I}_1) \cdot \prod_{r^k \notin \mathcal{I}_2} (1 - \mu^k(\mathcal{I}_1)).$$

For a given information set $\mathcal{I}_2$, the probability that buyer 2 selects seller i is $\sigma^i(\mathcal{I}_2)$. Thus, we have:

$$P_+^i = \mu^i(\mathcal{I}_1) \left[1 - \sum_{\substack{\mathcal{I}_2 \text{ s.t.} \\ r^i \in \mathcal{I}_2}} \sigma^i(\mathcal{I}_2) \cdot \prod_{\substack{r^j \in \mathcal{I}_2 \\ j \neq i}} \mu^j(\mathcal{I}_1) \cdot \prod_{r^k \notin \mathcal{I}_2} (1 - \mu^k(\mathcal{I}_1)) \right]. \tag{3}$$

In a symmetric equilibrium, all sellers but seller i always choose prices and advertising intensities at the equilibrium level, (r^*, q^*). Then, buyers should believe that $\mu^k(\mathcal{I}_1) = q^*$

for all $k \neq i$ and (3) simplifies to:

$$P_+^i = \mu^i(\mathcal{I}_1)\left[1 - \sum_{\substack{\mathcal{I}_2 \text{ s.t.} \\ r^i \in \mathcal{I}_2}} \sigma^i(\mathcal{I}_2) \cdot q^{*|\mathcal{I}_2|-1} \cdot (1-q^*)^{M-|\mathcal{I}_2|}\right]. \tag{4}$$

The restrictions imposed on the selecting strategy, together with the fact that $r^k = r^*, \forall k \neq i$, implies that, conditional on both containing r^i, the two information sets with the same size must induce similar selecting behavior.[15] Therefore, (4) becomes

$$P_+^i = \mu^i(\mathcal{I}_1)\left[1 - \sum_{z=1}^{M} \binom{M-1}{z-1} \cdot \sigma^i(\mathcal{I}_2 || \mathcal{I}_2| = z, r^i \in \mathcal{I}_2) \cdot q^{*z-1} \cdot (1-q^*)^{M-z}\right], \tag{5}$$

where $\sigma^i(\mathcal{I}_2||\mathcal{I}_2| = z, r^i \in \mathcal{I}_2)$ is the probability that buyer 2 selects seller i conditional on his information set containing r^i and the size of his information set being z. To simplify notation, we define buyer 2's expected probability of selecting seller i conditional on $r^i \in \mathcal{I}_2$ as:

$$\Lambda^i(\sigma^i) = \sum_{z=1}^{M} \binom{M-1}{z-1} \cdot \sigma^i(\mathcal{I}_2||\mathcal{I}_2| = z, r^i \in \mathcal{I}_2) \cdot q^{*z-1} \cdot (1-q^*)^{M-z}$$

Buyer 1's expected payoff from selecting seller i can then be written as:

$$U_1^i = (1-r^i)(P_0^i + P_+^i) = (1-r^i)(1 - \mu^i(\mathcal{I}_1)\Lambda^i(\sigma^i)).$$

We can similarly compute buyer 1's expected payoff from selecting any observed seller $k \neq i$, $U_1^k = (1-r^*)\left(P_0^k + P_+^k\right)$, with $P_0^k = 1 - q^*$ and

$$\begin{aligned} P_+^k = q^*\Bigg[1 - \mu^i(\mathcal{I}_1) \sum_{z=2}^{M} \binom{M-2}{z-2} \cdot \sigma^k(\mathcal{I}_2||\mathcal{I}_2| = z, r^i \in \mathcal{I}_1) \cdot q^{*z-2} \cdot (1-q^*)^{M-z} \\ - (1-\mu^i(\mathcal{I}_1)) \sum_{z=2}^{M} \binom{M-2}{z-2} \cdot \sigma^k(\mathcal{I}_2||\mathcal{I}_2| = z-1, r^i \notin \mathcal{I}_1) \cdot q^{*z-2} \cdot (1-q^*)^{M-z}\Bigg]. \end{aligned} \tag{6}$$

From the symmetry assumption, $\sigma^k(\mathcal{I}_2||\mathcal{I}_2| = z, r^i \in \mathcal{I}_1) = \frac{1-\sigma^i(\mathcal{I}_2||\mathcal{I}_2|=z, r^i \in \mathcal{I}_1)}{z-1}$ and $\sigma^k(\mathcal{I}_2||\mathcal{I}_2| = z-1, r^i \notin \mathcal{I}_1) = \frac{1}{z-1}$. As in the definition of Λ^i, we can denote the sum of the two last terms in the square bracket in (6) as Λ^k. Buyer 1's expected payoff from selecting seller k is then

[15] For example, if $\mathcal{I}_2 = \{r^i, r^k\}$ and $\mathcal{I}_2' = \{r^i, r^l\}$, then $\sigma^i(\mathcal{I}_2) = \sigma^i(\mathcal{I}_2')$ and $\sigma^k(\mathcal{I}_2) = \sigma^l(\mathcal{I}_2')$ since $r^k = r^l = r^*$.

$U_1^k = (1 - r^*)(1 - q^* \Lambda^k)$.

Now, it is straightforward to extend our analysis to the general case where $N \geq 2$ buyers are present. Since all sellers except i use symmetric strategies, let $\Lambda^j = \Lambda^k \equiv \Lambda$ and $U^k = U^l \equiv U$ for $j, k \neq i$. Facing $N-1$ potential competitors, the probability that buyer 1 is alone in seller i's store is then $(1 - \mu^i(\mathcal{I})\Lambda^i)^{N-1}$, and the expected payoff from selecting seller i is $(1 - r^i)(1 - \mu^i(\mathcal{I})\Lambda^i)^{N-1}$.

Buyer 1 selects by comparing expected payoffs from selecting seller i, U^i, relative to expected payoffs from selecting any other observed seller, U. We now formally define the equilibria in the subgame where buyers make selections.

Definition 1 *A symmetric equilibrium in a buyer's selecting game is a common strategy profile* $\{\sigma(\mathcal{I})\}_{\mathcal{I} \in \mathbb{P}(\{r_1, \ldots\ldots, r^M\})}$ *used by all buyers and satisfying these: (i)* $\sigma^i(\mathcal{I}) = 0, \forall\, i \in \mathcal{M}$ *if* $\mathcal{I} = \emptyset$*; (ii)* $\sigma^i(\mathcal{I}) = 1$ *if* $\mathcal{I} = \{r^i\}$*; (iii)* $\forall\, r^i, r^k \in \mathcal{I}$ *with* $\sigma^i(\mathcal{I}) > 0$ *and* $\sigma^k(\mathcal{I}) > 0$, $U^i = U^k = \max_{r^l \in \mathcal{I}} U^l$*; (iv) for* $r^i \in \mathcal{I}$ *with* $\sigma^i(\mathcal{I}) = 0$, $U^i \leq \max_{r^l \in \mathcal{I}} U^l$.

The next lemma provides a condition ensuring consistency of buyers' beliefs and it must be fulfilled in any buyers' selecting game equilibrium. All proofs are in Appendix A.

Lemma 1 *In the buyers' selecting game, the conditional expected probabilities of selecting the deviator,* Λ^i*, and of selecting a non-deviator,* Λ*, must satisfy these conditions:*

$$\mu^i(\mathcal{I})\Lambda^i + (M-1)q^*\Lambda = 1 - \left(1 - \mu^i(\mathcal{I})\right)(1 - q^*)^{M-1}, \ \forall \mathcal{I}. \tag{7}$$

Lemma 1 illustrates an intuitive relationship between Λ^i and Λ. If we multiply both sides of (7) by the number of buyers, N, the left-hand side of (7) is just the expected number of active buyers searching in the market. The right-hand side is the expected number of informed buyers receiving advertisements from at least one seller. Thus, equation (7) simply equates the expected number of active buyers with the expected number of informed buyers. This relationship is believed by any buyer receiving advertisements from seller i.[16] The next lemma shows that, in any subgame following a pair of reserve prices announced by a deviator and non-deviator, the induced pair of equilibrium probabilities of selecting sellers is unique.

Lemma 2 *In the buyers' selecting game, any* (r^i, r^*) *induces a unique* (Λ^i, Λ) *as the equilibrium in the buyers' selecting game, provided the belief* $\mu^i(\mathcal{I})$ *is unique for any* $\mathcal{I}$.

In the standard directed-search model with perfect observability of prices, any pair of reservation prices (r^i, r^*) induces a unique symmetric equilibrium-selecting strategy in the

[16]Note that $\mathcal{I}_1$ appears in both sides of (7). The market-clearing condition is a subjective concept and holds for every buyer observing r_i.

buyers' selecting game. In the current environment, with imperfect observability, the uniqueness of selecting strategy following a given (r^i, r^*) is not guaranteed except when $N = 2$.[17] The expected probability of selecting sellers, which aggregates the selecting probabilities over the buyers' information set, is, however, unique provided the induced belief is unique, which is all market participants care about.

Note $\Lambda^i = 0$ (1) iff $\sigma^i(\mathcal{I} \| \mathcal{I}| = z, r^i \in \mathcal{I}) = 0$ (1) for all $z = 1, ..., M$. Now suppose that, for any pair of (r^i, r^*), $\mu^i(\mathcal{I}) \in (0, 1)$ and it is unique. Then, from the proof of Lemma 2, the induced expected selecting probability is a function of sellers' announcements and buyers' beliefs:

$$\Lambda^i = \begin{cases} 1, & \text{if } \left[\frac{M-1-(1-\mu^i(\mathcal{I}))[1-(1-q^*)^{M-1}]}{(M-1)(1-\mu^i(\mathcal{I}))}\right]^{N-1} < \frac{1-r^i}{1-r^*}; \\ \frac{1}{\mu^i(\mathcal{I})} - \frac{(M-1)+\left(1-\mu^i(\mathcal{I})\right)(1-q^*)^{M-1}}{\mu^i(\mathcal{I})[(M-1)(\frac{1-r^i}{1-r^*})^{\frac{1}{N-1}}+1]}, & \text{if } \begin{array}{l} \left[1-\frac{1-(1-\mu^i(\mathcal{I}))(1-q^*)^{M-1}}{(M-1)}\right]^{N-1} \leq \frac{1-r^i}{1-r^*} \\ \leq \left[\frac{M-1-(1-\mu^i(\mathcal{I}))[1-(1-q^*)^{M-1}]}{(M-1)(1-\mu^i(\mathcal{I}))}\right]^{N-1} \end{array}; \\ 0, & \text{if } \frac{1-r^i}{1-r^*} < \left[1 - \frac{1-(1-\mu^i(\mathcal{I}))(1-q^*)^{M-1}}{(M-1)}\right]^{N-1} \end{cases} \tag{8}$$

which completely characterizes buyers' selection choices.

3.2 Sellers' choices

We now return to the game's first stage and study seller i's incentive to deviate. Her expected profit is:

$$\Pi^i = r^i \cdot \Pr[n^i = 1] + 1 \cdot \Pr[n^i \geq 2] - C(q^i). \tag{9}$$

Seller i understands buyers' searching patterns in the game's second stage and expects buyers receiving her advertisements, on average, to select according to Λ^i. To calculate $\Pr[n^i = 1]$, these events must be certain: (1.) no buyer receives advertisements from seller i (with probability $(1 - q^i)^N$); and (2.) one buyer receives advertisements from seller i, but does not visit seller i (with probability $Nq^i(1 - q^i)^{N-1}(1 - \Lambda^i)$); ....; (N+1.) all buyers receive advertisements from seller i, but none visit seller i (with probability $(q^i)^N(1 - \Lambda^i)^N$). Summing the above probabilities yields:

$$\Pr[n^i = 0] = \sum_{k=0}^{N} \binom{N}{k} \left[q^i(1 - \Lambda^i)\right]^k \left(1 - q^i\right)^{N-k} = \left(1 - q^i\Lambda^i\right)^N. \tag{10}$$

[17] Suppose $N = 2$. Since any buyer observing only one seller's advertisement must visit this seller with probability 1, $\{r^1, r^2\}$ is the only information set (buyer type) that matters for assessing selecting strategy. Then, the analysis degenerates to finding the selecting strategy of buyers with type $\{r^1, r^2\}$. The uniqueness of the equilibrium-selecting strategy can be established following Peters (1984).

To calculate $\Pr[n^i = 2]$, these events must be certain: (1.) only one buyer receives seller i's advertisements and visits seller i (with probability $Nq^i(1-q^i)^{N-1}\Lambda^i$); and (2.) two buyers receive seller i's advertisements but only one visits seller i (with probability $N(N-1)(q^i)^2(1-q^i)^{N-2}[2\Lambda^i(1-\Lambda^i)]$);.......(N.) N buyers receive seller i's advertisements but only one visits seller i (with probability $(q^i)^N N\Lambda^i(1-\Lambda^i)^{N-1}$). Adding the above probabilities yields:

$$\Pr[n^i = 1] = \ q^i\Lambda_i \sum_{k=0}^{N-1} \binom{N}{k+1} (k+1) \left[q^i(1-\Lambda^i)\right]^k \left(1-q^i\right)^{N-k-1} = \ Nq^i\Lambda^i \left(1-q^i\Lambda^i\right)^{N-1}. \tag{11}$$

Seller i then sets r^i and q^i so as to solve this profit-maximization problem:

$$\max_{r^i,q^i} \Pi^i = \ 1-(1-r^i)Nq^i\Lambda^i\left(1-q^i\Lambda^i\right)^{N-1} - \left(1-q^i\Lambda^i\right)^N - C(q^i) \\ \text{s.t.} \quad \text{Equation (8).} \tag{12}$$

So far, we have not imposed any restriction on buyers' beliefs when they observe a deviating price $r^i \neq r^*$, so the results are general. In principle, a reasonable belief about the off-equilibrium can be derived using certain refinements. In particular, if an observable action signals an agent's unobservable action (as seller i does here), one can utilize the wary-belief' argument (McAfee and Schwartz, 1994, Rey and Verge, 2004, and In and Wright, 2012). It is well known, however, that deriving such beliefs explicitly is extremely difficult. Appendix B provides a two-buyer-and-two-seller example to illustrate derivation of equilibrium advertising under a wary-belief specification. In the main text, we hereafter focus on passive beliefs so that $\mu^i(\mathcal{I}) = q^*, \forall \mathcal{I}$. Passive beliefs are used widely in literature studying price's quality-signaling effect (e.g., Klein and Leffler, 1981, Wolinsky, 1983, and Bester, 1998) and the secret contracting between manufactures and retailers (e.g., Hart and Tirole, 1990). Most of our results are derived for large markets, in which individual participants understand that they are too small to influence the equilibrium allocation. Thus, their response to unexpected prices might not be as strategic as one would observe when participants trade in small markets (e.g., firms in oligopoly markets). Moreover, simultaneously, large-market participants may still have a relatively good idea about prevailing prices. Then, passive beliefs are not as restrictive as it might appear *a priori*.

For ease of exposition, we define $\tau \equiv 1-(1-q)^M$. Proposition 1 characterizes the SPBE when buyers hold passive beliefs.

Proposition 1 *In a finite market with trades settled by auctions, a symmetric equilibrium with positive advertising intensity exists when* $C''(0) < MN(N-1)/(M-1)$*. The equilibrium* *choice of r^* and q^* are characterized by these conditions:*

$$r^* = \frac{(\phi - \frac{1}{M})\tau}{(M-1) + (\phi - 1)\tau}, \tag{13}$$

$$C'(q^*) = r^* \frac{\phi\tau}{q^*}\left(1 - \frac{\tau}{M}\right)^{N-1} + (1 - r^*)\frac{\phi(\phi - \frac{1}{M})\tau^2}{q^*}\left(1 - \frac{\tau}{M}\right)^{N-2}. \tag{14}$$

The boundary condition, $C''(0) < MN(N-1)/(M-1)$, ensures the marginal benefit of increasing advertising intensity is strictly higher than the marginal cost at least for advertising intensity, q, that is small enough. Also note that, if equilibrium advertising intensity is $q^* = 1$ and thus seller-buyer communication is perfect, r^* equals the equilibrium reserve price in the standard directed-search models for finite markets. The advertising intensity is, however, determined endogenously in our model and may not always equal 1.

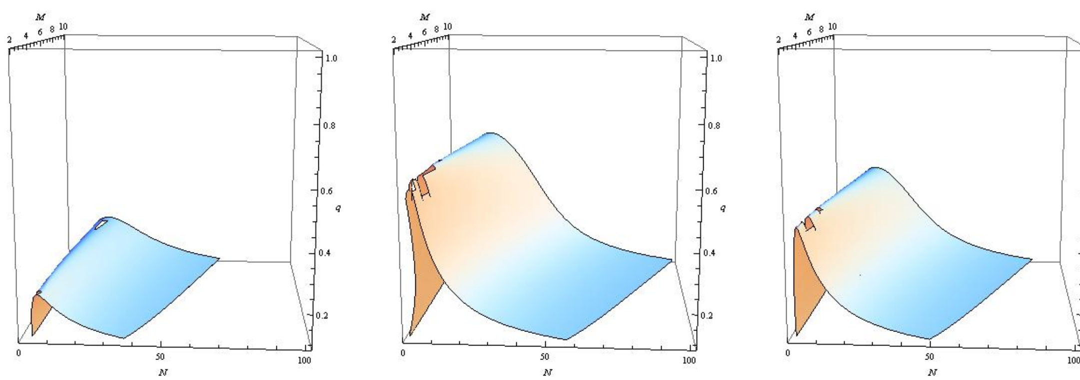

Figure 1: Equilibrium advertising intensity in the finite market: $x = 0.3$, $y = 0.5$; $x = 0.8$, $y = 0.5$; $x = 0.8$, $y = 0.9$ (from the left to the right).

For further intuition, we consider a CRIR cost function ($C(q) = y\ln(1-q)/\ln(1-x)$) and plot in Figure 1 equilibrium advertising intensity q^* as a function of M and N. For a fixed M, q^* first increases with N but eventually decreases when buyer numbers are relative large. The impact, however, of changing M on q^* when holding N fixed is not obvious, as Figure 1 reveals. Thus, we plot the level sets (iso-advertising intensity curves), the combination of M and N yielding the same q^*, in Figure 2. Figure 2 suggests the change in M may have a similar impact on q^* as the change in N.

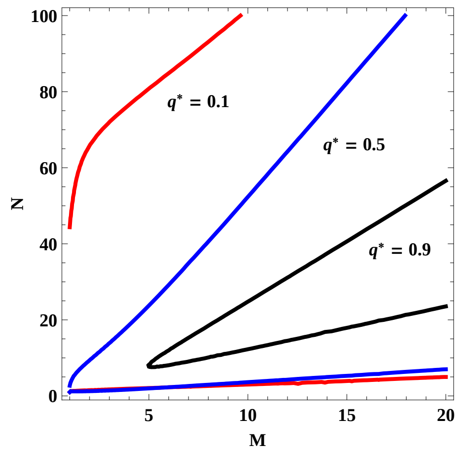

Figure 2: Level sets with $x = 0.4$ and $y = 0.6$.

3.3 Competitive advertising in large markets

Following directed-search literature convention, we extend our analysis to the large-market case where both seller and buyer numbers go to infinity but the buyer-to-seller ratio (market tightness) is fixed at some finite level, $\phi = N/M$.

From the finite market analysis, we know a buyer's expected utility of selecting seller i is $U^i = (1 - q^*\Lambda^i)^{N-1}(1 - r^i)$ and of selecting seller $k \neq i$ is $U^k = (1 - q^*\Lambda^k)^{N-1}(1 - r^*)$. We define the expected queue length at seller i as $\eta^i = Nq^*\Lambda^i$ from a buyer's perspective, and as $\theta^i = Nq^i\Lambda^i$ from seller i's perspective. Similarly, we define the expected queue length at any seller $k \neq i$'s store as $\eta^k = Nq^*\Lambda^k$. When participant numbers tend to infinity, buyers' expected utilities of selecting seller i and seller $k \neq i$ approach $U^i(r^i, \eta^i) = e^{-\eta^i}(1 - r^i)$ and $U^k(r^*, \eta^*) = e^{-\eta^k}(1 - r^*)$, respectively.

Define $\overline{U} \equiv \max\{U^i(r^i, \eta^i), U^k(r^*, \eta^k)\}$, which represents the market utility of a buyer receiving advertisements from at least two sellers, including seller i's. Then, $\overline{U}$ is the maximum expected utility a buyer can obtain. He will select a seller with positive probability only if the expected utility of doing so is at least $\overline{U}$.[18] Let $\tilde{\eta}^j(r^j, U)$ equal the unique value of η^j satisfying $U^j(r^j, \eta^j) = \overline{U}$ for $1 - r^j \geq \overline{U} > 0$, and zero for $1 - r^j < \overline{U}$ for $j = 1, ..., M$. This structure holds for all sellers, including both deviators and non-deviators.

Then, given (r^i, r^*), a symmetric equilibrium in the buyer's selecting game is characterized by market utility $\overline{U}^*$ and expected queue length η^{k*} such that $\eta^{k*} = \tilde{\eta}^k$ for all $k \in \mathcal{M}$. This

[18]Faced with infinite sellers, the probability a buyer receives only seller i's advertisements is zero. Therefore, to attract any buyer, seller i must provide an expected utility no lower than U. That is, the market-utility property as in Peters (2000) is revoked.

symmetric equilibrium in the subgame assigns expected queue lengths such that any buyer with $|\mathcal{I}| \geq 2$ is indifferent between selecting any seller k such that $\eta^k > 0$.

In the game's first stage, seller i understands that posting r^i will induce the corresponding equilibrium in the buyers' selecting game. The choice of r^i and q^i yields seller i's expected queue length $\theta^i(r^i, q_i, \overline{U})$. Seller i's profit-maximization problem is:

$$\Pi^{i*}(U) = \Bigg\{ \max_{r^i,q^i} \; \Pi^i(r^i, q^i, \overline{U}) = \theta^i(r^i, q^i, \overline{U}) e^{-\theta^i(r^i,q^i,\overline{U})} r^i \\ + \left(1 - e^{-\theta^i(r^i,q^i,\overline{U})} - \theta^i(r^i, q^i, \overline{U}) e^{-\theta^i(r^i,q^i,\overline{U})}\right) - C(q^i) \Bigg\}. \tag{15}$$

One can use the market-utility constraint, $e^{-\theta^i}(1 - r^i) = \overline{U}$, to replace r^i in seller i's objective function. Seller i's maximization problem becomes:

$$\max_{r^i,q^i} \; 1 - e^{-\theta^i(r^i,q^i,\overline{U})} - \left[\frac{\theta^i(r^i, q^i, \overline{U}) e^{-\theta^i(r^i,q^i,\overline{U})}}{e^{-\eta^i(r^i,q^i,\overline{U})}}\right] \overline{U} - C(q^i). \tag{16}$$

Then, seller i's choice of r^i can be more conveniently solved using θ^i as the choice variable. The first-order condition w.r.t. θ^i is then

$$e^{-\theta^i} - \left[\frac{e^{-\theta^i}}{e^{-\eta^i}} + \theta^i \left(\frac{d(e^{-\theta^i}/e^{-\eta^i})}{d\theta^i}\right)\right] \overline{U} = 0. \tag{17}$$

Since r^i is observable to buyers receiving seller i's advertisement, η^i adjusts at the same rate as θ^i if the change's cause is the change in r^i. Thus, when evaluating the above first-order condition at $r^i = r^*$ and $q^i = q^*$, we have $\frac{d(e^{-\theta^i}/e^{-\eta^i})}{d\theta^i} = \frac{\partial(e^{-\theta^i}/e^{-\eta^i})}{\partial r^i} = 0$. Then, the equilibrium queue length is:

$$e^{-\theta^*} = \overline{U}^*. \tag{18}$$

Using the market-utility constraint $\overline{U}^* = e^{-\eta^*}(1 - r^*) = e^{-\theta^*}(1 - r^*)$, it is clear that $r^* = 0$.

Now consider the advertising intensity equilibrium choice. We still utilize the maximization problem in (16) and use θ^i as the choice variable. Note that since now θ^i changes because of q^i, η^i is treated as a constant as buyers do not observe q^i. The first-order condition then yields:

$$e^{-\theta^i} - \left[\frac{e^{-\theta^i}}{e^{-\eta^i}} - \theta^i \left(\frac{e^{-\theta^i}}{e^{-\eta^i}}\right)\right] \overline{U} - C'(q^i) \frac{dq^i}{d\theta^i} = 0. \tag{19}$$

Since $\theta^i = Nq^i\Lambda^i$, we have $dq^i/d\theta^i = 1/(N\Lambda^i)$. Substitute this expression into the above first-order condition and evaluate it at $r^* = 0$ and $q^i = q^*$. Equilibrium advertising intensity is determined by:

$$e^{-\phi} - e^{-\phi}(1-\phi) - \frac{q^*C'(q^*)}{\phi} = 0 \;\Leftrightarrow\; \frac{\phi^2 e^{-\phi}}{q^*} = C'(q^*). \tag{20}$$

Note that $qC'(q)$ is a strictly increasing function equal to 0 when $q = 0$. Thus, the equilibrium is unique. Moreover, both r^* and q^* are the limits of the corresponding ones in the finite-market equilibrium.

Proposition 2 *In a large market with market tightness ϕ, there exists a unique symmetric equilibrium with positive advertising intensity. The equilibrium reserve price $r^* = 0$ and equilibrium advertising intensity is characterized by $\phi^2 e^{-\phi} = q^*C'(q^*)$.*

As in the standard directed-search model, competition between sellers in attracting buyers drives the reserve price down to zero. Then, a seller makes positive profits only if at least two buyers select her. The probability of seller i having at least two visitors when other sellers choose q^* is $1 - e^{-q^i\phi/q^*} - (q^i\phi/q^*)e^{-q^i\phi/q^*}$. If seller i increases her advertising intensity by dq, the probability of having at least two buyers increases by $[(\phi^2/q^*)e^{-\phi}]dq$ and so does her expected revenue. In equilibrium, this additional benefit should equal the additional cost seller i incurs, which is $C'(q^*)dq$.

In most goods markets, capacity constraints are not binding and each seller sells multiple units of goods. Further, these markets are often oligopolies, as sellers are relatively big. Then, market concentration or competitiveness is better captured by each seller's market share. One research question often considered by industrial organization literature is the relationship between equilibrium advertising intensity and market concentration. In labor markets, however, where all market participants are relatively small and capacity constraints are always binding, a better measure of market concentration is the buyer-to-seller ratio, or market tightness, ϕ. We explore this in the next proposition.

Proposition 3 *When market tightness ϕ increases, equilibrium advertising intensity q^* first increases and eventually drops. In particular, when $\phi < 2$, $dq^*/d\phi > 0$ and $\phi > 2$, $dq^*/d\phi < 0$.*

To better understand Proposition 3, consider a scenario where M is fixed but new buyers are entering the market so that ϕ becomes larger. When N is relatively small ($\phi < 2$), competition among sellers is fierce. Sellers are less willing to invest in advertising, as the probability of attracting buyers is small. Arrival of new buyers provides an opportunity for sellers to make profits by attracting at least two buyers, so they invest more in advertising. In contrast, when N is sufficiently large ($\phi > 2$), each seller can almost guarantee being visited by at least two buyers at her initial advertising intensity. When new buyers are entering the market, even maintaining the initial advertising intensity is too costly. The newly arrived buyers can compensate for a decrease in advertising intensity. Therefore, equilibrium advertising intensity drops.

The advertising pattern we just have derived is very different from previous findings on commodity-goods-markets advertising.[19] In particular, Grossman and Shapiro (1984) show that, because the increased competition tends to reduce the yield on a given firm's advertising expenditure as consumers are more likely to be informed of a product better suited to their tastes, the impact of a decrease in market concentration on advertising intensity is always negative.[20] In contrast, our model shows that the impact of increased market tightness on advertising intensity is non-monotonic. It is initially positive but eventually negative. Our finding is important to understand advertising patterns and design labor market policies. For example, if governments want to enhance firms' incentives to advertise in labor markets, the policy of firm-subsidization might be equally achieved by improving firms' access to the labor market. This policy is effective only when market tightness is low.

4 Price-posting

Price-posting, another mechanism widely used in real-world business and popular in directed-search literature, reflects, in contrast to auctions, sellers' full commitment power.[21] A seller will sell her good at the posted price no matter how many visitors she receives. If more than one visitor arrives, the good is allocated according to an equal-chance rationing rule, so both buyers and sellers expect positive surpluses.

By studying advertising equilibrium with price-posting, we can examine whether the advertising pattern we have derived in the last section is robust. We can also examine how different degrees of commitment affect sellers' incentives to advertise. In principle, this problem can be studied by constructing a general mechanism which allows sellers to choose the degree of commitment. Appendix B discusses use of a general mechanism, but here we focus only on auctions and price-posting. All other mechanisms can be seen as a convex

[19] Nelson (1974) and Tesler (1964) provide an alternative explanation for the inverted-U shape result. They interpret it from a reciprocal and dynamic perspective: more efficient firms tend to use advertising as a competitive instrument, which in turn increases concentration by directing sales to the most efficient firms.

[20] Several empirical studies, however, find evidence of an inverted-U shape relationship between advertising intensity and market concentration in consumption good markets (e.g., Kaldor and Silverman, 1948, Buxton, Davies, and Lyons, 1984, and Uri, 1987.

[21] Price-posting is thoroughly analyzed in Peters (1984, 2000) and Burdett, Shi, and Wright (2001) in environments where posting prices is costless and prices are perfectly observable to all buyers.

combination of these two and so are the induced advertising intensities.[22] Finally, since many studies only consider price-posting as the trading protocol, we can compare our prediction directly with these (see Section 5).

In what follows, we consider a similar environment except that now sellers simultaneously post prices p^j and commit to selling at p^j no matter how many buyers visit. For any *ex-post* demand level, $n^j \geq 1$, a good is allocated to each buyer with equal probability $1/n^j$.

4.1 Buyer's choices

In the following, we still treat seller i as the only potential deviator. Without loss of generality, we consider the selecting problem for a buyer, called buyer 1, who receives advertisements from seller i and at least one non-deviating seller. He needs to assess the probability of obtaining goods at seller i's store. As discussed before, buyer 1 believes seller i's advertising intensity is $\mu^i(\mathcal{I}_1)$ and any other buyer selects seller i with expected probability Λ^i conditional on receiving seller i's advertisements. Then, buyer 1 should believe that the probability he will obtain the good at seller i's store is:

$$\Omega^i = \frac{1 - (1 - \mu^i(\mathcal{I}_1)\Lambda^i)^N}{N\mu^i(\mathcal{I}_1)\Lambda^i}. \tag{21}$$

Note that the numerator of (21) is the probability that seller i makes a sale while the denominator is seller i's store's expected queue length (the average number of buyers selecting seller i). Since each buyer selecting seller i has an equal chance to obtain the good, matching consistency requires that the probability that seller i makes a sale equals the probability each buyer gets the good times the average number of buyers selecting seller i. Thus, buyer 1's expected payoff of selecting seller i is $U^i = (1 - p^i)\Omega^i$. Similarly, buyer 1's expected payoff of selecting a non-deviating seller is $U = (1 - p^*)\Omega$ where Ω is the probability of obtaining goods from selecting a non-deviator.

As in the previous section, we can show that, for any given pair of (p^i, p^*), a unique pair of (Λ^i, Λ) is induced as the equilibrium expected probability in the buyers' selecting

[22] More generally, one can study a general mechanism that specifies the price and trading rule for every possible realization of the number of buyers that a seller is matched with, as in Eeckhout and Kircher (2010), Virag (2011), and Geromichalos (2012). The general mechanism simply reflects that the degree of lack of commitment can occur somewhere between full commitment (price-posting) and no commitment (auctions) when more than one buyer selects the seller. This, however, substantially complicates the analysis without providing much more insight in terms of equilibrium advertising beyond what we derive here. If equilibrium advertising intensity with auctions is higher than with price-posting, then equilibrium advertising intensity with a general mechanism will be somewhere in-between. Instead, therefore, we focus on identifying which mechanism yields higher equilibrium advertising intensity

game, provided $\mu^i(\mathcal{I})$ is unique.[23] To obtain the explicit expression of Λ_i, we can utilize the indifference condition $(1 - p^i)\Omega^i = (1 - p^*)\Omega$ and Lemma 1. For the sake of obtaining the equilibrium, however, we need not derive the explicit expression of Λ_i (see the proof of Proposition 4).

4.2 Seller's choices

Focus now on seller i's choice of advertising and pricing. Under price-posting, she only cares about whether she gets any visitors and solves this maximization problem:

$$\max_{q^i, p^i} p^i \left[1 - (1 - q^i \Lambda^i)^N\right] - C(q^i),$$

subject to buyers' best responses in the induced subgame. Note that $1 - (1 - q^i\Lambda^i)^N$ is the probability that at least one buyer selects seller i. To simplify exposition, define $\rho = 1 - q^*\Lambda = 1 - (\tau/M)$. Proposition 4 characterizes the SPBE with positive advertising intensity in a finite market.

Proposition 4 *In a finite market where sellers commit to posted prices, there exists a unique SPBE with positive advertising intensity. If (p^*, q^*) is an interior solution, it is characterized by these conditions:*

$$p^* = \frac{M(1 - \rho^N) - \rho^{N-1}(1 - \rho)MN}{M(1 - \rho^N) - N\rho^{N-1}(1 - \rho)}, \tag{22}$$

$$\phi\tau\rho^{N-1}p^* - q^*C'(q^*) = 0. \tag{23}$$

Under price-posting, the unique SPBE always exhibits positive advertising intensity. Sellers choose either $q^* = 1$ or some intermediate advertising-intensity level. When every seller chooses $q = 1$ so $\rho = 1 - (1/M)$, the market becomes perfectly transparent in that every buyer observes all posted prices. Consequently, equilibrium price is:

$$p^* = \frac{(M - 1) - (\frac{M-1}{M})^{M\phi}(M - 1 + M\phi)}{(M - 1) - (\frac{M-1}{M})^{M\phi}(M - 1 + \phi)}, \tag{24}$$

just as in the standard directed-search model with price-posting.[24]

[23] Since the proof uses a similar argument as in Lemma 2, we omit it here.
[24] See Burdett, Shi, and Wright (2001).

4.3 Competitive advertising equilibrium

Now we analyze the equilibrium properties of advertising in the large market by taking M and N to infinity while holding their ratio constant at $\phi = N/M$. It is tedious to calculate the limit of seller i's matching probability and then calculate her optimal choice. It is easy to verify that the p^* and q^* determined by equation (22) and (23) converge to their corresponding large-market counterparts, just as in the auction case. We therefore simply take the limits of p^* and q^* using equation (22) and (23). Proposition 5 summarizes the large-market equilibrium.

Proposition 5 *When price-posting is used in the large market, the unique equilibrium, (p^*, q^*), is characterized by:*

$$p^* = 1 - \frac{\phi}{e^{\phi} - 1}, \tag{25}$$

$$q^* C'(q^*) = \frac{\phi e^{-\phi}(1 - e^{-\phi} - \phi e^{-\phi})}{1 - e^{-\phi}}. \tag{26}$$

Equilibrium advertising intensity q^* has an inverted U-shape in market tightness ϕ.

Proposition 5 shows the equilibrium price to be a strictly increasing function of ϕ. Equilibrium advertising intensity increases in ϕ when ϕ is relatively small ($\phi < 1.669$), but decreases in ϕ when ϕ is relatively large ($\phi > 1.669$). Thus, the advertising pattern derived from the auction case is robust. Worth noting is that maximum advertising intensity is reached with lower market tightness when compared to the auction case (in which the peak appears at $\phi = 2$). This difference reflects that, with price-posting, sellers need only attract one buyer to ensure positive profits. Thus, their incentive to advertise is lower at a smaller level of market tightness.

4.4 Robustness of canonical directed search models

In equilibrium, with both auctions and price-posting and passive beliefs, each seller sells her good with probability $1 - e^{-\phi}$, while each buyer obtains a good with probability $(1 - e^{-\phi})/\phi$. Surprisingly, under these matching rates (sellers' and buyers' probabilities of trade), market participants trade as if they all participated and were randomly matched, just as the standard directed-search model predicts (see Julien, Kennes, and King, 2000 and Burdett, Shi, and Wright, 2001). One would naturally expect that ϕ should influence the matching rates in a somewhat different fashion as here most buyers do not observe all prices. The endogenous adjustment of equilibrium advertising intensity drives our result. For instance, consider sellers' matching rates in auctions when $\phi < 2$. To exclude the direct impact of an increase of ϕ, we only consider how the induced higher q^* affects matching rates through two opposing channels. On the one hand, a seller informs more buyers, which may raise her matching rate. On the other hand, for buyers who indeed receive this seller's advertisements, their information sets should expand because of higher q^*. Given they now have more choices, their probability of selecting a particular seller is lower. These two effects cancel out on the equilibrium path, which leaves the increase of ϕ, the factor directly affecting matching rates, as the only effect at work.

Previous sections showed that equilibrium (reserve) prices are identical to corresponding equilibrium (reserve) prices in the canonical model. Since matching rates are also identical, we can conclude easily that both sellers' and buyers' gross revenues remain the same as in the canonical model. Thus, we say that the standard directed-search model's predictions regarding prices, matching rates, and gross revenues are robust to environments without perfectly observable prices.

5 Trading protocols, market transparency, and efficiency

Here, we explore three implications of our model. Firstly, which of the two mechanisms lead to higher advertising intensity and which do sellers prefer? In our model, information possessed by buyers, market transparency, is endogenously determined. Unlike previous models, which assume exogenous market transparency, we study the relationship between the equilibrium price under price-posting and market transparency. Finally, we study whether the constrained matching efficiency is attained when sellers incur entry costs.

5.1 Auction versus price posting

The main difference between auctions and price-posting is that auctions allow for exploitation of *ex-post* opportunities. With auctions, however, a seller needs to attract at least two visitors to make positive profits. In contrast, one visitor will suffice with price-posting. This fact is critical for understanding the difference between the advertising patterns under the two mechanisms. To secure positive profits, a seller using auctions needs to convince more than one buyer to select her to trigger the bidding process.

Proposition 6 *For any market tightness $\phi > 0$ equilibrium advertising intensity in a large market is strictly higher when sellers use auctions rather than price-posting.*

Figure 3 depicts the equilibrium advertising intensities under both mechanisms with CRIR advertising technology.

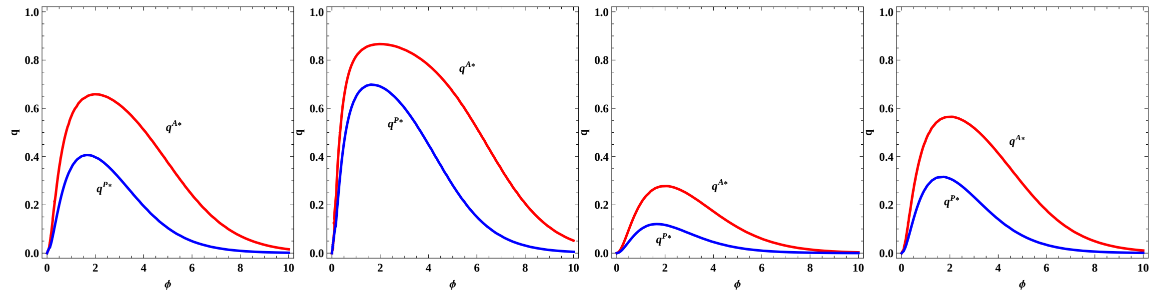

Figure 3: Comparison of q^{A*} and q^{P*}: $x = 0.1$, $y = 0.7$; $x = 0.3$, $y = 0.7$; $x = 0.3$, $y = 0.5$; $x = 0.5$, $y = 0.7$ (from the left to the right).

With auctions, we have that $0 = r^* < p^*$ and sellers can fully extract surplus from a match if they encounter more than one buyer. In contrast, sellers always leave positive surplus to buyers if they ex-ante commit to the posted prices. Recall also that, whichever mechanism is used, sellers' matching rates are identical in the large market. Consequently, we can write sellers' profits under an auction mechanism as:

$$\Pi^{A*} = 1 \cdot \Pr[n \geq 2] - C(q^{A*}) = 1 - e^{-\phi} - \phi e^{-\phi} - C(q^{A*}), \tag{27}$$

where q^{A*} denotes the equilibrium advertising intensity under auctions. The equilibrium profit under a price-posting mechanism is given by:

$$\Pi^{P*} = p^* \cdot \Pr[n \geq 1] - C(q^{P*}) = 1 - e^{-\phi} - \phi e^{-\phi} - C(q^{P*}), \tag{28}$$

where q^{P*} denotes equilibrium advertising intensity under price-posting. From Proposition 6, we can immediately draw the following result:

Corollary 1 *For any market tightness ϕ we have that $\Pi^{P*} > \Pi^{A*}$.*

This result is important because it shows how different mechanisms in a competitive environment can yield different expected profits in large markets. The previous findings, including both the finite-market and large-market analysis, assert that price-posting cannot do better than auctions from sellers' viewpoints when there is competition between sellers. To our knowledge, our paper is the first to show that price-posting can dominate auctions in a competitive environment.[25] This is because sellers incur strictly higher advertising costs under auctions while revenues are the same as under price-posting.

[25] Our result complements Julien, Kennes, and King (2001) and Kultti (2003), who show that in the finite market, sellers earn higher profits with auctions. In larger markets, however, auctions and price-posting

5.2 Equilibrium price and market transparency

Here, we illustrate another of our model's important implications regarding the relationship between price and market transparency. Conventional wisdom suggests prices should fall when consumers are better able to observe and compare prices. In other words, market transparency improvements impose downward pressure on prices. We observe this theoretical prediction in environments with homogenous buyers or where competition only occurs among sellers. Some recent work, including Anderson and Renault (2000) and Lester (2011), however, shows this insight may not hold when buyers hold heterogenous information and/or there is also competition among buyers.[26] One limitation of previous work is that authors assume exogenous information heterogeneity and analyze only finite markets. In our model, buyers' information heterogeneity is endogenously induced by advertising technology. This important feature allows us to extend the insight in Anderson and Renault (2000) and Lester (2011) to large markets.

Traditionally, the number of informed buyers measures market transparency. Moreover, informed buyers are also assumed to know all posted prices so that the size of informed buyers' information set equals the number of sellers. In our model, an informed buyer's information set size can take any value between 1 and M (for finite markets). In particular, its value is determined by the equilibrium advertising intensity q^*. Thus, we use q^* to measure market transparency.[27]

In our environment, market transparency can be improved through two channels: (i) more buyers become informed; and (ii) each informed buyer observes more sellers. The first channel can be viewed as the extensive margin by increasing advertising intensity, and the second as the intensive margin where sellers increase advertising intensity. From Proposition 5, we know the equilibrium price p^* is an increasing function in ϕ. The equilibrium advertising intensity, q^*, however, increases in ϕ when ϕ is relatively small, but decreases in ϕ when ϕ is relatively large. Thus, both equilibrium price and advertising intensity can co-move

generate equal profits see Kultti (1999), Kultti (2003), Julien, Kennes, and King (2000), and Burdett, Shi, and Wright (2001).

[26]Anderson and Renault (2000) analyze a duopoly sequential search model with differentiated products. Uninformed buyers create a positive externality for informed buyers, as sellers tend to charge low prices to prevent uninformed buyers from searching further. Thus, when more uninformed buyers enter the market, market transparency worsens but prices can fall. Lester (2011), on the other hand, focuses on homogeneous products using a directed-search framework. His model assumes that informed buyers observe all posted prices while uninformed buyers observe no price. Each uninformed buyer randomly selects one seller. With more informed buyers, there is downward pressure on price because sellers must reduce prices to compete for informed buyers. There is also, however, upward pressure on prices because informed buyers know that any low price will attract large numbers of informed competitors. Overall, price can fall with more informed buyers.

[27]Note that using Mq^*, the mean value of the number of advertisements a buyer receives in the finite-market equilibrium, to measure market transparency will deliver the same result.

positively when ϕ increases if ϕ is relatively low. If we consider an exogenous increase in ϕ, it will be *ad hoc* to discuss the extensive margin as there are newly arrived buyers. The corresponding change of q^*, however, improves existing buyers' information through the intensive margin.

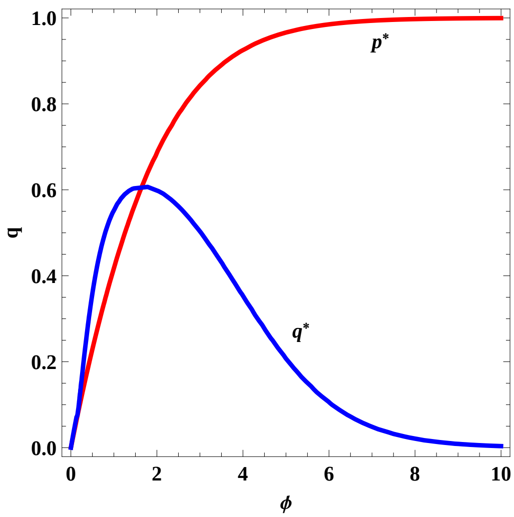

Figure 4: The joint increase of p^* and q^*.

Using the result from the large-market analysis in Section 4.3, we can depict p^* and q^* in Figure 4. In particular, we consider a CRIR advertising technology with $x = 0.8$ and $y = 0.2$. Figure 4 shows clearly the positive correlation between market transparency and price when ϕ is relatively low. This analysis also holds even with infinite buyers and sellers. This then complements Lester's (2011) findings. In the relevant range of ϕ, sellers find it optimal to advertise more aggressively to attract buyers. High advertising intensity, q^*, may cause buyers to observe multiple sellers but simultaneously increases the number of buyers making purchases. Since $q^* < 1$ in the relevant range of ϕ, most buyers do not observe all prices. This, in turn, allows sellers to increase prices. Thus, in large markets when market transparency is endogenously determined, it is possible to observe price increases together with market transparency improvements.

5.3 Efficiency

One salient feature of directed-search models is that the induced allocation is typically constrained efficient.[28] This is because buyers observe all posted prices and make trade-offs

[28]The constrained-efficiency property is found in Julien, Kennes, and King (2000), Burdett, Shi, and Wright (2001), Shi (2001), Shimer (2005), and Kircher (2009). Inefficiency may occur when buyers' search intensities are explicitly modeled (number of job applications). See, for example, Albrecht, Gautier, and

between price and expected queues. This induces matches with sellers in the same way a social planner would organize the market. Given the matching efficiency, entry decisions are also efficient if sellers must incur entry costs to obtain market access. When buyers cannot perfectly observe all prices, as in our model, we may expect inefficiency to arise with respect to entry decisions. Moreover, there is an additional margin in our model regarding efficiency, which is advertising intensity.

We first consider the social planner's problem. The social planner maximizes total surplus by controlling entry decisions, which is captured by the number of sellers M, and advertising intensity q. Let each seller's entry cost be $F > 0$. The social planner faces the same frictions as do market participants. As described in Section 2, we restrict buyers' strategies to be symmetric. We start with a finite number of participants and solve the social planner's problem in the limit. When the social planner chooses advertising intensity q, each buyer should receive at least one advertisement with probability $1-(1-q)^M$ — i.e., the probability that a buyer is active in the market. Moreover, a buyer should receive advertisements from Mq sellers on average. Each buyer is instructed to select each seller in his information set with equal probability. Thus, a seller reaching a buyer is expected to be selected by this buyer with probability $[1-(1-q)^M]/(Mq)$. Each seller expects to reach Nq buyers on average. Then, the expected surplus generated by each seller is:

$$1-\left(1-\frac{1-(1-q)^M}{Mq}\right)^{Nq}. \tag{29}$$

Denote the efficient choice of (q,ϕ) as (q^E,ϕ^E). The social planer maximizes total surplus per buyer (the number of buyers is fixed), so the maximization problem can be written as:

$$\max_{M,q}\ \frac{M}{N}\left[1-\left(1-\frac{1-(1-q)^M}{Mq}\right)^{Nq}\right]-\frac{M(C(q)+F)}{N}. \tag{30}$$

Take M and N to infinity with $N/M=\phi$. Note that if $q>0$, the first term in (30) converges to $(1-e^{-\phi})/\phi$, which is independent of q. Then, the social planner always wants to decrease q as $C'(q)>0$. This leads to a contradiction. Therefore, the social planner will choose $q^E=0$. $\lim_{M\to\infty} Mq^E$, however, must be positive as otherwise there will be no match.

Although $q^E=0$, the social planner's objective function's value depends on whether $\lim_{M\to\infty} Mq^E \in (0,\infty)$ or $\lim_{M\to\infty} Mq^E=\infty$. This limit property is, however, undefined in our model, since we cannot derive a closed-form solution of q^E for a finite market. We

Vroman (2006) and Galenianos and Kircher (2009). We, however, are modeling search intensity from the seller's side.

therefore assume q^E decreases at a much faster rate than the rate of increase of M, so that $\lim_{M\to\infty} Mq^E = \lambda < \infty$. Then, the social planner's objective function becomes:

$$\max_{\phi} \frac{1-\left(1-\frac{1-e^{-\lambda}}{\lambda}\right)^{\lambda\phi}}{\phi} - \frac{F}{\phi}. \tag{31}$$

The efficient ϕ is then determined by the first-order condition:

$$-1+\left(\frac{-1+e^{-\lambda}+\lambda}{\lambda}\right)^{\lambda\phi}\left[1-\lambda\phi\ln(\frac{-1+e^{-\lambda}+\lambda}{\lambda})\right]=F. \tag{32}$$

When $\lambda \geq 0.1$, we have $-1+\left(\frac{-1+e^{-\lambda}+\lambda}{\lambda}\right)^{\lambda\phi}\left[1-\lambda\phi\ln(\frac{-1+e^{-\lambda}+\lambda}{\lambda})\right] \approx 1-e^{-\phi^E}-\phi^E e^{-\phi^E}$. Then, condition (32) can be written as:

$$1-e^{-\phi^E}-\phi^E e^{-\phi^E}=F. \tag{33}$$

Recall that, in the equilibrium analysis with both auctions and price-posting, equilibrium advertising intensity is always strictly positive. This implies that equilibrium advertising intensity is excessive compared to the efficient level as $q^E = 0$. If sellers must incur F to enter the market, equilibrium market tightness ϕ^* is determined by this free-entry condition:

$$1-e^{-\phi^*}-\phi^* e^{-\phi^*}-C(q^*)=F. \tag{34}$$

From the comparison of (33) and (34), we know $\phi^* > \phi^E$ since $C(q^*) > 0$ and $1-e^{-\phi}-\phi e^{-\phi}$ is strictly increasing in ϕ. Therefore, equilibrium entry is insufficient. Moreover, the inefficiency is worse under an auction mechanism since $q^{A*} > q^{P*} > q^E$ so that $\phi^{A*} > \phi^{P*} > \phi^E$.

Proposition 7 *Compared to the efficient level, equilibrium advertising intensity is excessive and equilibrium entry is insufficient. The inefficiency is more severe when sellers use auctions.*

The decentralized equilibrium is then inefficient, with too much advertising and too little entry. This result can be understood as follows. By increasing advertising intensity, sellers inform more buyers about their goods but simultaneously expand each informed buyer's information set. When buyers have many choices, their selections tend to overlap, which harms matching efficiency. When sellers choose advertising intensities, they ignore this negative externality and advertise more than efficiency requires. High advertising intensity raises advertising costs and lowers the expected profit of market entry. Thus, the equilibrium entry level is insufficient.

6 Conclusion

Capacity constraints are important features of many large markets. In labor (housing) markets, most workers (buyers/renters) search for jobs (houses) by browsing websites, yet economists have not fully explored the strategic use of advertising to reach such workers (buyers). We establish the first directed-search model with stochastic and costly advertising. Advertising endogenously creates buyer heterogeneity. In this frictional environment, buyers' perceptions about advertising intensity matter. The suspicion of a high advertising intensity may restrain them from selecting an observed seller as they expect a high probability of being rationed. An immediate implication is that high advertising intensity might not be always beneficial for sellers, even with costless advertising.

We provide a full characterization for the equilibrium advertising pattern and the terms of trade. Equilibrium advertising intensity has an inverted U-shape in market tightness. We show that sellers are better off with price-posting as sellers advertise more under auctions. Another novel result is that even in large markets, equilibrium prices can increase when buyers' information improves.

The welfare analysis is also of interest. It departs from canonical directed models by showing that there exists excessive advertising and insufficient entry in equilibrium because advertising may cause more overlap in buyers' selection decisions and therefore amplify market friction. The highest coordination friction appears when all buyers observe all sellers. Thus, imposing information friction to limit buyers' choice may enhance welfare.

The analysis raises several avenues for future research. One is to examine empirically the model predictions as it provides several interesting, testable comparative statics. Another would be to investigate the effect of seller heterogeneity on equilibrium advertising intensity. It is not a priori clear that large firms will advertise more. Another possibility would be to explore trade-offs between information and coordination friction to then determine whether market transparency is always good for efficiency.

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Appendix A: proofs.

Proof of Lemma 1.

Consider the expected probability of selecting an arbitrary non-deviating seller, Λ^k with $k \neq i$. Use the expression in (6) but replace the subscript of $\mathcal{I}_2$ as it now denotes the information set of any of the other $N-1$ buyers. Replace z by $y = z-2$ and use the fact that $\sigma^k(\mathcal{I}_2||\mathcal{I}_2| = z, r^i \in \mathcal{I}_1) = \frac{1-\sigma^i(\mathcal{I}_2||\mathcal{I}_2|=z, r^i \in \mathcal{I}_1)}{z-1}$ and $\sigma^k(\mathcal{I}_2||\mathcal{I}_2| = z-1, r^i \notin \mathcal{I}_1) = \frac{1}{z-1}$, Λ^k can be rewritten as

$$\begin{aligned}\Lambda^k = \mu^i(\mathcal{I}_1) \sum_{y=0}^{M-2} \binom{M-2}{y} \cdot \frac{1-\sigma^i(\mathcal{I}||\mathcal{I}| = y+2, r^i \in \mathcal{I}_1)}{y+1} \cdot q^{*y} \cdot (1-q^*)^{M-y-2} \\ - (1-\mu^i(\mathcal{I}_1)) \sum_{y=0}^{M-2} \binom{M-2}{y} \cdot \frac{1}{y+1} \cdot q^{*y} \cdot (1-q^*)^{M-y-2}.\end{aligned} \tag{35}$$

The second term in (35) can be further simplified to

$$(1-\mu^i(\mathcal{I}_1)) \sum_{y=0}^{M-2} \left[\binom{M-2}{y} \cdot (q^*)^y \cdot (1-q^*)^{M-y-2} \cdot \frac{1}{y+1} \right] = (1-\mu^i(\mathcal{I}_1)) \left[\frac{1-(1-q^*)^{M-1}}{(M-1)q^*} \right], \tag{36}$$

For the first term in (35), we have that

$$\begin{aligned}&\mu^i(\mathcal{I}_1) \sum_{y=0}^{M-2} \binom{M-2}{y} \cdot \frac{1-\sigma^i(\mathcal{I}||\mathcal{I}| = y+2, r^i \in \mathcal{I}_1)}{y+1} \cdot q^{*y} \cdot (1-q^*)^{M-y-2} \\ =&\mu^i(\mathcal{I}_1) \sum_{y=0}^{M-2} \left[\binom{M-2}{y} \cdot (q^*)^y \cdot (1-q^*)^{M-y-2} \cdot \frac{1}{y+1} \right] \\ &\quad - \mu^i(\mathcal{I}_1) \sum_{y=0}^{M-2} \left[\binom{M-2}{y} \cdot (q^*)^y \cdot (1-q^*)^{M-y-2} \cdot \frac{\sigma^i(\mathcal{I}||\mathcal{I}| = y+2, r^i \in \mathcal{I}_1)}{y+1} \right]\end{aligned} \tag{37}$$

Summing up the two terms, we obtain

$$\begin{aligned}
\Lambda^k =& - \ \mu^i(\mathcal{I}_1) \sum_{y=0}^{M-2} \left[\binom{M-2}{y} \cdot (q^*)^y \cdot (1-q^*)^{M-y-2} \cdot \frac{\sigma^i(\mathcal{I}||\mathcal{I}| = y+2, r^i \in \mathcal{I}_1)}{y+1} \right] \\
& + \frac{1-(1-q^*)^{M-1}}{(M-1)q^*} \\
=& - \mu^i(\mathcal{I}_1) \Bigg[\frac{1}{(M-1)q^*} \sum_{y=0}^{M-1} \left[\binom{M-1}{y} \cdot (q^*)^y \cdot (1-q^*)^{M-y-1} \cdot \sigma^i(\mathcal{I}||\mathcal{I}| = y+2, r^i \in \mathcal{I}_1) \right] \\
& - \frac{(1-q^*)^{M-1}}{(M-1)q^*} \Bigg] + \frac{1-(1-q^*)^{M-1}}{(M-1)q^*} \\
=& \ - \frac{\mu^i(\mathcal{I}_1)\Lambda^i}{(M-1)q^*} + \frac{1-(1-\mu^i(\mathcal{I}_1))(1-q^*)^{M-1}}{(M-1)q^*}.
\end{aligned} \tag{38}$$

The second inequality comes from the fact that $\sigma^i(\mathcal{I}||\mathcal{I}| = y+2, r^i \in \mathcal{I}_1) = 1$ when $y = 0$. The third inequality comes from the definition of Λ^i . Rearrange the above equation, then the claim in Lemma 1 follows. ■

Proof of Lemma 2.

The proof closely follows Peters (1984). Define the expected payoffs of selecting seller i, the deviator, and selecting seller k, an arbitrary non-deviator, by $U^i(x) = (1-r^i)(1-\mu^i(\mathcal{I})x)^{N-1}$ with the domain of $[0,1]$ and $U^k(x) = (1-r^*)(1-q^*x)^{N-1}$ with the domain of $[0,1]$. Suppose that both $\mu^i(\mathcal{I})$ and q^* belong to $(0,1)$. Both $U^i(x)$ and $U^k(x)$ are continuous and strictly decreasing function. Define the inverse functions of $U^i(x)$ and $U^k(x)$ as $[U^i]^{-1}(y)$ and $[U^k]^{-1}(y)$, respectively. Clearly, $[U^i]^{-1}(y)$ is strictly decreasing on $R^i \equiv [(1-r^i)(1-\mu^i(\mathcal{I}))^{N-1}, 1-r^i]$ from 1 to 0, and $[U^k]^{-1}(y)$ is strictly decreasing on $R^k \equiv [(1-r^*)(1-q^*)^{N-1}, 1-r^*]$ from 1 to 0. Define the following new function:

$$[\overline{U}^i]^{-1}(y) = \begin{cases} 1 & \text{if } \ y < (1-r^i)(1-\mu^i(\mathcal{I}))^{N-1}; \\ [U^i]^{-1}(y) & \text{if } \ 0 \le y \le 1-r^i; \\ 0 & \text{if } \ y > 1-r^i. \end{cases} \tag{39}$$

Similarly, we can define

$$[\overline{U}^k]^{-1}(y) = \begin{cases} 1 & \text{if } \ y < (1-r^*)(1-q^*)^{N-1}; \\ [U^k]^{-1}(y) & \text{if } \ (1-r^*)(1-q^*)^{N-1} \le y \le 1-r^*; \\ 0 & \text{if } \ y > 1-r^*. \end{cases} \tag{40}$$

Finally, define $[\overline{U}]^{-1}(y) \equiv \mu^i(\mathcal{I})[\overline{U}^i]^{-1}(y) + (M-1)q^*[\overline{U}^k]^{-1}(y)$. The domain of $[\overline{U}]^{-1}(y)$ is $R \equiv R^i \bigcup R^k$. Denote j^* as the seller who offers r^{j^*} so that $r^{j^*} = \min\{r^i, r^k\}$.

First, note that if $R^{j^*} \bigcap R^j = \emptyset$ for $j \neq j^*$, then a buyer gets strictly higher surplus by visiting seller j^* no matter what other buyers select. Then, $\Lambda^{j^*} = 1$.

Now, suppose that $R^{j^*} \bigcap R^j \neq \emptyset$ for $j \neq j^*$. Note that $[\overline{U}]^{-1}(y)$ is continuous and strictly decreasing over R. Also, $[\overline{U}]^{-1}(\inf R) > \mu^i(\mathcal{I}) + (M-1)q^* > 1-(1-\mu^i(\mathcal{I}))(1-q^*)^{M-1}$ and $[\overline{U}]^{-1}(\sup R) = 0$. From Lemma 1, we know that $[\overline{U}]^{-1}(y) = 1-(1-\mu^i(\mathcal{I}))(1-q^*)^{M-1}$. Then, there exists a unique $y^* \in R$ so that $[\overline{U}]^{-1}(y^*) = 1-(1-\mu^i(\mathcal{I}))(1-q^*)^{M-1}$. Then, assigning $\Lambda^i = [U^i]^{-1}(y^*)$ and $\Lambda^k = [U]^{-1}(y^*), \forall k \neq i$ gives the equilibrium expected selecting probability in the buyers' selecting game. ■

Proof of Proposition 1.

Suppose that buyers are using completely mixed strategy in equilibrium and select each observed seller with equal probability. To derive the equilibrium reserve price, we first write down the first-order condition with respect to r^i as follows

$$q^i\Lambda^i(1-q^i\Lambda^i) + [(Nq^i\Lambda^i - 1)(1-r^i) + (1-q^i\Lambda^i)]\frac{\partial q^i\Lambda^i}{\partial r^i} = 0.$$

With passive belief, the expected probability for buyers to select seller i can be explicitly derived from condition (8). In particular, we have

$$q^i\Lambda^i = 1 - \frac{(M-1) + (1-q^*)^M}{(M-1)(\frac{1-r^i}{1-r^*})^{\frac{1}{N-1}} + 1}.$$

Evaluating $q^i\Lambda^i$ and $\frac{\partial(q^i\Lambda^i)}{\partial r^i}$ at $r^i = r^*$ and $q^i = q^*$ yields

$$q^i\Lambda^i|_{r^i=r^*, q^i=q^*} = \frac{1-(1-q^*)^M}{M},$$

and

$$\frac{\partial(q^i\Lambda^i)}{\partial r^i}\Big|_{r^i=r^*, q^i=q^*} = \frac{-(M-1)^2 - (M-1)(1-q^*)^M}{M^2(N-1)(1-r^*)}.$$

Substitute the previous two expressions into the first-order condition with respect to r^i, then the equilibrium reserve price r^* is given by

$$r^* = \frac{(N-1)[1-(1-q^*)^M]}{M(M-1)+(N-M)[1-(1-q^*)^M]}. \tag{41}$$

The first-order condition with respect to q^i (note that Λ^i does not depend on q^i) is given by

$$r^i N\Lambda^i(1-q^i\Lambda^i)^{N-1} + (1-r^i)N(N-1)q^i(\Lambda^i)^2(1-q^i\Lambda^i)^{N-2} = C'(q^i).$$

Evaluate it at $r^i = r^*$ and $q^i = q^*$, the equilibrium advertising intensity q^* is then characterized by

$$\begin{pmatrix} r^* \frac{N[1-(1-q^*)^M]}{Mq^*}[1-\frac{1-(1-q^*)^M}{M}]^{N-1} + \\ (1-r^*)\frac{N(N-1)[1-(1-q^*)^M]^2}{M^2q^*}[1-\frac{1-(1-q^*)^M}{M}]^{N-2} \end{pmatrix} = C'(q^*), \tag{42}$$

Substitute in $\tau = 1-(1-q)^M$ and $\phi = N/M$, the above condition simplifies to

$$r^* \frac{\phi\tau}{q^*}\left(1-\frac{\tau}{M}\right)^{N-1} + (1-r^*)\frac{\phi(\phi-\frac{1}{M})\tau^2}{q^*}\left(1-\frac{\tau}{M}\right)^{N-2} = C'(q^*), \tag{43}$$

which is the condition we give in Proposition 1.

Seller i can deviate by increasing reserve price to a level at which only buyers with $\mathcal{I} = \{r^i\}$ select her with positive probability (buyers with information set $\mathcal{I}$ so that $r_i \in \mathcal{I}$ and $|\mathcal{I}| \geq 2$ no longer use completely mixed strategies). Since this type of buyers do not have any other choices, it is optimal for seller i to set $r^i = 1$. Seller i can make profits equal to 1 whenever $n^i \geq 1$. The corresponding revenue $1-[1-q^i(1-q^*)^{M-1}]^N$. Since we cannot explicitly write down the profit in the candidate equilibrium, we are unable to compare equilibrium profit with this deviating profit. So we need to the large market argument. Since $1-[1-q^i(1-q^*)^{M-1}]^N \approx 0$ when M and N are relatively large while the equilibrium profit Π^* is always strictly positive (see the convergence result in Section 3.3), seller i will not deviate in this way.

We now establish the equilibrium existence. Note that the right-hand side and left-hand side of equation (43) are continuous over [0,1]. Furthermore, $\lim_{q\to 0}\tau/q = M$ and $\lim_{q\to 1}\tau/q = 1$. The L.H.S. of equation (43) goes to 0 when q goes to zero. Moreover, if we differentiate the L.H.S. of equation (43) with respect to q again and evaluate it at $q = 0$, it equals to $MN(N-1)/(M-1)$. Thus, $C'(0) = 0$ and $C''(0) < MN(N-1)/(M-1)$ guarantee that the L.H.S. of equation (43) intersects the R.H.S. from above. Then, a SPNE with positive advertising intensity exists.

Finally, when q goes to 1, the L.H.S. of equation (43) goes to $\phi(\phi-1/M)(M-1)(1-$

$1/M)^{N-2}/(M-2+\phi)$. Then, as long as $C'(1) > \phi(\phi-1/M)(M-1)(1-1/M)^{N-2}/(M-2+\phi)$, the left-hand side of equation (43) intersects with the right-hand side at some $q^* \in (0,1)$. Thus, an interior solution exists. ■

Proof of Proposition 3.
Rewrite equation (20) as

$$\phi^2 e^{-\phi} = q^* C'(q^*).$$

Differentiating both sides with respect to ϕ yields

$$\phi e^{-\phi}(2-\phi) = C''(q^*)q^* + C'(q^*)\frac{dq^*}{d\phi}.$$

Rearranging it gives the the following relationship between q^* and ϕ

$$\frac{dq^*}{d\phi} = \frac{(q^*)^2\phi e^{-\phi}(2-\phi)}{C''(q^*)q^* + C'(q^*)}.$$

The claim in Proposition 3 then follows immediately. ■

Proof of Proposition 4.
First, it is possible for seller i to deviate to a price so that only buyers with $\mathcal{I} = \{p_i\}$ will select her. In this case, there is no reason for seller i to charge price lower than 1. Then, $p^i = 1$. Seller i then chooses q^i to maximize $1 - [1 - q^i(1-q^*)^{M-1}]^N$, which is also the probability that there exists at least one buyer with $\mathcal{I} = \{p_i\}$. As we argue in the proof of Proposition 2, when M and N are relatively large, this deviating profit is approximately zero, while in the equilibrium we show below sellers make strictly positive profit no matter how large are M and N. So we will focus on the equilibrium where sellers choose $p^* \in (0,1]$.

Assume the interior solution exists. The first-order conditions of seller i's maximization problem w.r.t p_i and q_i are then given by

$$1 - \left(1 - q^i\Lambda^i\right)^N + p^i N q^i \left(1 - q^i\Lambda^i\right)^{N-1}\frac{d\Lambda_i}{dp^i} = 0, \tag{44}$$

$$p^* N\Lambda^* \left(1 - q^*\Lambda^*\right)^{N-1} = C'(q^*). \tag{45}$$

Assume passive belief. Use the market-clearing condition in Lemma 1 to replace Λ in the indifference condition $(1-p^i)\Omega^i = (1-p^*)\Omega$, and then totally differentiate it with respect to p, we obtain

$$\frac{d\Lambda}{dp} = \frac{(1-\rho)(1-\rho^N)}{q(1-p)\left[\frac{M}{M-1}(1-\rho^N) - \frac{MN}{M-1}\rho^{N-1}(1-\rho)\right]},$$

where $\rho = 1 - q\Lambda = 1 - (\tau/M)$. Substitute $d\Lambda/dp$ into (44) and set $q^i = q^*$ and $p^i = p^*$, the equilibrium price p^* can be written as a function of q^*.

$$p^* = \frac{M(1-\rho^N) - \rho^{N-1}(1-\rho)MN}{M(1-\rho^N) - N\rho^{N-1}(1-\rho)}. \tag{46}$$

We now use equation (45) and (46) to show the existence of $q^* > 0$. If $q^* \to 0$, then $\Lambda \to 1$, and from (46) we have $p^* \to 1$. Thus, the L.H.S. of (45) converges to N when $q^* \to 0$. If $q^* \to 1$, then $\Lambda \to 1/M$, and equation (46) shows that p^* converges to a positive number

$$\hat{p} \equiv \frac{(M-1) - (\frac{M-1}{M})^{M\phi}(M-1+M\phi)}{(M-1) - (\frac{M-1}{M})^{M\phi}(M-1+\phi)} \in (0,1).$$

Then, the L.H.S. of (45) converges to a positive number, $\hat{p}N(1/M)(1-1/M)^N$. Since $C'(0) = 0$ and $C'(q) > 0$, the R.H.S. of (45) is either always lower than the L.H.S. of (45), or it intersects with the L.H.S. of (45) at some $q^* \in (0,1]$. ■

Proof of Proposition 5.

Take the limit of (22) and (23), we can directly obtain (25) and (26). So what left to show is that q^* is of inverted U-shape in ϕ. Define $A = \phi e^{-\phi}$ and $B = 1 - e^{-\phi}$. Totally differentiate (26) with respect to ϕ, we have

$$\frac{dq^*}{d\phi} = \frac{B(B-2A)A' + A^2B'}{[C''(q)q + C'(q)]\,B^2}.$$

Since $C''(q)q + C'(q) > 0$ and $B^2 > 0$, the sign of $dq^*/d\phi$ is equal to the sign of $B(B-2A)A' + A^2B'$. It is easy to verify the following holds

$$\begin{aligned} B(B-2A)A' + A^2B' \gtreqless 0 \;\Leftrightarrow\; & (A-B)^2 \gtreqless \phi B(B-2A), \\ \Leftrightarrow\; & \frac{1}{\phi(1-e^{-\phi})} \gtreqless \frac{1-e^{-\phi}-2\phi e^{-\phi}}{(1-e^{-\phi}-\phi e^{-\phi})^2}. \end{aligned}$$

Define $F(\phi) \equiv \frac{1}{\phi(1-e^{-\phi})}$ and $G(\phi) \equiv \frac{1-e^{-\phi}-2\phi e^{-\phi}}{(1-e^{-\phi}-\phi e^{-\phi})^2}$. We have

$$F'(\phi) = -\frac{e^{\phi}(-1+e^{-\phi}+\phi)}{(1-e^{-\phi})^2\phi^2} < 0,$$

$$G'(\phi) = \frac{e^{\phi}(1-e^{\phi}+\phi+2\phi^2)}{(-1+e^{\phi}-\phi)^3} > 0.$$

Thus, $F(\phi)$ is a strictly decreasing function while $G(\phi)$ is a strictly increasing function.

Moreover, we have $\lim_{\phi \to 0} G(\phi) = \infty$, $\lim_{\phi \to 0} F(\phi) = -\infty$, $\lim_{\phi \to \infty} G(\phi) = 0$, and $\lim_{\phi \to \infty} F(\phi) = 1$. Therefore, there exists a unique $\hat{\phi}(\approx 1.669)$ so that $F(\phi) > G(\phi)$ when $\phi < \hat{\phi}$ and $F(\phi) < (\phi)$ when $\phi > \hat{\phi}$. This implies that the sign of $dq^*/d\phi$ is initially positive but eventually becomes negative. The result then follows. ■

Proof of Proposition 6.

Compare the first-order conditions with respect to q under auction and under price posting in the large market. Since $qC'(q)$ is a strictly increasing function of q, the equilibrium advertising intensity will be strictly higher under auctions if

$$\begin{aligned} & \phi^2 e^{-\phi} \geq \frac{\phi e^{-\phi}(1 - e^{-\phi} - \phi e^{-\phi})}{1 - e^{-\phi}} \\ \Leftrightarrow\ & \phi \geq \frac{(1 - e^{-\phi} - \phi e^{-\phi})}{1 - e^{-\phi}} \\ \Leftrightarrow\ & \phi - 1 + e^{-\phi} \geq 0. \end{aligned}$$

First, notice that, when $\phi = 0$, this weak inequality holds as an equality. Second, $\phi - 1 + e^{-\phi}$ is continuous in ϕ and

$$\frac{d}{d\phi}(\phi - 1 + e^{-\phi}) = 1 + e^{-\phi} > 0.$$

Then, $\phi - 1 + e^{-\phi}$ is a strictly increasing function and thus is strictly positive for any $\phi > 0$. So we can conclude that the equilibrium advertising intensity under auction mechanism is strictly higher for any $\phi > 0$. ■