Strategic vote trading in proportional representation systems

Dimitrios Xefteris*
University of Cyprus

Nicholas Ziros†
University of Cyprus

Abstract

This paper introduces the trading norms of strategic market games (à la Shapley and Shubik, 1977) to a vote trading model (à la Casella et al., 2012). In particular we study a two-party election in a proportional representation framework (the utility of a voter is proportionally increasing in the vote-share of her favorite party) considering that prior to the voting stage, voters are free to trade votes for money. We assume *a)* that voters are heterogeneous not just in terms of ordinal preferences but also as far as cardinal preferences are concerned and *b)* that a voter's preferences (both ordinal and cardinal) are her private information, and we prove generic existence of a *unique full trade equilibrium* (an equilibrium in which nobody refrains from vote trading). Unlike equilibria of other vote trading models which predict a small number of vote buyers (at most two), this equilibrium is characterized by *many buyers and many sellers*: voters who are relatively indifferent between the two parties sell their votes for money and voters who have relatively more asymmetric valuations of the two parties buy these votes. We moreover argue that vote trading before elections in such systems *unambiguously improves voters' welfare* and, therefore, it should receive law-makers' attention.

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*Department of Economics, University of Cyprus, P.O. Box 20537, Nicosia 1678, Cyprus. E-mail: xefteris.dimitrios@ucy.ac.cy

†Department of Economics, University of Cyprus, P.O. Box 20537, Nicosia 1678, Cyprus. E-mail: n.ziros@ucy.ac.cy

1 Introduction

Despite many critiques on ethical and philosophical grounds, vote markets have been a research issue for economists for quite a long time, because they reflect voters' intensity of preferences over policies in the same way that regular markets reflect consumers' preferences over commodities. However, vote trading models are not immune to criticism on economics' grounds as well, because of votes' distinct characteristics when compared to other commodities. For example, vote trading embodies externalities on third parties and demands are interdependent. Moreover, decentralized allocations of votes are rarely supported by a market-clearing price (see Ferejohn, 1974; Philipson and Snyder, 1996). These characteristics have prevented the wide adoption of a general model of decentralized vote trading. This paper contributes to the emerging literature on vote markets in a dual manner; *a)* by introducing to a vote trading model the rules of strategic market games and *b)* by analyzing the market for votes in the framework of a proportional representation system.

Concerning *a)*, strategic market games are a class of non-cooperative games that are characterized by an explicit market mechanism, which maps individuals' actions to trades and prices (see, for example, Shapley and Shubik, 1977, Peck et al., 1992). Our approach, instead of assuming price-taking behavior with all its resultant inadequacies, allows individuals to strategically trade votes for money, with their market actions having a clear impact on prices. Given the variety of issues in political economics in which vote trading has been used for, we believe that the development of a strategic version of the trading process is very important as it helps us to evaluate which of the conclusions of the literature, if any, carry over to non-competitive markets. Moreover, the introduction of the mechanisms of these games allows us to study vote trading using a conventional solution concept (Bayesian Nash equilibrium) that is more familiar to voting literature. Earlier papers (Casella et al., 2012; Casella et al., 2014; Casella and Turban, 2014) employed a price-taking approach and, hence had to adopt solution concepts that associate with such a framework (for example, ex ante competitive equilibrium in Casella et al., 2012).

Concerning *b)*, a study of proportional representation systems makes our analysis be relevant to real world elections in which policy outcomes are not solely determined by the simple plurality winner. To be more precise, we consider that the utility of a voter is proportionally increasing in the vote-share of her favorite party. This captures the fact that in many parliamentary democracies the whole distribution of parliament seats is crucial for the determination of policy outcomes and hence a voter should prefer that her favorite party elects as many parliament members as possible. Such a framework has been employed to analyze a variety of issues in political economics literature[1] and provides us with all the necessary tools to avoid the deficiencies involved in simple plurality rule. In particular, our model does not involve severe discontinuities in utilities at the point at which the winner changes and hence, at the right price, there is always demand for any single vote offered in exchange for money.

Combining the above features we study a two-stage game, in which prior to the voting stage voters are free to trade votes for money. We assume that voters are heterogeneous not just in terms of ordinal preferences but also as far as cardinal preferences are concerned and that a voter's preferences (both ordinal and cardinal) are her private information. We moreover consider that if a voter decides to sell her vote she has to sell it as a whole, and that if a voter decides to buy votes she has to bid a predetermined amount of money - which we fix to one dollar without loss of generality. These assumptions, combined with the market mechanism of strategic market games, suggest that the price of a vote is equal to the ratio of the number of vote buyers over the number of vote sellers. Of course, since voters' preferences are private information at the vote trading stage and all players have to decide simultaneously whether to buy, sell or do nothing it is straightforward that when they make their choices they are uncertain regarding how many of them choose what and hence they are uncertain about the price of a vote. In this setup, we wish to address the following questions: Is a Bayesian Nash equilibrium with positive trade guaranteed to exist? Does the persistent small number of vote buyers in equilibrium as reported in the price-taking literature (in Casella et al., 2012 for example there are at most two vote buyers and everybody else is a vote seller) carry over to this strategic trading context? What are the welfare implications of these equilibrium outcomes, and in particular, is the vote trading option more efficient than the no-trade alternative, in which individuals simply cast their votes?

[1] See, for instance, Lijphart (1984), Ortuno-Ortin (1997), Grossman and Helpman (1999), Llavador (2006), Sahuguet and Persico (2006), Merrill and Adams (2007), De Sinopoli and Iannantuoni (2007), Herrera et al. (2014), Saporiti (2014), Iaryczower and Mattozzi (2013) and Matakos et al. (2013) among others.

Our results give explicit answers to these questions. Initially, we establish in Proposition 1 that vote trading occurs in every almost strict Bayesian Nash equilibrium.[2] In such an equilibrium, voters, who are relatively indifferent between the two parties, offer their votes in exchange for money, whereas voters who have relatively more asymmetric valuations of the parties offer their money in exchange for votes. Concerning the number of voters who take part in trading, we also demonstrate that there is always a unique almost strict Bayesian Nash equilibrium in which all individuals engage in vote trading, that is, no single voter prefers to abstain from it. Indeed, Proposition 2 defines the unique preference intensity parameter threshold that groups individuals into vote sellers and vote buyers. Finally, given that vote trading is preferred to the *just cast your vote* alternative by all individuals, we show in Proposition 3 that allowing for vote trading *unambiguously improves social welfare.*

It should be noted that our results offer a novel positive perspective on vote trading since they stand in profound contrast to existing ones in the literature. For instance, let us consider the pioneering study of Casella et al. (2012). That paper considers an election under a simple majority rule, that individuals place stochastic demands for votes and that the market clears ex ante in expectation, whereas ex post an anonymous rationing rule determines which trades are executed in case of imbalance between the realized demands and supplies. The questions of Casella et al. (2012) are similar to ours, their results though are quite different: they demonstrate that an equilibrium with active vote trading is always characterized by small number of buyers (i.e., only the two voters with the most intense preferences are willing to buy votes and all other voters sell their votes). This fact leads to dictatorship, as a single voter acquires a majority position. Moreover, due to dictatorship, if the number of voters is large (or if the distribution of valuations is not very skewed) vote trading is welfare decreasing when compared to simple majority rule without vote trading. Similar results in decentralized competitive vote markets also appear in Casella and Turban (2014) and Casella et al. (2014), which consider two groups of unequal size supporting different policies and show that only the most intense voter of each group demands votes, with the minority's voter being more aggressive. As a result, the minority's favorite policy is implemented with higher probability than the efficient level, which suggests that vote trading can be welfare inferior to simple

[2] A Bayesian Nash equilibrium in which almost all players' types strictly prefer their equilibrium strategy to any other.

majority rule without vote trading. These strong differences though between the current and earlier approaches to vote trading should come at no surprise since a) the transition from price-taking to strategic behavior has non-trivial implications on equilibrium outcomes in most trade environments and b) proportional systems involve a completely distinct set of strategic incentives as far as voters' behavior is concerned compared to simple majority rule.

We proceed to develop the model in Section 2. Next we present the results in Section 3. Some further comments follow in Section 4.

2 The model

We consider a society $Q = \{1, 2, ..., n\}$ of $n > 2$ voters and two fixed alternatives, A and B. Each voter $i \in Q$ is characterized by her ordinal preferences, $t_i \in \{(A \succ B), (B \succ A)\}$, and an intensity parameter, $w_i \in [0, +\infty)$.[3] Each voter is assumed to have one vote (which she can trade for money) and one unit of money (numeraire).[4] If we denote by $v_A \in [0, 1]$ the vote share of alternative A and by $v_B = 1 - v_A$ the vote share of alternative B, then the utility of voter i after the election is given by

$$u_i = \begin{cases} v_A \times w_i + m_i & \text{if } t_i = (A \succ B) \\ v_B \times w_i + m_i & \text{if } t_i = (B \succ A) \end{cases},$$

where $m_i \geq 0$ is the amount of money voter i ends up having. Notice that this formulation of voters preferences is perfectly compatible with other papers studying proportional representation systems (see, for example, Iaryczower and Mattozzi, 2013). What we do is simply to normalize the utility one receives from her least preferred party to zero. As it will be evident in the following section, this is very helpful in conducting a tractable analysis and is obviously without any loss of generality.

Each voter $i \in Q$ knows t_i and w_i but is uncertain about the ordinal preferences and the intensity parameters of her fellow citizens. The beliefs of voter $i \in Q$ regarding the ordinal

[3] When $t_i = (A \succ B)$ and $w_i = 0$ or when $t_i = (A \prec B)$ and $w_i = 0$, voter i is essentially indifferent between the two alternatives.

[4] This is assumed only for simplicity - our analysis extends to a case of asymmetric initial wealth.

preferences of $j \in Q - \{i\}$ are modelled by a Bernoulli distribution with parameter $\frac{1}{2}$ and support $\{(A \succ B), (B \succ A)\}$ and beliefs of voter $i \in Q$ regarding the intensity parameter of $j \in Q - \{i\}$ are given by an absolutely continuous distribution F with support $[0, +\infty)$ and which is twice differentiable in its support.

As far as the timing of the game is concerned we make the following assumptions: in stage 1 vote trading takes place; in stage 2 the players who haven't sold their votes vote strategically in order to maximize their payoffs, while players who sold their votes remain inactive; in stage 3 the payoffs of all players are computed. In particular, as far as stage 1 is concerned, we assume that each player chooses an action x_i from the set $\{s, b, y\}$ where s stands for "sell vote", b stands for "buy votes" and y stands for "neither sell nor buy votes". Players who choose action s sacrifice their whole vote and players who choose b sacrifice the whole unit of money that they have. If we denote by B the set of players who choose action b and by S the set of players who choose action s then, following the trading technology of strategic market games and conditional on both B and S being non-empty, the amount of money that is assigned to each player who chose s is

$$p = \frac{\#B}{\#S}$$

and the amount of extra votes that are assigned to each player who chose b is

$$h = \frac{\#S}{\#B}.$$

In our framework, votes are perfectly divisible and, hence, a player might end up having a non-integer number of votes. Notice that we study a model of proportional representation - all that matters is the share of votes that each alternative receives - and hence voting is merely an attribution of additional weight to one's preferred alternative. That is, there is no reason at all why these weights cannot be non-integers. What we should stress though is that the above is strictly conditional on both B and S being non-empty. When at least one of these sets is empty then *no trade takes place*; all players keep their money and their vote and all are allowed to vote in stage 2.

Since the behavior of players who can vote in stage 2 (the ones who haven't sold their votes in stage 1) is completely unambiguous - attributing all votes that one has to her preferred alternative is a dominant strategy - we essentially have a one-shot game and, hence, we define an equilibrium only in terms of players' strategies and beliefs in the first stage of the game. The obvious solution concept for such one-shot game of private information with a continuum of types is Bayesian Nash equilibrium (BNE) in pure strategies. We will focus on almost strict BNE in pure strategies, that is on equilibria such that a measure one of players' types strictly prefer their equilibrium strategy to any other. Hence, when we use the term equilibrium we refer to this particular subset of BNE.

3 Results

We directly proceed to the statement of the formal results of the paper.

Proposition 1 *In every equilibrium trade occurs with positive probability.*

Proof. Assume that there exists an almost strict BNE such that trade does not occur with positive probability. If in such an equilibrium a measure zero of types chooses b and a non-degenerate measure, $z \in (0,1]$, of types chooses s then there exists a $\hat{w}$ such that all players with $w_i > \hat{w}$ are better off choosing b than any other action. This is so because in such a case the expected utility of a player $i \in Q$ with intensity parameter w_i when choosing s or y is given by

$$Eu_i(s) = Eu_i(y) = (\tfrac{1}{2} + \tfrac{1}{2n})w_i + 1$$

while when choosing b it is given by

$$Eu_i(b) = (1-z_s)^{n-1}[(\tfrac{1}{2}+\tfrac{1}{2n})w_i + 1] + \textstyle\sum_{k=1}^{n-1}[\binom{n-1}{k} z^k (1-z)^{n-1-k} E(v_{J_i} \mid x_i = b, \#B = 1 \text{ and } \#S = k)]w_i$$

where $J_i = A$ if $t_i = (A \succ B)$ and $J_i = B$ if $t_i = (B \succ A)$.

We notice that

$$E(v_{J_i} \mid x_i = b,\ \#B = 1 \text{ and } \#S = k) = \frac{k + 1 + \frac{1}{2}(n - k - 1)}{n}$$

and hence we get that

$$Eu_i(b) = (1 - z_s)^{n-1} + \frac{w_i(1 + n + (n-1)z_s)}{2n}$$

Standard algebraic manipulations yield that for every $w_i > \hat{w} = \frac{2n((1-z_s)^n + z_s - 1)}{(n-1)(z_s-1)z_s}$ (which is strictly positive for any $n > 1$ and $z_s > 0$)[5] it is the case that $Eu_i(b) > Eu_i(s) = Eu_i(y)$. Obviously, the measure of types for which $w_i > \hat{w}$ is equal to $1 - F(\hat{w})$ and it is hence non-degenerate for any $z_s \in (0, 1]$. Thus, it is not possible that in an almost strict BNE a measure zero of types chooses b and a positive measure of types chooses s. One can similarly show that in an almost strict BNE it is not possible that a measure zero of types chooses s and a positive measure of types chooses b. Finally, it is trivial to see that no trade (a measure one of players' types choose y) is a BNE. If a measure zero of players' types is expected to choose s (b) then nobody strictly prefers b (s) over y. In fact every player is absolutely indifferent among all actions. Precisely because all players' types are indifferent among all actions, no trade is not an almost strict BNE. These prove that in every almost strict BNE it must be the case that trade occurs with positive probability. ■

The above proposition establishes that vote trading takes place in every equilibrium; it does not guarantee that an equilibrium actually exists. The next proposition does that by proving existence of a unique full trade equilibrium for every possible parameter values. In a full trade equilibrium almost no one refrains from vote trading (a measure one of players' types choose either to sell or to buy votes) and hence only players who decided to buy votes actually get to vote. This means that each of these voters carries as many votes as any other and thus the vote trading game that we analyze essentially leads to a pay-to-vote mechanism: players who pay one dollar get the right to vote and players who do not pay to vote get an equal share of the amount gathered by those who paid to vote.

Proposition 2 *For any admissible F there exists a unique full trade equilibrium and it is such that all players with intensity parameters smaller than $\ddot{w}$ sell their votes and all players with intensity parameters larger than $\ddot{w}$ buy votes, where $\ddot{w}$ is uniquely defined by* $2n([1 - F(\ddot{w})]^n -$

[5] For $z_s \to 1$ we have that $\hat{w} \to \frac{2n}{n-1} > 0$.

$1) = \dddot{w}F(\dddot{w})^n - (2n + \dddot{w})F(\dddot{w})$.

Proof. **Step 1** We first argue that in every almost strict BNE there exists a) $\dot{w}$ such that all players with $w_i < \dot{w}$ are better off choosing s than y and all players with $w_i > \dot{w}$ are better off choosing y than s and b) $\ddot{w}$ such that all players with $w_i > \ddot{w}$ are better off choosing b than y and all players with $w_i < \ddot{w}$ are better off choosing y than b. Since in every almost strict BNE trade occurs with positive probability it follows that a measure $z_s > 0$ of types choose s and a measure $z_b > 0$ of types choose b. If this is the case, then the expected utility of a player $i \in Q$ with intensity parameter w_i when choosing s is given by

$$Eu_i(s) = (1 - z_b)^{n-1}[(\tfrac{1}{2} + \tfrac{1}{2n})w_i + 1] + [1 - (1 - z_b)^{n-1}][\tfrac{1}{2}w_i + E(p \mid x_i = s \text{ and } \#B > 0) + 1]$$

when choosing b it is given by

$$Eu_i(b) = (1 - z_s)^{n-1}[(\tfrac{1}{2} + \tfrac{1}{2n})w_i + 1] + [1 - (1 - z_s)^{n-1}]E(v_{J_i} \mid x_i = b \text{ and } \#S > 0)w_i$$

while when choosing y it is given by

$$Eu_i(y) = (\tfrac{1}{2} + \tfrac{1}{2n})w_i + 1$$

where $J_i = A$ if $t_i = (A \succ B)$ and $J_i = B$ if $t_i = (B \succ A)$.

We observe that all the above expected utilities are linear functions of intensity parameter w_i. Hence, since $E(p \mid x_i = s \text{ and } \#B > 0) > 0$ we have that all players with $w_i < \dot{w} = 2nE(p \mid x_i = s \text{ and } \#B > 0)$ are better off choosing s than y and all players with $w_i > \dot{w}$ are better off choosing y than s. Similarly, since $E(v_{J_i} \mid x_i = b \text{ and } \#S > 0) > (\frac{1}{2} + \frac{1}{2n})$ we have that all players with $w_i < \ddot{w} = 1/[E(v_{J_i} \mid x_i = b \text{ and } \#S > 0) - (\frac{1}{2} + \frac{1}{2n})]$ are better off choosing y than b and all players with $w_i > \ddot{w}$ are better off choosing b than y.

Step 2 In a full trade almost strict BNE it must be the case that $0 < \ddot{w} \leq \dot{w}$ and hence that $z_s = F(\dddot{w}) = 1 - z_b$ where $\dddot{w} > 0$ is the solution of $Eu_i(s) = Eu_i(b)$. This is so because if we had $0 < \dot{w} < \ddot{w}$ then a set of types of positive measure $F(\ddot{w}) - F(\dot{w}) > 0$ would prefer action y over b and over s.

Assume that such an equilibrium exists. Then all players with $w_i > \dddot{w}$ choose b and that all players with $w_i < \dddot{w}$ choose s. If the posited behavior is an equilibrium then no player should have incentives to deviate. In this case we have that

$$E(p \mid x_i = s \text{ and } \#B > 0) = \frac{\sum_{k=0}^{n-2}[\frac{n-1-k}{k+1}\binom{n-1}{k}F(\dddot{w})^k[1-F(\dddot{w})]^{n-1-k}]}{1-F(\dddot{w})^{n-1}}$$

and

$$E(v_{J_i} \mid x_i = b \text{ and } \#S > 0) = \frac{\sum_{k=0}^{n-2}[(\frac{1}{2}+\frac{1}{2+2k})\binom{n-1}{k}[1-F(\dddot{w})]^k F(\dddot{w})^{n-1-k}]}{1-[1-F(\dddot{w})]^{n-1}}.$$

The above help us write the expected utilities in a much more convenient form. That is,

$$Eu_i(s) = \frac{2-2[1-F(\dddot{w})]^n + w_i F(\dddot{w})}{2F(\dddot{w})} + \frac{w_i F(\dddot{w})^{n-1}}{2n}$$

and

$$Eu_i(b) = \frac{w_i[nF(\dddot{w}) + F(\dddot{w})^n - n - 1]}{2n[F(\dddot{w})-1]} + [1-F(\dddot{w})]^{n-1}.$$

We observe that a) $Eu_i(s) = Eu_i(b)$ if and only if $w_i = \frac{2n([1-F(\dddot{w})]^n+F(\dddot{w})-1)}{F(\dddot{w})^n-F(\dddot{w})}$, b) $Eu_i(s) = Eu_i(y)$ if and only if $w_i = \frac{2n([1-F(\dddot{w})]^n+F(\dddot{w})-1)}{F(\dddot{w})^n-F(\dddot{w})}$ and c) $Eu_i(b) = Eu_i(y)$ if and only if $w_i = \frac{2n([1-F(\dddot{w})]^n+F(\dddot{w})-1)}{F(\dddot{w})^n-F(\dddot{w})}$. These show that in a full trade almost strict BNE it must be the case that $0 < \dot{w} = \ddot{w} = \dddot{w} = \frac{2n([1-F(\dddot{w})]^n+F(\dddot{w})-1)}{F(\dddot{w})^n-F(\dddot{w})}$. So if we show the existence of a unique $\dddot{w} > 0$ such that $\dddot{w} = \frac{2n([1-F(\dddot{w})]^n+F(\dddot{w})-1)}{F(\dddot{w})^n-F(\dddot{w})}$, we essentially establish both existence and uniqueness of a full trade almost strict BNE.

We define $R(x) = \frac{2n((1-x)^n+x-1)}{x^n-x}$ and observe that a) $\lim_{x\to 0} R(x) = 2n(n-1) > 0$, b) $\lim_{x\to 1} R(x) = 2n/(n-1) \in (0, \lim_{x\to 0} R(x))$ and $\frac{\partial R(x)}{\partial x} < 0$ for every $x \in (0,1)$. That is, $\dddot{w} = \frac{2n([1-F(\dddot{w})]^n+F(\dddot{w})-1)}{F(\dddot{w})^n-F(\dddot{w})}$, which may be re-written as $2n([1-F(\dddot{w})]^n-1) = \dddot{w}F(\dddot{w})^n-(2n+\dddot{w})F(\dddot{w})$, is guaranteed to have a unique solution for every admissible F and hence the game admits a unique full trade almost strict BNE. ■

As far as comparative statics of this equilibrium are concerned we note that the threshold value $\dddot{w}$ need not be monotonic to n and hence the expected share of vote sellers (vote buyers) in this unique full trade equilibrium need not monotonically increase or decrease in the cardinality of the population. In Figure 1 we plot $\dddot{w}$ for various values of n considering that $F = \ln N(0,1)$ (F is a log-normal distribution with mean $e^{1/2}$ and variance $(e-1)e$) and we see that $\dddot{w}$ is initially decreasing in n and then increasing.

[Insert Figure 1 about here]

We finally argue that in any equilibrium of this game players' welfare is unambiguously larger compared to the no trading scenario. This holds both in social and in individual terms and most importantly it is true both under the veil of ignorance and when players are fully aware of their preferences. The intuition why allowing for vote trading in the framework of a proportional representation system unambiguously improves welfare lies in the fact that the no trade action delivers to an individual the same expected utility *independently of what other players choose to do.* Hence, a player that neither sells nor buys votes expects the same utility both when vote trading is allowed (that is, when some players might be expected to engage in it) and when vote trading is not allowed (that is, when nobody is expected to engage in vote trading).

Proposition 3 *All (a positive measure of) players' types expect weakly (strictly) larger utility in every equilibrium of this game compared to when vote trading is not allowed.*

Proof. The proof is straightforward. We know that the expected utility of a player $i \in Q$ with intensity parameter w_i when choosing y is given by $Eu_i(y) = (\frac{1}{2} + \frac{1}{2n})w_i + 1$. We notice that this is independent of what other players' types might be expected to do. Hence, this should be the expected utility of this player even when nobody is expected to trade and thus in the variation of the game in which trade is not allowed. By proposition 1 we know that in every almost strict BNE it has to be the case that a positive measure of types trades and by proposition 2 we know that an almost strict BNE actually exists. Therefore, in every almost strict BNE it is the case that a positive measure of players' types expect strictly larger utility than $Eu_i(y)$ and no player expects lower utility than $Eu_i(y)$. ■

This establishes that once a voter knows her preferences and given the equilibrium expectations regarding what other players will do, she is better off (all players' types weakly and a positive measure of them strictly) when vote trading is allowed compared to when it is not. Since this holds for every possible player type it should trivially extend a) to the society as a whole and b) to a possible constitutional design pre-stage in which voters are not yet aware of their preferences. That is, if voters were somehow asked to choose under a veil of ignorance whether they would like to allow vote trading or not, they would unanimously approve vote trading.

4 Concluding remarks

This paper is an attempt to provide a better understanding of vote markets by studying a simple proportional representation environment with strategic players. We provide clear-cut results by showing that when exchange of votes for money is allowed, voters are willing to engage in such a trading. Moreover, concerning social welfare, vote trading makes voters better off, so the superiority of vote trading compared to the no trade alternative is easily justified. We believe that our approach has been able to deal with many of the criticisms that have been made to other models of decentralized vote trading; yet there are still many central questions in the literature, so we hope that our approach will pave the way for new studies. An interesting way to go forward would possibly be to study vote trading mechanisms such that prices are determined as in this paper - via the strategic market games mechanism - but which might allow different players to bid different amounts. Finally, given that experimental strategic market games have already been implemented (e.g., Huber et al., 2010; Duffy et al., 2011), we believe that it is challenging for future research to test our theoretical results by conducting laboratory experiments in the same way as Casella et al., (2012) and Casella et al., (2014).

References

[1] Casella, A., Llorente-Saguer, A., Palfrey, T., (2012). Competitive equilibrium in markets for votes. Journal of Political Economy 120, 593-658.

[2] Casella, A., Palfrey, T., Turban, S., (2014). Vote trading with and without party leaders. Journal of Public Economics 112, 115-128.

[3] Casella, A., Turban, S., (2014). Democracy undone. Systematic minority advantage in competitive vote markets. Games and Economic Behavior 88, 47-70.

[4] De Sinopoli, F., Iannantuoni, G., (2007). A spatial voting model where proportional rule leads to two-party equilibria. International Journal of Game Theory 35, 267-286.

[5] Duffy, J., Matros, A., Temzelides, T., (2011). Competitive behavior in market games: Evidence and theory. Journal of Economic Theory 146, 1437-1463.

[6] Ferejohn, J.A., (1974). Sour notes on the theory of vote trading. Social Science Working Paper no 41, California Institute of Technology.

[7] Grossman, G.M., Helpman, E., (1999). Competing for endorsements. American Economic Review 89, 501-524.

[8] Herrera, H., Morelli, M., Palfrey, T., (2014). Turnout and power sharing. The Economic Journal 124, F131-F162.

[9] Huber, J., Shubik, M., Sunder, S., (2010). Three minimal market institutions with human and algorithmic agents: Theory and experimental evidence, Games and Economic Behavior 70, 403-424.

[10] Iaryczower, M., Mattozzi, A., (2013). On the nature of competition in alternative electoral systems. The Journal of Politics, 75, 743-756.

[11] Llavador, H., (2006). Electoral platforms, implemented policies, and abstention. Social Choice and Welfare 27, 55-81.

[12] Lijphart, A., (1984) Democracies: Patterns of Majoritarian and Consensus Government in Twenty-One Countries. Yale University Press; New Haven.

[13] Matakos, K., Troumpounis, O., Xefteris, D., (2013). Electoral rule disproportionality and platform polarization. EPSA 2013 Annual General Conference Paper 809. Available at SSRN: http://ssrn.com/abstract=2225152

[14] Merrill, S., Adams, J., (2007). The effects of alternative power-sharing arrangements: do "moderating" institutions moderate party strategies and government policy outputs? Public Choice 131, 413-434.

[15] Ortuno-Ortin, I., (1997). A spatial model of political competition and proportional representation. Social Choice and Welfare 14, 427-438.

[16] Peck, J., Shell, K., Spear, S., (1992). The market game: Existence and structure of equilibrium. Journal of Mathematical Economics 21, 271-299.

[17] Philipson, T.J., Snyder, J.M., (1996). Equilibrium and efficiency in an organized vote market. Public Choice 89, 245-265.

[18] Sahuguet, N., Persico, N., (2006). Campaign spending regulation in a model of redistributive politics. Economic Theory 28, 95-124.

[19] Saporiti, A., (2014). Power sharing and electoral equilibrium. Economic Theory 55, 705-729.

[20] Shapley, L., Shubik, M., (1977). Trade using one commodity as a means of payment. Journal of Political Economy 85, 937-968.

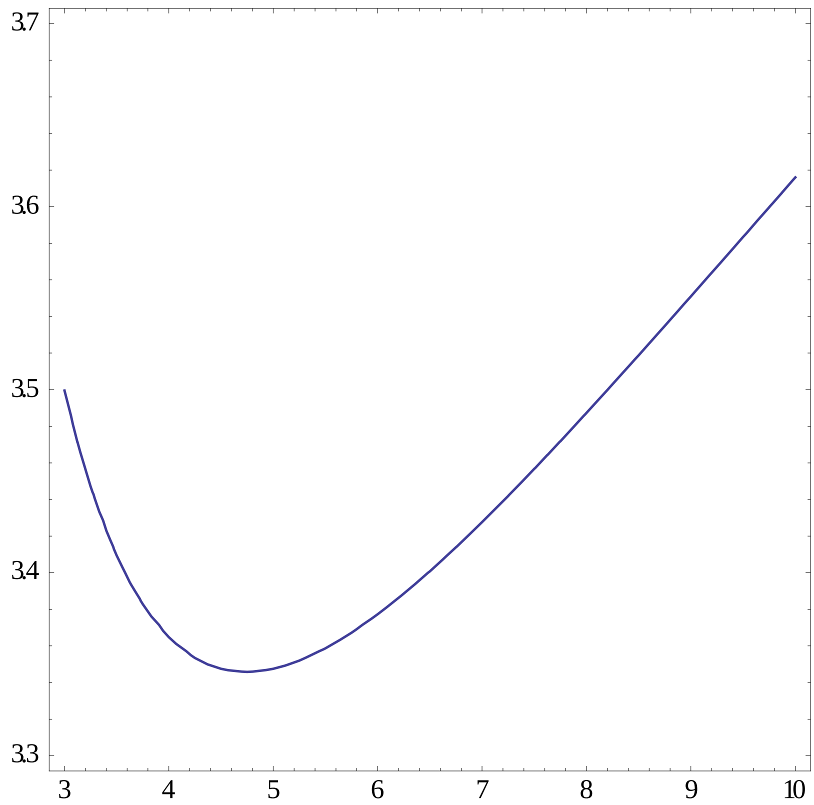

Figure 1. The threshold value, $\ddot{w}$, as a function of *n* considering that $F = lnN(0,1)$.