# Price Rigidities in a Productive Network

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PRELIMINARY AND INCOMPLETE

**Abstract**

In a multi-sector New-Keynesian model where input-output linkages between firms create a productive network, we study the interaction between sectoral heterogeneity of price rigidities and sectoral heterogeneity of input-output linkages. Our goal is to explore the implications of these two forms of heterogeneity on (1) the real effects of monetary policy, (2) the aggregate propagation of sectoral idiosyncratic shocks, and (3) the pass-though to aggregate inflation of an oil price shock. We show that the effects of this interaction is significant after calibrating our model to the US economy.

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# 1 Introduction

There is an incipient revival of interest in the literature to explore the macroeconomic implications of conceptualizing the economy as a productive network. This network is composed by sectoral input-output linkages when firms in one economic sector use as intermediate inputs the goods produced by other firms in other economic sectors and their own. Two prominent directions which the literature has taken are (1) the amplification of the real effects of monetary policy due to the network (Nakamura and Steinsson, 2010), and (2) the aggregate propagation of idiosyncratic shocks through the network (Acemoglu et al, 2012). This paper contributes in these two directions by studing the interaction between sectoral heterogeneity of price rigidities and sectoral heterogeneity in input-output linkages. In addition, we are also interested in (3) the effect of the interaction between these two forms of heterogeneity on the pass-through of an oil price shock to aggregate inflation.

To contribute in these three directions, we use a multi-sector new-Keynesian model that allows for intermediate inputs (as in Basu, 1995, and Carvalho and Lee, 2011) and a general sectoral input-output structure of the productive network. We seek to theoretically study the mechanisms involved in the model and also provide quantitative assessments after using micro data and standard parametrizations to calibrate the model.

Regarding our first direction, Nakamura and Steinsson (2010) show that the need of intermediate inputs for production creates a form of strategic complementarity that amplifies the real effect of monetary shocks. Importantly, this is a "macro" source of strategic complementarity that is not affected by the criticism of Bils, Klenow and Malin (2012) and Klenow and Willis (2006). These authors argue that some forms of "micro" sources of strategic complementarity, such as high elasticity of substitution across goods, yield counterfactual predictions. To make their point, Nakamura and Steinsson (2010) assume that all the heterogeneity in sectoral input-output linkages is due to transitory heterogeneity in relative prices, so the productive network in the economy is symmetric in steady state. The question we address in this paper is: How does a deeper form of heterogeneity in sectoral input-output linkages that survives in steady state affect this mechanism of amplification of the real effects of monetary policy?

Regarding our second direction, Acemoglu et al (2012) show that when sectoral input-output linkages are homogeneous across sectors, so the productive network is symmetric, idiosyncratic shocks wash out in the aggregate. However, this result does not hold in a asymmetric productive network arising from heterogeneity in sectoral input-output linkages. In particular, shocks to sectors that take inputs from few sectors but supply inputs to many sectors have a disproportionate aggregate effect. Acemoglu et al (2012) make this point in a frictionless economy. One if not the most, important lession of monetary economics is that price rigidities have the ability to substantially change the propagation of shocks. Thus, we ask: Under what conditions does the sectoral heterogeneity of price rigities amplify the aggregate propagation of idiosyncratic shocks?

Finally, regarding our third direction, we do not have a clear reference point in the literature, but the large decrease in the oil price experienced in the last few months gives us strong empirical motivation. In particular, we are interested in the ways the sectoral heterogeneity in price rigidities and input-output linkages determine the persistence and cross-sectional propagation of an oil price shock.

# 2 Baseline model

## 2.1 Households

Consider a representative household whose utility is

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \sum_{k=1}^{K} \int_{\Im_k} g_k \frac{L_{kjt}^{1+\varphi}}{1+\varphi} dj \right)$$

subject to

$$P_t C_t = \sum_{k=1}^{K} W_{kt} \int_{\Im_k} L_{kjt} dj + \sum_{k=1}^{K} \Pi_{kt} + I_{t-1} B_{t-1} - B_t$$

where $C_t$ and $P_t$ respectively are aggregate consumption and aggregate prices to be specified below, $L_{kjt}$ and $W_{kt}$ are labor employed and wages paid by firm $j$ in sector $k = 1, ..., K$, $\Pi_{kt}$ is transfers from a firm in sector $k$, $I_{t-1}$ is the gross interest rate paid by bonds holding at the beginning of period $t$, $B_{t-1}$.

Aggregate consumption is

$$C_t \equiv \left[ \sum_{k=1}^{K} \omega_{ck}^{\frac{1}{\eta}} C_{kt}^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where $C_{kt}$ is the aggregation of sectoral consumption

$$C_{kt} \equiv \left[ n_k^{-1/\theta} \int_{\Im_k} C_{kjt}^{1-\frac{1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}.$$

where $C_{kjt}$ is the amount of goods produced by firm $j$ in sector $k$ that is demanded for consumption of households.

Note that the elasticity of substitution across sectors $\eta$ is allowed to be different than the elasticity of substitution within sectors $\theta$. The aggregation weights $\{\omega_{ck}\}$ will determine the share of a sector $k$ in total consumption. We assume that $\sum_{k=1}^{K} \omega_{ck} = 1$. The set of firms in sector $k$ is denoted as $\Im_k$ which has total measure $n_k$ such that $\sum_{k=1}^{K} n_k = 1$.

The aggregate consumption price $P_t^c$ aggregates sectoral prices $\{P_{kt}\}$ according to

$$P_t^c = \left[ \sum_{k=1}^{K} \omega_{ck} P_{kt}^{1-\eta} \right]^{\frac{1}{1-\eta}},$$

where

$$P_{kt} = \left[ \frac{1}{n_k} \int_{\Im_k} P_{kjt}^{1-\theta} dj \right]^{\frac{1}{1-\theta}}.$$

Households' demand for sectoral composite goods $C_{kt}$ and for each firm' goods $C_{kjt}$ respectively are

$$\begin{aligned} C_{kt} &= \omega_{ck} \left( \frac{P_{kt}}{P_t^c} \right)^{-\eta} C_t, \\ C_{kjt} &= \frac{1}{n_k} \left( \frac{P_{kjt}}{P_{kt}} \right)^{-\theta} C_{kt}. \end{aligned}$$

Other key optimality conditions from households are

$$\text{(labor supply) } \frac{W_{kt}}{P_t^c} = g_k L_{kjt}^{\varphi} C_t^{\sigma} \text{ for all } k, j,$$

$$\text{(Euler equation) } \mathbb{E}_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} I_t \frac{P_t^c}{P_{t+1}^c} \right] = 1$$

## 2.2 Firms

There is a continuum of firms in the economy with total measure one divided in sectors $k = 1, ..., K$. The set of firms in sector $k$ is denoted as $\Im_k$ which has total measure $n_k$ such that $\sum_{k=1}^{K} n_k = 1$.

The production function of firm $j$ in sector $k$ is

$$Y_{kjt} = A_t A_{kt} L_{kjt}^{1-\delta} Z_{kjt}^{\delta},$$

where $Z_{kjt}$ is an aggregator of all goods used by firm $k, j$ as intermediate inputs:

$$Z_{kjt} \equiv \left[ \sum_{r=1}^{K} \omega_{kr}^{\frac{1}{\eta}} Z_{kjt}\left(r\right)^{1-\frac{1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

where, in turn, $Z_{kjt}\left(r\right)$ is the amount of goods from sector $r$ used by firm $j$ in sector $k$. The aggregator weight $\omega_{kr}$ is the weight of goods from sector $r$ in the intermediate inputs aggregator used by firms in sector $k$ such that $\sum_{r=1}^{K} \omega_{kr} = 1$ These weights are allowed to be different across sectors.

$Z_{kjt}\left(r\right)$ is also an aggregator according to

$$Z_{kjt}\left(r\right) \equiv \left[ n_r^{-1/\theta} \int_{\Im_r} Z_{kjt}\left(r, j'\right)^{1-\frac{1}{\theta}} dj' \right]^{\frac{\theta}{\theta-1}}$$

where $Z_{kjt}\left(r, j'\right)$ is the amount of goods produced by firm $j'$ in sector $r$ that is used as input by firm $j$ in sector $k$.

From the point of view of firms in sector $k$, the aggregate intermediate input price $P_t^k$ and the sectoral price $P_{rt}^k$ are

$$\begin{aligned} P_t^k &= \left[ \sum_{r=1}^{K} \omega_{kr} P_{rt}^{1-\eta} \right]^{\frac{1}{1-\eta}}, \\ P_{rt} &= \left[ \frac{1}{n_r} \int_{\Im_r} P_{rj't}^{1-\theta} dj' \right]^{\frac{1}{1-\theta}}. \end{aligned}$$

Note that the aggregate prices $\left\{P_t^k\right\}$ relevant for sector $k$ are in general different across sectors and with the aggregate consumption price $P_t^c$ since all sectors have different I/O linkages. Sectoral prices $P_{kt}$ however are all identical for all sectors and for consumption.

Demands by firm $j$ in sector $k$ of $Z_{kjt}(r)$ and $Z_{kjt}(r, j')$ are

$$\begin{aligned} Z_{kjt}(r) &= \omega_{kr} \left( \frac{P_{rt}}{P_t^k} \right)^{-\eta} Z_{kjt}, \\ Z_{kjt}\left(r, j'\right) &= \frac{1}{n_r} \left( \frac{P_{rj't}}{P_{rt}} \right)^{-\theta} Z_{kjt}(r) \end{aligned}$$

where $P_{rj't}$ is the price charged by firm $j'$ in sector $r$.

There is also sectoral heterogeneity in firms' Calvo parameter. The objective of firm $j, k$ is

$$\max_{P_{kjt}} \mathbb{E}_t \sum_{s=0}^{\infty} Q_{t,t+s} \alpha_k^s \left[ P_{kjt} Y_{kjt+s} - MC_{kjt+s} Y_{kjt+s} \right]$$

where $MC_{kjt} = \frac{1}{1-\delta} \left( \frac{\delta}{1-\delta} \right)^{-\delta} A_t^{-1} A_{kt}^{-1} W_{kt}^{1-\delta} \left( P_t^k \right)^{\delta}$ in reduced form after imposing the efficiency condition:

$$\delta W_{kt} L_{kjt} = (1 - \delta) P_t^k Z_{kjt}.$$

In principle we should distinguish firms in sectors $k$ by the type of customers they have (households or productive firms from a given sector). However, since the across-sectors and within-sectors elasticities of substitution are the same for consumption and intermediate inputs in all sectors, we can simply write the optimal price as

$$\sum_{s=0}^{\infty} Q_{t,t+s} \alpha_k^s Y_{kjt+s} \left[ P_{kt}^* - \frac{\theta}{\theta - 1} MC_{kjt+s} \right] = 0$$

Note that the optimal price conditioning on adjusting is the homogeneous within sectors. Thus, aggregating within sectors,

$$P_{kt} = \left[ (1 - \alpha_k) P_{kt}^{*1-\theta} + \alpha_k P_{kt-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

## 2.3 Monetary policy

Monetary policy controls $I_t$ which is set according to:

$$\text{(Taylor rule) } I_t = \frac{1}{\beta} \left( \frac{P_t}{P_{t-1}} \right)^{\phi_\pi} \left( \frac{C_t}{\overline{C}} \right)^{\phi_y} e^{\mu_t}$$

where $\mu_t$ is a monetary shock which follows an AR(1) process with persistence $\rho_\mu$.

Note that monetary policy reacts to aggregate consumption inflation and aggregate consumption that here is the best proxy of value-added production.

### 2.4 Equilibrium conditions and definitions

$$
\begin{aligned}
B_t &= 0, \\
L_{kt} &= \int_{\Im_k} L_{kjt} dj, \\
Y_{kjt} &= C_{kjt} + \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{k'j't}\left(k,j\right) dj', \\
W_t &\equiv \sum_{k=1}^{K} n_k W_{kt}, \\
L_t &\equiv \sum_{k=1}^{K} L_{kt}
\end{aligned}
$$

The first of these equations is the equilibrium in the assets market. The second equation simply aggregate labor within sectors by summing up hours worked in firms belonging to a given sector. The last equation is Walras law for the outpur produced by firm $j$ in sector $k$. I use from here on notation $j'$ and $k'$ for firms/sectors from the standing point of a given firm $j$ in sector $k$. The fourth and fifth equations are definitions of the aggregate wage (which is a weighted average of sectoral wages) and aggregate labor (which linearly sums up hours worked in all sectors).

## 3 Steady state

All shocks are set to zero, so $A = A_k = 1$. The weight $\{g_k\}$ of sectors in the disutility of labor for households and the measure $\{n_k\}$ of sectors in the total of firms will be set in steady state wages faced by all firms the same, production of all firms the same, inputs used by all firms are the same, and prices charged by all firms the same:

$$W_k = W,\ \ Y_{kj} = Y,\ \ L_{kj} = L,\ \ Z_{kj} = Z,\ \ P_{kj} = P \text{ for all } k, j.$$

Since $P_{kj} = P$, then

$$P^c = P^k = P_k = P,$$

i.e., consumption aggregate price $P^c$, the aggregate prices relevant for all sectors $\{P^k\}$ and sectoral prices $\{P_k\}$ are all identical to $P$.

Similarly,

$$C_k = \omega_{ck} C, \;\; C_{kj} = \frac{\omega_{ck}}{n_k} C,$$

$$Z_{k'j'}(k) = \omega_{k'k} Z, \;\; Z_{k'j'}(k,j) = \frac{\omega_{k'k}}{n_k} Z.$$

In words, sectoral consumption $C_k$ is a fraction of total consumption $C$, and the demand of firm $k, j$ for consumption is equally shared by all firms within sector $k$. Similarly, the sectoral demand of firm $k', j'$ from goods in sector $k$ (denoted by $Z_{k'j'}(k)$) is a fraction of the composite intermediate input firm $k', j'$ uses (denoted by $Z_{k'j'} = Z$). The total demand of firm $k, j$ from firm $k', j'$ (denoted by $Z_{k'j'}(k,j)$) is the same for all firms in sector $k$.

Note that in steady state the following holds:

$$C_k = \int_{\Im_k} C_{kj}, \;\; C = \sum_{k=1}^{K} C_k,$$

$$Z_{kj} = \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{kj}(k', j')\, dj' = Z$$

Therefore, in stetady state it holds

$$Y_t = \sum_{k=1}^{K} \int_{\Im_k} Y_{kj} dj.$$

Next, labor supply in steady state is given by

$$\frac{W}{P} = g_k L_k^{\varphi} C^{\sigma}$$

so $W_k = W$ if $g_k = n_k^{-\varphi}$ since $L_k = n_k L$ where

$$\frac{W}{P} = L^{\varphi} C^{\sigma} \tag{1}$$

From Walras's law at the firm level,

$$\begin{aligned} Y_{kj} &= C_{kj} + \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{k'j'}(k,j)\, dj' \\ Y &= \frac{\omega_{ck}}{n_k} C + \frac{1}{n_k} \left( \sum_{k'=1}^{K} n_{k'} \omega_{k'k} \right) Z \text{ for all } k \end{aligned}$$

therefore

$$n_k = \omega_{ck} \frac{C}{Y} + \left( \sum_{k'=1}^{K} n_{k'} \omega_{k'k} \right) \frac{Z}{Y} \text{ for all } k$$

If we impose that $\sum_{k=1}^{K} n_k = 1$ and $\sum_{k=1}^{K} \omega_{k'k} = 1$, summing up this expression across sectors we get

$$Y = C + Z \tag{2}$$

so the assumptions made in steady state are consistent with $Y_{kj} = Y$.

The sectoral de composition of output is as follows

$$\begin{aligned} Y_{kj} &= C_{kj} + \sum_{k'=1}^{K} \int_{\Im_{k'}} Z_{k'j'}(k,j)\, dj' \\ Y_{kj} &= n_k^{-1} \left[ \omega_{ck} C + \sum_{k'=1}^{K} \int_{\Im_{k'}} \omega_{k'k} Z_{k'j'} dj' \right] \\ Y_k &= \int_{\Im_k} Y_{kj} dj = \omega_{ck} C + \sum_{k'=1}^{K} \int_{\Im_{k'}} \omega_{k'k} Z_{k'j'} dj' \end{aligned}$$

The efficiency condition for production is

$$\text{(efficiency condition)} \;\; (1-\delta) P Z_{kj} = \delta W L_{kj} \tag{3}$$

Combining this expression with the production function yields

$$Z_{k'j'} = \left[ \frac{\delta W}{(1-\delta) P} \right]^{1-\delta} Y_{k'j'} = \chi Y_{k'j'}$$

so

$$Y_k = \omega_{ck} C + \chi \sum_{k'=1}^{K} \omega_{k'k} Y_{k'} \tag{4}$$

In vectoral representation

$$\mathbb{Y}_k = (\mathbb{I} - \chi \Omega)^{-1} \Omega_c C$$

where $\Omega$ is the matrix of all $\{\omega_{k'k}\}$ weights and $\Omega_c$ is the vector of all $\{\omega_{ck}\}$ weights.

Other equations are

$$\text{(production function) } Y = L^{1-\delta} Z^{\delta} \tag{5}$$

$$\text{(budget constraint) } C = \left(\frac{W}{P}\right) L + \frac{\Pi}{P} \tag{6}$$

$$\text{(profits def) } \frac{\Pi}{P} = Y - \left(\frac{W}{P}\right) L - Z \tag{7}$$

$$\text{(optimal pricing) } P = \frac{\theta}{\theta - 1} \xi W^{1-\delta} P^{\delta} \tag{8}$$

where $\xi = \frac{1}{1-\delta}\left(\frac{\delta}{1-\delta}\right)^{-\delta}$. This latter expression implies that assumptions made in this sector are consistent with $P_{kj} = P$.

**Solution.** From equation (8) we obtain

$$\frac{W}{P} = \left(\frac{\theta - 1}{\theta \xi}\right)^{\frac{1}{1-\delta}} . \tag{*}$$

Using equation (3), eqs (5) and (7) respectively become

$$\begin{aligned} Y &= \left(\frac{\delta}{1-\delta}\right)^{\delta} \left(\frac{W}{P}\right)^{\delta}, \\ \frac{\Pi}{P} &= Y - \frac{1}{1-\delta}\left(\frac{W}{P}\right) L \end{aligned}$$

Combining (*) and these two equations, we get

$$\frac{\Pi}{P} = \frac{1}{\theta} Y.$$

Plugging this expression in (6) and using (2), we get

$$\begin{aligned} C &= \left(1-\delta\left(\frac{\theta-1}{\theta}\right)\right)Y=(1-\psi)\,Y, \qquad (**) \\ Z &= \psi Y. \end{aligned}$$

It may be verified that $\psi=\chi^{-1}$ in equation (4) after imposing that $Y=\sum_{k=1}^{K}Y_k$.

With these expressions, we solve for $n_k$:

$$n_k=\omega_{ck}\,(1-\psi)+\left(\sum_{k'=1}^{K}n_{k'}\omega_{k'k}\right)\psi \text{ for all } k.$$

From $Z=\psi Y$ and eq. (3) we get

$$L=\left[\delta\left(\frac{\theta-1}{\theta}\right)\right]^{-\frac{\delta}{1-\delta}}Y \qquad (***)$$

and from plugging (*), (**), and (***) into equation (1), we get $Y$.

Note that $Y$ depends on $\delta$, the share of intermediate inputs in production, but it does not depend on the exact I/O structure of the economy summarized by $\Omega_c$ and $\Omega$. However, as we will see next, the I/O structure of the economy will be important for the propagation of shocks.

# 4 Log-linear system

**Aggregation:**

$$\text{Prices: } p_t^c=\sum_{k=1}^{K}\omega_{ck}p_{kt},\;\; p_t^k=\sum_{k'=1}^{K}\omega_{k'k}p_{k't},\;\; p_{kt}=\frac{1}{n_k}\int_{\Im_k}p_{kj}dj.$$

$$\text{Consumption: } c_t=\sum_{k=1}^{K}\omega_{ck}c_{kt},\;\; c_{kt}=\frac{1}{n_k}\int_{\Im_k}c_{kj}dj.$$

$$\text{Intermediate inputs} \quad : \quad z_{kjt} = \sum_{r=1}^{K} \omega_{kr} z_{kjt}(r), \;\; z_{kjt}(r) = \frac{1}{n_r} \int_{\Im_r} z_{kj}(r, j') \, dj',$$

$$z_t \;\; = \;\; \sum_{r=1}^{K} n_k z_{kt}, \;\; z_{kt} = \frac{1}{n_k} \int_{\Im_k} z_{kjt} dj$$

$$\text{Aggregate demand: } y_t = (1-\psi) c_t + \psi z_t \tag{9}$$

**Demands:**

$$c_{kt} - c_t = \eta (p_t^c - p_{kt}), \;\; c_{kjt} - c_{kt} = \theta (p_{kt} - p_{kjt}) \tag{10}$$

$$z_{kjt}(r) - z_{kjt} = \eta \left(p_t^k - p_{rt}\right), \;\; z_{kjt}(r, j') - z_{kjt}(r) = \theta \left(p_{rt} - p_{rj't}\right) \tag{11}$$

$$\text{Walras' law: } y_{kjt} = (1-\psi) c_{kjt} + \psi \sum_{k'=1}^{K} \int_{I_{k'}} z_{k'j't}(k, j) \, dj' \tag{12}$$

**IS and labor supply:**

$$c_t = E_t[c_{t+1}] - \sigma^{-1} \left(i_t - E_t \pi_{t+1}^c\right)$$

$$w_{kt} - p_t^c = \varphi l_{kjt} + \sigma c_t$$

**Taylor rule:**

$$i_t = \phi_\pi \pi_t^c + \phi_c c_t + \mu_t$$

which includes the autoregressive monetary shock

$$\mu_t = \rho_\mu \mu_{t-1} + \varepsilon_{\mu t}$$

**Firms:**

$$\text{Production fn: } y_{kjt} = a_t + a_{kt} + (1-\delta) l_{kjt} + \delta z_{kjt}$$

where

$$a_t \;\; = \;\; \rho_a a_{t-1} + \varepsilon_{at},$$

$$a_{kt} \;\; = \;\; \rho_k a_{kt-1} + \varepsilon_{kt.}$$

$$\text{efficiency condition: } w_{kt} - p_t^k = z_{kjt} - l_{kjt}$$

$$\text{marginal cost: } mc_{kt} = (1-\delta)\, w_{kt} + \delta p_t^k - a_t - a_{kt}$$

$$\text{opt price: } p_{kt}^* = (1-\alpha_k \beta)\, mc_{kt} + \alpha_k \beta \mathbb{E}_t \left[p_{kt+1}^*\right]$$

$$\text{sectoral price: } p_{kt} = (1-\alpha_k)\, p_{kt}^* + \alpha_k p_{kt-1}$$

**System in reduced form:** Combining the log-linear expressions for Walras' law (equation 12) with demands for consumption and intermediate inputs (eqs. 10 and 11) and aggregate demand (eq. 9),

$$y_{kjt} = y_t + \eta \left[(1-\psi)\, p_t^c + \psi \sum_{k'=1}^{K} n_{k'} p_t^{k'}\right] + (\theta - \eta)\, p_{kt} - \theta p_{kjt}^*$$

Combining the efficiency condition, labor supply I get an expression for $z_{kjt}$ which I replace into the production function:

$$y_{kjt} = a_t + a_{kt} + (1+\delta\varphi)\, l_{kjt} + \delta\sigma c_t + \delta \left(p_t^c - p_t^k\right)$$

which I use to replace $l_{kjt}$ from the labor supply equation to get

$$w_{kt} = \frac{\varphi}{1+\delta\varphi}\left[y_{kjt} - a_t - a_{kt} - \delta\sigma c_t - \delta\left(p_t^c - p_t^k\right)\right]$$

Therefore, the reduced form system is

$$p_{kt}^* = (1-\alpha_k\beta)\, mc_{kt} + \alpha_k\beta\mathbb{E}_t\left[p_{kt+1}^*\right]$$

$$mc_{kt} = \frac{(1-\delta)\,\varphi}{1+\delta\varphi}\left[y_{kjt} - \delta p_t^c - \delta\sigma c_t\right] - \frac{1+\varphi}{1+\delta\varphi}\left[a_t + a_{kt} - \delta p_t^k\right]$$

$$y_{kjt} = y_t + \eta \left[(1-\psi)\, p_t^c + \psi \sum_{k'=1}^{K} n_{k'} p_t^{k'}\right] + (\theta - \eta)\, p_{kt} - \theta p_{kjt}^*$$

$$p_{kt} = (1-\alpha_k)\, p_{kt}^* + \alpha_k p_{kt-1}$$

plus the aggregation equations

$$\begin{aligned}
p_t^c &= \sum_{k=1}^{K} \omega_{ck} p_{kt} \\
p_t^k &= \sum_{k'=1}^{K} \omega_{k'k} p_{k't}
\end{aligned}$$

All these equations may be combined in one for each sector (so $K$ equations total) with $K+2$ unknowns: $\{p_{kt}\}$ , $c_t$ and $y_t$. To get rid of $y_t$ I use

$$\begin{aligned}
y_t &= (1-\psi) c_t + \psi z_t, \\
z_t &= (1+\varphi) l_t + \sigma c_t + \left( p_t^c - \sum_{k=1}^{K} n_k p_t^k \right), \\
y_t &= a_t + \sum_{k=1}^{K} n_k a_{kt} + (1-\delta) l_t + \delta z_t.
\end{aligned}$$

so I just need to add

$$\text{IS+Taylor rule: } c_t = E_t [c_{t+1}] - \sigma^{-1} \left( \phi_\pi \pi_t^c + \phi_c c_t + \mu_t - E_t p_{t+1}^c + p_t^c \right)$$

to solve for sectoral prices $\{p_{kt}\}$ and value-added production $c_t$.

# 5 Benchmark models

To understand the effect of the I/O structure in this economy, I produce two simplified versions of this model: In the first model (benchmark 1), sectors have the same relative size than in the model above, but there are no intermediate inputs. In the second model (benchmark 2), sectors have the same relative size than in the model above, there are intermediate inputs, but there are no differences across sectors regarding how much they buy from other sectors. In the following I label as "baseline model" the setup that we have solved so far.

Before going to these benchmark models, I am depicting a repice to solve our baseline model. The solution of the log-linear system of the two benchmark models follow the same recipe. The idea is to transparently show the variations implied by the two benchmarks.

I start by writing down two key equations for sectoral prices:

$$\begin{aligned} p_{kt} &= (1-\alpha_k)\, p^*_{kt} + \alpha_k p_{kt-1}, \\ p^*_{kt} &= (1-\alpha_k\beta)\, mc_{kt} + \alpha_k\beta \mathbb{E}_t \left[p^*_{kt+1}\right] \end{aligned}$$

where

$$mc_{kt} = (1-\delta)\, w_{kt} + \delta p^k_t - a_t - a_{kt}$$

The expression for $mc_{kt}$ can be put in terms of $y_t$ and prices by solving for $w_{kt}$ using

$$\begin{aligned} w_{kt} - p^c_t &= \varphi l_{kjt} + \sigma c_t, \\ y_{kjt} &= a_t + a_{kt} + l_{kjt} + \delta \left(w_{kt} - p^k_t\right), \\ y_{kjt} &= y_t + \eta \left[(1-\psi)\, p^c_t + \psi \sum_{k'=1}^{K} n_{k'} p^{k'}_t\right] + (\theta - \eta)\, p_{kt} - \theta p^*_{kjt} \end{aligned}$$

where the second equation is obtained combining the production function and the efficiency condition.

The model is closed by using the aggregate expressions

$$y_t = (1-\psi)\, c_t + \psi z_t,$$

where

$$z_t = \frac{1+\varphi}{1+\delta\varphi}\left[y_t - a_t - \sum_{k'=1}^{K} n_{k'} a_{kt}\right] + \frac{(1-\delta)\,\sigma}{1+\delta\varphi} c_t + \frac{(1-\delta)}{1+\delta\varphi}\left(p^c_t - \sum_{k'=1}^{K} n_{k'} p^{k\prime}_t\right),$$

and the equations

$$c_t = E_t\left[c_{t+1}\right] - \sigma^{-1}\left[\phi_c c_t - \left(E_t p^c_{t+1} - (1+\phi_\pi)\, p^c_t + p^c_{t-1}\right) + \mu_t\right]$$

$$
\begin{aligned}
p_t^c &= \sum_{k=1}^{K} \omega_{ck} p_{kt}, \\
p_t^k &= \sum_{k'=1}^{K} \omega_{k'k} p_{k't}
\end{aligned}
$$

and the constants

$$
n_k = (1-\psi)\,\omega_{ck} + \psi \left( \sum_{k'=1}^{K} n_{k'} \omega_{k'k} \right).
$$

The solution yields sectoral prices $\{p_k\}$ and value added output $c_t$.

## 5.1 Benchmark 1: No I/O linkages

This model is obtained by setting $\delta = 0$, so $Z_{kjt} = 0$ and $Y_{kjt} = C_{kjt}$ for all $k, j, t$. Thus the log-linear system to solve this model is

$$
\begin{aligned}
p_{kt} &= (1-\alpha_k)\, p_{kt}^* + \alpha_k p_{kt-1}, \\
p_{kt}^* &= (1-\alpha_k \beta)\, mc_{kt} + \alpha_k \beta \mathbb{E}_t \left[ p_{kt+1}^* \right]
\end{aligned}
$$

where

$$
mc_{kt} = w_{kt} - a_t - a_{kt}
$$

The first difference wrt the baseline model is that marginal costs here vary only with wages and productivity. In the baseline model,the I/O structure creates a weighted average between wages and aggregate prices $p_t^k$. Following the same recipe above, we can get an expression for $w_{kt}$ in terms of $y_t$ and prices by using

$$
\begin{aligned}
w_{kt} - p_t^c &= \varphi l_{kjt} + \sigma c_t, \\
y_{kjt} &= a_t + a_{kt} + l_{kjt}, \\
y_{kjt} &= y_t + \eta p_t^c + (\theta - \eta)\, p_{kt} - \theta p_{kjt}^*
\end{aligned}
$$

Without I/O, the production function (second equation) does not vary with wages and prices, as in the baseline model. Therefore, the I/O structure creates a loop between wages and production that is not present here. Besides, looking at the third equation, the I/O structure creates a weighted average between all aggregate prices $p_t^c$ and $\left\{p_t^k\right\}$ as determinant of a firm's total demand.

The model is closed by using the aggregate expressions

$$y_t = c_t$$

Here it is clear that the I/O structure creates a loop between $y_t$ and $c_t$ that is also affected by $a_t$, $\{a_{kt}\}$ and aggregate prices $p_t^c$ and $\left\{p_t^k\right\}$.

Finally, the model is closed by the same equations as above

$$c_t = E_t\left[c_{t+1}\right] - \sigma^{-1}\left[\phi_c c_t - \left(E_t p_{t+1}^c - (1+\phi_\pi)\, p_t^c + p_{t-1}^c\right) + \mu_t\right]$$

$$p_t^c = \sum_{k=1}^{K} \omega_{ck} p_{kt}.$$

with the only modification in the constants

$$n_k = \omega_{ck}.$$

Summing up, no I/O structure implies that variations in sectoral prices do not affect marginal costs, there is no feedback between firms' production and wages they pay, and there is no heterogeneity in firms' demand as a response to the variation of a given sectoral price. At the aggregate level, no I/O structure implies that

### 5.2 Benchmark 2: Equal I/O structure across sectors

This model is obtained by setting $\omega_{kr} = \omega_{cr}$ for all $r$, so $n_k = \omega_{ck}$ and $p_t^k = p_t^c = p_t$ for all $k$.

Following the same recipe as above, the solution of the log-linear version of the model is obtained from the equations:

$$\begin{aligned} p_{kt} &= (1-\alpha_k)\, p_{kt}^* + \alpha_k p_{kt-1}, \\ p_{kt}^* &= (1-\alpha_k\beta)\, mc_{kt} + \alpha_k\beta\mathbb{E}_t\left[p_{kt+1}^*\right] \end{aligned}$$

where

$$mc_{kt} = (1-\delta)\, w_{kt} + \delta p_t - a_t - a_{kt}$$

In the baseline model, it is the price relevant for sector $k$ what enters in marginal costs, so only vartiations in a given sectoral price affects the marginal costs of firms in sector $k$ according to its weights in the I/O matrix for sector $k$. In contrast, in the benchmark 2 model, variations in a given sectoral price equally affect the marginal cost of all firms. Besides, bigger sectors have bigger impact on marginal costs.

The expression for $mc_{kt}$ can be put in terms of $y_t$ and prices by solving for $w_{kt}$ using

$$\begin{aligned} w_{kt} - p_t &= \varphi l_{kjt} + \sigma c_t, \\ y_{kjt} &= a_t + a_{kt} + l_{kjt} + \delta \left( w_{kt} - p_t \right), \\ y_{kjt} &= y_t + \eta p_t + \left( \theta - \eta \right) p_{kt} - \theta p^*_{kjt} \end{aligned}$$

Looking at the reduced form of the production function (second equation above), in the benchmark 2 model it is the unique price index what affects production. In the baseline model, the price aggregator that enters in this expression is $p_t^k$, so heterogeneous I/O weights imply that variations in a given sectoral price have heterogeneous effect in variations in production across sectors.

Similarly, looking at the demand of firm $kj$ (third equation above), in the benchmark 2 model $p_t^k$ plays no role, so variations in prices of a given sector have the same effect on demand for all sectors. This is again not true in the baseline model where variations in sectoral prices have heterogeneous effects on firms' demands according to their I/O weights.

Following the recipe of solution, the model is closed by a relationship betwee $y_t$ and $c_t$:

$$y_t = \left(1 - \psi\right) c_t + \psi z_t,$$

where

$$z_t = \frac{1 + \varphi}{1 + \delta \varphi} \left[ y_t - a_t - \sum_{k'=1}^{K} n_{k'} a_{kt} \right] + \frac{\left(1 - \delta\right) \sigma}{1 + \delta \varphi} c_t.$$

In the baseline model, prices enter into the the expression for $z_t$ because of heterogeneity in I/O weights. This effect is not present here.

Then, following the recipe, we need a relationship between $c_t$ and sectoral prices, which is given by

$$c_t = E_t\left[c_{t+1}\right] - \sigma^{-1}\left[\phi_c c_t - \left(E_t p_{t+1} - \left(1+\phi_\pi\right) p_t + p_{t-1}\right) + \mu_t\right]$$

$$p_t = \sum_{k=1}^{K} \omega_{ck} p_{kt},$$

which remain unchanged when comparing the baseline model and the benchmark 2 model.

Summing up, heterogeneity in the I/O structure of sectors enters into heterogeneity in variations of marginal costs, production and demand for a given firm, and as a wedge in the aggregate relationship between production $y_t$ and value-added output $c_t$.

# 6 Numerical exercises

This section conducts a number of IRF exercises with out baseline model and benchmarks 1 and 2. The goal is to shed light on the four issues this paper studies regarding the interaction between sectoral heterogeneity in price stickiness and sectoral heterogeneity in I/O linkages regarding four issues:

1. The strength and persistance of real effects of monetary shocks.
2. The aggregate propagation of idiosyncratic shocks.
3. The pass-through in aggregate and sectoral inflation of an oil price shock.
4. The implications of a monetary policy rule that ignores I/O linkages and interpret any aggregate fluctuation as result of aggregate shocks.

TO BE CONTINUED....