Sequential Auctions, Price Trends, and Risk Preferences

Audrey Hu   Liang Zou

University of Amsterdam/ Tinbergen Institute

15 February, 2015

Introduction

- Sequential auctions are market institutions where multiple units of (nearly) identical goods are sold one by one using the same auction rule

Introduction

- Sequential auctions are market institutions where multiple units of (nearly) identical goods are sold one by one using the same auction rule
- In real life, sequential auctions frequently take place to sell

Introduction

- Sequential auctions are market institutions where multiple units of (nearly) identical goods are sold one by one using the same auction rule
- In real life, sequential auctions frequently take place to sell
  - Flowers (van den Berg et al., 2001), tuna fish, cattle, livestock (Buccola, 1982), fine wine (Ashenfelter, 1989; McAfee and Vincent, 1993)

Introduction

- Sequential auctions are market institutions where multiple units of (nearly) identical goods are sold one by one using the same auction rule
- In real life, sequential auctions frequently take place to sell
  - Flowers (van den Berg et al., 2001), tuna fish, cattle, livestock (Buccola, 1982), fine wine (Ashenfelter, 1989; McAfee and Vincent, 1993)
  - Stamps (Thiel and Petry, 1995), rare books, art (Pesando and Shum, 1996; Beggs and Graddy, 1997), antiques (Ginsburgh and van Ours, 2007), jewelry (Chanel et al.,1996)

Introduction

- Sequential auctions are market institutions where multiple units of (nearly) identical goods are sold one by one using the same auction rule
- In real life, sequential auctions frequently take place to sell
  - Flowers (van den Berg et al., 2001), tuna fish, cattle, livestock (Buccola, 1982), fine wine (Ashenfelter, 1989; McAfee and Vincent, 1993)
  - Stamps (Thiel and Petry, 1995), rare books, art (Pesando and Shum, 1996; Beggs and Graddy, 1997), antiques (Ginsburgh and van Ours, 2007), jewelry (Chanel et al.,1996)
  - Licenses, commercial real estate (Lusht, 1994), timber, oil, gas and mineral rights

Introduction

- Sequential auctions are market institutions where multiple units of (nearly) identical goods are sold one by one using the same auction rule
- In real life, sequential auctions frequently take place to sell
    - Flowers (van den Berg et al., 2001), tuna fish, cattle, livestock (Buccola, 1982), fine wine (Ashenfelter, 1989; McAfee and Vincent, 1993)
    - Stamps (Thiel and Petry, 1995), rare books, art (Pesando and Shum, 1996; Beggs and Graddy, 1997), antiques (Ginsburgh and van Ours, 2007), jewelry (Chanel et al.,1996)
    - Licenses, commercial real estate (Lusht, 1994), timber, oil, gas and mineral rights
    - Blocks of shares of IPO firms

Introduction

- Sequential auctions are market institutions where multiple units of (nearly) identical goods are sold one by one using the same auction rule
- In real life, sequential auctions frequently take place to sell
  - Flowers (van den Berg et al., 2001), tuna fish, cattle, livestock (Buccola, 1982), fine wine (Ashenfelter, 1989; McAfee and Vincent, 1993)
  - Stamps (Thiel and Petry, 1995), rare books, art (Pesando and Shum, 1996; Beggs and Graddy, 1997), antiques (Ginsburgh and van Ours, 2007), jewelry (Chanel et al.,1996)
  - Licenses, commercial real estate (Lusht, 1994), timber, oil, gas and mineral rights
  - Blocks of shares of IPO firms
  - Treasury bills and notes

Empirical Regularity

- In November 1981, Sotheby's (New York) sold seven leases on RCA-owned satellite-based telecommunications transponders.

Empirical Regularity

- In November 1981, Sotheby's (New York) sold seven leases on RCA-owned satellite-based telecommunications transponders.
  - The sequence of prices generated in the RCA transponder lease auction, from first round to seventh, was

$$\$14.4, \$14.1, \$13.7, \$13.5, \$12.5, \$10.7, \$11.2m.$$

Empirical Regularity

- In November 1981, Sotheby's (New York) sold seven leases on RCA-owned satellite-based telecommunications transponders.
    - The sequence of prices generated in the RCA transponder lease auction, from first round to seventh, was

$$\$14.4, \$14.1, \$13.7, \$13.5, \$12.5, \$10.7, \$11.2m.$$

- Ashenfelter (1989): declining price patterns in fine wine sequential auctions

Empirical Regularity

- In November 1981, Sotheby's (New York) sold seven leases on RCA-owned satellite-based telecommunications transponders.
  - The sequence of prices generated in the RCA transponder lease auction, from first round to seventh, was

$$\$14.4, \$14.1, \$13.7, \$13.5, \$12.5, \$10.7, \$11.2m.$$

- Ashenfelter (1989): declining price patterns in fine wine sequential auctions
- A large number of empirical work reported a similar declining price phenomenon thereafter

Declining Price Anomaly

- The declining price pattern clearly violates the law of one price

Declining Price Anomaly

- The declining price pattern clearly violates the law of one price
- Opposite to the predictions of the standard models:

Declining Price Anomaly

- The declining price pattern clearly violates the law of one price
- Opposite to the predictions of the standard models:
    - Risk neutral bidders with private values: price sequence should be a martingale (Milgrom and Weber, 1982)

Declining Price Anomaly

- The declining price pattern clearly violates the law of one price
- Opposite to the predictions of the standard models:
  - Risk neutral bidders with private values: price sequence should be a martingale (Milgrom and Weber, 1982)
  - Risk neutral bidders with affiliated values: price sequence should be upward drifting (Weber, 1983; Milgrom and Weber, 2000)

Declining Price Anomaly

- The declining price pattern clearly violates the law of one price
- Opposite to the predictions of the standard models:
  - Risk neutral bidders with private values: price sequence should be a martingale (Milgrom and Weber, 1982)
  - Risk neutral bidders with affiliated values: price sequence should be upward drifting (Weber, 1983; Milgrom and Weber, 2000)
- Question: Why?

Existing Explanations

- Ashenfelter's conjecture (1989): risk aversion leads to declining prices

Existing Explanations

- Ashenfelter's conjecture (1989): risk aversion leads to declining prices
- McAfee and Vincent (1993): risk aversion explains declining prices, but the logic rests on *nondecreasing absolute risk aversion (NDARA)*.

Existing Explanations

- Ashenfelter's conjecture (1989): risk aversion leads to declining prices
- McAfee and Vincent (1993): risk aversion explains declining prices, but the logic rests on *nondecreasing absolute risk aversion (NDARA)*.
- Mezzetti (2011): risk aversion explains declining prices if bidders care only about price risk

Existing Explanations

- Ashenfelter's conjecture (1989): risk aversion leads to declining prices
- McAfee and Vincent (1993): risk aversion explains declining prices, but the logic rests on *nondecreasing absolute risk aversion (NDARA)*.
- Mezzetti (2011): risk aversion explains declining prices if bidders care only about price risk
- Declining prices have to do with institutional details that are abstracted away in the standard model (20 plus theoretical papers)

Hu and Zou, 2014

- **Key Assumption**: marginal utilities of bidders must be log-submodular in income and type
- **New Insights:** *background risk* and *nonincreasing absolute risk aversion* are perfectly consistent with declining prices
- Equilibrium is characterized for general m-period sequential Dutch and Vickrey auctions
- Results are obtained in a much more general environment:
  - If bidders are risk neutral (averse, risk-loving), then the price trend is martingale (super-martingale, sub-martingale);
  - Both auctions are ex post efficient.

Background Risk

On December 12, 2012, Forbes published a commentary on the then upcoming UK 4G auctions:

"...put simply, the current mobile network operators cannot afford to lose the auction. Without 4G in their inventory they will be **left behind** *[emphasis original] as customers sign on to new contracts with the 'modern' 4G networks in 2013 and 2014."*

- When bidders' objective is to avoid "pain" rather than to seek "pleasure," they face *background (or status-quo) risk*.
- Bidders' willingness-to-pay can be directly related to the severity of their background risk *should they lose* in the auctions

Bidder Preferences

- m (risky) objects or assets are for sale to n ($> m$) bidders, each having a unit demand
- Each bidder has a private type t, ex ante distributed according to F with positive density f on $[0, H]$
- The preference of a typical bidder with type t is represented by

$$\begin{cases} w(x,t) & \text{if he wins the object and receives } x \\ u(x,t) & \text{if he loses and receives } x \end{cases}$$

- Examples: (i) bi-attribute utility, (ii) heterogeneous utility for income x, and (iii) Bernoulli utility for income $t + x$.
- Other non-expected utility interpretations are possible, e.g., reference dependence.

Assumptions

- A1. The partial derivatives $w_1(x,t) > 0$ and $w_2(x,t) > u_2(0,t)$ for all x and t such that $w(x,t) \geq u(0,t)$.
- A2. $w_1(x,t)$ and $w(x,t) - u(0,t)$ are log-submodular in (x,t) for all x and t such that $w(x,t) > u(0,t)$.
  - For all $x < x'$ and $t < t'$,

$$w_1(x',t')w_1(x,t) \leq w_1(x',t)w_1(x,t')$$

- No restriction is made on the signs of the partial derivatives $u_2(0,t)$ and $w_2(x,t)$.

Examples: Case 1

$u(0,t) \equiv U(0)$ and $w(x,t) = U(v(t)+x)$, with $v'(t) > 0$.

- $v(t) = t$: reduces to the private values model of McAfee and Vincent (1993).
- $v' > 0$, $w(x,t)$ is log-submodular iff U exhibits nondecreasing absolute risk aversion (NDARA), a condition required for the existence of a pure strategy symmetric equilibrium.

Examples: Case 2

$u(0, t) \equiv U(0)$ and $w(x, t) = U(v(t) + \varphi(x))$, with $v'(t) > 0$,and $\varphi'(x) > 0$.

- Bi-attribute utility: the object is of certain quality $v(t)$ that contributes to the utility.
- But the object may not have an equivalent monetary value (e.g., Case 2 of Maskin and Riley, 1984).
- For U risk neutral, this case reduces to Mezzetti (2011) for his private-values case.
- For U nonlinear, A2 continues to hold for the NDARA class of functions U.

Examples: Case 3

$u(0,t) \equiv 0$ and $w(x,t) = \int \max(v + x - B, 0) dQ(v|t)$, where a higher t shifts Q to the right in the sense of first-order stochastic dominance.

- This case captures the effect of limited liability, where B can be interpreted as the bidder's liability or face value of debt.
- Because w is now convex in x, we have $w_{11} > 0$ so the bidder's induced utility w is risk preferring.
- Suppose the density $Q_1(v|t)$ exists and is positive on the support of v. Then A2 holds if the reverse hazard rate $Q_1(v|t)/Q(v|t)$ is nondecreasing in t (e.g., Board, 2007)

Examples: Case 4

A bidder's income v has a distribution $K(v|t)$ if losing and $H(v|t)$ if winning, i.e.,

$$
\begin{aligned}
u(0,t) &= \int_{-\infty}^{\infty} U(v)dK(v|t) \\
w(x,t) &= \int_{-\infty}^{\infty} U(v+x)dH(v|t) \text{ with } H(v|t) < K(v|t)
\end{aligned}
$$

- Winning provides a more favorable income distribution $H(v|t)$, which dominates the status-quo income distribution $K(v|t)$ in the sense of FOSD.
- A bidder is exposed to *background risk* if $u(0,t)$ cannot be "normalized" as zero without losing generality.
- Bidders have exposures to both *ensuing risk*, since v remains uncertain to the winner, and *background risk*.

Sequential Dutch Auction

- At the start, a bidding strategy for a bidder with type t is a collection of m bid functions $b_1, ..., b_m$ where $b_k(t|p_1, ..., p_{k-1})$ denotes his bid in the kth auction given that he has lost the first $k-1$ auctions, conditional on observing the winning prices $p_1, ..., p_{k-1}$.
- By symmetry, w.l.o.g. we focus on analyzing the optimal strategies of bidder 1.
  - Let the random variable Y_k denote the kth highest type from among the $n-1$ bidders other than bidder 1, so that if bidder 1 with type t wins the kth auction, in an increasing equilibrium it must be the case that

$$Y_k < t < Y_{k-1}, \quad k = 1, ..., m$$

where $Y_0 = \infty$ (by default).

Bidder Payoffs

- In equilibrium, the conditional expected payoff for bidder 1, when he lost the previous $k-1$ auctions and observed $Y_{k-1} = y_{k-1}$, can be specified recursively for all k by

$$W_I^k(t|y_{k-1}) = \underbrace{w(-b_k(t), t) F_k(t|y_{k-1})}_{Y_k < t < y_{k-1}}$$
$$+ \underbrace{\int_t^{y_{k-1}} W_I^{k+1}(t|y) dF_k(y|y_{k-1})}_{t < Y_k < y_{k-1}.}$$

- The final period expected payoff is given by

$$W_I^m(t|y_{m-1}) =$$
$$w\left(-b_m(t), t\right) F_m(t|y_{m-1}) + u(0,t)(1 - F_m(t|y_{m-1}))$$

Equilibrium Bidding Strategies (Theorem 1)

Theorem

Suppose A1-A2 hold. Then there exists a unique increasing symmetric equilibrium of the Dutch sequential auctions $\{b_k : k = 1, ..., m\}$ *characterized by*

$$\begin{aligned} b_m'(t) &= (n-m)\frac{w(-b_m(t),t) - u(0,t)}{w_1(-b_m(t),t)}\frac{f(t)}{F(t)} \\ b_k'(t) &= (n-k)\frac{w(-b_k(t),t) - w(-b_{k+1}(t),t)}{w_1(-b_k(t),t)}\frac{f(t)}{F(t)}, \\ k &= 1, ..., m-1 \end{aligned}$$

with the initial conditions $b_k(0) = b_0$ *that solves* $w(-b_0, 0) = u(0,0)$.

Sequential Vickrey Auction Equilibrium (Theorem 2)

Theorem

Suppose A1-A2 hold, and that winning bids are announced. Then there exists a unique increasing symmetric equilibrium of the Vickrey sequential auctions $\{a_k : k = 1, ..., m\}$ *satisfying*

$$\begin{aligned} w(-a_m(t), t) &= u(0, t) \\ w(-a_k(t), t) &= \int_0^t w(-a_{k+1}(y), t)\, dF_{k+1}(y|t) \end{aligned}$$

Predictions under CARA

- Sequential Dutch auction:

$$
\begin{aligned}
b_m(t) &= \frac{1}{\lambda} \ln \int_0^t \exp(\lambda x) d\left(\frac{F(x)^{n-m}}{F(t)^{n-m}}\right) \\
b_k(t) &= \frac{1}{\lambda} \ln \int_0^t \exp(\lambda b_{k+1}(x)) d\left(\frac{F(x)^{n-k}}{F(t)^{n-k}}\right), \\
k &= 1, ..., m-1
\end{aligned}
$$

- Sequential Vickrey auction:

$$
\begin{aligned}
a_m(t) &= t \\
a_k(t) &= \frac{1}{\lambda} \ln \int_0^t \exp(\lambda a_{k+1}(x)) d\left(\frac{F(x)^{n-k}}{F(t)^{n-k}}\right), \\
k &= 1, ..., m-1
\end{aligned}
$$

Price Trends (Theorem 3)

Theorem

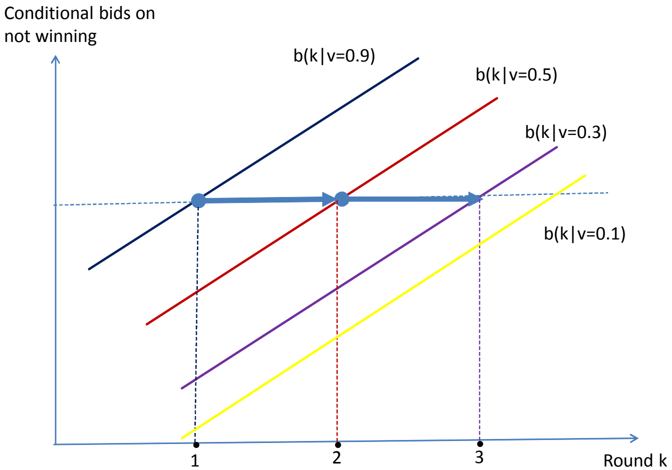
Suppose A1-A2 hold. Let $p_1, ..., p_m$ be the prices the objects are sold in periods $1, ..., m$ of the Dutch or Vickrey auctions, respectively. Then, for all $k = 1, ..., m-1$,

(i) if $w_{11} < 0$, then $E(\tilde{p}_{k+1}|p_k) < p_k$;

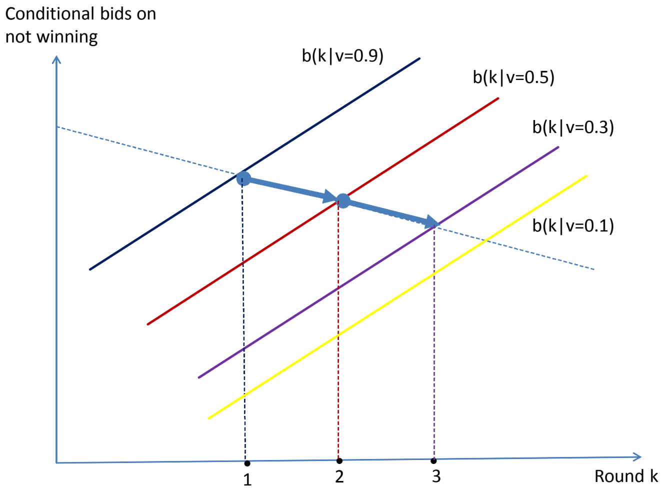
(ii) if $w_{11} = 0$, then $E(\tilde{p}_{k+1}|p_k) = p_k$;

(iii) if $w_{11} > 0$, then $E(\tilde{p}_{k+1}|p_k) > p_k$.

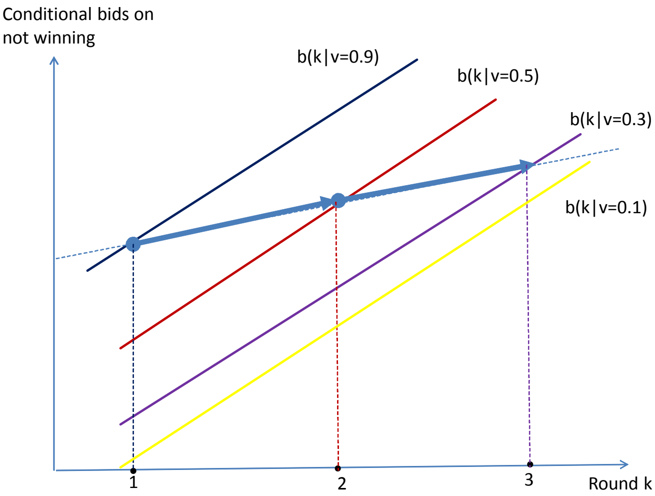
Martingale Price Trend under Risk Neutrality



Super-martingale Price Trend under Risk Aversion



Sub-martingale Price Trend under Risk Loving



Results on Revenue Comparison and Pareto Efficiency

- Sequential Dutch raises more (less) expected revenue than the Vickrey when bidders are risk averse (loving)

$$b_k(t) \geq (\leq) \int_0^t a_k(y) dF_k(y|t)$$

- Bidders are indifferent between the sequential Dutch and Vickrey auctions (extension of Matthew's payoff equivalence theorem)
- If both the seller and bidders are risk averse, then sequential Dutch Pareto dominates Vickrey

$$V(b_k(t)) \geq \int_0^t V(a_k(y)) dF_k(y|t)$$

implies

$$E\left[V(b_k(t))|y_k = t\right] \geq E\left[\int_0^t V(a_k(y)) dF_k(y|t)|y_k = t\right]$$

Conclusion

- Although much has been done in auctions for single unit object, in practice, auctions are rarely conducted that way.
- Typically, multiple objects are sold sequentially.
- In light of the preponderant evidence that people are not risk neutral, we believe much research is yet to be done.
- This paper provides only a first step. The second step can be a generalization toward interdependent valuations and affiliated signals.
- Given the present paper, we believe that the key still remains to be the assumption of log-submodularity, extended to N dimensional private signals.