On Cost Overruns in Procurement with Budget Constrained Contractors*

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Abstract

We consider auctions/tenders for the procurement of goods and services in settings where potential contractors have limited funds and face ex-post risks, e.g. cost overruns, which cannot be contracted away or insured at the time of tender. We examine a family of payment schemes in which a fraction of the award is paid immediately after the tender with the rest paid only after successful completion of the project. We identify the trade-offs the procurement agency faces when choosing among such payment schemes, and show that it is optimal to pay a positive fraction of the award at the allocation time. We also examine the practice of requiring surety bonds, which is common in construction industry in the US. We show that in the setting where the contractors may not have enough funds to finance the cost overruns, such practice may be detrimental to the expected cost of the project.

1 Introduction

The sheer volume of goods and services procured by the public and private sector is enormous. Government procurement represents 17.4% of GDP on average for OECD countries.[1] The use of competitive bidding, specifically auctions, is encouraged in such procurement whenever possible.[2] Large corporations procure most of their supplies via auctions. Households solicit (several) quotes from the contractors when building

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[1] From "Government at a Glance 2009," OECD, http://dx.doi.org/10.1787/724227300453.

[2] See, i.e. the USA Federal Acquisition Regulation https://www.acquisition.gov/far/ ADD EU REF. Often procurement auctions are called tenders.

or renovating houses, fixing cars, or relocating. Schools and ceremony organizers procure lunches and catering services in a similar way.

Procurement auction is often conducted before the procured item is produced. The bids at the auction are then necessarily based on the estimates of the costs. With a complex production process, the contractors are unable to foresee or describe all of the costs involved, which leads to significant cost overruns.[3] Importantly, many contractors are small businesses that cannot finance substantial cost overruns. The contractors who find their projects either impossible or unprofitable to complete given the tender price may default leaving behind an unfinished project.[4] The procurer suffers from such defaults as she is left to finish the project at an extra cost, such as the cost of adapting the technology, the cost of running a new tender, time delays, litigation costs, etc. Hence, the procurer who minimizes the expected cost necessarily cares not only about the auction price but also about the likelihood and the consequences of default by the auction winner.

We analyse procurement auctions with ex post risks (cost overruns) and budget constrained contractors. Both of these features impact on the expected cost of the procurer. It is important to consider them simultaneously, since addressing each in isolation leads to opposite recommendations. Advance payments ease budget constraints, but reduce incentives to cover cost overruns. In contrast, paying the entire award after the project completion and requiring a performance or surety bond posted immediately after the auction provides incentives to cover cost overruns, but exacerbates budget constraints.

In our model, the contractors have private information both about the cost of the project and about their own budgets. The total cost of the project is subject to

[3] Ashley & Workman (1986) in a survey of contractors and buyers in USA building industry report that project engineering must be 40-60% complete to establish a reasonable estimate for the cost. Flyvbjerg, Holm & Buhl (2002) in a study of public infrastructure procurement auctions report that in 90% of the projects the cost exceeds the tender price, with the average cost overrun of 28%.

The "Big Dig" – a mega transport infrastructure project in Boston, including building a bridge and a network of highway tunnels under the central part of the city, has been constructed at the cost of 8.6 billion dollars over the original estimate. Opera House in Sydney took extra ten years to complete, went through numerous design revisions with an intent to lower the cost, and yet exceeded the original budget by more than 14 times.

Cost overruns are also prevalent in defence procurement. Scherer & Peck (1962) estimate that development costs of U.S. defence programs exceed the original predictions by 220 percent on average. According to a recent report from the US Congressional audit agency almost 70% of the 96 largest defence programs suffered from huge cost overruns with a combined total of nearly 300 billion dollars, see Drew (2009). Joint Strike Fighter, the most expensive defence project in history, has projected cost overruns of over 1 trillion dollars.

[4] More than 80,000 contractors filed for bankruptcy leaving behind unfinished private and public construction projects with liabilities exceeding $21 billion in the United States during 1990-1997 (Dun & Bradstreet Business Failure Record REF**?**). The failure rate in the US construction industry also steadily increases, see Kangari (1988). More recently, Ganaway (2006) estimates that only 43% of U.S. construction firms remain in business for longer than 4 years.

the shock which is realized only after substantial (initial) investment has been made. Whether the shock hits or not is also private information of the winning contractor, the event is not verifiable and cannot be contracted upon.[5] The contractors are protected by limited liability and may wish to declare bankruptcy in case of the cost overrun, in which case the procurer would have to complete the project on their own at an extra cost.

For analytical simplicity we suppose that the total award to be paid to the winning contractor is determined by a second lowest-bid auction.[6] The problem of the procurer is to decide what share of the award to pay in advance, at the time of the initial investment into the project, and what share to pay upon project completion. The shares are announced to the contractors before the auction. Thus the procurer's instrument encompasses both advance payments and surety bonds once we allow for the negative advance share.

We find that bidding strategies of the contractors and their default decisions are shaped by three kinds of constraints. First, the contractors have to have enough funds to initiate the project. As a result the bidders with insufficient funds bid higher so that, should they win, the advance share of the award covers their deficit. The decision whether to default is driven by the *willing* to finish and *able* to finish constraints. The contractors are unwilling to finish the project if the completion share of the award is smaller than the cost overrun. Respectively the contractors are unable to finish the project if they have insufficient funds to continue when the cost overrun occurs. The likelihood of default affects the bids as well because the completion share is retained by the procurer in case of default.

The procurer's expected costs come from two sources: she pays full award to the winner if the project is completed and pays him only the advance share if the winner is in default. In such case the procurer pays extra completion cost to the third party. We show that when the contractors are not budget constrained the overall expected cost is a sum of the expected second lowest cost, the expected value of the cost overrun and the expected extra cost of completion when the original contractor is in default. The minimization of the expected cost of the procurer amounts to the minimization of the probability of default. As the result, when the contractors are not budget constrained, it is optimal to pay a negative share of the award as an advance and return this payment to the winner together with the entire auction award upon successful completion. In practice this scheme is implemented with the use of the

[5]If the shocks were verifiable or contractible the procurer could absorb the risks of cost overruns by offering cost-plus contracts, see Bajari & Tadelis (2001). We focus on the so called fixed-price contracts. This form of the contract is most commonly used in the government defence procurement, see Fox (1974), McAfee & McMillan (1986), and in the public infrastructure procurement, see Tadelis & Bajari (2006).

[6]While this format is rarely used for procurement, it serves as a notationally simpler equivalent of open descending bid formats that are commonly used. Lowest-bid sealed-bid auctions with budget constrained bidders do not have readily available analytical solutions. We expect though that qualitatively similar conclusions hold for them as well.

surety bonds.

When the bidders are budget constrained, they bid less aggressively, which increases award levels and lead to more often defaults due to inability to finance cost overruns. As the result it may be optimal to pay a non-zero fraction of the award ex ante. The ex ante fraction increases if the deficits of the contractors increase. Advance payments are recommended in many standard building contracts. They are part of B141, which is the home owner-architect agreement designed by the American Institute of Architects. Home building contracts in Australia and Canada also allow for advance payments.[7,8] The common rational is that they partially insulate the contractor when the *buyer* goes bankrupt. However, the buyer can typically extend the mortgage when undertaking house renovations and is certainly required to provide financial guarantees when contracting to build the house. Similar advance payment clauses are used in the government procurement contracts. Our results show that advance payments are optimal when the contractors are budget constrained.

We assume that ex post risks cannot be contracted away either due to limited liability provisions or complexities of forecasting the risks and writing complete contracts. Of course, in practice some of the risks can be appropriately forecasted and resolved. For instance, cost-plus contracts are extensively used in the situations when the technology of the project is well established, e.g. construction of a building or a bridge according to a known prototype, and main risks come from fluctuating material prices. In these contracts, the buyer pays the material costs and so bears the burden of a possible cost overrun (see Bajari & Tadelis (2001)). We effectively assume that all cost components that are observable, become observable, or can be foreseen and contracted upon are appropriately managed and focus on the rest of the costs, which are privately known or observed by the contractors.

The rest of the paper is organized as follows. In Section 2 we describe in greater detail the related literature. The model is presented in Section 3. In Section 4 we analyze the case when the contractors do not face any budget constraints. In Section 5 we describe equilibrium strategies for budget constrained contractors. In Section 6 we characterize the optimal (the buyer's expected cost minimizing) split of the award into the advance and ex-post payments. Section 7 concludes.

2 Literature

The economic literature on procurement is quite large, books by Laffont & Tirole (1993) and McAfee & McMillan (1988) offer excellent overviews. Procurement problems combine elements of adverse selection stemming from contractors private information about the elements of goods or services to be procured and moral hazard

[7] See for example, www.fairtrading.nsw.gov.au/pdfs/About_us/Publications /Home_building_contract_over_5000.pdf

[8] Advance payment is not a progress payment. Progress payment is made when a specific stage of construction is completed, which is verifiable. In our model such stage is the entire porject.

stemming from imperfectly observed efforts of contractors. Adverse selection on the individual contractor level is addressed by the usual screening methods with the procurement agency offering a menu of contracts to a particular contractor. On the industry level it is addressed by relying on competitive bidding procedures. Moral hazard is addressed by selecting specific kinds of contracts.

In the simple incomplete information procurement setting without any ex post risk, the characterization of the optimal procurement procedure — the one that minimizes the costs to the procurement agency — is similar to the characterization of the auction maximizing the expected revenue to the seller and can be done along the lines of Myerson (1981) or Riley & Samuelson (1981) optimal auction (see also Manelli & Vincent (1995) for optimal procurement mechanism when goods vary in quality). Similarly, procurement auctions with budget constrained bidders can be addressed as common auctions by Che & Gale (1998), Che & Gale (2000), Che, Gale & Kim (2013), Pai & Vohra (2014). One should be careful in translating the results of the analysis of common auctions to procurement settings. For instance, the seller would prefer first-price sealed-bid auctions to open ascending price (English) or second-price sealed bid auctions when faced with budget constrained buyers as buyers are less likely to hit the constraints in the first-price auctions. In procurement tenders the opposite is true. The bidders are less likely to hit the constraints in open descending price tenders or second-lowest bid tenders than in the lowest bid ones, thus the lowest bid tenders are less preferred when contractors are budget constrained .

Laffont & Tirole (1986), Laffont & Tirole (1987) and McAfee & McMillan (1986) consider a model complementary to ours where the costs are subject to ex-post shocks but may be reduced by the winner's unobservable effort. The buyer therefore faces both adverse selection and moral hazard and auctions off a menu of contracts. McAfee & McMillan (1986) consider n risk averse contractors that bid for a linear incentive contract that factors in both the cost (assumed observable ex-post) and the winning bid and derive the optimal contract in this class. Laffont & Tirole (1986) with one agent and Laffont & Tirole (1987) with n risk neutral contractors derive the optimal selling procedure for an incentive contract. The optimal contract is shown to be linear in ex-post cost with the burden of the cost overrun shared between the winner and the buyer. The winner of the tender in Laffont & Tirole (1987) optimally exerts the same amount of effort as the sole agent in Laffont & Tirole (1986) (and so moral hazard is separated from adverse selection), but enjoys lower informational rent.

The contracts in these papers combine the elements of the fixed price and cost plus contracts. Such contracts are infeasible in our setting as both the original estimate and the final value of the cost remain the private information of the winner.[9] In addition in our model the contractors are protected by limited liability therefore cost sharing schemes even when feasible provide limited incentives.

Parlane (2003) considers a model with limited liability similar to ours, but in her model the ex post shock occurs before any investment takes place. She shows

[9] See also Tirole (1986) on why the cost may be unverifiable even after completion.

that among all efficient mechanisms in which only the winner gets paid, the lowest bid tender leads to the highest expected price, thus minimizing the chances of bankruptcy. Our model better captures the feature of the reality that contractors have to actually work on the project and invest in it substantially before finding out the overall costs, see Footnote ?.

In Tirole (1986) the (sole) contractor can invest unobservable effort into cost reduction. Later an unverifiable shock to the cost is realized and the buyer and the contractor negotiate the terms of trade. If the contractor's effort is not observed by the buyer, in a vast majority of renegotiation schemes, the contractor exerts less effort than in the first best case with complete contract. If the effort is observable and is jointly determined (the renegotiation rule is unconstrained) the equilibrium level of effort may be lower than under unobservability, higher than the first-best level, or intermediate between the two values.

Piccione & Tan (1996) consider a model in which ex-ante symmetric potential contractors invest in the cost reduction technology and then compete for the procurement contract. The cost can be further reduced by exerting effort with no exogenous shocks. If the buyer is able to commit to the procurement mechanism before the investment stage the first best solution can be implemented by either lowest or second lowest bid tender under quite general conditions. If the buyer chooses the mechanism after the initial investment stage the level of investment is suboptimal. Arozamena & Cantillon (2004) consider a similar model, however, the firms are ex-ante heterogeneous, investment is only made by one firm and the level of investment is observable which brings asymmetry to the competition. If investment affects which firm is the most efficient then the lowest bid tender will induce less investment than the second lowest bid tender. In all of these papers the contractors have sufficient budgets to cover all possible costs and it is *assumed* that the award is paid ex-post.

Zheng (2001) and further Calveras, Ganuza & Hauk (2004) and Burguet, Ganuza & Hauk (2012) study the settings where the bidders are symmetric with respect to both ex ante and ex post costs, but have privately known budgets and can borrow at exogenously given rate. With limited liability and with the budget being the only screening variable incentive compatibility implies that winning probabilities are monotonically decreasing with budgets, thus an auction allocates the contract to the least financially solvent contractor.[10] In our setting, naturally, privately known budgets affect the bidding strategies but those mainly depend on the privately known costs, see the equilibrium construction in Section ?.

Rhodes-Kropf & Viswanathan (2005) like this paper consider a setting where the bidders privately know their budgets and estimates of the values. The values are subject to the ex-post shock after the bidding is over. Rhodes-Kropf & Viswanathan (2005) study the first-price auction and allow financing of the bids via broad spectrum of instruments: borrowing at exogenous interest rate, issuing equity or debt, contingent securities. The main finding is that even though access to competitive

[10] Zheng (2001)'s model also have an increasing in budgets equilibrium when interest rate is high.

financial markets reduces the problem to a single-index auction, that auction does not lead to efficient allocation. We in contrast, take a stand that unlimited borrowing is not possible given the risky nature of the business the contractors are engaged and study a genuine two dimensional problem.

Waehrer (1995) and further Board (2007) consider an auction model (with unlimited budgets) where after the auction but before the settlement a verifiable shock to all the values is realized.[11] The winner can default on her bid and lose her bond. Further the winner and the seller may negotiate a new price. Rhodes-Kropf & Viswanathan (2000) consider a twist to Waehrer (1995)'s model where they allow the bidders to bid in securities whose value is derived from the future revenue of the firm being auctioned. They consider a number of financial instruments and show that in many cases non-cash auctions lead to higher expected revenue than cash auctions. Esö & White (2004) derive the equilibrium bidding strategies in Waehrer (1995)'s setting where the bidders are risk averse and their values are interdependent.

Contract theory literature traditionally deals with one-on-one relationship between the buyer and the contractor. Here we concentrate on the papers that deal with procurement and cost overruns. Again with few exceptions all of this literature assumes no credit constraints. Bajari & Tadelis (2001) build the model in which there is no role for the ex ante asymmetric information, however, the design may be subject to ex-post changes. The contractor possesses private information about the cost of the change of the original design. The buyer ex ante decides on the completeness of the design. With a more complete design the likelihood that it will be renegotiated ex-post is smaller, which reduces the ex post cost to the buyer. Providing more complete designs is, however, costly ex ante. Bajari & Tadelis (2001) compare cost-plus and fixed-price contracts and show that simple projects will be procured via fixed-price contracts and will have a high level of design completeness. More complex projects will be procured at cost-plus contracts and will have low level of completeness. Crocker & Reynolds (1993) provide an empirical study of the effects of various types of contracts used in defence procurement.

In Riordan & Sappington (1988) the buyer either uses contingent prices or can commit to negotiate a price in the second period once uncertainty about the shock is realized. It is often difficult to provide a complete contract that will cover any contingency and impossible to do so if the costs are not verifiable like in our model. Committing to negotiate a price in the future may be suboptimal since the contractors will be submitting very low bids trying to lock in the contract before the shock is realized and take advantage of the situation to increase the price at the renegotiation stage.

[11] Contracts with auction prices contingent on the value of the shock are assumed out.

3 The Model

We consider a conventional procurement auction model in an independent private value setting with three additional features: a possibility of an ex post shock (a cost overrun), limited liability and budget constraints of potential contractors.

The procurer (also referred to as the buyer) wants to realize a project at the minimal possible cost and runs an auction to determine the contractor to work on the project. We suppose that the buyer uses a second-lowest sealed bid tender to allocate the contract, that is, invites each potential contractor to make a bid, awards the contract to the one with the lowest bid, and sets the value of the award W equal to the second lowest bid. The only choice of the buyer is to decide on the advance and completion payments: respectively, the parts of the award to be transferred to the winning contractor immediately after the auction and after the project is completed. We denote by $r \in [0, 1]$ the share of the award to be paid ex post. Thus, once the tender finishes and the award is set at W, the buyer pays to the winner $(1 - r) W$, and once the project is completed, she pays the remaining rW. We refer to contractors also as bidders, and use personal pronouns she for the buyer and he for any bidder.

There are N potential contractors that have capacity to complete the project. Each contractor i is characterized by a pair (c_i, m_i), where c_i is i's cost of the initial investement representing all capital, labor, and managerial resources to complete the project as seen at the time of the auction, and m_i is i's cash holdings. Each contractor's type is his private information, which is independently drawn from cumulative distribution F with support $\left[C^{\min}, C^{\max}\right] \times \left[M^{\min}, M^{\max}\right]$.

Any contractor that is awarded the contract and invests in it faces ex-post risk of a cost overrun. After the initial investment has been made the winner may have to incur extra cost $Z > 0$ with probability $p > 0$ (and no extra costs otherwise). These costs can arise due to a management oversight, an adverse shock to input costs, or some other unforeseen contingencies. The incidence of a cost overrun is private information of the winning contractor.

Each contractor faces a "hard" budget constraint, that is, he has to have sufficient funds on hand to make both the initial investment and to cover the cost overrun if any. That is, for instance, contractor i with costs c_i and cash m_i must have $m_i + (1 - r) W \geq c_i$ to make the initial investment. We denote with $d_i = c_i - m_i$ the (initial) deficit of contractor i. For each contractor i it is useful to introduce $\underline{d}_i$, and $\bar{d}_i$, the lower and the upper ends of the support of the distribution of his deficit d_i.

We assume that the winner of the auction has to make the initial investment (and so has to bid sufficiently high if short on cash) but has limited liability regarding cost overruns. If the overrun occurs, the winning contractor may decide whether to continue with the project. He may choose to default on the project if he does not have sufficient funds to cover the extra costs or if the remaining payment, rW, is lower than the extra costs. If the original contractor defaults on the project, the completion payment is not made. If the buyer would like to complete the project she would have to cover the cost overrun Z and extra cost A, where $A > 0$ represents additional adjustment and organizational costs.

Finally, both the buyer and the contractors are risk-neutral profit-maximizers and have the same value for money. The values of A, Z, p, and c.d.f. F are commonly known. We focus on the equilibrium in dominant strategies.

The timeline of the interaction between the procurer and contractors is as follows. At time 0, the buyer announces that she wants to procure the project and reveals the split of the award into the advance and completion payments. Each contractor i learns (c_i, m_i). At time 1, the tender is being conducted, the contractors submit bids, the winner, say j, and the size of the award W are determined. Shortly thereafter, the advance payment $(1-r)\,W$ is made and the initial investment of c_j is undertaken. At time 2, the shock z—the need of an extra investment—may hit ($z = Z$ with probability p or $z = 0$, otherwise). If it does not, the project is completed and the completion payment rW is made. If the shock hits, the winner either makes the extra investment Z and receives rW or defaults on the project.

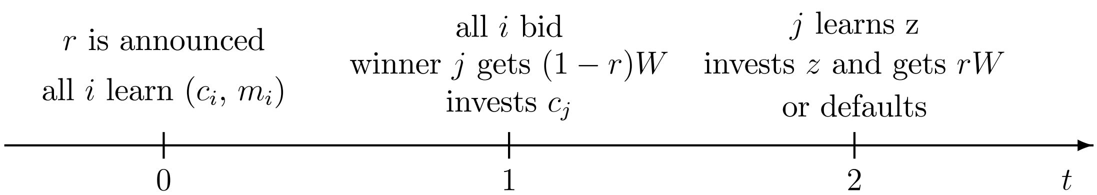

Figure 1: Timeline.

The model we consider is the simplest one that allows to study bidding and the procurer's decision problem under ex post shocks and budget constraints. There are several aspects of the model that we would like to comment on.

In practice, some of the cost overruns are observable by both the procurer and the contractor or publicly verifiable, e.g. unforeseen increases in costs of materials or labor given a specific construction plan. Being observable or verifiable such costs can be contracted upon in advance. Examples of such contracts are cost-plus contracts used in construction industry. Similarly, if the project can be divided into stages and partial progress is observable, often the contract is structured so that progress payments are made after each stage of the project, see i.e. par. 5 in A101, Standard Form of Agreement Between Owner and Contractor, issued by the American Institute of Architects. Such payments alleviate the budget constraints for the contractors and potentially provide a better incentive structure to complete the project, but do not eliminate the main trade-off of the procurer: pay more at the end to incentivize the contractor to deal with the overruns or pay more at the beginning to ease the budget constraints. To capture the essense of this tradeoff we suppose that substantial investment is made initially, no partial progress is observable, and the overruns are not publicly observable or verifiable. We assume that an overrun can take only one value solely to simplify the exposition and higlight the incentive constraints for bidders.

Most procurement procedures in practice are of two forms: lowest-bid tenders or decreasing price open auctions. We choose to analyze the second lowest-bid tender for analytical simplicity. In a private values environment, the dominant strategy equilibrium we analyse is also a dominant strategy equilibrium of the decreasing price auction. We believe that qualitatively our results would be similar for the lowest-bid auction, but due to the lack of analytic solutions one will have to compute the bidding strategies and the optimal split into advance and completion payments numerically.

Finally, we bypass the issue of a possible renegotiation between the procurer and the winning contractor when the latter defaults on the project.[12] This is a complicated separate issue. The very possibility of renegotiation may provide incentives for contractors to hold up the procurer: during the works on the project ask for higher compensation threatening to default. This, in turn, may change the bidding at the tender and/or cause the procurer to commit not to renegotiate the contract. Information structure may play a crucial role for the outcome of renegotiation. Contractors' private information on the costs, cash holdings, and cost overruns and the procurer's private information on the value of the project (irrelevant for the analysis here) and adaptation costs A imply a complicated model of the barganing game between the procurer and the winning contractor. Renegotiation is unlikely to resolve efficiently, and some projects will be in default after the initial investment (due to impossibility of the efficient bargaining under private inormation, see Myerson & Satterthwaite (1983)).

Side comment: we consider an extreme information treatment where everything related to a buyer is his private (unverifiable and non-contractible) private information. This, in particular, includes costs of the project; money holdings; and the incidence of the cost overrun. In practice, some of these may be observable and contractible, but then, likely, delivery contracts can be written accordingly. E.g. cost plus contracts are typically used when the material costs and possible overruns are observable to all the parties. We abstract away from many other realistic features. For instance, surety companies that process and provide surety bonds on behalf of bidders can work as screening agencies having superior relative to the procurer information about the bidders, e.g. their capacities, ability to provide goods or servives in time and of sufficient quality. Also, often complex contracts are split in stages, so intermediate payments are made as certain progress targets are made.

[12]Such renegotiation may be prohibited by law when the buyer is a government agency. Gil and Oudot (2010) **ABSENT REFERNCE** report some cases where the bidders who lost at the tender filed law suits against the buyer after the latter attempted to renegotiate the contract terms with the winner.

4 Unlimited budgets

In this Section we examine the simplified problem, where for each contractor the budget constraint never binds, $m > c + Z$ or $d < -Z$. (To ease notation we omit lower index i, as functional forms for profits and bidding strategies are going to be the same for all contractors.) We show that for this problem it is optimal to pay only after completion of the project, that is, $r^* = 1$.

Even though the contractors are not budget constrained they will optimally default if the ex-post portion of the award is too low, $rW < Z$. Thus, the expected payoff to the contractor with cost c who wins award W is

$$\pi(c, W) = \begin{cases} (1-p)W + p(1-r)W - c = (1-pr)W - c, \text{ if } rW \leq Z, \\ W - pZ - c, \text{ otherwise.} \end{cases}$$

As in the common second-price sealed bid auction the payoff conditional on winning is independent of the own bid. The bid determines when you win and so it is optimal to set it equal to the minimal award level that leads to non-negative expected profit conditional on winning. To find the equilibrium bidding strategy note that $\pi(c, W)$ is ing in W and decreasing in c. Consider award $W^* = Z/r$ at which the winning contractor is indifferent between continuing and defaulting and threshold type $c^*(r)$ such that $\pi(c^*, W^*) = 0$ for given r,

$$c^*(r) = Z/r - pZ. \tag{1}$$

Then, for $c < c^*(r)$, $\pi(c, W^*) > 0$, and so optimal $b^*(c) < W^*$; for $c > c^*(r)$, $\pi(c, W^*) > 0$ and optimal $b^*(c) > W^*$. The optimal bid for any c solves $\pi(c, b^*(c)) = 0$,

$$b^*(c) = \begin{cases} c/(1-pr), \text{ if } c < c^*(r), \\ c + pZ, \text{ if } c \geq c^*(r). \end{cases} \tag{2}$$

The bidding function is presented on Figure 2. Note that it has a kink at $c^*(r)$. Note also that without cost overruns (for $p = 0$) the contractors bid their cost c. When $p > 0$, bid $c + pZ$ is the expected cost of a contractor willing to complete the project despite the cost overruns. Contractors with low c bid more aggressively as they are taking the risk of not receiving the ex-post share of the award. Since they take this risk their bid exceeds c when $r > 0$. The higher is the share of the award paid ex ante, $1 - r$, the flatter is the part of the bidding strategy that corresponds to $c < c^*(r)$ and the higher is the threshold $c^*(r)$.

To compute the expect cost of the project to the buyer note that in equilibrium the second lowest bid comes from the contractor with the second lowest cost. Let $b_{(2)}$ and $c_{(2)}$ denote, respectively, the second lowest bid and cost. Let $H_{(2)}(c)$ be the distribution of the second lowest cost among N independent draws from distribution F.

With probability $1-p$ no shock occurs, the project is completed, and the award is paid in full, $W = b_{(2)}$. With probability p, the shock occurs, and the total payment is

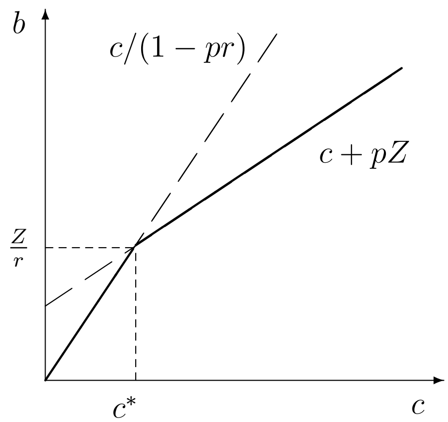

Figure 2: Bidding strategy when the bidder faces "willing" to finish constraint.

still $b_{(2)}$ if $b_{(2)} \geq Z/r$, and $(1-r)\,b_{(2)} + Z + A$, otherwise. Note that $\Pr(b_{(2)} < Z/r) = \Pr\left(c_{(2)} < c^*(r)\right) = H_{(2)}(c^*(r))$, where $H_{(2)}(\cdot)$ is the c.d.f of the distribution of the second lowest cost. If $c_{(2)} < c^*(r)$, the overall payment is $(1-p)b_{(2)} + p\,(1-r)\,b_{(2)} + pZ + pA = c_{(2)} + pZ + pA$. The equality here follows from $b_{(2)} = c_{(2)}/\,(1-pr)$. If $c_{(2)} \geq c^*(r)$, the project is finished by the winner, and the payment is $b_{(2)} = c_{(2)} + pZ$. Thus, the total expected costs to the buyer are:

$$\mathsf{E}C\,(r) = \mathsf{E}c_{(2)} + pZ + pA \cdot H_{(2)}(c^*(r)). \tag{3}$$

Note that r only affects the expected cost through the probability of default $H_{(2)}(c^*(r))$. For any $A > 0$, the expected cost is minimized when $H_{(2)}(c^*(r))$ is minimized. Since $H_{(2)}$ is increasing and $c^*(r)$ is decreasing it is optimal to set $r = 1$. Note also that $c^*(1) = (1-p)\,Z$. We, therefore, have established

Proposition 1 *With unlimited budgets it is optimal to pay the entire award ex-post. With such optimal* $r = 1$ *the expected cost is given by*

$$\mathsf{E}C = \mathsf{E}c_{(2)} + pZ + pA \cdot H_{(2)}\left[(1-p)\,Z\right].$$

This finding is intuitive. Indeed, by paying only ex-post, the buyer puts all the risk onto the winning contractor. But as the contractors are risk-neutral and do not face budget constraints, they can fully internalize the risks without imposing extra costs on the buyer or affecting the efficiency of the allocation. Nevertheless, the ways in which r affects the bidding strategies and the expected cost of the buyer is worth clarifying. Start with some $r < 1$ and consider slightly increasing it. Such increase not only lowers the probability of default but also raises the bids of all the contractors with $c < c^*(r)$, see (2). Remarkably, the latter *has no* effect on the expected cost of the buyer. The bid of the contractor with $c_{(2)}$ determines both the award paid to the winner and whether the winner will default after the shock. Suppose $c_{(2)} < c^*(r)$.

Then the contractor with the second lowest cost (and the winner) expects to default and only receive the advance share of the award. Higher r decreases the advance share and for that reason raises the bids for those with $c < c^*(r)$. Since after the default only the advance share is paid by the buyer the increase in the bid of $c_{(2)}$ is exactly cancelled by the lower advance share. If, in contrast, $c_{(2)} > c^*(r)$ then the increase in r does not affect the award and the decrease in the advance share itself is inconsequential, the default does not occur and the award is paid in full. **end of new**

Surety Bonds

In the analysis above default occures in equilibrium, when two of the bidders have costs below $(1-p)Z$. Recall that so far the liability levels were set at zero. It is possible to further incentivize the bidders by increasing their liability in case of default. One specific way to increase liability levels is with surety bonds — the common practice in the U.S. construction industry.[13] A surety bond is the amount, often a share of the award, that the contractor has to put aside at the beginning of the project.[14] If the project is completed, this amount is returned to the contractor, if not, the procurer captures it.

It is straigtforward to add surety bonds to our model. Consider surety bond set at sW for some share $s \geq 0$. Then the model with surety bonds effectively has an advance payment of $(1-r-s)W$ and a completion payment of $(r+s)W$. Therefore, the optimal choice of a pair (r, s) reduces to optimization over $r+s$, and so the overall problem is equivalent to the optimal choice of r in the original model, allowing however for $r > 1$.

With $r+s > 1$ one can obtain lower expected costs than in Proposition 1 under unlimited budgets. Specifically, the costs (3) are minimized when $H_{(2)}(c^*(r+s)) = 0$, or when $c^*(r+s) = C^{\min}$. By solving, $Z/(r+s) - pZ = C^{\min}$, we obtain

$$r^* + s = \frac{Z}{C^{\min} + pZ}. \tag{4}$$

[13] Surety bonds were introduced in the U.S. by the Heard Act of 1894 and are also quite common in Canada and Japan. The Heard Act was replaced in 1935 by the Miller Act. The Miller Act requires the contractor to provide surety bond for any Federal construction contract over \$100k in value. All the U.S. states have since adopted similar legislation, through the acts known as "Little Miller Acts." The American Institute of Architects also recommends the use of the surety bond in its standard building contract. For the easy exposition on the surety bonds see **??**, more detailed aspects of the relevant legislation and of the implementation practices are covered in Houston, Monaghan & Shahinian (2005) and Schwartzkopf & Tasker (2005).

[14] In reality the surety bond is posted by the surety company on behalf of the contractor. Surety company is not an insurance company, however. Before the surety issues the bond the contractor or her indemnitors file a General Indemnity Agreement (GIA) with the surety and jointly identify the assets to serve as collateral on the bond.

For the rest of the paper we stick to the original assumption of zero liability, commenting on the effects of surety bonds where appropriate.

5 Optimal Bidding Strategies for Budget Constrained Contractors

In this Section we derive the optimal bidding strategies for the contractors with general two-dimensional types.

There are three constraints that the winning contractor faces. First, the initial investment of c is *feasible* for the contractor of type (c, m) with deficit $d = c - m$ and ex-ante portion of the award $(1 - r) W$, if

$$m + (1 - r) W \geq c \text{ or } (1 - r) W \geq d.$$

In case shock Z realizes, the contractor will continue with the project if the following two constraints are satisfied. The *willing* to finish constraint requires the ex-post part of the award to exceed the shock,

$$W \geq Z/r.$$

The *able* to finish constraint requires that the contractor has enough funds to cover the shock,

$$m + (1 - r) W - c \geq Z \text{ or } W \geq (Z + d) / (1 - r).$$

As in the case of unlimited budgets, the optimal bid for the contractor of type (c, m) is the lowest award level that guarantees her non-negative profits. That is, for every type c the optimal bidding strategy $b^* (c)$ solves $\pi (c, b^* (c)) = 0$. Feasibility constraint requires $b^* \geq d/ (1 - r)$ for any c. Willing and able to finish constraints jointly determine the threshold award level W^{**} that satisfies both, $W^{**} = \max \{Z/r, (Z + d) / (1 - r)\}$. Given r introduce threshold

$$d^* (r) = Z \left(\frac{1 - r}{r} - 1 \right),$$

which is the level of the deficit that balances the terms in W^{**}.

Note that $d^* (r)$ is decreasing in r. The expected profit of the contractor (c, m) who wins award W is

$$\pi (c, W) = \begin{cases} (1 - p)W + p (1 - r) W - c = (1 - pr)W - c, & \text{if } W \in [d/ (1 - r), W^{**}]; \\ W - c - pZ, & \text{if } W > W^{**}. \end{cases}$$

When $d < d^*(r)$,

$$Z/r > (Z + d) / (1 - r).$$

This implies that whenever the able to finish constraint is violated the willing to finish constraint is violated as well. To check whether the winner will default only the willing to finish constraint needs to be verified. Further we say in such situations that for such (c, d) and r, the willing to finish constraint is the one that matters for defaults.

Further with $d < d^*(r)$, $W^{**} = Z/r$. Similarly to the unlimited budgets case (with $d = -\infty$),

$$b^*(c) = \begin{cases} \max\{c/(1-pr), d/(1-r)\}, & \text{if } c < c^*; \\ \max\{c + pZ, d/(1-r)\}, & \text{if } c \geq c^*. \end{cases}$$

The only difference with the unlimited budgets is that if the deficit is positive, $d > 0$, every bid has to be at least $d/(1-r)$ to enable the initial investment.

Otherwise, when $d > d^*(r)$, the able to finish, not the willing to finish constraint matters for defaults, $W^{**} = (Z + d)/(1-r)$. For such d, let $c^1(d, r)$ be the type for which $c/(1-pr) = W^{**} = (Z+d)/(1-r)$. Also let $c^2(d, r)$ be the type for which $c + pZ = (Z + d)/(1-r)$. Therefore,

$$c^2(d,r) = \frac{Z+d}{1-r} - pZ,$$
$$c^1(d,r) = (Z+d)\frac{1-pr}{1-r} = \frac{Z+d}{1-r} - p(Z+d)\frac{r}{1-r}.$$

Since $d > d^*(r)$ and hence $(Z+d)\frac{r}{1-r} > Z$, it follows that

$$c^2(d,r) > c^1(d,r) > c^*(r).$$

Note also that both $c^1(d, r)$ and $c^2(d, r)$ are increasing in r. It is useful to think of $(c^*(r), Z/q)$ as of the intersection of $c/(1-pq)$ with $c+pZ$. Then $c^1(d, r)$ and $c^2(d, r)$ are the intersections of these two lines with the level $(Z + d)/(1-r)$, whenever this level is above Z/r, see Figure 3.

The bidding strategies for $c < c^1(d,r)$ and $c > c^2(d,r)$ are determined from $\pi(c, b^*(c)) = 0$ as before. For $c \in (c^1(d,r), c^2(d,r))$, $\pi(c, W^{**}) > 0$, but for all $W < W^{**}$, $\pi(c, W) < 0$ as they are unable to finish the project and earn positive profits from it. Therefore, the optimal bid for all c in the interval $(c^1(d,r), c^2(d,r))$ is $b^*(c,d) = W^{**} = (Z+d)/r$. Altogether, for $d > d^*(r)$, the optimal bidding strategy

$$b^*(c,d) = \begin{cases} \max\{c/(1-pq), d/(1-r)\}, & \text{if } c < c^1(d,r); \\ (Z+d)/(1-r), & \text{if } c \in (c^1(d,r), c^2(d,r)); \\ c + pZ, & \text{if } c \geq c^2(d,r). \end{cases} \tag{5}$$

This optimal bidding strategy (for $d > 0$ and $d > d^*(r)$) is presented in Figure 3.

Note that compared to the case of unlimited budgets there are two extra components to the bidding strategy, one coming from the feasibility constraint (only for $d > 0$) and the other from the able to finish constraint. These components for a given

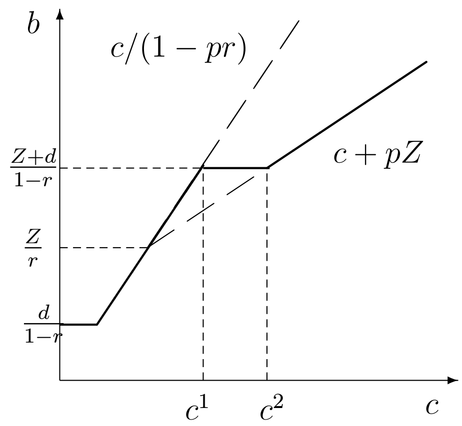

Figure 3: Bidding strategy when the bidder faces "able" to finish constraint (for a fixed deficit d).

deficit level result in flat portions of the bidding strategy reflecting the "resource," not incentive nature of the two constraints: the need to have sufficient amount of funds.

The following comparative statics are important for the analysis of the expected cost in the following section. Given r, if d increases, the flat portions of the bidding function rise. Now for given d suppose r decreases, that is, the advance share increases. The line $c + pZ$ is not affected. Both of the flat portions fall down. The flat segment at the level $(Z + d) / (1 - r)$ shrinks with both ends of the segment $c^1(d, r)$ and $c^2(d, r)$ shifting to the left. The steeper $c/(1 - pr)$ segment of the bidding strategy rotates around 0 and gets flatter. Thus, for all the contractors, bids either stay the same or decrease when r decreases.

6 Optimal Advance Share with Budget Constrained Contractors

It is clear from our analysis that in the two-dimensional setting even the second-lowest bid auction may not allocate the contract to the bidder with the lowest cost. The contractor with higher cost and lower deficit can bid more aggressively than the more efficient contractor with higher deficit. Moreover the bids of different contractors can be shaped by different types of constraints.

To study these constraints and describe how they influence the bidding strategies and the expected cost of the buyer, we first consider simplified "fixed deficit" problems, where for every contractor i, his deficit $d_i = d$. In such problems the contractor with the lowest cost always wins the auction, and the expression for the expected cost of the buyer can be derived analytically.

As we show, for $d \leq 0$ optimal $r^*(d)$ is unique. For $d > 0$ optimal $r^*(d)$ may not be unique, yet it is bounded from above.

6.1 Fixed Deficit Problems

In this Section we consider simplified "fixed deficit" problems. Formally,

Definition 1 *A fixed deficit problem with d is a procurement problem with $d_i = d$ for every contractor i.*

In any fixed deficit problem the winner is always the contractor with the lowest cost. The optimal bidding strategy for each i is, of course, described by (5). Introduce $r^*(d)$—the completion payment share that is *optimal* in the fixed deficit problem with d. (Such $r^*(d)$ is not necessarily unique.) We proceed with a series of Lemmata that characterize $r^*(d)$. The Lemmata are of independent interest and are further used in Proposition 2 that characterizes r^* —the optimal completion share in the general procurement problem where both costs and budgets are arbitrary.

The first two Lemmata characterize $r^*(d)$ in the fixed deficit problem with d. We show that $\hat{r}(d)$, the solution to

$$(Z + d)/(1 - r) = Z/r, \tag{6}$$

plays the central role in our characterization. Note that $\hat{r}(d)$ is unique and is given by

$$\hat{r}(d) = Z/(2Z + d), \tag{7}$$

which strictly decreases in d. Note also that $\hat{r}(d) > 0$ for any d, and $\hat{r}(d) < 1$ when $d > -Z$. For $d \leq -Z$ the feasibility and the able to finish constraints never bind. Therefore, from the argument that leads to Proposition 1, $r^*(d) = 1$ for such fixed deficit problems. We now consider $d \in (-Z, 0]$. For such d, the feasibility constraint does not bind. The next Lemma shows that $\hat{r}(d)$ is then optimal.

Lemma 1 *Consider the fixed deficit problem with $d \in (-Z, 0]$. Then, $r^*(d) = \hat{r}(d)$ and (at the optimum) the expected cost of the buyer satisfies*

$$\mathsf{E}C(d) = \mathsf{E}c_{(2)} + pZ + pA \cdot H_{(2)}[(2 - p)Z + d]. \tag{8}$$

The proof is given in Appendix 7. Here we provide the essense of the argument. Since all the contractor use the same bidding strategy, the bid of the contractor with $c_{(2)}$ determines both the award paid to the winner and whether the winner subsequently defaults. For $r < \hat{r}(d)$ the willing to finish constraint determines the bidding strategy of every contractor, and the expected cost of the buyer is given by 3. Similarly to the unlimited budgets case the only effect of an increase in r is a lower probability of default. Hence optimal $r^*(d) \geq \hat{r}(d)$.

For $r > \hat{r}(d)$ again all the contractors use the same bidding strategy, given by 5, which incorporates the "step" at the level $(Z + d) / (1 - r)$. The expected cost of the buyer

$$\mathsf{E}C(d, r) = \mathsf{E}c_{(2)} + pZ + pA \cdot H_{(2)}\left(c^1(d, r)\right) + \mathsf{E}C^{AtF}(d, r),$$

correspondingly incorporates the term $\mathsf{E}C^{AtF}(d, r)$ which reflects the extra payments expected to be made to the contractors who bid $(Z + d) / (1 - r)$ due to the able to finish constraint. Reducing r to $\hat{r}(d)$, that is increasing the advance share of the award, the buyer relaxes the able to finish constraint and eliminates $\mathsf{E}C^{AtF}(d, r)$. Since with $r > \hat{r}(d)$ the able to finish constraint is the one that matters for defaults, reducing r to $\hat{r}(d)$ also decreases the probability of default. Hence optimal $r^*(d) = \hat{r}(d)$.

Just as with the unlimited budgets in the fixed deficit problems the minimization of the expected cost amounts to the minimization of the probability of default. If the willing to finish constraint matters for default, this probability is lower with higher r. If the able to finish constraint matters for default, the probability of default is lower with lower r. Since default can result from the violation of either the able to finish or the willing to finish constraints, setting $r^*(d) = \hat{r}(d)$ maximizes the likelihood of both constraints being satisfied simultaneously and hence decreases the probability of default.

We next consider fixed deficit problems with $d > 0$, where the feasibility constraint may bind. The following Lemma shows that for such problems $\hat{r}(d)$ is an upper bound on $r^*(d)$.

Lemma 2 *Consider the fixed deficit problem with $d > 0$. Then $r^*(d) \leq \hat{r}(d)$.*

The proof is given in Appendix 7. When $r \geq \hat{r}(d)$ and $d > 0$ both the feasibility and the able to finish constraints may bind. The expected cost of the buyer

$$\mathsf{E}C(d, r) = \mathsf{E}c_{(2)} + pZ + pA \cdot H_{(2)}\left(c^1(d, r)\right) + \mathsf{E}C^F(d, r) + \mathsf{E}C^{AtF}(d, r).$$

In the above $\mathsf{E}C^F(d, r)$ is the extra payment expected to be made to the contractors who bid $d / (1 - r)$ due to the feasibility constraint; $\mathsf{E}C^{AtF}(d, r)$ is the extra payments expected to be made to the contractors who bid $(Z + d) / (1 - r)$ due to the able to finish constraint; and $pA \cdot H_{(2)}(c^1(d, r))$ is the term proportional to the probability of default. All of these three terms decrease when r approaches $\hat{r}(d)$ from above. Thus $r^*(d) \leq \hat{r}(d)$ and $\mathsf{E}C^{AtF}(d, r^*(d)) = 0$.

Optimal $r^*(d)$ may not be unique and may differ from $\hat{r}(d)$ for $d > 0$. For any $r < \hat{r}(d)$, only the willing to finish and the feasibility constrains bind. Then the expected cost

$$\mathsf{E}C(d, r) = \mathsf{E}c_{(2)} + pZ + pA \cdot H_{(2)}(c^*(r)) + \mathsf{E}C^F(d, r). \tag{9}$$

In this expression, $H_{(2)}(c^*(r))$ is decreasing and $\mathsf{E}C^F(d, r)$ is increasing in r, so the minimum need not be reached at $\hat{r}(d)$.

Allowing for surety bonds, that is, for $r+s>1$, the above analysis stays essentially the same. For the fixed deficit problems with $d<C^{\min}-(2-p)Z$, similarly to unlimited budget case, surety bonds allow to achieve no defauls. It suffices to set $r^{*}+s$ as in (4). For such r^{*} and s, the winning contractor effectively pays $(r^{*}+s-1)W$ to the buyer upfront and always completes the project. The deficit d is low enough that the able to finish constraint never binds.

For $C^{\min}-(2-p)Z\leq d<-Z$, the able to finish constraint may bind when $r+s>1$. By Lemma 1, $r^{*}+s=\hat{r}(d)$. Since for such d, $\hat{r}(d)>1$, surety bonds are effective in reducing the expected costs.

For the fixed deficit problems with $d\geq -Z$, $r^{*}(d)+s$ follows from Lemmata 1 and 2. Since for such d, $r^{*}(d)+s\leq 1$, surety bonds are ineffective, as even with $s=0$, that is with no surety bonds, the lowest expected cost can be reached. If $s<r^{*}(d)$, then the lowest expected cost can be reached by setting $r=r^{*}(d)-s$. Note that higher s leads to higher advance payment share, effectively advance payments are offsetting surety bonds. If $s>r^{*}(d)$, then surety bonds are strictly suboptimal as they cannot be fully offset by the advance payments.

to delete

The next Lemma compares $r^{*}(d)$ across fixed deficit problems with different d and argues that $r^{*}(d)$ is decreasing in d.

Lemma 3 *Consider fixed deficit problems with d and $d'>d$. If $r^{*}(d)$ and $r^{*}(d')$ are unique then $r^{*}(d')<r^{*}(d)$. If $r^{*}(d)$ or $r^{*}(d')$ are not unique then for any selection from the corresponding optimal sets $r^{*}(d')<r^{*}(d)$.*

The proof is given in Appendix 7.

to say in conclusion In practice the surety bond is often set at the award level, that is $s=1$. When contractors are budget constrained so that $r^{*}<1$ such practice is damaging. Dispensing with surety bonds and relying on optimally chosen advance and completion shares would lower the expected cost of the procurer.

Importantly, the finding that surety bonds may have damaging effects when the contractors are budget constrained is robust to the specifications of the model. When surety bonds are set exogenously (not in proportion to the award) similar results follow. When the contractors are not budget constrained surety bonds are effective in discouraging defaults and when set high enough lead to the first best levels of the expected cost. When the resources of the contractors are limited surety bonds lose their effectiveness. Defaults occur because the contractors cannot finance the cost overruns. Advance payments can offset such adverse effects of the surety bonds but only to an extent. When the budgets of the contractors are tight enough, the use of surety bonds is suboptimal.[15] The important assumption that drives such results is that posting surety bonds ties up the resources of the winning contractor, exacerbating his able to finish constraint, which seem to be an institutional feature of the surety bond practice, see the references in Footnote REF???

[15] See our working paper for a detailed exposition.

6.2 Bounds on the Optimal Advance Share

This subsection uses the fixed deficit problems studied in Subsection 6.1 to characterize optimal r^* in the general two dimensional procurement problem. Note that if $d_i \leq -Z$ for all i, then any contractor may only face the willing to finish constraint and by Proposition 1 optimal $r^* = 1$. We further concentrate on the setting where the contractors may face any type of the constraints. Recall that $\underline{d}_i$ and $\bar{d}_i$ denote the lower and the upper ends of the support of the distribution of the deficit of contractor i.

Proposition 2 *a) Suppose $\bar{d}_i < \overline{d} \leq 0$ for every contractor. Then optimal $r^* > \hat{r}\left(\overline{d}\right)$, where $\hat{r}(d)$ solves (6). b) Alternatively, suppose $\underline{d}_i > \underline{d}$ for every contractor. Then optimal $r^* < \hat{r}\left(\underline{d}\right)$.*[16]

The proof is in Appendix. Proposition 2 implies, in particular, that if the possible deficits of all the contractors are negative and can be "sandwiched" between the bounds $\underline{d}$ and $\overline{d}$, then optimal r in the two-dimensional problem is in between the $\hat{r}$'s, see 7, for the corresponding fixed deficit problems.

If r is chosen below $\hat{r}\left(\overline{d}\right)$, then the willing to finish constraint is the one that matters for every contractor. Increasing r (until the able to finish constraint binds for some of the contractors) would then decrease the expected cost of the buyer, as with the unlimited budgets in Section 4. On the other hand, if r exceeds $\hat{r}\left(\underline{d}\right)$, then the feasibility and the able to finish constraints determine the bidding strategy of every contractor. Reducing r decreases the bids of all the contractors. A more subtle point, made formally in the proof, is that such reduction also decreases the probability of default of the winning contractor.

Proposition 2 implies:

Corollary 1 *Suppose $\underline{d}_i > -Z$ for all i. Then $r^* < 1$.*

This finding is intuitive but hardly trivial, given the complicated way in which r factors in the bidding strategies and the expected cost. Corollary 1 conveys one of the main messages of the paper that paying the entire award ex-post, which is a hidden assumption in most of the procurement literature, is suboptimal when the contractors may not have enough funds to cover the cost overrun.

Expand: under these conditions, every contractor is subject to the AtF constraint (under $r^* = 1$)..

Comment: this is indeed one of the main messages.. Need to say better/ add to intro/conclusion.

Message: info about financial constraints/ stability of contractors (all, not just the winner) is highly desirable for the careful/ proper/ choice of r. .. Possible sources are surety companies/ banks issuing credits to contractors../ surveys ...

[16] Note that statement $b)$ applies also to $\underline{d} \geq 0$.

Clearly the more is known about the deficits of the contractors, the better the buyer can finetune her choice of r^* relying on Proposition 2. The companies that post surety bonds for the contractors screen the winner and sometimes all the bidders and provide the buyer with better information about their financial stability. Our findings provide the rational for such screening and suggest the way to use the information on $\underline{d}$ and $\overline{d}$. However, this information is more useful when it is provided to the buyer *before* the tender and when all of the potential contractors are screened, not just the winner.

7 Conclusion

This paper concerns with the issue of cost overruns in public and private procurement projects which rightfully attracts the attention of the regulators. A typical prescription to deal with possible substantial cost overruns is to offer cost plus contracts or various schemes to split the extra costs. This prescription relies on the ability to observe or verify the extra costs. Similarly, if the original cost estimates are likely to differ significantly between potential contractors and are not observable, a typical prescription is to rely on competitive bidding — specify a type of the contract to be signed and run a tender to select the best contractor according to a specified criterion typically offering to complete the contract at the lowest price). The Russian Government, for instance, requires all (non secret) public procurement to be done via competitive bidding. These two prescriptions that deal with two realistic scenarios — the contractors being different in their efficiency and the ex post shocks to the costs — cannot be easily combined. For instance, bidding for the cost plus contracts results in winning by the contractor who wishes to complete the project at the lowest margin not the one who can do it at the lowest cost. On the other hand, bidding for the fixed term contracts does not overcome cost overruns.

In this paper we address both problems simultaneously and analyze a simple tool in the hands of procurement agencies, namely, a possibility to split the award, pay a fraction of it at the initial investment stage and the rest after the completion. We show that it may be optimal to pay a positive share of the award ex ante so as to ease the financial constraints of the contractors when they may be hit with a cost overrun.

Proofs of Lemmata in Subsection 6.1

Proof of Lemma 1. For any $r \leq \hat{r}(d)$, the equilibrium bidding strategy is exactly the same as in the case of unlimited budgets. The expected cost is also the same, given by (3). Importantly, the expected cost is strictly decreasing in r. Therefore, on the segment $r \leq \hat{r}(d)$ it is optimal to set $r = \hat{r}(d)$.

For any $r > \hat{r}(d)$, the project is completed despite the cost overruns when

$b_{(2)} \geq (Z+d)/(1-r)$ and is only completed with probability $1-p$ when $b_{(2)} < (Z+d)/(1-r)$, i.e. when $c_{(2)} \leq c_1(d,r)$. If case of default the contractor only collects the ex-ante portion of the award $(1-r)\,b_{(2)}$ and the buyer finishes the project at cost $Z+A$. Therefore, conditional on $c_{(2)} \leq c^1(d,r)$ the expected cost of the project is $c_{(2)} + pZ + pA$. The contractor finishes the project when $c_{(2)} \geq c^1(d,r)$. When $c_{(2)} \in (c^1(d,r), c^2(d,r))$ the buyer pays $(Z+d)/(1-r)$ and when $c_{(2)} \geq c^1(d,r)$ the buyer pays $c_{(2)} + pZ$. The expected cost of the project

$$\begin{aligned}\mathsf{E}C(d,r) = H_{(2)}\left(c^1(d,r)\right)\left[E\left[c_{(2)}|c_{(2)} \leq c^1(d,r)\right] + pZ + pA\right] + \\ + \left(H_{(2)}\left(c^2(d,r)\right) - H_{(2)}\left(c^1(d,r)\right)\right)(Z+d)/(1-r) + \\ + \left(1 - H_{(2)}\left(c^2(d,r)\right)\right)\left[E\left[c_{(2)}|c_{(2)} \geq c^2(d,r)\right] + pZ\right].\end{aligned}$$

By adding and subtracting the expectation of $c_{(2)} + pZ$ to the middle term, and combining conditional expectations into the unconditional, the expected cost simplifies to

$$\mathsf{E}C(d,r) = \mathsf{E}c_{(2)} + pZ + pA \cdot H_{(2)}\left(c^1(d,r)\right) + \int_{c^1(d,r)}^{c^2(d,r)} \left(\frac{Z+d}{1-r} - pZ - c\right) dH_{(2)}(c) \quad (10)$$

Since H and $c^1(d,r)$ are strictly increasing, $pA \cdot H_{(2)}(c^1(d,r))$ strictly increases in r. Note that the last term in $\mathsf{E}C(d,r)$ (driven by the binding able to finish constraint) can be written as

$$\mathsf{E}C^{AtF}(d,r) = \int_{c^1(d,r)}^{c^2(d,r)} \left(c^2(d,r) - c\right) dH_{(2)}(c). \quad (11)$$

Since the integrand is strictly positive whenever $c \neq c^2(d,r)$, then $\mathsf{E}C^{AtF}(d,\hat{r}(d)) = 0$ when $c^2(d,\hat{r}(d)) = c^1(d,\hat{r}(d))$, and $\mathsf{E}C^{AtF}(d,r) > 0$ for any $r > \hat{r}(d)$. Therefore, on the segment $r \geq \hat{r}(d)$, it is also optimal to set $r = \hat{r}(d)$.

Since $c^1(d,\hat{r}(d)) = (2-p)Z + d$ the expected cost at the optimal $r = \hat{r}(d)$ increases in d and is given by

$$\mathsf{E}C(d,r) = \mathsf{E}c_{(2)} + pZ + pA \cdot H_{(2)}\left[(2-p)Z + d\right].$$

■

Proof of Lemma 2. With $d > 0$ the feasibility constraint binds and increases the bids of the contractors with

$$c < c^F(d,r) = d(1-pr)/(1-r). \quad (12)$$

Note that $c^F(d,r) < c^*(r)$ when $(Z+d)/(1-r) \leq Z/r$, and $c^F(d,r) < c^1(d,r)$ when $(Z+d)/(1-r) > Z/r$. Suppose $r > \hat{r}(d)$, where $\hat{r}(d)$ solves (6).[17] Then the

[17] Indeed, $c^F(d,r)$ is the intersection of the line $c/(1-pr)$ with $d/(1-r)$. This intersection is always on the left of the intersection of $c/(1-pr)$ with $(Z+d)/(1-r)$ which defines $c^1(d,r)$. When $(Z+d)/(1-r) \leq Z/r$, $c^*(r) \geq c^1(d,r)$.

able to finish constraint binds. Introduce

$$\mathsf{E}C^F(d,r) = \int_0^{c^F(d,r)} \left(c^F(d,r) - c\right) dH_{(2)}(c). \tag{13}$$

Note that $c^F(d,r)$ strictly increases in r and therefore $\mathsf{E}C^F(d,r)$ also strictly increases in r. With this notation the expected cost can be written as

$$\mathsf{E}C(d,r) = \mathsf{E}c_{(2)} + pZ + pA \cdot H_{(2)}\left(c^1(d,r)\right) + \mathsf{E}C^F(d,r) + \mathsf{E}C^{AtF}(d,r).$$

The term $\mathsf{E}C^F(d,r)$ captures the effect on the expected cost of the bids $d/(1-r)$ that are above $c/(1-pr)$ that come from the contractors with $c < c^F(d,r)$. These types, since $c^F(d,r) < c^1(d,r)$ default on the project after the cost overrun and receive in expectation $(1-pr)\,d/(1-r) > c$. The inequality follows from the definition of $c^F(d,r)$.

Thus in $\mathsf{E}C(d,r)$ two terms — $pA \cdot H_{(2)}(c^1(d,r))$ and $\mathsf{E}C^F(d,r)$ strictly decrease when r approaches $\hat{r}(d)$ from above. At the same time $\mathsf{E}C^{AtF}(d,r)$ is reduced to 0 in the process. ■

Proof of Lemma 3. For $d \leq 0$ the result follows from Lemma 1. Now consider the fixed deficit problem with $d > 0$. Suppose that, contrary to the statement, there exist fixed deficit problems with d_1 and d_2 such that $d_2 > d_1 > 0$, but $r_2 = r^*(d_2) > r^*(d_1) = r_1$. Clearly $\mathsf{E}C(d_1,r_1) \leq \mathsf{E}C(d_1,r_2)$. Since $\mathsf{E}C(d,r)$ given by (9) is continuous and differentiable in d, we can reconstruct the expected cost from its derivative,

$$\mathsf{E}C(d_2,r) = \mathsf{E}C(d_1,r) + \int_{d_1}^{d_2} \frac{\partial \mathsf{E}C(x,r)}{\partial x} dx = \mathsf{E}C(d_1,r) + \int_{d_1}^{d_2} \frac{\partial \mathsf{E}C^F(x,r)}{\partial x} dx,$$

for all $r \leq r_2 = r^*(d_2) \leq \hat{r}(d_2) < \hat{r}(d_1)$ (so the expression (9) is the correct expression for the expected costs). From (13),

$$\frac{\partial \mathsf{E}C^F(d,r)}{\partial d} = \int_0^{c^F(d,r)} \frac{\partial c^F(d,r)}{\partial d} dH_{(2)}(c) = \frac{1-pr}{1-r} H_{(2)}(c^F(d,r)),$$

where c^F is given by (12). Then, we can express

$$\begin{aligned}
\mathsf{E}C(d_2,r_2) - \mathsf{E}C(d_2,r_1) &= \mathsf{E}C(d_1,r_2) - \mathsf{E}C(d_1,r_1) + \\
&\quad + \int_{d_1}^{d_2} \left(\frac{\partial \mathsf{E}C^F(x,r_2)}{\partial x} - \frac{\partial \mathsf{E}C^F(x,r_1)}{\partial x}\right) dx \\
&> \int_{d_1}^{d_2} \left(\frac{1-pr_2}{1-r_2} H_{(2)}(c^F(d,r_2)) - \frac{1-pr_1}{1-r_1} H_{(2)}(c^F(d,r_1))\right) dx > 0.
\end{aligned}$$

The last inequality is due to $\frac{1-pr_2}{1-r_2} > \frac{1-pr_1}{1-r_1}$ and $c^F(d, r_2) > c^F(d, r_1)$ as $r_2 > r_1$. Thus, we obtain that $\mathsf{E}C(d_2, r_1) < \mathsf{E}C(d_2, r_2)$, and so expected costs are not minimized at $r_2 = r^*(d_2)$ — a contradiction. ■

Proof of Proposition 2. First we argue that in the general procurement problem with $d_i \leq 0$ for all i it is not optimal to choose $r < \frac{1}{2}$. Indeed, if $r < \frac{1}{2}$ then for every i only the willing to finish constraint may bind. Hence all the contractors bid according to (2) an by the argument that leads to Proposition 1, r should be increased. Note that, for $d \leq 0$, $\hat{r}(d) \in \left[\frac{1}{2}, 1\right]$.

Now consider statement a) of the Proposition. Suppose by way of contradiction that optimal r^* satisfies $\hat{r}\left(\overline{d}\right) \geq r^* \geq \frac{1}{2}$. Choose d' such that $r^* = \hat{r}(d')$. We have argued that there exists such fixed deficit problem with $d' \leq 0$. Since by the supposition $\hat{r}\left(\overline{d}\right) \geq \hat{r}(d')$, and $\hat{r}$ is decreasing $\overline{d} \leq d'$. Now consider the actual realizations of the deficits, $d_1, d_2, \ldots, d_N$. Clearly for all i, $d_i < d'$. Since by construction $r^* = \hat{r}(d')$ and all the deficits are negative in the setting with such r^* and actual $\mathbf{d}$ only the willing to finish constraint may bind for any contractor, hence the expected cost is strictly decreased when r is increased slightly.[18] Hence optimal $r^* > \hat{r}\left(\overline{d}\right)$.

Next consider statement b) and suppose that to the contrary optimal $r^* \geq \hat{r}\left(\underline{d}\right)$.[19] Again choose d' so that $r^* = \hat{r}(d')$. Since $\hat{r}$ is decreasing, $d' \leq \underline{d}$ and by the supposition all actual $d_i > d'$. Consider a small decrease in r starting from r^*. Such small decrease has a direct effect on the expected cost of the buyer since it decreases the bids of all the contractors with $c < c^2(d', r^*)$, recall the equilibrium construction in Section 5. Such decrease in r, however, also affects the likelihood of default. Importantly, as we argue below, for the contractors with $d_i > d'$ when $r^* = \hat{r}(d')$ the two effects have the same sign: these contractors only default because the able to finish constraint is violated. Further we argue that with lower r the probability of default decreases. **There are two sources of the default risk that we need to be concerned about: (1) the probability of default by the contractor under consideration (own default, direct effect); (2) increased probability of default by other contractors due to decreased awards (due to decreased bids).**

Specifically we show that in all the realizations of the types of the contractors with $d_i > \underline{d}$ in which there has been no default with r^*, there will be no default with $\tilde{r} < r^*$, for $r^* - \tilde{r}$ sufficiently small. Let i and j be the contractors with the lowest and the second-lowest bids given r^*, $b_i = b_{(1)} \leq b_j = b_{(2)}$. With r^* there will be no default if: (i) $c_i \geq c^1(d_i, r^*)$; or (ii) if $c_i < c^1(d_i, r^*)$ and $b_j \geq (Z + d_i) / (1 - r^*)$. In case (i), as $c^1(d_i, r)$ increases in r, there will be no default if i still wins as $c_i > c^1(d_i, \tilde{r})$ still holds. If i does not win, it must be the case that $c_i > c^2(d_i, \tilde{r})$ while the new winner k (possibly $k = j$ but not necessarily) has $c_k \in [c^1(d_k, \tilde{r}), c^2(d_k, \tilde{r}))$, as his bid must decrease *more* than that of bidder i. Note that for $\tilde{r}$ close to r^* $c_k < c^1(d_k, \tilde{r})$ would contradict $c_i > c^1(d_i, r^*)$ and bidder i winning at r^*. Since $c_k \geq c^1(d_k, \tilde{r})$ there will

[18] The feasibility constraint cannot be binding since all $d_i < d' \leq 0$.

[19] The rest of the proof obviously applies when $\hat{r}\left(\underline{d}\right) < 1$.

be no default with $\tilde{r}$.

In case (ii), if i is not the winner given $\tilde{r}$, then without loss of any generality the new winner is contractor j who had to reduce her bid by more than contractor i did. If all $d_i \leq 0$ since feasibility constraint is never binding the bid of contractor j must belong to the flat segment $(Z + d_j) / (1 - \tilde{r})$ and so bidder j will not default. Hence the expected cost is reduced contradicting the supposition that $r^* \geq \hat{r}(\underline{d})$ was chosen optimally. In some $d_j > 0$ the (significantly reduced) bid of j can also belong to the flat segment $d_j/(1 - \tilde{r})$. However, by the supposition, j used to bid $b_j \geq (Z + d_i)/(1 - r^*)$ with r^*, while $b_i < (Z + d_i)/(1 - r^*)$ as $c_i < c^1(d_i, r^*)$. If $b_j > (Z + d_i)/(1 - r^*)$ then by the continuity of the bidding strategies there must exist $\tilde{r}$ close enough to r^* such that i is still the winner, the bid of j still determines the award and $b_j > (Z + d_i)/(1 - \tilde{r})$. If $b_j = d_j/(1 - r^*) = (Z + d_i)/(1 - r^*)$ then $d_j = Z + d_i$ while $b_i(r^*) < (Z + d_i)/(1 - r^*)$. Hence when r decreases slightly from r^* to $\tilde{r}$ it follows that $d_j/(1 - \tilde{r}) = (Z + d_i)/(1 - \tilde{r})$ while $b_i(\tilde{r}) < (Z + d_i)/(1 - \tilde{r})$ by continuity. Hence i wins with $\tilde{r}$, the award is $(Z + d_i)/(1 - \tilde{r})$ and i does not default. ■

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