# The Value of Information with an Endogenous Public Signal

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**Abstract**

I analyze equilibrium and welfare properties of an economy where agents have access to private information and to an endogenous public signal, which is a noisy aggregate of individual actions. I characterize how the interaction of payoff and information externalities impinges on the use of signals by agents, and on the social value of information. The endogenous public signal causes an information externality, which makes agents underweight private information in a larger payoff parameter region compared to exogenous public information. I find that previous results regarding the social value of private information may be overturned if the public signal is endogenous. In addition, it is empirically more plausible that providing more accurate public information decreases welfare in beauty contests. The model is general and nests several applications in the literature.

*Keywords*: Endogenous public information, externalities, welfare analysis.

*JEL codes:* D62, D82, G14.

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# 1 Introduction

Expectations play a key role in economies characterized by uncertainty and payoff externalities. Public predictions are influenced by expectations. In today's information age, data collection, processing and aggregation are becoming prevalent, and non-price mechanisms which combine dispersed information are widespread[1]. Disclosed public statistics, which are noisy, increasingly forecast the aggregate action rather than fundamentals. In other words, public information is mostly endogenous. In addition, agents have access to private information as a result of their observation of local market conditions, private communication, or local interactions. In this paper, I study the effects of varying the quality of public or private information on welfare in large markets. The main result is that the sign of the social value of private information may be overturned when public information is endogenous in relation to when public information is exogenous. This is attributed to the way in which the different inefficiencies interact.

The welfare effects of the social value of information are not well understood. Various models have analyzed the potentially detrimental effects of public information on welfare and have associated it with different mechanisms. For example, Morris and Shin (2002) examine a model with exogenous public information, and explain that, under certain conditions, public information may have a negative social value due to the wedge between individual and social incentives to coordinate. On the other hand, Amador and Weill (2010) consider that agents, in an economy with no payoff externalities, learn from prices (which are endogenous) and find that releasing more precise public signals can have adverse effects on welfare. In this article, I analyze how endogenous public information combines with payoff externalities in order to disentangle their joint effect on the social value of information.

To address this question, I consider a framework based on the payoff structure of Angeletos and Pavan (2007), which is general enough to accommodate a variety of applications. The model is static and of the linear-quadratic-Gaussian family. The economy is populated by a large number of agents, as in a monopolistically competitive market. Each agent's utility depends on fundamentals, such as a demand or cost shocks, and exhibits payoff externalities: strategic complementarity or substitutability. An agent uses private information and the noisy public signal to form expectations about fundamentals and aggregate statistics, both of which affect an agent's utility function. The welfare benchmark is the ex ante utility of a team of agents, subject to the constraint that private information cannot be transferred from one agent to another, or to a center.

When public information aggregates the economy's dispersed information, the precision of the

[1] See, for example, Einav and Levin (2014).

endogenous public statistic depends positively on the agents' response to private information, which influences how agents form expectations about fundamentals and aggregate statistics. Agents in equilibrium do not take into account that their response to private information affects the informativeness of the public signal, which increases non-fundamental volatility and augments equilibrium welfare losses. Therefore, endogenous public information generates an information externality. Due to the information externality, I find firstly that agents underweight private information in a larger payoff parameter region compared to when public information is exogenous, and secondly that the efficient weight given to private information is larger with endogenous than with exogenous public information. Besides information externalities, there may also be payoff externalities with complete or incomplete information. The way in which the different types of externalities interact allows a characterization of the regions where agents over-, under- or equally weight private information in relation to the efficient strategy.

For example, I show that in economies with no payoff externalities agents always underweight private information, such as where firms compete in the homogeneous product market and total surplus is the welfare benchmark. If public information was exogenous, the equilibrium and efficient strategies would coincide. In economies in which the ratio of social to individual incentives to align actions is less than one, such as in beauty contests, agents can underweight private information when actions are strategic substitutes. Therefore, the results obtained with exogenous public information (e.g. Morris and Shin (2002), Angeletos and Pavan (2007)) may be reversed with endogenous public information. Otherwise, agents can underweight private information when actions are strategic complements, which could not occur if public information was exogenous (e.g. Hellwig (2005), Angeletos and Pavan (2007)).

What are the welfare consequences of releasing a more precise public signal? Reducing the noise in the public signal increases the total precision of the endogenous public statistic, which has an effect on equilibrium welfare. In this article, I find the necessary and sufficient conditions for equilibrium welfare to increase with a reduction in the noise in the endogenous public signal. The sign of the welfare effect depends on a combination of payoff relevant parameters and on the ratio of public to private information precisions. Suppose that the full information equilibrium response to the fundamental is less than or equal to the efficient response. Then, the social value of public information is positive if the reduction in dispersion is larger than the combination of an increase in non-fundamental volatility and of an increase in the the covariance between the social return to the aggregate action and the difference in the aggregate action with incomplete and complete information.

Comparing the social value of public information with endogenous and exogenous public information, I obtain two differences and one similarity. Consider economies in which the equilibrium full information response to the fundamental is less than or equal to one in absolute value. First, the rate at which welfare changes is slower with endogenous than with exogenous public information. Second, the ratio of public to private precisions is larger with exogenous than with endogenous public information, which has implications for the transparency debate between Morris and Shin (2002) and Svensson (2006) in the beauty contest. Compared to when public information is exogenous, I find that endogenous public information makes it empirically more plausible that public information is detrimental, thus favoring the conclusions of Morris and Shin (2002) in the transparency debate. If the equilibrium full information response to the fundamental is greater than one in absolute value, then these results may be reversed. Finally, the similarity is that the combination of payoff relevant parameters which determine the social value of public information is the same with both exogenous and endogenous public information.

The social value of private information differs significantly between exogenous and endogenous public information. Increasing the precision of the private signal causes two effects on equilibrium welfare which combine additively. First, there is a partial effect in which the precision of the public signal is fixed. The second effect, which is only present when public information is endogenous, is as follows: increasing the precision of the private signal increases the precision of the endogenous public statistic, and as a result, increasing the precision of the public statistic changes welfare. In this article, I find the necessary and sufficient conditions for the total effect of increasing the precision of the private signal on welfare to be positive. The total effect on welfare is positive if both the partial effect and the social value of public information are positive. An example of such economy is where firms compete in a homogeneous product market with total surplus as the welfare benchmark. The total welfare effect is negative if both the partial effect and the social value of public information are negative. Importantly, I find that compared to when public information is exogenous, increasing the precision of the private signal may overturn the sign of the total welfare effect if the partial effect has a different sign from the endogenous public precision effect. In particular, endogenous public information changes the payoff parameter combination that determines the social value of private information. As a result it also changes the cutoff degree of strategic complementarity that determines whether the total welfare effect of increasing the precision of private information is positive or negative. For example, I will illustrate that this occurs in the anti-beauty contest model, which presents a similar payoff structure as in competition à la Bertrand with product differentiation.

An essential mechanism of this paper is the information externality, which was first studied in the literature of social learning, such as in Banerjee (1992), Bikhchandani et al. (1992), and later studied by Vives (1997). Their main contribution is to provide a rational explanation of herding focusing on the insufficient response to private information in agents' decisions. In contrast to this paper, herding models are typically dynamic and do not consider payoff externalities. Morris and Shin (2005) were among the first to show that, in economies that can be described by the beauty contest, public signals are less informative when central bankers disclose their forecasts than when they do not disclose anything. I obtain the same result in a static model, and in addition extend their findings by incorporating a welfare analysis in a general class of quadratic economies that exhibit payoff externalities.

Information externalities have also been studied in the literature of rational expectations (e.g. Grossman and Stiglitz (1980), Diamond and Verrechia (1981)), where agents learn from prices. Related to this line of research, Amador and Weill (2010) consider a micro-founded macroeconomic model in which prices constitute an endogenous source of information. In a model with no payoff externalities, they find that more precise public information can be detrimental. In contrast to them, I find that if there are no payoff externalities, more precise public and private information are both beneficial. In Amador and Weill (2010) there is an additional source of strategic complementarity, generated by the way agents learn from prices, which is responsible for the negative welfare result. In contrast to Vives (2013), I consider that the endogenous public statistic does not directly affect an agent's payoff and therefore it does not have an allocation role. It has only an information role. This is motivated by the empirical observation that non-price procedures to aggregate information are common. In addition, this paper presents a theoretically parsimonious way to introduce information externalities which are independent of the market structure, in the spirit of herding models.

In contexts of incomplete and exogenous information sources, payoff externalities have been analyzed by Morris and Shin (2002) for the beauty contest; Hellwig (2005) for Bertrand competition; and Angeletos and Pavan (2007) with a general model which encompasses many of the previous applications. The recent paper of Ui and Yoshizawa (2015) provides a categorization of the social value of information in quadratic games. In relation to this literature, I introduce endogenous public information and find that some previous results regarding the comparison between equilibrium and efficient allocations, and the social value of information results may be overturned due to the way information externalities combine with payoff externalities. Colombo et al. (2014) introduce endogenous *private* information and find that the magnitude and sign of the social value of public information may be overturned. In contrast, I focus on how endogenous public information affects the social value of private information.

The paper is organized as follows. Section 2 describes the model. Section 3 presents the equilibrium and efficient allocations, and draws a comparison with the benchmark of exogenous public information. Section 4 displays the social value of information results. Section 5 illustrates the previous results in several applications of interest. Section 6 concludes. Proofs are derived in the Appendix.

# 2 The model

I present a static linear-quadratic-Gaussian model, based on the preference structure of Angeletos and Pavan (2007). The information structure however differs. Public information arises from imperfectly aggregating the individual decisions of each agent in the economy, and is therefore endogenous.

## 2.1 Preferences

The economy is composed of a continuum of agents[2] distributed uniformly over the unit interval and indexed by $i$. Simultaneously, each agent chooses an action, $q_i$, to maximize the utility function

$$U(q_i, Q, \sigma_q, \theta), \tag{1}$$

where $\theta$ represents the fundamentals which exogenously affect an agent's utility function; $Q$ is the average of agents' actions given by $Q = \int q_i di$; and $\sigma_q^2$ is the variance of the agents' actions across the population defined as $\sigma_q^2 = \int (q_i - Q)^2 di$. Assume that $U$ is a quadratic polynomial[3] in the variables $q_i, Q, \sigma, \theta$. The dispersion in actions, $\sigma$, has only second-order effects, *i.e.* $U_{q\sigma} = U_{Q\sigma} = U_{\theta\sigma} = 0$ and $U_\sigma(q_i, Q, 0, \theta) = 0$ for all triplets $(q, Q, \theta)$ and that $U$ is symmetric across agents. Furthermore, I assume that the utility function is concave with respect to $q_i$. Without loss of generality, let

$$\alpha = -\frac{U_{qQ}}{U_{qq}} \tag{2}$$

be the degree of strategic complementarity. Actions are strategic complements whenever $\alpha > 0$, strategic substitutes whenever $\alpha < 0$ and strategically independent whenever $\alpha = 0$. To ensure uniqueness, I assume that $\alpha < 1$. Note that the payoff function described above admits payoff externalities with respect to the mean action whenever $U_Q \neq 0$ and with respect to the dispersion of actions whenever $U_\sigma \neq 0$.

[2]In market contexts, this assumption is as in monopolistic competition models. Ui and Yoshizawa (2015) consider a similar model with a finite number of agents.

[3]We can also view $U$ as a second order approximation of a concave function.

## 2.2 Information & Timing

The model is static and the information structure is symmetric. First the fundamentals, $\theta$, are drawn according to the prior distribution $\theta \sim N(\bar{\theta}, \sigma_\theta^2)$, with mean $\bar{\theta}$ and variance $\sigma_\theta^2$. Each agent surveys local market conditions and forms a private belief about fundamentals given by $s_i = \theta + \epsilon_i$. Simultaneously, agents have access to a noisy[4] endogenous public signal given by

$$w = \int q_i di + u, \tag{3}$$

where the noise in the endogenous public signal is normally distributed with $u \sim N(0, \sigma_u^2)$. Fundamentals and error terms of private and public signals are mutually independent and identically distributed across agents. For ease of interpretation, I shall often work with the precision of a random variable, which is the inverse of its variance. For any random variable *e.g.* $x$, with a non-zero variance, its precision is given by $\tau_x = 1/\sigma_x^2$. Then, each agent sets a strategy, $q(s_i, w)$, which is a function from the signal space to the action space of each agent. Finally payoffs are collected.

The seminal papers of Morris and Shin (2002) and Angeletos and Pavan (2007) have considered that the public signal is exogenous, defined as $y = \theta + v$, where $v \sim N(0, \tau_v^{-1})$. The case of exogenous public information will serve as a benchmark.

## 2.3 The Endogenous Public Signal

The public signal is endogenous since it emerges from within the system. Due to the static nature of the model, the public statistic aggregates the dispersed actions of the population and simultaneously influences these actions through its information role. Further, I provide three explanations for this formulation.

First, this is reminiscent of Rational Expectations's (RE) model since an agent forecasts the future according to the correct distribution of future events. Consequently, an agent's expectations are equal to the true statistical expected value conditional on the agent's information set, which includes the endogenous public signal. The classic framework of RE extensively identifies the price with the endogenous public signal, which has both information and allocation roles (e.g. starting from Grossman and Stiglitz (1980), Diamond and Verrechia (1981), and further developed by many others). The more recent papers of Vives (2011, 2014) apply this concept to market games where firms compete in demand or supply schedules (e.g.

[4]The noise in the public signal means that the average action does not fully reveal fundamentals.

Klemperer and Meyer (1989)). In contrast to models previously mentioned, in this paper I consider that the public statistic only has an information role, and not an allocation role, since it does not affect an agent's utility function. There is ample evidence that firms increasingly use strategies which are contingent on non-price aggregate statistics that accumulate dispersed information. For example, algorithmic trading uses strategies which are contingent on financial forecasts; marketing strategies which progressively use projections emerging from social media and search engines; and airline pricing strategies which use indicators of future sales[5].

Second, several papers have shown that the main properties of equivalent fully specified dynamic models are preserved in the steady state (or static counterparts). Examples of such models can be found in Morris and Shin (2005), with a special focus on the beauty contest application; Angeletos and Werning (2006) who apply it to the context of global games and equilibrium selection; and in Angeletos and Pavan (2009) where it is applied to contingent taxation. In relation to these models, I focus on the social welfare implications of disclosing more precise public or private signals.

Third, I provide alternative interpretations. The first is related to forecasting. Suppose that an information agency[6] forms an unbiased public forecast about the aggregate action which contains a forecasting error. Endogenous public information would arise when the public forecast aggregates the dispersed information about individual actions, such as prices or quantities, while exogenous public information arises when the public forecast is about exogenous fundamentals, such as demand or supply shocks. The second focuses on the connection between the market structure and the type of information environment. Avdjiev, McGuire and Tarashev (2012) argue that information tends to be exogenous in segmented markets, where price discovery is slow, while information tends to be endogenous in integrated markets, where price discovery is fast.

## 3 Equilibrium and Efficiency

Considering that public information is endogenous, first I solve an agent's maximization problem and find the equilibrium strategy. Second, I find the efficient strategy, or equivalently, the optimal weights that a welfare planner who maximizes ex ante social welfare would give to public and private information. Third, I compare the efficient and equilibrium weights to

[5]For algorithmic trading, see for example Chaboud et al. (2014). For marketing strategies based on social media predictions, see for example Naylor et al. (2012). For pricing in the airline industry see for example McAfee and Te Velde (2006).

[6]Such as a central bank, statistical research agency or information company.

the two sources of information with exogenous and endogenous public information.

### 3.1 Equilibrium

The equilibrium concept used is Bayesian Nash Equilibrium. First, a strategy is conjectured for each agent. Given the linear-quadratic-Gaussian structure of the model, I focus on linear strategies. Second, beliefs about fundamentals are updated using conditional expectations using these conjectured strategies. Third, the conjectured strategy must be self-fulfilling, thus I find the fixed point of the best responses. These determine the coefficients of the equilibrium strategy. Finally, the precision of the public statistic is determined.

Each agent maximizes expected utility conditional on the agent's own information set

$$max_{q_i} E\left[U(q_i, Q, \sigma_q, \theta)|s_i, w\right]. \tag{4}$$

Since the information structure is symmetric, I look for symmetric strategies. Therefore, an agent's strategy can be written as $q(s_i, w) = b' + as_i + c'w$, which is contingent on both the private signal and the endogenous noisy public statistic, $w$. Using the definition of the public statistic $w$, I note that $w = \int q_i + u = b' + a\theta + c'w + u$, whose informational content is given by the random variable, $z = a\theta + u$, so that $E\left[\theta|s_i, w\right] = E\left[\theta|s_i, z\right]$. Therefore, an agent's strategy can be written as

$$q(s_i, z) = b + as_i + cE\left[\theta|z\right]. \tag{5}$$

Importantly, the precision of the endogenous public signal is[7]

$$\tau = \left(var\left[\theta|z\right]\right)^{-1} = \tau_\theta + a^2\tau_u. \tag{6}$$

The precision of the public signal depends quadratically on the response an agent gives to private information. The more agents respond to private information, the more informative the public signal becomes. The results of the paper cannot be understood without taking into account the feedback effect between an individual strategy and the precision of the endogenous public signal. This effect is not present when public information is exogenous. Proposition 1 spells out the equilibrium strategy.

[7] The appendix of Vives (2008) summarizes the statistical properties of this model.

***Proposition 1 (Equilibrium).*** *For finite and non-zero precisions of information and $\alpha < 1$, there exists a unique linear Bayesian Nash Equilibrium, which is given by $q(s_i, z) = b + as_i + cE[\theta|z]$, where $a = k_1\gamma$, $b = k_0$, $c = k_1(1-\gamma)$ and $k_0, k_1$ define the full information equilibrium strategy given by $k(\theta) = k_0 + k_1\theta$, where $k_0 = \frac{-U_q(0,0,0,0)}{U_{qq}+U_{qQ}}$ and $k_1 = \frac{-U_{q\theta}}{U_{qq}+U_{qQ}}$. The weight given to private information, $\gamma$, can be written implicitly as*

$$\gamma = \frac{(1-\alpha)\tau_\epsilon}{(1-\alpha)\tau_\epsilon + \tau_\theta + k_1^2\tau_u\gamma^2}. \tag{7}$$

*Define $\gamma^m$ to be the unique real solution to a cubic equation associated with (7), where $\gamma^m$ satisfies $0 < \gamma^m < 1$.*

Henceforth, $a$ refers to the equilibrium response to private information, while $\gamma$ refers to the equilibrium weight given to private information. Similarly for $c$ and $1-\gamma$ with respect to public information.

If actions were strategically independent, the weights given to public and private information would correspond to Bayesian weights according to the relative precisions of the signals. When actions are strategic complements, agents place a higher weight on the public signal since it helps to predict better the aggregate action. The converse happens when actions are strategic substitutes. Endogenous public information modifies the precision of the public statistic in relation to the case of exogenous public information since it depends on the weight agents give to private information: higher weight given to private information makes the public statistic more informative. The precision of the public signal affects the size of the Bayesian weight, which modifies the equilibrium weights an agent gives to public and private information.

When the precision of the private signal tends to zero, agents do not give any weight to the private signal. As the precision of this signal increases, agents increase the weight given to the private signal until they place all the weight on the private signal, corresponding to an infinite precision of it. The weight given to the private signal decreases with the precision of fundamentals and with the precision of noise in the public statistic. At the limit, when the noise in the public signal is infinitely precise, agents give all the weight to the endogenous public statistic. These comparative statics are summarized in the corollary below.

***Corollary 1.*** *The equilibrium weight given to the private signal increases with the precision of the private signal, while it decreases with the precision of fundamentals and the precision of the noise in the public statistic. Furthermore, the equilibrium weight given to the private signal decreases with a larger degree of strategic complementarity.*

## 3.2 Efficiency

Using the ideas of Radner (1962), Vives (1988) and Angeletos and Pavan (2007), the welfare planner maximizes ex ante utility subject to the constraint that information can be neither transferred from one agent to another, nor does the welfare planner have access to the agent's private information. Following Angeletos and Pavan (2007), a Taylor expansion of ex ante utility can be written as

$$E\left[u\right] = E\left[W(k(\theta),0,\theta)\right] + E[W_Q(k(\theta),0,\theta)(Q-k(\theta))] + \frac{W_{QQ}}{2}E\left[(Q-k(\theta))^2\right] + \frac{W_{\sigma\sigma}}{2}E\left[(Q-q)^2\right], \tag{8}$$

where $W$ is defined as $W(Q,\sigma_q,\theta) = \int U(q_i,Q,\sigma_q,\theta)di$, where $W_{QQ} = U_{qq} + 2U_{qQ} + U_{QQ}$ and $W_{\sigma\sigma} = U_{qq} + U_{\sigma\sigma}$ and assume that $W_{QQ} < 0$ and $W_{\sigma\sigma} < 0$.

The expression of ex ante utility is the sum of four terms: (1) The expected aggregate utility at the full information equilibrium strategy. Notice that this term will only depend on one information parameter, which is the precision of fundamentals, $\tau_\theta$. (2) The covariance between social return to the aggregate action, $W_Q$, and the difference between the equilibrium aggregate action with incomplete and complete information, $Q - k(\theta)$. Notice that this term is zero when the full information action is efficient since the full information strategy, $k^*(\theta)$, is defined to be equal to $W_Q(k^*(\theta),0,\theta) = 0$. (3) The welfare loss due to non-fundamental volatility. (4) The welfare loss due to the dispersion of actions in the cross-section of the population. Notice that if the full information equilibrium strategy is efficient, $k^*(\theta) = k(\theta)$, the ex ante utility is then a sum of the expected aggregate utility at the full information efficient strategy, $W(k^*(\theta),0,\theta)$, and welfare losses due to non-fundamental volatility and dispersion.

The relative trade-off between volatility and dispersion, $\frac{W_{QQ}}{W_{\sigma\sigma}}$, is denoted by $1-\alpha^*$, where $\alpha^*$ was named as the socially optimal degree of strategic complementarity (with exogenous public information) by Angeletos and Pavan (2007). Without loss of generality, throughout the text I shall generically call $\alpha^*$ the socially optimal degree of strategic complementarity, even though $\alpha^*$ can be positive, negative or zero, corresponding to the optimal level of strategic complementarity, substitutability and independence, respectively.

Comparative statics of the ex ante utility evaluated at the equilibrium strategy with respect to $\tau_\epsilon, \tau_u$ will be equivalent to the opposite comparative statics to those of equilibrium welfare losses, given by

$$WL(\gamma^m) = \frac{\mid W_{\sigma\sigma} \mid (k_1)^2}{2} \left( \frac{(1-\alpha)(1-\alpha+2\phi(1-\alpha^*))\tau_\epsilon + (1-\alpha^*)(1+2\phi)\tau}{((1-\alpha)\tau_\epsilon+\tau)^2} \right), \quad (9)$$

where $\phi = \frac{k_1^*-k_1}{k_1}$ which is a measure of the degree of inefficiency with full information, and is equal to zero if the full information equilibrium action is efficient.

With incomplete information, the efficient strategy, $q^*(s_i, z)$, can be found by maximizing the ex ante utility. Given the structure of the model, I also focus on efficient strategies which are linear in the signals, as given by the next proposition.

***Proposition 2 (Efficiency):*** *For finite and non-zero precisions of information and $\alpha^* < 1$, the efficient linear strategy can be written as $q^*(s_i, z) = a^* s_i + b^* + c^* E[\theta \mid z]$, where $a^* = k_1^* \gamma^*$, $b^* = k_0^*$, $c^* = k_1^*(1-\gamma^*)$ and the full information strategy is a unique linear function of fundamentals given by $k^*(\theta) = k_0^* + k_1^*\theta$, where $k_0^* = \frac{-W_Q(0,0,0)}{W_{QQ}}$* and $k_1^* = \frac{-W_{Q\theta}}{W_{QQ}}$. *The efficient weight to private information, $\gamma^*$, is the unique real and positive root of a fifth degree equation which can be re-written implicitly as*

$$\gamma^* = \frac{(1-\alpha^*)\tau_\epsilon}{(1-\alpha^*)\tau_\epsilon + \tau^* - (1-\alpha^*)(1-\gamma^*)^2 \frac{\tau_u \tau_\epsilon k_1^{*2}}{\tau^*}}, \quad (10)$$

*where $0 < \gamma^* < 1$ and the efficient informativeness of the public statistic is $\tau^* = \tau_\theta + (a^*)^2\tau_u$.*

Henceforth, $a^*$ refers to the efficient response to private information, while $\gamma^*$ refers to the efficient weight to private information.

Notice that as the precision of the private signal increases, the efficient weight to private information increases, ranging from 0 to 1, as the precision of the private signal goes from zero to infinity. The efficient weight to the private signal is decreasing in the precision of the ex ante parameter and the exogenous degree of strategic complementarity. When the planner's weight on volatility relative to dispersion increases ($\alpha^*$ decreases), the efficient weight to private information increases. However, the effects of increasing the precision of the noise in the public statistic are ambiguous. These comparative statics are summarized in the corollary below.

***Corollary 2:*** *The efficient weight to the private signal increases with the precision of the private signal, while it decreases with the precision of fundamentals and with the exogenous optimal degree of strategic complementarity. The comparative statics of the efficient weight to the private signal with respect to the precision of the noise in the public statistic are ambiguous.*

## 3.3 Comparison with benchmarks

A comparison of the equilibrium and efficient strategies allows the inefficiencies that may be present in the equilibrium allocation to be identified. First, there may be a full information inefficiency if $k^*(\theta) \neq k(\theta)$. Second, in equilibrium, agents may not have the same individual incentives to align their actions as is collectively efficient (trade-off between volatility and dispersion). This inefficiency is summarized by $\alpha^* - \alpha$. Third, there is an information externality which occurs because public information is endogenous. In contrast to the welfare planner, agents in equilibrium do not take into account that a higher weight on private information increases the precision of the endogenous public statistic. The information externality vanishes if the noise in the endogenous public statistic is infinitely volatile ($\tau_u \to 0$) or if the private signal is infinitely noisy ($\tau_\epsilon \to 0$). The next corollary summarizes how these inefficiencies combine.

***Proposition 3 (Comparison of Equilibrium and Efficient Strategies):*** *The equilibrium weight to private information is larger than the efficient weight ($\gamma^m \geq \gamma^*$) if and only if the difference between $\alpha^* - \alpha$ is larger than the information externality ($\triangle < 0$).*

*There are three cases to consider:*

*a) If $\alpha = \alpha^*$ then agents always give an inefficiently low weight to private information for all values of the degree of strategic complementarity.*

*Otherwise, suppose for simplicity that $\alpha^* = \xi\alpha$, $\xi \geq 0$ and $\xi \neq 1$. If*

*b) $\xi < 1$ then there exists a level of strategic complementarity, $\alpha^0 < 0$, such that the equilibrium weight is efficient, $\gamma(\alpha^0) = \gamma^*(\alpha^0)$, and agents over-, under-, or equally weight private information according to: $sign(\gamma - \gamma^*) = sign\,(\alpha^0 - \alpha)$.*

*c) $\xi > 1$ then there exists a level of strategic complementarity, $\alpha^0 > 0$, such that the equilibrium weight is efficient, $\gamma(\alpha^0) = \gamma^*(\alpha^0)$, and agents over-, under-, or equally weight private information according to: $sign(\gamma - \gamma^*) = sign\,(\alpha - \alpha^0)$.*

Notice that if there is only the information externality ($\alpha = \alpha^*$), agents always give an inefficiently low weight to private information. When the are additional externalities, the magnitude of $\xi$ determines: (1) whether the level of strategic complementarity that internalizes both payoff and information externalities, $\alpha^0$, is positive or negative; (2) whether agents over-, under- or equally respond to private information when actions are strategic complements or substitutes.

The information externality is not present when public information is exogenous, and it is the key driver of the differences between the equilibrium and efficient weights to private information, as summarized in the next corollary.

***Corollary 3:*** *i) Agents underweight private information ($\gamma^* > \gamma$) for a larger range of degrees of strategic complementarity ($\alpha$) with endogenous than exogenous public information.*
*ii) For finite and non-zero equal precisions of endogenous and exogenous public information, i.e. $\tau^* = \tau_\theta + \tau_v$, the efficient weight to private information is always larger with endogenous public information than with exogenous public information: $\gamma^* > \gamma^{*exo}$.*

The first comparison between the endogenous and exogenous public information benchmark concerns the equilibrium strategy. Due to the information externality, agents in equilibrium underweight private information in a larger payoff-parameter region (represented by a larger range of values of $\alpha$) compared to when public information is exogenous. The second comparison is related to the efficient weight to private information with endogenous and exogenous public information. If the precisions of the exogenous and endogenous public statistics were the same, the efficient weight to private information would always be larger with endogenous than with exogenous public information. This is due to the fact that the welfare planner internalizes the feedback effect between the weight to private information and the precision of the endogenous public statistic.

Section 5, which presents several applications of the general model, and Figure 1 illustrates further how information and payoff externalities combine, and how endogenous public information modifies the results of the benchmark model of exogenous public information.

# 4 Social Value of Public and Private Information

In this section, I first analyze the welfare effects of releasing a more precise public signal in the context where the public signal is endogenous. Increasing the precision of the noise in the public signal can be interpreted as reducing the variance of the distribution of forecasting errors. I assume that the welfare planner cannot change the precision of the ex ante fundamental, $\tau_\theta$. Second, I analyze the welfare effects of increasing the precision of the private signal. The welfare planner can support policies which make private forecasts about the fundamentals more precise, such as fostering local communication and interactions, or facilitating the observation of local market conditions. Most of these insights can be obtained by analyzing the comparative statics of the ex ante utility evaluated at the equilibrium strategy with respect to the information parameters.

## 4.1 Social value of public information

When public information is a noisy statistic of the aggregate action, the welfare effect of changing the precision of the disclosed public signal can be written as

$$\frac{d(E[u(\gamma^m)])}{d\tau_u} = \left(\frac{\partial(E[u(\gamma^m)])}{\partial\tau}\right)_{\gamma\, cons.} \frac{d\tau}{d\tau_u}. \tag{11}$$

There are two effects on equilibrium welfare: (1) The effect of changing the overall precision of the public signal on equilibrium welfare, keeping the weight on the private signal fixed. This is a partial effect; (2) The effect of changing the precision of the noise in the public signal on the overall precision of the endogenous public signal.

Let me discuss the first effect. Keeping the weight on the private signal fixed, the welfare effect of increasing the overall precision of the endogenous public statistic is ambiguous and depends on the relationship between $\alpha, \alpha^*$ and on the degree of inefficiency with full information, $\phi$. When $\phi \geq 0$, equilibrium welfare increases with a more precise public signal if $\alpha^* \geq \alpha$, since the reduction in dispersion is larger than both the increase in non-fundamental volatility and the increase in the covariance term in equilibrium welfare loss. When $\phi \geq \frac{-1}{2}$, it is necessary that $\frac{(2\alpha-\alpha^*-1)}{2(1-\alpha^*)} > \phi$ for equilibrium welfare to decrease with a more precise public signal.

The second effect is always positive since increasing the precision of the noise in the public signal increases the overall precision of the endogenous public statistic. This can be explained by decomposing this effect into two. A *direct effect:* For a fixed weight on the private signal, increasing the precision of the noise in the public signal ($\tau_u$) increases the overall precision of the endogenous public signal ($\tau$). An *indirect effect* which is only present when public information is endogenous. Increasing the precision of the public signal makes agents place less weight on the private signal, which reduces the overall precision of the endogenous public statistic. Overall, the direct effect dominates the indirect effect, and increasing the precision of the noise in the public signal increases the overall informativeness of the public statistic. As a result the sign of the welfare effect will be determined exclusively by the first effect, given by $\frac{\partial(E[u(\gamma^m)])}{\partial\tau}\,|_{\gamma\, cons.}$. Proposition 4 summarizes the welfare effects of changing the noise in the endogenous public signal.

***Proposition 4 (Social Value of Public Information):*** *For non-zero and finite precisions of $\tau_\theta, \tau_\epsilon$, equilibrium welfare increases with respect to the precision of the noise in the public signal, $\tau_u$ if and only if*

$$\frac{(1-\alpha)(2\alpha-\alpha^*-1-2\phi(1-\alpha^*))}{(1-\alpha^*)} \leq \frac{\tau(1+2\phi)}{\tau_\epsilon}. \tag{12}$$

Proposition 4 finds necessary and sufficient conditions for equilibrium welfare to increase with the precision of the noise in the endogenous public statistic. The proposition shows how the welfare effects of varying the noise in the endogenous public signal depend on the ratio of public to private precisions ($\frac{\tau}{\tau_\epsilon}$), which is different with exogenous and endogenous public information; and on a combination of payoff relevant parameters $\frac{(1-\alpha)(2\alpha-\alpha^*-1-2\phi(1-\alpha^*))}{(1-\alpha^*)(1+2\phi)}$, which is invariant to the type of public information.

## 4.2 Social value of private information

The welfare planner can conduct policies that increase the precision of private information, whose effect on equilibrium welfare can be written as

$$\frac{d(E[u(\gamma^m)])}{d\tau_\epsilon} = \left(\frac{\partial(E[u(\gamma^m)])}{\partial\tau_\epsilon}\right)_{\tau\, cons.} + \left(\frac{\partial(E[u(\gamma^m)])}{\partial\tau}\right)_{\gamma\, cons.} \frac{d\tau}{d\tau_\epsilon}. \tag{13}$$

The welfare effect of changing the precision of private information can be separated into two additive effects: (1) A '*partial effect'*: The effect of changing the precision of the private signal on equilibrium welfare, whilst keeping the precision of the public signal fixed. (2) The *'endogenous public precision effect'*: As the precision of private information increases, the overall precision of the endogenous public signal also increases, which has an effect on equilibrium welfare.

The sign of the first effect is ambiguous and depends on the following. For a fixed precision of the public signal, equilibrium welfare increases with a more precise private signal if $\phi \geq 0$ and $\alpha \geq \alpha^*$, since the increase in dispersion is smaller than the reduction in non-fundamental volatility and the reduction in the covariance term in equilibrium welfare loss. The second effect is composed of two terms: a) increasing the precision of private information increases the overall precision of the endogenous public signal ($\frac{d\tau}{d\tau_\epsilon} > 0$); b) increasing the overall precision of the endogenous public signal affects equilibrium welfare, as described in Proposition 4. Notice that this second effect may change the sign and the magnitude of the first effect.

The next proposition summarizes the welfare effects of increasing the precision of private information.

***Proposition 5 (Social Value of Private Information):*** *For non-zero and finite precisions of $\tau_\theta, \tau_u$, equilibrium welfare increases with the precision of private information if and only if*

$$\varphi\tau_\epsilon + \chi\tau \leq 0 \tag{14}$$

*where* $\varphi = -(1-\alpha)^2(1-\alpha+2\phi(1-\alpha^*)) + \upsilon(1-\alpha)(2\alpha-1-\alpha^*-2\phi(1-\alpha^*))$
*and* $\chi = (1-\alpha)(2\alpha^*-\alpha-1) - 2\phi(1-\alpha)(1-\alpha^*) - \upsilon(1-\alpha^*)(1+2\phi)$
*where* $\nu = \frac{d\tau}{d\tau_\epsilon} = 2k_1^2\tau_u\gamma^m\frac{d\gamma^m}{d\tau_\epsilon} > 0$.

Proposition 5 spells out the sufficient and necessary conditions for equilibrium welfare to increase with respect to the precision of private information. The previous proposition summarizes four possible cases: (1) If welfare increases with the precision of public information (Proposition 4) and if the '*partial effect'* is positive, then welfare also increases with the precision of private information. (2) If welfare decreases with the precision of public information and if the '*partial effect'* is negative, then welfare also decreases with the precision of private information. (3) and (4) refer to cases in which the social value of public information has a different sign than the *'partial effect'* of increasing the precision of private information on welfare. In these two cases, it is possible that endogenous public information can overturn the previous welfare results of varying the precision of private information. Besides changing the ratio of public to private precisions, endogenous public information also changes the payoff parameter combination that determines the social value of private information (since $\nu \neq 0$). As a result, it changes the cutoff degree of strategic complementarity that determines whether the total welfare effect of increasing the precision of private information is possible or negative. The applications section will further feature how Proposition 4 and 5 combine.

## 4.3 Comparison with benchmarks

The next corollary compares social value of information with endogenous and exogenous public information. It shows the importance of the full information equilibrium response to the fundamental, $k_1$, in differentiating the effects of endogenous and exogenous public information on the social value of information. This is because the square of the full information equilibrium response to the fundamental, $k_1^2$, determines the magnitude of the response to private information, which is used to form the public signal from the dispersed information in the economy. The corollary below shows the comparison with benchmarks.

***Corollary 4.*** *i) With endogenous public information, if $k_1^2 \leq 1$ then reducing the noise in the endogenous public signal changes welfare at a slower rate compared to when public information is exogenous.*

*ii) Compared to when public information is exogenous, increasing the precision of the private signal may overturn the sign of the total welfare effect if the 'partial effect' has a different sign from the 'endogenous public precision effect'.*

*iii) Whenever the noise in the endogenous public signal is the same as the noise in the exogenous public signal (i.e. $\tau_u = \tau_v$), the ratio of overall public to private information precision is larger with endogenous compared to exogenous public information if and only if the response to private information is larger than one ( $\frac{\tau}{\tau_\epsilon} \geq \frac{\tau^{exo}}{\tau_\epsilon} \iff k_1^2\gamma^2 \geq 1$).*

There are three main differences in the social value of information with endogenous versus exogenous public information. First, if $k_1^2 \leq 1$, changing the precision in the noise in the endogenous public statistic changes equilibrium welfare at a slower rate with endogenous than exogenous public information since $\left| \frac{d(E[u(\gamma^m)])}{d\tau_u} \right| \leq \left| \frac{\partial(E[u(\gamma^m)])}{\partial\tau_u} \right|$. In other words, equilibrium welfare as a function of the precision of the noise in the endogenous public signal is flatter with endogenous than exogenous public information.

The second implication concerns the social value of private information. The sign of the social value of private information may be overturned with endogenous public information in relation to when public information is exogenous. This occurs when welfare increases (decreases) with the precision of public information and the '*partial effect*' is negative (positive). Then, welfare effect of increasing the precision of the private signal is ambiguous and depends on which effect is larger. Whether the sign of the social value of private information is overturned depends on the magnitude of the *'endogenous public precision effect',* given by $\frac{\partial(E[u(\gamma^m)])}{\partial\tau}\frac{d\tau}{d\tau_\epsilon}$, in relation to the *'partial effect'* of increasing the precision of the endogenous public statistic on welfare, i.e. $\left(\frac{\partial(E[u(\gamma^m)])}{\partial\tau_\epsilon}\right)_{\tau\ cons.}$.

The third implication concerns the ratio of overall public to private precision. If $k_1^2\gamma^2 \geq 1$ ($k_1^2\gamma^2 < 1$), then the precision of the public signal is larger (smaller) with exogenous than with endogenous public information. Notice that both Proposition 4 and 5 show that welfare increases with the precision information if and only if a condition is satisfied. This condition depends on parameters of the utility function $(\alpha, \alpha^*, \phi)$, which are the same with endogenous and exogenous public information, and on a ratio of public to private information precision, $\frac{\tau}{\tau_\epsilon}$ , which is larger with endogenous than exogenous public information if and only if $k_1^2\gamma^2 \geq 1$.

This third part of the corollary will bring insights for comparing the transparency implications with endogenous and exogenous public information.

# 5 Applications

This section describes a few applications of interest, which illustrate how, due to differences in the payoff structure, the endogeneity of public information is reflected in the equilibrium and efficient allocations, and in the social value of information. The information structure is as described in sections 2.2 and 2.3. This section illustrates with examples the results of the previous Propositions and Corollaries.

Table 1 shows the values of each of the three parameters which characterize the applications: the degree of inefficiency of the full information equilibrium ($\phi$), the relationship between $\alpha, \alpha^*$, and the full information equilibrium response to the fundamental, $k_1^2$. For each application, I discuss: (1) the differences between equilibrium and efficient allocations with endogenous and exogenous public information. For each type of application, Figure 1 illustrates the equilibrium and efficient weights to private information as a function of the degree of strategic complementarity, i.e. $\gamma(\alpha), \gamma^*(\alpha^*)$. The figures on the left hand side correspond to exogenous public information, while the figures on the right hand side correspond to endogenous public information. (2) The social value of information results for each application. Figure 2 illustrates the contour lines of ex ante welfare as a function of the precision of private information (vertical axis) and the precision in the noise in the public statistic (horizontal axis); and (3) Other interpretations of each model.

| Application | $\phi$ | $\alpha, \alpha^*$ | $k_1^2$ |
|---|---|---|---|
| Competition in a homogeneous product market[1] | 0 | $\alpha=\alpha^*$ | $\text{sign}(1-(\lambda+\beta))^2$ |
| Beauty Contest | 0 | $\alpha^*=0$, $\alpha\epsilon(-1,1)$ | 1 |
| Anti-Beauty Contest | 0 | $\alpha=0$, $\alpha^*\epsilon(-1,1)$ | 1 |
| Competition with product differentiation à la Cournot[2] | <0 | $0>\alpha>\alpha^*$ | >1 |

Table 1: Summary of Applications
*Notes:*
1. *Using total surplus as welfare benchmark.*
2. *Using producer surplus as welfare benchmark. In the sub-section, I also discuss this application with total surplus as welfare benchmark.*

## 5.1 *Firms Competing in a Homogeneous Product Market.*

This model was first studied by Vives (1988) and further developed by Angeletos and Pavan (2007). Suppose that an economy is composed of a continuum of households, each consisting of a producer and a consumer which make production choices with quadratic production costs. Each agent is uncertain about the intercept of his own marginal cost, which is represented by fundamentals. Each household chooses quantities $q_{1i}, q_{2i}$ of the two goods in the economy by maximizing the utility given by: $u_i = \delta q_{1i} + q_{2i}$, subject to the budget constraint $lq_{1i} + q_{2i} = \pi_i$, where $l$ is the price of good 1, good 2 is the numeraire and $\pi_i$ are the profits of producer $i$ given by: $\pi_i = (l-\theta)q_i - \frac{\lambda q_i^2}{2}$, where $q_i$ is the quantity produced by household $i$. The inverse demand for good 1 is $l = \delta - \beta Q$, where $Q$ is the total amount produced by all households in the economy. Imposing symmetry of production choices and market clearing, the maximization problem subject to the budget constraint is equivalent to:

$$U(q, Q, \sigma_q, \theta) = (\delta - \beta Q)q_i + \frac{\beta Q^2}{2} - \theta q_i - \frac{\lambda q_i^2}{2}, \tag{15}$$

assuming that $\beta + \lambda > 0$ and $2\beta + \lambda > 0$. The degree of strategic complementarity is $\alpha = \frac{-\beta}{\lambda}$, while $W = \int U(q, Q, \sigma_q, \theta)di = (\delta - \frac{\beta Q}{2})Q - \int(\theta q_i + \frac{\lambda}{2}q_i^2)di$ is equal to total surplus $TS = W(Q, \sigma_q, \theta) = \delta Q - \frac{(\beta+\lambda)Q^2}{2} - \frac{\lambda\sigma_q^2}{2}$. With full information the economy is efficient with $k(\theta) = k^*(\theta) = \frac{\delta-\theta}{\lambda+\beta}$. Note that $\xi = 1$ since $\alpha^* = \alpha = \frac{-\beta}{\lambda}$.

#### 5.1.1 Equilibrium and Efficient Allocations

With exogenous public information, the equilibrium allocation would be efficient since there are no externalities. However, when public information is endogenous and since $\alpha^* = \alpha$, agents in equilibrium give an inefficiently low weight to private information for all degrees of strategic complementarity due to the information externality (Proposition 3). Figures 1a. and 1b. illustrate the comparison of the equilibrium and efficient weights with exogenous and endogenous public information.

#### 5.1.2 Social Value of Information

The next corollary applies Propositions 4 and 5 to the payoff structure of firms that are competing in a homogeneous product market, characterized by $\alpha^* = \alpha$ and $\phi = 0$. Figures 2a. and 2b. illustrate the welfare contour lines, which summarize the social value of public and private information with exogenous and endogenous public information, respectively.

***Corollary 5:*** *Welfare increases with both the precision of public and private information.*

The difference in welfare contour lines between endogenous and exogenous public information show the following. First consider the case where $k_1^2 \leq 1$ (or equivalently $(\lambda + \beta)^2 \geq 1$). (1)

For a constant precision of public information and when the precision of private information is small, a higher precision of private information increases welfare at a faster rate with endogenous than exogenous public information. As the precision of private information becomes large, the rate of increase in welfare due to increasing private information becomes closer with endogenous and exogenous public information. (2) For a constant precision of private information, a higher precision of the noise in the public statistic increases welfare at a slower rate with endogenous versus exogenous public information. These conclusions may be reversed if $k_1^2 > 1$. These differences are entirely due to the information externality since there are no other externalities ($\alpha^* = \alpha$ and $\phi = 0$).

### 5.1.3 Other Interpretations

Another model with a similar payoff structure is the pure prediction model, which can be understood as a static reduced form of the model used in the herding literature. Suppose that agents have the objective of predicting the value of fundamentals, whose utility function is: $U(q, Q, \sigma_q, \theta) = -(q_i - \theta)^2$. Actions are strategically independent from an individual perspective, since $\alpha = 0$, and from a social perspective since $\alpha^* = 0$ because $W(Q, \sigma_q, \theta) = -(Q - \theta)^2 - \sigma_q^2$. Additionally, the full information equilibrium action is efficient: $k(\theta) = k^*(\theta) = \theta$ and hence $\phi = 0$. The conclusions of 5.1.1. and 5.1.2. apply to the pure prediction model as shown by Bru and Vives (2002).

## 5.2 Beauty Contest

Morris and Shin (2002) considered a utility function which formalizes Keynes' beauty contest metaphor for how financial markets work: Agents are not only concerned about predicting the fundamentals but also about outguessing the likely actions of others. An agent's utility function can be expressed as:

$$U(q, Q, \sigma_q, \theta) = -(1 - r)(q_i - \theta)^2 - r(q_i - Q)^2 + r\sigma_q^2, \tag{16}$$

where[8] $r \in (-1, 1)$. The degree of strategic complementarity is $\alpha = r$. Social welfare is $W(Q, \sigma_q, \theta) = -(1-r)(Q-\theta)^2 + r\sigma_q^2$, which implies that $\alpha^* = 0$. With full information, the equilibrium action is efficient and given by: $k(\theta) = k^*(\theta) = \theta$, which implies that $\phi = 0$ and $k_1 = 1$.

[8] Morris and Shin (2002) only considered the case $r \in (0, 1)$.

### 5.2.1 Equilibrium and Efficient Allocations

Refer to Figures 1c. and 1d. With exogenous public information, agents give an inefficiently low (high) weight to private information when actions are strategic complements (strategic substitutes). With endogenous public information, agents give an inefficiently low weight to private information when actions are strategic complements. However, when actions are strategic strategic substitutes, agents also underweight private information if $0 > \alpha > \alpha^0$, while agents overweight private information if $\alpha < \alpha^0 < 0$ and[9] $\alpha^0 > -1$, where $\alpha^0$ is the level of strategic complementarity that internalizes both payoff and information externalities and $\gamma(\alpha^0) = \gamma^*(\alpha^0)$. Therefore, in contrast to the benchmark case of exogenous public information: (1) Agents may underweight private information when actions are strategic substitutes. (2) Agents underweight private information for a larger range of $\alpha$ with endogenous than exogenous public information. This contrasts with the results of Morris and Shin (2002) and Angeletos and Pavan (2007).

### 5.2.2 Social Value of Information

The next Corollary applies Propositions 4 and 5 to the beauty contest.

***Corollary 6:*** *Case I: If $\alpha \leq \frac{1}{2}$ then welfare increases with the precision of both public information and private information.*

*Case II: If $\alpha > \frac{1}{2}$ and*

*i) If $(1-\alpha)(2\alpha-1) \leq \frac{\tau}{\tau_\epsilon}$ and $\frac{-(1-\alpha)^3+\upsilon(1-\alpha)(2\alpha-1)}{1-\alpha^2+\upsilon} \leq \frac{\tau}{\tau_\epsilon}$ then welfare increases with the precision of both public information and private information.*

*ii) If $(1-\alpha)(2\alpha-1) > \frac{\tau}{\tau_\epsilon}$ and $\frac{-(1-\alpha)^3+\upsilon(1-\alpha)(2\alpha-1)}{1-\alpha^2+\upsilon} \leq \frac{\tau}{\tau_\epsilon}$ then welfare decreases with the precision of public information while welfare increases with the precision of private information.*

Corollary 6 shows that there are two regions that characterize the social value of information in the beauty contest. In Case I welfare increases with the precision of both public and private information, as shown in Figure 2a. and 2b. If the degree of strategic complementarity is less than or equal to $\frac{1}{2}$ then the social value of information is always positive.

If the degree of strategic complementarity is larger than $\frac{1}{2}$ then there are two regions (refer to Figures 2c. and 2d.): i) Welfare increases with both the precision of public and private information. This region is characterized by a high ratio of public to private precisions in relation to a combination of payoff parameters, $(1-\alpha)(2\alpha-1)$. ii), welfare increases with

[9] $\alpha^0 > -1$ is needed since I have restricted $\alpha = r \in (-1, 1)$. If $\alpha^0 < -1$ then agents would underweight private information for all the admissible range.

the precision of private information but it decreases with the precision of public information, which occurs when the ratio of public to private precisions is smaller than $(1-\alpha)(2\alpha-1)$. Notice that since $\alpha > -1$, this means that the partial welfare effect of increasing the precision of private information is always positive. However, when public information is detrimental there are two possibilities for the total welfare effect: (1) Welfare increases with the precision of private information if the positive *'partial effect'* is greater than the negative *'endogenous public precision effect'*. (2) Welfare decreases with the precision of private information if the positive *'partial effect'* is smaller than the negative *'endogenous public precision effect'*. This second effect would overturn the main conclusions of the social value of private information with exogenous public information. However, the parameter configuration of the beauty contest rules this possibility, and only the first possibility prevails[10].

The social value of information with endogenous and exogenous public information presents some differences. In the beauty contest we have that $k_1^2\gamma^2 \leq 1$, and as a result, for a constant precision of private information, increasing the precision of the noise in the endogenous public statistic increases welfare at a faster rate with exogenous than endogenous public information. This has implications for public policy. To achieve a certain welfare change, the precision of the noise in the endogenous public statistic needs to be larger with endogenous than exogenous public information.

### 5.2.3 A further interpretation

The beauty contest application has steered a transparency debate between Morris and Shin (2002) and Svensson (2006), both of which considered that public information was exogenous. Morris and Shin (2002) argued that, in beauty contests, public information may be detrimental (Case II of Corollary 6), while Svensson claimed that the it is empirically implausible that public information is detrimental since it is required that private information is at least 8 times more precise than public information. This seems to contradict the observation that there are many resources spent by central banks and information agencies in analyzing and processing information which then becomes public.

When public information is endogenous and $k_1^2\gamma^2 \leq 1$, Corollary 4 shows that the ratio of public to private precisions is larger with exogenous than endogenous public information, i.e. $\frac{\tau^{exo}}{\tau_\epsilon} > \frac{\tau}{\tau_\epsilon}$. For fixed payoff parameter values and for a fixed precision of the private signal, welfare decreases for a larger range of $\tau_u$ with endogenous than exogenous public information. Therefore, it is empirically more plausible that providing more precise public information decreases welfare, thus favoring the concussions of Morris and Shin (2002) in the

[10] I can show that in the range $\alpha \in (-1, 1)$, we have that $-(1-\alpha)^3 + \upsilon(1-\alpha)(2\alpha-1) < 0$ since $\upsilon > 0$.

transparency debate.

## 5.3 Anti-Beauty Contest

This application is a new formalization of an economy where actions are strategically independent from an individual perspective, while there is an incentive to coordinate or anti-coordinate at the social level, as shown by

$$U(q_i, Q, \sigma_q, \theta) = -(1-r)(q_i - \theta)^2 - r\sigma_q^2. \tag{17}$$

An agent has the objective to take an action close to fundamentals, $\theta$, but there is a negative (positive) payoff externality due to the dispersion in actions of the population when $r > 0$ ($r < 0$). Regularity conditions require that $r < 1$. When there is incomplete information, actions are strategically independent since $\alpha = 0$. Welfare under the utilitarian aggregator is given by $W(Q, \sigma_q, \theta) = -(1-r)(Q-\theta)^2 - \sigma_q^2$. The economy is efficient with full information since $k(\theta) = k^*(\theta) = \theta$, which implies that $\phi = 0$ and $\alpha^* = r$. For example, when $r > 0$, it would be socially optimal to coordinate in order to reduce the cross-sectional dispersion which negatively affects the utility function of each agent.

### 5.3.1 Equilibrium and Efficient Allocations

Refer to Figures 1e and 1f. With exogenous public information, agents place too much (too little) weight on private information if $r > 0$ ($r < 0$) in relation to the efficient level. When public information is endogenous, agents also underweight the private signal in relation to the efficient level if $r < 0$, but the magnitude of the difference between the equilibrium and efficient weights is larger with endogenous than with exogenous public information. When $r > 0$, agents overweight the private signal if $\alpha > \alpha^0 > 0$.

### 5.3.2 Social Value of Information

The next Corollary applies Propositions 4 and 5 to the parameters $\alpha^* = r, \alpha = 0$ and $\phi = 0$.

***Corollary 7:*** *Case I:If* $\alpha^* \leq \frac{1+\upsilon}{2+\upsilon}$ *then welfare increases with both public and private information.*

Case II: If $\alpha^* > \frac{1+\upsilon}{2+\upsilon}$, *and*

i) *If* $\frac{(2\alpha^*-1)-\nu(1-\alpha^*)}{(1+\nu(1+\alpha^*)} < \frac{\tau_\epsilon}{\tau}$ *then welfare increases with both the precision of public information and private information.*

ii) *If* $\frac{(2\alpha^*-1)-\nu(1-\alpha^*)}{(1+\nu(1+\alpha^*)} > \frac{\tau_\epsilon}{\tau}$ *then welfare increases with the precision of public information and decreases with the precision of private information.*

Corollary 7 shows that if the socially optimal degree of strategic complementarity is less or equal to $\frac{1+\upsilon}{2+\nu}$, then welfare increases with the precision of both public and private information, as represented by Figures 2a and 2b. However, if the socially optimal degree of strategic complementarity is greater than $\frac{1+\upsilon}{2+\nu}$ the situation differs. Welfare always increases with the precision of public information but it may increase (decrease) with the precision of private information if the negative *'partial effect'* is larger (smaller) than the positive *'endogenous public precision effect'.*

Notice that if public information was exogenous the cutoff $\alpha^*$ between Case I and Case II would be equal to $\frac{1}{2}$, while the cutoff $\alpha^*$ with endogenous public information is equal to $\frac{1+\upsilon}{2+\nu}$, which is a strictly increasing function of $\upsilon$. Therefore, with endogenous public information, there is a smaller range of $\alpha^*$ that fall into Case I than when public information is exogenous. Figures 2e. and 2f. illustrate how for the same fixed parameter values, endogenous public information may overturn the sign of social value of information results with exogenous public information due to: (1) different combination of payoffs that determines the social value of private information ($\frac{(2\alpha^*-1)-\nu(1-\alpha^*)}{(1+\nu(1+\alpha^*)}$ with endogenous public information vs. $2\alpha^* - 1$ with exogenous public information), and as a result, a different cutoff value of $\alpha^*$ which distinguishes Case I and Case II; (2) different ratio of public to private information precisions since $k_1^2\gamma^2 < 1$, and $\tau_u = \tau_v$, implies that $\frac{\tau_\epsilon}{\tau} > \frac{\tau_\epsilon}{\tau^{exo}}$.

#### 5.3.3 Other Interpretations

The monopolistic competition à la Bertrand with product differentiation presents a similar structure to the anti-beauty contest with $r > 0$, since the payoff characteristics which describe the Bertrand model are $\alpha^* > \alpha > 0$ and $\phi > 0$. This model has been previously studied by Angeletos and Pavan (2007) and Ui and Yoshizawa (2015) in the context of exogenous public information. Since both models have a similar structure, analogous results[11] apply as in 5.3.1 and 5.3.2.

[11] The main difference between both models is due to the different payoff structure, but all the qualitative conclusions are identical.

With exogenous public information, Ui and Yoshizawa (2015) show that the Bertrand game is either a Type I game, meaning that welfare increases with the precision of both public and private information, or Type -IV game, meaning that welfare always increases with the precision of public information, while it may decrease or increase with the precision of private information. Considering that public information is endogenous and applying Corollary 7, I find that endogenous public information makes it more likely that the Bertrand game is of Type I.

## 5.4 Monopolistic Competition à la Cournot with Product Differentiation.

This formulation is based on the model of Vives (1990). Suppose a large number of firms compete à la Cournot and sell a differentiated product. The inverse demand that a firm faces is linear and random given by $l_i = \theta - (1-\delta)\kappa_i - \delta K$ where $(l_i, \kappa_i)$ are a price-quantity pairs for firm $i$, and $\delta$ can be interpreted as the degree of product differentiation with $\delta \in (0, 1)$. In the limit, when $\delta \to 0$ firms are isolated monopolies, while when $\delta \to 1$ there is perfect competition. For simplicity suppose that marginal costs are constant and equal to zero. Setting $\kappa_i = q_i$, the profit of each firm can be written as:

$$U(q, Q, \sigma_q, \theta) = \theta q_i - (1-\delta)q_i^2 - \delta q_i Q. \tag{18}$$

With full information, the profit maximizing quantity is equal to $k(\theta) = \frac{\theta}{2-\delta}$ and the degree of strategic substitutability is $\alpha = \frac{-\delta}{2(1-\delta)} < 0$. The social welfare, $W$, corresponds to aggregate profits since it is equal to: $\Pi = W(Q, \sigma_q, \theta) = \theta Q - Q^2 - (1-\delta)\sigma_q^2$, which implies that the optimal degree of strategic substitutability is $\alpha^* = 2\alpha < 0$. With full information, the quantity which maximizes total profits shows that firms over-respond to the fundamental in equilibrium since $k^*(\theta) = \frac{\theta}{2} < k(\theta)$. This implies that the full-information inefficiency is $\phi = \frac{-\delta}{2} = \frac{\alpha}{1-2\alpha} < 0$ and $k_1 = \frac{2(1-\alpha)}{1-2\alpha}$

### 5.4.1 Equilibrium and Efficient Allocations

Figures 1g and 1h show that firms in competition à la Cournot firms always under-respond to private information in relation to the efficient level. The magnitude of the under-response is larger with endogenous than exogenous public information due to the information externality. This is because $\alpha < 0 < \alpha^0$ and Proposition 3 shows that $sign(\gamma - \gamma^*) = sign(\alpha - \alpha^0) < 0$.

### 5.4.2 Social Value of Information

The next Corollary applies Propositions 4 and 5 to the parameters $\alpha^* = 2\alpha$ and $\phi < 0$ with $k_1 > 1$.

**Corollary 8:** *Case I: If $-\frac{1}{2} \leq \alpha < 0$ then total profits increase with the precision of both public and private information.*
*Case II: If $\upsilon - \sqrt{\upsilon^2 + \upsilon + 1} \leq \alpha < \frac{-1}{2}$ and*
*i) If $-(1+2\alpha)(1-\alpha) \leq \frac{\tau}{\tau_\epsilon}$ and $\frac{-((1-\alpha)^2(1+\alpha)+\upsilon(1-\alpha)(1+2\alpha))}{\upsilon+(1-\alpha)^2} \leq \frac{\tau}{\tau_\epsilon}$ then total profits increase with the precision of both public and private information.*
*ii) If $-(1+2\alpha)(1-\alpha) > \frac{\tau}{\tau_\epsilon}$ and $\frac{-((1-\alpha)^2(1+\alpha)+\upsilon(1-\alpha)(1+2\alpha))}{\upsilon+(1-\alpha)^2} \leq \frac{\tau}{\tau_\epsilon}$ then total profits decrease with the precision of public information and increase with the precision of private information.*
*Case III: If $\alpha < \upsilon - \sqrt{\upsilon^2 + \upsilon + 1}$ and*
*i) If $-(1+2\alpha)(1-\alpha) \leq \frac{\tau}{\tau_\epsilon}$ and $\frac{-((1-\alpha)^2(1+\alpha)+\upsilon(1-\alpha)(1+2\alpha))}{\upsilon+(1-\alpha)^2} \leq \frac{\tau}{\tau_\epsilon}$ then total profits increase with the precision of both public and private information.*
ii) *If $-(1+2\alpha)(1-\alpha) > \frac{\tau}{\tau_\epsilon}$ and $\frac{-((1-\alpha)^2(1+\alpha)+\upsilon(1-\alpha)(1+2\alpha))}{\upsilon+(1-\alpha)^2} \leq \frac{\tau}{\tau_\epsilon}$ then total profits decrease with the precision of public information and increase with the precision of private information.*
*iii) If $-(1+2\alpha)(1-\alpha) > \frac{\tau}{\tau_\epsilon}$ and $\frac{-((1-\alpha)^2(1+\alpha)+\upsilon(1-\alpha)(1+2\alpha))}{\upsilon+(1-\alpha)^2} > \frac{\tau}{\tau_\epsilon}$ then total profits decrease with the precision of both public and private information.*

Refer to Figure 2a. and 2b. for Case I; to Figures 2c. and 2d. for Case II; and to Figures 2g. and 2h. for Case III with endogenous and exogenous public information, respectively. Corollary 8 shows that the social value of information of Cournot competition can be described in three cases. Given the parameter configuration for Cournot, it is not possible that welfare increases with the precision of public information while it decreases with the precision of private information. Notice that the cutoff $\alpha$ between Case II and Case III changes with endogenous public information. If public information was exogenous, this cutoff would be equal to $-1$ (which would correspond to $\upsilon = 0$), as shown by Ui and Yoshizawa (2015). Endogenous public information decreases the range of $\alpha$ that fall in Case II versus Case III. Additionally, endogenous public information changes the payoff parameter combination that determines the social value of private information ($-(1+\alpha)$ with exogenous vs. $\frac{-((1-\alpha)^2(1+\alpha)+\upsilon(1-\alpha)(1+2\alpha))}{\upsilon+(1-\alpha)^2}$ with endogenous public information).

### 5.4.3 Another welfare benchmark

Another welfare benchmark worth analyzing is total surplus. Suppose that a representative consumer maximizes $V(q_i) - \int l_i q_i di = \int (\theta q_i - \frac{Q^2}{2} - \frac{(1-\delta)(q_i^2 - Q^2)}{2} - l_i q_i) di$. Then, total surplus can be shown to be equal to $TS = V(q_i) = \theta Q - \frac{Q^2}{2} - \frac{(1-\delta)\sigma_q^2}{2}$, where the relationship between the representative consumer's utility function and total surplus is given by: $\alpha^* = 2\alpha$ and the full-information inefficiency is: $\phi = 1 - \delta = \frac{1}{1-2\alpha} > 0$. Like in the case of exogenous public information (e.g. Vives (1990)), I obtain that total surplus is always increasing with the precision of both private and public information.

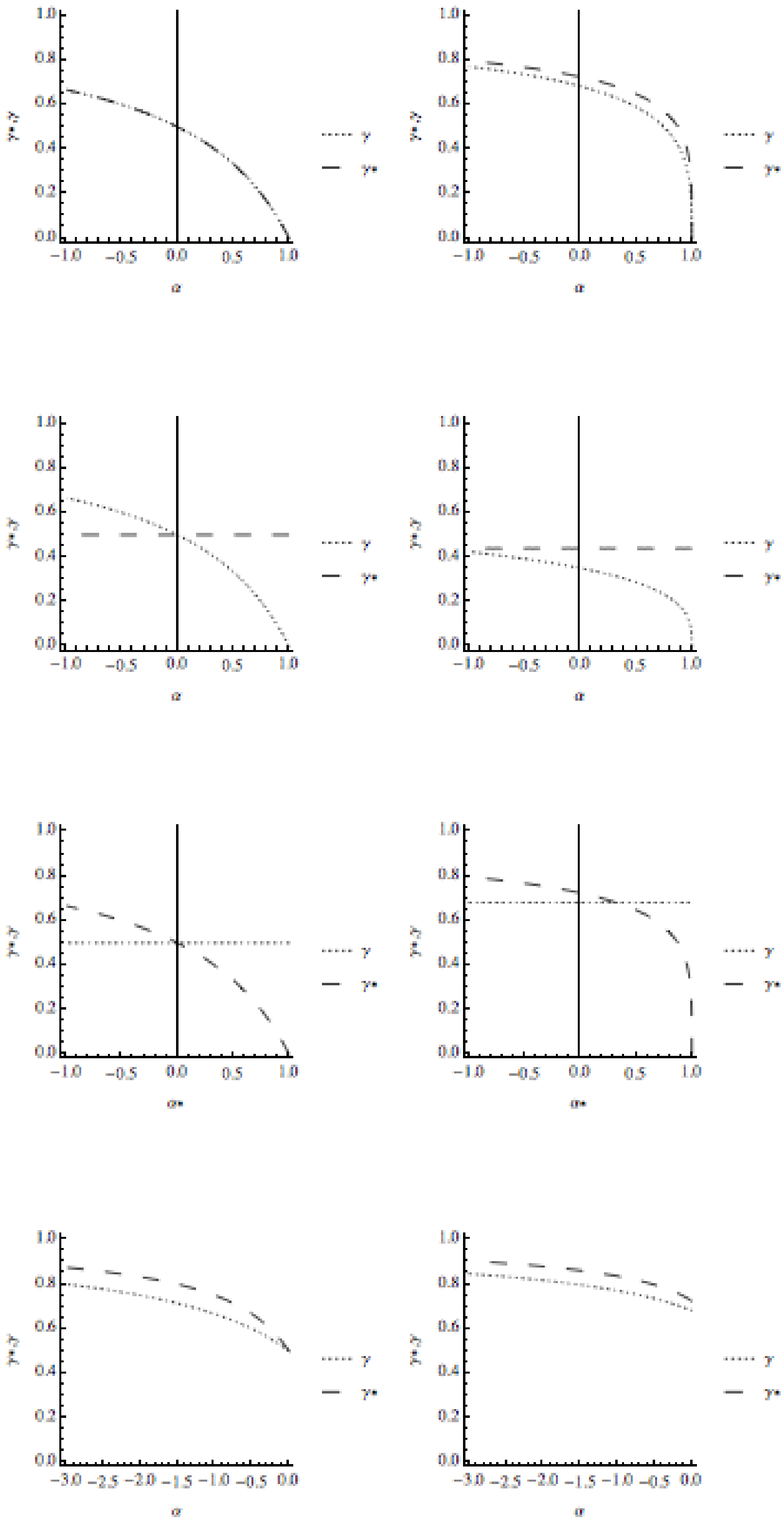

Figure 1: *Equilibrium and efficient weights to private information with exogenous (left) and endogenous (right) public information.* The horizontal axis displays the degree of strategic complementarity ($\alpha$) and the vertical axis the weight to private information ($\gamma$ corresponds to the equilibrium weight to private information and $\gamma^*$ to the efficient weight to private information). If it exists within the range, the graph also displays $\alpha^0$ which corresponds to the level of strategic complementarity at which the equilibrium and efficient weights coincide, i.e. $\gamma(\alpha) = \gamma^*(\alpha^*(\alpha))$.

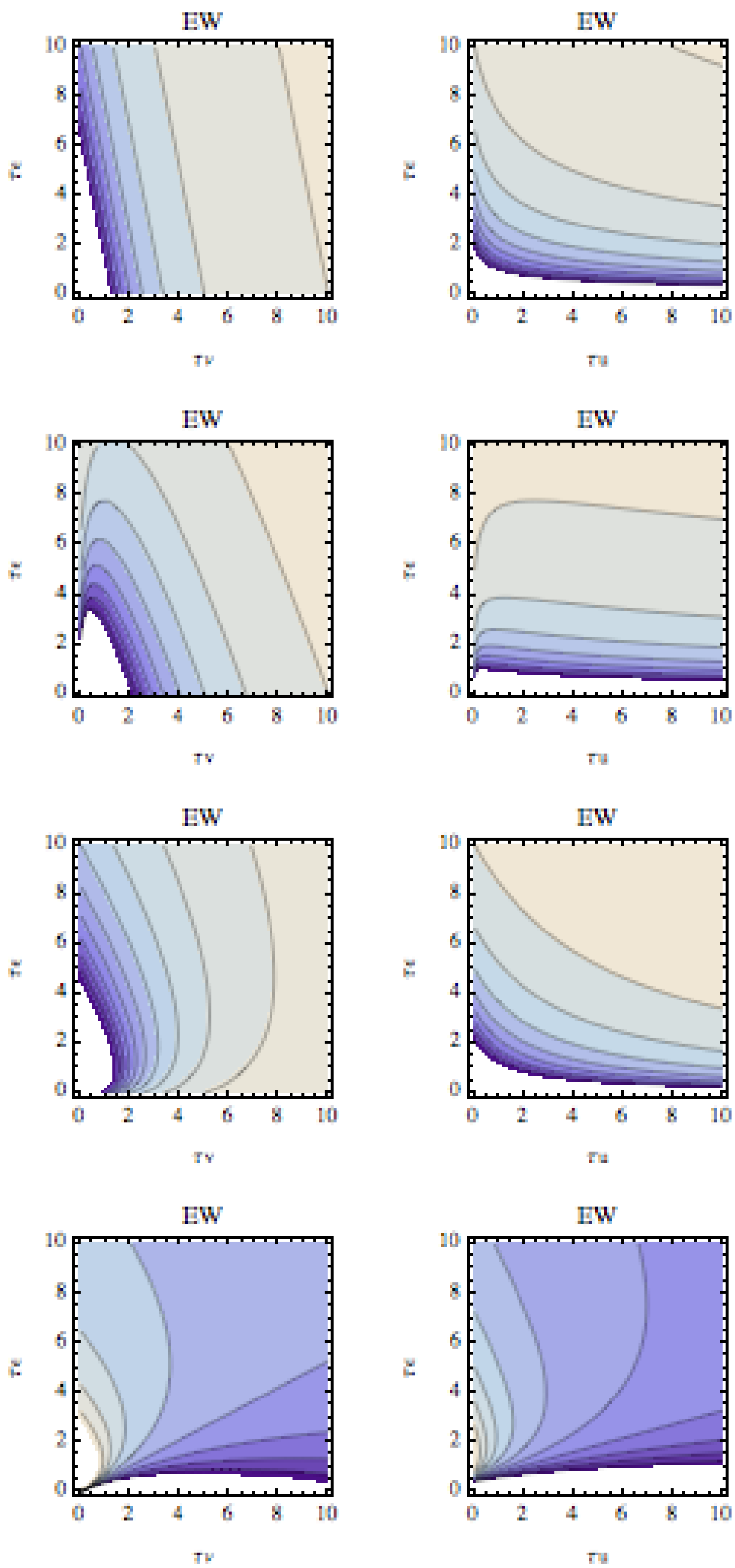


Figure 2: *Social welfare contours with exogenous (left) and endogenous (right) public information.* The horizontal axis displays the precision of the noise in the endogenous public statistic ($\tau_v$ or $\tau_u$) and the vertical axis displays the precision of private information ($\tau_\epsilon$). Welfare is larger (smaller) when the color is lighter (darker).

# 6 Concluding Remarks

I have investigated the social value of information with an endogenous public signal, in markets which are characterized by payoff externalities and heterogeneous information about fundamentals. I have considered that the public signal is a noisy aggregate statistic of the actions taken by the population, and hence is endogenous. This has been motivated by the observation that non-price systems to aggregate information are nowadays ubiquitous, and by the theoretical need to study information externalities which are independent of the market structure. I have found that endogenous public information causes an information externality because an agent in equilibrium does not take into account how his action influences the informativeness of the public signal. Consequently, I have shown that agents place an inefficiently low weight on private information over a larger payoff parameter region compared to exogenous public information, and that the efficient weight to private information is larger with endogenous than with exogenous public information. The way in which the different externalities interact influences the social value of public and private information. Importantly, I have demonstrated that the results of the social value of private information may be overturned when public information is endogenous in relation to when public information is exogenous. For example this occurs in the anti-beauty contest model, which presents a similar payoff structure as in competition à la Bertrand with product differentiation.

This paper could be extended in a number of ways. It would be interesting to ask how information externalities relate to different market structures, such as in Avdjiev, McGuire and Tarashev (2012) or Vives (2013). Another natural question to explore is the robustness of the analysis presented in this paper for more general information structures such as in Myatt and Wallace (2012) or Bergemann and Morris (2013), or in the rational inattention literature (e.g. Sims (2006) or Veldkamp (2011)).

# Appendix

**Proof of Proposition 1** First, I find the equilibrium strategy with full information, which will satisfy $U_q = 0$. Note that $U_{qq} < 0$ and therefore the second order condition is satisfied. Because of full information $q_i = Q = k(\theta)$, and since $U$ is quadratic, the full information strategy will be a linear function of fundamentals $k(\theta) = k_0 + k_1\theta$. A Taylor expansion of $U_q(q, q, 0, \theta)$ around $q = 0$ and $\theta = 0$ gives $U_q(k, k, 0, \theta) = U_q(0, 0, 0, 0) + U_{qq}k + U_{qQ}k + U_{q\theta}\theta$. Therefore, $k(\theta) = -\frac{U_q(0,0,0,0)}{U_{qq}+U_{qQ}} - \frac{U_{q\theta}}{U_{qq}+U_{qQ}}\theta$, and notice that $U_{qq} + U_{qQ} \neq 0$ since I have assumed that $\alpha < 1$.

Second, I find the equilibrium strategy with incomplete information and an endogenous public signal. A best response is a strategy $q'$, satisfies $E\left[U_q(q', Q, \sigma_q, \theta) \mid s_i, z\right] = 0$, where a Taylor expansion of $U_q$ around the full information equilibrium is given by: $U_q(q', Q, \sigma_q, \theta) = U_q(k, k, 0, \theta) + U_{qq}(q' - k) + U_{qQ}(Q - k)$, where $U_q(k, k, 0, \theta) = 0$ and $U_{qq} < 0$.

Therefore, using the definition of $\alpha$, I can write the expression for the best response strategy as

$$q'(s_i, z) = E\left[(1 - \alpha)k(\theta) + \alpha Q \mid s_i, z\right]. \tag{19}$$

Setting an arbitrary strategy of the form $q(s_i, z) = b + as_i + cE\left[\theta \mid z\right]$ and noting that $E\left[\theta|z, s_i\right] = \frac{\tau_\epsilon}{\tau_\epsilon+\tau}s_i + \frac{\tau}{\tau_\epsilon+\tau}E\left[\theta \mid z\right]$, I substitute in the best response strategy and match coefficients to find the fixed point and I obtain:

$$b = k_0,\ a = \frac{k_1(1-\alpha)\tau_\epsilon}{(1-\alpha)\tau_\epsilon+a^2\tau_u+\tau_\theta} = k_1\gamma \text{ and } c = \frac{k_1\tau}{(1-\alpha)\tau_\epsilon+a^2\tau_u+\tau_\theta} = k_1(1 - \gamma).$$

Note that the equation for $a$ is defined implicitly and I note that $\gamma$ will be the solution of a cubic equation given by

$$\pi(\gamma) = \gamma^3 k_1^2 \tau_u + \gamma((1 - \alpha)\tau_\epsilon + \tau_\theta) - (1 - \alpha)\tau_\epsilon, \tag{20}$$

where we are interested in finding $\pi(\gamma^m) = 0$. By the Descartes' Rule of signs, I note that there is only one sign change and therefore, there will be only a positive real root. If I also check $\pi(-\gamma)$, I notice that there are no sign changes, and therefore, there are no negative real roots. To conclude, this equation will have one positive real root and 2 imaginary real roots. I consider the positive real root. Additionally, I note that $\pi(0) = -(1 - \alpha)\tau_\epsilon < 0$ and $\pi(1) = k_1^2\tau_u + \tau_\theta > 0$, and therefore, the positive real root will be between 0 and 1. Therefore, the weight to private information will be a positive number between 0 and 1. ■

**Proof of Corollary 1** Differentiating $\pi(\gamma^m)$ with respect to $\tau_\epsilon$, I obtain that

$$\frac{\partial\gamma^m}{\partial\tau_\epsilon} = \frac{(1-\alpha)(1-\gamma^m)}{(1-\alpha)\tau_\epsilon+\tau_\theta+3(\gamma^m)^2k_1^2\tau_u} > 0,$$

since $0 < \gamma^m < 1$. Differentiating $\pi(\gamma^m)$ with respect to $\tau_\theta$,

$$\frac{\partial\gamma^m}{\partial\tau_\theta} = \frac{-\gamma^m}{(1-\alpha)\tau_\epsilon+\tau_\theta+3(\gamma^m)^2k_1^2\tau_u} < 0,$$

since $0 < \gamma^m < 1$. Similarly,

$$\frac{\partial\gamma^m}{\partial\tau_u} = \frac{-(\gamma^m)^3k_1^2}{(1-\alpha)\tau_\epsilon+\tau_\theta+3(\gamma^m)^2k_1^2\tau_u} < 0.$$

and differentiating $\pi(\gamma^m)$ with respect to $\alpha$:

$$\frac{\partial\gamma^m}{\partial\alpha} = \frac{-(1-\gamma^m)\tau_\epsilon}{(1-\alpha)\tau_\epsilon+\tau_\theta+3(\gamma^m)^2k_1^2\tau_u} < 0,$$

since $0 < \gamma^m < 1$, then $\frac{\partial\gamma^m}{\partial\alpha} < 0$. ■

**Proof of Proposition 2** First, I find the efficient strategy with full information,$k^*(\theta)$, which satisfies $W_Q(k^*, 0, \theta) = 0$, and the second order condition is satisfied since $W_{QQ} < 0$. A Taylor expansion of $W_Q$ around $k^*(\theta) = 0$ gives $W_Q(k^*, 0, \theta) = W_Q(0, 0, 0)+W_{QQ}k^*+W_{Q\theta}\theta = 0$. By the same argument as in Proposition 1, $k^*(\theta)$ will be a linear function of $\theta$, which can be written as $k^*(\theta) = k_0^* + k_1^*\theta$. Therefore, $k^*(\theta) = -\frac{W_Q(0,0,0)}{W_{QQ}} - \frac{W_{Q\theta}}{W_{QQ}}\theta$.
Second, we need to find

$$E[u] = \\ E[W(k(\theta), 0, \theta)] + E[W_Q(k(\theta), 0, \theta)(Q - k(\theta))] + \frac{W_{QQ}}{2}E\left[(Q - k(\theta))^2\right] + \frac{W_{\sigma\sigma}}{2}E\left[(Q - q)^2\right],$$

for a candidate equilibrium strategy. We first note that $Q-k(\theta) = k_1(1-\gamma)(E[\theta \mid z]-\theta))$ and $q - Q = k_1\gamma\epsilon_i$. The term $E[W_Q(k(\theta), 0, \theta)(Q - k(\theta))]$ is more involved. A Taylor expansion of $W_Q(k(\theta), 0, \theta))$ around $k(\theta) = k^*(\theta)$ is equivalent to $W_Q(k(\theta), 0, \theta)) = W_Q(k^*(\theta), 0, \theta)) + W_{QQ}(k(\theta) - k^*(\theta))$, which is equal to $W_{QQ}(k(\theta) - k^*(\theta))$ since $W_Q(k^*(\theta), 0, \theta)) = 0$. Now, the expectation term is equal to:

$$E[W_Q(k(\theta), 0, \theta)(Q - k(\theta))] = \frac{- \mid W_{QQ} \mid k_1^2\phi(1 - \gamma)}{\tau}, \tag{21}$$

since $E[(E[\theta \mid z] - \theta), \theta] = \frac{-1}{\tau}$ and $W_{QQ} < 0$, where the full information inefficiency is $\phi = \frac{k_1^*-k_1}{k_1}$. Therefore, ex ante utility at a candidate equilibrium strategy is equivalent to

$$E\left[u\right] = E\left[W(k(\theta),0,\theta)\right] - \frac{\mid W_{\sigma\sigma}\mid k_1^2}{2}\left(\frac{\gamma^2}{\tau_\epsilon} + \frac{(1-\alpha^*)(1-\gamma)^2}{\tau} + \frac{2(1-\alpha^*)\phi(1-\gamma)}{\tau}\right), \quad (22)$$

Third, I find the welfare planner's efficient strategy with incomplete information and an endogenous public signal. Note that the welfare planner internalizes all externalities, including payoff externalities with full information, which imply that $k^*(\theta) = k(\theta)$ and hence $\phi = 0$. Maximizing ex ante utility is equivalent to minimizing welfare losses at a candidate equilibrium strategy. The first order condition can be computed as $\frac{dWL}{d\gamma} = \left(\frac{\partial WL}{\partial\gamma}\right)_{\tau\, const} + \frac{\partial WL}{\partial\tau}\frac{\partial\tau}{\partial\gamma} = 0$, and hence $\frac{dWL}{d\gamma} = 0$ at $\gamma = \gamma^*$. The second order condition is satisfied since $\frac{d^2WL}{d\gamma^2} > 0$. Computing this derivative at $\gamma^*$, we notice that the first order condition is equivalent to

$$\left(\frac{\gamma^*}{\tau_\epsilon} - \frac{(1-\alpha^*)(1-\gamma^*)}{\tau^*} - \frac{(k_1^*)^2\tau_u\gamma^*(1-\alpha^*)(1-\gamma^*)^2}{(\tau^*)^2}\right) = 0. \quad (23)$$

which defines $\gamma^*$ implicitly as

$$\gamma^* = \frac{(1-\alpha^*)\tau_\epsilon}{(1-\alpha^*)\tau_\epsilon + \tau^* - (1-\alpha^*)(1-\gamma^*)^2\frac{\tau_u\tau_\epsilon(k_1^*)^2}{\tau^*}},$$

which defines $\gamma^*$ as a solution of a quintic equation which can be written as $\psi(\gamma^*) = 0$, where

$$\psi(\gamma) = (\gamma)^5\tau_u^2k_1^4 + 2(\gamma)^3k_1^2\tau_u\tau_\theta + (\gamma)^2k_1^2\tau_u\tau_\epsilon(1-\alpha^*) + \gamma(\tau_\theta^2 + \tau_\epsilon(1-\alpha^*)(\tau_\theta - \tau_u k_1^2)) - \tau_\epsilon\tau_\theta(1-\alpha^*).$$

Applying Descartes' Rule of signs to find out the number of real roots, I notice that there is only one change in sign, and therefore, this polynomial will have only one real positive root. By looking at the number of roots of $\psi(-\gamma^*)$, I notice that there are two sign changes and therefore, this polynomial will have either 2 or 0 negative real roots (and 2 or 4 imaginary roots, respectively). The second order condition requires that $\gamma^* > 0$ and hence I do not consider the negative roots. Notice that $\psi(0) = -\tau_\epsilon\tau_\theta(1-\alpha^*) < 0$, while $\psi(1) = \tau_u^2k_1^4 + 2k_1^2\tau_u\tau_\theta + \tau_\theta^2 > 0$. Therefore, the unique positive real root will be between 0 and 1. ■

**Proof of Corollary 2** Using $\chi(\gamma)$ to derive the derivative of $\frac{\partial\gamma^*}{\partial\tau_\epsilon}$ I note that:

$$\frac{\partial\gamma^*}{\partial\tau_\epsilon} = \frac{\gamma^*((1-\alpha^*)(\tau_u k_1^2))(1-\gamma^*)}{5(\gamma^*)^4\tau_u^2k_1^4 + 6(\gamma^*)^2k_1^2\tau_u\tau_\theta + 2\gamma^*k_1^2\tau_u\tau_\epsilon(1-\alpha^*)} > 0,$$

since $0 < \gamma^* < 1$. And

$$\frac{\partial\gamma^*}{\partial\tau_\theta} = \frac{-2\gamma^*\tau^* + \tau_\epsilon(1-\alpha^*)(1-\gamma^*)}{5(\gamma^*)^4\tau_u^2k_1^4 + 6(\gamma^*)^2k_1^2\tau_u\tau_\theta + 2\gamma^*k_1^2\tau_u\tau_\epsilon(1-\alpha^*)} < 0,$$

can be easily shown to be negative by the definition of $\gamma^*$. I find that

$$\frac{\partial \gamma^*}{\partial \tau_u} = k_1^2 \gamma^* \left( \frac{-2(\gamma^*)^2\tau^* + \tau_\epsilon(1-\alpha^*)(1-\gamma^*)}{5(\gamma^*)^4\tau_u^2 k_1^4 + 6(\gamma^*)^2 k_1^2 \tau_u \tau_\theta + 2\gamma^* k_1^2 \tau_u \tau_\epsilon (1-\alpha^*)} \right),$$

whose sign of this is is ambiguous. Finally,

$$\frac{\partial \gamma}{\partial \alpha^*} = \frac{-(\gamma\tau_\epsilon\tau_u k_1^2 + \tau_\epsilon\tau_\theta)(1-\gamma^*)}{5(\gamma^*)^4\tau_u^2 k_1^4 + 6(\gamma^*)^2 k_1^2 \tau_u \tau_\theta + 2\gamma^* k_1^2 \tau_u \tau_\epsilon (1-\alpha^*)} < 0,$$

since $0 < \gamma^* < 1$. ■

**Proof of Proposition 3** Under the conditions defined in the Proposition 2, $WL$ is strictly convex in $\gamma$, and $\frac{WL(\gamma)}{d\gamma} = 0$ at $\gamma = \gamma^*$. Therefore $sign\left(\frac{dWL}{d\gamma} \mid_{\gamma=\gamma^m}\right) = sign\left(\gamma^m - \gamma^*\right)$, which implies that

$$sign\left(\gamma^m - \gamma^*\right) = sign\left(\alpha^* - \alpha - \frac{(1-\alpha^*)(1-\alpha)(k_1^2\tau_u\tau_\epsilon)}{((1-\alpha)\tau_\epsilon + \tau)^2}\right), \tag{24}$$

where the information externality is $\triangle = \frac{(1-\alpha^*)(1-\alpha)(k_1^2\tau_u\tau_\epsilon)}{((1-\alpha)\tau_\epsilon+\tau)^2} < 0$. For the three cases considered, note that:

a) If $\alpha = \alpha^*$ then $sign\left(\gamma^m - \gamma^*\right) = sign\left(-\frac{(1-\alpha^*)(1-\alpha)(k_1^2\tau_u\tau_\epsilon)}{((1-\alpha)\tau_\epsilon+\tau)^2}\right) < 0$.
Now suppose that $\alpha^*(\alpha) = \xi\alpha$, where $\xi \geq 0$. Notice that the equilibrium and efficient weights to private information are both strictly decreasing function of $\alpha$, since $\gamma(\alpha)$ and $\gamma^*(\alpha^*(\alpha))$, which range from 1 to 0 as $\alpha \to -\infty$ to $\alpha \to 1$ and $\alpha^* \to 1$, respectively. Therefore, using the intermediate value theorem, I can show that there exists a unique intersection between the two curves $-\infty < \alpha^0 < 1$, except for the case when $\xi = 1$, which is equal to: $\alpha^0 = \frac{(1-\alpha^*)(1-\alpha)(k_1^2\tau_u\tau_\epsilon)}{(\xi-1)((1-\alpha)\tau_\epsilon+\tau)^2}$. Notice that

$$sign(\alpha^0) = sign\left(\frac{(1-\alpha^*)(1-\alpha)(k_1^2\tau_u\tau_\epsilon)}{(\xi-1)((1-\alpha)\tau_\epsilon+\tau)^2}\right) = sign(\xi - 1).$$

Using the result of the first part of the Corollary, it follows immediately that

$$sign\left(\gamma - \gamma^*\right) = sign\left(\alpha(\xi-1) - (\xi-1)\alpha^0\right) = sign\left((\xi-1)(\alpha-\alpha^0)\right).$$

and results b) and c). ■

**Proof of Corollary 3** i) This is an immediate consequence of the first part of Corollary 3.

ii) Note that $\gamma^*$ is the solution of $\frac{dWL}{d\gamma} = \frac{\partial WL}{\partial\gamma} \mid_{\tau=const} + \frac{\partial WL}{\partial\tau}\frac{\partial\tau}{\partial\gamma} = 0$, which implies that $\frac{\partial WL}{\partial\gamma} \mid_{\tau=const} > 0$ since and $\frac{\partial WL}{\partial\tau}\frac{\partial\tau}{\partial\gamma} < 0$ (because $\frac{\partial WL}{\partial\tau} < 0$ and $\frac{\partial\tau}{\partial\gamma} > 0$). Also note that $\gamma^{*exo}$ is the solution of $\frac{\partial WL}{\partial\gamma} \mid_{\tau=const}$. Therefore, because $WL$ is strictly convex, then $\gamma^* > \gamma^{*exo}$. ■

**Proof of Proposition 4** Comparative statics of ex ante utility with respect to $\tau_u$ are given by (11), which are equivalent to the opposite comparative statics of equilibrium welfare loss given by (9) with respect to $\tau_u$. I obtain the following two expressions

$$\left(\frac{\partial WL}{\partial \tau}\right)_{\gamma\, cons.} = \frac{\mid W_{\sigma\sigma} \mid (k_1)^2}{2}\left(\frac{(1-\alpha)((2\alpha-1-\alpha^*)-2\phi(1-\alpha^*))\tau_\epsilon-(1-\alpha^*)(1+2\phi)\tau}{(\tau_\epsilon(1-\alpha)+\tau)^3}\right), \tag{25}$$

and

$$\frac{d\tau}{d\tau_u} = k_1^2\gamma^2 + 2k_1^2\tau_u\gamma^m\frac{\partial\gamma^m}{\partial\tau_u} = k_1^2\gamma^2\left(\frac{(1-\alpha)\tau_\epsilon+\tau_\theta+(\gamma^m)^2k_1^2\tau_u}{(1-\alpha)\tau_\epsilon+\tau_\theta+3(\gamma^m)^2k_1^2\tau_u}\right). \tag{26}$$

Notice that $0 < \frac{d\tau}{d\tau_u} < k_1^2\gamma^2$. Therefore,

$$\frac{d(E[u(\gamma^m)])}{d\tau_u} \geq 0 \Longleftrightarrow \frac{(1-\alpha)(2\alpha-\alpha^*-1-2\phi(1-\alpha^*))}{(1-\alpha^*)} \leq \frac{\tau(1+2\phi)}{\tau_\epsilon}. \blacksquare$$

**Proof of Proposition 5** Comparative statics of ex ante utility with respect to $\tau_\epsilon$ are given by (13), which are equivalent to the opposite comparative statics of equilibrium welfare loss given by (9) with respect to $\tau_\epsilon$. I obtain the following expressions:

$$\left(\frac{\partial WL(\gamma^m)}{\partial \tau_\epsilon}\right)_{\tau\, cons.} = \frac{\mid W_{\sigma\sigma} \mid (k_1)^2}{2}\left(\frac{-(1-\alpha)(1-\alpha+2\phi(1-\alpha^*))\tau_\epsilon+((2\alpha^*-\alpha-1)-2\phi(1-\alpha^*))\tau}{(\tau_\epsilon(1-\alpha)+\tau)^3}\right), \tag{27}$$

$\left(\frac{\partial WL(\gamma^m)}{\partial \tau}\right)_{\gamma\, cons.}$ is given in expression (24), and

$$\nu = \frac{d\tau}{d\tau_\epsilon} = (2k_1^2\tau_u\gamma^m\frac{\partial\gamma^m}{\partial\tau_\epsilon}) = \frac{2k_1^2\tau_u(1-\alpha)(1-\gamma^m)\gamma^m}{(1-\alpha)\tau_\epsilon+\tau_\theta+3(\gamma^m)^2k_1^2\tau_u} > 0 \tag{28}$$

Combining expressions (26) and (27), I obtain that $\frac{d(E[u(\gamma^m)])}{d\tau_u} \geq 0 \Longleftrightarrow \varphi\tau_\epsilon + \chi\tau \leq 0$, where

$$\varphi = -(1-\alpha)^2(1-\alpha+2\phi(1-\alpha^*)) + \upsilon(1-\alpha)(2\alpha-1-\alpha^*-2\phi(1-\alpha^*))$$

and

$$\chi = (1-\alpha)(2\alpha^*-\alpha-1) - 2\phi(1-\alpha)(1-\alpha^*) - \upsilon(1-\alpha^*)(1+2\phi) \blacksquare$$

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