The Distributional Effects of Monetary Policy in a Life Cycle Model *

Tobias Cwik
Swiss National Bank

Aeimit Lakdawala
Michigan State University

William B Peterman
Federal Reserve Board of Governors

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Abstract

We explore the distributional effects of monetary policy in an overlapping generation heterogeneous agent model. In the life cycle model, agents endogenously determine consumption, savings, and their portfolio in the face of idiosyncratic income shocks, uncertain mortality, aggregate productivity shocks, and monetary policy shocks, which gives rise to inequality both within and between the cohorts. This life cycle diversity, along with nominal rigidities create a rich environment where monetary policy can differentially affect agents through three channels. In particular: (i) young agents rely more heavily on labor income relative to the old, (ii) agents tend to transition from net borrowers to net savers as they age, and (iii) agents' portfolio allocation tends to change over the life cycle. This paper aims to quantitatively assess the relative importance of these channels by considering the short-run and long-run effects of monetary policy on different agents within the model. In particular, we assess the different implications of a monetary policy shock for agents both across and between cohorts. Additionally, we determine how overall inequality and welfare change in the long run when monetary policy focuses more on inflation. Overall, our results indicate that due to these channels monetary policy can have quantitatively different effects on agents both between and within cohorts.

Keywords: Monetary Policy, Inequality, Overlapping Generations, Portfolio Choice, Life Cycle
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*Views expressed in this paper are the authors' own and do not reflect the view of the Federal Reserve System or the SNB. Emails: william.b.peterman@frb.gov, aeimit@msu.edu, and tobias.cwik@snb.ch.

1 Introduction

> "The extent of and continuing increase in inequality in the United States greatly concerns me."
>
> Janet Yellen, Federal Reserve Chairman, October 17^{th} 2014.

Among both academics and policymakers there is a growing interest in the potential relationship between monetary policy and inequality. Specifically, monetary policy may affect agents differently depending on where they stand in their life cycle, the wealth they are endowed with or the income prospects they face. Moreover, in light of increasing income and wealth inequality in the US in the past few decades, various economic commentators and policymakers have linked monetary policy and overall inequality.[1] There is some recent empirical evidence that suggests that monetary policy actions affect overall inequality (see, for example, Coibion et al. (2012)). But until recently, the theoretical models analyzing monetary policy mostly focused on representative agent models with no role for distributive effects or consequences for overall inequality. This paper adds to this literature by examining how the implications of monetary policy vary for different agents in a heterogenous agent life cycle model and how this variation affects overall inequality.

We consider a life cycle model that combines a standard heterogenous agent overlapping generations model with the nominal frictions of a New Keynesian setup. The firm dynamics are based on the textbook sticky price models as in Galí (2008) and Woodford (2003), where monetary policy has real effects. On the household side, idiosyncratic income shocks and a life cycle setting creates a rich environment with heterogeneity both within and across cohorts. Additionally, households are allowed to borrow or save and they optimally choose the proportion of safe and risky assets in their portfolio, adding another channel that introduces heterogeneity. Monetary policy follows an interest rate rule a la Taylor (1993), where the nominal interest rate responds to inflation and output. Within the context of this model, we analyze whether the effects of monetary policy vary over the life cycle and how this contributes to overall inequality.

Our analysis focuses on three potential channels through which monetary policy can differentially affect households over the life cycle. First, as households age they tend to differ in the composition of their income. Younger households tend to rely more heavily on

[1] See the speech by Fed Chair Janet Yellen on "Perspectives on Inequality and Opportunity from the Survey of Consumer Finances", speech by St. Louis Fed President James Bullard titled "Income Inequality and Monetary Policy: A Framework with Answers to Three Questions",

labor income while older households receive a bigger share of their income from returns on asset holdings. Therefore, if expansionary monetary policy boosts the return to savings more than wages, then older households would benefit more than younger households. Second, as households age they tend to transition from net borrowers to net savers. Thus an unexpected decrease in interest rates or increase in inflation will benefit borrowers and hurt savers.[2] In contrast to the first channel, the second channel may potentially lead expansionary monetary policy to be relatively more beneficial for younger households. Third, as households age, they tend to change their portfolio composition. Since monetary policy can affect the returns to risky and risk free assets differently, it can have a differential effect on young versus old households. Given that these three channels cause expansionary monetary policy to be more beneficial for different agents, who could benefit most from expansionary monetary policy is uncertain. Thus, we use our life cycle model to determine in detail how households are differentially affected by monetary policy and decompose the effects of each of these three channels. Moreover, we determine whether this potentially unequal affect of monetary policy contributes to overall inequality.

We begin by analyzing the stochastic steady state of our model and compare the model's predictions for consumption and wealth over the life cycle to the data. Additionally, we use data from the Survey of Consumer Finances and confirm that, in many dimensions, the model also matches the pattern of the portfolio holdings over the life cycle. This gives us confidence that we have a reasonable model to explore the distributive effects of monetary policy through the channels described above. After confirming that these predictions match the data, we next perform various exercises that examine both the short run and long run effects of monetary policy over the life cycle and the implication for inequality.

We focus on the dynamic effects of monetary policy in three different exercises. First, we study the response to an unanticipated monetary policy shock, which affects households' income and wealth differentially based on their asset holdings and their position in the life cycle through the channels discussed above. Our preliminary results suggest that a contractionary monetary policy shock leads to an overall increase in welfare. The welfare increase is primarily due to the return on risky assets increasing for many subsequent periods after initially dropping. The rise in the return to risky assets after the initial shock is generated in our model because aggregate capital falls dramatically after the shock leading to an increase in the marginal product of capital. This exaggerated effect is because we have not yet calibrated the adjustment cost parameters leading agents to be too responsive in

[2]See Doepke and Schneider (2006) for empirical evidence of this channel.

their savings decisions. Thus, these results should be viewed as a illustrative example of the potential implications of monetary policy.[3]

This overall welfare increase is felt differentially. Agents in their 50s tend to benefit more than younger workers and retirees. In particular, we find that the welfare benefits from the shock are approximately twice as large as the benefits for younger workers, because older agents hold a larger stock of the risky asset. Moreover, we find that households who are endowed with a higher ability level tend to hold a smaller share of their wealth in the risky asset. Thus, the welfare benefit from the shock tends to be smaller for these higher ability households, compared to lower ability households, who are more invested in risky assets. These preliminary results indicate that all three channels have a quantitatively important role in causing a monetary policy shock to have an uneven affect across agents.

Next, we plan on studying the effect of monetary policy in a downturn induced by an adverse TFP shock. In particular, during this adverse business cycle episode, we will determine how the differential effect of monetary policy over the life cycle changes under different monetary policy regimes. Moreover, we will examine whether the differential effects leads to more or less overall inequality after the shock. In addition, we plan on considering the long run implications of changes in the endogenous component of monetary policy actions. In particular, we will examine the differences in both inequality and welfare in the stochastic steady states under different formulations of the monetary policy rule, by allowing for a higher or lower response to inflation.

Our paper is related to the recent research on the uneven effect of monetary policy across individuals and its overall implications for inequality. On the empirical side, Coibion et al. (2012) study the impact of monetary policy shocks on inequality. Using the CEX database to construct inequality measures, they find that contractionary monetary policy shocks raise inequality due to uneven effects across individuals. Higher interest rates lead financial income to increase, which disproportionately benefits agents with higher wealth. Moreover, using the same database Wong (2014) finds that young households tend to respond more to contractionary monetary policy shocks than older ones. These empirical findings establish a need for a clear theoretical analysis that will help shed light on the channels through which their results work. In a recent paper Auclert (2014) argues that the heterogeneity in the marginal propensity to consume leads to a "redistribution" channel through which monetary policy actions can have aggregate effects . Two additional related empirical stud-

[3]We are currently working on calibrating the model, however given the computational intensity of this model we were unable to completely calibrate the model prior to the submission deadline.

ies that include a life cycle model in a New Keynesian framework are Sterk and Tenreyro (2013) and Kara and Von Thadden (2010). Sterk and Tenreyro (2013) examine alternative mechanisms for monetary policy to affect economic activity that operates in the absence of nominal rigidities due to the life cycle setting. Kara and Von Thadden (2010) examine what affect changes in demographics have on the equilibrium interest rates.

On the theoretical side, perhaps the paper most similar to ours is the recent work by Gornemann et al. (2014). They consider the intergenerational redistribution of monetary policy in a stochastic aging model. While they tackle a similar question, their focus is more on the labor decision of households and the unemployment process. Their rich setup of the labor market comes at the expense of life cycle heterogeneity, where they use a parsimonious stochastic aging setup. Thus we view their paper as providing interesting complementary results to our work. In a related paper, Gornemann et al. (2012) show in a heterogenous agent model with infinitely living households that an unanticipated monetary policy contraction raises income and welfare of the wealthiest 5 percent, while the other agents see a fall in their income and welfare.

Our paper is also related to a growing theoretical literature, which studies distributional effects in heterogenous agent overlapping generations models under flexible prices. Two examples include Hur (2013) and Glover et al. (2010) which analyze the impact of the recent Great Recession on income and wealth of different generations in a life cycle model. Hur (2013) finds that younger generations suffered the largest welfare decline due to a drop in labor income, while older generations encountered the largest decline in wealth due to the fall in asset prices. Glover et al. (2010) show that young generations can be better off, if older generations have a strong incentive to sell assets in downturns to smooth consumption. This leads asset prices to fall more strongly than wages, benefiting younger generations. But these papers tend to exclude any role for monetary policy.

2 Model

For the analysis we employ a general equilibrium heterogenous agent life cycle model. Agents enter the economy when they start working and face an age-dependent probability of dying at each age. They supply labor inelastically and can smooth their consumption over time by saving in two different assets, bonds with a risk free return (risk free asset) and a risky asset. They face adjustment costs, when they change their level of risky asset holdings.

The risky asset is invested in physical capital, which is rented out to intermediate good firms for production. We assume there is a continuum of intermediate good producers operating under monopolistic competition and being constrained in price setting by quadratic price adjustment costs, following Rotemberg (1982). They produce using a Cobb-Douglas production function. Competitive final goods firms combine intermediate goods to provide final goods for private and government consumption and investment into the physical capital stock.

The government finances its consumption and social security benefits using the revenues from taxing labor income of the households. Monetary policy determines the risk free nominal interest rate at which the agents can borrow or lend. The way it sets the interest rate is characterized by an interest rate rule a la Taylor (1993). In the following we give a formal exposition of the optimization problem of the individuals, the firms and the government.

2.1 Demographics

Time is assumed to be discrete, and the model period is equal to one year. The economy is populated by S overlapping generations of ages $j = 20, 21, ..., J$, with J being the maximum possible age an agent can live until. The size of each new cohort entering the economy grows at a constant rate n. Lifetime length is uncertain with mortality risk varying over the lifetime. Conditional on being alive at age s, Ψ_j is the probability of an agent living to age $j+1$. Since agents are not certain how long they will live, they may die while still holding savings. If an agent dies with savings, the assets are confiscated by the government and distributed equally to all the living agents as transfers (tr_t). An agent works until age s_r, at which point they are exogenously forced to retire.

2.2 Individuals

Each period, agent's labor earnings are given by $w_t l_t$, where w_t represents a wage rate per efficiency unit of labor and l_t is an agents total labor units. These labor units can be represented by $l_t = \omega_{i,j} h$. In particular, h are the hours spent providing labor services, which are exogenous set at $h = \overline{h}$. Moreover, $\omega_{i,j}$ is the idiosyncratic labor productivity of agent i at age j which follows:

$$\log \omega_{i,j} = \theta_j + \alpha_i + \nu_{i,t}. \tag{1}$$

In this specification, θ_j governs the average age-profile of wages (or age-specific human capital), $\alpha \sim NID(0, \sigma_\alpha^2)$ is an individual-specific fixed effect (or ability) that is observed at birth and stays fixed for an agent over the life cycle, and $\nu_{i,t}$ is a persistent shock, also received each period, which follows a first-order autoregressive process:

$$\nu_{i,t} = \rho\nu_{i,t-1} + \psi_{i,t} \text{ with } \psi_{i,t} \sim NID(0, \sigma_\nu^2) \text{ and } \nu_1 = 0. \tag{2}$$

Thus, an agent works a stochastic amount of time in each period which is exogenous.

Agents are to invest using two different assets. First, agents can save using a bond (or risk-free asset), $b_{i,j,t}$, with a gross risk free real return of r_t. Additionally, agents can save using a a risky asset, $a_{i,j,t}$. The risky asset is invested in the physical capital stock. By renting the risky assets to intermediate goods firms for production they receive a return of r_t^k. Agents are allowed to borrow using the risk free asset up to a certain limit, $\underline{b}$, such that $b_{i,j,t} \geq \underline{b}$ in any period. If agents borrow they are required to pay a return of $\frac{r}{\Psi_j}$ to account for the probability of death. An agent's total savings is represented by $s_{i,j,t} = a_{i,j,t} + b_{i,j,t}$, and the share of the total savings invested in the risky asset is represented by $\mu_{i,j,t}^a$. In each period the risky asset depreciates at a rate of δ. Moreover, there is an adjustment cost to increasing or decreasing the level of asset holdings after replacing depreciated capital in each period, $\varphi(s_{i,j,t+1}\mu_{i,j,t}^a - s_{i,j,t}\mu_{i,j,t}^a)$.

Therefore, in each period, after receive their labor income a pre-retired agent chooses how much to consume, how much to save in bonds, and how much to save in risky assets. The agent maximizes the following value function prior to retirement,

$$W(X_t, \omega_{i,j,t}, s_{i,j,t}, \mu_{i,j,t}^a) = \max_{c_{i,j,t}, s_{i,j,t+1}, \mu_{i,j,t}^a} \left\{ u(c_{i,j,t}) + \Psi_j \mathbb{E}_t W(X_{t+1}, \omega_{i,j,t+1}, s_{i,j,t+1}, \mu_{i,j,t+1}^a) \right\} \tag{3}$$

s.t.

$$c_{i,j,t} + s_{i,j,t+1} = s_{i,j,t}\mu_{i,j,t}^a(1 + r_t^k - \delta) + s_{i,j,t}(1 - \mu_{i,j,t}^a)r_t + w_t\omega_{i,j,t}h(1-\tau) \tag{4}$$

$$-\varphi(s_{i,j,t}\mu_{i,j,t+1}^a - s_{i,j,t}\mu_{i,j,t}^a) + tr_t + div_t \tag{5}$$

where $c_{i,j,t}$, $w_t\omega_{i,j,t}h$, and τ are consumption, labor income, and the tax rate, respectively. After an agent retires, they no longer receive labor income. However, they are eligible to receive a social security benefit b_{ss} which is the same for all living agents. A retired agent maximizes the following value function

$$W(X_t, \omega_{i,j,t}, s_{i,j,t}, \mu_{i,j,t}^a) = \max_{c_{i,j,t}, s_{i,j,t+1}, \mu_{i,j,t}^a} \left\{ u(c_{i,j,t}) + \Psi_j \mathbb{E}_t W(X_{t+1}, \omega_{i,j,t+1}, s_{i,j,t+1}, \mu_{i,j,t+1}^a) \right\} \tag{6}$$

s.t.

$$
\begin{aligned}
c_{i,j,t} + s_{i,j,t+1} &= s_{i,j,t}\mu^a_{i,j,t}(1 + r^k_t - \delta) + s_{i,j,t}(1 - \mu^a_{i,j,t})r_t && (7)\\
&\quad -\varphi(s_{i,j,t}\mu^a_{i,j,t+1} - s_{i,j,t}\mu^a_{i,j,t}) + b_{ss} + tr_t + div_t && (8)
\end{aligned}
$$

2.3 Firms

2.3.1 Final Goods Producer

Final goods are composites of intermediate goods produced by a continuum of monopolistic competitive firms and are used for domestic consumption c_t, government consumption g_t and investment into physical capital. We use $i \in [0, 1]$ to index intermediate good firms as well as their products and prices. Final goods firms operate under perfect competition and purchase intermediate goods, $y_t(i)$. They use the following Dixit-Stiglitz aggregation technology

$$
y_t = \left(\int_0^1 y_t(i)^{\frac{1}{1+\lambda_p}} di \right)^{1+\lambda_p} \tag{9}
$$

where $(1 + \lambda_p = \frac{\epsilon}{\epsilon - 1})$ denotes the price markup and ϵ the elasticity of substitution. The representative final goods firms produce y_t while minimizing expenditures and take each input price $P_t(i)$ and the price of its final good P_t as given. The problem of the representative final good producer looks as follows:

$$
\max P_t y_t - \int_0^1 P_t(i) y_t(i) \tag{10}
$$

s.t.

$$
y_t = \left(\int_0^1 y_t(i)^{\frac{1}{1+\lambda_p}} di \right)^{1+\lambda_p} \tag{11}
$$

The resulting demand function for an individual intermediate good i is given by

$$
y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{\frac{-(1+\lambda_p)}{\lambda_p}} y_t \tag{12}
$$

2.3.2 Intermediate Good Producer

Each intermediate good producer i buys labor l and capital services k_t at the competitive real rates w_t and r^k_t and sells its output to final goods producers under monopolistic competition at the nominal price $P_t(i)$. The production of intermediate goods, $y_t(i)$, is governed by a Cobb-Douglas production function, where $k_t(i)$ denotes capital used for production of firm i and l denotes labor.

$$y_t(i) = Z_t(k_t(i))^{\alpha}(l)^{(1-\alpha)} \tag{13}$$

Z_t is a total factor productivity shock, which follows a first-order autoregressive process:

$$\ln Z_t = (1-\rho_Z)\ln Z + \rho_Z \ln Z_{t-1} + \epsilon_{Z,t} \tag{14}$$

Each intermediate good firm minimizes their cost of production given their production technology

$$\min r_t^k k_t(i) + w_t l \tag{15}$$

s.t.

$$y_t = Z_t(k_t(i))^{\alpha}(l)^{(1-\alpha)} \tag{16}$$

Minimizing w.r.t. capital and labor delivers the following first order conditions

$$w_t = (1-\alpha) mc_t(i) \frac{y_t(i)}{l} \tag{17}$$

$$r_t^k = \alpha mc_t(i) \frac{y_t(i)}{k_t(i)} \tag{18}$$

where $mc_t(i)$ is the lagrange multiplier on the production function and denotes real marginal costs. In equilibrium each intermediate good producer faces the same marginal costs and chooses the same factor inputs. Therefore $mc_t(i) = mc_t$ and $k_t(i) = k_t$.

Solving the first order conditions for real marginal costs implies

$$mc_t = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} w_t^{1-\alpha} \left(r_t^k\right)^{\alpha} Z_t^{-1} \tag{19}$$

We assume that each firm has the opportunity to change its price every period given Rotemberg (1982) quadratic price adjustment costs. The optimal price $\widetilde{P}_t(i)$ set by firm i at time t results from the following optimization problem

$$\max E_t \sum_{s=0}^{\infty} Q(X_t, X_{t+s}) \left[\frac{\widetilde{P}_t(i)}{P_t} - mc_{t+s}\right] y_{t+s}(i) - \frac{\phi_\pi}{2}\left(\frac{\widetilde{P}_t(i)}{\widetilde{P}_{t-1}(i)} - \pi\right)^2 \tag{20}$$

subject to the demand function defined by (12). Price adjustments are costly for the firms, if they change their price more or less than the steady-state inflation rate π. ϕ_π characterizes the size of the price adjustment costs and thus governs the degree of price stickiness in the model. We assume that intermediate good firms are owned by the agents and profits are distributed to them in form of dividends div_t.[4] Therefore future profits of intermediate good firms are discounted with the agents stochastic discount factor $Q(X_t, X_{t+1})$, which is defined as follows

$$1 = \mathbb{E}_t \left(Q(X_t, X_{t+1}) \frac{R_t}{\pi_{t+1}} \right) \tag{21}$$

The gross nominal interest rate R_t is set by the central bank and the gross risk free real rate the agents receive or pay on holding or borrowing in bonds is given by

$$r_t = \frac{R_t}{\mathbb{E}_t \pi_{t+1}} \tag{22}$$

Maximizing the price setting problem of the intermediate good producer w.r.t. the optimal price and imposing that due to symmetry in equilibrium all intermediate good producers set the same price $P_t(i) = P_t$, it follows

$$(1-\epsilon)Y_t + mc_t \epsilon Y_t - \phi_\pi \pi_t (\pi_t - \pi) + E_t Q(X_t, X_{t+1}) \pi_{t+1} \phi_\pi (\pi_{t+1} - \pi) \tag{23}$$

2.4 Government

2.4.1 Monetary authorities

Empirical evidence shows that interest rate rules a la Taylor (1993) describe monetary policy making in recent decades well. Taylor type rules raise the nominal interest rate R_t, when inflation π_t is above the inflation target π or when output exceeds the non-stochastic steady-state output. The parameters $\rho_\pi > 1$ and $\rho_Y \geq 0$ determine how fast monetary policy stabilizes inflation and output, respectively.

$$\ln \left(\frac{R_t}{R} \right) = \rho_\pi \ln \left(\frac{\pi_t}{\pi} \right) + \rho_Y \ln \left(\frac{y_t}{y} \right) + D_t \tag{24}$$

D_t is a first order autoregressive process, capturing persistent deviations of monetary policy from systematic behavior.

$$\ln D_t = \rho_D \ln D_{t-1} + \epsilon_{D,t} \tag{25}$$

[4] For simplicity we assume in the current draft of the life cycle model that profits are dumped into the ocean.

2.4.2 Fiscal authorities

Finally, we assume that the government runs a balanced budget every period and uses the revenues from labor income taxes to fund social security benefits and government consumption g_t, which is considered non-productive. Therefore the governments budget constraint is

$$g_t + bss = \tau y_t \tag{26}$$

3 Calibration

Prior to solving the model, it is necessary to choose functional forms and calibrate the model's parameters. Calibrating the model involves a two-step process. The first step is choosing parameter values for which there are direct estimates in the data. Second, to calibrate the remaining parameters, values are chosen so that certain targets in the model match the values observed in the U.S. economy (Simulated Method of Moments). Table 1 summarizes the parameter values.[5]

3.1 Demographics

In the model, agents are born at a real-world age of 20 that corresponds to a model age of 1. Agents are exogenously forced to retire at a real-world age of 66. If an individual survives until the age of 100, he dies the next period. We set the conditional survival probabilities in accordance with the estimates in Bell and Miller (2002). We adjust the size of each cohort's share of the population to account for a population growth rate of 1.1 percent.

3.2 Individuals

Agents have time-separable preferences over consumption and labor services. Conditional on survival, agents discount their future utility by β. We use $u(c_{i,j,t}) = \frac{c_{i,j,t}^{1-\sigma_1}}{1-\sigma_1}$, as the utility function, where σ_1 is the parameter that controls risk aversion. We follow previous studies and set σ_1 equal to 2. We determine β such that steady-state interest rate is 4% at an annualized rate.

[5]Since this is a general equilibrium model, changing one parameter will alter all the values in the model that are used as targets. However, we present targets with the parameter that they most directly correspond to.

Table 1: Calibrated Parameters

Parameter	Value	Description
		Households
σ_1	2	Relative risk aversion
ρ	0.958	Individual labor productivity
σ_ν^2	0.017	Std individual labor productivity
σ_α^2	0.065	Individual-specific fixed effect
φ_1	0.06	Risky asset adjustment costs
φ_2	1.6	Risky asset adjustment costs
δ	8.3%	Depreciation rate
		Firms
ϵ	7	Elasticity of substitution
α	0.36	Capital share
ϕ_π	172	Quadratic price adjustment costs
ρ_z	.75	Total factor productivity shock
σ_z	.00016	Std total factor productivity shock
		Government
π	2%	Inflation target
ρ_π	1.5	Inflation response monetary policy rule
ρ_Y	1	Output response monetary policy rule
ρ_d	.7	Monetary policy shock
σ_D	.0000625	Std monetary policy shock
τ_k	.2	Capital tax
λ_h	.15	Labor tax rate

An agent's idiosyncratic labor productivity $\omega_{i,j}$ is composed of three components: (i) age specific human capital (θ_j), (ii) an individual-specific fixed effect (α_i), and (iii) a persistent shock ($\nu_{i,t}$). The idiosyncratic labor productivity process is calibrated based on the estimates from the PSID data in Kaplan (2012).[6] Both the permanent and persistent shocks to individuals' productivity are distributed normal with a mean of zero where $\rho = 0.958$, $\sigma_\alpha^2 = 0.065$, and $\sigma_\nu^2 = 0.017$.[7] We discretize these shocks in order to solve the model, using two states to represent the permanent shock and five states for the persistent shock.[8] For expositional convenience, we refer to the two different states of the permanent shock as high and low ability.

Agents are able to adjust their holdings of the risky asset each period, however they pay an adjustment cost. This adjustment cost is set such that the general portfolio allocation of the agents over their lifecycle is in line with the data and to match the ratio of the variance of the gross investment to the variance of output of 3.5, consistent with the data (see Gornemann et al. (2012)). In particular, agents pay the adjustment costs $\varphi(s_{i,j,t+1}\mu^a_{i,j,t} - s_{i,j,t}\mu^a_{i,j,t}) = \varphi_1(s_{i,j,t+1}\mu^a_{i,j,t} - s_{i,j,t}\mu^a_{i,j,t})^{\varphi_2}$. As a preliminary calibration, we set $\varphi_1 = 0.06$ and $\varphi_2 = 1.6$. These values are too low and thus the model predicts that the ratio of the variance of gross investment to output will be 14.3, significantly higher than the data. These calibration parameters are preliminary values which we used to calculate rough results.[9] The model's over prediction of the relative volatility of investment will have implications for the economic dynamics. Therefore, the interpretation of the results in the next section should be considered illustrative. Finally, we set $\delta = 8.3\%$ which is consistent with many life cycle studies (for example see Conesa et al. (2009)).[10]

3.3 Firms

On the firm side we set the elasticity of substitution between intermediate good firms to $\epsilon = 7$. This implies a steady state markup λ_p of 17%, a value in the middle of the range typically assumed in the literature. We set $\alpha = 0.36$, which corresponds to a steady-state

[6]For details on estimation of this process, see Appendix E in Kaplan (2012).

[7]We choose to ignore the iid idiosyncratic risk that is included in Kaplan (2012) to reduce the state space of the model.

[8]We use the Rouwenhorst method in order to discretize the persistent shock since the autocorrelation is high.

[9]Calculating the model takes approximately one month so doing a rigorous calibration will take additional time. Hence these values were used to provide illustrative results.

[10]The life cycle studies tend to target an investment to output ratio in steady state in order to pin down depreciation.

share of capital income roughly equal to 36 percent (see for example Christiano et al. (2005)). The parameter governing the quadratic price adjustment costs is calibrated to $\phi_\pi = 172$, in order to introduce price stickiness comparable to a slope of the New Keynesian Phillips Curve estimated on data for the Great Moderation.[11] We set the parameters of the aggregate total factor productivity shock, $\rho_z = .75$ and $\sigma_z = .00016$, to yield reasonable fluctuations in aggregate output.

3.4 Government

The parameters in the monetary policy rule are set to the Taylor (1999) rule. We assume an inflation target of 2%. The response of the nominal interest rate to the deviation of inflation from its inflation target is set to $\rho_\pi = 1.5$ and it adjusts one for one to deviations of output from its stochastic steady state ($\rho_Y = 1$). For the monetary policy shock we use $\rho_d = .7$ and calibrate the standard deviation of the monetary policy shock, $\sigma_D = .0000625$, such that a one-standard-deviation monetary shock has a size of 25 basis points annualized. Finally, both labor and capital income are taxed at separate flat tax rates. We set the capital tax rate, $\tau_k = .2$, and the labor tax rate, $\lambda_h = .15$.

4 Preliminary Results

We begin by examining asset holdings over the life cycle in the data and comparing it to the predictions from our model. It is important to asses our model's predictions with regards to both savings and portfolio choices because both will be important for determining the strength of the three potential channel that may cause an unequal affect from monetary policy over the life cycle. Next, we assess the aggregate effects of a monetary policy shock in our model. Finally, we discuss some preliminary results which determine the distribution of the effect of a monetary policy shock across different agents in our model.

4.1 Asset holdings over the life cycle

The Survey of Consumer Finances (SCF), sponsored by the Federal Reserve Board is a rich data source on the cross-sectional asset holdings of U.S. households. We use the data from the publicly available SCF data and broadly categorize households' assets and debts

[11] It is important to note that we calibrate our life cycle model to an annual not quarterly frequency.

into “risky” vs. “risk-free”.[12] Since we are comparing the data to the predictions from our model, we want this measure to correspond as closely as possible to the concepts of the risk-free and the risky asset holdings of households over their life cycle in our model.

Consistent with the definitions in Bricker et al. (2014), we quantify total net worth in three broad categories: (i) debt, (ii) nonfinancial assets, (iii) and financial assets. We consider all debt to be in the riskless category since in our model agents can only borrow using the bond. We categorize all non-financial assets into the risky category since most of the overall return to these assets are either related to their appreciation in price (residential property) or stream of revenues (businesses) which are uncertain. For financial assets, we consider equity to be a risky asset and the remainder of financial assets to be risk-free.[13]

Figure 1 uses data from the 2013 SCF survey to plot the total value of risky assets, risk-free assets, and net worth using these classifications. Several features are noteworthy. When workers enter the labor force, they tend to have debt at the same time as they hold risky assets. While the holdings of risky assets grow steadily till households retire, the accumulation of debt peaks in the late 30s, after which they start paying off some of the debt. Sometime in the early-50s households have a net positive holding of safe assets, which peaks at retirement. Total wealth also peaks around the age of 70, near the typical retirement age for most individuals. Once wealth peaks, households tend to dissave more quickly in risky assets compared to risk-free assets.[14]

Figure 2 plots the life cycle pattern of asset holdings generated in the stochastic steady state of our model. One difference is that we plot the predicted profiles from our model from the age of 20 to 100, while size of the sample limits us from examining households older than 80 in the data. Overall for the ages in which we can compare (20 - 80), our model does a decent job of matching the properties of the asset holdings over the life cycle described above. Similar to the data, our model predicts that agents will begin their lifetime by holding negative levels of bonds and positive levels of the risky asset. Similar to the data, we find that the level of debt peaks in the late thirties. Finally, our model also predicts that total wealth will peak around the age of retirement. However, our model predicts that households

[12] For more detail on the calculations and survey see Bricker et al. (2014).

[13] Again, we follow the classification of wealth into the broad categories discussed in Bricker et al. (2014). Examples of these financial assets that are not equity include certificates of deposits or savings bonds. Some categories of assets in the SCF may contain equities but may not be entirely equities, such as managed assets. We follow the procedures in Bricker et al. (2014) to apportion the appropriate shares of these assets to equities and non-equities assets.

[14] We have also tried alternative classifications of risky vs. riskless in the SCF and found that the qualitative pattern of assets holdings over the life cycle remains intact.

wind down their holdings of both risky and risk-free assets a bit faster than is observed in the data.[15] Moreover, unlike the data, the model predicts that agents will dissave more quickly in bonds than the risky asset. Overall, our model predicts similar life cycle patterns of wealth compared to the data.

Next we compare the data and the model's prediction for how portfolio choice will vary across agents. Figure 3 plots the share of risky vs. risk-free assets in the SCF data separated into two education categories. The low education group includes individuals who have less than a college education. The high education group is for individuals who have at least a college education. Workers with lower education hold a lower share of their portfolio in risk-free assets and correspondingly a larger share in the risky-asset. This is mostly because risky assets like houses and cars tend to account for a large share of their wealth. For both groups the share in risk-free assets peaks around retirement and falls after that. Tuning to our model's prediction, figure 4 shows this share for the high type and the low type workers over the life cycle.[16] The model matches the fact that high type workers tend to hold a higher share of their wealth in the risk-free asset. Additionally the model produces similar dynamics, with the share rising before retirement and falling afterwards. However, our model over predicts the share of both high and low ability agent's portfolio allocated to risk-free assets.

4.2 Transmission of a monetary policy shock

In this section we look at how a surprise increase in the nominal interest rate affects aggregate prices and quantities in the life cycle model. We model the contractionary monetary policy shock as half a standard deviation, exogenous increase in D that prevails one period.[17] Figures 5 and 6 present the dynamic fluctuations of inflation, the risk-free return, risky return, and wages.

The contractionary monetary policy shock raises the risk free interest rate (shown in figure 6). The real risk free interest rate affects the household's marginal rate of substitution between current and future consumption and households save more and postpone consump-

[15] This is a common problem in life cycle models that do not include either risky medical expenses at the end of life or a bequest motive.

[16] Education in the data can be thought of as a proxy for ability in our model.

[17] We show in the current draft the transmission of a one period monetary policy shock for computational feasibility as a first pass. In the current formulation the shock completely dissipates after the first period. Since the shock process is modelled as persistent, the complete dissipation after the first period can be viewed as a follow on smaller expansionary monetary policy shock in the second period. With a persistent monetary policy shock the transmission would be more persistent.

tion in light of higher expected returns. At the same time households change their portfolio allocation and decide to save more in the risk free bond instead of the risky asset. These decision lead capital to be lower in the next period, however the size of the fall in risky asset holdings is attenuated by the adjustment costs.[18] Because there are price adjustment costs for the firm, firms react to the drop in demand by producing less and lowering demand for the production factors capital and labor. Lower factor demand leads factor prices, the risky return on holding physical capital and the wage rate, to drop substantially in the first period (see figure 5). With both wages and the risky return on holding physical capital falling and technology unchanged, marginal costs also declines (the dynamics of marginal cost is controlled by equation 19). When marginal costs fall, intermediate goods firms adjust prices downwards given price adjustment costs leading to a downward pressure on inflation. In figure 6 inflation falls by slightly less than 0.2 % in the first period after a half a standard deviation monetary policy shock. Due to the fall in output and inflation, monetary policy lowers the nominal risk free interest rate immediately according to the Taylor (1999) rule, but still the equilibrium nominal risk free interest rate stays positive.

As mentioned, these dynamics lead to a significant reduction in aggregate capital in the second period (one period after the shock). This leads to a significant increase in the return to the risky asset in the second period.[19] This large decline in capital leads the model to predict more variance in gross investment over the business cycle than is consistent with the data. We are currently working on calibrating the model such that the variance of investment over the business cycle matches the data which should eliminate this increase. Thus, the interpretation of the results should be viewed more as an illustrative example of how the effect of monetary policy may vary over the life cycle.

4.3 Welfare effects of a monetary policy shock

In figure 7, the black line shows the welfare effects of the monetary policy shock measured as the percent change in expected future consumption necessary to make an agent indifferent between experiencing the shock and living in a world without the shock. The red line shows the average welfare effect for agents of high ability and the blue line highlights for agents who

[18]Currently, this attenuation appears to be too small as the variance of aggregate capital over the business cycle is larger in our model than the data. Hence, we plan on calibrating the model such that our model predicts a similar amount of variance of aggregate capital as exists in the data.

[19]The increase is further enhanced because as opposed to a persistent shock, we assume the shock dissipating after one period. These dynamics also lead to an increase in marginal costs and wages in the second period.

are of low ability.[20] There are competing effects of the monetary policy shock but overall agents in the model benefit from this shock. On the negative side, wages and the risky return decline at the onset of the shock. However, wages and the risky return increase in subsequent periods. In addition, the risk-free rate rises for numerous periods as a consequence of the shock which will have a different welfare effect on agents depending on whether they hold a positive or negative level of savings in the risk free asset. Overall, we find the positive effects tend to dominate and on average welfare increases for agent of all ages at the time of the shock.[21]

Generally, for agents who have not retired at the time of the shock, the benefit from the monetary policy shock tends to be higher than for agents who are older. And among working cohorts, agents who are 50 at the time of the shock tend to benefit more than agents who are 30 at the time of the shock. The increase in the welfare benefit for these older working agents is because agents tend to hold more savings as they age. Thus, the overall increase in the return to savings is more beneficial for these agents with higher levels of savings. Turning to agents who are retired at the time of the shock, the benefit from the monetary policy shock tends to be less for agents who are older at the time of the shock. The relatively smaller benefit for agents who are older is because these agents are in a point in their life cycle in which they are dissaving, thus, older agents tend to have lower levels of savings. The large drop at age 64 is because these agents work one period with lower wages but do not benefit from the higher wages. Retired agents tend to benefit less since as they have relatively fewer risky assets and earn no wage income.

Since our model involves within cohort heterogeneity as well, we can separate the welfare effects by productivity type. We divide each cohort into high productivity and low productivity, shown by the red and blue lines respectively. Interestingly, the lower productivity agents benefit more from the monetary policy shock as they tend to hold a higher share of their portfolio in the risky asset. Figure 4 plots the average percent of agents portfolio that is in the risk-free asset. Generally, lower ability agents tend to have a lower share of their portfolio in risk-free assets and subsequently enjoy a larger welfare benefit from the increase in the return to the risky asset after the shock. Thus even though overall welfare increases in the economy as a result of a monetary policy shock, these welfare effects are felt unevenly both by age and type. These results also indicate that all three channels are

[20] High and low ability agents refer to the permanent productivity shock agents receive at birth.

[21] We believe this dynamic is an artifact of the model not being properly calibrated. We are currently working on calibrating the model, however due to the heavy computational demand needed to solve the model we were not able to fully calibrate the model in time for submission.

causing monetary policy to have a different effect on agents depending on their stage of the life cycle. Hence, these illustrative results demonstrate that after calibrating our model there is potential for monetary policy to have an uneven affect on welfare both across and within cohorts. Moreover, these uneven effects could lead to a change in overall inequality in the economy.

5 Conclusion

This paper examines the effect of monetary policy in a life cycle model. In particular, it aims to determine if monetary policy has a quantitatively different affect on agents depending on where they are in the life cycle. The illustrative example indicates that due to differences in sources of income, level of savings versus debt, and portfolio composition over the life cycle, monetary policy has an uneven impact on older and younger agents.

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Figure 1: Asset Holdings from the SCF

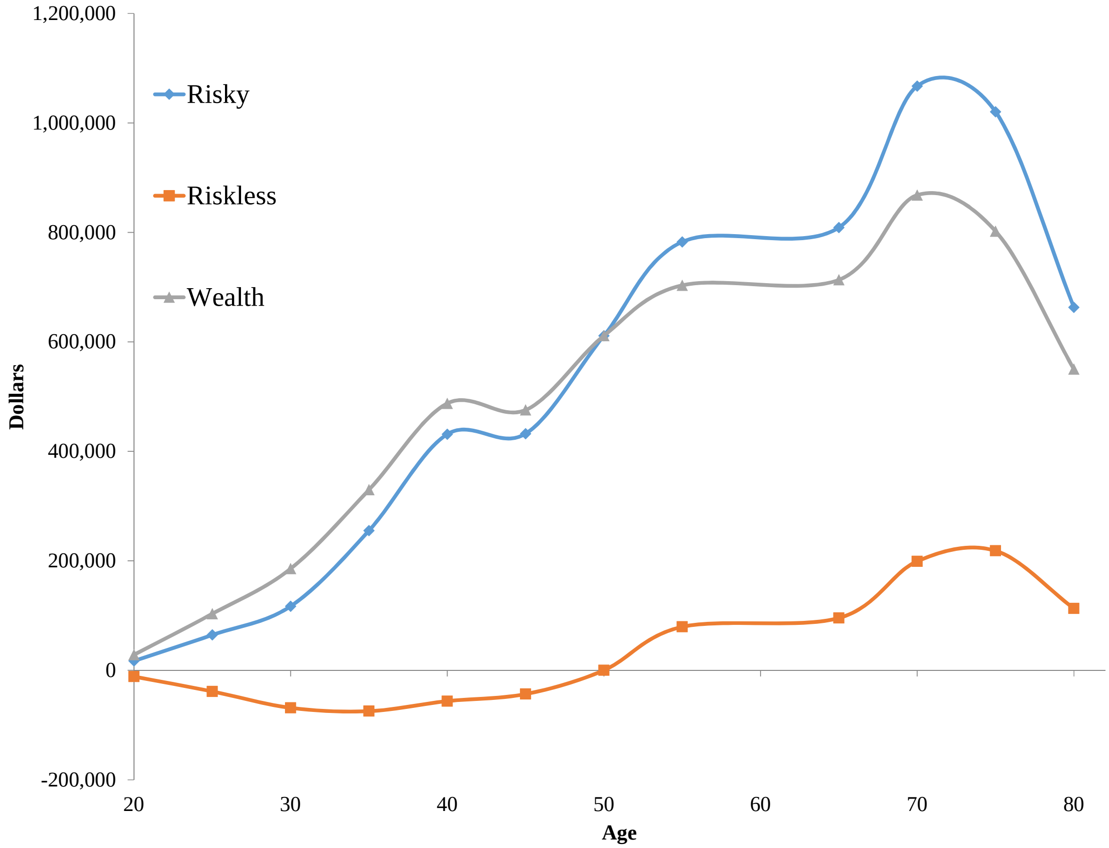

Figure 2: Asset Holdings from the model (stochastic steady state)

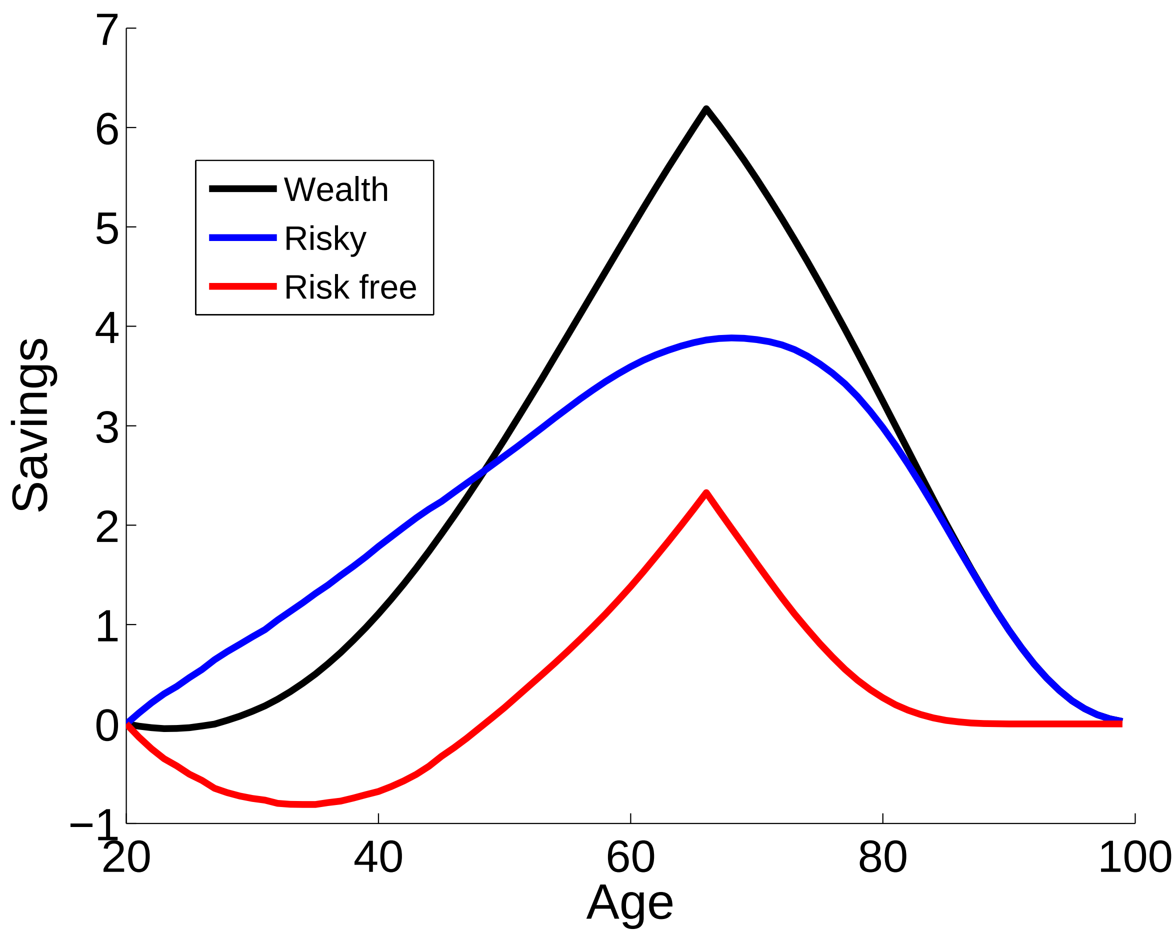

Figure 3: Share of wealth in risk-free from the SCF

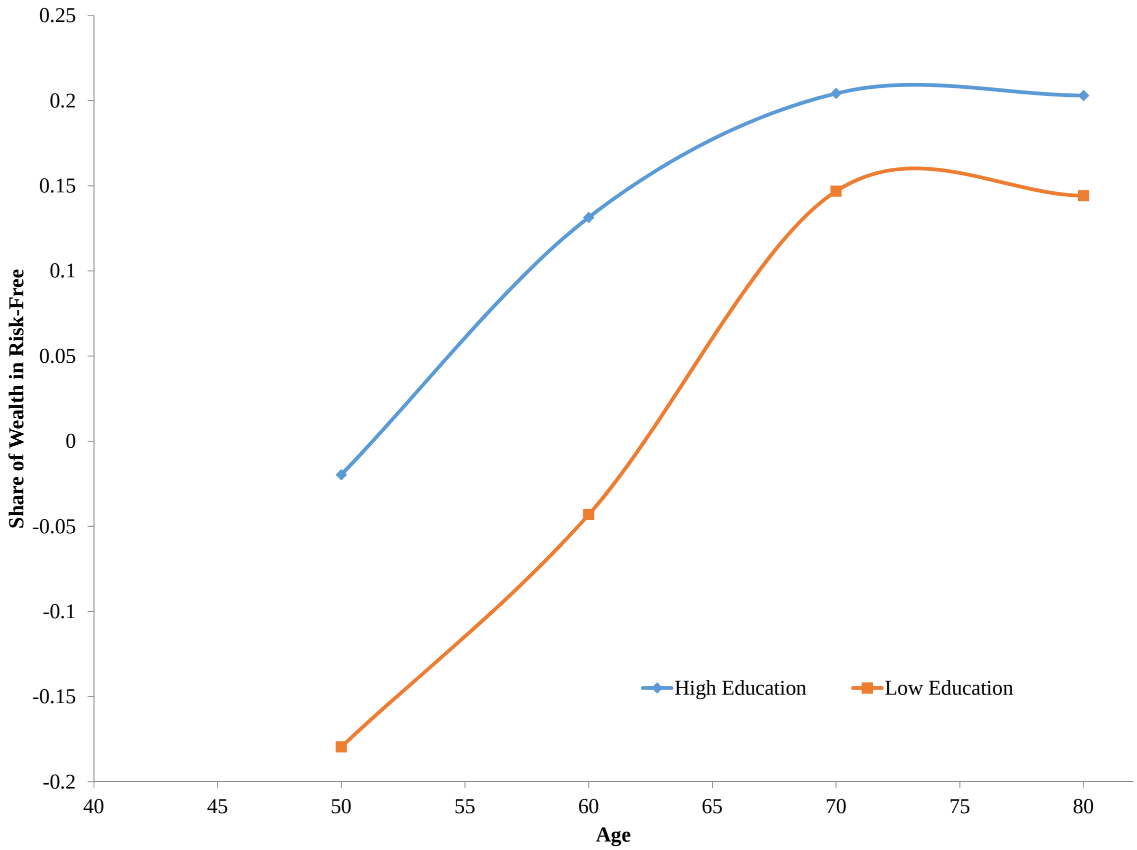

Figure 4: Share of wealth in risk-free asset from the model

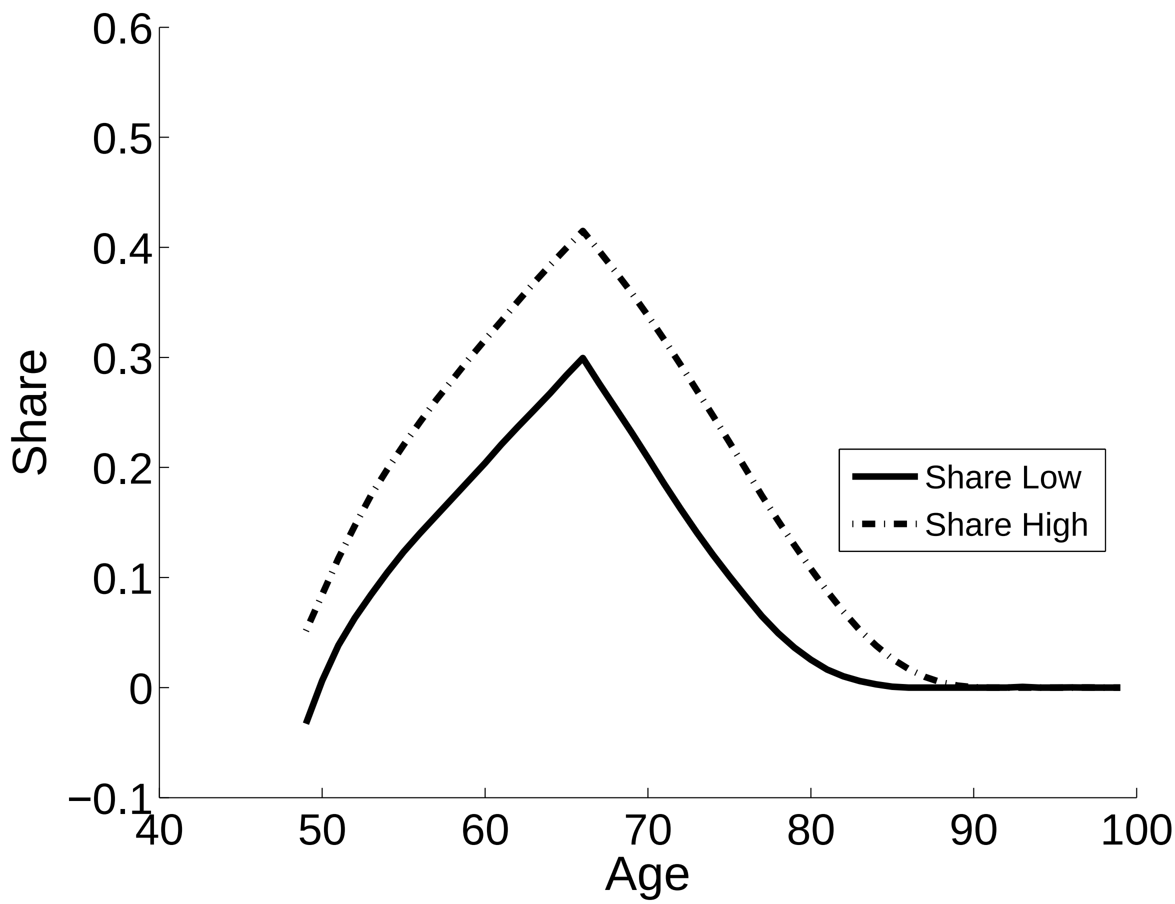

Figure 5: Impulse responses to a one-half std monetary policy shock

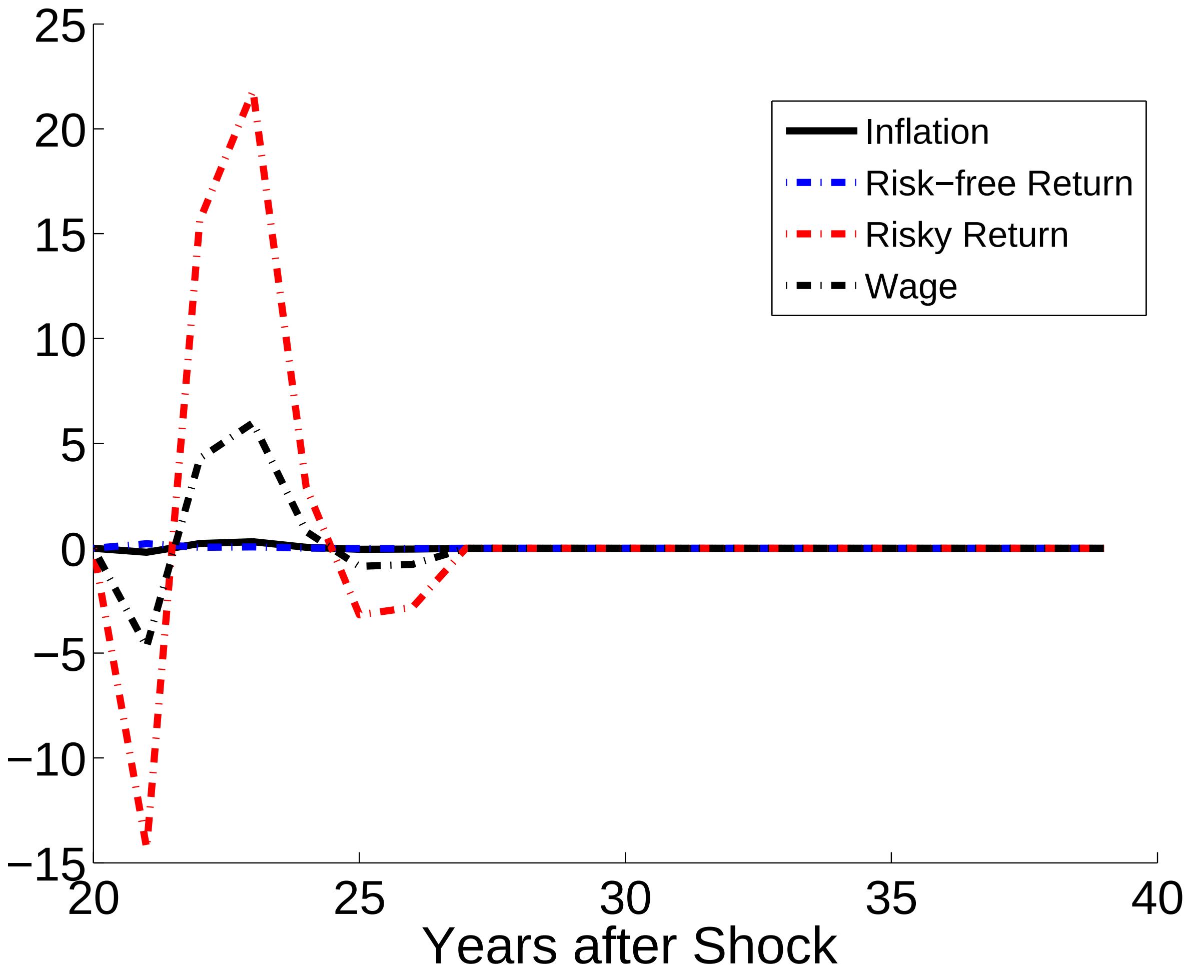

Figure 6: Impulse responses to a one-half std monetary policy shock

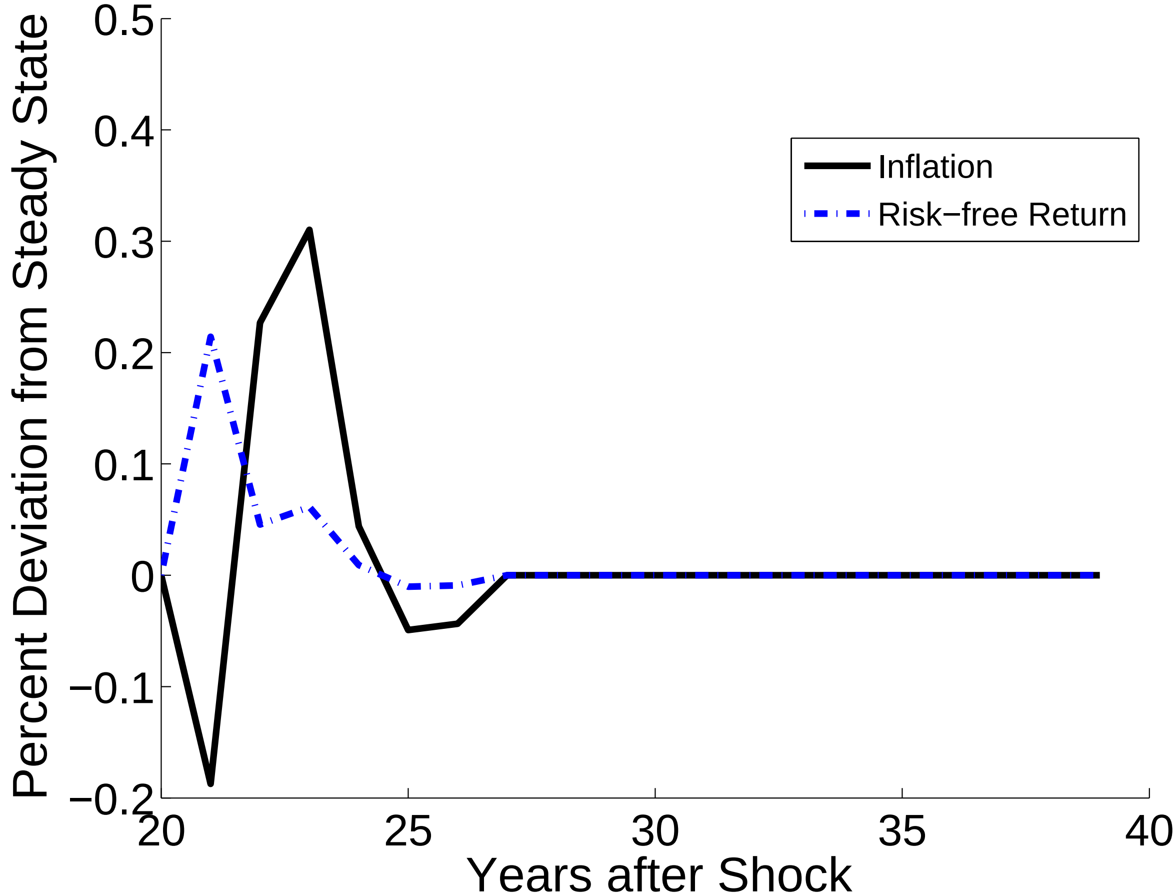

Figure 7: Welfare effects of monetary policy shock