Implementation and Detection[1]

Hitoshi Matsushima[2]

Faculty of Economics, University of Tokyo

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Abstract

We investigate implementation of social choice functions, where we assume the presence of information that may be verifiable after the allocation decision. We require a mechanism to have the unique iteratively undominated strategy profile and the same profile to uniquely survive through iterative eliminations irrespective of specifications such as prior distribution. By permitting small side payments, we demonstrate a mild sufficient condition under which a social choice function is implementable. The verifiable information needs not to be relevant to the social choice function at all. Through roundabout routes of detection, just a marginal verifiability is sufficient to incentivize all players.

Keywords: Unique Implementation with Small Side Payments, Limited Knowledge, Verifiability, Strict Iterative Dominance, Detection.

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[2] Department of Economics, University of Tokyo, Hongo, Bunkyo-ku, Tokyo 113-0033, Japan. E-mail: hitoshi[at]e.u-tokyo.ac.jp

1. Introduction

This paper investigates implementation of a social choice function under incomplete information, where the central planner attempts to achieve the desirable allocation as the state-contingent value of a social choice function, but does not know which state actually occurs. Even if the central planner knows that at least three players receive the entire information, he (or she) does not know who actually receive such information.

The purpose of this paper is to clarify the possibility that the central planner succeeds to incentivize all players to announce truthfully by permitting just a small side payments. We demonstrate a concrete method of how to design such a well-behaved incentive mechanism. The key assumption is that there exists information that some players expect to be verifiable after making the allocation decision.

This paper applies strict iterative dominance as the equilibrium concept, where we require the uniqueness of strategy profile that survives through the iterative elimination steps of strictly dominated strategies. Importantly, this paper additionally requires that the same strategy profile uniquely survives through such steps irrespective of detailed specifications such as prior distribution. Hence, this paper will develop a new implementation theory on the basis of severe knowledge limitation[3].

To be more precise, we assume that players' prior distributions over states are not common knowledge, and that what kind of information channels (information partitions, or information functions) each player accesses is unknown to the central planner. Hence, we cannot utilize proper scoring rule or its variant for incentivizing players in the same manner as Jackson et.al. (1990), Cremer and McLean (1988), and Matsushima (1990). We even assume that players' payoff functions are not common knowledge. Hence, we cannot expect the replacement of 'exact' with 'virtual' to make the implementation problem easier to solve in the same manner as Matsushima (1988) and Abreu and Sen (1991).

Despite of the above-mentioned limitation, this paper can demonstrate a rather mild sufficient condition under which a social choice function is uniquely

[3] For the surveys on implementation theory, see Moore (1992), Palfrey (1992), Osborne and Rubinstein (1994), Maskin and Sjostrom (2002), and others.

implementable in strict iterative dominance. Surely the presence of verifiable information is indispensable; the sufficient condition, however, indicates that the verifiable information needs not to be relevant to the value of the social choice function at all.

The key concepts for understanding the sufficient condition are *detection* and *self-detection*; an information channel is said to detect another information channel if a player, who pretends to access the latter channel even if he fails, cannot ignore the possibility that another player, who accesses the former channel as the detector, obtains some information that is inconsistent with what the player announces, i.e., the intersection between the detector's information and what the player announces is empty. Hence, in order to prevent the player from lying, it would be effective to penalize him whenever the detector finds out his lying.

We, however, have the following two remaining incentive problems. The first problem is the detector's incentive to tell the truth; we need the detector's detector, the detector's detector's detector, the detector's detector's detector's detector, and so on. We need a finite but possibly long chain of detectors. In this case, the first player on this chain must be the central planner, whose information is verifiable by assumption, i.e., needs no more detector. Note that the role of the verifiable information is just to detect the second player. The central planner steps forward to such a first tiny step; by a roundabout way, all players come to announce truthfully.

It is also crucial for each player to receive information not all but a little at once through time; a single player appears several times in this chain. He can be well-detected at some stage because of lack of knowledge about his detector's information, which he may come to know at the later stage. It is often the case that a player's detector at some stage is detected by another player who was detected by this player at the earlier stage.

The second remaining incentive problem is to incentivize a player to announce truthfully about whether he actually accesses the demanded information channels. In order to overcome this problem, we need a further careful mechanism design; otherwise, uninformed players have no chance but to pretend to know something, or even well-informed players is willing to announce incorrectly that he failed to access demanded information channels.

We define a sequence of information channels as describing the manner in which each player gradually accesses information channels through time. We call a sequence of information channels *self-detective* if it is accompanied by the above-mentioned chain of detectors in an appropriate manner. Based on these notations, the main theorem proves that any social choice function is uniquely implementable in strict iterative dominance whenever there is a self-detective sequence of information channels.

This theorem is permissive, not only because self-detection is itself a weak requirement, but also because of the following reasons; we can permit that most of possible sequences of information channels are not self-detective. We can permit that the probability to access self-detective sequences of information channels is as close to zero as possible. We even permit any self-detective sequence to be inaccessible at all. All we need is to assume that each player cannot ignore the possibility that the other players detects his lying even if it should be just a tiny probability. We can even permit any player whose detector is not the central planner to recognize that the central planner fails to access any non-degenerate information channel.

The constructed mechanism is detail-free in that it does not depend on specifications such as prior distribution and payoff functions. This mechanism requires any player to make just a tiny monetary payment to the central planner. We make this payment as close to zero as possible, according to a variant of the method that originates in Abreu and Matsushima (1992a, 1992b, 1994).

2. Literatures

The pioneering work by Maskin (1999) investigates Nash implementation of a social choice correspondence in which the mechanism satisfies that at any state the set of all Nash equilibrium outcomes is equivalent to the value of the social choice correspondence. In contrast, this paper investigates not a social choice correspondence but a social choice function, and requires not only the uniqueness of equilibrium outcome but also the uniqueness of equilibrium strategy profile.

Maskin assumes that the central planner never accesses any verifiable information channel, and shows that monotonicity is a necessary condition for Nash implementation.

Note that monotonicity is a very restrictive condition for social choice functions. In contrast, this paper will permit the central planner to access verifiable information channels. To the best of my knowledge, this paper would be the first serious attempt to investigate implementation with this permission.

There are a number of previous works that provided their respective permissive results in the implementation literature. Matsushima (1988) and Abreu and Sen (1991) showed that any social choice function is virtually implementable in Nash equilibrium; the permission of slight deviation from the optimal allocation gives a substantial impact on players' incentives. In contrast, this paper concerns, not virtual, but exact, implementation. Moore and Repullo (1988) and Palfrey and Srivastava (1990) replace Nash equilibrium with more restrictive equilibrium concepts such as subgame perfect equilibrium and un-dominated Nash equilibrium. In contrast, this paper replaces Nash equilibrium even with a weaker equilibrium concept.

The previous works such as Maskin (1999) innovate different methods from the Abreu-Matsushima mechanism, namely the modulo game and the integer game. According to the modulo game method, we can design a mechanism that implements a social choice correspondence in terms of pure strategy Nash equilibrium, while there still exist multiple mixed strategy Nash equilibria that induce undesirable allocations. According to the integer game method, we can design a mechanism that implements a social choice correspondence in terms of not only pure but also mixed strategy Nash equilibrium, while the integer game method has a serious drawback of unboundedness that the size of the message spaces is infinite and any announcement cost is not permitted, which makes the mechanism impossible to apply in practice (Jackson (1992)).

In contrast, the Abreu-Matsushima method (Abreu and Matsushima (1992a, 1992b, 1994)) is bounded and (at least virtually) implements any social choice function in mixed strategy Nash equilibrium. This paper will apply the Abreu-Matsushima method to the last K rounds of our mechanism design.

There are the previous works such as Gibbard (1973), Satterthwaite (1975), Jackson (1991), Matsushima (1991, 1993, 2005, 2008a), Abreu and Matsushima (1992b), Duggan (1997), and Cheny and Kunimoto (2014), which investigate implementation on the assumption of incomplete information. Many of these works explore the Bayesian framework that crucially depends on particular specifications of prior distributions and information structures. The papers by Bergemann and Morris (2009, 2012) investigate universal type spaces that include the description of priors. In contrast, this paper considers an arbitrarily fixed set of possible states that do not include information about priors. This paper would investigate unique implementation that is robust with respect to specifications of universal type space[4].

The weak knowledge assumption in this paper makes the problem with verifiable information substantial. In fact, whenever the prior distributions are common knowledge and the distribution of verifiable information depends on each player's private information in probability, then, according to proper scoring rules (Jackson et.al. (1990), Cremer and McLean (1988), and Matsushima (1990)), joint with the Abreu-Matsushima method, it is clear that any social choice function is uniquely implementable in strict iterative dominance with almost no side payments.

This paper assumes that each player $i's$ payoff is quasi-linear, risk-neutral, and satisfy the expected utility hypothesis, but does not much depend on these assumptions. It is often the case that Abreu-Matsushima method is not almighty once we consider un-expected utilities (Jackson (2001), for instance). This paper, however, will argue that this criticism is wrong. We can drastically weaken these assumptions with no substantial change but at the expense of irrelevant complexity. All we need for our unique implementation is to assume that each player's payoff function is continuous across lotteries and side payments.

Since players' prior distributions are unspecified, this paper defines an information channel as a partition of the state space in the same manner as models of higher-order knowledge[5]. In such models, it has often been discussed as 'the puzzle of the hats' that a tiny information release gives a big influence on players' reasoning. In contrast, this paper focuses more on the impact of information release on strategic interaction; thanks to such releases a player can find a manner of undetected lying that is beneficial to him but may be harmful to the public. Hence, we assume *imperfect information* on the mechanism that the central planner hides a player's message from the other players in

[4] There are exceptions such as Chung and Ely (2003) and Oury and Tercieux (2012), which investigated their respective robustness with respect to slight uncertainty about specifications. In contract, this paper investigates the entire uncertainty about specifications.

[5] See Osborne and Rubinstein (1994, Chapter 5).

order to prevent them from utilizing his message for finding undetected lies. Information cascade such as Bikhchandani et. al. (1992) and Anderson and Holt (1997) is also relevant; the observation of the other players' activities motivates a player to hide his private information and follow only the public information implied by their activities, resulting in inefficiency.

The organization of this paper is as follows. Section 3 shows the basic model. Section 4 introduces sequence of information channels. Section 5 defines unique implementation in strict iterative dominance. Section 6 shows the impossibility theorem on the assumption that the central planner cannot access information channels. Sections 7 and 8 define detection and self-detection, respectively. Section 9 demonstrates the main theorem of this paper. Section 10 shows the complete proof of this theorem. Section 11 considers non-expected utility. Section 12 concludes.

3. The Model

This paper investigates the situation in which the central planner selects an allocation and makes side payments with other players according to a sequential revelation mechanism, which will be defined later. Let $N \equiv \{1,...,n\}$ denote the nonempty and finite set of all players, where $n \geq 3$. Let A denote the nonempty and finite set of all allocations. Let Δ denote the set of all lotteries over allocations.

Let Ω denote the nonempty and finite set of possible states. Let $p:\Omega \to [0,1]$ denote a probability function on Ω with full support, where $\sum_{\omega\in\Omega} p(\omega)=1$, and $p(\omega)>0$ for all $\omega\in\Omega$. For every non-negative real number $\varepsilon_1 \geq 0$, let $P(\varepsilon_1)$ denote the set of all probability functions on Ω with full support such that $p(\omega)>\varepsilon_1$ for all $\omega\in\Omega$. We regard p as a player's prior distribution over states, where this player believes that any state in Ω occurs at least with probability ε_1. This paper considers ε_1 to be positive but as close to zero as possible, implying that we make almost no restriction on the set of possible prior distributions. We do not assume that each player's prior distribution on Ω is common knowledge.

Each player $i's$ payoff is quasi-linear, risk-neutral, and satisfy the expected utility hypothesis. Player i obtains the payoff $v_i(a,\omega)-r_i$ when the central planner selects the allocation $a\in A$ and player i makes the side payment $r_i\in R$ to the central planner. Let $v_i(\alpha,\omega)\equiv\sum_{a\in A}v_i(a,\omega)\alpha(a)$ for all lottery $\alpha\in\Delta$. Let $v\equiv(v_i)_{i\in N}$.

For each positive real number $\eta>0$, let $V_i(\eta)$ denote the set of all payoff functions $v_i:A\times\Omega\to R$ for player $i\in N$ satisfying

$$\max_{\substack{(a,a')\in A^2\\ \omega\in\Omega}}\left|v_i(a,\omega)-v_i(a',\omega)\right|\leq\eta .$$

Let $V(\eta)\equiv\prod_{i\in N}V_i(\eta)$. We consider η to be as large as possible, implying that we make almost no restriction on the set of payoff functions. We do not assume that each player's payoff function is common knowledge.

4. Information Channels

Let Ψ denote the set of all partitions of Ω. A generic element of Ψ is denoted by $\psi\equiv(\psi(\omega))_{\omega\in\Omega}\in\Psi$, where $\psi(\omega)\subset\Omega$, $\omega\in\psi(\omega)$, and for every $\omega\in\Omega$ and $\omega'\in\Omega$, either $\psi(\omega)=\psi(\omega')$ or $\psi(\omega)\bigcap\psi(\omega')=\phi$.

Each player receives information not all but a little at once from round 1 through round T as follows. For each $t\in\{1,...,T\}$, let $C_{i,t}\subset\Psi$ denote the set of possible partitions for player i at round t. A generic element of $C_{i,t}$ implies an *information channel* for player i at round t, denoted by $c_{i,t}\equiv(c_{i,t}(\omega))_{\omega\in\Omega}$. Let

$$C_i\equiv\prod_{\tau=1}^{T}C_{i,\tau},\quad c_i\equiv(c_{i,\tau})_{\tau=1}^{T}\in C_i,\quad C_i^t\equiv\prod_{\tau=1}^{t}C_{i,\tau},\quad c_i^t\equiv(c_{i,\tau})_{\tau=1}^{t}\in C_i^t,$$

$$C_t\equiv\prod_{i\in N}C_{i,t},\quad c_t\equiv(c_{i,t})_{i\in N}\in C_t,\quad C^t\equiv\prod_{i\in N}C_i^t,\quad c^t\equiv(c_i^t)_{i\in N}\in C^t,$$

$$c_i^t\equiv\bigvee_{\tau=1}^{t}c_{i,\tau},\quad c_i^T\equiv c_i,\text{ and }\ c_i^t(\omega)=\bigvee_{\tau=1}^{t}c_{i,\tau}(\omega)=\bigcap_{\tau=1}^{t}c_{i,\tau}(\omega),$$

where the partition $\overset{t}{\underset{\tau=1}{\vee}} c_{i,\tau}$ denotes the coarsest common refinement of $\{c_{i,\tau}\}_{\tau=1}^{t}$. For each $i \in N$, fix an arbitrary subset $\hat{C}_i \subset C_i$, where for every $t \in \{1,...,T\}$ and $c_{i,t} \in C_{i,t}$, there exists $\tilde{c}_i \in \hat{C}_i$ such that $c_{i,t} = \tilde{c}_{i,t}$. We regard $\hat{C}_i$ the set of possible sequence of information channels for player i.

It is important in this paper to assume that the central planner can access an information channel denoted by a partition $c_0 \in \Psi$, his (or her) observation through which becomes *verifiable* after he selects the allocation $a \in A$. Let us fix an arbitrary subset $\hat{C}_0 \subset \Psi$, which is regarded as the set of possible information channels for the central planner.

Let us call $c \in \prod_{i \in N \cup \{0\}} \hat{C}_i$ a *sequence of information channels*. Let us fix an arbitrary subset of sequences of information channels, denoted by

$$\hat{C} \subset \prod_{i \in N \cup \{0\}} \hat{C}_i ,$$

where we assume that for every $i \in N \bigcup \{0\}$ and $c_i \in \hat{C}_i$, there exists $c_{N \cup \{0\} \setminus \{i\}}$ such that $c = (c_i, c_{N \cup \{0\} \setminus \{i\}}) \in \prod_{i \in N \cup \{0\}} \hat{C}_i$ belongs to $\hat{C}$. We regard $\hat{C}$ the set of possible sequences of information channels.

Let $q : \hat{C} \to [0,1]$ denote a probability function on $\hat{C}$ with full support, where $\sum_{c \in \hat{C}} q(c) = 1$, and $q(c) > 0$ for all $c \in \hat{C}$. With probability $q(c)$, the sequence of information channels $c \in \hat{C}$ becomes accessible to the players and the central planner. At each round $t \in \{1,...,T\}$, each player $i \in N$ accesses the information channel $c_{i,t}$ and observes its element that includes the true state $\omega \in \Omega$, i.e., $c_{i,t}(\omega) \subset \Omega$.

We assume that only a single sequence of information channels in $\hat{C}$ is accessible, any sequence that does not belong to $\hat{C}$ is never accessible, and no player accesses any more information channel after round T. The central planner accesses an information channel c_0 and observes $c_0(\omega) \subset \Omega$, which becomes verifiable only after his allocation decision.

For every non-negative real number $\varepsilon_2 \geq 0$, let $Q(\varepsilon_2)$ denote the set of all probability functions q on C such that $q(c) > \varepsilon_2$ for all $c \in \hat{C}$. We regard q as a player's prior distribution on the set of possible sequences of information channels $\hat{C}$; he believes that any sequence in $\hat{C}$ become accessible at least with probability ε_2. This paper considers ε_2 to be positive but as close to zero as possible, implying that a sequence of information channels in $\hat{C}$ is accessible only with a tiny probability. We do not assume that each player's prior distribution on $\hat{C}$ is common knowledge.

5. Unique Implementation in Strict Iterative Dominance

Fix arbitrary integers $T \geq 0$ and $K > 0$. We define a *sequential revelation mechanism*, or shortly a *mechanism*, as a $(T+K)$ – round decentralized decision procedure, which is denoted by a triplicate $G = (M, M_0, g, x)$, where

$$M \equiv \prod_{i \in N} M_i \,,\quad M_i \equiv \prod_{t=1}^{T+K} M_{i,t} \,,$$

$M_{i,t}$ denotes the non-empty and finite set of all messages of each player $i \in N$ at round $t \in \{1,...,T+K\}$, $g : M \to \Delta$ denotes an *allocation rule*, $x \equiv (x_i)_{i \in N}$, and $x_i : M \times M_0 \to R$ denotes a *payment rule for each player* $i \in \{1,...,n\}$. Let

$$m_i \equiv (m_{i,t})_{t=1}^{T+K} \in M_i \,,\quad M_t \equiv \prod_{i \in N} M_{i,t} \,,\quad m_t \equiv (m_{i,t})_{i \in N} \in M_t \,,\quad M_i^t \equiv \prod_{\tau=1}^{t} M_{i,\tau} \,,$$

$$m_i^t \equiv (m_{i,1},...,m_{i,t}) \in M_i^t \,,\quad M^t \equiv \prod_{i \in N} M_i^t \,,\text{ and }\ m^t \equiv (m_i^t)_{i \in N} \in M^t \,.$$

According to the mechanism G, each player $i \in N$ makes an announcement $m_{i,t} \in M_{i,t}$ at each round $t \in \{1,...,T+K\}$. At the end of the final round $T+K$, based on their announcement profile $m \in M$, the central planner selects an allocation according to the lottery $g(m) \in \Delta$. After his allocation selection, the central planner makes an announcement $m_0 \in M_0$, and then each player $i \in N$ makes the side payment $x_i(m,m_0)\in R$ to the central planner. Note that $g(m)$ does not depend on m_0, while $x(m,m_0)$ depends on m_0.

We assume *imperfect information* on the mechanism in that any player $i's$ message $m_{i,t}$ at any round $t\in\{1,...,N+K\}$ is unknown to the other players until the end of the final round $T+K$. Hence, we define a *strategy for each player* $i\in N$ as $s_i=(s_{i,t})_{t=1}^{T+K}$, where for every $t\in\{1,...,T-1\}$,

$$s_{i,t}:\Omega\times C_i^t\rightarrow M_{i,t} \text{ for all } t\in\{1,...,T-1\},$$

(1) $$s_{i,t}(\omega,c_i^t)=s_{i,t}(\omega',c_i'^t) \text{ if } c_i^t(\omega)=c_i'^t(\omega'),$$

and for every $t\in\{T,...,T+K\}$,

$$s_{i,t}:\Omega\times C_i\rightarrow M_{it} \text{ for all } t\in\{T,...,T+K\},$$

and

(2) $$s_{i,t}(\omega,c_i)=s_{i,t}(\omega',c_i') \text{ if } c_i(\omega)=c_i'(\omega').$$

The equalities (1) and (2) imply that each player $i's$ strategy at each round t depends only on the information that he has observed up to round t. Let S_i denote the set of all strategies for player i. Let $S\equiv\prod_{i\in N}S_i$ and $s\equiv(s_1,...,s_n)\in S$. For every $i\in N$, $\omega\in\Omega$, and $c_i\in C_i$, we denote

$$m_i(s_i,\omega,c_i)=(m_{i,t}(s_i,\omega,c_i^t))_{t=1}^{T+K}\in M_i,$$

where

$$m_{i,t}(s_i,\omega,c_i)=s_{i,t}(\omega,c_i^t) \text{ for all } t\in\{1,...,T+K\},$$

which implies the message announced by player i at round t at state ω when he accesses c_i^t. Let

$$m(s,\omega,c)=(m_i(s_i,\omega,c_i))_{i\in N}\in M.$$

We further define a *strategy for the central planner* as $s_0:\Omega\times C_0\rightarrow M_0$, where

$$s_0(\omega,c_0)=s_0(\omega',c_0') \text{ if } c_i(\omega)=c_i'(\omega').$$

According to s_0, the central planner announces $s_0(\omega,c_0)\in M_0$ when he accesses $c_0(\omega)\subset\Omega$. Let S_0 denote the set of all strategies for the central planner.

Fix an arbitrary strategy $s_0 \in S_0$ for the central planner. The central planner can commit himself to announce according to this particular strategy s_0, because his observation $c_0(\omega)$ becomes verifiable. With this commitment, we define the equilibrium concept, namely, *strict iterative dominance*, in the following manner. Let

$$S_i(0) = S_i \text{ for all } i \in N .$$

Recursively, for each $h \geq 1$, we define a subset of strategies for each player $i \in N$, $S_i(h) \subset S_i$, in the manner that $s_i \in S_i(h)$ if and only if there exists no $s_i' \in S_i(h-1)$ that strictly dominates s_i irrespective of (v, p, q), i.e., there exists no $s_i' \in S_i(h-1)$ such that for every $v \in V(\eta)$, $p \in P(\varepsilon_1)$, $q \in Q(\varepsilon_2)$, and $s_{-i} \in S_{-i}(t-1)$,

$$E[v_i(a,\omega) - t_i \mid G, p, q, s_0, s] < E[v_i(a,\omega) - t_i \mid G, p, q, s_0, s_i', s_{-i}],$$

where $E[\cdot \mid G, p, q, s_0, s]$ denotes the expectation operator conditional on (G, p, q, s_0, s). Hence, we eliminate a strategy if and only if there exists another strategy that strictly dominates it irrespective of specifications $(v, p, q) \in V(\eta) \times P(\varepsilon_1) \times Q(\varepsilon_2)$.

We define

$$S_i^*(G, s_0, \eta, \varepsilon_1, \varepsilon_2) \equiv \bigcap_{h=0}^{\infty} S_i(h) .$$

A strategy $s_i \in S_i$ for player i is said to be *strictly iteratively undominated* with respect to $(G, s_0, \eta, \varepsilon_1, \varepsilon_2)$ if

$$s_i \in S_i^*(G, s_0, \eta, \varepsilon_1, \varepsilon_2) .$$

We define

$$S^*(G, s_0, \eta, \varepsilon_1, \varepsilon_2) \equiv \prod_{i \in N \cup \{0\}} S_i^*(G, s_0, \eta, \varepsilon_1, \varepsilon_2) .$$

A strategy profile $s \in S$ is said to be *strictly iteratively undominated* with respect to $(G, s_0, \eta, \varepsilon_1, \varepsilon_2)$ if

$$s \in S^*(G, s_0, \eta, \varepsilon_1, \varepsilon_2) .$$

The above-mentioned concept of strict iterative dominance is *detail-free* in that a strategy must be strictly iteratively undominated irrespective of (v, p, q).

We define a *social choice function* as a mapping from states to lotteries $f:\Omega \to \Delta$, where $f(\omega)$ implies the desirable allocation that the central planner would like to select at the state $\omega \in \Omega$. Let F denote the set of all social choice functions.

We denote by $f^{G,s,c}:\Omega \to \Delta$ the social choice function induced by the strategy profile $s \in S$ in the mechanism G when players access the sequence of information channels $c \in \hat{C}$, i.e.,

$$f^{G,s,c}(\omega) = g(m(s,\omega,c)) \text{ for all } \omega \in \Omega .$$

A mechanism G is said to *uniquely implement a social choice function* $f \in F$ *in strict iterative dominance with respect to* $(s_0,\eta,\varepsilon_1,\varepsilon_2)$ if there exists the unique strictly iteratively undominated strategy profile s with respect to $(G,s_0,\eta,\varepsilon_1,\varepsilon_2)$, i.e.,

$$S^*(G,s_0,\eta,\varepsilon_1,\varepsilon_2) = \{s\},$$

and it induces the social choice function f irrespective of which sequence of information channels to be accessible, i.e.,

$$f = f^{G,s,c} \text{ for all } c \in \hat{C} .$$

A social choice function f is said to be *uniquely implementable in strict iterative dominance with respect to* $(\eta,\varepsilon_1,\varepsilon_2)$ if there exist such G and s_0. A social choice function f is said to be *uniquely implementable in strict iterative dominance with respect to* $(\eta,\varepsilon_1,\varepsilon_2)$ *with almost no side payments* if for every $\xi > 0$, there exists such a mechanism $G=(M,g,x)$ and a strategy s_0 for the central planner, and G satisfies

$$\max_{\substack{i\in N \\ m\in M, m_0 \in M_0}} |x_i(m)| \le \xi .$$

A social choice function f is said to be *uniquely implementable in strict iterative dominance with almost no side payments* if for every $(\eta,\varepsilon_1,\varepsilon_2)$, f is uniquely implementable in strict iterative dominance with respect to $(\eta,\varepsilon_1,\varepsilon_2)$ with almost no side payments.

6. Impossibility

This section introduces unique implementation in strict Nash equilibrium as follows. A mechanism G is said to *uniquely implement a social choice function* $f \in F$ *in strict Nash equilibrium with respect to* (s_0, v) if there exists a strategy profile $s \in S$ such that for every $c \in \hat{C}$,

$$f = f^{G,s,c} \text{ for all } c \in \hat{C},$$

and, for every $\omega \in \Omega$, $m(s,\omega,c) \in M$ is the unique Nash equilibrium with respect to (v, s_0) in that it is the unique message profile $m \in M$ satisfying that for every $c_0 \in C_0$, $i \in N$, and $m_i' \in M_i \setminus \{m_i\}$,

$$v_i(g(m),\omega) - x_i(m, s_0(\omega, c_0)) \geq v_i(g(m_i', m_{-i}),\omega) - x_i(m_i', m_{-i}, s_0(\omega, c_0)),$$

and $m = m(s,\omega,c)$ is a strict Nash equilibrium in that for every $c_0 \in C_0$, $i \in N$, and $m_i' \in M_i \setminus \{m_i\}$,

$$v_i(g(m),\omega) - x_i(m, s_0(\omega, c_0)) > v_i(g(m_i', m_{-i}),\omega) - x_i(m_i', m_{-i}, s_0(\omega, c_0)).$$

A social choice function f is said to be *uniquely implementable in strict Nash equilibrium with respect to* v if there exist such G and s_0. A social choice function f is said to be *uniquely implementable in strict Nash equilibrium with respect to* v *with almost no side payments* if for every $\xi > 0$, there exists such a mechanism $G = (M, g, x)$ and a strategy s_0 for the central planner, and G satisfies $\max\limits_{\substack{i \in N \\ m \in M, m_0 \in M_0}} |x_i(m)| \leq \xi$.

Provided that ε_1 is sufficiently close to zero, it is clear that if a social choice function f is uniquely implementable in strict iterative dominance with respect to $(\eta, \varepsilon_1, \varepsilon_2)$, then, for every $v \in V(\eta)$, it is uniquely implementable in strict Nash equilibrium with respect to v. It is also clear that if a social choice function f is uniquely implementable in strict iterative dominance with respect to $(\eta, \varepsilon_1, \varepsilon_2)$ with almost no side payments, then, for every $v \in V(\eta)$, it is uniquely implementable in strict Nash equilibrium with respect to v with almost no side payments.

A social choice function $f \in F$ is said to be weakly *monotonic* with respect to v if for every $\omega \in \Omega$ and $\omega' \in \Omega$, either $f(\omega) = f(\omega')$ or there exist $a \in A$ and $i \in N$ such that

$$v_i(f(\omega),\omega) - v_i(a,\omega) > v_i(f(\omega),\omega') - v_i(a,\omega').$$

A social choice function $f \in F$ is said to be *strictly monotonic* with respect to v if for every $\omega \in \Omega$ and $\omega' \in \Omega$, either $f(\omega) = f(\omega')$ or there exist $a \in A$ and $i \in N$ such that

$$v_i(f(\omega),\omega) > v_i(a,\omega) \text{ and } v_i(f(\omega),\omega') < v_i(a,\omega').$$

Strict monotonicity is more restrictive than the standard definition of monotonicity, which is a necessary condition for dominant strategy implementation such as Gibbard (1973) and Satterthwaite (1975); a social choice function $f \in F$ is said to be *monotonic* with respect to v if for every $\omega \in \Omega$ and $\omega' \in \Omega$, either $f(\omega) = f(\omega')$ or there exist $a \in A$ and $i \in N$ such that

$$v_i(f(\omega),\omega) \geq v_i(a,\omega) \text{ and } v_i(f(\omega),\omega') < v_i(a,\omega').$$

Under the full domain of payoff functions, any monotonic social choice function f is *dictatorial*, i.e., there exist a player $i \in N$ such that for every $\omega \in \Omega$ and $\omega' \in \Omega$,

$$v_i(f(\omega),\omega) \geq v_i(f(\omega'),\omega).$$

We define the *degenerate* partition $\underline{\psi} \in \Psi$ by

$$\underline{\psi}(\omega) = \Omega \text{ for all } \omega \in \Omega.$$

Proposition 1: *Suppose that the degenerate partition* $\underline{\psi}$ *is the only possible information channel for the central planner at all times, i.e.,*

(3) $\qquad C_{0,t} = \{\underline{\psi}\}$ *for all* $t \in \{1,\ldots,T\}$.

Then, if a social choice function $f \in F$ *is not weakly monotonic with respect to* v*, then it is not uniquely implementable in strict Nash equilibrium with respect to* v*. If it is not strictly monotonic with respect to* v*, then it is not uniquely implementable in strict Nash equilibrium with respect to* v *with almost no side payments.*

Proof: Suppose that G uniquely implements f in strict Nash equilibrium with respect to (v, s_0). Then, there exists $s \in S$ such that for every $\omega \in \Omega$ and $c \in \hat{C}$,

$$g(m(s,\omega,c)) = f(\omega),$$

and $m(s,\omega,c)$ is a strict Nash equilibrium with respect to (v, s_0); for every $i \in N$ and $m_i \in M_i \setminus \{m_i(s_i,\omega,c_i)\}$,

$$v_i(g(m(s,\omega,c)),\omega) - x_i(m(s,\omega,c), s_0(\omega,c_0))$$
$$> v_i(g(m_i,m_{-i}(s,\omega,c)),\omega) - x_i(m_i,m_{-i}(s,\omega,c), s_0(\omega,c_0)) .$$

For every $\omega' \in \Omega \setminus \{\omega\}$, if $f(\omega) \neq f(\omega')$, then $m(s,\omega,c) \in M$ is not a Nash equilibrium; there exist $i \in N$ and $m_i \in M_i \setminus \{m_i(s_i,\omega,c_i)\}$ such that

$$v_i(g(m(s,\omega,c)),\omega') - x_i(m(s,\omega,c), s_0(\omega,c_0))$$
$$< v_i(g(m_i,m_{-i}(s,\omega,c)),\omega') - x_i(m_i,m_{-i}(s,\omega,c), s_0(\omega,c_0)) .$$

These inequalities imply that f is weakly monotonic.

Next, suppose additionally that $\max_{\substack{i\in N\\ m\in M}} |x_i(m)|$ is close to zero, i.e., side payments are almost negligible. Then, the property of strict Nash equilibrium can be rewritten as follows: for every $i \in N$ and $m_i \in M_i \setminus \{m_i(s_i,\omega,c_i)\}$,

$$v_i(g(m(s,\omega,c)),\omega) > v_i(g(m_i,m_{-i}(s,\omega,c)),\omega) .$$

For every $\omega' \in \Omega \setminus \{\omega\}$, if $f(\omega) \neq f(\omega')$, then there exist $i \in N$ and $m_i \in M_i \setminus \{m_i(s_i,\omega,c_i)\}$ such that

$$v_i(g(m(s,\omega,c)),\omega') < v_i(g(m_i,m_{-i}(s,\omega,c)),\omega') .$$

This implies that f is strictly monotonic.

Q.E.D.

In order to implement non-trivial social choice functions, it is inevitable that the central planner has the opportunity to access non-degenerate verifiable information channels.

7. Detection

This section introduces a key concept for incentivizing each player to make the honest announcements under the weak knowledge assumptions, namely, *detection*. As a generalization of partition, we define an *information function* by $\psi : \Omega \to 2^{\Omega}$ by assuming that

$$\omega \in \psi(\omega) \text{ for all } \omega \in \Omega .$$

Note that a partition of Ω, $\psi \in \Psi$, is an information function that satisfies

$$[\omega' \in \psi(\omega)] \Leftrightarrow [\psi(\omega') = \psi(\omega)].$$

Fix an arbitrary information function ψ and two arbitrary partitions $\psi' \in \Psi$ and $\tilde{\psi} \in \Psi$. Consider a situation in which a player accesses the information channel $\tilde{\psi}$, but pretends to access ψ'. There exists another player as the detector, who can receive the information implied by the information function ψ. The information function ψ is said to *detect the partition* $\psi' \in \Psi$ *against the partition* $\tilde{\psi} \in \Psi$ if for every $\tilde{\omega} \in \Omega$ and $\omega' \in \Omega$ satisfying

$$\tilde{\psi}(\tilde{\omega}) \not\subset \psi'(\omega'),$$

there exists $\omega \in \tilde{\psi}(\tilde{\omega})$ such that

$$\psi(\omega) \cap \psi'(\omega') = \phi .$$

Detection implies that if the player tells a lie by announcing what he does not know, i.e., $\psi'(\omega')$, the detector can find out his lie with a positive probability according to the information function ψ.

For convenience of our explanation, let us fix an arbitrary $\tilde{\omega} \in \Omega$ as the true state and an arbitrary $\omega' \in \Omega$ as a possibly false state. A player observes $\tilde{\psi}(\tilde{\omega})$, but tells a lie by announcing $\psi'(\omega')$ instead of $\tilde{\psi}(\tilde{\omega})$. Assume that the player does not know whether $\psi'(\omega')$ actually occurs, i.e.,

$$\tilde{\psi}(\tilde{\omega}) \not\subset \psi'(\omega').$$

Detection implies that the player expects some state $\omega \in \tilde{\psi}(\tilde{\omega})$ to occur with a positive probability, at which the intersection between what he announces, $\psi'(\omega')$, and what the detector knows, $\psi(\omega)$, is empty, i.e., $\psi(\omega) \cap \psi'(\omega') = \phi$. In this case, the player cannot ignore the possibility that his lie comes out by the detector, because $\psi(\omega)$ and $\psi'(\omega')$ never takes place at the same time. In order to prevent this player from lying, it would be an effective method to penalize him whenever the detector's observation is inconsistent with the player's announcement, i.e., $\psi(\omega)\cap\psi'(\omega')=\phi$.

We, however, have the following two remaining incentive problems. The first problem concerns the detector's incentive to make the honest announcement. In order to incentivize not only the player but also the detector, we need the detector's detector, the detector's detector's detector, and so on. The detail of this problem will be discussed in the next section. The second problem concerns the incentive of a player when he fails to access the demanded information channel. We need an incentive device to motivate such a player to announce truthfully that he fails to access it. This problem will be discussed in Section 9.

Example 1: Consider the situation in which a state is expressed by an $(L+1)-$tuple $\omega=(\omega_0,\omega_1,...,\omega_L)$, where

$$\omega_l\in\{1,2,3\}\quad\text{for all}\quad l\in\{0,1,...,L\}.$$

There exist $L+1$ distinct firms, i.e., firms 0, 1, …., and L. The $l-th$ component ω_l describes the financial condition of firm l. Condition '1' implies 'bad', condition '2' implies 'normal', and condition 3 implies 'good'.

For each $l\in\{0,...,L\}$, we define $\tilde{\psi}^l\in\Psi$ as the partition that identifies the financial condition of firm l; for every $\omega\in\Omega$ and $\omega'\in\Omega$,

$$\tilde{\psi}^l(\omega)=\tilde{\psi}^l(\omega')\quad\text{if and only if}\quad\omega'_l=\omega_l.$$

We specify Ω as a 'moth-eaten' state space in the manner that $\omega\in\Omega$ if and only if

(4) $$\omega_l\neq\omega_{l-1}\quad\text{for all}\quad l\in\{1,...,L\}.$$

The inequalities (4) imply that it is common knowledge that the financial condition of a firm is not exactly the same as that of its neighbor firm; the financial condition of a firm is in some degree informative to its neighbor firm's financial condition.

Fix an arbitrary firm $l\in\{1,...,L\}$. Assume that

$$\psi=\tilde{\psi}^{l-1}\quad\text{and}\quad\psi'=\tilde{\psi}^l.$$

We further assume that $\tilde{\psi}$ are not finer than $\underset{l\notin\{l-2,l-1\}}{\vee}\tilde{\psi}^l$. These assumptions and (4) guarantee that the player never identifies the financial condition of firm $l-1$. This player, however, pretends to know the financial condition of firm l by announcing $\psi'(\omega') = \tilde{\psi}^l(\omega')$, i.e., ω'_l. The detector accesses the information channel $\psi = \tilde{\psi}^{l-1}$ and observes the financial condition of its neighbor firm $l-1$.

We can show that $\psi = \tilde{\psi}^{l-1}$ detects $\psi' = \tilde{\psi}^l$ against $\tilde{\psi}$. Suppose $\tilde{\psi}(\tilde{\omega}) \not\subset \psi'(\omega')$, i.e., $\tilde{\psi}(\tilde{\omega}) \not\subset \tilde{\psi}^l(\omega')$. This supposition, along with the fact that the player never identifies the financial condition of firm $l-1$, implies that there exist $\overline{\omega} \in \tilde{\psi}(\tilde{\omega})$ and $\hat{\omega} \in \tilde{\psi}(\tilde{\omega})$ such that $\overline{\omega} \notin \tilde{\psi}^l(\omega')$, $\hat{\omega} \notin \tilde{\psi}^l(\omega')$, $\overline{\omega}_l = \hat{\omega}_l$, and $\overline{\omega}_{l-1} \neq \hat{\omega}_{l-1}$, i.e.,

$$\overline{\omega}_l \neq \omega'_l,\ \hat{\omega}_l \neq \omega'_l,\ \overline{\omega}_l = \hat{\omega}_l, \text{ and } \overline{\omega}_{l-1} \neq \hat{\omega}_{l-1},$$

which along with (4) implies that

$$\text{either } \omega'_l = \overline{\omega}_{l-1} \text{ or } \omega'_l = \hat{\omega}_{l-1}.$$

Hence, the player expects that both $\overline{\omega}$ and $\hat{\omega}$ can be considered as the true state, but one of these states, say, $\omega \in \{\overline{\omega}, \hat{\omega}\}$, are inconsistent with his announcement $\psi'(\omega') = \tilde{\psi}^l(\omega')$, i.e.,

$$\omega'_l = \omega_{l-1}.$$

This implies that there exists $\omega \in \tilde{\psi}(\tilde{\omega})$ such that $\psi(\omega) \cap \psi'(\omega') = \phi$. Hence, we have shown that $\tilde{\psi}^{l-1}$ detects $\tilde{\psi}^l$ against $\tilde{\psi}$. It is crucial to assume that the set of states Ω is a moth-eaten subset of $\{1,2,3\}^{L+1}$, which is necessary condition for a firm's financial condition to be informative to other firms' financial conditions.

8. Self-Detection

This section introduces a key concept, namely, *self-detection,* for incentivizing all players to make the honest announcements during the first T rounds. An *order of detection* is defined as (ι, κ), where $\iota : \{1,...,nT\} \to N$, $\kappa : \{1,...,nT\} \to \{1,...,T\}$,

$$\left|\{l \in \{1,...,nT\} \mid \iota(l) = i\}\right| = T \text{ for all } i \in N,$$

and for every $l \in \{1,...,nT\}$ and $l' \in \{1,...,nT\}$,

$$[\, l < l' \text{ and } \iota(l) = \iota(l')\,] \Rightarrow [\, \kappa(l) < k(l')\,].$$

In the order of (ι,κ), player $\iota(l)'s$ lie at round $\kappa(l)$ is detected by the other players' previous observations joint with the central planner's verifiable information.

To be precise, for every $i\in N$, $c_i\in\hat{C}_i$, $c_i'\in\hat{C}_i$, and $t\in\{1,...,T\}$, we define an information function $c_{i,t}(c_i'):\Omega\to 2^{\Omega}$ in the manner that for every $\omega\in\Omega$,

$$c_{i,t}(c_i')(\omega)=c_{i,t}(\omega) \qquad \text{if } c_i'^{t}(\omega)\subseteq c_{i,t}(\omega),$$

and

$$c_{i,t}(c_i')(\omega)=\Omega \qquad \text{if } c_i'^{t}(\omega)\not\subseteq c_{i,t}(\omega),$$

where we denoted

$$c_i'^{t}=\mathop{\vee}_{\tau=1}^{t}c_{i,\tau}'.$$

Suppose that player i accesses the sequence of information channels $c_i'\in\hat{C}_i$, and is required to tell whether he knows the information implied by another sequence $c_i\in\hat{C}_i$. If he does not recognize that $c_{i,t}(\omega)$ takes place at round t, then the resultant value of the information function $c_{i,t}(c_i')(\omega)=\Omega$ indicates that he does not know $c_{i,t}(\omega)$. Otherwise, $c_{i,t}(c_i')(\omega)=c_{i,t}(\omega)$, indicating that he knows $c_{i,t}(\omega)$.

For every $c\in\prod_{i\in N\cup\{0\}}\hat{C}_i$, $c'\in\prod_{i\in N\cup\{0\}}\hat{C}_i$, and (ι,κ), we define

$$\psi^1(c,c',\iota,\kappa)\equiv c_0',$$

and for every $l\in\{2,...,nT\}$, we define the associated information function $\psi^l(c,c',\iota,\kappa):\Omega\to 2^{\Omega}$ by

$$\psi^l(c,c',\iota,\kappa)(\omega)=c_0'(\omega)\cap\{\bigcap_{\substack{l'<l\\ \iota(l')\neq\iota(l)}}c_{\iota(l'),\kappa(l')}(c_{\iota(l')}')(\omega)\} \quad \text{for all } \omega\in\Omega.$$

The information function $\psi^l(c,c',\iota,\kappa)$ implies the aggregate of information implied by the sequence of information channels (c_0',c_N) that all players except for player $\iota(l)$ could observes according to another sequence of information channels c' before player $\iota(l)$ observes $c_{\iota(l),\kappa(l)}(\omega)$ in the order of (ι,κ).

Based on the above notations, a sequence of information channels $c \in \prod_{i \in N \cup \{0\}} \hat{C}_i$ is said to be *self-detective* in $\hat{C}$ if there exists (ι, κ) such that for every $l \in \{1,...,nT\}$ and $c'_{\iota(l)} \in \hat{C}_{\iota(l)}$, there exists $(c'_0, c'_{N\setminus\{\iota(l)\}}) \in \hat{C}_0 \times \prod_{j \in N\setminus\{\iota(l)\}} \hat{C}_j$ satisfying the following properties:

$$c' = (c_0, c'_{\iota(l)}, c'_{N\setminus\{\iota(l)\}}) \text{ belongs to } \hat{C},$$

and

$$\psi^l(c, c', \iota, \kappa) \text{ detects } c_{\iota(l),\kappa(l)} \text{ against } c'^{\kappa(l)}_{\iota(l)}.$$

Self-detection implies that at every round $t \in \{1,...,T\}$, for every player $i \in N$ who observed c'^t_i up to round t, the resultant information function $\psi^l(c, c', \iota, \kappa)$ detects this player's lie $m_{i,t} = c_{i,t}(\omega)$, where we denoted $\iota(l) = i$ and $\kappa(l) = t$. Hence, in order to prevent player $\iota(l)$ from lying at round $\kappa(l)$, it would be an effective method to penalize him whenever $\psi'(\omega') = c_{\iota(l),\kappa(l)}(\omega')$ is inconsistent with $\psi(\omega) = \psi^l(c, c', \iota, \kappa)$, i.e.,

$$\psi^{l-1}(\omega) \cap c_{\iota(l),\kappa(l)}(\omega') = \phi.$$

It is important to note that in order to incentivize the first player, i.e., player $\iota(1)$, it is necessary that the central planner accesses non-negligible verifiable information channels, i.e., $C_0 \neq \{\underline{\psi}\}$.

Since $\hat{C}$ would be a proper subset of $\prod_{i \in N \cup \{0\}} \hat{C}_i$, the self-detective sequence of information channels does not necessarily belong to $\hat{C}$. It is often the case that a player recognizes with certain that the central planner fails to access any non-degenerate information channel. On the other hand, if $\hat{C}$ is equivalent to $\prod_{i \in N \cup \{0\}} \hat{C}_i$ and a self-detective sequence c belongs to $\hat{C}$, then the definition of self-selection is simplified in the manner that there exists (ι, κ) such that for every $l \in \{1,...,nT\}$ and $c'_{\iota(l)} \in \hat{C}_{\iota(l)}$,

$$c_0 \vee \{ \bigvee_{\substack{l'<l \\ \iota(l')\neq\iota(l)}} c_{\iota(l'),\kappa(l')} \} \text{ detects } c_{\iota(l),\kappa(l)} \text{ against } c'^{\kappa(l)}_{\iota(l)} .$$

In this case, each player always expects the other players and the central planner to access the self-detective sequence with a positive probability.

Example 2: Consider a special case of Example 1, where we assume

$$L = nT - 2 .$$

The central planner attempts to make the appropriate financial rescue for firm $nT-2$. The central planner, however, can only identify the financial condition of firm 0, which includes no information that is related to the financial condition of the target firm $nT-2$. Despite of this, the central planner can motivate each player to reveal more related information through the following roundabout route.

Consider a sequence of information channels c^* and (ι,κ) such that

(5) $$c_0^* = \tilde{\psi}^0 ,$$

(6) $$c^*_{\tau(l),\kappa(l)} = \tilde{\psi}^l \text{ for all } l \in \{1,...,nT\} ,$$

and

(7) $$\iota(l) \neq \iota(l-1) \text{ and } \iota(l) \neq \iota(l-2) \text{ for all } l \in \{2,...,nT\} ,$$

where we denoted

(8) $$\tilde{\psi}^{nT} = \tilde{\psi}^{nT-1} = \tilde{\psi}^{nT-2} .$$

The equality (5) implies that according to c^*, the central planner only observes the financial condition of firm 0, i.e., ω_0. The equalities (6) imply that for each $l \in \{1,...,nT\}$, player $\iota(l) \in N$ observes the financial condition of firm l, i.e., ω_l, at round $\kappa(l)$. The inequalities (7) imply that for each $l \in \{2,...,nT-2\}$, player $\iota(l)$ never observes the financial conditions of firms $l-1$ and $l-2$, i.e., ω_{l-1} and ω_{l-2}. The equalities (8) imply that three distinct players, i.e., player $\iota(nT-2)$, player $\iota(nT-1)$, and player $\iota(nT)$, commonly observe the financial condition of the target firm $nT-2$, i.e., ω_{nT-2}.

We further assume that for every $c \in \hat{C}$, $t \in \{1,...,T\}$, $t' \in \{1,...,t-1\}$, and $i \in N$,

(9) $$c_{i,t'}(\omega) = c_{i,t'}(\omega') \text{ if } \omega_{l'} = \omega'_{l'} \text{ for all } l' \notin \{l-2, l-1\} ,$$

where $\iota(l)=i$ and $\kappa(l)=t$. The assumption (9) implies that irrespective of which sequence of information channels in $\hat{C}$ becomes accessible, player $\iota(l)$ never accesses any information channel concerning $(\omega_{l-2},\omega_{l-1})$.

We can show that c^* is self-detective. The information channel c_0^* reveals $\omega_0\in\{1,2,3\}$, implying $\omega_1\neq\omega_0$, because of (4). For every $l\in\{1,...,nT-3\}$, the information channel $c^*_{\iota(l),\kappa(l)}$ reveals $\omega_l\in\{1,2,3\}$, implying $\omega_{l+1}\neq\omega_l$, because of (4). For every $l\in\{1,...,nT-3\}$, player $\iota(l)$ does not know either of ω_{l-1} and ω_{l-2}, because of (9). According to the same method as in Section 7, for each $l\in\{1,...,nT\}$, whenever player $\iota(l)$ tells a lie, then he cannot ignore the possibility that his lie is detected by $c^*_{\iota(l-1),\iota(l-1)}$. Hence, for every $\tilde{c}\in\hat{C}$, c_0^* detects $c^*_{\iota(1),\kappa(1)}$ against $\tilde{c}^{\kappa(1)}_{\iota(1)}$, and for every $l\in\{2,...,nT\}$, $c^*_{\iota(l-1),\kappa(l-1)}$ detects $c^*_{\iota(l),\kappa(l)}$ against $\tilde{c}^{\kappa(l)}_{\iota(l)}$. From these observations, we have shown that c^* is self-detective in $\hat{C}$.

The central planner is the detector to player $\iota(1)$, player $\iota(1)$ is the detector to player $\iota(2)$, …, and player $\iota(nT-1)$ is the detector to player $\iota(nT)$. According to these chain, the central planner succeeds to incentivize three players, i.e., player $\iota(nT-2)$, player $\iota(nT-1)$, and player $\iota(nT)$, to make the honest announcement about the financial condition of the target firm $nT-2$.

It is important to note that the self-detective sequence c^* need not to be included in $\hat{C}$. All we need for the above arguments is that each player $\iota(l)$ expects player $\iota(l-1)$ to access the information channel $\tilde{\psi}^{l-1}$ with a positive probability. We even permit each player in $\{1,2,...,n-3\}$ to access at most one non-degenerate information channel, and each player in $\{n-2,n-1,n\}$ to access at most two non-degenerate information channels.

For simplicity, we assume that $T=1$,

$$\iota(i)=i \quad \text{for all} \quad i\in\{1,...,n\},$$

$$\hat{C}_i=\{\tilde{\psi}^{n-2}\} \quad \text{for all} \quad i\in\{n-2,n-1,n\},$$

and

$\hat{C}_i = \{\tilde{\psi}^i, \underline{\psi}\}$ for all $i \in \{0,...,n-3\}$.

Let us specify $\hat{C}$ as a proper subset of $\prod_{i \in N \cup \{0\}} \hat{C}_i$ such that for every $i \in \{0,...,n-3\}$, there is a sequence of information channels c in $\hat{C}$ such that

$c_i = \tilde{\psi}^i$ and $c_{i+1} = \tilde{\psi}^{i+1}$,

and for every $i \in \{0,...,n-3\}$, whenever $c_i = \tilde{\psi}^i$ and $c_{i+1} = \tilde{\psi}^{i+1}$, then

$c_j = \underline{\psi}$ for all $j \notin \{i, i+1, n-2, n-1, n\}$.

It is clear that $c^* = (\tilde{\psi}^i)_{i \in N \cup \{0\}}$ is self-detective in $\hat{C}$ in this case. Importantly, each player $i \in N$, who observes firm $i's$ financial condition $\tilde{\psi}^i(\omega) = \omega_i$, automatically recognizes that any player $j \notin \{i-1, i, i+1, n-2, n-1, n\}$ fails to observe firm $j's$ financial condition. In particular, any player $i \in \{2,...,n-3\}$, who observes firm $i's$ financial condition $\tilde{\psi}^i(\omega) = \omega_i$, automatically recognizes that the central planner fails to observe firm 0's financial condition $\tilde{\psi}^0(\omega) = \omega_0$.

9. Main Theorem

We introduce the following three conditions on f and $\hat{C}$, which are sufficient for unique implementation in strict iterative dominance.

Condition 1: There exists a sequence of information channels c^* that is self-detective in $\hat{C}$.

Condition 1 is a quite mild requirement, even if it is very crucial for our possibility theorem; it permits that there exist many sequences of information channels in $\hat{C}$ that are *not* self-detective. It is often the case that with a very high probability, only the degenerate partition $\underline{\psi}$ is accessible to the central planner. We even permit the self-detective sequence of information channel to be inaccessible at all.

Fix an arbitrary partition $\psi^* \in \Psi$, and focus on social choice functions that depend only on this partition.

Condition 2: A social choice function $f \in F$ is measurable with respect to ψ^* in the sense that for every $\omega \in \Omega$ and $\omega' \in \Omega$,

$$f(\omega) = f(\omega') \quad \text{whenever} \quad \psi^*(\omega) = \psi^*(\omega').$$

We further require that there always exist at least three players who identify the information implied by ψ^*.

Condition 3: For every $c \in \hat{C}$ and $\omega \in \Omega$, there exist at least three players $i \in N$ such that

$$c_i(\omega) \subseteq \psi^*(\omega).$$

Condition 3 would be necessary for unique implementation with almost no common knowledge assumptions. For instance, let us consider the case that there exists a sequence of information channels $c \in \hat{C}$, according to which, at most two players know $f(\omega)$. Such an informed player is willing to tell a lie, whenever he expect this sequence to be accessible with a sufficient probability.

Theorem 2: *Suppose that Conditions 1, 2, and 3 hold. Then, any social choice function is uniquely implementable in strict iterative dominance with almost no side payments.*

The complete proof of Theorem 2 will be shown in Section 10. In this proof, we design a mechanism G that is detail-free in the sense that irrespective of the specifications $(v, p, q) \in V(\eta) \times P(\varepsilon_1) \times Q(\varepsilon_2)$, it uniquely implements the social choice function f in strict iterative dominance. This permissive result holds irrespective of the specifications $(\eta, \varepsilon_1, \varepsilon_2, \xi)$.

In the designed mechanism, at any round from round 1 to round T, each player announces about what he observes at this round. At round $T+1$, each player announces about what he knows about the information related to the social choice function, i.e., $\psi^*(\omega)$. From round $T+2$ to round $T+K$, each player repeatedly announces about what he knows about $\psi^*(\omega)$. At the end of the final round $T+K$, the central planner randomly picks up a profile $m_t = (m_{i,t})_{i\in N}$ among the last $K-1$ profiles $\{m_t\}_{t=T+2}^{T+K}$, and then selects the allocation according to a variant of the majority rule.

According to the Abreu-Matsushima method, the central planner requires any player to make a side payment as a penalty whenever he is one of the first deviant from the previous announcements after round $T+1$. We only require this payment to be greater than $\eta/_{K-1}$. By selecting a sufficiently large K, we can make this penalty payment as close to zero as possible.

Example 3: Let us consider a simple version of Example 1, where we assume that $L=1$, i.e., a state is expressed by $\omega = (\omega_0, \omega_1)$. Assume that a player accesses an information channel $\tilde{\psi}$ specified by

$$\tilde{\psi}(\omega) = \tilde{\psi}(\omega') \text{ if and only if either } \omega_1 = \omega_1' = 1 \text{ or } \{\omega_1, \omega_1'\} \subset \{2,3\}.$$

This player can distinguish between the case that the financial condition of firm 1 is 1 and the case that it belongs to $\{2,3\}$; he cannot distinguish between 2 and 3. He pretends to access the information channel that identifies the financial condition of firm 1, i.e., $\psi' = \tilde{\psi}^1$.

The central planner, as the detector to this player, accesses the information channel that identifies the financial condition of firm 0, i.e., $\psi = \tilde{\psi}^0$. Clearly, $\psi = \tilde{\psi}^0$ detects $\psi' = \tilde{\psi}^1$ against $\tilde{\psi}$.

The player is required to announce either 0, 1, 2, or 3 as his opinion about firm 1's financial condition. In this case, the announcement of '$\omega_1 = 0$' implies that he fails to identify firm 1's financial condition. Let us specify the following incentive scheme, $\chi = (\chi^{\omega_0})_{\omega_0 \in \{1,2,3\}}$; for each $\omega_0 \in \{1,2,3\}$,

$$\chi^{\omega_0} : \{0,1,2,3\} \to R,$$

and for each $\omega_1 \in \{0,1,2,3\}$,

$$\chi^{\omega_0}(\omega_1) = 1 \qquad \text{if } \omega_0 = \omega_1,$$

$$\chi^{\omega_0}(\omega_1) = 0 \qquad \text{if } \omega_0 \neq \omega_1 \text{ and } \omega_1 \neq 0,$$

and

$$\chi^{\omega_0}(\omega_1) = \varepsilon_1\varepsilon_2 / 2 \qquad \text{if } \omega_1 = 0.$$

According to χ, the player makes the side payment $\chi^{\omega_0}(\omega_1')$ to the central planner when he announces $\tilde{\psi}^1(\omega')$, i.e., ω_1', and the central planner observes $\tilde{\psi}^0(\omega)$.

We can show that the player is willing to announce honestly. If he observes $\omega_1 = 1$, i.e., he identifies the financial condition of firm 1, he can escape from the penalty payment by announcing $\omega_1' = 1$ truthfully. If the player fails to identify it, i.e., only knows $\omega_1 \in \{2,3\}$, he can best save the penalty payment in expectation by announcing '$\omega_1 = 0$' instead of any value in $\{1,2,3\}$; it is at least with probability $\varepsilon_1\varepsilon_2$ that the central planner finds out his lie. By announcing '$\omega_1 = 0$' he pays just $\varepsilon_1\varepsilon_2 / 2$, while by announcing any value in $\{1,2,3\}$, he must pay at least $\varepsilon_1\varepsilon_2$ in expectation, which is greater than $\varepsilon_1\varepsilon_2 / 2$.

11. Proof of the Theorem

Assume Conditions 1, 2, and 3. Fix an arbitrary $K > 0$, which is selected sufficiently large. We specify a sequential revelation mechanism $G = (M, g, x)$ as follows. For every $t \in \{1,...,T\}$ and $i \in N \cup \{0\}$,

$$M_{i,t} = \{m_{i,t} \subset \Omega : m_{i,t} = c_{i,t}^*(\omega) \text{ for some } \omega \in \Omega\} \cup \{\Omega\}.$$

For every $t \in \{T+1,...,T+K\}$ and $i \in N \cup \{0\}$,

$$M_{i,t} = \{m_{i,t} \subset \Omega : m_{i,t} = \psi^*(\omega) \text{ for some } \omega \in \Omega\} \cup \{\Omega\}.$$

At each round t from 1 to T, each player $i \in N \bigcup \{0\}$ announces either an element of $c_{i,t}^*$ or Ω, where the announcement of Ω implies that he failed to receive the information implied by $c_{i,t}^*$. At each round from $T+1$ to $T+K$, each player $i \in N \bigcup \{0\}$ repeatedly announces about what he knows about the information implied by ψ^*.

Let us select an arbitrary allocation $a^* \in A$ as the status quo. For every $m \in M$ and $t \in \{T+2,...,T+K\}$,

$$g(m) = g_t(m_t) \text{ with probability } \frac{1}{K-1},$$

where we specify $g_t(m_t)$ as a variant of majority rule as follows: for every $t \in \{T+2,...,T+K\}$, $m_t \in M_t$, and $\omega \in \Omega$,

$$g_t(m_t) = f(\omega) \qquad \text{if } m_{i,t} = \psi^*(\omega) \text{ for at least two players in } N \text{, and}$$

$$m_{i,t} \notin \{\psi^*(\omega), \Omega\} \text{ for at most one player in } N,$$

and

$$g_t(m_t) = a^* \qquad \text{otherwise.}$$

According to g, the central planner randomly picks up round t among $\{T+2,...,T+K\}$, and then selects the allocation $g_t(m_t)$. If at least three players in N announce the same opinion, i.e., $\psi^*(\omega)$ for some $\omega \in \Omega$, and at most one player in N announces a conflicting opinion, i.e., $\psi^*(\omega') \notin \{\psi^*(\omega), \Omega\}$, then the central planner accepts the first opinion and then selects $g_t(m_t) = f(\omega)$. Otherwise, the central planner selects the status quo $g_t(m_t) = a^*$.

Fix arbitrary positive real numbers $\gamma > 0$, $\varphi_t > 0$, and $\lambda_t > 0$. Let $\rho(i,t)$ denote the integer in $\{n(t-1)+1,...,nt\}$ such that

$$\iota(\rho) = i \text{ and } \kappa(\rho) = t,$$

where ι and κ were the functions in the definition of self-detection.

We specify x_i by

$$x_i(m) = \sum_{t=1}^{T+1} x_{i,t}(m) + y_i(m),$$

where we specify $x_{i,t} : M \to R$ and $y_i : M \to R$ in the following manner; for every $m \in M$ and $t \in \{1,...,T+1\}$,

$$x_{i,t}(m) = \gamma \quad \text{if} \quad m_{i,t} = \Omega,$$

$$x_{i,t}(m) = 0 \quad \text{if} \quad m_{i,t} \neq \Omega, \text{ and}$$

(11) $$(\bigcap_{\tau=1}^{T} m_{0,\tau}) \cap (\bigcap_{\substack{\iota(\rho') \neq i \\ \rho' < \rho(i,t)}} m_{\iota(\rho'),\kappa(\rho')}) \cap m_{i,t} \neq \phi,$$

and

$$x_{i,t}(m) = \varphi_t \quad \text{if} \quad m_{i,t} \neq \Omega, \text{ and}$$

(12) $$(\bigcap_{\tau=1}^{T} m_{0,\tau}) \cap (\bigcap_{\substack{\iota(\rho') \neq i \\ \rho' < \rho(i,t)}} m_{\iota(\rho'),\kappa(\rho')}) \cap m_{i,t} = \phi.$$

At each round t from 1 to $T+1$, any player $i \in N$ is fined by $\gamma > 0$ if he announces Ω. He is never fined if the inequality (11) holds, i.e., if his announcement $m_{i,t}$ is consistent with the other players' previous announcements in the order of (ι, κ), i.e., $\bigcap_{\substack{\iota(\rho') \neq i \\ \rho' < \rho(i,t)}} m_{\iota(\rho'),\kappa(\rho')}$, as well as player 0's announcements during the entire rounds from 1 to T, i.e., $\bigcap_{\tau=1}^{T} m_{0,\tau}$. He is fined by $\varphi_t > 0$ otherwise, i.e., if the equality (12) holds.

For every $m \in M$ and $t \in \{T+2,...,T+K\}$,

$$y_i(m) = \lambda_t \quad \text{if} \quad m_{i,t} \neq m_{i,T+1}, \text{ and}$$

$$m_{j,\tau} = m_{j,T+1} \text{ for all } j \in N \text{ and } \tau \in \{T+2,...,t-1\},$$

and

$$y_i(m) = 0 \quad \text{if} \quad \text{there exists no such } t \in \{T+2,...,T+K\}.$$

The design of y_i originates in Abreu and Matsushima (1992a). According to y_i, player $i \in N$ is fined by $\lambda_t > 0$ if he is one of the first players in N who make different announcements from their respective $(T+1)$-round announcements, where t denotes the round at which they make such announcements.

Let us specify s_i^* as the *honest strategy* for each player $i \in N \cup \{0\}$ in the manner that for every $\omega \in \Omega$ and $c_i \in C_i$, for every $t \in \{1,...,T\}$,

$$s_{i,t}^*(\omega, c_i^t) = c_{i,t}^*(\omega) \quad \text{if} \quad c_i^t(\omega) \subseteq c_{i,t}^*(\omega),$$

$$s_{i,t}^*(\omega, c_i^t) = \Omega \quad \text{if} \quad c_i^t(\omega) \not\subseteq c_{i,t}^*(\omega),$$

and for every $t \in \{T+1,...,T+K\}$,

$$s_{i,t}^*(\omega, c_i) = \psi^*(\omega) \quad \text{if} \quad c_i(\omega) \subseteq \psi^*(\omega),$$

and

$$s_{i,t}^*(\omega, c_i) = \Omega \qquad \text{if} \quad c_i(\omega) \not\subseteq \psi^*(\omega).$$

According to s_i^*, at any round t from round 1 to round T, each player i announces truthfully about what he observes about $c_{i,t}^*$. Whenever he fails to access $c_{i,t}^*$ or receive any finer information, he announces Ω. At any round from round $T+1$ to round $T+K$, each player i announces truthfully about what he observes about ψ^*. Whenever he fails to know such information, he announces Ω.

Under Condition 3, according to the honest strategy profile s^*, the central planner can certainly select $f(\omega)$ at all times, where ω denotes the true state. This implies $f = f^{G,s^*,c}$ for all $c \in \hat{C}$.

Let us specify $\gamma > 0$, $\varphi_t > 0$, and $\lambda_t > 0$ by

(13) $$\lambda_{T+K} > \eta \big/ K - 1,$$

(14) $$\lambda_t > \lambda_{t+1} \quad \text{for all} \quad t \in \{T+2,...,T+K\},$$

(15) $$\varepsilon_1 \varepsilon_2 \varphi_{T+1} > \lambda_{T+2} + \gamma,$$

(16) $$\varepsilon_1 \varepsilon_2 \varphi_t > \gamma \quad \text{for all} \quad t \in \{1,...,T\},$$

and

(17) $$\xi > \sum_{t=1}^{T+1} \varphi_t + \lambda_{T+2}.$$

Note that for every $\xi > 0$, there exist $\gamma > 0$, $\varphi_t > 0$, and $\lambda_t > 0$ that satisfy (13), (14), (15), (16), and (17). Note also that the side payment that the mechanism G requires each player $i \in N$ to make is less than $\sum_{t=1}^{T+1} \varphi_{i,t} + \lambda_{i,T+2}$. This, along with (17), implies that $\max_{i \in N, m \in M} |x_i(m)| \le \xi$. Hence, all we have to do for the completion of the proof is to prove that s^* is the unique strictly iteratively undominated strategy profile with respect to $(v, s_0^*, \varepsilon_1, \varepsilon_2)$.

Fix an arbitrary $\omega \in \Omega$ as the true state. Assume that player 0 announces truthfully according to s_0^*. Firstly, consider round 1. Suppose that a player $i \in N$ observes $c_{i,1}^*(\omega)$ or any finer information. In this case, by announcing $c_{i,1}^*(\omega)$, he is not fined through $x_{i,1}$. If he announces Ω, he is fined through $x_{i,1}$ by $\gamma > 0$. If he announces any element of $c_{i,1}^*$ other than $c_{i,1}^*(\omega)$, it follows from Condition 1, $p \in P(\varepsilon_1)$, $q \in Q(\varepsilon_2)$, and the specification of $x_{i,1}$ that with at least probability $\varepsilon_1 \varepsilon_2 > 0$, he is fined through $x_{i,1}$ by $\varphi_1 > 0$. Since his announcement at round 1 never influences the allocation selection, player i is willing to announce $c_{i,1}^*(\omega)$.

Suppose that player i fails to observe $c_{i,1}^*(\omega)$ or any finer information at round 1. In this case, if he announces any element of $c_{i,1}^*$, it follows from Condition 1, $p \in P(\varepsilon_1)$, $q \in Q(\varepsilon_2)$, and the specification of $x_{i,1}$ that he is fined through $x_{i,1}$ by $\varphi_1 > 0$ with at least probability $\varepsilon_1 \varepsilon_2$. Since his announcement at round 1 never influences the allocation selection, it follows from (16) that player i is willing to announce Ω.

Next, consider any round $t \in \{2, ..., T\}$. Suppose that from round 1 to round $t-1$, all players announce truthfully according to s^*. Suppose that a player $i \in N$ observes $c_{i,t}^*(\omega)$ or any finer information. By announcing $c_{i,t}^*(\omega)$ truthfully, he is never fined through $x_{i,t}$. If he announces Ω, he is fined through $x_{i,t}$ by $\gamma > 0$. If he announces any element of $c_{i,t}^*$ other than $c_{i,t}^*(\omega)$, it follows from Condition 1, $p \in P(\varepsilon_1)$, $q \in Q(\varepsilon_2)$, and the specification of $x_{i,t}$ that he is fined through $x_{i,t}$ by $\varphi_t > 0$ with at least probability $\varepsilon_1 \varepsilon_2$. Since his announcement at round t never influences the allocation selection, player i is willing to announce $c_{i,t}^*(\omega)$.

Suppose that player i fails to observe $c_{i,t}^*(\omega)$ or any finer information. If he announces any element of $c_{i,t}^*$, it follows from Condition 1, $p \in P(\varepsilon_1)$, $q \in Q(\varepsilon_2)$, and the specification of $x_{i,t}$ that he is fined through $x_{i,t}$ by $\varphi_t > 0$ with at least probability $\varepsilon_1\varepsilon_2$. Since his announcement at round t never influences the allocation choice, it follows from (16) that player i is willing to announce Ω.

Next, consider round $T+1$. Suppose that from round 1 to round T, all players announce truthfully according to s^*. Suppose that a player $i \in N$ has observed $c_i^T(\omega) \subseteq \psi^*(\omega)$. By announcing $\psi^*(\omega)$, he is never fined through $x_{i,T+1}$. If he announces Ω, he is fined through $x_{i,T+1}$ by $\gamma > 0$. If he announces any element of ψ^* other than $\psi^*(\omega)$, it follows from Condition 1, $p \in P(\varepsilon_1)$, $q \in Q(\varepsilon_2)$, and the specification of $x_{i,T+1}$ that he is fined through $x_{i,T+1}$ by $\varphi_{T+1} > 0$ with at least probability $\varepsilon_1\varepsilon_2$. Since his announcement at round $T+1$ never influences the allocation selection but influences the possibility of being fined through y_i, it follows from (14) and (15) that player i is willing to announce $\psi^*(\omega)$.

Suppose that player i has failed to observe $\psi^*(\omega)$ or any finer information up to round T, i.e., $c_i^T(\omega) \not\subseteq \psi^*(\omega)$. If he announces any element of ψ^* at round $T+1$, it follows from Condition 1, $p \in P(\varepsilon_1)$, $q \in Q(\varepsilon_2)$, and the specification of $x_{i,T+1}$ that he is fined through $x_{i,T+1}$ by $\varphi_{T+1} > 0$ with at least probability $\varepsilon_1\varepsilon_2$. Since his announcement at round $T+1$ never influences the allocation selection but influences the possibility of being fined through y_i, it follows from (14) and (15) that player i is willing to announce Ω.

Finally, consider any round $t \in \{T+2,...,T+K\}$. We apply the method of Abreu-Matsushima mechanism. Suppose that from round 1 to round $t-1$, all players announce according to s^*. Suppose that there exists another player $j \in N \setminus \{i\}$ who announces $m_{j,t} \neq m_{j.T+1}$ at round t. If player i announces $m_{i,t} \neq m_{i.T+1}$, he is fined by $\lambda_{i,t}$ through y_i. If player i announces $m_{i,t} = m_{i.T+1}$, he is never fined through y_i. His announcement at round t influences the allocation selection through $g_t(m_{0,T+1}, m_t) = f(\omega)$, but this influence on his welfare is at most within the limit

$\eta/_{K-1}$. This, along with (13) and (14), implies that player i is willing to announce $m_{i,t} = m_{i.T+1}$.

Suppose that any other player $j \in N \setminus \{i\}$ announces $m_{j,t} = m_{j.T+1}$. If player i announces $m_{i,t} \neq m_{i.T+1}$, he is fined by $\lambda_t > 0$ through y_i. Even if player i announces $m_{i,t} = m_{i.T+1}$, he may be fined through y_i, which, however, is smaller than λ_t, because of (14). From Condition 3 and the specification of g_t, his announcement at round $t \in \{T+2,...,T+K\}$ gives no influence on the allocation selection. Hence, player i is willing to announce $m_{i,t} = m_{i.T+1}$.

From the above observations, we has shown that s^* is the unique strictly iteratively undominated strategy profile with respect to $(v, s_0^*, \varepsilon_1, \varepsilon_2)$,

11. Abreu-Matsushima Method Revisited: Non-Expected Utility

Throughout this paper, we assumed quasi-linearity, risk-neutrality, and the expected utility hypothesis. We, however, can replace these assumptions with much weaker conditions. We define a payoff function for each player $i \in N$ as a function

$$u_i : \Delta \times R \times \Omega \to R,$$

where $u_i(\alpha, t_i, \omega)$ implies the payoff for player i when the allocation is determined according to $\alpha \in \Delta$ and player i receives a private good payment $t_i \in R$ at state $\omega \in \Omega$. This section will assume that $u_i(\alpha, t_i, \omega)$ is continuous with respect to $\alpha \in \Delta$ and $t_i \in R$, and that $u_i(\alpha, t_i, \omega)$ is increasing in t_i.

For simplicity of arguments, we assume that $T = 1$, and all players as well as the central planner certainly observe the true state $\omega \in \Omega$; for each $i \in N \bigcup \{0\}$,

$$\hat{C}_i = \{\overline{c}_i\}, \text{ where } \overline{c}_i(\omega) = \{\omega\} \text{ for all } \omega \in \Omega.$$

We slightly modify the definition of strict iterative dominance; let

$$M_i(0, \omega) = M_i \text{ for all } i \in N \text{ and all } \omega \in \Omega.$$

Recursively, for each $h \geq 1$, we define $M_i(h,\omega) \subset M_i$ in the manner that $m_i \in M_i(h,\omega)$ if and only if there exists no $m_i' \in M_i(h-1,\omega)$ that strictly dominates m_i, i.e., for every $m_{-i} \in M_{-i}(h-1,\omega)$,

$$v_i(g(m),-x_i(m),\omega) < v_i(g(m'_i, m_{-i}),-x_i(m'_i, m_{-i}),\omega),$$

We define

$$M_i^*(\omega) \equiv \bigcap_{h=0}^{\infty} M_i(h,\omega).$$

We specify G in the same manner as in Theorem 2. Let

$$M_i = \prod_{k=1}^{K} M_i^k .$$

We define $g^k : M^k \to \Delta$ as follows; for each $\omega \in \Omega$,

$$g^k(m^k)(f(\omega)) = 1 \qquad \text{if } m_i^k = \omega \text{ for } n-1 \text{ players,}$$

and

$$g^k(m^k)(a^*) = 1 \qquad \text{if there exists no such } \omega .$$

Let

$$g(m) = \frac{\sum_{k=1}^{K} g^k(m^k)}{K}.$$

Let

$$x_i(m,\omega) = \frac{\xi}{2}(1+\frac{r}{K}) \qquad \text{if there exists } k \in \{1,...,K\} \text{ such that}$$

$m_j^{k'} = \omega$ for all $k' \leq k$ and $j \in N \setminus \{i\}$, and

$m_i^k \neq \omega$,

and there exist r rounds such that

$m_i^k \neq \omega$.

Since $u_i(\alpha,t_i,\omega)$ is continuous with respect to $\alpha \in \Delta$ and $t_i \in R$, and it is increasing in t_i, we can select K sufficiently large to satisfy that

(18) $$u_i(\alpha,0,\omega) > u_i(\alpha',-\xi/2,\omega) \text{ for all } \omega \in \Omega \text{ if } \max_{a\in A}|\alpha(a)-\alpha'(a)| \leq \frac{1}{K}.$$

With such a sufficiently large K, in the same manner as in Theorem 2, the honest strategy profile s^* is the unique strictly iteratively undominated strategy profile. He is fined by paying $\xi/2$ whenever he is the first deviant and there is another first deviant; from (18), he dislikes lying at round 1, because his lying gives only a tiny influence on the allocation selection.

He dislike lying even when there is no other first deviant: his lying gives no influence on the allocation at all, while he is fined by $\xi/2K$. Recursively, at any round $t \geq 2$, provided all players tell the truth at all previous rounds, any player clearly dislikes lying.

12. Conclusion

This paper investigated unique implementation, where we made the very weak knowledge assumptions that prior distributions on state space and information accessibility, as well as payoff functions, are not common knowledge. We designed a detail-free mechanism and applied a detail-free version of strict iterative dominance. Importantly, we permitted the central planner to access verifiable information channels; otherwise it was impossible to implement a social choice function. This would be the first paper to investigate such weak knowledge assumptions and verifiable information accessibility.

We showed a sufficient condition for unique implementation in strict iterative dominance; a tiny possibility of self-detection was sufficient. This result is very permissive. The central planner's channels could be marginal and even unrelated to the social choice function. The probability of each player as well as the central planner accessing non-degenerate channels is as close to zero as possible. We even permit each player to recognize that the central planner fails to access non-degenerate channels.

References

Abreu, D. and H. Matsushima (1992): "Virtual implementation in iteratively undominated strategies: Incomplete information," unpublished manuscript, Princeton University.

Abreu, D. and H. Matsushima (1992): "Virtual Implementation in Iteratively Undominated Strategies: Complete Information," *Econometrica* 60, 993–1008.

Abreu, D. and H. Matsushima (1994): "Exact Implementation," *Journal of Economic Theory* 64 (1), 1-19.

Abreu, D. and A. Sen (1991): "Virtual implementation in Nash equilibrium," *Econometrica* 59, 997–1021.

Anderson, L. and C. Holt (1997): "Information Cascades in the Laboratory," *American Economic Review* 87, 847-862.

Bergemann, D. and S. Morris (2009): "Robust virtual implementation, *Theoretical Economics* 4, 45–88.

Bergemann, D. and S. Morris (2012): *Robust Mechanism Design*, World Scientific Press, 2012.

Bikhchandani, S., D. Hirshleifer, and I. Welch (1992): "A Theory of Fads, Fashion, Custom, and Cultural Change as Informational Cascades," *Journal of Political Economy* 100(5), 992-1026.

Cheny, Y.-C. T. Kunimoto, and T. Sun (2014): "Implementation with Transfers," mimeo.

Chung, K.-S. and J. Ely (2003): "Implementation with Near-Complete Information," *Econometrica* 71, 857-871.

Cremer, J. and R. McLean (1988): "Full extraction of the surplus in Bayesian and dominant strategy auctions", *Econometrica* 56, 1247–1257.

Duggan, J. (1997): "Virtual Bayesian implementation," *Econometrica* 65, 1175–1199.

Dutta, B. and A. Sen (2012): "Nash Implementation with Partially Honest Individuals," *Games and Economic Behavior* 74 (1), 154-169.

Eliaz, K. (2002): "Fault Tolerant Implementation," *Review of Economic Studies* 69, 589–610.

Gibbard, A. (1973): "Manipulation of Voting Schemes," *Econometrica* 41, 587-601.

Jackson, M. (1991): "Bayesian Implementation," *Econometrica* 59, 461-477.

Jackson, M. (1992): "Implementation in Undominated Strategies: A Look at Bounded Mechanisms," *Review of Economic Studies* 59, 757–775.

Jackson, M. (2001): "A Crash Course in Implementation Theory," *Social Choice and Welfare 18*, 655-708.

Johnson, S., J. Pratt, and R. Zeckhauser (1990): "Efficiency despite mutually payoff-relevant private information: The finite case", *Econometrica* 58, 873-900.

Kartik, N. and O. Tercieux (2012): "Implementation with Evidence," *Theoretical Economics* 7, 323-356.

Maskin, E. (1999): "Nash equilibrium and Welfare Optimality," *Review of Economic Studies* 66, 23–38.

Maskin, E. and T. Sjostrom (2002): "Implementation Theory," *Handbook of Social Choice and Welfare 1*, *Vol. 1*, ed. by K. Arrow, A. Sen, and K. Suzumura, Elsevier.

Matsushima, H. (1988): "A New Approach to the Implementation Problem," *Journal of Economic Theory* 45, 128–144.

Matsushima, H. (1991): "Incentive Compatible Mechanisms with Full Transferability," *Journal of Economic Theory* 54 (1), 198-203.

Matsushima, H. (1993): "Bayesian Monotonicity with Side Payments," *Journal of Economic Theory* 59, 107-121.

Matsushima, H. (2005): "On Detail-Free Mechanism Design and Rationality," *Japanese Economic Review* 56 (1), 41-54.

Matsushima, H. (2008a): "Detail-Free Mechanism Design in Twice Iterative Dominance," *Journal of Economic Theory* 56 (1), 41-54.

Matsushima, H. (2008b): "Role of Honesty in Full Implementation," *Journal of Economic Theory* 127, 353–359.

Matsushima, H. (2008c): "Behavioral Aspects of Implementation Theory," *Economics Letters* 100, 161–164.

Matsushima, H. (2013): "Process Manipulation in Unique Implementation," *Social Choice and Welfare* 41 (4), 883-893.

Moore, J. (1992): "Implementation in Environments with Complete Information," in *Advances in Economic Theory: Sixth World Congress*, ed. by J.J. Laffont. Cambridge University Press.

Moore, J. and R. Repullo (1988): "Subgame Perfect Implementation", *Econometrica* 56, 1191-1220

Osborne, M. and A. Rubinstein (1994): *A Course in Game Theory*, MIT Press.

Oury, M., and O. Tercieux (2012): "Continuous Implementation," *Econometrica* 80, 1605-1637.

Palfrey, T. (1992): "Implementation in Bayesian Equilibrium: The Multiple Equilibrium Problem in Mechanism Design," in *Advances in Economic Theory: Sixth World Congress*, ed. by J.J. Laffont, Cambridge University Press.

Palfrey, T. and S. Srivastava (1991): "Nash Implementation Using Undominated Strategies," Econometrica 59, 479-501.

Satterthwaite, M. (1975): "Strategy-Proofness and Arrow's Conditions: Existence and Correspondence Theorems for Voting Procedures and Social Welfare Functions," *Journal of Economic Theory* 10, 187-217.