Profitability and Concentration of Competitive Insurance Markets with Better Informed Principals

Preliminary Version

Laura Abrardi[*], Luca Colombo[†], Piero Tedeschi[‡]

February 2015

Abstract

We study a competitive insurance market in which insurers have an imperfect informative advantage over policy holders. We show that the presence of insurers privately and heterogeneously informed about risk can explain the persistent profitability observed in some insurance markets. Furthermore, we find that a lower market concentration may entail an increase in insurance premia. Finally, we find that pooling and separating equilibria may coexist for a given market structure.

Keywords: Insurance markets, Asymmetric information, Risk assessment, Market concentration.
JEL codes: D43, D82, G22

1 Introduction

The issue of damage insurance markets profitability has been largely debated by the theoretical and the empirical literature. While the traditional approach (see e.g. Rothschild and Stiglitz, 1976) predict that the efficient competitive mechanism rules out the possibility of positive profits for insurance companies, the empirical evidence seems to suggest the

[*]Politecnico di Torino, Corso Duca degli Abruzzi 24, 10129 Torino. E-mail: laura.abrardi@polito.it
[†]Università Cattolica del Sacro Cuore, Department of Economics and Finance, Largo A. Gemelli 1, 20123 Milano. E-mail: lucava.colombo@unicatt.it
[‡]Università Cattolica del Sacro Cuore, Department of Economics and Finance, Largo A. Gemelli 1, 20123 Milano. E-mail: piero.tedeschi@unicatt.it

contrary, with prices not reflecting insurance costs (Robinson, 2004). A natural explanation for the observed profitability of the industry lies in its market concentration. However, the available empirical studies have not found a significant relationship between the recent rise in insurance prices and market concentration (Dafny *et Al.*, 2009; Hyman and Kovacic, 2004). As Hyman and Kovacic (2004) point out, concentration does not indicate *per se* the presence of anti-competitive behavior.

This puzzle has raised considerable interest, and has been addressed by a number of theoretical studies. Bisin and Guaitoli (2004) provide an explanation based on the presence of moral hazard in a context of non-exclusive contracts, Wambach (2000) focuses on the role of heterogeneity in risk aversion, while Villeneuve (2005) emphasizes the effects of the existence of better informed insurers. None of these papers, however, can rule out the existence of a zero profit equilibrium. The persistent profitability of the insurance industry is therefore challenging for existing theories.

This paper contributes to the literature by providing a theoretical explanation for the observed profitability of specific insurance markets. Namely, we show that when insurers are better informed than policyholders and their assessment of risk is imperfect profits are not only possible, but actually necessary for the existence of an equilibrium, so that actuarially fair outcomes are impossible. Furthermore, we show that a regulatory intervention aimed at the reduction of market concentration can turn out to be ineffective or even counter-productive in terms of curtailing profits, possibly leading to an increase of insurance prices.

Traditional models of asymmetric information in insurance markets (affected by moral hazard, adverse selection, or both) typically assume better informed policyholders. However, the elusive nature of risk and the skills required to estimate it make it plausible that the informative advantage is held not (or not only) by policyholders, but by insurers. Insurers are indeed better qualified, for their greater expertise and access to data, to obtain a more precise assessment of risk. This assumption is not new in the literature, as the policyholders' inability to correctly estimate risk is, as a matter of fact, highlighted by a large number of studies on policyholders' overconfidence, or unrealistic pessimism.[1]

Better informed policyholders are also at the center stage in Villeneuve (2000, 2005), Seog (2009), and Fombaron (1997), and it is tested by Cawley and Philipson (1999), Finkelstein and McGarry (2006), and Smith *et al.* (2001). In particular, Villeneuve (2005) studies a competitive insurance market finding that insurers' profits may be positive when they are

[1] On policyholders' overconfidence, see e.g. Camerer (1997), Fang and Moscarini (2005), Garcia, Sangiorgi and Urosevic (2007), Hoelzl and Rustichini (2005), Kőszegi (2006), Menkhoff *et al.* (2006), Noth and Weber (2003), Sandroni and Squintani (2007), Van den Steen (2004), and Zábojník (2004). As for unrealistic pessimism, refer to the seminal contribution by Kahneman and Tversky (1979).

better informed than policyholders about risk.

While the idea that insurers are better equipped than policyholders to estimate risk is appealing, the idea that risk can be perfectly assessed, even by professionals, may be far-fetched. The same elusive nature of risk implies that its estimation may be subject to mistakes by individual insurers, due to imperfect knowledge or biased attitudes toward it. The difficulties underlying the process of risk estimation, even for experts, may be the cause of heterogeneity among insurers, compounded by the insurers' unwillingness (or impossibility) to share with competitors their beliefs about risk.[2]

Taking this into account, we consider a competitive insurance market in which insurers are better able than policyholders to estimate risk, but despite their expertise, their estimation of risk is imperfect. More precisely, we assume that each insurer estimates policyholders' risk on the basis of a private, imperfect signal. By doing so, we depart from Villeveuve's framework in two directions. First, by assuming that insurers are imperfectly able to assess risk (their estimations are private and cannot be observed neither by the policyholders, nor by other competitors). second, by assuming that the private information held by the insurers concerns the riskiness of the environment in which they operate and not the characteristics of the policyholders. Importantly, both differences with respect to Villeneuve's setup allow us to build a theoretical framework that is consistent with Harsanyi's approach to incomplete information games, where each player is more informed about her own type than any other player.

We find that the competitive mechanism is unable to guarantee the existence of actuarially fair insurance for all policyholders. This follows from the fact that truthful revelation of insurers' private information requires positive profits in equilibrium, which obviously cannot be achieved under actuarially fair contracts. Insurers' beliefs about the riskiness of the environment in which they operate have important implications on the informational rent needed to induce truthful revelation. More precisely, insurers with a lower assessment of risk may have an incentive to lie, pretending to expect a riskier environment, in order to charge higher premia to customers. Hence, a truthful disclosure about the riskiness of the environment requires an higher informational rent the safer is the insurer's estimation of risk.

We show that incentive compatible contracts may be the outcome both of pooling and

[2] For example, many laws require that medical records cannot be released to outsiders without the consent of the patient. This increases the probability of mistakes in the estimation of risk by insurers. Moreover (see e.g. Fombaron, 1997) companies may learn about the risk of their insureds by observing claims records and contract choices, but will not share these private informations freely with rival firms. As a consequence, the rival firms do not have access to accident histories.

separating equilibria, the existence of which depends on the distribution of insurers and policyholders' preferences, as well as on policyholders' out of equilibrium beliefs about insurers' assessments of risk.[3] Due to the truthful revelation condition discussed above, positive profits are observed both for pooling and separating equilibria, and in both cases they are shown to be larger for insurers' expecting a safer market environment. Although this result has already been shown by Villeneuve (2005) for pooling equilibria, as far as separating equilibria are concerned, it is a novel contribution of this paper.

Interestingly, market concentration plays an important role in determining which type of equilibria are likely to emerge. In particular, pooling equilibria are shown to exist when the market is sufficiently concentrated, while the existence of separating equilibria requires a sufficiently dispersed industry. Furthermore, although it turns out to be impossible characterizing analytically the parameters constellations for which pooling and separating equilibria exist, numerical analysis with a CARA utility specification shows that there exist a parameter region in which both types of equilibria coexist, and another one for which the two sets of equilibria are disjoint.

The rest of the paper is organized as follows. Section 2 describes the model and introduces the equilibrium concepts and the main definitions. Section 3 characterizes pooling and separating equilibria and focuses on their profitability for generic utility specifications. Section 4 studies the correlation between market concentration and profits both for pooling and separating equilibria, also providing numerical results based on a CARA utility specification. Section 5 concludes. All proofs and technical details that are not crucial for the understanding of the underlying logic of the model are relegated into an Appendix.

2 The model

Consider an insurance market with N insurers ($i = 1...N$) and a continuum of policyholders of unit mass. Each policyholder is endowed with initial wealth $\bar{W}$ and faces the possibility of incurring a loss d. A policyholder's wealth is equal to W_N in the event of no-accident, and it reduces to W_A if the loss occurs.

We denote the true state of the world (e.g. the riskiness of the environment in which a policyholder's lives) by θ and we restrict it to be either high (D) or low (S): $\theta \in \Theta = \{S, D\}$. The state of the world θ influences the loss probability p_θ faced by each policyholder. Both θ and p_θ are not observable by the policyholder. Conversely, each insurer i receives

[3] The existence of pooling equilibria is consistent with the findings of the literature on more informed principals in insurance markets, see e.g. Villeneuve (2005).

independently and privately an imperfect signal $\hat{\theta} \in \hat{\Theta} = \left\{\hat{S}, \hat{D}\right\}$ about θ, such that $1/2 < \Pr(\theta = \hat{\theta}_i) = \alpha < 1$. While signal $\hat{\theta}_i$ is private information of insurer i, the probability α that the signal is correct is common knowledge for both insurers and policyholders. In our setup, the signal $\hat{\theta}_i$ identifies the insurer's type and it is not observable by the other insurers. The set of signals received by all insurers is denoted with the state $\sigma = (n_{\hat{\theta}})_{\hat{\theta} \in \hat{\Theta}}$, where $n_{\hat{\theta}}$ is the number of insurers of type $\hat{\theta}$. We let Σ be the set of all possible states.

Insurers compete on offers, which define an insurance premium and a reimbursement in case the loss occurs. We denote the space of all possible offers, including no insurance, with Ω. We denote with $C^i \in \Omega$ insurer i's offer, and by $\mathbf{C} = \{C^i\}_{i=1...N}$ the vector of offers by all insurers. An offer C defines a menu of contracts $c = (W_A, W_N)$, each one of which specifies the policyholder's wealth in the two possible states of the world: loss and no loss. The policyholder maps the vector of offers into the set of probability distributions over types: $\tilde{p} : \mathbf{C} \rightarrow \hat{\Theta}$. Offers are exclusive and cannot be withdrawn. If the policyholder is indifferent between different offers, she randomly selects one of them.

The timing of the model is as follows.

1. At the beginning of the game, nature moves and chooses the realization of θ, and subsequently draws independently and from a common distribution the signals $\hat{\theta}_i$, conditional on θ;

2. insurer i, for all $i = 1...N$, privately observes $\hat{\theta}_i$ and, conditional on $\hat{\theta}_i$, updates her prior on θ;

3. insurers simultaneously offer a contract;

4. the policyholder observes all offers, updates her beliefs and selects exclusively the preferred offer or the reservation option (no-insurance contract);

5. the accepted contract is implemented and payoffs are received.

Players' payoff functions depend on the specific contract c that is implemented and on the expected loss probability. In particular, policyholders are assumed to be risk averse and their preferences are represented by a Von Neumann-Morgenstern utility function $U(W)$. Thus, the expected utility of a policyholder in a generic contract $c = (W_A, W_N)$ given a belief $\tilde{p}$ about loss probability is given by

$$EU_{\tilde{p}}(c) = \tilde{p}U(W_A) + (1 - \tilde{p})U(W_N).$$

Note that for most of our analysis, it will not be necessary to specify the form of the utility function, as our results will hold true for any continuous and concave utility function. However, it will sometimes be convenient to rely on a specific functional form for policyholders' utility function. In these cases, we will focus on the following CARA specification

$$U(W;\beta) := \beta_0 - \beta_0 e^{-\beta W} \tag{1}$$

that, besides being analytically tractable, lends itself to parameterizations that will be useful later on in analyzing market concentration.

Insurers are assumed to be risk neutral and profit maximizers. Their profit function in a generic contract $c = (W_A, W_N)$ with an expected loss probability equal to p is given by

$$E\pi_p(c) = p(\bar{W} - d - W_A) + (1-p)(\bar{W} - W_N).$$

The observation of the insurers' offers provides some market information to the policyholder, in that it signals insurers' types. This market information is a belief on the set of signals received by all insurers, namely it is a belief on the state σ.

The loss probability p_σ when the state is σ can be defined as

$$p_\sigma = \sum_{\theta\in\Theta} p_\theta \Pr(\theta|\sigma),$$

where $\Pr(\theta|\sigma)$ denotes the probability that the state of the world is θ (and hence the loss probability is p_θ) given the set of signals received by all insurers defined by σ.

We focus on Perfect Bayesian Equilibria (PBE), which in our setup are defined as follows.

Definition 1 *A Perfect Bayesian Equilibrium is defined by: (a) a set of offers* $C^e = (C^e_{\hat{S}}, C^e_{\hat{D}})$*, where* $C^e_{\hat{\theta}}$ *is the menu of contracts offered in equilibrium by insurers with type* $\hat{\theta}$*, and (b) a belief mapping* $\tilde{p}$ *such that:*

1. *actions are sequentially rational: for any insurer i, for any type* $\hat{\theta}$*, the offer* $C^{ie}_{\hat{\theta}}$ *maximizes the profit of insurer i having type* $\hat{\theta}$ *given (i)* $\mathbf{C}^{-ie}$ *(the actions of all other insurers), (ii)* $\Pr(\Theta|\hat{\theta})$ *(the probability distribution over types) and (iii)* $\tilde{p}(\mathbf{C}^{-ie}, \cdot)$ *(its impact on beliefs given the others' actions);*

2. *beliefs are consistent with Bayes rule when relevant: if* $C^e_{\hat{S}} \neq C^e_{\hat{D}}$ *then* $\tilde{p}(\mathbf{C}^e) = \sum_{\theta\in\Theta} p_\theta \Pr(\theta|\sigma)$*, else* $\tilde{p}(\mathbf{C}^e) = \sum_{\theta\in\Theta} p_\theta \Pr(\theta)$*.*

Contract c is acceptable by a policyholder if and only if it satisfies her participation constraint that, given the state σ and the reservation (no-insurance) contract C_0, can be written as

$$EU_{p_\sigma}(c) \geq EU_{p_\sigma}(C_0).$$

Hence, a contract is acceptable by the policyholder if it is preferred to the no-insurance option C_0 according to the preferences of a policyholder who has updated her beliefs over risk given the state σ.

The concept of Perfect Bayesian Equilibrium requires the definition of a belief system formed through Bayesian updating in equilibrium, and arbitrarily defined off the equilibrium path. Following the literature on informed principals (Villeneuve, 2005; Seog, 2009), we characterize the beliefs off the equilibrium path by distinguishing between 'optimistic' and 'pessimistic' policyholders. The former are customers who underestimate their risk off the equilibrium path. This optimistic attitude can be given several interpretations, based on the vast literature on overconfidence,[4] or on the empirical literature providing evidence of cross-subsidization in insurance markets.[5] In particular, an optimistic belief may be determined by the observation that a policyholder who is offered a deviation contract may interpret it as an insurer's attempt to capture a low risk and profitable customer. Conversely, pessimistic policyholders are customers who estimate realistically or even pessimistically their risk off the equilibrium path, so that their beliefs are correct or overestimating risk. In our framework, an optimistic belief system can be defined formally as follows (beliefs are taken to be pessimistic in all cases in which they are non-optimistic).

Definition 2 *Policyholders are optimistic off the equilibrium path whenever their belief system $\tilde{p}(C)$ is such that they assign a lower loss probability than the equilibrium one, $\tilde{p}(C) < \tilde{p}(\mathbf{C}^e)$, for any deviation $C \in \Omega$.*

Note that this definition is fulfilled if policyholders think that the deviating insurer received a safe signal $\hat{S}$. As in Villeneuve (2005), policyholders' beliefs off the equilibrium path are common knowledge. A crucial notion for the characterization of equilibria is that of distance between types (see again Villeneuve, 2005), which we define as follows.

[4] See e.g. Kahneman and Tversky (1979), Weinstein (1980), Kreuter and Strecher (1995), and Robb *et al.* (2004) for underestimation of health risk, as well as Groeger and Grande (1996), Svenson (1981), or Walton and McKeown (2001) for overconfidence on driving ability.

[5] See e.g. Weisberg and Derrig (1991), Cummins and Tennyson (1992), and Nielson and Kleffner (2003).

Definition 3 *Types are distant if all contracts that are actuarially fair or profitable given the market information summarized by the state σ are not acceptable by an optimistic policyholder, that is, they do not satisfy her participation constraint.*

According to this definition, the notion of distance between types implies that an optimistic policyholder would be willing to accept solely contracts that an insurer would offer only to individuals characterized by lower risk than the one she faces. In this case, undercutting strategies fail because insurers cannot meet policyholders' overly optimistic expectations: all offers that are acceptable by an optimistic policyholder would in fact result in a loss for the insurer. As a consequence, profitable equilibria are needed as policyholders would reject any possible alternative offer that insurers could afford.

A simple formal argument helps further illustrating the notion of distance. Consider the indifference curve of a policyholder whose participation constraint is binding in some states. If the first derivative of such curve computed in the no-insurance contract C_0 is greater (in absolute value) than the slope of the isoprofit line of a state in which one further insurer received signal $\hat{D}$, then types are distant (by the convexity of the indifference curves). It follows that types are distant when, for example, risk aversion is sufficiently low.

3 Equilibrium characterization

The characterization of equilibria implicitly requires to solve two asymmetric information problems. The first is the standard more informed principle problem stemming from the fact that insurers have private information about the riskiness of the environment in which they operate. The second follows from the observation that policyholders gather market information on the contracts offered by all insurers operating in the industry. The latter problem can be solved by using the delegation principle proposed by Martimort and Stole (2002) for common agency games with adverse selection, under the assumption that contracts of contracts are not available. The former problem instead is typically addressed by means of the revelation principle, the applicability of which to the present setup cannot however be taken for granted. We addressed both issues in a companion paper (see Abrardi et al., 2015), showing both that – in the absence of contracts of contracts – the delegation principle can be extended to multi-principal games with better informed principals, and that principals' types are truthfully revealed in direct mechanisms.[6]

[6] This result bears important implications, as it provides a theoretical foundation to the widespread practice of restricting attention to direct revelation equilibria in multi-principal signaling games.

A full characterization of the set of equilibria emerging in our setup proves extremely difficult, as it depends on a number of factors, such as the distance between types (as in Villeneuve, 2005), or the set of beliefs off the equilibrium path. In order to ease the exposition, we investigate pooling and separating equilibria independently, and we focus exclusively on equilibria whose outcomes convey interesting economic messages in terms of profitability or market concentration.

Pooling equilibria. In pooling equilibria, no revelation of information occurs.[7] Insurers, regardless of the signal received, offer the contract c^e, for all $i = 1...N$. As no information is revealed in equilibrium, insurers simply update their beliefs on the state based on their own signal $\hat{\theta}$. The loss probability estimated by insurers when signal $\hat{\theta}$ is received can thus be written as $p_{\hat{\theta}} = \sum_{\theta=S,D} p_\theta \Pr(\theta|\hat{\theta})$.

The expected loss probability estimated by policyholders in equilibrium corresponds instead to the ex-ante loss probability $p = \sum_{\theta=S,D} p_\theta \Pr(\theta)$, so that the policyholder's expected utility in a generic contract $c = (W_A, W_N)$ can be written as $EU_p(c) = pU(W_A) + (1-p)U(W_N)$.

The equilibrium contract c^e must have the following characteristics. First, it must be profitable or fair for all insurers' types; i.e., insurers' participation constraint $E\pi_{p_{\hat{\theta}}}(c^e) \geq 0$ must be satisfied for all $\hat{\theta}$. Second, it must be acceptable by a policyholder whose expectations are based solely on the prior; i.e., the policyholder's participation constraint $EU_p(c^e) \geq EU_p(C_0)$ must be satisfied. Third, the equilibrium must be more profitable than any deviation acceptable by a policyholder given her beliefs; i.e., no deviation is possible. In particular, denoting with c^d the most profitable deviation acceptable by an optimistic policyholder, expected profits in equilibrium must be greater than the profits in c^d; i.e. $\frac{1}{N} E\pi_{p_{\hat{\theta}}}(c^e) \geq E\pi_{p_{\hat{\theta}}}(c^d)$, for all $\hat{\theta}$. The contract $c^d = (W_A^d, W_N^d)$ can be defined as follows.[8]

$$
\begin{aligned}
c^d &= \arg\max_{c^d} E\pi^e_{p_{\hat{S}}}(c) \\
&\quad s.t. EU_{p_{\hat{S}}}(c^d) \geq EU_{p_{\hat{S}}}(C_0)
\end{aligned}
\tag{2}
$$

By construction, the solution of problem (2) is a full insurance contract that satisfies the constraint $EU_{p_{\hat{S}}}(c^d) \geq EU_{p_{\hat{S}}}(C_0)$ with equality. Namely, contract c^d is such that $W_A^d = W_N^d$ and $EU_{p_{\hat{S}}}(c^d) = EU_{p_{\hat{S}}}(C_0)$.

[7] Recall that information is revealed through the differences in the contracts offered by different types of insurers.

[8] We define c^d for the optimistic policyholder only. This restriction is justified since, as we show in the Appendix, a pooling equilibrium is not consistent with non-optimistic beliefs.

Proposition 1 *A pooling equilibrium exists if and only if types are distant and* $\frac{1}{N}E\pi^e_{p_{\hat{S}}} \geq E\pi_{p_{\hat{S}}}(c^d)$.

Proof. See Appendix. ■

Note that Proposition 1 establishes that in our framework pooling equilibria do exist for distant types only. This is a well known result in the literature on better informed principals (see e.g., Villeneuve, 2009), which depends crucially on policyholders' beliefs off the equilibrium path. Indeed, we show in the proof of the proposition that pooling equilibria do not exist if agents hold pessimistic beliefs (see the proof in the Appendix). The result is summarized in the following Remark.

Remark 1 *No pooling equilibrium with pessimistic beliefs exists.*

Pooling equilibria must necessarily be associated to an optimistic belief system. The intuition behind this finding relies on the idea that when a policyholder is overly optimistic about her own risk, she expects larger rebates in the contract than insurers are willing to make – given the distance between types. Thus, marginal undercuts from pooling equilibria are rejected by the policyholder, who demands optimistically a higher discount on the offer. The combination of optimistic beliefs and distance between types hinders the effectiveness of the competitive mechanism, thus preventing the emergence of an actuarially fair outcome.

In particular, the assumption of distant types combined with an optimistic belief system is a sufficient condition to prevent deviations by $\hat{D}$ insurers. In fact, any deviation that is acceptable by an optimistic policyholder entails a loss for a $\hat{D}$ insurer by the notion of distant types.[9] This does not hold true for $\hat{S}$ insurers. The deviation contract c^d is at the same time acceptable by the optimistic policyholder and profitable for the $\hat{S}$ insurer. It follows that the distance between types is no longer a sufficient condition to prevent deviations by $\hat{S}$ insurers, and condition $\frac{1}{N}E\pi^e_{p_{\hat{S}}} \geq E\pi_{p_{\hat{S}}}(c^d)$ becomes necessary. Notice that, if $E\pi_{p_{\hat{D}}}(c^e) \geq 0$, then it must be $E\pi_{p_{\hat{S}}}(c^e) > 0$ due to $p_{\hat{S}} < p_{\hat{D}}$. Given that $\hat{S}$ insurers have lower estimations on risk than $\hat{D}$ insurers, a pooling equilibrium is always strictly profitable for $\hat{S}$ insurers.

In order to understand the implications of the distant type requirement in Proposition 1 on the characteristics of pooling equilibria, we need to better characterize the notion of distance between types. To do so, it is convenient to resort to the CARA utility specification (1), relying on which we can easily state the following Lemma.

Lemma 1 *Types are distant for pooling contracts if* $e^{-\beta d} \geq \frac{(1-p)/p}{(1-p_{\hat{S}})/p_{\hat{S}}}$.

[9] Recall that all contracts acceptable by a policyholder who evaluates optimistically her risk as $\tilde{p} = p_{\hat{S}}$ is not profitable for an insurer when the latter's estimation of risk is equal to $p_{\hat{D}}$, like in the case of $\hat{D}$ insurers.

Proof. See Appendix. ■

Note that, since $p > p_{\hat{S}}$, then $(1-p)/p < (1-p_{\hat{S}})/p_{\hat{S}}$. It follows that the condition in Lemma 1 is verified when βd is sufficiently small or when p is sufficiently higher than $p_{\hat{S}}$, that is $p_{\hat{D}}$ is sufficiently high. hence, in markets where policyholders' risk aversion is sufficiently low or the size of the damage is relatively small, there is more scope for pooling equilibria.

Separating equilibria. To investigate separating equilibria it is important to evaluate the equilibrium profits for insurers receiving signal $\hat{S}$. In order to pursue this objective, it is convenient to focus on the case of pessimistic beliefs, where competition between insurers (and consequently the pressure towards actuarially fair outcomes) is strongest. Insurers' declarations identify the state σ and the vector of offers C^e that is implemented, implying that separating contracts induce the revelation of market information. As the equilibrium outcome depends on insurers' declarations about their signals, we need to specify an appropriate incentive compatibility constraint for the insurers. More precisely, the incentive compatibility constraint for an insurer of type $\hat{\theta}$ is

$$\sum_{\sigma \in \Sigma} E\pi_{p_\sigma}(C^{ie}_{\hat{\theta}}) \Pr(\sigma|\hat{\theta}) \phi_\sigma(C^{ie}_{\hat{\theta}}) \geq \sum_{\sigma \in \Sigma} E\pi_{p_\sigma}(C^{ie}_{\hat{\theta}'}) \Pr(\sigma|\hat{\theta}) \phi_{\sigma'}(C^{ie}_{\hat{\theta}'}), \tag{3}$$

where $\hat{\theta}$ is the true signal, $\hat{\theta}'$ is the declared signal and $\phi_\sigma(C^{ie}_{\hat{\theta}})$ is the probability that in equilibrium, in the state σ, the customer accepts the offer $C^{ie}_{\hat{\theta}}$ of insurer i. The left hand-side of Condition 3 represents the expected profits for an insurer who has received signal $\hat{\theta}$ if she truthfully reveals her signal; the right hand-side represents the expected profits of an insurer who received signal $\hat{\theta}$ but declares instead signal $\hat{\theta}'$.

Let $\sigma(n_{\hat{\theta}} \geq x)$ be the set of states in which the number $n_{\hat{\theta}}$ of insurers with signal $\hat{\theta}$ is greater or equal than x. Recall that the characterization of the equilibrium depends on out of equilibrium beliefs.[10] In order to investigate the profitability of insurance markets, it is useful focusing on pessimistic beliefs, as they deliver the lowest possible profits (see Proposition 3 below), putting as in the most disadvantageous situation to build our case. The following proposition characterizes a separating equilibrium with pessimistic beliefs.

Proposition 2 *If a separating equilibrium with pessimistic beliefs exists, it is unique and consists of the following strategies:*

- *insurers having received signal $\hat{D}$ offer the menu of actuarially fair contracts that is incentive compatible in all states in which there are one or more $\hat{D}$ types;*

[10] It can be shown that separating equilibria with pessimistic beliefs can exist both for distant and non-distant types.

- *insurers having received signal $\hat{S}$ offer a menu of incentive compatible contracts such that: (i) in states in which $n_{\hat{S}} \geq 2$ contracts are actuarially fair, and (ii) in the state in which $n_{\hat{S}} = 1$, the contract maximizes $\hat{S}$'s expected profits subject to it being strictly preferred to $\hat{D}$'s offers.*

Proof. See Appendix. ■

From Proposition 2, it follows that in a separating equilibrium insurers offer a menu of contracts for each possible state of the world. This occurs because in a separating equilibrium an insurer's offer signals her type. Consequently, by observing all offers, the policyholder is able to infer information about the set of insurers' signals. This market information is not available to insurers when they make the offer, since at that moment they only know their own type (namely, their private signal). As a consequence, insurers' offers should be designed in such a way to allow a policyholder to self-select on the basis of her market information. This is achieved through a menu of contracts for each possible state, which constitutes in this framework the equivalent of the policyholder's 'type'.[11]

The characterization of the equilibrium illustrated by Proposition 2 is based on the following intuitive argument. Due to the policyholder's pessimistic updating of the risk expectations off the equilibrium path, a profitable outcome should in general be impossible. Since the policyholder is always willing to accept marginal deviations from a contract, Bertrand competition between insurers makes an undercutting strategy always profitable. For this reason, the only possible outcome is the actuarially fair one in all states in which more than one insurer with the same risk assessment is present. However, a different outcome emerges in the state where one insurer receives signal $\hat{S}$ and all other $N-1$ insurers receive signal $\hat{D}$. In this case, the lower estimation of risk by the $\hat{S}$ insurer allows her to benefit from a competitive advantage with respect to her $\hat{D}$ competitors. The offer of a full insurance, profitable contract by the $\hat{S}$ insurer cannot be countered by the $\hat{D}$ type without giving up the incentive compatibility of her menu. In particular, from Proposition 2 it follows that an insurer $\hat{D}$ is never able to sell the contract in the state where $n_{\hat{S}} = 1$. In other words, the contract offered by $\hat{D}$ in equilibrium for the state with $n_{\hat{S}} = 1$ remains latent and it is always dominated by the offer of the type $\hat{S}$.

Equilibrium profits. Notice that the equilibrium described in Proposition 2 is always strictly profitable for $\hat{S}$ insurers. This stems immediately from the observation that the expected profits of a $\hat{S}$ insurer are equal to $Pr(\sigma(n_{\hat{S}} = 1)|\hat{S}) E\pi_{p_{\sigma(n_{\hat{S}}=1)}}(c^e_{n_{\hat{S}}=1})$, which are

[11] The focus on the direct mechanism represented by the offer of the menu of contracts is allowed by the delegation principle (or Menu Theorem; see Martimort and Stole, 2002), valid in multi-principal settings.

positive since in state $n_{\hat{S}} = 1$ the only insurer receiving signal $\hat{S}$ has a competitive edge. This follows from the fact that the $\hat{S}$ insurer has an ex ante lower estimate of risk than her competitors, hence having the opportunity to design insurance contracts offering better conditions to policyholders.

The following proposition shows that the level of profits in separating equilibria is characterized by a lower bound given by the profits of the equilibrium illustrated by Proposition 2.

Proposition 3 *If a separating equilibrium exists, its profits both for $\hat{D}$ and $\hat{S}$ insurers are not lower than those in a separating equilibrium with pessimistic beliefs.*

Proof. See Appendix. ■

The following corollary follows then immediately from Proposition 3.

Corollary 1 *The equilibrium expected profits for $\hat{S}$ insurers are always strictly positive.*

Proof. See Appendix. ■

The corollary highlights an important property of equilibria in this framework, namely that the insurance contracts offered by $\hat{S}$ insurers always provide positive expected profits. The source of profits for $\hat{S}$ insurers is differentiated on the basis of the nature (pooling or separating) of the equilibrium. On the one hand, from Proposition 2 it follows that pooling equilibria are strictly profitable for $\hat{S}$ insurers in order to prevent deviations that would be acceptable by an optimistic policyholder. On the other hand, from Proposition 3 it follows that separating equilibria are strictly profitable for $\hat{S}$ insurers in order to induce the truthful revelation of the insurer's type and to guarantee that the adoption of the equilibrium strategy of $\hat{D}$ types is not profitable for them.

Observe that the corollary holds true for all possible set of out-of-equilibrium beliefs, and regardless of the distance between types. Indeed, even when competitive pressure is high – due to, for example, to the presence of types that do not meet the conditions for being distant, or to that of pessimistic beliefs off the equilibrium path, insurers with a low estimation of risk will always be able to achieve positive profits in equilibrium. The reason is twofold. First, as already argued, when all her competitors have a more pessimistic estimation of risk ex-ante (i.e., all her competitors are $\hat{D}$), an $\hat{S}$ insurer has a competitive advantage over them and she can therefore offer more attractive contracts to customers, given the same ex-post estimation of risk (i.e. the state $\hat{S}\hat{D}$). Second, as required by incentive compatibility, an $\hat{S}$ insurer requires an informative rent to truthfully reveal her signal.

4 Market concentration

An important issue in the evaluation of insurance contracts is related to their effects on market concentration, as well as to the relationships between market concentration and profitability. To address this issue, we now investigate how the number of firms operating in the market affects the characteristics of the equilibrium.

According to Proposition 1, a necessary condition for the existence of pooling equilibria is that expected profits in equilibrium are larger than those in the most profitable deviation c^d for a $\hat{S}$ insurer; i.e.

$$\frac{1}{N} E\pi^e_{p_{\hat{S}}} \geq E\pi_{p_{\hat{S}}}(c^d). \tag{4}$$

From (4), it follows that the number of insurers consistent with the existence of a pooling symmetric equilibrium has the upper bound

$$N \leq \frac{E\pi^e_{p_{\hat{S}}}}{E\pi_{p_{\hat{S}}}(c^d)}. \tag{5}$$

Note that the upper bound defined in (5) is specific to the proposed equilibrium. Since pooling equilibria are not unique, it is convenient to characterize the equilibrium with the highest possible number of firms. From (5) it follows immediately that the larger are the expected profits associated to the equilibrium contract, the larger is the upper bound.

In order to better understand the nature of the upper bound on the number of firms in a pooling equilibrium, it is convenient to focus on condition (4). When the number of competitors in the market increases and all firms are offering the same contract, the probability of attracting a customer decreases (i.e., the l.h.s. of (4) is reduced). Hence, expected profits in equilibrium decrease and the deviation becomes increasingly tempting. It follows that, with a large number of insurers, only contract with higher premia can be sustained as equilibria.

Turning now to separating equilibria, it is easy to show that the maximum number of firms consistent with the equilibrium must have a lower bound $\underline{N}$ in order to induce insurers to truthfully disclose their information (this is especially so if insurers receive the safe signal $\hat{S}$). A more precise definition of $\underline{N}$ requires the specification of the incentive compatibility constraint guaranteeing truthful information disclosure. In order to do so, we use the following simplifying notational conventions: σ_1 denotes the state where only one insurer receives the safe signal, $\sigma(n_{\hat{s}} = 1)$, and analogously σ_0 denotes the state where all insurers received signal $\hat{D}$, $\sigma(n_{\hat{s}} = 0)$. The incentive compatibility constraint can then be written as[12]

$$\pi^e_{\sigma_1}(c^{\hat{S}}_{\sigma_1}) \geq \frac{\pi_{\sigma_1}(c^{\hat{D}}_{\sigma_0})}{N} \Rightarrow N \geq \frac{\pi_{\sigma_1}(c^{\hat{D}}_{\sigma_0})}{\pi_{\sigma_1}(c^{\hat{S}}_{\sigma_1})}, \tag{6}$$

where c^{θ}_{σ} is the contract offered in equilibrium by an insurer of type θ ($\theta = \left\{\hat{S}, \hat{D}\right\}$) to be implemented in the state σ. When beliefs are pessimistic, equilibrium contracts are uniquely defined (see Proposition 2). As a consequence, the value of N in (6) is unique. Conversely, when beliefs off the equilibrium path are optimistic and types are distant, there exists a multiplicity of equilibria, each one characterized by a different N consistent with (6).

To better understand the characteristics of the upper bound for pooling equilibria and of the lower bound for separating equilibria, in the following we rely on the CARA utility specification (1), which lends itself to parameterizations that will be useful in the numerical analysis conducted below. Given such specification, the upper bound on the number of firms consistent with the existence of a pooling equilibrium, which we label as $\bar{N}$, is characterized by the following Lemma.

Lemma 2 *There exists an upper bound $\overline{N}$ for the existence of pooling equilibria, such that*

$$\overline{N} = \frac{-p_{\hat{S}}d + \frac{1}{\beta}\ln\left(1 + p\left(e^{\beta d} - 1\right)\right)}{-p_{\hat{S}}d + \frac{1}{\beta}\ln\left(1 + p_{\hat{S}}\left(e^{\beta d} - 1\right)\right)}. \tag{7}$$

Proof. See Appendix. ■

Note that the r.h.s. of (7) is that of (5) under the CARA utility specification (1) for the pooling equilibrium entailing the highest expected profits (i.e. the full insurance contract in which policyholders participation constraint is binding). By inspection of (7), it is immediate to see that $\overline{N}$ depends on a number of factors: namely, the size of the damage, the level of risk, and the degree of risk aversion. Observe, in particular, that the higher is the value of p with respect to $p_{\hat{S}}$, the higher is $\overline{N}$

In the same way, given the CARA specification in (1), the value of $\underline{N}$ in the set of separating equilibria, is characterized by the following Lemma.

Lemma 3 *There exists a lower bound $\underline{N}$ for the existence of separating equilibria, such that*

$$\underline{N} = \frac{(p_{\sigma_0} - p_{\sigma_1})d}{-p_{\sigma_1}d + \frac{1}{\beta}\ln\left(1 + p_{\sigma_1}\left(e^{\beta d} - 1\right)\right)}.$$

[12] Profits in all states different from those in which $n_{\hat{S}} = 0$ and $n_{\hat{S}} = 1$ are equal to zero due to competition between insurers. Thus, Condition (3) can be simplified into (6).

Proof. See Appendix. ■

A numerical analysis helps investigating the link between market concentration and profits. In particular, expected profits have been considered both at the aggregate level (i.e., the expected profits generated by the contract accepted by all customers), and at the individual firm level (i.e., the ex ante profits that would be generated by the acceptance of a specific contract times the probability of such a contract being purchased by customers). Figures 1 and 2 below represent, for any level of N, the highest possible profits achievable in both pooling and separating equilibria.[13]

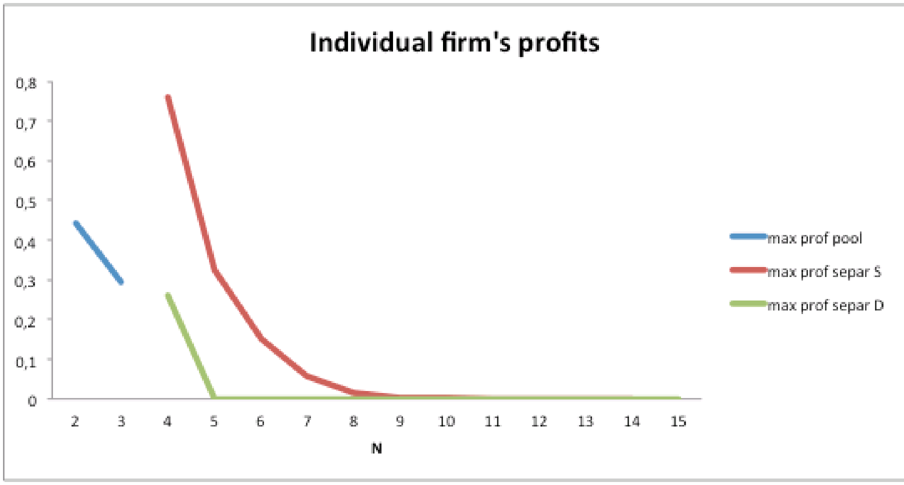

Figure 1: Disjoint sets of pooling and separating equilibria: firm expected profits

Note that Figures 1 and 2 are constructed under the assumption of a large damage (20% of wealth). If this is the case, the largest possible number of firms consistent with a pooling equilibrium is 3 while the lowest possible number of firms consistent with a separating equilibrium is 4. Hence, if we consider only the equilibria with the highest possible level of expected profits, the two sets of equilibria are totally disjoint. Focusing on the setup of Figure 1, it can be seen that a low number of competitors has a positive effect both on the equilibrium expected profits in the pooling contract and on the expected profits in the optimal deviation from the separating equilibrium. In fact, as the number of firms operating in the industry grows, the expected profits in the pooling contract are reduced.[14] Turning now to Figure 2, it is interesting to note that the expected profits generated by the accepted contracts in

[13] For the precise expression of the highest expected profits in pooling and separating equilibria see (10) and (11) in the Appendix, respectively. Figures 1 and 2 are drawn using the following parameters constellation: $p_{\hat{S}} = 0.001$; $p_{\hat{D}} = 0.005$; $\Pr(S) = 0.9$; $\Pr(D) = 0.1$; $\alpha = 0.8$; $\bar{W} = 10000$; $d = 2000$; $\beta_0 = 1$; $\beta = 0.0001$.

[14] Note that in the numerical simulations, differently than in the theoretical analysis above – where it wasn't necessary, we need to focus also on separating equilibria where both $\hat{D}$ and $\hat{S}$ types can hold optimistic beliefs.

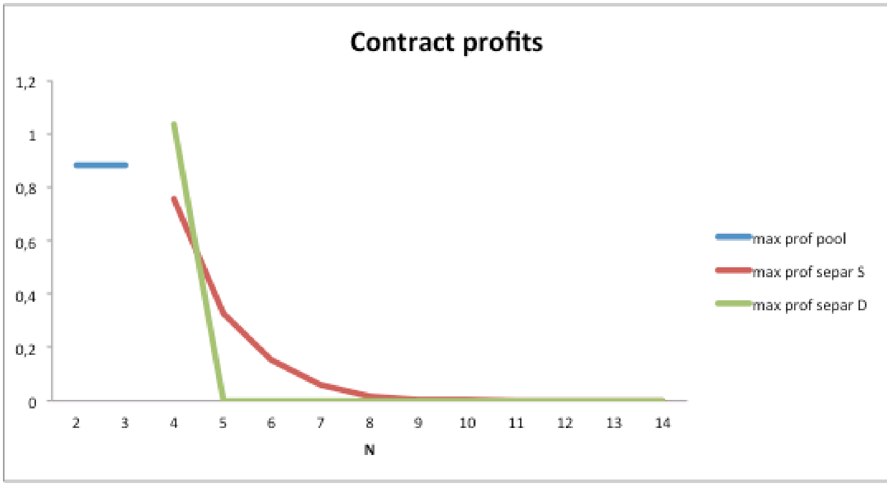

Figure 2: Disjoint sets of pooling and separating equilibria: insurer's profits at the accepted equilibrium contract

a pooling equilibrium remain constant for increasing market concentration, suggesting that an increase in market concentration is not necessarily welfare detrimental for customers; a finding which is consistent with the available empirical literature.

Note finally that pooling and separating equilibria can coexist for a given level of market concentration, as it shown by Figures 3 and 4, which differ from the previous ones just for the assumption of a damage accounting for a much lower fraction of wealth.[15] It is immediate to see that pooling and separating equilibria now coexist even for a fairly dispersed industry ($N \leq 14$) and that almost all qualitative features of equilibrium profits already highlighted for Figures 1 and 2 continue to hold. Furthermore and quite important, for the given set of parameter values we consider, pooling equilibria dominates separating ones in terms of expected profits; a finding that seems to be consistent with the widespread diffusion of pooling contracts in damage insurance markets.

5 Concluding Remarks

Insurance markets are significantly affected by asymmetric information problems, which may be related to both the characteristics of policyholders and the ways in which risk is assessed.

In this case, differently than in that of pessimistic beliefs, the profits accruing to a $\hat{D}$ type in a separating equilibrium can be positive. The numerical simulations in Figures 1 and 2 show that this occurs for $N \leq 5$.

[15] The parameter set for which Figures 3 and 4 are drawn differs from that used in Figures 1 and 2 just for a lower value of the damage, which is now assumed to be equal to 2% of wealth.

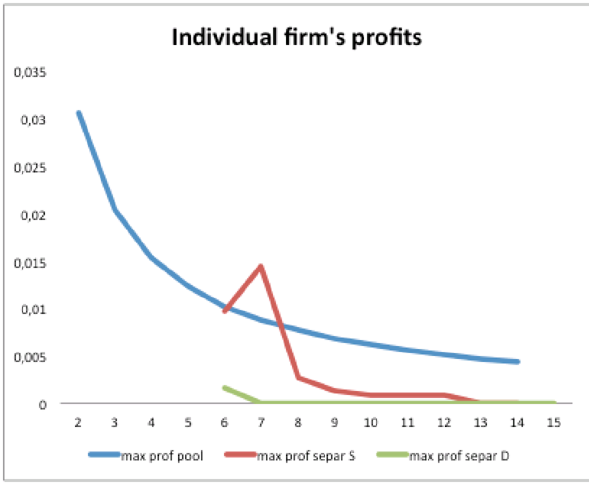

Figure 3: Dominant pooling equilibria: firm expected profits

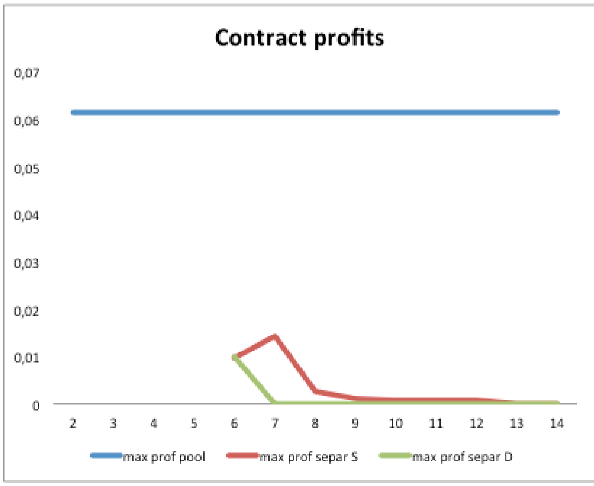

Figure 4: Dominant pooling equilibria: profits at the accepted equilibrium contract

Because of their expertise and access to appropriate data, insurers can reasonably be expected to be better equipped than policyholders to accurately assess the level of risk associated to a specific environment. Still, a precise assessment of risk is not straightforward even for practitioners, and some degree of heterogeneity in insurers' evaluations is unavoidable. This paper focuses specifically on the role of information asymmetries in the assessment of risk. In doing so, it neglects entirely the role of the private information held by policyholders about their own quality, which has however extensively been studied in the literature.

One of the key results of our analysis is that insurers assessing risk to be low need necessarily to make positive profits in equilibrium, even in a competitive insurance industry. Indeed, while a low assessment of risk should lead to actuarially fair insurance in a competitive setting, fair allocations do not provide sufficient incentives to reveal information. Policyholders being over-optimistic on the low riskiness of the environment would in fact only accept 'cheap' contracts, that would in turn determine a loss for the insurers offering them. Therefore, in equilibrium insurers' profits need necessarily to be positive in order to induce truthful revelation of information by insurers, which hinders the effectiveness of the competitive mechanism.

We show that incentive compatible contracts may result both in pooling and separating equilibria, the existence of which depends on the notion of distance between policyholders' types and on their out-of-equilibrium beliefs about insurers' assessments of risk. As already argued, positive profits are observed both in pooling and separating equilibria as they are needed to guarantee the truthful revelation of insurers' private assessment of risk. In both types of equilibria, profits are shown to be larger for insurers' expecting a safer market environment. This result is new to the literature, as far as separating equilibria are concerned.

We also find that market concentration plays an important role in determining which type of equilibria are likely to emerge. In particular, we show that pooling equilibria exist when the market is sufficiently concentrated, while separating equilibria arise only in sufficiently dispersed industry. Furthermore, some numerical exercises, based on a CARA utility specification, highlight the existence both of parameter regions where pooling and separating equilibria coexist, and of parameters constellations for which the two sets of equilibria are disjoint. Moreover, we also find that pooling equilibria may dominate separating equilibria in terms of expected profits.

The current setup of the paper can be extended in at least two directions. First of all, a more systematic numerical and theoretical analysis is needed to assess the driving forces behind the findings illustrated above, and better highlight the roles played by market concentration, policyholders' risk aversion, as well as the precision of insurers' assessment of risk. Second, an important goal of future research is to extend the current setup to encompass private information by policyholders as well, allowing for double-sided information asymmetries.

References

[1] Abrardi, L., Colombo, L. and P. Tedeschi (2015), Signaling Games in Multi-Principal Settings. *Mimeo, Università Cattolica del Sacro Cuore, Milano.*

[2] Bisin, A. and D. Guaitoli (2004), Moral Hazard and Non-Exclusive Contracts. *Rand Journal of Economics,* 2:306-328.

[3] Camerer, C. (1997), Progress in Behavioral Game Theory. *Journal of Economic Perspectives*, 11(4): 167-88.

[4] Cawley, J. and T. Philipson (1999), An Empirical Examination of Information Barriers to Trade in Insurance. *American Economic Review,* 89(4): 827-846.

[5] Cummins, J. D. and S. Tennyson (1992), Controlling Automobile Insurance Costs. *Journal of Economic Perspectives*, 6: 95-115.

[6] Dafny, L., Duggan, M. and Ramanarayanan, S. (2009), Paying a Premium on Your Premium? Consolidation in the U.S. Health Insurance Industry. *NBER Working Paper No. 15434.*

[7] Fang, H. and G. Moscarini, (2005), Morale Hazard. *Journal of Monetary Economics*, 52(4): 749-777.

[8] Finkelstein, A. and K. McGarry (2006), Multiple Dimensions of Private Information: Evidence from the Long-Term Care Insurance Market. *American Economic Review*, 96(4): 938-958.

[9] Fombaron, N. (1997), No-Commitment and Dynamic Contracts in Competitive Insurance Markets with Adverse Selection. *Working Paper Thema.*

[10] Garcia, D., Sangiorgi, F. and B. Urosevic (2007), Overconfidence and Market Efficiency with Heterogeneous Agents. *Journal of Economic Theory*, 30(2), 313-36.

[11] Groeger, J.A and G.E. Grande (1996), Self-preserving Assessments of Skill? *British Journal of Psychology*, 87: 61-79.

[12] Hyman, D. A. and Kovacic, W. E. (2004), Monopoly, Monopsony, and Market Definition: An Antitrust Perspective On Market Concentration Among Health Insurers. *Health Affairs*, 23(6): 25-28.

[13] Hoelzl, E. and A. Rustichini (2005), Overconfident: Do You Put Your Money on It? *Economic Journal*, 115: 305-18.

[14] Kahneman, D. and A. Tversky (1979), Prospect Theory: An Analysis of Decision Under Risk. *Econometrica*, 47(2), 263-291.

[15] Köszegi, B. (2006), Ego Utility, Overconfidence, and Task Choice. *Journal of the European Economic Association*, 4(4): 673-707.

[16] Kreuter, M.W. and V.J. Strecher (1995), Changing Inaccurate Perceptions of Health Risk. *Health Psychology*, 14: 56-63.

[17] Martimort, D. and L. Stole (2002), The Revelation and Delegation Principles in Common Agency Games. *Econometrica*, 70, 1659-1673.

[18] Menkhoff, L., Schmidt, U. and T. Brozynski (2006), The Impact of Experience on Risk Taking, Overconfidence, and Herding of Fund Managers: Complementary Survey Evidence. *European Economic Review*, 50(7), 1753-66.

[19] Nielson, N. L. and A. E. Kleffner (2003), Auto Insurance Reform for Canada's Tort Provinces" Working Paper, Haskayne School of Business, University of Calgary.

[20] Noth, M. and M. Weber (2003), Information Aggregation with Random Ordering: Cascades and Overconfidence. *Economic Journal*, 113(484), 166-89.

[21] Robb K.A., Miles, A. and J. Wardle (2004), Subjective and Cbjective Risk of Colorectal Cancer (UK). *Cancer Causes and Control* 15: 21-25.

[22] Robinson, J. (2004), Consolidation And The Transformation Of Competition. In Health Insurance, Health Affairs, 23(6): 11-24.

[23] Rothschild, M. and J.E. Stiglitz (1976), Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information. *Quarterly Journal of Economics*, 90: 629-649.

[24] Sandroni, A. and F. Squintani (2007), Overconfidence, Insurance and Paternalism. *American Economic Review*, 97(5): 1994-2004.

[25] Seog, S. H. (2009), Insurance Markets with Differential Information. *Journal of Risk and Insurance*, 76, 279-294.

[26] Smith, V. K., Taylor, D. H. Jr. and Sloan F. A. (2001), Longevity Expectations and Death: Can People Predict Their Own Demise? *American Economic Review.* 91(4):1126–1134.

[27] Svenson, O. (1981), Are we all less Risky Drivers and more Skillful than our Fellow Drivers? *Acta Psychologica*, 47: 143-148.

[28] Van den Steen, E. (2004), Rational Overoptimism. *American Economic Review*, 94(4), 1141-1151.

[29] Villeneuve, B. (2000), The Consequences for a Monopolistic Insurer of Evaluating Risk Better than Customers: The Adverse Selection Hypothesis Reversed. *The Geneva papers on Risk and Insurance Theory*, 25: 65-79.

[30] Villeneuve, B. (2005), Competition Between Insurers with Superior Information. *European Economic Review*, 49(2): 321-340.

[31] Walton, D. and P.C. McKeown (2001), Drivers' Biased Perceptions of Speed and Safety Campaign Messages. *Accident Analysis and Prevention*, 33: 629-640.

[32] Wambach, A. (2000), Payback Criterion, Hurdle Rates and The Gain of Waiting. *International Review of Financial Analysis,* 9(3):247-258.

[33] Weinstein, N.D. (1980), Unrealistic Optimism About Future Life Events. *Journal of Personality and Social Psychology*, 39: 806-820.

[34] Weisberg, H. I. and R. A. Derrig (1991), Fraud and Automobile Insurance: A Report on the Baseline Study of Bodily Injury Claims in Massachusetts. *Journal of Insurance Regulation*, 9: 497-541.

[35] Zábojník, J. (2004), A Model of Rational Bias in Self-Assessments. *Economic Theory*, 23(2), 259-82.

Appendix

Proof of Proposition 1. Proof of the necessary part. Suppose that beliefs are pessimistic. Recall that in equilibrium $\hat{D}$ needs to make non negative expected profits. Hence $\hat{S}$ surely makes strictly positive equilibrium expected profits. Hence $\hat{S}$ can undercut his competitors and capture the policyholder with probability one. Therefore we now assume that beliefs are optimistic. Hence, if types are not distant some insurer has a a profitable deviation, by the definition of distant types. Suppose that types are distant and $\frac{1}{N}E\pi^e_{p_{\hat{S}}} < E\pi_{p_{\hat{S}}}(c^d)$. Since c^d is acceptable for the policyholder by (2), a $\hat{S}$ insurer could profitably deviate by offering c^d.

Proof of the sufficient part. Consider a pooling equilibrium with optimistic beliefs; it must be such that $E\pi_{p_{\hat{D}}}(c^e) \geq 0$. Assume distance between types. An insurer's most profitable deviation acceptable by an optimistic policyholder, is by definition contract c^d defined in (2). For the definition of distant types, it must be $EU_{p_{\hat{D}}}(c^d) < EU_{p_{\hat{D}}}(C_0)$, therefore a deviation is not profitable for a $\hat{D}$ insurer. Now consider a $\hat{S}$ insurer. If condition $\frac{1}{N}E\pi^e_{p_{\hat{S}}} \geq E\pi_{p_{\hat{S}}}(c^d)$ holds, deviation c^d defined in (2) is not profitable. *A fortiori*, any other deviation c, is not profitable. ■

Proof of Lemma 1. Consider the indifference curve of the policyholder such that her participation constraint is binding in some state; if the first derivative of such curve computed in the no-insurance contract C_0 is greater (in absolute value) than the slope of the isoprofit line of the next riskier state, then types are distant (by the convexity of the indifference curves). The indifference curve in C_0 given state $\hat{S}$ is given by

$$U_{\hat{S}}(C_0) = p_{\hat{S}}(\beta_0 - \beta_0 e^{-\beta W_A}) + (1 - p_{\hat{S}})(\beta_0 - \beta_0 e^{-\beta W_N}),$$

and its derivative can be written as

$$\frac{\partial U_{\hat{S}}(C_0)/\partial W_N}{\partial U_{\hat{S}}(C_0)/\partial W_A} = \frac{(1 - p_{\hat{S}})\beta_0 \beta e^{-\beta W_N}}{p_{\hat{S}}\beta_0 \beta e^{-\beta W_A}}.$$

In C_0, $W_N = \bar{W}$ and $W_A = \bar{W} - d$. Hence, such derivative computed in C_0 can be rewritten as

$$\frac{\partial U_{\hat{S}}(C_0)/\partial W_N}{\partial U_{\hat{S}}(C_0)/\partial W_A} = \frac{(1 - p_{\hat{S}})}{p_{\hat{S}}} e^{-\beta(W_N - W_A)} = \frac{(1 - p_{\hat{S}})}{p_{\hat{S}}} e^{-\beta d}.$$

The slope of the state with expected loss p is (in absolute value) given by $\frac{(1-p)}{p}$.The condition for the distance between types for pooling equilibria thus requires

$$\frac{(1 - p_{\hat{S}})}{p_{\hat{S}}} e^{-\beta d} \geq \frac{(1 - p)}{p},$$

that is

$$e^{-\beta d} \geq \frac{(1 - p)/p}{(1 - p_{\hat{S}})/p_{\hat{S}}}, \tag{8}$$

which proves the lemma. ■

Proof of Proposition 2. To ease the exposition, the proof of the proposition is organized in a sequence of Lemmas.

Lemma 4 *All insurers' types offer a fully separating, incentive compatible menu with one contract for all states.*

Proof. Let us first consider $\hat{S}$ insurers. Suppose that $\hat{S}$ in equilibrium offers a menu in which pooling on some states occurs. Namely, $\hat{S}$ offers a menu $C^e_{\hat{S}} = \left\{c^e_{\hat{S},\sigma}\right\}_{\sigma(n_{\hat{S}}\geq 1)}$ such that for some states $\sigma'(n_{\hat{S}} \geq 1)$, $\sigma''(n_{\hat{S}} \geq 1)$, $c^e_{\hat{S},\sigma'} \equiv c^e_{\hat{S},\sigma''}$. Let us assume without loss of generality that the number of $\hat{S}$ insurers in the state σ'' is larger than the number of $\hat{S}$ insurers in the state σ'. Let us identify the sector Δ in the plane with all contracts c preferred to the equilibrium in state but not in (i.e., such that $U_{p_{\sigma'}}(c) > U_{p_{\sigma'}}(c^e_{\hat{S},\sigma'})$ and $U_{p_{\sigma''}}(c) < U_{p_{\sigma''}}(c^e_{\hat{S},\sigma'})$); by the single crossing property, sector Δ is non-empty. A $\hat{S}$ insurers could profitably deviate by offering the menu $C^{dev}_{\hat{S}} = \left\{c^{dev}_{\hat{S},\sigma}\right\}_{\sigma(n_{\hat{S}}\geq 1)}$, where $c^{dev}_{\hat{S},\sigma} \equiv c^e_{\hat{S},\sigma}$ for all states $\sigma \neq \sigma''$, and $c^{dev}_{\hat{S},\sigma''} \in \Delta$ For any state $\sigma \neq \sigma''$, profits are the same as in equilibrium (because the same contracts are offered); however, in state σ'', the policyholder prefers $c^{dev}_{\hat{S},\sigma''}$ by construction. If $c^{dev}_{\hat{S},\sigma''}$ is sufficiently close to $c^e_{\hat{S},\sigma''}$, the positive profits in $c^{dev}_{\hat{S},\sigma''}$ are similar to those in $c^e_{\hat{S},\sigma''}$ but obtained with probability 1. Hence, $C^{dev}_{\hat{S}}$ si a profitable deviation: in equilibrium for $\hat{S}$ pooling cannot occur for any state. The proof is symmetric for type $\hat{D}$. ■

Lemma 5 *In all states with $n_{\hat{S}} \geq 2$, contracts are actuarially fair.*

Proof. Consider all states in which more than one $\hat{S}$ insurer is present, namely $\sigma(n_S \geq 2)$. If more than one insurer received signal $\hat{S}$, competition can develop and, by undercutting, the only possible outcome is the actuarially fair one; in fact, the undercut deviation would be accepted by the policyholder, given her pessimistic beliefs off the equilibrium path. Hence in states $\sigma(n_{\hat{S}} \geq 2)$ the actuarially fair outcome is the only possible outcome:

$$E\pi_{p_{\hat{S}}}(c^e_{\hat{S},\sigma(n_S\geq 2)}) = 0, E\pi_{p_{\hat{D}}}(c^e_{\hat{D},\sigma(n_S\geq 2)}) = 0.$$

■

Lemma 6 *Insurers having received signal $\hat{D}$ offer the menu of actuarially fair contracts that is incentive compatible in all states in which there are one or more $\hat{D}$ types.*

Proof. Suppose that $c^e_{\hat{D},\sigma(n_{\hat{D}}=N)}$ is profitable in the riskiest state $\sigma(n_{\hat{D}} = N)$. Then, when the state is the riskiest, all competitors are offering $C^e_{\hat{D}}$ and one insurer could always profitably deviate with an undercutting strategy, which is accepted by the policyholder due to her pessimistic out of equilibrium beliefs. Hence, in the riskiest state the only possible equilibrium is the contract that maximizes the policyholder's utility under the insurer's participation constraint, namely the actuarially fair full insurance contract. Consider now any other state (except the riskiest) $\sigma(1 \leq n_{\hat{D}} < N)$. In this state $\sigma(1 \leq n_{\hat{D}} < N)$, there is at least one insurer $\hat{D}$ and one insurer $\hat{S}$. Assume that in $\sigma(1 \leq n_{\hat{D}} < N)$ the $\hat{D}$ insurer offers a profitable contract c; since c has to be incentive compatible with the offer in the riskiest state $\sigma(n_{\hat{D}} = N)$, and the latter is the actuarially fair full insurance contract, c must be an underinsurance contract. Given the underinsurance contract c, in $\sigma(1 \leq n_{\hat{D}} < N)$ the equilibrium strategy for an $\hat{S}$ insurer is to offer a contract on the next higher indifference curve of c but with more coverage (they are not constrained by incentive compatibility as contracts offered by $\hat{D}$ are), so as to capture the policyholder if the state is $\sigma(1 \leq n_D < N)$. Given this strategy of a $\hat{S}$ insurer, the $\hat{D}$ insurer can win the customer with an undercutting strategy. The only contract in which undercutting is no longer possible is the actuarially fair contract for the $\sigma(1 \leq n_D < N)$ state. Hence, in equilibrium $\hat{D}$ offers the Rothschild-Stiglitz actuarially fair menu that is incentive compatible for all states $\sigma(n_D \geq 1)$. This implies that the contract $c^e_{\hat{D},\sigma(n_{\hat{D}}=N)}$ accepted in case the state is the riskiest one is a full insurance contract and contract $c^e_{\hat{D},\sigma(n_{\hat{S}}=1)}$ is an underinsurance contract. ■

Lemma 7 *In the state with $n_{\hat{S}} = 1$, the contract offered in equilibrium by $\hat{S}$ is strictly preferred to $\hat{D}$'s equilibrium contract: i.e.,* $U_{\sigma(n_{\hat{S}}=1)}(c^e_{\hat{D},\sigma(n_{\hat{S}}=1)}) \leq U_{\sigma(n_{\hat{S}}=1)}(c^e_{\hat{S},\sigma(n_{\hat{S}}=1)})$.

Proof. Suppose instead that $U_{p_{\sigma_1}}(c^e_{\hat{D},\sigma_1}) > U_{p_{\sigma_1}}(c^e_{\hat{S},\sigma_1})$. Then, in equilibrium, in the state σ_1 the policyholder chooses the $\hat{D}$'s offer, namely $c^e_{\hat{D},\sigma_1}$ (which is the best contract of $C^e_{\hat{D}}$ in the state σ_1, given the incentive compatibility of the menu from Lemma 4). Since $c^e_{\hat{D},\sigma_1}$ is an underinsurance contract (to ensure the incentive compatibility with $c^e_{\hat{D},\sigma_0}$), we can identify the indifference curve for the state σ_1 passing through $c^e_{\hat{D},\sigma_1}$. Then, we identify the set of contracts for which the following characteristics hold simultaneously: (i) the contract lies above this indifference curve, and (ii) it lies under the isoprofit line in state σ_1. This set of contracts is non-void by the convexity of the indifference curves. Denote with $\tilde{c}$ a contract belonging to this set.

Consider the menu $\tilde{C} = \left\{\tilde{c}, c^e_{\hat{S},\sigma}\right\}_{\sigma(n_{\hat{S}}\geq 2)}$. Menu $\tilde{C}$ differs from $C^e_{\hat{S}}$ only for the contract in the state σ_1, since $\tilde{C}$ is still incentive compatible. By Lemma 4, $C^e_{\hat{S}}$ must be incentive compatible, so that $U_{p_{\sigma_1}}(c^e_{\hat{S},\sigma_1}) \geq U_{p_{\sigma_1}}(c^e_{\hat{S},\sigma(n_{\hat{S}}>1)})$. Recall that by hypothesis, one has that

$U_{p_{\sigma_1}}(\tilde{c}) > U_{p_{\sigma_1}}(c^e_{\hat{D},\sigma_1}) > U_{p_{\sigma_1}}(c^e_{\hat{S},\sigma_1})$. Then it follows that $U_{p_{\sigma_1}}(\tilde{c}) > U_{p_{\sigma_1}}(c^e_{\hat{S},\sigma(n_{\hat{S}}>1)})$, and we need only to study the gain in the state σ_1 for a $\hat{S}$ insurer who deviates and offers $\tilde{C}$ (because in all other states the outcome -and thus the profits- are the same). If an insurer $\hat{S}$ deviates and offers $\tilde{C}$, in the state σ_1, the policyholder with non-optimistic beliefs off the equilibrium path forms a beliefs $\tilde{p} = p_{\sigma(n_{\hat{S}}=1)}$ about the loss probability and chooses contract $\tilde{c}$ (because $U_{p_{\sigma_1}}(\tilde{c}) > U_{p_{\sigma_1}}(c^e_{\hat{D},\sigma_1})$), so that the deviation $\tilde{C}$ is profitable (because it lies below the isoprofit line and thus grants positive profits, while in equilibrium the policyholder would have chosen the $\hat{D}$'s insurer offer so that the profits for $\hat{S}$ in state σ_1 in equilibrium are zero). It follows that, in equilibrium, contract $c^e_{\hat{S},\sigma_1}$ must be preferred to $c^e_{\hat{D},\sigma_1}$. ■

Let us denote with $c^{\max}_{\sigma_1}$ the contract that maximizes $\hat{S}$'s profits and is incentive compatible with type $\hat{D}$'s offers in the state $\sigma(n_{\hat{S}} = 1)$ (contract $c^e_{\hat{D},\sigma_1}$). More formally:

$$\begin{aligned} c^{\max}_{\sigma_1} &= \arg\max_{c} E\pi^e_{p_{\sigma_1}}(c) \\ & \quad s.t. EU_{p_{\sigma_1}}(c) \geq EU_{p_{\sigma_1}}(c^e_{\hat{D},\sigma_1}). \end{aligned} \tag{9}$$

Contract $c^{\max}_{\sigma_1}$ is a full insurance contract. As a matter of fact, contract $c^{\max}_{\sigma_1}$ by definition maximizes the insurer $\hat{S}$ profits in the state σ_1 under the incentive compatibility constraint of a policyholder who believes that the state is σ_1; hence, it must entail full insurance. When the state is σ_1, contract $c^{\max}_{\sigma_1}$ (if accepted) is more profitable than $c^e_{\hat{S},\sigma_1}$ (if accepted). This is due to the fact that both contracts $c^{\max}_{\sigma_1}$ and $c^e_{\hat{S},\sigma_1}$ must be preferred by a policyholder believing that the state σ_1 to contract $c^e_{\hat{D},\sigma_1}$ (contract $c^{\max}_{\sigma_1}$ by definition and contract $c^e_{\hat{S},n_{\hat{S}}=1}$ by Lemma 7). Given this common constraint, contract $c^{\max}_{\sigma_1}$ is the profit maximizer for $\hat{S}$, so that profits in $c^{\max}_{\sigma_1}$ are greater than profits in equilibrium in the state σ_1 for $\hat{S}$.

Lemma 8 *In the state with $n_{\hat{S}} = 1$, the equilibrium contract of the $\hat{S}$ type maximizes $\hat{S}$'s expected profits and is incentive compatible with the $\hat{D}$'s offers in the state $\sigma(n_{\hat{S}} = 1)$; i.e., $c^e_{\hat{S},\sigma(n_{\hat{S}}=1)} = c^{max}_{\sigma(n_{\hat{S}}=1)}$.*

Proof. Let us suppose that $c^e_{\hat{S},\sigma_1} \neq c^{\max}_{\sigma_1}$. To prove that this cannot be true, we need to show that $c^{\max}_{\sigma_1}$ would always be a profitable deviation for $\hat{S}$. Now, $c^{\max}_{\sigma_1}$ is a profitable deviation if 1) it is accepted by the policyholder and 2) increases the profits in the state σ_1 without decreasing those in all other states when $n_{\hat{S}} > 1$. With respect to point 1), in turn, $c^{\max}_{\sigma_1}$ is accepted by the policyholder only if the policyholder prefers $c^{\max}_{\sigma_1}$ over the offer of all other insurers. Given that we are considering the behavior of $\hat{S}$ in the state σ_1 all other $N-1$ insurers have type $\hat{D}$. It follows that $c^{\max}_{\sigma_1}$ is accepted by the policyholder only if the policyholder prefers $c^{\max}_{\sigma_1}$ over $c^e_{\hat{D},\sigma_1}$. This is true by the definition of $c^{\max}_{\sigma_1}$ in (9), as $c^{\max}_{\sigma_1}$ is such that $U_{\sigma_1}(c^{\max}_{\sigma_1}) > U_{\sigma_1}(c^e_{\hat{D},\sigma_1})$. For what concerns point 2), $c^{\max}_{\sigma_1}$ is the profit maximizing contract by definition (9), so that it must be more profitable than $c^e_{\hat{S},\sigma_1}$. In all other states (i.e., one or more insurers offered $C^e_{\hat{S}}$), the policyholder prefers $c^e_{\hat{S},\sigma(n_{\hat{S}}>1)}$ to $c^{\max}_{\sigma_1}$ (namely $U_{\sigma(n_{\hat{S}}>1)}(c^e_{\hat{S},\sigma(n_{\hat{S}}>1)}) > U_{\sigma(n_{\hat{S}}>1)}(c^{\max}_{\sigma_1})$), so that $c^{\max}_{\sigma_1}$ is not chosen. The reason is the following (to lighten the notation, let us use the binary preference relation sign, so that, for example, $x \succeq_\sigma y$ means that a policyholder who believes that the state is σ prefers contract x to contract y).Consider the generic state $\sigma(n_{\hat{S}} > 1)$. Let us define contract $c^{af\max}_{\sigma(n_{\hat{S}}>1)}$ as the actuarially fair contract in the state $\sigma(n_{\hat{S}} > 1)$ such that $c^{\max}_{\sigma_1} \succeq_{\sigma_1} c^{af\max}_{\sigma(n_{\hat{S}}>1)}$ and contract $c^e_{\hat{S},\sigma(n_{\hat{S}}>1)}$ as the equilibrium contract offered by $\hat{S}$ for the state $\sigma(n_{\hat{S}} > 1)$. Since by Lemma 4 $c^e_{\hat{S},\sigma(n_{\hat{S}}>1)}$ must be incentive compatible in equilibrium, we have that $c^e_{\hat{S},\sigma_1} \succeq_{\sigma_1} c^e_{\hat{S},\sigma(n_{\hat{S}}>1)}$. Since by definition $c^{\max}_{\sigma_1} \succeq_{\sigma_1} c^{af\max}_{\sigma(n_{\hat{S}}>1)}$, then it must be (because $p_{\sigma_1} > p_{\sigma(n_{\hat{S}}>1)}$) that $c^{af\max}_{\sigma(n_{\hat{S}}>1)} \succeq_{\sigma(n_{\hat{S}}>1)} c^{\max}_{\sigma_1}$. We have that $c^e_{\hat{S},\sigma_1} \succeq_{\sigma_1} c^{\max}_{\sigma_1}$ (because they both are incentive compatible with $c^e_{\hat{D},\sigma_1}$ but $c^{\max}_{\sigma_1}$ is profit maximizer in the state σ_1). Then, it must be $c^e_{\hat{S},\sigma(n_{\hat{S}}>1)} \succeq_{\sigma(n_{\hat{S}}>1)} c^{af\max}_{\sigma(n_{\hat{S}}>1)}$ (they are both actuarially fair – $c^e_{\hat{S},\sigma(n_{\hat{S}}>1)}$ by Lemma 5 and $c^{af\max}_{\sigma(n_{\hat{S}}>1)}$ by definition – for $\sigma(n_{\hat{S}} > 1)$ but $c^{af\max}_{\sigma(n_{\hat{S}}>1)}$ has more underinsurance because of $c^e_{\sigma_1} \succeq_{\sigma_1} c^e_{\hat{S},\sigma(n_{\hat{S}}>1)}$). From all the previous binary relationship, we have: $c^e_{\hat{S},\sigma(n_{\hat{S}}>1)} \succeq_{\sigma(n_{\hat{S}}>1)} c^{af\max}_{\sigma(n_{\hat{S}}>1)} \succeq_{\sigma(n_{\hat{S}}>1)} c^{\max}_{\sigma_1}$. Hence, when $c^{\max}_{\sigma_1}$ is offered, but the state is such that $n_{\hat{S}} > 1$, the equilibrium contract $c^e_{\hat{S},\sigma(n_{\hat{S}}>1)}$ is chosen. Thus, the deviation $c^{\max}_{\sigma_1}$ is more profitable than the equilibrium $C^e_{\hat{S}}$. It follows that, if $c^e_{\hat{S},\sigma_1} \neq c^{\max}_{\sigma_1}$, a profitable deviation can always be found and in equilibrium we have that $c^e_{\hat{S},\sigma_1} = c^{\max}_{\sigma_1}$. ■

By Lemmas 4, 5, 6, 8 the equilibrium outcome is unique, which completes the proof of the proposition. ■

Proof of Proposition 3. Profits of $\hat{D}$ insurers in the equilibrium of Proposition 2 are equal to zero, which is a lower bound for the participation of the $\hat{D}$ type. Consider instead the $\hat{S}$ type. We want to show that the equilibrium of Proposition 2 sets a lower bound to the $\hat{S}$ profits among separating equilibria. From Proposition 2, for any out of equilibrium belief and any distance between types, type $\hat{S}$ makes positive profits only in the state with $n_{\hat{S}} = 1$, while in states in which $n_{\hat{S}} > 1$ the profits of $\hat{S}$ are zero. So, it is sufficient to focus on the state with $n_{\hat{S}} = 1$. From Proposition 2, the equilibrium contract $c^e_{\hat{D},\sigma(n_{\hat{S}}=0)}$ offered by $\hat{D}$ in state $n_{\hat{D}} = N$ is the full insurance, actuarially fair contract for the state $\sigma(n_{\hat{D}} = N)$. By construction, contract $c^e_{\hat{D},\sigma(n_{\hat{S}}=0)}$ maximizes the utility of a policyholder with beliefs on risk equal to $p_{\sigma(n_{\hat{D}}=N)}$. Moreover, from Proposition 2, the contract $(c^e_{\hat{D},\sigma(n_{\hat{S}}=1)})$ offered by $\hat{D}$ for the state $\sigma(n_{\hat{S}} = 1)$.must be incentive compatible with $c^e_{\hat{D},\sigma(n_{\hat{S}}=0)}$. It follows that contract $c^e_{\hat{D},\sigma(n_{\hat{S}}=1)}$ is an underinsurance contract. By the convexity of the indifference curves, the utility curve passing through $c^e_{\hat{D},\sigma(n_{\hat{S}}=1)}$ identifies a convex section Λ of the plane with the set of contracts preferred by a policyholder who estimates risk as $p_{\sigma(n_{\hat{S}}=1)}$. Any other contract c different from $c^e_{\hat{D},\sigma(n_{\hat{S}}=0)}$.identifies a new set Λ', with $\Lambda \subset \Lambda'$. As $c^e_{\hat{S},\sigma(n_{\hat{S}}=1)}$, from Proposition 2, must be incentive compatible with $c^e_{\hat{D},\sigma(n_{\hat{S}}=1)}$, it belongs to the set Λ. The generic equilibrium contract c would instead belong to Λ' with more profitable contracts. ■

Proof of Corollary 1. Follows directly from Propositions 2 and 3. ■

Proof of Lemma 2. The value of $\bar{N}$ is obtained by considering the most profitable equilibrium for $\hat{S}$, so that the numerator of (5) is maximized. Such equilibrium corresponds to the contract $\varkappa$ =$(W_A^{\varkappa}, W_N^{\varkappa})$ that maximizes the profits for an insurer with ex-ante loss probability p. The contract $\varkappa$ has to satisfy the following conditions: i) be acceptable by a policyholder who expects a loss with probability p and ii) maximize the profits for an insurer who expects a loss with probability p. Let $\beta_0 - \beta_0 e^{-\beta W}$ be the explicit expression of the utility function. In the contract that maximizes profits, the consumer's participation constraint is binding:

$$p(\beta_0 - \beta_0 e^{-\beta W_A^{\varkappa}}) + (1-p)(\beta_0 - \beta_0 e^{-\beta W_N^{\varkappa}}) = p(\beta_0 - \beta_0 e^{-\beta W_A^0}) + (1-p)(\beta_0 - \beta_0 e^{-\beta W N^0}).$$

Moreover, contract $\varkappa$ lies on the 45 degree line: $W_A^{\varkappa} = W_N^{\varkappa} = W^{\varkappa}$, which implies, given that $W_N^0 = \bar{W}$ and $W_A^0 = \bar{W} - d$

$$\beta_0 - \beta_0 e^{-\beta W^{\varkappa}} = p(\beta_0 - \beta_0 e^{-\beta(\bar{W}-d)}) + (1-p)(\beta_0 - \beta_0 e^{-\beta \bar{W}}).$$

Through simple algebra, we obtain:

$$e^{-\beta W^{\varkappa}} = e^{-\beta \bar{W}}(pe^{\beta d} + 1 - p)$$

that is

$$W^{\varkappa} = \bar{W} - \frac{1}{\beta}\ln(1 + p(e^{\beta d} - 1)).$$

The profits in a full insurance pooling contract $c = (W^c, W^c)$ for an insurer who expects a loss with probability p is $\pi_p(c) = \bar{W} - pd - W^c$. Thus,

$$\pi_p(\varkappa) = \bar{W} - pd - \bar{W} + \frac{1}{\beta}\ln(1 + p(e^{\beta d} - 1)) = -dp + \frac{1}{\beta}\ln(1 + p(e^{\beta d} - 1)). \tag{10}$$

Similarly, we can obtain the profits of $\hat{S}$ in the most profitable deviation acceptable by a policyholder who expects a loss with probability $p_{\hat{S}}$; i.e.

$$\pi_{\hat{S}}(dev) = -dp_{\hat{S}} + \frac{1}{\beta}\ln(1 + p_{\hat{S}}(e^{\beta d} - 1)),$$

so that from (5) $\bar{N}$ is defined by

$$\overline{N} = \frac{-p_{\hat{S}}d + \frac{1}{\beta}\ln\left(1 + p\left(e^{\beta d} - 1\right)\right)}{-p_{\hat{S}}d + \frac{1}{\beta}\ln\left(1 + p_{\hat{S}}\left(e^{\beta d} - 1\right)\right)}.$$

■

Proof of Lemma 3. The value of $\underline{N}$ can be obtained from (6) by focusing on the equilibrium in which contract $c_{\sigma_1}^{\hat{S}}$ maximizes $\hat{S}$'s profits (i.e. the denominator of (6)), and $c_{\sigma_0}^{\hat{D}}$ minimizes $\hat{S}$'s profits (i.e. the numerator of (6)). Observe that contract $c_{\sigma_1}^{\hat{S}}$ satisfies with equality the participation constraint of a policyholder who expects a loss with probability p_{σ_1}, and maximizes the profits of an insurer who expects a loss with probability p_{σ_1}. Given these conditions, contract $c_{\sigma_1}^{\hat{S}}$ is a full insurance contract whose profits, are

$$\pi_{\sigma_1}(c_{\sigma_1}^{\hat{S}}) = -p_{\sigma_1}d + \frac{1}{\beta}\ln\left(1 + p_{\sigma_1}\left(e^{\beta d} - 1\right)\right). \tag{11}$$

Conversely, contract $c_{\sigma_0}^{\hat{D}}$ maximizes the expected utility of a policyholder who expects a loss with probability p_{σ_0}, and minimizes the profits of an insurer who expects a loss with probability p_{σ_0}. Given these conditions, $c_{\sigma_0}^{\hat{D}}$ is a full insurance, actuarially fair, contract defined by

$$W_A(c_{\sigma_0}^{\hat{D}}) = W_N(c_{\sigma_0}^{\hat{D}}) = \bar{W} - p_{\sigma_0}d,$$

the profits of which are given by

$$\pi_{\sigma_1}(c_{\sigma_0}^{\hat{D}}) = \bar{W} - p_{\sigma_1}d - W_A(c_{\sigma_0}^{\hat{D}}) = (p_{\sigma_0} - p_{\sigma_1})d. \tag{12}$$

By substituting (11) and (12) in (6), $\underline{N}$ can then be defined as

$$\underline{N} = \frac{(p_{\sigma_0} - p_{\sigma_1})d}{-p_{\sigma_1}d + \frac{1}{\beta}\ln\left(1 + p_{\sigma_1}\left(e^{\beta d} - 1\right)\right)}. \tag{13}$$

■