The strategic value of partial vertical integration

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Abstract

We investigate the strategic incentives for partial vertical integration, namely, partial ownership agreements between manufacturers and retailers, when the retailers privately know their costs and engage in differentiated good price competition. The partial misalignment between the profit objectives within a partially integrated manufacturer-retailer hierarchy entails a higher retail price than under full integration. This “information vertical effect” translates into a “competition horizontal effect”: the partially integrated hierarchy’s commitment to a higher retail price induces the competitor to increase its price, which strategically relaxes competition. Our analysis provides theoretical support for the recently documented empirical evidence on partial equity holdings.

Keywords: asymmetric information, partial vertical integration, vertical mergers, vertical restraints.

JEL Classification: D82, L13, L42.

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1. Introduction

Most of the practical and theoretical debate about the firms’ organizational structure in vertically related markets has focused on two extreme alternatives: full vertical integration and separation. However, it is quite common to observe partial vertical integration, namely, partial ownership agreements in which a firm acquires less than 100% of shares in a vertically related firm (e.g., Allen and Phillips 2000; Fee et al. 2006; Reiffen 1998). Riordan (2008) reports that in 2003 News Corp., a major owner of cable programming networks in the US, acquired 34% of shares in Hughes Electronics, which operates via its wholly-owned subsidiary Direct TV in the downstream market of direct broadcast satellite services. Gilo and Spiegel (2011) provide empirical evidence that partial vertical integration is much more common than full integration in telecommunication and media markets in Israel. For instance, Bezeq operates in the broadband Internet infrastructure market and holds a share of 49.77% in DBS Satellite Services that competes in the downstream multi-channel broadcast market.

Despite the empirical relevance of this phenomenon, little theoretical attention has been devoted to partial vertical integration. The aim of this paper is to investigate the strategic incentives of vertically related firms to partially integrate and their competitive effects.

We address this question in a setting where two manufacturer-retailer hierarchies engage in differentiated good price competition and the retailers are privately informed about their production costs. The economic literature has emphasized since Crocker’s (1983) seminal contribution that a major problem within a supply hierarchy is that a firm can access privileged information about some relevant aspects of the market. In our framework, a manufacturer exclusively deals with its retailer, which is reasonable in the presence of product-specific investments that have to be sunk before production decisions take place. Moreover, in line with the main literature on competing hierarchies under asymmetric information (e.g., Martimort 1996; Martimort and Piccolo 2010), bilateral contracting within a supply hierarchy is secret. This reflects the natural idea that the trading rules specified in a contractual relationship are not observed by competitors and therefore cannot be used for strategic purposes. Alternatively, these rules can be easily (secretly) renegotiated if both parties agree to do so.

In the benchmark case of full information within a supply hierarchy, a manufacturer that uses non-linear (secret) contracts is indifferent about the ownership stake in its retailer. This

\footnote{1}{For theoretical justifications of exclusive dealing in a context of asymmetric information, we refer to Gal-Or (1991b).}
is because the manufacturer makes the retailer residual claimant for the hierarchy’s profits and appropriates these profits through a fixed fee. The outcome of vertical integration is achieved irrespective of the ownership stake, and therefore vertical ownership arrangements are inconsequential.

This well-known result does not hold in the presence of asymmetric information about the retail costs. To begin with, consider a successive monopoly framework where a manufacturer-retailer pair operates in isolation. It is well established in the economic literature (e.g., Gal-Or 1991c) that asymmetric information within a supply hierarchy entails a higher retail price in order to curb the (costly) informational rents to the retailer, which reduces the hierarchy’s profits. Full vertical integration internalizes the negative informational externality within the hierarchy and restores the outcome of full information.

This strict preference for full vertical integration does not carry over in a more competitive environment. In a setting where two manufacturer-retailer pairs engage in differentiated good price competition, we show that partial vertical integration can emerge in equilibrium. In line with the successive monopoly framework, a partial vertical ownership agreement entails an information vertical effect: the partial misalignment between the profit objectives of the manufacturer and the retailer leads to a higher retail price than under full integration in order to reduce the informational rents to the retailer. For a given retail price of the competitor, this form of double marginalization from asymmetric information reduces the hierarchy’s profitability relative to full integration. In a competitive environment, however, the information vertical effect translates into an opposite competition horizontal effect: the partially integrated hierarchy’s commitment to a higher retail price induces an accommodating behavior of the rival that increases its price as well. Therefore, partial vertical integration is profitable since it constitutes a strategic device to relax competition. The trade-off between the benefits of softer competition and the informational costs drives the equilibrium degree of vertical integration.

The possibility that partial acquisitions can lead to a dampening of competition is explicitly addressed in the US antitrust law. Section 7 of the Clayton Act of 1914 (now Section 18 of Title 15 of the US Code) provides that

"No person engaged in commerce or in any activity affecting commerce shall acquire, directly or indirectly, the whole or any part of the stock or other share capital and no person subject to the jurisdiction of the Federal Trade Commission shall acquire the whole or any part of the assets of another person engaged also
in commerce or in any activity affecting commerce, where in any line of commerce or in any activity affecting commerce in any section of the country, *the effect of such acquisition may be substantially to lessen competition*, or to tend to create a monopoly.²

The Department of Justice and the Federal Trade Commission have recognized the potential anticompetitive effects arising from a firm’s partial acquisition of a horizontal competitor. In the Horizontal Merger Guidelines revised in 2010 a section has been introduced, which is explicitly devoted to partial acquisitions. The economic literature has provided a formal foundation for this approach and it has shown that partial ownership arrangements between rivals can lead to a dampening of competition (e.g., Foros et al. 2011; Gilo et al. 2006).

The main contribution of this paper is to unveil the strategic incentives for partial ownership agreements between firms that do not compete with each other but are vertically related. Therefore, our results suggest that antitrust investigations of partial acquisitions should be extended to vertically related markets. Notably, this does not require any enlargement of antitrust powers in the US, in the light of the general provisions of the Clayton Act.

Our analysis is presented in a fairly general setting without making any particular assumption on functional forms. Remarkably, it provides theoretical support for the empirical evidence recently documented in Ouimet (2013) that partial equity stakes are more likely to be preferred to full integration in industries that require relationship-specific investments, such as vertically related markets. Hence, the predictions of our model may serve as guidance for the empirical work on the competitive effects of partial vertical integration.

2. Related literature

The private and social effects of partial ownership agreements in horizontally related markets have been extensively explored in the economic literature. For instance, Gilo et al. (2006) provide the conditions under which partial cross ownership arrangements facilitate tacit collusion. Foros et al. (2011) show that a firm can prefer to acquire a partial ownership stake in a rival rather than fully merge, if it obtains the corporate control over price decisions.

Conversely, the literature on partial ownership in vertically related markets is still in its infancy. A relevant contribution is Dasgupta and Tao (2000), which demonstrates that partial vertical ownership may perform better than take-or-pay contracts if the upstream firms make

²Quotation with emphasis added can be found at http://www.law.cornell.edu/uscode/text/15/18.
investments that benefit the downstream firms. However, Greenlee and Raskovich (2006) find that, under certain circumstances, partial vertical ownership interests do not have any effect on the price or output choices of downstream firms. In a setting with an upstream homogeneous product and downstream imperfect competition, Hunold et al. (2012) show that passive (non-controlling) ownership of downstream firms in upstream firms is more profitable than full merger. Gilo et al. (2014) find that partial ownership acquisitions may increase the risk of anticompetitive foreclosure relative to full integration. Our paper contributes to this strand of literature and shows that in the presence of asymmetric information partial vertical integration constitutes a strategic commitment device to relax competition.

Our analysis is also related to the literature about the strategic choice between vertical separation and integration when supply hierarchies compete. This issue has been investigated in a setting of complete information (e.g., Bonanno and Vickers 1988; Gal-Or 1991a; Jansen 2003) and, more relevantly for our purposes, in a context of asymmetric information. Caillaud and Rey (1995) provide an overview of the strategic use of vertical delegation. Gal-Or (1992) shows that, in the presence of asymmetric information about the retail costs, for intermediate costs of integration one firm finds it optimal to integrate while its rival remains vertically separated. Barros (1997) demonstrates that in an oligopolistic industry some firms may profit from a commitment to face asymmetric information about their agents’ operations, since they are prevented from extracting full surplus and can provide the agents with a credible incentive to invest. Along these lines, Gal-Or (1999) derives the conditions under which vertically related firms follow different strategies about the integration or separation of their sale functions when asymmetric information concerns consumer demand. Contrary to the aforementioned contributions, we allow for a partial degree of vertical integration and show that partial vertical ownership agreements trade off the benefits of softer competition against the informational costs.

Our paper also belongs to the literature on vertical restraints under asymmetric information. In a successive monopoly framework with adverse selection, Gal-Or (1991c) compares quantity fixing and resale price maintenance contracts. Martimort (1996) investigates the choice of competing manufacturers between a common or exclusive retailer and shows that this choice depends on the degree of product differentiation and on the magnitude of the adverse selection problem. In a model with adverse selection and moral hazard, Martimort and Piccolo (2007) qualify the results of Gal-Or (1991c) according to the retailers’ technology for providing ser-
vices. In a setting with competing manufacturer-retailer pairs, Martimort and Piccolo (2010) and Kastl et al. (2011) show that manufacturers may strategically prefer quantity fixing to resale price maintenance contracts and investigate the welfare consequences of these contractual relationships.

Our work shares some relevant similarities with the strand of literature that explores the scope for strategic delegation in a competitive environment. However, contrary to early contributions (e.g., Fershtman and Judd 1987), we consider a setting of asymmetric information with secret contracting (e.g., Martimort 1996), where the terms of trade cannot be used for strategic purposes.

The rest of the paper is structured as follows. Section 3 sets out the formal model. Section 4 considers the benchmark case of a manufacturer informed about the costs of its retailer. Section 5 shows that partial vertical integration is preferred to full integration in the presence of asymmetric information. Using explicit functions, Section 6 derives the equilibrium degree of partial vertical integration and performs a comparative static analysis. Section 7 investigates alternative assumptions and the robustness of the results. Section 8 discusses some antitrust policy implications. Section 9 concludes. All formal proofs are provided in the Appendix.

3. The model

Setting We consider a vertically related market where two upstream manufacturers, $M_1$ and $M_2$, provide symmetrically differentiated goods through two downstream retailers, $R_1$ and $R_2$, which engage in price competition. As discussed in the introduction, we assume that each manufacturer is in an exclusive relationship with one retailer. To make the analysis as sharp as possible for our aims, in the spirit of Martimort and Piccolo (2010) we consider a setting where manufacturer $M_1$ and retailer $R_1$ exclusively deal with each other, while manufacturer $M_2$ is fully integrated with retailer $R_2$.

Let $q_i(p_i, p_{-i})$ denote the (direct) demand function for good $i = 1, 2$, which is decreasing and (weakly) concave in its own price $p_i$, i.e., $\frac{\partial q_i(p_i, p_{-i})}{\partial p_i} < 0$ and $\frac{\partial^2 q_i(p_i, p_{-i})}{\partial p_i^2} \leq 0$. Goods are substitute, i.e., $\frac{\partial q_i(p_i, p_{-i})}{\partial p_{-i}} \geq 0$ (the equality holds if and only if demands are independent), and own-price effects are larger than cross-price effects, i.e., $\left|\frac{\partial q_i(p_i, p_{-i})}{\partial p_i}\right| > \frac{\partial q_i(p_i, p_{-i})}{\partial p_{-i}}$. We also assume $\frac{\partial^2 q_i(p_i, p_{-i})}{\partial p_i \partial p_{-i}} \geq 0$, which ensures strategic complementarity in prices.\(^4\) Manufacturing costs are

\[^3\text{As discussed in Section 7.2, our results carry over in a more symmetric setting where both supply hierarchies decide on the degree of vertical integration.}\]

\[^4\text{This means that the reaction functions are positively sloped (Bulow et al. 1985). We refer to Vives (1999,}\]
normalized to zero.

Manufacturer $M_1$ offers retailer $R_1$ a contract that specifies a retail price $p_1$ for the good and a fixed franchise fee $t_1$ paid by the retailer to the manufacturer for the right to sell the good.\(^5\) Let $\theta_1 \in \{\theta_l, \theta_h\}$ be the marginal retail cost, whose realization is private information of the retailer at the time the contract is signed. With probability $\nu \in (0, 1)$ costs are $\theta_l$, while with probability $1 - \nu$ costs are $\theta_h$, where $\Delta \theta \equiv \theta_h - \theta_l > 0$. Retailer $R_1$’s (interim expected) profits are

$$\pi_{R_1} = p_1 E_{\theta_2} (q_1 (p_1, p_2) | \theta_1) - \theta_1 E_{\theta_2} (q_1 (p_1, p_2) | \theta_1) - t_1,$$

(1)

where $E_{\theta_2} (q_1 (p_1, p_2) | \theta_1)$ represents the expected quantity of $R_1$. The expectation is taken over the retail costs $\theta_2$ of the competitor $M_2 - R_2$ and it is conditional on $R_1$’s retail costs $\theta_1$, since $R_1$ does not know the retail costs of the competitor when contracting with $M_1$ but the uncertainty depends on its cost realization. Specifically, in line with the main literature (e.g., Gal-Or 1991b, 1999; Martimort 1996), we allow for positive correlation between retail costs.\(^6\) In the example provided in Section 6, we consider the case of perfect cost correlation, which implies $\theta_1 = \theta_2$.

Manufacturer $M_1$’s (interim expected) profits are

$$\pi_{M_1} = t_1 + \rho [p_1 E_{\theta_2} (q_1 (p_1, p_2) | \theta_1) - \theta_1 E_{\theta_2} (q_1 (p_1, p_2) | \theta_1) - t_1],$$

(2)

which is a weighted sum of upstream profits from the franchise fee $t_1$ and downstream profits $\pi_{R_1}$ in (1) from retail operations. When offering a contract to $R_1$, $M_1$ is concerned about the profits in (2). The parameter $\rho \in [0, 1]$ represents the ownership stake acquired by $M_1$ in $R_1$. Following O’Brien and Salop (2000), $\rho$ captures the financial interest of the acquiring firm, which is entitled to receive a share of the profits of the acquired firm. If $\rho = 0$, the two firms are fully separated. If $\rho \in (0, 1)$, $M_1$ has a partial ownership stake in $R_1$, and therefore the two firms are partially integrated. If $\rho = 1$, $M_1$ wholly owns $R_1$ and they are fully integrated.

\(^5\)This contractual mode yields the manufacturer the highest profit. The practice of dictating the final price to a retailer is commonly known as resale price maintenance. As discussed in Section 7.3, our qualitative results are unaffected if we consider a two-part tariff contract that consists of a unit wholesale price and a fixed fee.

\(^6\)Positive correlation seems to be reasonable in competitive markets where costs are usually subject to common trends. Our results fully apply with independent costs.

Ch. 6) for a full characterization of the standard regularity and stability conditions that ensure the existence of a unique equilibrium.
Notably, $R_1$ maximizes its own profits in (1) irrespective of the ownership stake $\rho$ acquired by $M_1$. This is typically the case when the acquiring firm has a silent financial interest in the acquired firm or legal constraints such as “fiduciary obligation”, which are included in the corporate charter or antitrust law, induce the acquired firm to act as an independent, stand-alone entity. Clearly, a sufficiently high $\rho$ ensures $M_1$ the corporate control on $R_1$, which could influence $R_1$’s behavior in favor of $M_1$ so that the profit objectives of the two firms would be more aligned. Our qualitative results carry over in this alternative scenario, with the main difference being that they would depend on the threshold of the ownership stake that guarantees corporate control.

We wish to derive the equilibrium degree of vertical integration between manufacturer $M_1$ and retailer $R_1$, namely, the ownership stake $\rho$ that $M_1$ decides to acquire in $R_1$. Following the main literature on partial ownership (e.g., Foros et al. 2011; Greenlee and Raskovich 2006; Hunold et al. 2012), we assume that $M_1$ chooses the ownership stake $\rho$ in $R_1$ that maximizes the (expected) joint profits of the two firms. This ensures that $M_1$ can design an offer to $R_1$ which makes the shareholders in both firms better off, so that they will find it mutually beneficial to sign such an agreement. A joint profit maximizing ownership agreement does not leave any scope for mutually beneficial renegotiations and exhibits a commitment value. In Section 7.1, we qualify our results for the case where the manufacturer maximizes its own profits when deciding on the ownership stake.

In order to focus on the strategic effects of acquisition, we abstract from any cost saving that may arise from the ownership arrangement.

The (interim expected) profits of the vertical structure $M_2 - R_2$ are

$$\pi_2 = p_2 E_{\theta_1} (q_2 (p_1, p_2) | \theta_2) - \theta_2 E_{\theta_1} (q_2 (p_1, p_2) | \theta_2), \quad (3)$$

where $E_{\theta_1} (q_2 (p_1, p_2) | \theta_2)$ represents the (conditional) expected quantity of $M_2 - R_2$. The two competing supply hierarchies do not know the costs of each other but, as stressed before, their

\footnote{In a similar vein, Farrell and Shapiro (1990) suggest the criterion of joint profits to derive the equilibrium ownership stake. Notably, this approach seems to reflect the practice of takeovers and acquisitions. For instance, in the US a bidder that makes an offer to purchase less than 100% of the shares of a firm must accept all shares tendered on a pro-rated basis. For further discussion on this point, we refer to Foros et al. (2011).}

\footnote{In the baseline model we do not impose any particular restriction on $\theta_2$ that can take values either within a discrete set or an interval. Moreover, we only require standard regularity conditions on the probability distribution function for $\theta_2$.}
costs are positively correlated. As it will be clear in the sequel, since \( M_2 - R_2 \) is vertically integrated, our results fully apply when the upstream manufacturer \( M_2 \) does not know the costs of its downstream division \( R_2 \).

**Contracting** In line with the main literature on competing supply hierarchies (e.g., Gal-Or 1999; Kastl et al. 2011; Martimort 1996; Martimort and Piccolo 2010), bilateral contracting within a hierarchy is secret. We invoke the revelation principle (e.g., Myerson 1982) in order to characterize the set of incentive feasible allocations. In our setting, this means that, for any strategy choice of \( M_2 - R_2 \), there is no loss of generality in deriving the best response of \( M_1 \) within the class of direct incentive compatible mechanisms. Specifically, manufacturer \( M_1 \) offers retailer \( R_1 \) a direct contract menu \( \left\{ t_1 \left( \hat{\theta}_1 \right), \ p_1 \left( \hat{\theta}_1 \right) \right\}_{\hat{\theta}_1 \in \{\theta_l, \theta_h\}} \) that determines a fixed franchise fee \( t_1(.) \) and a retail price \( p_1(.) \) contingent on the retailer’s report \( \hat{\theta}_1 \in \{\theta_l, \theta_h\} \) about its costs. This contract menu must be incentive compatible, namely, it must induce the retailer to report truthfully its costs, which implies \( \hat{\theta}_1 = \theta_1 \) in equilibrium.\(^9\)

It is worth noting that this contract mechanism is incomplete, since manufacturer \( M_1 \) cannot contract upon the retail price of the competitor \( M_2 - R_2 \).\(^10\) This assumption, which has a solid foundation in the literature (e.g., Gal-Or 1991a, 1991b, 1992, 1999; Kastl et al. 2011; Martimort 1996; Martimort and Piccolo 2010), can be justified on several grounds. For instance, a contract contingent on the retail price of the competitor may be condemned as collusive practice by antitrust authorities.\(^11\)

**Timing** The sequence of events unfolds as follows.

(I) \( M_1 \) decides on which ownership stake \( \rho \in [0, 1] \) to acquire in \( R_1 \).

(II) \( R_1 \) and \( M_2 - R_2 \) privately learn their respective retail costs \( \theta_1 \in \{\theta_l, \theta_h\} \) and \( \theta_2 \).

(III) \( M_1 \) secretly makes an offer \( \left\{ t_1 \left( \hat{\theta}_1 \right), \ p_1 \left( \hat{\theta}_1 \right) \right\}_{\hat{\theta}_1 \in \{\theta_l, \theta_h\}} \) to \( R_1 \). The offer can be either rejected or accepted by \( R_1 \).\(^12\) If the offer is rejected, each firm obtains its outside option.

\(^9\)Since the manufacturer can obtain (a part of) the retailer’s profits, it might infer the value of the retail costs and implement a penalty that extracts all profits of the retailer arising from cost misreporting. However, this penalty is unfeasible under a range of reasonable circumstances. The profit realization may be affected by (independent) random shocks which, for instance, occur after the firms’ decisions. In this case, retail costs cannot be directly inferred from the retailer’s profits and, especially in the presence of limited liability, it would be unfeasible to design any penalty that deters cost misreporting. Furthermore, the fine implemented by the manufacturer would have the only effect of expropriating the profits of the other shareholders of the retailer. This would be interpreted as a violation of their rights and condemned by antitrust authorities.

\(^10\)Similarly, the contract cannot be made contingent on any report from \( M_2 - R_2 \) about its retail costs.

\(^11\)Alternatively, the retail price charged by the rival can be hard to observe or verify because of the lack of the proper auditing rights. We refer to Martimort (1996) for a discussion of this assumption.

\(^12\)This is usually justified in the literature by assuming that a retailer is selected from a very large population of (equally ex ante efficient) potential firms, so that the manufacturer is able to dictate the terms of trade in the
(normalized to zero), while $M_2 - R_2$ acts as a monopolist. If the offer is accepted, $R_1$ picks one element within the contract menu by sending a report $\hat{\theta}_1 \in \{\theta_1, \theta_h\}$ about its costs.

(IV) Competition takes place in the downstream market and payments are made.

The solution concept we adopt is Perfect Bayesian Equilibrium, with the additional “passive beliefs” refinement (e.g., Kastl et al. 2011; Martimort 1996; Martimort and Piccolo 2010). Whenever $R_1$ receives an unexpected offer from $M_1$, it does not change its beliefs about the equilibrium strategy of $M_2 - R_2$.

Proceeding backward, we first compute the retail prices in the competition stage for a given ownership stake. Afterwards, we derive the equilibrium ownership stake.

4. Benchmark: manufacturer fully informed about the retail costs

To better appreciate how the strategic value of partial ownership follows from the presence of asymmetric information, we first consider the benchmark case in which $M_1$ is fully informed about the costs of its retailer $R_1$.

We formalize the main results in the following lemma.

**Lemma 1** If manufacturer $M_1$ is fully informed about the costs $\theta_1$ of retailer $R_1$, the equilibrium retail price $\hat{p}_i$ charged by the supply hierarchy $M_i - R_i$, $i = 1, 2$, satisfies

$$E_{\theta_{-i}}(q_i(\hat{p}_i, \hat{p}_{-i})|\theta_i) + (\hat{p}_i - \theta_i) \frac{\partial E_{\theta_{-i}}(q_i(\hat{p}_i, \hat{p}_{-i})|\theta_i)}{\partial \hat{p}_i} = 0. \tag{4}$$

The equilibrium ownership stake that $M_1$ holds in $R_1$ is any $\hat{\rho} \in [0, 1]$.

The retail price of each supply hierarchy is set above marginal costs in order to equate (expected) marginal revenues with (expected) marginal costs from retailing. The problem of manufacturer $M_1$ coincides with the problem of the vertical structure $M_2 - R_2$. Since contracting is secret and cannot be used for strategic purposes, a fully informed manufacturer using non-linear contracts finds it optimal to remove the double marginalization problem by making its retailer residual claimant for the hierarchy’s profits, which are extracted via a fixed fee. Hence, the full integration outcome is achieved irrespective of the ownership stake $\rho$, and the choice of the degree of vertical integration is inconsequential. This well-known neutrality result (e.g., Caillaud and Rey 1995) does not hold anymore in the presence of asymmetric information.

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13 Throughout the analysis we assume interior solutions in the competition stage.
5. The case of asymmetric information

As discussed in Section 3, when retailer $R_1$ privately knows its costs, manufacturer $M_1$ can restrict attention to a direct incentive compatible contract menu $\{(t_{1l}, p_{1l}), (t_{1h}, p_{1h})\}$, where $(t_{1l}, p_{1l})$ and $(t_{1h}, p_{1h})$ are the contracts designed for the efficient and inefficient retailer, with costs $\theta_l$ and $\theta_h$ respectively.

5.1. The competition stage

We first derive the retail prices for a given ownership stake. In addition to the participation constraints $\pi_{R_{1l}} \geq 0$ and $\pi_{R_{1h}} \geq 0$, the contract offered by $M_1$ to $R_1$ must satisfy the following incentive compatibility constraints

\[
\pi_{R_{1l}} = p_{1l} E_{\theta_2} (q_1 (p_{1l}, p_2) | \theta_l) - \theta_l E_{\theta_2} (q_1 (p_{1l}, p_2) | \theta_l) - t_{1l} \\
\geq p_{1h} E_{\theta_2} (q_1 (p_{1h}, p_2) | \theta_l) - \theta_l E_{\theta_2} (q_1 (p_{1h}, p_2) | \theta_l) - t_{1h}
\]

(5)

\[
\pi_{R_{1h}} = p_{1h} E_{\theta_2} (q_1 (p_{1h}, p_2) | \theta_h) - \theta_h E_{\theta_2} (q_1 (p_{1h}, p_2) | \theta_h) - t_{1h} \\
\geq p_{1l} E_{\theta_2} (q_1 (p_{1l}, p_2) | \theta_h) - \theta_h E_{\theta_2} (q_1 (p_{1l}, p_2) | \theta_l) - t_{1l}.
\]

(6)

Conditions (5) and (6) ensure that $R_1$ does not benefit from misreporting its costs.

The participation constraint $\pi_{R_{1h}} \geq 0$ for the inefficient retailer and the incentive constraint (5) for the efficient retailer are binding at the optimal contract.\(^{14}\) Rewriting the binding incentive constraint (5) as

\[
\pi_{R_{1l}} = \pi_{R_{1h}} + p_{1h} [E_{\theta_2} (q_1 (p_{1h}, p_2) | \theta_l) - E_{\theta_2} (q_1 (p_{1h}, p_2) | \theta_h)] \\
+ \theta_h E_{\theta_2} (q_1 (p_{1l}, p_2) | \theta_h) - \theta_l E_{\theta_2} (q_1 (p_{1h}, p_2) | \theta_l) \\
= \pi_{R_{1h}} + \Delta \theta E_{\theta_2} (q_1 (p_{1h}, p_2) | \theta_h) - (p_{1h} - \theta_l) \\
\times [E_{\theta_2} (q_1 (p_{1l}, p_2) | \theta_h) - E_{\theta_2} (q_1 (p_{1h}, p_2) | \theta_l)],
\]

(7)

\(^{14}\)Otherwise, $M_1$ could increase the franchise fee and be better off. For further technical details we refer to the proof of Proposition 1 in the Appendix.
$M_1$’s problem of maximizing its (expected) profits in (2) becomes

\[
\max_{p_{11}, p_{1h}} \nu \{ p_{11} E_{\theta_2} (q_1 (p_{11}, p_2) | \theta_1) - \theta_1 E_{\theta_2} (q_1 (p_{11}, p_2) | \theta_1) - (1 - \rho) \times [\Delta \theta E_{\theta_2} (q_1 (p_{1h}, p_2) | \theta_h) - (p_{1h} - \theta_1) (E_{\theta_2} (q_1 (p_{1h}, p_2) | \theta_h) - E_{\theta_2} (q_1 (p_{1h}, p_2) | \theta_1))] \} + (1 - \nu) [p_{1h} E_{\theta_2} (q_1 (p_{1h}, p_2) | \theta_h) - \theta_h E_{\theta_2} (q_1 (p_{1h}, p_2) | \theta_h)]. \tag{8}
\]

Using (3), the problem of the vertical structure $M_2 - R_2$ is

\[
\max_{p_2} p_2 E_{\theta_1} (q_2 (p_1, p_2) | \theta_2) - \theta_2 E_{\theta_1} (q_2 (p_1, p_2) | \theta_2). \tag{9}
\]

Defining

\[
\Phi (p_{1h}, p_2) \equiv \Delta \theta \frac{\partial E_{\theta_2} (q_1 (p_{1h}, p_2) | \theta_h)}{\partial p_1} - E_{\theta_2} (q_1 (p_{1h}, p_2) | \theta_h) + E_{\theta_2} (q_1 (p_{1h}, p_2) | \theta_l) - (p_{1h} - \theta_1) \left( \frac{\partial E_{\theta_2} (q_1 (p_{1h}, p_2) | \theta_h)}{\partial p_1} - \frac{\partial E_{\theta_2} (q_1 (p_{1h}, p_2) | \theta_l)}{\partial p_1} \right), \tag{10}
\]

we can now derive the retail prices for a given ownership stake $\rho$.

**Proposition 1** If retailer $R_1$ is privately informed about its costs $\theta_1 \in \{ \theta_l, \theta_h \}$, the retail price charged by the supply hierarchy $M_1 - R_1$ is $p_1 \in \{ p_{1l}^*, p_{1h}^* \}$, where $p_{1l}^*$ and $p_{1h}^*$ satisfy respectively

\[
E_{\theta_2} (q_1 (p_{1l}^*, p_2^*) | \theta_l) + (p_{1l}^* - \theta_l) \frac{\partial E_{\theta_2} (q_1 (p_{1l}^*, p_2^*) | \theta_l)}{\partial p_{1l}} = 0 \tag{11}
\]

\[
E_{\theta_2} (q_1 (p_{1h}^*, p_2^*) | \theta_h) + (p_{1h}^* - \theta_h) \frac{\partial E_{\theta_2} (q_1 (p_{1h}^*, p_2^*) | \theta_h)}{\partial p_{1h}} - \phi (\nu) (1 - \rho) \Phi (p_{1h}^*, p_2^*) = 0, \tag{12}
\]

with $\phi (\nu) \equiv \frac{\nu}{1 - \nu}$. Furthermore, the retail price $p_2^*$ charged by the supply hierarchy $M_2 - R_2$ satisfies

\[
E_{\theta_1} (q_2 (p_1^*, p_2^*) | \theta_2) + (p_2^* - \theta_2) \frac{\partial E_{\theta_1} (q_2 (p_1^*, p_2^*) | \theta_2)}{\partial p_2} = 0. \tag{13}
\]

This yields the following lemma.

**Lemma 2** It holds that (i) $\frac{\partial p_{1l}^*}{\partial \rho} < 0$, (ii) $\frac{\partial p_{1h}^*}{\partial \rho} < 0$, (iii) $\frac{\partial p_2^*}{\partial \rho} < 0$. 

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Figure 1: Reaction functions under full and asymmetric information

Under asymmetric information the efficient retailer commands some informational rents defined by (7). The maximand in (8) reveals that the retailer’s rents are costly for manufacturer $M_1$ as long as it does not fully internalize these rents, i.e., $\rho < 1$. As Figure 1 illustrates, since $\Phi$ in (10) is negative the reaction function of the inefficient retailer shifts outward relative to the full information level. Hence, the price in (12) is higher than under full information in order to reduce the informational rents appropriated by the efficient retailer. The magnitude of this form of double marginalization from asymmetric information depends on the ownership stake $\rho$ that determines $M_1$’s degree of internalization of $R_1$’s profits. As Lemma 2 indicates, higher values for $\rho$ translate into a lower price of the inefficient retailer, since $M_1$ internalizes $R_1$’s profits to a larger extent. In particular, with a full acquisition of $R_1$ ($\rho = 1$), $M_1$ maximizes joint profits in (8) and fully internalizes $R_1$’s rents. This completely removes the informational costs and the retail price reflects the full information level in (4).

The reaction function of the efficient retailer coincides with that under full information. This is because the informational rents in (7) are independent of the price of the efficient retailer and therefore $M_1$ does not find it profitable to implement any price distortion. However, the price in (11) charged by the efficient retailer is generally higher than the full information price in (4) if $\rho < 1$. To understand the rationale for this result, consider the reaction function of

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15 The figure represents the case of linear demand. Moreover, for the sake of clarity, the reaction function of $M_2 - R_2$ is depicted for a given (and known) $\theta_2$.

16 This result is reminiscent of the rent extraction-efficiency trade-off in optimal regulation (e.g., Baron and Myerson 1982).
Since prices are strategic complements, a lower value for $\rho$, which shifts outward the reaction function of the inefficient retailer, induces $M_2 - R_2$ to set a higher price, as shown in Lemma 2. The efficient retailer also increases its price in response to the higher price charged by the competitor.

It is worth noting that the latter result depends on the fact that $M_2 - R_2$ cannot distinguish between the efficient and inefficient retailer and therefore it determines its price on the basis of the (conditional) expectation about the rival’s retail costs. As we will see in Section 6, when costs are perfectly correlated, $M_2 - R_2$ certainly knows the rival’s costs, and therefore it does not distort its price when the retailer is efficient. In this case, both the price of $M_2 - R_2$ and the price of the efficient retailer reflect their full information values.

5.2. The equilibrium ownership stake

Having derived the retail prices for a given ownership stake, we can go back to the first stage of the game and determine the equilibrium ownership stake. Since $M_1$ chooses how much of $R_1$ to acquire in order to maximize joint profits, $M_1$’s maximization problem is

$$
\max_{\rho \in [0,1]} \nu [p_{11}^* (\rho) E_{\theta_1} (q_1 (p_{11}^* (\rho), p_{2}^* (\rho)) | \theta_1)] - \theta_1 E_{\theta_2} (q_1 (p_{11}^* (\rho), p_{2}^* (\rho)) | \theta_1]) 
$$

$$
+ (1 - \nu) [p_{1h}^* (\rho) E_{\theta_2} (q_1 (p_{1h}^* (\rho), p_{2}^* (\rho)) | \theta_2)] - \theta_h E_{\theta_2} (q_1 (p_{1h}^* (\rho), p_{2}^* (\rho)) | \theta_2)].
$$

We are now in a position to show our main results.

**Proposition 2** If retailer $R_1$ is privately informed about its costs $\theta_1 \in \{\theta_1, \theta_h\}$, then partial vertical integration, i.e., $\rho \in (0, 1)$, is more profitable for the supply hierarchy $M_1 - R_1$ than full vertical integration whenever consumer demands are interdependent ($\frac{\partial q_i}{\partial p_{-i}} > 0$). The equilibrium ownership stake that manufacturer $M_1$ holds in retailer $R_1$ is $\rho^* < 1$. Full vertical integration, i.e., $\rho^* = 1$, arises in equilibrium if consumer demands are independent ($\frac{\partial q_i}{\partial p_{-i}} = 0$).

Under asymmetric information $M_1$ is no longer indifferent about the ownership stake in $R_1$. If demands are independent and therefore each supply hierarchy acts as a monopolist, full vertical integration is preferable. This corroborates the well-known result that vertical
integration removes the negative informational externality within the hierarchy and improves its profits. Notably, we obtain this result without needing to assume that a manufacturer exogenously acquires any relevant information about its retailer when they are fully integrated. It follows from Proposition 1 that under full integration a manufacturer fully internalizes the retailer’s informational rents and does not find it optimal to implement any price distortion.

Proposition 2 shows that this strict preference for full vertical integration does not carry over in a more competitive environment. In this case partial vertical integration gives the supply hierarchy $M_1 - R_1$ higher profits than full integration, and the equilibrium ownership stake that $M_1$ acquires in $R_1$ is strictly lower than full ownership, i.e., $\rho^* < 1$. We know from Proposition 1 that a partial misalignment between the profit objectives within the partially integrated hierarchy leads to a higher retail price of the inefficient retailer than under full integration in order to reduce the (costly) informational rents to the efficient retailer. This information vertical effect constitutes a form of double marginalization from asymmetric information that, for a given price charged by the competitor $M_2 - R_2$, reduces the profitability of the supply hierarchy $M_1 - R_1$. In the presence of price competition, the information vertical effect translates into an opposite competition horizontal effect. The commitment of the partially integrated hierarchy $M_1 - R_1$ to a higher retail price than under full integration induces the competitor $M_2 - R_2$ to increase its price as well, which strategically relaxes competition. The equilibrium degree of vertical integration trades off the benefits of softer competition against the informational costs.

In the sequel, using explicit functions, we derive the equilibrium degree of vertical integration.

6. An illustrative example

The consumer demand for good $i = 1, 2$ takes the following form

$$q_i = \alpha - \beta p_i + \gamma p_{-i}, \quad (15)$$

where $\alpha$ and $\beta$ are positive parameters, and $\gamma \in [0, \beta)$ denotes the degree of substitutability between goods.$^{18}$ The profits of $R_1$, $M_1$ and $M_2 - R_2$ are respectively given by (1), (2) and (3), with retail costs being now perfectly correlated, which implies $\theta_1 = \theta_2 \in \{\theta_1, \theta_h\}$.  

$^{18}$The demand system in (15) follows from the optimization problem of a unit mass of identical consumers with a quasi-linear utility function $y + U(q_1, q_2)$, where $y$ is the composite good and $U(q_1, q_2) = a(q_1 + q_2) - \frac{1}{2} (bq_1^2 + bq_2^2 + 2gq_1q_2)$, with $a > 0, b > g \geq 0$, and $\alpha = \frac{a(\beta - a)}{b - g}, \beta = \frac{b}{b - g}, \gamma = \frac{a}{b - g}$ (see, e.g., Vives 1999, Ch. 6).
The following lemma collects the main results with a fully informed manufacturer.

**Lemma 3** If manufacturer $M_1$ is fully informed about the costs $\theta_1$ of retailer $R_1$, the equilibrium retail price $\hat{p}_i$, $i = 1, 2$, is

$$\hat{p}_i = \frac{\alpha + \beta \theta_i}{2 \beta - \gamma}.$$  \hspace{1cm} (16)

The equilibrium ownership stake that $M_1$ holds in $R_1$ is any $\hat{\rho} \in [0, 1]$.

We know from Lemma 1 that under full information within each supply hierarchy retail prices are set to equalize marginal revenues and marginal costs. As costs are perfectly correlated, the prices in (16) of the two supply hierarchies coincide.

Now, we turn to the case of asymmetric information. As in Section 5, we first characterize the retail prices for a given ownership stake $\rho$.

**Lemma 4** If retailer $R_1$ is privately informed about its costs $\theta_1 \in \{\theta_l, \theta_h\}$, the retail price charged by the supply hierarchy $M_1 = R_1$ is $p^*_1 \in \{p^*_{1l}, p^*_{1h}\}$, where

$$p^*_{1l} = \frac{\alpha + \beta \theta_l}{2 \beta - \gamma}$$ \hspace{1cm} (17)

$$p^*_{1h} = \frac{(\alpha + \beta \theta_l) (4 \beta^2 - \gamma^2) + \phi (\nu) (1 - \rho) \left[ 4 \beta^2 \Delta \theta - \gamma^2 (\alpha + \beta \theta_h) \right]}{(2 \beta - \gamma) \left[ 4 \beta^2 - \gamma^2 (1 + \phi (\nu) (1 - \rho)) \right]},$$ \hspace{1cm} (18)

with $\phi (\nu) \equiv \frac{\nu}{1 - \nu}$. Furthermore, the retail price charged by the supply hierarchy $M_2 = R_2$ is $p^*_2 \in \{p^*_{2l}, p^*_{2h}\}$, where

$$p^*_{2l} = \frac{\alpha + \beta \theta_l}{2 \beta - \gamma}$$ \hspace{1cm} (19)

$$p^*_{2h} = \frac{(\alpha + \beta \theta_h) (4 \beta^2 - \gamma^2) + \gamma \phi (\nu) (1 - \rho) \left[ 2 \beta^2 \Delta \theta - \gamma (\alpha + \beta \theta_h) \right]}{(2 \beta - \gamma) \left[ 4 \beta^2 - \gamma^2 (1 + \phi (\nu) (1 - \rho)) \right]},$$ \hspace{1cm} (20)

Lemma 4 illustrates with explicit solutions the main insights gleaned from Proposition 1. With perfectly correlated costs, $M_2 - R_2$ certainly knows whether it faces the efficient retailer, whose price is not distorted for rent extraction purposes. Therefore, in this case the retail prices in (17) and (19) reflect their full information values in (16).

Conversely, the price in (18) of the inefficient retailer can be inflated above the full information level to reduce the informational rents to the efficient retailer. Denoting by $\hat{p}_{1h}$ the full information price in (16) of the inefficient retailer, we find
\[ p_{1h}^* (\rho) - \hat{p}_{1h} = \frac{4 \beta^3 \Delta \theta \phi (\nu) (1 - \rho)}{(2 \beta - \gamma) [4 \beta^2 - \gamma^2 (1 + \phi (\nu) (1 - \rho))]} \geq 0, \]

which vanishes if and only if \( \rho = 1 \). This expression measures the price distortion from asymmetric information and crucially depends on the ownership stake \( \rho \). Differentiating \( p_{1h}^* (\rho) \) in (18) yields

\[ \frac{\partial p_{1h}^*}{\partial \rho} = -\frac{4 \beta^3 (2 \beta - \gamma) \Delta \theta \phi (\nu)}{[4 \beta^2 - \gamma^2 (1 + \phi (\nu) (1 - \rho))]^2} < 0, \tag{21} \]

which indicates that a lower ownership stake \( \rho \) exacerbates the upward price distortion. As we know from the discussion following Lemma 2, this is because \( M_1 \) internalizes to a lesser extent the profits in (7) of the efficient retailer and therefore it is more inclined to curb these profits via a price increase. For a given price charged by the competitor \( M_2 - R_2 \), the upward price distortion driven by a lower ownership stake reduces the profits of \( M_1 - R_1 \) relative to full integration. However, in the presence of price competition, \( M_2 - R_2 \) reacts with an accommodating behavior. In particular, differentiating (20) yields

\[ \frac{\partial p_{2h}^*}{\partial \rho} = -\gamma \frac{4 \beta^2 (2 \beta - \gamma) \Delta \theta \phi (\nu)}{[4 \beta^2 - \gamma^2 (1 + \phi (\nu) (1 - \rho))]^2} < 0 \tag{22} \]

for \( \gamma > 0 \). A lower ownership stake of \( M_1 \) in \( R_1 \) translates into higher retail prices for both supply hierarchies. Note from (21) and (22) that the price response of \( M_2 - R_2 \) to a change in \( \rho \) is smoother than the price response of \( M_1 - R_1 \). Hence, even though the two supply hierarchies share the same retail costs, \( M_1 - R_1 \) sets a price in (18) which is higher than the price in (20) of the competitor \( M_2 - R_2 \) for \( \rho < 1 \).

The following proposition illustrates the result of the trade-off between the benefits of softer competition and the informational costs.

**Proposition 3** If retailer \( R_1 \) is privately informed about its costs \( \theta_1 \in (\theta_l, \theta_h) \), the equilibrium ownership stake that manufacturer \( M_1 \) holds in retailer \( R_1 \) is

\[ \rho^* = \max \left\{ 1 - \frac{\gamma^2 (4 \beta^2 - \gamma^2) (\alpha - (\beta - \gamma) \theta_h)}{\phi (\nu) [8 \beta^3 \Delta \theta (2 \beta^2 - \gamma^2) + \gamma^2 (\alpha - (\beta - \gamma) \theta_h)]}; 0 \right\}. \tag{23} \]
It holds that \( \rho^* < 1 \) whenever consumer demands are interdependent (\( \gamma \neq 0 \)). In particular, we have

(i) partial vertical integration, i.e., \( \rho^* \in (0, 1) \), if
\[
\phi \left( \nu \right) > \frac{\gamma^2 (4 \beta^2 - \gamma^2) (\alpha - \beta - \gamma) \theta_h}{8 \beta^2 \Delta \sigma (2 \beta^2 - \gamma^2) + \gamma^2 (\alpha - (\beta - \gamma) \theta_h)},
\]

(ii) full vertical separation, i.e., \( \rho^* = 0 \), otherwise.

Full vertical integration, i.e., \( \rho^* = 1 \), is preferred if consumer demands are independent (\( \gamma = 0 \)).

Proposition 3 reveals that, when demands are interdependent, \( M_1 \) finds it optimal to partially integrate with \( R_1 \) if the retailer is relatively likely to be efficient (\( \phi \left( \nu \right) \) increases with \( \nu \)). A higher probability \( \nu \) of the efficient retailer translates into larger expected (costly) informational rents and therefore a higher retail price of the inefficient retailer to curb these rents. If \( \nu \) is relatively high, a partial financial interest of \( M_1 \) in \( R_1 \) optimally trades off the benefits of softer competition against the informational costs. Otherwise, the double marginalization from asymmetric information is relatively small and the benefits of departing from full integration always dominate the associated informational costs. As a consequence, \( M_1 \) prefers to fully separate from \( R_1 \).

A pattern of full integration that removes any informational externality within the hierarchy is optimal if each supply chain acts as a monopolist, i.e., \( \gamma = 0 \), since there is no benefit of relaxing competition. As Figure 2 illustrates, the relation between the degree of product differentiation \( \gamma \) and the equilibrium ownership stake \( \rho^* \) is non-monotone. If \( \gamma \) increases, \( \rho^* \) declines over an initial range of \( \gamma \). As competition intensifies but it is not too severe, a reduction in the ownership stake yields benefits of softer competition that dominate the associated informational costs. However, above a certain threshold the pattern is reversed and a higher \( \gamma \) translates into a higher \( \rho^* \). This is because competition is relatively fierce and the two supply hierarchies are less willing to coordinate on high prices. At the extreme, when goods are close substitutes and competition tends to be perfect, the equilibrium organizational structure approaches the outcome of full integration (if \( \gamma \to \beta \), then \( \alpha \to 0 \) and \( \rho^* \to 1 \)). Since perfect competition strengthens undercutting incentives and prices reflect marginal costs, \( M_1 \) cannot induce any accommodating behavior of the rival.

Note from (23) that a higher spread of retail cost distribution \( \Delta \theta \) increases the equilibrium ownership stake. A more severe problem of asymmetric information exacerbates the increase

\[\text{Note that } \alpha - (\beta - \gamma) \theta_h \geq 0 \text{ ensures non-negative quantities under full information, which implies that the ratio in (23) is non-negative.}\]
in the price of the inefficient retailer and induces $M_1$ to mitigate the price distortion through a larger degree of vertical integration.

The result in Proposition 3 that the ownership stake of $M_1$ in $R_1$ is lower than 100% holds whenever demands are interdependent ($\gamma \neq 0$). Hence, partial vertical integration can emerge even with complementary goods ($\gamma < 0$). As (21) and (22) indicate, a higher price of the inefficient retailer $R_1$ arising from a lower ownership stake than under full integration translates into a lower price for the complementary good provided by $M_2 - R_2$, since prices are strategic substitutes. This stimulates the output of $M_1 - R_1$ and improves its profits.

7. Robustness

We discuss some assumptions of the model in order to gain insights into the robustness of the results.

7.1. Derivation of the equilibrium value for the ownership stake

Following the main literature, we derive the ownership stake of $M_1$ in $R_1$ from the joint profit maximization problem. As discussed in Section 3, this ensures that $M_1$ can design an offer to $R_1$ that makes the shareholders in both firms better off, so that they will find it mutually profitable to accept this offer. We now examine the case in which $M_1$, instead of caring about joint profits, chooses $\rho$ to maximize its profits in (2). Using (2) and (7), the equilibrium value for the ownership stake $\rho$ solves
max \begin{array}{c}
\nu \{ p^*_{11} (\rho) E_{\theta_2} (q_1 (p^*_{11} (\rho), p^*_2 (\rho)) | \theta_1 ) - \theta_1 E_{\theta_2} (q_1 (p^*_{11} (\rho), p^*_2 (\rho)) | \theta_1 ) \\
-(1-\rho) [ \Delta \theta E_{\theta_2} (q_1 (p^*_{1h} (\rho), p^*_2 (\rho)) | \theta_h ) - (p^*_{1h} (\rho) - \theta_1) \\
\times (E_{\theta_2} (q_1 (p^*_{1h} (\rho), p^*_2 (\rho)) | \theta_h ) - E_{\theta_2} (q_1 (p^*_{1h} (\rho), p^*_2 (\rho)) | \theta_h ))] \\
+(1-\nu) [ p^*_{1h} (\rho) E_{\theta_2} (q_1 (p^*_{1h} (\rho), p^*_2 (\rho)) | \theta_h ) - \theta_h E_{\theta_2} (q_1 (p^*_{1h} (\rho), p^*_2 (\rho)) | \theta_h )] \end{array}.

This yields the following result.

**Proposition 4** Suppose that manufacturer $M_1$ chooses the ownership stake $\rho$ in retailer $R_1$ to maximize the profits in (2) rather than joint profits. If the probability of the efficient retailer $\nu$ is relatively low and consumer demands are interdependent ($\frac{\partial q_i}{\partial p_{-i}} > 0$), then partial vertical integration is more profitable for $M_1$ than full vertical integration, and the equilibrium ownership stake that $M_1$ holds in $R_1$ is $\rho^* < 1$.

Proposition 4 shows that, under certain circumstances, partial integration is more profitable than full integration and can still arise in equilibrium even though it is not the result of joint profit maximization. Specifically, sufficient (but not necessary) condition for $M_1$ to acquire less than full ownership in $R_1$ is that the efficient retailer is relatively unlikely and therefore its (expected) informational rents are not too costly. Hence, the strategic incentives to partially integrate are weaker than under joint profit maximization. The rationale for this result is that a manufacturer which acquires an ownership stake in its retailer to maximize its own profits internalizes not only the allocative costs from asymmetric information but also the distributional costs arising from the inability to fully appropriate the profits of the retailer. The informational costs of partial integration are higher than under joint profit maximization and therefore full integration becomes more attractive.

### 7.2. Fully integrated competitor

Another assumption that deserves further discussion is that manufacturer $M_1$ faces the fully integrated competitor $M_2 - R_2$ when deciding on the degree of vertical integration. Notably, this modeling choice allows the investigation of the unilateral incentive to partially integrate in a setting biased in favor of the decision of full integration. The fully integrated hierarchy $M_2 - R_2$ does not suffer any negative externality from asymmetric information and can only
benefit from $M_1 - R_1$’s higher retail price. Nevertheless, $M_1 - R_1$ still prefers to partially integrate and commit to a higher retail price than under full integration. It follows that, if $M_2 - R_2$ could choose the degree of vertical integration, it would be also inclined to partially integrate, which strengthens the incentive of $M_1 - R_1$ for partial integration and further relaxes competition. As a consequence, a more symmetric framework in which both supply hierarchies are allowed to determine the ownership stake in their retailers reinforces the outcome of partial vertical integration.

7.3. Resale price maintenance

The contract that manufacturer $M_1$ offers to the retailer $R_1$ directly specifies the retail price, which is known as resale price maintenance. Even though this type of vertical arrangements is sometimes viewed with skepticism by the antitrust authorities, some countries (e.g., New Zealand) traditionally allow this practice if the beneficial effects can be shown to outweigh the anticompetitive harm. Remarkably, in the 2007 case “Leegin Creative Leather Products, Inc., vs. PSKS, Inc.” the US Supreme Court replaced the well-established doctrine of per se unlawfulness of resale price maintenance with a rule of reason that allows a firm to produce evidence that an individual resale price maintenance agreement is justified.\(^{20}\)

Our qualitative results do not depend on the use of resale price maintenance agreements. To fix ideas, consider a contract that stipulates a two-part tariff specifying a unit wholesale price and a fixed fee the retailer pays to the manufacturer. As shown in Gal-Or (1991c), under asymmetric information a manufacturer increases the unit price for the inefficient retailer in order to reduce the informational rents to the efficient retailer, which translates into a higher retail price. In our setting the magnitude of the upward price distortion depends on the ownership stake acquired by the manufacturer in its retailer. Partial vertical integration still induces a trade-off between the benefits of softer competition and the informational costs.

8. Antitrust policy implications

In line with the main theoretical literature, most empirical research on vertical integration (exhaustively surveyed in Lafontaine and Slade 2007) has focused on the binary choice between separation and integration. However, as documented in some relevant empirical works (e.g., Allen and Phillips 2000; Fee et al. 2006; Reifen 1998), partial vertical ownership is a common

\(^{20}\)For some empirical evidence on resale price maintenance in Europe, we refer to Bonnet and Dubois (2010).
phenomenon. In particular, consistently with our results, Ouimet (2013) finds that partial equity stakes tend to be preferred to full integration in industries which require relationship-specific investments, such as vertically related markets. The predictions of our model about the impact of partial vertical integration on competition lend themselves to empirically testable validations.

As discussed in the introduction, the harmful effects of partial acquisitions between competitors are comprehensively addressed in the US antitrust law. In other countries, such as Austria, Germany, the UK, Australia, Canada, Japan and New Zealand, antitrust authorities are also entitled to scrutinize acquisitions of partial ownership. However, the European Commission does not have any explicit competence in this area under the current merger control rules.

Partial acquisitions have recently received great attention in merger control. The US Horizontal Merger Guidelines have recognized since 2010 the relevance of partial acquisitions. In Europe, current proposals aim at expanding the remit of the merger control function to enable the European Commission to examine partial acquisitions.\textsuperscript{21} Our results suggest that antitrust investigations should be extended to partial ownership agreements between vertically related firms. Antitrust authorities could block partial ownership agreements in the light of their anticompetitive effects if they anticipate that the firms would prefer full merger to separation. Along the same lines, takeover regulations could be implemented, which facilitate full acquisitions relative to partial equity holdings. A well-known caveat is that full integration might entail anticompetitive behavior in the form of rivals’ foreclosure. Interestingly enough, Gilo et al. (2014) show that under certain conditions partial integration is more likely to lead to input foreclosure than full integration. Therefore, our conclusions provide further corroboration for the anticompetitive effects of partial integration.

Our model predicts that partial ownership agreements for strategic purposes will typically emerge when firms compete in prices. This mode of competition can naturally arise in the sectors with relationship-specific investments (such as vertically related markets) investigated in Ouimet (2013), where the number of patents is a proxy for the relevance of these investments. Conversely, we do not generally expect any strategic partial ownership when severe capacity constraints induce quantity competition. This is because the partially integrated hierarchy’s output reduction to curb informational rents triggers a more aggressive behavior of the rivals.

\textsuperscript{21}The interested reader is referred to the consultation document “Towards more effective EU merger control” issued by the European Commission in 2013, which is available at http://ec.europa.eu/competition/consultations/2013_merger_control/merger_control_en.pdf.
9. Concluding remarks

In this paper we investigate the strategic incentives for partial ownership agreements in vertically related markets where two manufacturer-retailer pairs engage in differentiated good price competition and the retailers are privately informed about their production costs. A partial ownership stake of a manufacturer in its retailer introduces a misalignment between the profit objectives of the two firms and entails an upward price distortion for the inefficient retailer in order to reduce the (costly) informational rents to the efficient retailer. This information vertical effect generates a form of double marginalization from asymmetric information that reduces the supply hierarchy’s profitability for a given price of the rival. In the presence of competition, this information vertical effect translates into an opposite competition horizontal effect. The partially integrated hierarchy’s commitment to a higher price than under full integration induces the rival to increase its price as well, which relieves competition and makes partial integration profitable. The equilibrium degree of vertical integration trades off the benefits of softer competition against the informational costs.

Our analysis provides theoretical support for the empirical evidence on partial vertical integration and formulates antitrust policy recommendations for mergers and acquisitions in vertically related markets.

Acknowledgments  I thank Helmut Bester, Giacomo Calzolari, Matthias Dahm, Vincenzo Denicolò, Mario Gilli, Martin Peitz, Klaus M. Schmidt, Nicolas Schutz, Giancarlo Spagnolo, Konrad Stahl and Roland Strausz for valuable comments and suggestions. I also thank the participants at the SFB-TR15 Conference 2013 in Munich, the MaCCI Competition and Regulation Day Workshop 2013 in Mannheim, the EARIE Conference 2014 in Milan, the UECE Conference 2014 in Lisbon and the EIEF-UNIBO-IGIER Bocconi Workshop on Industrial Organization 2014 in Bologna.

Appendix

This Appendix collects the proofs.

Proof of Lemma 1. Substituting $t_1$ with $\pi_{R_1}$ from (1), $M_1$’s problem of maximizing (2) becomes

$$\max_{p_1, \pi_{R_1}} p_1 E_{\theta_2} (q_1 (p_1, p_2) | \theta_1) - \theta_1 E_{\theta_2} (q_1 (p_1, p_2) | \theta_1) - (1 - \rho) \pi_{R_1} \quad s.t. \quad \pi_{R_1} \geq 0,$$
where the constraint ensures the participation of $R_1$ in the contractual relationship with $M_1$.

Since the maximand decreases with $\pi_{R_1}$ for any $\rho \in [0, 1]$, we find $\pi_{R_1} = 0$ in equilibrium.\footnote{Indeed, $\pi_{R_1}$ vanishes for $\rho = 1$ and any $\pi_{R_1} \geq 0$ can be supported in equilibrium. However, this does not affect the first-order condition for $p_1$.}

Taking the first-order condition for $p_1$ yields $E_{\theta_2} (q_1 (p_1, p_2) \mid \theta_1) + (p_1 - \theta_1) \frac{\partial E_{\theta_2} (q_1 (p_1, p_2) \mid \theta_1)}{\partial p_1} = 0$.

Using (3), the problem of $M_2 - R_2$ writes as

$$\max_{p_2} p_2 E_{\theta_1} (q_2 (p_1, p_2) \mid \theta_2) - \theta_2 E_{\theta_1} (q_2 (p_1, p_2) \mid \theta_2),$$

which entails from the first-order condition $E_{\theta_1} (q_2 (p_1, p_2) \mid \theta_2) + (p_2 - \theta_2) \frac{\partial E_{\theta_1} (q_2 (p_1, p_2) \mid \theta_2)}{\partial p_2} = 0$.

Solving the system of the first-order conditions for $M_1$ and $M_2 - R_2$ yields the expression in (4).

**Proof of Proposition 1**. The results in the proposition immediately follow from the first-order conditions for $p_{hl}$ and $p_{lh}$ in the maximization problem in (8), and from the first-order condition for $p_2$ in the maximization problem in (9). We now show that the incentive constraint (6) is satisfied in equilibrium. Manipulating terms in (6) yields

$$\pi_{R_{1h}} = p_{1h} E_{\theta_2} (q_1 (p_{1h}, p_2) \mid \theta_h) - \theta_h E_{\theta_2} (q_1 (p_{1h}, p_2) \mid \theta_h) - t_{1h}$$

$$\geq p_{1l} E_{\theta_2} (q_1 (p_{1l}, p_2) \mid \theta_h) - \theta_h E_{\theta_2} (q_1 (p_{1l}, p_2) \mid \theta_h) - t_{1l}$$

$$= \pi_{R_{1l}} + p_{1l} [E_{\theta_2} (q_1 (p_{1l}, p_2) \mid \theta_h) - E_{\theta_2} (q_1 (p_{1l}, p_2) \mid \theta_l)]$$

$$+ \theta_l E_{\theta_2} (q_1 (p_{1l}, p_2) \mid \theta_l) - \theta_h E_{\theta_2} (q_1 (p_{1l}, p_2) \mid \theta_h)$$

$$= \pi_{R_{1l}} - \Delta \theta E_{\theta_2} (q_1 (p_{1l}, p_2) \mid \theta_l) + (p_{1l} - \theta_h)$$

$$\times [E_{\theta_2} (q_1 (p_{1l}, p_2) \mid \theta_h) - E_{\theta_2} (q_1 (p_{1l}, p_2) \mid \theta_l)].$$

Using (7) evaluated in equilibrium, we obtain

$$0 \geq \Delta \theta [E_{\theta_2} (q_1 (p_{1h}^*, p_2^*) \mid \theta_h) - E_{\theta_2} (q_1 (p_{1l}^*, p_2^*) \mid \theta_l)] - (p_{1h}^* - \theta_h) [E_{\theta_2} (q_1 (p_{1h}^*, p_2^*) \mid \theta_h) - E_{\theta_2} (q_1 (p_{1l}^*, p_2^*) \mid \theta_l)].$$

Since we have from Taylor’s expansion

$$E_{\theta_2} (q_1 (p_{1h}^*, p_2^*) \mid \theta_h) \approx E_{\theta_2} (q_1 (p_{1l}^*, p_2^*) \mid \theta_k) + \frac{\partial E_{\theta_2} (q_1 (p_{1l}^*, p_2^*) \mid \theta_k)}{\partial p_1} (p_{1h}^* - p_{1l}^*), \; k = l, h,$$
this condition can be rewritten as

\[ 0 \geq -E_{\theta_2} (q_1 (p_{1l}^*, p_{2}^*) | \theta_h) (p_{1l}^* - p_{1l}^*) - \frac{\partial E_{\theta_2} (q_1 (p_{1l}^*, p_{2}^*) | \theta_h)}{\partial p_{1l}} (p_{1l}^* - \theta_l) (p_{1l}^* - p_{1l}^*) + E_{\theta_2} (q_1 (p_{1l}^*, p_{2}^*) | \theta_h) (p_{1l}^* - p_{1l}^*) + \frac{\partial E_{\theta_2} (q_1 (p_{1l}^*, p_{2}^*) | \theta_l)}{\partial p_{1l}} (p_{1l}^* - \theta_l) (p_{1l}^* - p_{1l}^*) = - (p_{1l}^* - p_{1l}^*) [E_{\theta_2} (q_1 (p_{1l}^*, p_{2}^*) | \theta_h) - E_{\theta_2} (q_1 (p_{1l}^*, p_{2}^*) | \theta_l) + \frac{\partial E_{\theta_2} (q_1 (p_{1l}^*, p_{2}^*) | \theta_h)}{\partial p_{1l}} (p_{1l}^* - \theta_h) - \frac{\partial E_{\theta_2} (q_1 (p_{1l}^*, p_{2}^*) | \theta_l)}{\partial p_{1l}} (p_{1l}^* - \theta_l)] \]

The expression in square brackets is (strictly) positive, since \( \frac{\partial q_1}{\partial p_{1l}} \geq 0 \), \( \frac{\partial^2 q_1}{\partial p_{1l} \partial p_{2l}} \geq 0 \) and costs are positively correlated. As \( p_{1l}^* - p_{1l}^* > 0 \), the constraint (6) is satisfied in equilibrium. Finally, we check that the participation constraint \( \pi_{R_{1l}} \geq 0 \) is also satisfied in equilibrium. Using the incentive constraint (7), sufficient (but not necessary) condition for this to be the case is that either the degree of cost correlation or the level substitutability is not too high. ■

**Proof of Lemma 2.** Denoting by \( z \) the left-hand side of (11), the implicit function theorem yields \( \frac{\partial p_{1l}^*}{\partial p} = - \frac{\partial z (p_{1l}^*, \rho)}{\partial (p_{1l}^*, \rho) / \partial p_{1l}} \). Standard computations entail

\[ \frac{\partial z (p_{1l}^*, \rho)}{\partial p} = E_{\theta_2} \left( \frac{\partial q_1 (p_{1l}^*, p_{2}^*)}{\partial p_{1l}} | \theta_h \right) + (p_{1l}^* - \theta_l) E_{\theta_2} \left( \frac{\partial^2 q_1 (p_{1l}^*, p_{2}^*)}{\partial p_{1l} \partial p_{2l}} | \theta_l \right) < 0, \]

where the inequality follows from the assumptions on the parameters of the model, \( p_{1l}^* - \theta_l > 0 \), and \( \frac{\partial q_1}{\partial p_{1l}} < 0 \) (see below). Since \( \frac{\partial z (p_{1l}^*, \rho)}{\partial p_{1l}} < 0 \) (second-order condition for \( p_{1l}^* \)), it follows that \( \frac{\partial p_{1l}^*}{\partial p} < 0 \).

Denoting by \( f \) the left-hand side of (12), we have \( \frac{\partial p_{1l}^*}{\partial p} = - \frac{\partial f (p_{1l}^*, \rho)}{\partial f (p_{1l}^*, \rho) / \partial p_{1l}} < 0 \), where the inequality follows from \( \frac{\partial f (p_{1l}^*, \rho)}{\partial p_{1l}} < 0 \) (\( \Phi \) defined in (10) is negative) and \( \frac{\partial f (p_{1l}^*, \rho)}{\partial p_{1l}} < 0 \) (second-order condition for \( p_{1l}^* \)).

Denoting by \( g \) the left-hand side of (13), we have \( \frac{\partial g (p_{1l}^*, \rho)}{\partial p} = - \frac{\partial g (p_{1l}^*, \rho)}{\partial g (p_{1l}^*, \rho) / \partial p_{1l}} \). Then,

\[ \frac{\partial g (p_{1l}^*, \rho)}{\partial p} = E_{\theta_1} \left( \frac{\partial q_2 (p_{1l}^*, p_{2}^*)}{\partial p_{1l}} | \theta_2 \right) + (p_{2}^* - \theta_2) E_{\theta_1} \left( \frac{\partial^2 q_2 (p_{1l}^*, p_{2}^*)}{\partial p_{1l} \partial p_{2l}} | \theta_2 \right) < 0, \]

where the inequality follows from the assumptions on the parameters of the model, \( p_{2}^* - \theta_2 > 0 \) and, in addition, \( \frac{\partial q_2}{\partial p_{1l}} = 0 \) and \( \frac{\partial q_2}{\partial p_{2l}} < 0 \) for a given \( p_{2}^* \). Since \( \frac{\partial g (p_{1l}^*, \rho)}{\partial p_{2l}} < 0 \) (second-order condition for \( p_{2}^* \)), it follows that \( \frac{\partial p_{2l}^*}{\partial p} < 0 \). ■

**Proof of Proposition 2.** Differentiating the objective function (14) with respect to the
ownership stake \( \rho \) yields

\[

\nu \left[ \frac{\partial \rho_{t1}}{\partial \rho} E_{\theta_2} \left( q_1 (p_{t1}^* (\rho), p_2^* (\rho)) | \theta_t \right) + (p_{t1}^* (\rho) - \theta_t) \frac{\partial E_{\theta_2} \left( q_1 (p_{t1}^* (\rho), p_2^* (\rho)) | \theta_t \right)}{\partial \rho} \right] + (1 - \nu) \\
\times \left[ \frac{\partial p_{t1}^*}{\partial \rho} E_{\theta_2} \left( q_1 (p_{t1h}^* (\rho), p_2^* (\rho)) | \theta_h \right) + (p_{t1h}^* (\rho) - \theta_h) \frac{\partial E_{\theta_2} \left( q_1 (p_{t1h}^* (\rho), p_2^* (\rho)) | \theta_h \right)}{\partial \rho} \right].
\]

Applying the chain rule yields

\[

\frac{\partial E_{\theta_2} \left( q_1 (p_{t1}^* (\rho), p_2^* (\rho)) | \theta_k \right)}{\partial \rho} = \frac{\partial p_{t1k}^*}{\partial \rho} \frac{\partial E_{\theta_2} \left( q_1 (p_{t1}^* (\rho), p_2^* (\rho)) | \theta_k \right)}{\partial p_{t1k}} + E_{\theta_2} \left( \frac{\partial q_1 (p_{t1}^* (\rho), p_2^* (\rho))}{\partial p_{t2}} \frac{\partial p_{t2}^*}{\partial \rho} | \theta_k \right), \quad k = l, h.
\]

(25)

Using (25), we find after some manipulation

\[

\nu \left\{ \frac{\partial \rho_{t1}}{\partial \rho} \left[ E_{\theta_2} \left( q_1 (p_{t1}^* (\rho), p_2^* (\rho)) | \theta_t \right) + (p_{t1}^* (\rho) - \theta_t) \frac{\partial E_{\theta_2} \left( q_1 (p_{t1}^* (\rho), p_2^* (\rho)) | \theta_t \right)}{\partial \rho} \right] \\
+ (p_{t1}^* (\rho) - \theta_t) E_{\theta_2} \left( \frac{\partial q_1 (p_{t1}^* (\rho), p_2^* (\rho))}{\partial p_{t2}} \frac{\partial p_{t2}^*}{\partial \rho} | \theta_t \right) \right\} + (1 - \nu) \\
\times \left\{ \frac{\partial p_{t1h}^*}{\partial \rho} \left[ E_{\theta_2} \left( q_1 (p_{t1h}^* (\rho), p_2^* (\rho)) | \theta_h \right) + (p_{t1h}^* (\rho) - \theta_h) \frac{\partial E_{\theta_2} \left( q_1 (p_{t1h}^* (\rho), p_2^* (\rho)) | \theta_h \right)}{\partial \rho} \right] \\
+ (p_{t1h}^* (\rho) - \theta_h) E_{\theta_2} \left( \frac{\partial q_1 (p_{t1h}^* (\rho), p_2^* (\rho))}{\partial p_{t2}} \frac{\partial p_{t2}^*}{\partial \rho} | \theta_h \right) \right\}.
\]

(26)

Note from (11) that the expression in the first line of (26) is zero, while the expressions in the second and fourth line are negative as \( p_{t1}^* - \theta_t > 0, p_{t1h}^* - \theta_h > 0, \frac{\partial q_1}{\partial p_{t2}} > 0, \) and \( \frac{\partial p_{t2}^*}{\partial \rho} < 0 \) (see Lemma 2). We find from (12) that the expression in square brackets in the third line is zero at \( \rho = 1 \), which implies that \( \rho < 1 \) is optimal. If \( \frac{\partial q_1}{\partial p_{t2}} = 0 \) (independent consumer demands), the first-order condition (26) is zero for \( \rho = 1 \), which is optimal (given that the second-order conditions hold).

\textbf{Proof of Lemma 3.} Using (1), \( M_1 \)’s problem of maximizing (2) can be written as

\[

\max_{p_{t1k}, \pi_{R_{1k}}} p_{t1k} (\alpha - \beta p_{t1k} + \gamma p_{2k}) - \theta_k (\alpha - \beta p_{t1k} + \gamma p_{2k}) - (1 - \rho) \pi_{R_{1k}} \quad \text{s.t.} \quad \pi_{R_{1k}} \geq 0, \quad k = l, h,
\]

where the constraint ensures that \( R_1 \) (whose costs are \( \theta_l \) or \( \theta_h \)) is willing to participate in the contractual relationship with \( M_1 \). Since the maximand decreases with \( \pi_{R_{1k}} \) for any \( \rho \in [0, 1] \), we have \( \pi_{R_{1k}} = 0 \) in equilibrium. Taking the first-order condition for \( p_{t1k} \) yields \( \alpha - 2\beta p_{t1k} + \gamma p_{2k} + \beta \theta_k = 0. \)
Using (3), the problem of $M_2 - R_2$ is

$$
\max_{p_{2k}} p_{2k} (\alpha - \beta p_{2k} + \gamma p_{1k}) - \theta_k (\alpha - \beta p_{2k} + \gamma p_{1k}) , \ k = l, h,
$$

which yields $\alpha - 2\beta p_{2k} + \gamma p_{1k} + \beta \theta_k = 0$. The system of the first-order conditions for $M_1$ and $M_2 - R_2$ yields the expression in (16). ■

**Proof of Lemma 4.** Replacing (15) into (5) and (6), the incentive constraints can be written as

$$\begin{align*}
\pi_{R_{1l}} &= p_{1l} (\alpha - \beta p_{1l} + \gamma p_{2l}) - \theta_l (\alpha - \beta p_{1l} + \gamma p_{2l}) - t_{1l} \\
&\geq p_{1h} (\alpha - \beta p_{1h} + \gamma p_{2l}) - \theta_l (\alpha - \beta p_{1h} + \gamma p_{2l}) - t_{1h} \\
&= \pi_{R_{1h}} + \gamma p_{1h} (p_{2l} - p_{2h}) + \theta_h (\alpha - \beta p_{1h} + \gamma p_{2h}) - \theta_l (\alpha - \beta p_{1h} + \gamma p_{2l}) \\
&= \pi_{R_{1h}} + \Delta \theta (\alpha - \beta p_{1h} + \gamma p_{2l}) - \gamma (p_{2h} - p_{2l}) (p_{1h} - \theta_l) \tag{27}
\end{align*}$$

$$\begin{align*}
\pi_{R_{1h}} &= p_{1h} (\alpha - \beta p_{1h} + \gamma p_{2h}) - \theta_h (\alpha - \beta p_{1h} + \gamma p_{2h}) - t_{1h} \\
&\geq p_{1l} (\alpha - \beta p_{1l} + \gamma p_{2h}) - \theta_h (\alpha - \beta p_{1l} + \gamma p_{2h}) - t_{1l} \\
&= \pi_{R_{1l}} + \gamma p_{1l} (p_{2h} - p_{2l}) - \theta_h (\alpha - \beta p_{1l} + \gamma p_{2h}) + \theta_l (\alpha - \beta p_{1l} + \gamma p_{2l}) \\
&= \pi_{R_{1l}} - \Delta \theta (\alpha - \beta p_{1l} + \gamma p_{2l}) + \gamma (p_{2h} - p_{2l}) (p_{1l} - \theta_h). \tag{28}
\end{align*}$$

Standard arguments imply that the participation constraint $\pi_{R_{1h}} \geq 0$ for the inefficient retailer and the incentive constraint (27) for the efficient retailer are binding at the optimal contract. Substituting them in (2) and using (15), $M_1$’s maximization problem becomes

$$\begin{align*}
\max_{p_{1l}, p_{1h}} &\nu \left\{ p_{1l} (\alpha - \beta p_{1l} + \gamma p_{2l}) - \theta_l (\alpha - \beta p_{1l} + \gamma p_{2l}) - (1 - \rho) |\Delta \theta (\alpha - \beta p_{1h} + \gamma p_{2l}) \\
&- \gamma (p_{2h} - p_{2l}) (p_{1h} - \theta_l) | \right\} + (1 - \nu) \left\{ p_{1h} (\alpha - \beta p_{1h} + \gamma p_{2h}) - \theta_h (\alpha - \beta p_{1h} + \gamma p_{2h}) \right\}.
\end{align*}$$

The first-order conditions for $p_{1l}$ and $p_{1h}$ are respectively $\alpha - 2\beta p_{1l} + \gamma p_{2l} + \beta \theta_l = 0$ and $\alpha - 2\beta p_{1h} + \gamma p_{2h} + \beta \theta_h + \phi (\nu) (1 - \rho) |\beta \Delta \theta + \gamma (p_{2h} - p_{2l}) | = 0$.

Substituting (15) into (3), the maximization problem of $M_2 - R_2$ becomes

$$\begin{align*}
\max_{p_{2k}} &p_{2k} (\alpha - \beta p_{2k} + \gamma p_{1k}) - \theta_k (\alpha - \beta p_{2k} + \gamma p_{1k}) , \ k = l, h,
\end{align*}$$

27
which yields $\alpha - 2\beta p_{2k} + \gamma p_{1k} + \beta \theta_k = 0$. The first-order conditions for the maximization problems of $M_1$ and $M_2 - R_2$ yield the results in the proposition.

We now check that the two omitted constraints in $M_1$’s problem are satisfied in equilibrium. Substituting the binding constraint (27) into (28) yields after some manipulation $0 \geq - (p_{1h}^* - p_{1l}^*) [\beta \Delta \theta + \gamma (p_{2h}^* - p_{2l}^*)]$, which is fulfilled since $p_{1h}^* - p_{1l}^* > 0$ and $p_{2h}^* - p_{2l}^* > 0$. Moreover, the binding constraint (27) implies that sufficient (but not necessary) condition for the participation constraint $\pi_{R_{il}} \geq 0$ to be satisfied is that $\gamma$ is not too high. ■

**Proof of Proposition 3.** The maximization problem of $M_1$ is

$$\max_{\rho \in [0,1]} \nu \left[ p_{1l}^* (\alpha - \beta p_{1l}^* + \gamma p_{2l}^*) - \theta_l (\alpha - \beta p_{1l}^* + \gamma p_{2l}^*) \right]$$

$$+ (1 - \nu) \left[ p_{1h}^* (\alpha - \beta p_{1h}^* + \gamma p_{2h}^*) - \theta_h (\alpha - \beta p_{1h}^* + \gamma p_{2h}^*) \right].$$

Using the results in Lemma 4, the first-order condition for $\rho$ can be written as

$$\left\{ -\phi (\nu) \left[ 4 \beta^3 \Delta \theta - \gamma^2 (\alpha + \beta \theta_h) \right] \left[ 4 \beta^2 - \gamma^2 (1 + \phi (\nu) (1 - \rho)) \right] \right. \right.$$

$$\left( 2 \beta - \gamma \right) \left[ 4 \beta^2 - \gamma^2 (1 + \phi (\nu) (1 - \rho)) \right]^2$$

$$\left. - \gamma^2 \phi (\nu) \left[ (\alpha + \beta \theta_h) \left( 4 \beta^2 - \gamma^2 \right) + \phi (\nu) (1 - \rho) \right] \left[ 4 \beta^3 \Delta \theta - \gamma^2 (\alpha + \beta \theta_h) \right] \right\} \left( 2 \beta - \gamma \right) \left[ 4 \beta^2 - \gamma^2 (1 + \phi (\nu) (1 - \rho)) \right]^2$$

$$\left. + \left\{ \alpha - \frac{(\beta - \gamma) \left( 4 \beta^2 - \gamma^2 \right) (\alpha + \beta \theta_h)}{(2 \beta - \gamma) \left[ 4 \beta^2 - \gamma^2 (1 + \phi (\nu) (1 - \rho)) \right]} - \phi (\nu) (1 - \rho) \right. \right.$$

$$\left. \times 2 \beta^2 \left( 2 \beta^2 - \gamma^2 \right) \Delta \theta - \gamma^2 (\alpha + \beta \theta_h) (\beta - \gamma) \right\} \left( 2 \beta - \gamma \right) \left[ 4 \beta^2 - \gamma^2 (1 + \phi (\nu) (1 - \rho)) \right] \left. \right.$$

$$\left. + \left\{ (\alpha + \beta \theta_h) \left( 4 \beta^2 - \gamma^2 \right) + \phi (\nu) (1 - \rho) \right\left[ 4 \beta^3 \Delta \theta - \gamma^2 (\alpha + \beta \theta_h) \right] - \theta_h \right\} \phi (\nu) \left( 2 \beta^2 \left( 2 \beta^2 - \gamma^2 \right) \Delta \theta - \gamma^2 (\alpha + \beta \theta_h) (\beta - \gamma) \right) \right.$$}

$$\left( 2 \beta - \gamma \right) \left[ 4 \beta^2 - \gamma^2 (1 + \phi (\nu) (1 - \rho)) \right]^2$$

$$\left. + \gamma^2 \phi (\nu) \left[ (\beta - \gamma) \left( 4 \beta^2 - \gamma^2 \right) (\alpha + \beta \theta_h) \right. \right.$$

$$\left( 2 \beta - \gamma \right) \left[ 4 \beta^2 - \gamma^2 (1 + \phi (\nu) (1 - \rho)) \right]^2$$

$$\left. + \phi (\nu) (1 - \rho) \right\} \left. \times 2 \beta^2 \left( 2 \beta^2 - \gamma^2 \right) \Delta \theta - \gamma^2 (\alpha + \beta \theta_h) (\beta - \gamma) \right\} \left( 2 \beta - \gamma \right) \left[ 4 \beta^2 - \gamma^2 (1 + \phi (\nu) (1 - \rho)) \right]^2$$

$$\left. \right] = 0.$$  

Combining terms implies

$$\phi (\nu) (1 - \rho) \left( 4 \beta^2 - \gamma^2 \right) \left\{ 4 \alpha \beta^3 \gamma^2 (2 \beta - \gamma) \Delta \theta + \left[ 2 \beta^2 \left( 2 \beta^2 - \gamma^2 \right) \Delta \theta - \gamma^2 (\alpha + \beta \theta_h) (\beta - \gamma) \right] \right. \right.$$

$$\left. \left. \times \left( 4 \beta^3 \Delta \theta - \gamma^2 (\alpha + \beta \theta_h) \right) + \gamma^2 \left[ 2 \beta^2 \left( 2 \beta^2 - \gamma^2 \right) \Delta \theta - \gamma^2 (\alpha + \beta \theta_h) (\beta - \gamma) \right] (\alpha + \beta \theta_h) \right) \right. \right.$$

$$\left. \right\} \left( 2 \beta - \gamma \right) \left[ 4 \beta^2 - \gamma^2 (1 + \phi (\nu) (1 - \rho)) \right]^2$$
+2\beta^2 (2\beta^2 - \gamma^2) \left[ 4\beta^3 \Delta \theta - \gamma^2 (\alpha + \beta \theta_h) \right] \Delta \theta + 2\beta^2 \gamma^2 (2\beta - \gamma) (2\beta^2 - \gamma^2) \Delta \theta \theta_h \right) \\
+ (4\beta^2 - \gamma^2)^2 \left[-4\alpha \beta^3 (2\beta - \gamma) \Delta \theta + 4\beta^3 (\beta - \gamma) (\alpha + \beta \theta_h) \Delta \theta \right. \\
+2\beta^2 (2\beta^2 - \gamma^2) (\alpha + \beta \theta_h) \Delta \theta - 2\beta^2 (2\beta - \gamma) (2\beta^2 - \gamma^2) \Delta \theta \theta_h \right] = 0,

which gives after further manipulation \( \phi (\nu) (1 - \rho) \left[ 8\beta^3 (2\beta^2 - \gamma^2) \Delta \theta + \gamma^4 (\alpha - (\beta - \gamma) \theta_h) \right] - \\
\gamma^2 (4\beta^2 - \gamma^2) (\alpha - (\beta - \gamma) \theta_h) = 0. \) This yields the optimal ownership stake in (23). Points (i) and (ii) of the proposition follow from standard computations. ■

**Proof of Proposition 4.** Differentiating (24) with respect to \( \rho \) yields

\[
\nu \left\{ \frac{\partial p^*_{1l}}{\partial \rho} \left[ E_{\theta_2} \left( q_1 \left( p^*_{1l} (\rho) , p^*_{2} (\rho) \right) | \theta_l \right) + (p^*_{1l} (\rho) - \theta_l) \frac{\partial E_{\theta_2} \left( q_1 \left( p^*_{1l} (\rho) , p^*_{2} (\rho) \right) | \theta_l \right)}{\partial p_{1}} \right] \right. \\
+ (p^*_{1l} (\rho) - \theta_l) E_{\theta_2} \left( \frac{\partial q_1 \left( p^*_{1l} (\rho) , p^*_{2} (\rho) \right) }{\partial p_{2}} \frac{\partial p^*_{2}}{\partial \rho} | \theta_l \right) \right\} + (1 - \nu) \\
\times \left\{ \frac{\partial p^*_{1h}}{\partial \rho} \left[ E_{\theta_2} \left( q_1 \left( p^*_{1h} (\rho) , p^*_{2} (\rho) \right) | \theta_h \right) + (p^*_{1h} (\rho) - \theta_h) \frac{\partial E_{\theta_2} \left( q_1 \left( p^*_{1h} (\rho) , p^*_{2} (\rho) \right) | \theta_h \right)}{\partial p_{1}} \right] \right. \\
+ (p^*_{1h} (\rho) - \theta_h) E_{\theta_2} \left( \frac{\partial q_1 \left( p^*_{1h} (\rho) , p^*_{2} (\rho) \right) }{\partial p_{2}} \frac{\partial p^*_{2}}{\partial \rho} | \theta_h \right) \left. \right\} + \nu \pi_{R_{1l}} - \nu (1 - \rho) \frac{\partial \pi^*_{R_{1l}}}{\partial \rho},
\]

where \( \pi_{R_{1l}}^* \) is defined by (7) evaluated in the competition stage equilibrium. Since for \( \rho = 1 \) the expressions in the two curly brackets are negative (see the proof of Proposition 2), while the last term in the fourth line vanishes, sufficient (but not necessary) condition for the entire expression to be negative and therefore \( \rho < 1 \) be optimal is that \( \nu \) is low enough. ■

**References**


