Liquidity Freezes Under Adverse Selection*

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Abstract

This paper analyses how adverse selection prevents liquidity from flowing from liquid to illiquid firms. Such market segmentation impairs the transmission mechanism of monetary policy, and requires specific policies to rebuild the liquidity channels throughout the economy. We show that the optimal policy requires a combination of contingent subsidies to promote ex ante insurance against liquidity shocks, and taxes on investment to alleviate moral hazard problems.

Keywords: liquidity; adverse selection; monetary policy; funding liquidity risk; macro-prudential supervision; financial market freezes

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1 Introduction

The effective implementation of monetary policy requires that the liquidity injected by the central bank flow throughout the economy to those firms which need it. Yet, recent examples are reminders that liquidity conditions can vary substantially across firms:

- During the summer 2007 subprime market crisis, banks faced a liquidity freeze in the asset backed commercial paper market. The freeze began by affecting special purpose vehicles which had issued short term asset backed commercial paper, and ended up putting severe funding stress on sponsoring banks which had provided backup credit lines to their vehicles.
- Interbank markets stopped functioning well after the bankruptcy of Lehman Brothers. The European Central Bank broadened the range of eligible collateral in its open market operations so as to offset the shortage of good collateral. As a result, many banks substituted from the secondary market to the primary one, leading the central bank to effectively replace the interbank market in the allocation of liquidity among banks.
- During the crisis, the Federal Reserve set up an emergency program to buy corporate short term debt with the aim of supporting the orderly functioning of the commercial paper market, as many corporations were no longer able to roll over their maturing commercial paper.
- The European sovereign debt crisis provoked heterogeneous liquidity conditions across the euro area. With the growing fear of a euro area break up, banks in core economies trimmed exposures to members under stress. Liquidity dried up in the periphery of the monetary union, leading local banks to scale back credit and sell their assets.

Such heterogeneity in liquidity conditions is hard to reconcile with the existence of sophisticated financial markets which provide a variety of instruments to insure against liquidity shocks. If firms are able to insure against these shocks, then liquidity will flow freely across the economy and only the aggregate amount of available liquidity should matter.

This is the conclusion of liquidity models such as the framework developed in Holmström and Tirole (2013). To address this limitation, we extend their model by adding adverse selection. In this extended model, we are able to analyze the conditions in which liquidity dries up in financial markets, and compare the effectiveness of liquidity policies.

We find that adverse selection raises the cost of insurance, and our results suggest that firms tend to overinvest and obtain too little insurance against liquidity shocks. Ex post, this allocation leads to the inability to transfer funds among firms and to oversized projects being liquidated. It is optimal to rescue firms which did not obtain insurance, but ex post bailouts create moral hazard as firms do not seek insurance ex ante.

The underlying assumptions of our model are described in Section 2. As in Holmström and Tirole (2013), illiquidity ultimately stems from limits to pledgeable income. Firms have stochastic needs for liquidity, and they want to plan their liquidity in advance so as to avoid credit rationing at a later stage.[1] Firms must choose the size of investment projects to take account of a tradeoff with the level of insurance against liquidity shocks.

We start with the analysis of the optimal liquidity choices of the firm, and obtain two liquidity regimes. In the first regime, which we could associate with normal times,

[1] The Holmström and Tirole (2013) framework has the advantage of not restricting to particular financial contracts, while the standard dynamic stochastic general equilibrium model with financial frictions often relies on standard debt contracts, as for example in Bernanke, Gertler and Gilchrist (1999).

liquidity flows from liquidity-long to liquidity-short firms. In the second regime, which we associate with market segmentation, liquidity does not flow ex post, thus making the shadow value of liquidity different across socially useful projects. Good firms with high liquidity shocks and no insurance are closed down. Reductions in the value of collateral, more serious adverse selection problems, and dearer liquidity make market segmentation more likely.

We then compare firms' choices with the allocation that a planner who could transfer liquidity costlessly would implement. The planner is constrained in the same way as firms to offer contracts where the payment to outside investors cannot be larger than the pledgeable income of the firm. Allocations are different because good entrepreneurs obtain more benefits from choosing higher initial investment and less insurance than the central planner. This analysis is carried out in Section 3 for idiosyncratic shocks and Section 4 for aggregate shocks.

Section 5 considers government policies. Ex post, it is optimal to rescue socially useful firms which did not buy insurance. Yet, ex post bailouts create perverse incentives, as entrepreneurs anticipate interventions and decide to overinvest and refuse to buy insurance. As a result, the insurance market unravels. Still, incentives to overinvest can be mitigated with taxes on investment.

We contrast bailout policies with policies in which public authorities can credibly commit (that they will not deviate to the optimal ex post policies). In our simple environment, it is enough to consider subsidies to firms which suffer high liquidity shocks and taxes on initial investment. Subsidies induce entrepreneurs to get insurance, but also induce them to increase leverage and investment; taxes reduce the perverse incentives for overinvestment. With this type of policies, the private and the public sectors share the burden of financing firms with liquidity needs and, as a result, the amount of subsidies needed to achieve the second-best is lower than in the case of bailout policies.

Although it is convenient to interpret the two policy instruments in our model as taxes and subsidies, it is also interesting to map them into the actual policies that have been implemented in the EU and in the US. These policies include conventional monetary policies—that is, changing the interest rate faced by all agents in the economy—unconventional policies such as relaxing collateral standards, and prudential policies, such as restricting financial leverage. Taxing initial investment and rebating the proceeds to firms ex post turns out to be equivalent to adjusting the intertemporal rate of substitution for those firms in the model. Thus the policies described can be reinterpreted as interest rate policies. Bailouts and the provision official liquidity against collateral which would not be accepted elsewhere can be naturally thought of as targeted subsidies, whereas upfront taxes can be interpreted as a restriction on debt financing. Our framework shows how to coordinate these tools, and under which circumstances these policies are useful or not. In the final section we discuss these interpretations in greater detail.

The segmentation that can arise in our model helps explain why monetary policy can be ineffective when markets are fragmented. Unless individual firms plan their liquidity in advance, raising the amount of aggregate liquidity does not guarantee that liquidity flow to firms with liquidity needs. Ex post, the private sector lacks the means to transfer liquidity, and the shadow value of liquidity will be different across firms as a result of credit rationing. Aggregate liquidity policies must be accompanied by measures which entice entrepreneurs to plan their liquidity in advance, thus rebuilding the channels of liquidity throughout the economy.

Literature review. Krishnamurthy and Vissing-Jorgenssen (2012) identify the demand for US Treasuries, and Krishnamurthy and Vissing-Jorgenssen (2013) use US data to identify the demand for inside liquidity. The financial sector creates most of the liquidity supplied by the private sector, and Krishnamurthy and Vissing-Jorgenssen (2013) document the crowd out of the supply of inside liquidity following increases in the supply of US Treasuries (except for checking accounts, which are often backed by Treasuries).

A number of studies (see, for example, Garcia-Appendini and Montoriol-Garriga 2013, and Kashyap and Stein 2000) document that firms and banks face adverse shocks better when they hold more liquid securities in their balance sheet. However, the definition of liquidity has to be used with caution because liquidity is essentially a form of insurance which may show up in more subtle ways as assets in a balance sheet. Since a firm can meet liquidity needs by borrowing using its projects as collateral, the amount of funding that can be raised ex post is determined by the pledgeable income of the firm, which is hard to measure and often neglected in empirical studies.

Still, funding liquidity can be measured indirectly. Cassola, Hortaçsu and Kastl (2013) use bidding data from the European Central Bank's auctions for one-week loans during the summer 2007 subprime market crisis, and identify a significant number of bidders whose willingness-to-pay for this type of liquidity increased substantially after August 2007, suggesting that liquidity did not flow among banks and that the shadow price of liquidity was different across banks. The European crisis provided an opportunity to analyze the behavior of European global banks in the US. Acharya, Afonso, and Kovner (2012), Correa, Sapriza, and Zlate (2013) and Ivashina, Scharfstein, Stein (2012) document that these banks cut their lending by more than US banks because they relied more on short term funding and had fewer sources of liquidity insurance.

Our contribution is at the intersection of two strands in the theoretical literature: the segmentation of liquidity markets, and the distinction between ex ante and ex post provision of liquidity. Regarding the first strand, our motivation stems from the need to understand why liquidity doesn't flow among firms, and how this segmentation impairs monetary policy.

In the context of relationship banking in which firms have access to funds through a unique bank, Freixas and Jorge (2008) distinguish the pledgeable income of the firm from the pledgeable income of the bank. As a result of private benefits, banks are rationed in the interbank market, thus causing a shortage of funding among bank dependent borrowers. Firms which hold relationships with illiquid banks are more likely to be liquidated. The authors highlight the role of T-bills and bank deposits in coping with the liquidity shocks of banks' clients, and use the results to address the role of market segmentation in the monetary policy transmission mechanism. Still, the authors do not offer normative implications.

Also concerned with adverse selection, Freixas and Holthausen (2005) consider peer monitoring in a model in which cross-border information about banks is less precise than home country information, and show that there is segmentation in the uninsured interbank market. Bruche and Suarez (2010) also analyze how money markets allocate funds across banks from different regions, and suggest that banks with abundant retail deposit funding can remain marginally financed at relatively low rates (paid on insured deposits), while the rest have to pay high interest rates (either on uninsured wholesale funding from other regions, or to attract insured deposits from their own region).

A second strand of the literature that is directly relevant for our analysis is concerned with the distinction between the provision of liquidity ex ante and ex post. According to Kahn and Wagner (2012), aggregate shortages of liquidity can arise for two reasons: insufficient availability of ex ante liquidity, or insufficient ability to obtain ex post liquidity. The role of the central bank in a crisis depends crucially on the type of liquidity shortage experienced. While the central bank can address the ex ante availability problem through liquidity injection (thus allowing banks to hoard liquid assets), there is no role for the central bank in the ex post problem (except for bailing out banks).

In a model with aggregate liquidity shocks, Farhi and Tirole (2012) show that firms privilege leverage and scale when they anticipate authorities will bail them out (even though firms would choose to fully insure against liquidity shocks if there were no government). Like Farhi and Tirole, we also argue that moral hazard should be contained ex ante through prudential policies which limit the overinvestment problem and reduce liquidity risks. There are important differences with our paper: first, in Farhi and Tirole (2012) the inefficiencies stem from the lack of commitment of the authorities, while in our setup adverse selection is responsible for malfunctioning financial markets which are unable to redistribute existing liquidity (because liquidity insurance is too expensive). Second, they do not emphasize the role of policies in which authorities can commit not to bail out firms ex post. We contrast time-consistent policies with policies with commitment. Even with commitment, we show that government action to solve the adverse selection problem also ends up creating moral hazard problems. Third, they emphasize the maturity mismatch responsible for the subprime crisis, while we highlight the segmentation in liquidity markets which occurred in the aftermath of the American and European crises.

Our paper also contributes to the literature on policy rates. For Farhi and Tirole (2012) the use of the interest rate policy is advantageous because it allows for screening opportunistic institutions. Freixas, Martin, Skeie (2011) distinguish ex ante from ex post interbank interest rates, and show that the optimal ex post interbank rate should be low during liquidity crises so as to facilitate the redistribution of liquidity. Confronting this view, Allen, Carletti, and Gale (2009) suggest that the central bank should stabilize interest rates through open market operations. Aggregate uncertainty about liquidity demand leads to volatile interest rates, which is inefficient because it leads to volatile consumption (for risk averse consumers).

2 The Model

The model is based on Holmström and Tirole (2013). There are three dates $t = 0, 1, 2$, and a single good that can be used for consumption or investment at each date. Consumers are risk neutral and value consumption according to

$$c_0 + c_1 + c_2.$$

The good cannot be stored from one period to the next. Consumers cannot promise to fund future investments because their future endowments are not pledgeable.

There is a positive fixed supply L_S of an asset, which acts a store of value. We think of the store of value as government bonds backed by the government's ability to tax consumers.

Definition *Government bonds are risk free assets issued at $t = 0$, which pay one unit of the good at date $t = 1$.*[2] *The government is able to commit to make future payments by taxing consumers.*

The price of government bonds at date 0 is q, and $q \geq 1$ since consumers are indifferent between consumption in dates 0 and 1 (if $q < 1$, consumers would demand an infinite amount of government bonds); the value of q may be greater than one, since the income of consumers is not pledgeable and they cannot supply liquidity. This asset enables agents to transfer wealth across periods, thus providing outside liquidity to the corporate sector.

There is a continuum of firms. At date 0, each firm chooses the scale of the project I. At date 1, each firm suffers a liquidity shock which can take one of two values. The liquidity shock can either be low, ρ_L, or high, ρ_H. The value of the liquidity shock

[2] It does not matter if the good is delivered at date 1 or date 2, because consumers are indifferent between consumption in both dates.

determines how much more needs to be invested per unit for the project to continue. It is possible to continue at a smaller scale than I, and the continuation scale is i with $0 \leq i \leq I$. Thus if the project continues at scale i the total investment equals $I + i\rho_L$ when the liquidity shock is low, and $I + i\rho_H$ when the liquidity shock is high. Firms have no alternative projects, so funding is only useful to cope with liquidity shocks.

Returns are realized at date 2, and there are no returns from the portion of the project that is not carried forward. The project yields a pledgeable return $\rho_0 i$, and an illiquid private return $(\rho_1 - \rho_0)\, i$ to the entrepreneur. Throughout we assume

$$0 \leq \rho_L < \rho_0 < \rho_H < \rho_1.$$

In other words, the low liquidity shock does not require pre-arranged financing; the firm has enough pledgeable assets to pay for period 1 investing. However, the high liquidity shock is not self-financing. Since the initial investment I is a sunk cost, it is efficient to continue the project ex post. Let f_L and $f_H = 1 - f_L$ denote the probabilities of a low and a high liquidity shock.

All firms have a date 0 endowment of goods $A > 0$, and no endowments at dates 1 and 2. They need $I - A$ in external funds to be able to invest. Outside investors are risk neutral, competitive and are willing to lend at a zero interest rate. Entrepreneurs are protected by limited liability, and so their pledgeable income cannot take negative values.

To the Holmström and Tirole model we add heterogeneity in firm's expected liquidity needs. There is a measure α of good firms, and a measure $(1 - \alpha)$ of bad firms, with $0 < \alpha < 1$. The two types of firms are indistinguishable, and differ only in their probabilities of liquidity shocks. For good firms $f_L = f_{LG}$ and $f_H = f_{HG}$, and for bad firms $f_L = f_{LB}$ and $f_H = f_{HB}$, with $f_{LG} > f_{LB}$.

We impose a set of conditions on the returns of the good, and the bad projects. Let $\overline{f}_L = \alpha f_{LG} + (1-\alpha) f_{LB}$ and $\overline{f}_H = \alpha f_{HG} + (1-\alpha) f_{HB}$ or, in other words, $\overline{f}_L$ and $\overline{f}_H$ are population averages.

Assumption 1

$$\rho_0 < 1 + f_{LG}\rho_L + f_{HG}\rho_H < \frac{1 + f_{LG}\rho_L}{f_{LG}} \tag{1a}$$

$$\rho_1 < \min\left\{1 + f_{LB}\rho_L + f_{HB}\rho_H, \frac{1 + f_{LB}\rho_L}{f_{LB}}\right\} \tag{1b}$$

$$\frac{1 + \overline{f}_L\rho_L}{\overline{f}_L} < \rho_1. \tag{1c}$$

The middle and the right-hand side terms in expression (1a) represent the expected cost of one unit of the good project (adjusted by the probability of completing the project) when the project is continued in both states and when it is abandoned in the high shock state, respectively. The left-hand side inequality implies that good projects are not self-financing. Expression (1a) implies *a fortiori* that the average project is not self-financing and, from the social point of view, continuing in both states is better than continuing only in the low shock state, that is

$$\rho_0 < 1 + \overline{f}_L\rho_L + \overline{f}_H\rho_H < \frac{1 + \overline{f}_L\rho_L}{\overline{f}_L}. \tag{2}$$

Expression (1b) states that bad firms are not socially useful, and outside investors will not finance bad firms if they identify them. The only possibility for bad entrepreneurs of getting finance is a pooling equilibrium, in which they mimic the good entrepreneurs. Expression (1c) states that the average project is socially useful, and implies *a fortiori* that the good project is socially useful. For convenience in later calculations, we define

$$\Omega = 1 - \overline{f}_L(\rho_0 - \rho_L) = 1 - f_{LB}(\rho_0 - \rho_L) - (1 - f_{LB} - f_{HG})(\rho_0 - \rho_L)\alpha$$

with $0 < \Omega < 1$ by expression (2); Ω turns out to be the capital ratio when liquidity is expensive.

It is helpful to consider two special cases. In Section 3 there is only idiosyncratic liquidity shocks, and no aggregate risk. In Section 4 there are aggregate liquidity shocks, but no idiosyncratic uncertainty.

3 Idiosyncratic liquidity shocks

With idiosyncratic liquidity shocks, the system is able to generate sufficient liquidity internally, so that there is no need for an outside source of liquidity (in contrast to the aggregate liquidity shocks case of Section 4).

Still, the corporate sector may be unable to distribute the liquidity internally when the adverse selection problem is serious or there is insufficient pledgeable income. Hence, there are two regimes. In the first regime, firms with low liquidity shocks channel their excess liquidity to firms with liquidity shortages, such that the ex post shadow value of liquidity is equal for all firms. In the second regime, financial markets are unable to redistribute excess liquidity, exposing firms to refinancing problems in case of a bad shock. In the second regime, illiquid firms are terminated.

3.1 Pooling equilibrium

This section establishes sufficient conditions for the existence of a pooling equilibrium. Suppose the continuation scale be i_L when the liquidity shock is low, and i_H when the liquidity shock is high, with $0 \leq i_L, i_H \leq I$. A contract specifies the initial investment I and the continuation scales i_L and i_H, as well as the payments to each party for each date and liquidity shock. In a pooling equilibrium, bad entrepreneurs mimic the good entrepreneurs, and all entrepreneurs choose the same I, i_L and i_H.

The next lemma shows that entrepreneurs appropriate the nonpledgeable income and outside investors appropriate the pledgeable income. Hence, a pooling contract may be accurately represented by the triplet (I, i_L, i_H).

Lemma 1 *Under Assumption 1 and pooling, the expected profit of each type of entrepreneur is given by*

$$\pi (I, i_L, i_H; f_L, A) = f_L (\rho_1 - \rho_0) i_L + f_H (\rho_1 - \rho_0) i_H - A \tag{3}$$

with $f_L \in \{f_{LG}, f_{LB}\}$ *and* $f_H \in \{f_{HG}, f_{HB}\}$. *The participation constraint of outside investors is given by*

$$\overline{f}_L (\rho_0 - \rho_L) i_L + \left(\overline{f}_H + q - 1\right) (\rho_0 - \rho_H) i_H \geq I - A,$$

and the participation constraints of good and bad entrepreneurs are

$$\pi (I, i_L, i_H; f_L, A) \geq 0, \qquad \text{with } f_L \in \{f_{LG}, f_{LB}\}. \tag{4}$$

Proof. See appendix. ■

Since entrepreneurial capital has a higher rate of return than the cost of outside capital, it is optimal for the entrepreneur to commit all of the firm's pledgeable income to the outside investors and keep the illiquid portion of the return (the nonpledgeable return associated with $\rho_1 - \rho_0$). This specification of payments maximizes the return on the entrepreneur's initial assets A.

A good entrepreneur ought to anticipate that he will not be able to raise enough funds in the capital market to face the high liquidity shock. When $i_H > 0$, liquidity must be planned in advance. Firms should not wait until the liquidity shock occurs, as they would not be able to finance the high liquidity shock.

With outside liquidity, firms can hoard government bonds in order to be able to absorb the liquidity shock by selling these assets when needed. The firm buys ℓ government bonds at date 0, and can continue at a scale i_H in the high shock state, if it satisfies the liquidity constraint $(\rho_H - \rho_0)\, i_H \leq \ell$. Recall that the price of government bonds is denoted by q, and the existence of liquidity at a price $q > 1$ makes investment i_H comparatively more expensive. Also, firms can use inside liquidity and sign contingent contracts with other firms at date 0, to exchange liquidity at date 1 between liquidity long firms (which suffered a shock $\rho_L < \rho_0$) and liquidity short firms (which experienced a shock $\rho_H > \rho_0$).[3]

A good way of thinking about managing liquidity is in terms of insurance. Outside investors provide insurance against liquidity shocks, by providing liquidity in the high shock state (since they deliver $\rho_H - \rho_0 > 0$ to the firm), for which they are compensated in the low shock state (since they receive $\rho_0 - \rho_L > 0$). Thus, liquidity insurance provides firms with a cross-subsidy from good states to bad ones.

Good firms solve the following problem:

$$
\begin{aligned}
&\max_{\{I, i_L, i_H\}} f_{LG}\,(\rho_1 - \rho_0)\, i_L + f_{HG}\,(\rho_1 - \rho_0)\, i_H - A \\
&subject \quad to \\
&\overline{f}_L\,(\rho_0 - \rho_L)\, i_L + \left(\overline{f}_H + q - 1\right)(\rho_0 - \rho_H)\, i_H \geq I - A \\
&0 \leq i_L, i_H \leq I
\end{aligned}
\tag{5}
$$

The participation constraint of outside investors binds; let $x = \frac{i_H}{I}$, and rewrite this constraint as

$$
I\,(x, q) = \frac{A}{\Omega - \left(\overline{f}_H + q - 1\right)(\rho_0 - \rho_H)\, x}. \tag{6}
$$

[3] Equivalently, if a bank redistributes liquidity across the corporate sector, each firm could secure a nonrevocable credit line at $t = 0$, which provides enough funds when the firm experiences a high liquidity shock.

This expression makes clear that the optimal policy for the good entrepreneur trades off the scale of the initial investment against the ability to withstand high liquidity shocks. The condition for continuation in the high shock state is

$$q \leq \widehat{q} \equiv \frac{(1 - f_{LG})\,\Omega}{f_{LG}\,(\rho_H - \rho_0)} + \overline{f}_L. \tag{7}$$

The value of $\widehat{q}$ parametrizes the ex ante shadow value of liquidity; condition (7) compares the market price of liquidity with its ex ante shadow value. When the shadow value of liquidity exceeds the market price, good entrepreneurs seek insurance and set $i_H = I$.

It remains to show that $q = 1$ is indeed the equilibrium price of government bonds. These bonds provide outside liquidity to the corporate sector. Yet, the corporate sector's long term investment creates enough inside liquidity (in the form of tradable rights at $t = 1$) to cope with the liquidity needs.[4] As a result of the excess supply of liquidity, the price of government bonds is driven down to 1.

To see this, consider first the case $\widehat{q} < 1$. In this case, the gross demand for liquidity by the corporate sector is zero, which drives the price of government bonds down to 1. When $\widehat{q} \geq 1$, the corporate sector can provide enough inside liquidity for those firms with large liquidity needs. Because shocks are drawn independently across firms, the total liquidity created by the corporate sector equals $\overline{f}_L\,(\rho_0 - \rho_L)\,i_L$, and the total liquidity needs by the corporate sector are $\overline{f}_H\,(\rho_H - \rho_0)\,i_H$. The participation constraint of outside investors guarantees that there is positive net inside liquidity, since $I - A > 0$. Since the net supply of liquidity is positive, the price of government bonds is again driven down to 1. Hence, the price of government bonds q equals 1 in all possible cases.

[4]Thus, although we have assumed that there are government bonds, our results in the idiosyncratic shocks case would hold without outside liquidity. Government bonds will provide actual benefits in the aggregate shocks case discussed below.

A pooling equilibrium exists provided the following assumption holds.

Assumption 2 $f_{LB}(\rho_1 - \rho_0) \geq \Omega$

This assumption is more restrictive than Assumption 1 (expression 1c), and it ensures that the pooling contract satisfies the following features:

- It is a contract that the outside investors want to accept, that is it satisfies the participation constraint of outside investors.
- Bad and good entrepreneurs prefer this contract to becoming outside investors.
- Good entrepreneurs have no temptation to signal their type so as to reduce their funding costs. Good entrepreneurs can signal their type by offering a contract different from the equilibrium pooling contract. The gain from signaling their type depends on the off the equilibrium path beliefs of outside investors. We assume that outside investors do not revise their convictions and retain their prior beliefs if they observe an unexpected contract offer by an entrepreneur, unless the bad type makes negative profit with the new contract being offered. If bad entrepreneurs make negative profit with the unexpected contract being offered, then outside investors update their beliefs and place zero probability on the contract being offered by a bad entrepreneur. Formally, we strengthen the assumption of passive beliefs off the equilibrium path with the requirement that outside investors place zero probability on types which make negative profit with the unexpected contract.[5]

The next result shows that there is a regime in which all entrepreneurs seek liquidity insurance.

[5] We assume that a contract can be unambiguously associated with the good type if and only if the bad type makes negative profit. This is weaker than the intuitive criterion, which states that a signal can be unambiguously associated with the good type when the bad type makes less profit by deviating from the pooling equilibrium than in the pooling equilibrium.

Proposition 1 *(Insurance case) For $\widehat{q} \geq 1$, under Assumptions 1 and 2, there is a pooling equilibrium in financial markets in which entrepreneurs set $i_H = i_L = I = \frac{A}{\Omega - \overline{f}_H(\rho_0 - \rho_H)}$.*

Proof. See appendix. ■

A pooling equilibrium also exists when the ex ante shadow value of liquidity is lower than its market price, and the next result characterizes the regime in which entrepreneurs do not insure.

Proposition 2 *(No-insurance case) For $\widehat{q} < 1$, under Assumptions 1 and 2, there is a pooling equilibrium in financial markets in which entrepreneurs set $i_H = 0$ and $i_L = I = \frac{A}{\Omega}$.*

Proof. See appendix. ■

There are two regimes in the financial market. In the first regime (described in Proposition 1), liquidity flows throughout the economy, the ex post shadow value of liquidity is equal across firms, and no firm defaults. In the second regime (described in Proposition 2), insurance is too expensive so that firms prefer to increase the size of their projects instead of obtaining liquidity insurance, thus exposing them to a potential refinancing problem in case of a bad shock. Liquidity does not flow from liquidity-long to liquidity-short firms and projects with positive social value are terminated, thereby making the ex post shadow value of liquidity different across firms.

Pledgeable income ρ_0 and the measure of good firms α influence the threshold $\widehat{q}$, so that there may be a change in regime when ρ_0 or α shift. The pledgeable income ρ_0 might be interpreted as the value of collateral. When the value of the aggregate collateral falls, liquidity stops flowing among firms. During the subprime and the European crises, many borrowers were unable to obtain liquidity in money markets. In some cases, these difficulties were preceded by large falls in the value of the collateral backing the loans.[6]

The value of $1-\alpha$ measures the degree of adverse selection. The impact of α on $\widehat{q}$ is positive, for sufficiently low values of f_{HG}. In this case, liquidity may stop flowing when informational problems become more serious. In other words, declines in firm quality in the presence of asymmetric information among banks can be responsible for a freeze in interbank lending.

Finally, our model also shows the dangers of market fragmentation for the implementation of monetary policy. When $\widehat{q}<1$, the ex post shadow value of liquidity differs across firms although the market price of liquidity is the same for all firms. Importantly, there is plenty of liquidity, so that injecting liquidity will not solve the inefficiencies.

3.2 Welfare

We assume that the central planner can transfer liquidity costlessly, but cannot transform the nonpledgeable income of the corporate sector into pledgeable income. Moreover, the central planner cannot directly distinguish good from bad firms (although it can elicit revelation through self-selection). Because of Assumption 1 (expression 1c) and expression (2), the optimal continuation rule with a pooling equilibrium prescribes never abandoning the projects. It remains to show that a strategy of pooling both types of entrepreneur is less costly than trying to identify the good entrepreneurs.

[6]Kocherlakota (2000) presents a similar argument to explain banking crises in the US and in Japan, suggesting that aggregate shocks affect the value of pledgeable income. Like us, Kocherlakota assumes that shocks do not necessarily influence the projects' social value - only the ability to share the social value.

3.2.1 The second-best solution

Suppose the central planner proposes an allocation which pools good and bad entrepreneurs.[7] The second-best solution solves

$$\begin{aligned}&\max_{\{I,i_L,i_H\}} \overline{f}_L(\rho_1-\rho_L)\,i_L+\overline{f}_H(\rho_1-\rho_H)\,i_H-I\\&\textit{subject to}\\&\overline{f}_L(\rho_0-\rho_L)\,i_L+\overline{f}_H(\rho_0-\rho_H)\,i_H\geq I-A\\&0\leq i_L,i_H\leq I.\end{aligned}$$

The participation constraint of outside investors binds. Substitute it into the objective function, to obtain

$$\overline{f}_L(\rho_1-\rho_0)\,i_L+\overline{f}_H(\rho_1-\rho_0)\,i_H-A. \tag{8}$$

Comparing with the profit function in the problem of good entrepreneurs in expression (3), we see that the benefits of the central planner of financing the high liquidity shock are higher than for the good entrepreneur (since $\overline{f}_H > f_{HG}$). As a result, the incentives of the entrepreneurs and of the central planner are not aligned. Continuation in the high liquidity shock is socially desirable, but it does not imply that it is profit maximizing for entrepreneurs.

The participation constraint of outside investors is identical in the central planner's and in the good entrepreneur's problems. From this participation constraint, we obtain the same investment function as in the problem of good entrepreneurs (that is, expression 6), and we can write social welfare as a function of x

$$U(x)=\left[\frac{\overline{f}_L(\rho_1-\rho_0)+\overline{f}_H(\rho_1-\rho_0)\,x}{\Omega-\overline{f}_H(\rho_0-\rho_H)\,x}-1\right]A.$$

[7] We maximize ex ante social welfare *before* entrepreneurs know their types and, as a result, we include the payoffs of good and bad entrepreneurs in the computation of social welfare.

The central planner sets $i_H = I$ if and only if

$$U(0) < U(1) \Leftrightarrow \rho_H < \frac{1 + \overline{f}_L \rho_L}{\overline{f}_L},$$

and expression (2) guarantees that this is the case. It is optimal to get insurance in the high liquidity shock state (and choose low initial investment I).

Proposition 3 *Under Assumptions 1 and 2, and α sufficiently large, the second-best is a pooling contract with $i_H = i_L = I = \frac{A}{\Omega - \overline{f}_H(\rho_0 - \rho_H)}$.*

Proof. See appendix. ■

The central planner uses a pooling contract with insurance in the high liquidity shock state; it does so at the cost of lower initial investment, as $I(1) < I(0)$. The value of $1 - \alpha$ measures the degree of adverse selection. For a large value of α, the adverse selection problem is not serious and pooling both types of entrepreneur is the best option from a planner's perspective. For low values of α, the adverse selection problem is too serious and the central planner would prefer to separate the two types of entrepreneur. The boundary for α is provided in the appendix.

In conclusion, the second-best prescribes continuation (regardless of the liquidity shock) when the adverse selection problem is not too serious, but financial markets achieve the second-best if and only if $\widehat{q} \geq 1$. When the market value of liquidity exceeds its ex ante shadow value $\widehat{q}$, continuation in the high liquidity shock is socially desirable but is not profit maximizing for entrepreneurs. Projects with high liquidity shocks are terminated, as entrepreneurs do not insure against these shocks because they consider insurance too expensive. As result, liquidity is not channeled from firms with excess liquidity to firms with liquidity shortage, making the ex post shadow value of liquidity different across projects. The case $\widehat{q} < 1$ is the interesting case from a policy perspective.

3.3 Separating equilibrium

We have shown that a pooling equilibrium exists under Assumption 2. In this subsection we show that, when they exist, the separating equilibrium is Pareto dominated by the pooling equilibrium. We also consider the case when Assumption 2 does not hold, and a pooling equilibrium may not exist. Markets achieve the second-best allocation in the absence of a pooling equilibrium.

3.3.1 The pooling equilibrium Pareto dominates the separating equilibrium

In the pooling equilibrium, the good entrepreneur solves problem (5). Using the result $i_L = I$, one can represent the problem in the space (I, i_H). The participation constraint of outside investors can be seen as a budget constraint for good entrepreneurs, and becomes

$$i_H = \frac{1 - \overline{f}_L (\rho_0 - \rho_L)}{\overline{f}_H (\rho_0 - \rho_H)} I + \frac{A}{\overline{f}_H (\rho_H - \rho_0)},$$

and the isoprofit curves for the good entrepreneur have slope $-\frac{f_{LG}}{f_{HG}}$. When $\widehat{q} > 1$, the budget constraint is steeper than the isoprofit curves. Figure 1 plots the budget constraint with the pooling contract (the left downward sloping solid line) and an isoprofit line for the good entrepreneur (dashed line). The 45^o line depicts the restriction $i_H \leq I$. Point E_P in Figure 1 represents the combination (I, i_H) which maximizes the profit to the good entrepreneur, and it implies full insurance (hence we are in the case of Proposition 1).

Still, in a pooling equilibrium the good entrepreneur cannot have incentives to separate from the pool of good and bad entrepreneurs so as to benefit from lower financing costs. Signaling his type would imply offering a contract which does not

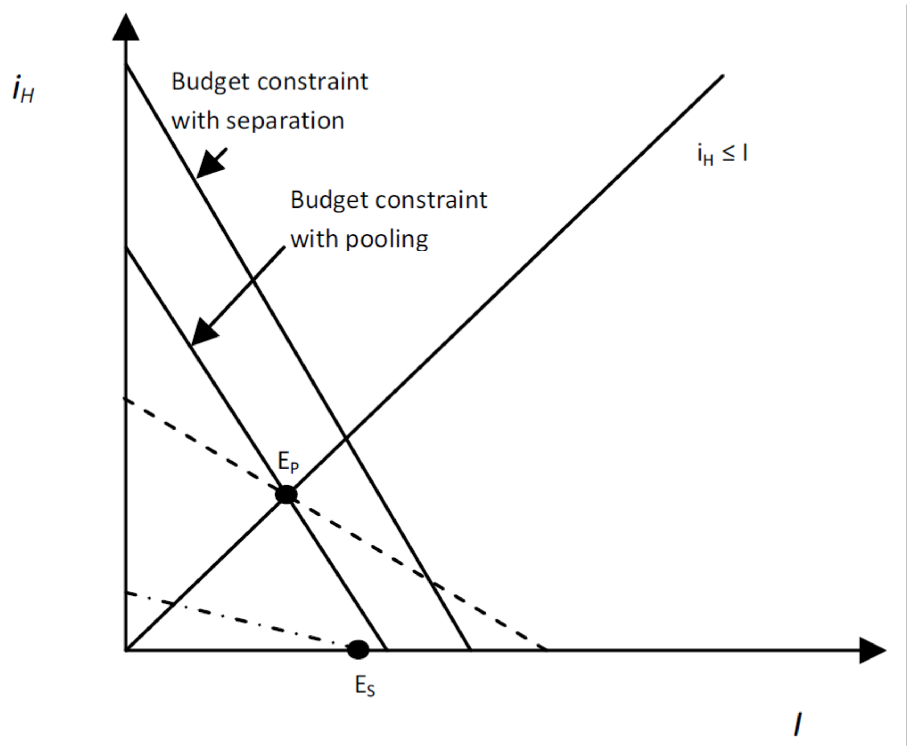

Figure 1: The idiosyncratic liquidity shocks case with $\hat{q} > q = 1$ and the pooling equilibrium. Budget constraints (solid lines with negative slope), isoprofit line for the good entrepreneur (dashed line), constraint $i_H \leq I$ (solid line with positive slope). The dot-dashed line represents the signalling constraint, that is the set of points in which the participation constraint of bad entrepreneurs holds with equality. Both types of entrepreneur make 0−profit at the point where the $i_H = I$ line intersects the signaling line.

appeal to bad entrepreneurs, that is, it would have to satisfy the signalling constraint

$$f_{LB}\,(\rho_1-\rho_0)\,i_L+f_{HB}\,(\rho_1-\rho_0)\,i_H-A\leq 0.$$

Signaling by the good entrepreneur implies partial liquidation ex post. Since $f_{HB} > f_{HG}$, it is easier for good entrepreneurs to signal their type (so as to separate from bad entrepreneurs) by liquidating in the high shock state. Additionally, signalling his type would shift the budget constraint of the good entrepreneur to the right. The budget constraint when the good entrepreneur is identified as a the good type becomes

$$f_{LG}\,(\rho_0-\rho_L)\,i_L+f_{HG}\,(\rho_0-\rho_H)\,i_H=I-A.$$

This budget constraint is steeper than the isoprofit lines of the good entrepreneur.[8] Figure 1 represents the signaling constraint (the dot-dashed line) and the budget constraint of a good entrepreneur when outside investors identify him (the right downward sloping solid line).

Assumption 2 guarantees that the signaling constraint is below the budget constraint with pooling, thus making signaling very costly. Indeed, liquidating the project in the high shock state is not enough for the good entrepreneur to signal his type, so that the good entrepreneur must signal his type by partial liquidating in the low shock state (that is, by setting $i_H = 0$ and $i_L < I$).

Since the signaling constraint yields the $0-$isoprofit curve for bad entrepreneurs and the profits of both types of entrepreneur are identical along the $i_H = I$ line, then both types of entrepreneur make positive profits in the pooling equilibrium. Outside investors make $0-$profit along the budget line, and point E_P satisfies all conditions for a pooling equilibrium.

[8] It is not clear if the budget constraint with separation is steeper than the budget constraint with pooling.

Point E_S in Figure 1 represents a separating equilibrium in which bad entrepreneurs are identified and do not obtain funding. The separating contract $\left(I^S, i_L^S, i_H^S\right)$ has the following features:

- It must be a contract that the outside investors want to accept, that is $f_{LG}\left(\rho_0 - \rho_L\right) i_L^S + f_{HG}\left(\rho_0 - \rho_H\right) i_H^S \geq I^S - A$.
- It cannot appeal to bad entrepreneurs, that is it satisfies the signalling constraint $f_{LB}\left(\rho_1 - \rho_0\right) i_L^S + f_{HB}\left(\rho_1 - \rho_0\right) i_H^S - A \leq 0$.
- Good entrepreneurs must prefer this contract to becoming outside investors, that is $f_{LG}\left(\rho_1 - \rho_0\right) i_L^S + f_{HG}\left(\rho_1 - \rho_0\right) i_H^S \geq A$.

The separating equilibrium E_S implies complete liquidation in the high shock state and partial liquidation in the low shock state, and it is Pareto dominated by the pooling equilibrium. Good entrepreneurs are better off in the pooling equilibrium than in the separating equilibrium, and bad entrepreneurs are also better off when their type is not revealed. Hence, the pooling equilibrium Pareto dominates the separating equilibrium.[9]

3.3.2 No need for economic policy in absence of a pooling equilibrium

We now present an example in which Assumption 2 is violated, and a pooling equilibrium may not exist. In this case, the signaling constraint shifts upwards as illustrated in Figure 2. Point E_P represents the possible pooling contract. The figure shows that it is tempting for good entrepreneurs to deviate from E_P to contract E_S since it yields higher profit. With partial liquidation in the high shock state, the good entrepreneur signals himself and obtains positive gains from deviating from the pooling contract. Hence there is no pooling equilibrium.

[9] We have illustrated the case of Proposition 1. Under the conditions of Proposition 2, the isoprofit curves are steeper than the budget constraint, and the pooling equilibrium also Pareto dominates the separating equilibrium.

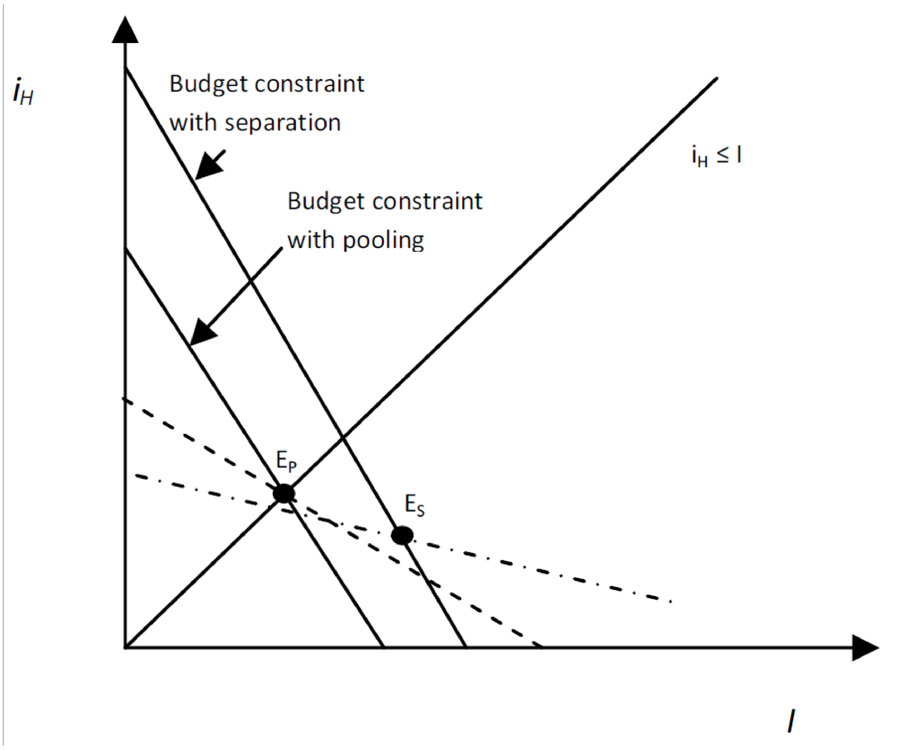

Figure 2: The idiosyncratic liquidity shocks case with $\widehat{q} > q = 1$ and the separating equilibrium. Budget constraints (solid lines with negative slope), isoprofit line for the good entrepreneur (dashed line), constraint $i_H \leq I$ (solid line with positive slope). The dot-dashed line represents the signalling constraint, that is the set of points in which the participation constraint of bad entrepreneurs holds with equality.

Although there is no pooling equilibrium in this case, there is a separating equilibrium in which bad entrepreneurs are identified and do not obtain funding. Point E_S in Figure 2 represents the separating equilibrium. The separating contract yields partial liquidation in the high shock state and no liquidation in the low shock state, thus showing that the signaling cost is low.

Welfare. If the central planner wishes to separate good from bad entrepreneurs, it does not need to implement policies to improve the market allocation. Let us sketch the argument behind this result. One can show that the indifference curves of the social planner are:

- Shallower than the budget constraint with separation (because of Assumption 1 expression 1a).
- Shallower than the isoprofit curves of good entrepreneur.
- Steeper than the signaling constraint (which is the $0-$isoprofit curve for bad entrepreneurs).

In this case, decentralized financial markets achieve the allocation that the central planner would choose if he was restricted to separating equilibrium. Point E_S in Figure 3 represents the market equilibrium and the choice of the central planner. The market equilibrium achieves constrained efficiency in the absence of a pooling equilibrium. Hence, it is not interesting to study economic policy for the separating equilibrium as in one case the separating equilibrium is Pareto dominated, whereas in the other case it reaches constrained efficiency. Since it may be possible to improve the market allocation when there is a pooling equilibrium, we will focus on the pooling equilibrium when we study economic policies.

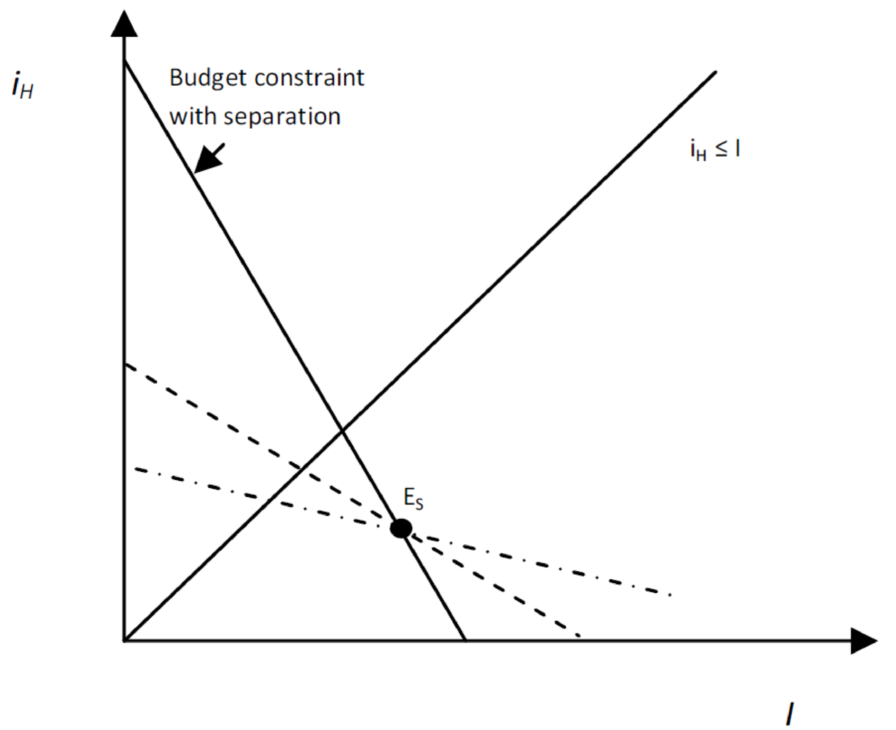

Figure 3: The second-best in the idiosyncratic liquidity shocks case with a separating equilibrium. Budget constraint (solid line with negative slope), indifference curve for the social planner (dashed line), constraint $i_H \leq I$ (solid line with positive slope), signalling constraint (dot-dashed line).

4 Aggregate liquidity shocks

As in the previous section, there are two regimes in the aggregate liquidity shocks case, one in which firms seek insurance and other in which firms do not seek insurance.

Unlike the idiosyncratic liquidity shocks case, however, when firms seek insurance the corporate sector is unable to generate liquidity to cope with high liquidity shocks. With perfectly correlated liquidity shocks, it is impossible to redistribute liquidity within the corporate sector ex post. Even if entrepreneurs wanted to buy insurance against high liquidity shocks, no firm would be able to offer such a contract since all firms suffer the same shock.

The corporate sector needs the provision of outside liquidity when shocks are perfectly correlated. When outside liquidity is insufficient, liquidity will command a premium or, eventually, force some firms to terminate their projects.

In order to model the problem of aggregate liquidity shocks, consider three states, one in which both types of firms have low liquidity shocks $\{\rho_L\rho_L\}$, one in which both types have high shocks $\{\rho_H\rho_H\}$ and, finally, one state in which good firms have low shocks and bad firms have high shocks $\{\rho_L\rho_H\}$. We assume that state $\{\rho_L\rho_L\}$ occurs with probability f_{LB}, $\{\rho_H\rho_H\}$ with probability f_{HG}, and $\{\rho_L\rho_H\}$ with probability $1-f_{HG}-f_{LB}$. Bad entrepreneurs are identifiable when their firms suffer a large shock, and the good entrepreneurs receive a low shock - that is, in state $\{\rho_L\rho_H\}$.

As before, we assume that good and average firms are socially useful but are not self-financing, and bad firms are not socially useful. These condition are guaranteed by Assumption 1. Expression (1b) implies outside investors will not finance bad firms if they identify them. Hence, bad firms are not funded in state $\{\rho_L\rho_H\}$ because, (i) ex ante, these projects have negative net present value, and outside investors do not insure them in state $\{\rho_L\rho_H\}$ and, (ii) ex post, bad firms will not be able to obtain finance in state $\{\rho_L \rho_H\}$, as they have insufficient pledgeable income.

Expressions (1c) and (2) imply

$$\frac{1+\left[f_{LB}+\left(1-f_{LB}-f_{HG}\right)\alpha\right]\rho_L+f_{HG}\rho_H}{f_{LB}+\left(1-f_{LB}-f_{HG}\right)\alpha+f_{HG}} < \frac{1+\left[f_{LB}+\left(1-f_{LB}-f_{HG}\right)\alpha\right]\rho_L}{f_{LB}+\left(1-f_{HG}-f_{LB}\right)\alpha} < \rho_1. \tag{9}$$

The left-hand side and the middle terms in this expression represent the expected costs of one unit of the average project (adjusted by the probability of completing the project), when good projects are never abandoned and when all projects are abandoned in state $\{\rho_H \rho_H\}$, respectively. The term $(1 - f_{LB} - f_{HG})$ is multiplied by α because outside investors only finance a measure α of good firms in state $\{\rho_L \rho_H\}$. The above expression implies that the average project is socially useful (that is, α is large enough), and it is better to continue the average project in state $\{\rho_H \rho_H\}$. Recall that expression (1a) implies *a fortiori* that the average project is not self-financing.

4.1 The provision of outside liquidity

In the absence of outside liquidity, all entrepreneurs must liquidate their projects in state $\{\rho_H \rho_H\}$. Government bonds provide outside liquidity to the corporate sector, and they are the only source of liquidity insurance in state $\{\rho_H \rho_H\}$ since consumers cannot make promises on their future income. With outside liquidity, firms can hoard liquid securities that can be resold when needed. The firm buys ℓ government bonds at date 0, and can continue at a scale i_H in state $\{\rho_H \rho_H\}$, if it satisfies the liquidity constraint

$$(\rho_H - \rho_0)\, i_H \leq \ell. \tag{10}$$

Without loss of generality, we assume that bad firms return their liquidity ℓ to outside investors in state $\{\rho_L \rho_H\}$.

Recall that the price of government bonds is denoted by q. In the idiosyncratic shocks case, the price of government bonds is always equal to 1 in equilibrium. In the aggregate shocks case, however, the price of government bonds q can be above 1. The existence of liquidity at a price $q > 1$ makes investment i_H comparatively more expensive.

The demand by entrepreneurs for liquidity depends on the value of continuing in the high liquidity shock state. The ex ante shadow value of an initial unit of liquidity can be calculated to be

$$\overline{q} = \frac{(1 - f_{LG})\,\Omega}{f_{LG}\,(\rho_H - \rho_0)} + f_{LG}.$$

This $\overline{q}$ is the maximum value to pay for a bond for use for liquidity insurance. When liquidity is too expensive, entrepreneurs do not wish to continue their projects in the high shock state. When the price q is above $\overline{q}$, the entrepreneur does not wish to insure.

The value $\overline{q}$ is calculated by equating the expected profit with continuation in state $\{\rho_H \rho_H\}$ to the expected profit with termination in this state (detailed calculations are available in the appendix). In other words, the parameter $\overline{q}$ is the threshold price at which good entrepreneurs are indifferent between continuing or not when the firm suffers a high liquidity shock.

4.2 Pooling equilibrium

We calculate the pooling equilibrium for the aggregate shocks case, and establish sufficient conditions under which it exists.

Since consumers are indifferent between consumption in dates 0 and 1, the price of government bonds is $q \geq 1$. There are two regimes. In the first regime, $\overline{q} \geq 1$ which implies that firms seek insurance and wish to continue if there is sufficient outside liquidity. In the other regime, $\overline{q} < 1$ and entrepreneurs do not seek insurance since the price of government bonds is above the critical threshold $\overline{q}$.

The next assumption strengthens Assumption 1 (expressions 1a and 1c), and guarantees that all pledgeable income is given to outside investors. It implies that α must be large enough (it is automatically satisfied for $\alpha = 1$).

Assumption 3 $f_{LG}\,(\rho_1 - \rho_0)\,\alpha > \Omega$.

Let i_L and i_H represent the continuation scales in states $\{\rho_L \rho_L\}$ and $\{\rho_H \rho_H\}$, respectively, and let i_{LH} represent the continuation scale of good entrepreneurs in state $\{\rho_L \rho_H\}$.

4.2.1 Insurance case: the case $\overline{q} \geq 1$

When $\overline{q} \geq 1$, entrepreneurs seek liquidity insurance if $q < \overline{q}$, and do not insure if $q > \overline{q}$. In the aggregate shocks case, Assumption 2 guarantees that bad entrepreneurs want to participate in a pooling equilibrium. Still, there is the temptation for good entrepreneurs to deviate from a pooling equilibrium.

Assumption 4 $f_{LG}\,(\rho_1 - \rho_0)\,\frac{1-(f_{LG}-f_{LB})(\rho_0-\rho_L)}{1+(f_{LG}-f_{LB})(\rho_1-\rho_0)} \geq \Omega$.

This is a new additional assumption which was not needed before. It complements Assumption 2 and guarantees that pooling is a best strategy for good entrepreneurs. To see this, note that any deviation from the pooling equilibrium must satisfy the signaling constraint

$$f_{LB}\,(\rho_1 - \rho_0)\,i_L + f_{HG}\,(\rho_1 - \rho_0)\,i_H - A \leq 0$$

which implies that the profit of the good entrepreneur in any separating contract equals $(f_{LG} - f_{LB})\,(\rho_1 - \rho_0)\,i_{LH}$. The good entrepreneur only obtains positive profit in the state that unambiguously identifies him, that is state $\{\rho_L \rho_H\}$. This state has probability $f_{LG} - f_{LB}$, and the profit of the deviation decreases as this probability decreases.

When $f_{LG} - f_{LB}$ is sufficiently small, Assumption 4 holds and the profit from a separating contract is lower than the profit from a pooling contract.

Proposition 4 *(Insurance case) Under Assumptions 1, 2, 3, 4, and* $\overline{q} \geq 1$*, there is a pooling equilibrium in financial markets. When* $\overline{q} > q \geq 1$*, all entrepreneurs set investment equal to* $I = \frac{A}{\Omega - (f_{HG}+q-1)(\rho_0 - \rho_H)}$*, good entrepreneurs set* $i_L = i_{LH} = i_H = I$*, and bad entrepreneurs set* $i_L = i_H = I$ *and do not continue in state* $\{\rho_L \rho_H\}$*. When* $q = \overline{q}$*, good entrepreneurs are indifferent between continuing their projects in state* $\{\rho_H \rho_H\}$ *or not.*

Proof. See appendix. ■

4.2.2 No-insurance case: the case $\overline{q} < 1$

In this case, entrepreneurs do not seek liquidity insurance because $q \geq 1$.

Proposition 5 *(No-insurance case) Under Assumptions 1, 2, 3, 4, and* $\overline{q} < 1$*, there is a pooling equilibrium in financial markets. Entrepreneurs do not seek insurance, and set* $I = \frac{A}{\Omega}$*. Good entrepreneurs set* $I = i_L = i_{LH}$*, and bad entrepreneurs set* $I = i_L$ *and do not continue in state* $\{\rho_L \rho_H\}$*. All entrepreneurs set* $i_H = 0$*.*

Proof. See appendix. ■

There are two regimes in the market for government bonds. In the regime described in Proposition 4, the corporate sector demands government bonds for $q = 1$. Figure 4 depicts the aggregate demand for liquidity (we denote the demand for liquidity by the corporate sector by $L_D(q)$). The corporate sector does not demand liquidity when the price of government bonds is above $\overline{q}$. The demand curve is downward sloping for prices in the interval $(1, \overline{q})$, and the demand for liquidity is infinitely elastic for $q = 1$ as consumers are willing to accept any amount of liquidity at this price. When the supply of outside liquidity equals L_{S2}, the corporate sector takes an amount equal to $L_D(1)$ and consumers take the rest; the price of liquidity is one. When the supply of outside liquidity equals L_{S1}, the corporate sector buys all government bonds, and there is a positive liquidity premium $q-1$. A reduction in the supply of outside liquidity will drive the liquidity premium up. Firms continue to insure and and reduce investment I, as long as the supply of outside liquidity is larger or equal to $L_D(\overline{q})$. When the supply of outside liquidity is lower than $L_D(\overline{q})$, then the price of liquidity will reach $\overline{q}$ and some good firms do not obtain insurance.

Outside supply of liquidity could be reduced by policy. Factors that might influence such policies might be the fear of sovereign default, which impedes the ability of the government to issue new bonds. A crisis of confidence has a negative effect on the amount of outside liquidity a government can back, leading to an increase in the liquidity premium and, eventually, to a liquidity freeze in financial markets. The appendix contains, together with the proof of Proposition 4, the expressions for the aggregate demand of liquidity by the corporate sector which underlie the diagram.

The regime described in Proposition 5 is depicted in Figure 5. Entrepreneurs do not find it attractive to insure for a price of liquidity equal to one, so that the demand for liquidity by the corporate sector disappears at the equilibrium price $q=1$. In this case, consumers define an infinitely elastic demand for liquidity. The injection of aggregate liquidity will not induce firms to seek liquidity insurance, as all outside liquidity is absorbed by the consumers.

Changes in pledgeable income ρ_0 can cause a shift between the two regimes. Since the cutoff $\overline{q}$ increases with ρ_0, an increase in pledgeable income eases continuation in the high shock state, suggesting that liquidity stops flowing among firms when the value of collateral falls.

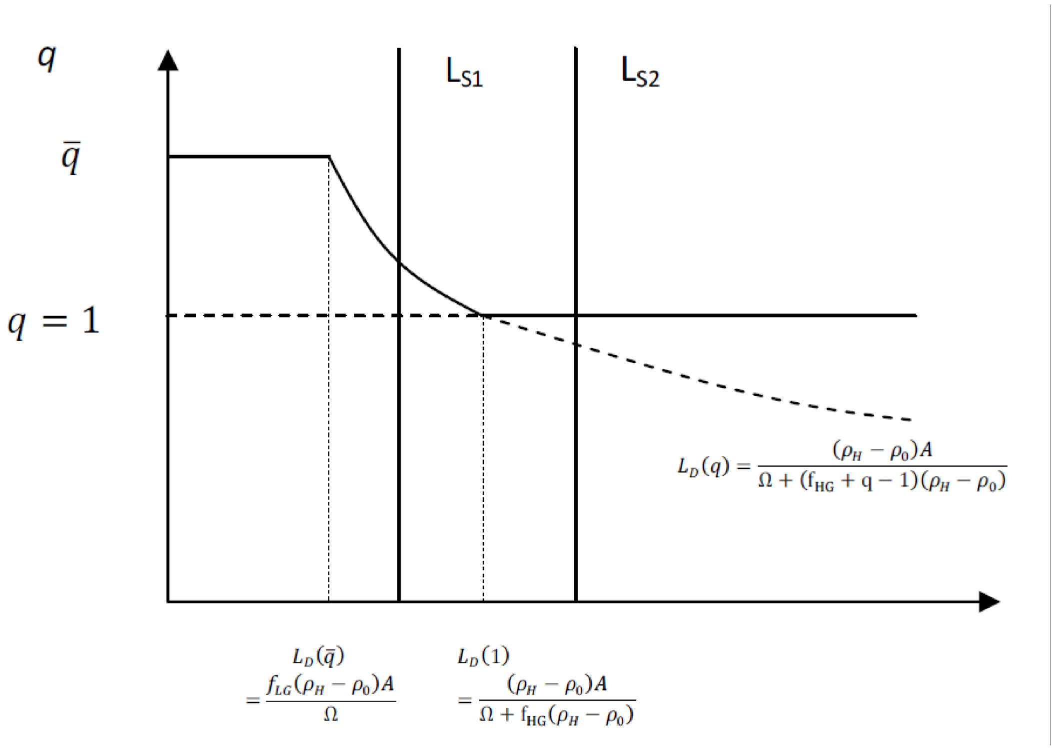

Figure 4: The aggregate shocks case with $\overline{q} > 1$. The aggregate demand for liquidity and the supply of outside liquidity are represented by the solid lines. The demand for liquidity by the corporate sector is represented by $L_D(q)$.

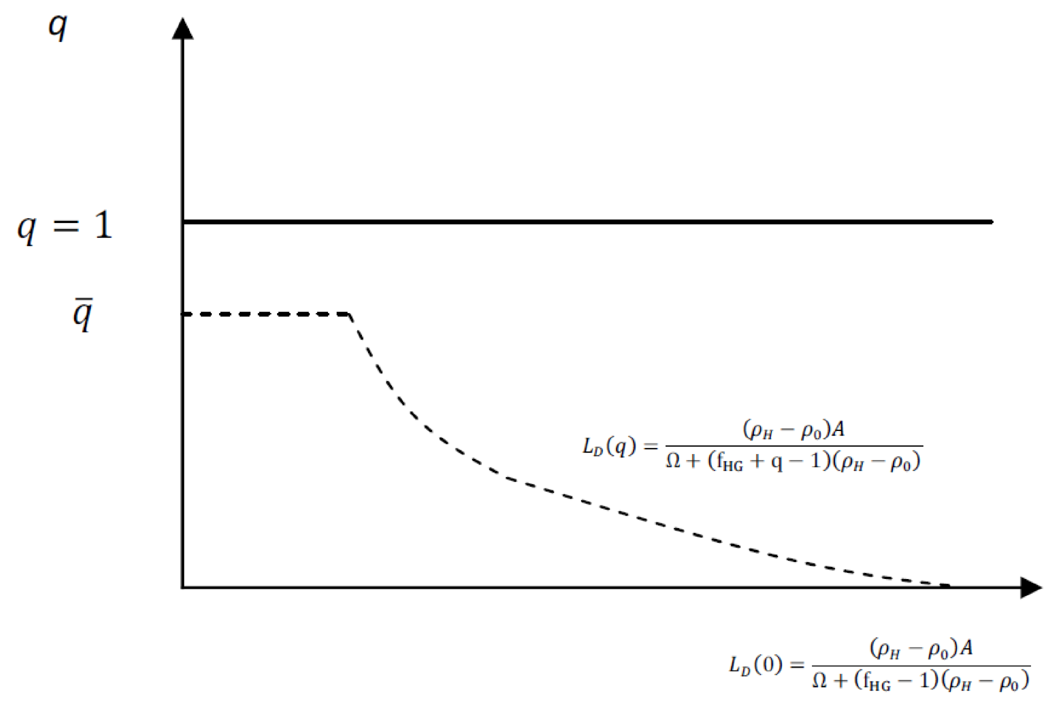

Figure 5: The aggregate shocks case with $\overline{q} < 1$. The aggregate demand for liquidity is represented by the horizontal solid line. The demand for liquidity by the corporate sector is represented by the dashed line.

Welfare results are qualitatively the same as in the idiosyncratic shocks case, with the second-best level of investment equal to $\frac{A}{\Omega - f_{HG}(\rho_0 - \rho_H)}$ (the results are available in the appendix). The central planner can create liquidity in state $\{\rho_H \rho_H\}$, as it can use its taxation power to transfer income from consumers to firms in this state. The central planner would like to insure state $\{\rho_H \rho_H\}$, but financial markets may not be willing to provide unsubsidized insurance in this state.

To sum up, there are three possibilities. When $\overline{q} > 1$, and the supply of outside liquidity L_{S1} is less than $L_D(1)$, entrepreneurs take all liquidity; when $\overline{q} > 1$ and supply L_{S2} is greater than $L_D(1)$, entrepreneurs take $L_D(1)$ and consumers take the rest. When $\overline{q} < 1$, entrepreneurs do not find liquidity useful and consumers take all liquidity. The second-best prescribes the corporate sector to obtain an amount of liquidity equal to $L_D(1)$.

Separating equilibrium. A pooling equilibrium exists under Assumptions 2 and 4. When they exist, the separating equilibrium is Pareto dominated by the pooling equilibrium. Results are qualitatively the same as in the idiosyncratic shocks case when Assumption 2 does not hold.

5 Economic policies

Ex post, it is always optimal to rescue firms that did not get insurance and suffered high liquidity shocks. Since the initial investment is a sunk cost and $\rho_1 > \rho_H$, it is not efficient to close down these firms. One possible ex post policy would be to provide a subsidy equal to $\rho_H - \rho_0$, and let outside investors lend an amount equal to the pledgeable income ρ_0.

Such bailout policy creates moral hazard at the initial date, as entrepreneurs would anticipate ex post interventions, and would invest too much without getting insurance ex ante. With bailouts, the insurance market unravels at the initial date.

We contrast the bailout case with the case in which public authorities have the ability to commit (at the initial date) to a policy in which they can (credibly) promise not to bail out firms. We consider two instruments which affect the pledgeable income of the corporate sector: (i) a contingent subsidy rate s, which is transferred when firms suffer a high liquidity shock, and (ii) a tax rate t on initial investment.[10]

We do not consider taxes and subsidies on nonpledgeable income. If the public authorities were able to tax nonpledgeable income, and use these taxes to subsidize pledgeable income, then the authorities would be able to effectively overcome the fundamental constraint of our model: that not all income is pledgeable.[11]

We describe several policies which implement the second-best, and have an expected zero tax revenue. When liquidity flows throughout the economy, the best policy is to inject liquidity such that the price of liquidity equals 1. When firms do not seek insurance, the government should implement other alternative policies. The first policy is a combination of bailouts and taxes on investment, such that the government bears the whole cost of saving firms. The second policy subsidizes insurance and taxes initial investment, thereby enticing entrepreneurs to seek liquidity. With this policy, the private and the public sectors share the cost of rescuing firms.

Note that we do not include the policy rate among the tools of the government. As in other recent models, interest rate policies are equivalent to a subset of policies with taxes and subsidies.

Again, it is useful to distinguish idiosyncratic shocks case from aggregate shocks

[10] One could consider a more tailored tax strategy as, for example, a contingent tax rate which is paid when the firm suffers a low liquidity shock. This might be useful in a more complex environment, but does not provide additional advantages in our simple environment. We assume that the government is able to prevent consumers from accessing subsidies.

[11] Suppose there was no adverse selection, as in Holmström and Tirole (2013). If the central planner were able to transform nonpledgeable income into pledgeable income, then the central planner would be able to implement the first-best.

case.[12] With taxes and subsidies, we impose a more restrictive version of the assumptions used earlier. The details and derivations are available in the appendix; here we simply describe the results.

5.1 Idiosyncratic liquidity shocks

Financial markets achieve the second-best when $\widehat{q} \geq 1$. In this case, firms obtain insurance and no public intervention is required. We restrict the study to the case $\widehat{q} < 1$, as the government does not wish to intervene otherwise.[13]

Subsidies and taxes can only affect the amount of pledgeable income, as the government cannot tax nonpledgeable income. The objective function of good entrepreneurs contains the nonpledgeable income, so taxes and subsidies cannot affect it.[14] The participation constraint of outside investors consists of pledgeable income, and this income can be taxed or subsidized. Appendix A.8 shows that good entrepreneurs solve

$$\begin{aligned} &\max_{\{I,i_L,i_H\}} f_{LG}\left(\rho_1-\rho_0\right)i_L+f_{HG}\left(\rho_1-\rho_0\right)i_H-A \\ &subject \quad to \\ &\overline{f}_L\left(\rho_0-\rho_L\right)i_L+\overline{f}_H\left(\rho_0-\rho_H+s\right)i_H-tI\geq I-A \\ &0\leq i_L,i_H\leq I \end{aligned} \tag{11}$$

[12]Recall that the central planner prefers not to separate good form bad entrepreneurs, so that the second-best implies pooling good and bad entrepreneurs. The policies that we will be studying consist of subsidizing firms in states with high liquidity shocks and taxing initial investment. Bad entrepreneurs will benefit from these policies, but good entrepreneurs might consider the possibility of separation. We assume that the government would tax a separating contract, such that it would become unappealing.

[13]The policy section focusses on the pooling equilibrium, as we have seen in the absence of pooling equilibrium the economy reaches constrained efficiency.

[14]Variable A represents an opportunity cost and is not affected by taxes or subsidies. One could remove A from the objective function and the solution would not change.

5.1.1 Time-consistent policies with ex post bailout

An ex post bailout policy is equivalent to setting a subsidy $s_b = \rho_H - \rho_0$. This policy distorts the choice of initial investment (it would be too big) and prevents insurance.

Still, the bailout policy could be accompanied by taxes which help to fix initial investment I at the right level. To assess this possibility, we evaluate the maximization problem (11) with $s = s_b$. The solution to program (11) implies $i_L = i_H = I$, and

$$I = \frac{A}{\Omega + t}$$

The optimal level of investment that would be chosen by the central planner is given by Proposition 3, and the tax rate t can be calibrated to achieve the optimal level of investment. Formally,

$$\frac{A}{\Omega + t} = \frac{A}{\Omega - \overline{f}_H (\rho_0 - \rho_H)} \Leftrightarrow t = \overline{f}_H (\rho_H - \rho_0)$$

Given the law of large numbers, the value of the tax is equal to the value of the bailouts. The ex post bailout can replicate the second-best, with the following features:

- The government provides insurance, instead of outside investors. The bailout policy prevents financial markets from working properly.
- The government uses taxes to reduce the incentives to overinvest.
- The net revenue of the bailout policy is zero.

5.1.2 Policies with commitment

The government commits not to bail out firms (although it may subsidize firms which undergo high liquidity shocks). Good entrepreneurs solve program (11), in which the subsidy s might be different from s_b. The profit of good entrepreneurs equals

$$\Pi(x) = \left(\frac{f_{LG}(\rho_1 - \rho_0) + f_{HG}(\rho_1 - \rho_0)x}{\Omega + t - \overline{f}_H(\rho_0 - \rho_H + s)x} - 1 \right) A.$$

and we obtain $\Pi(0) \leq \Pi(1)$, when

$$1 \leq \frac{(\Omega + t)(1 - f_{LG})}{f_{LG}(\rho_H - \rho_0 - s)} + \overline{f}_L \equiv q_i \tag{12}$$

This expression is equivalent to expression (7) when there are taxes and subsidies (and $q = 1$). The cutoff q_i depends on the value of taxes and subsidies, and the effects are as expected: the cutoff increases with s and t.

Optimal policy with commitment The government computes the optimal values of the contingent subsidy s^* and tax t^* under commitment. These would be the values which set the cutoff q_i equal to one, that is

$$1 = \frac{(\Omega + t^*)(1 - f_{LG})}{f_{LG}(\rho_H - \rho_0 - s^*)} + \overline{f}_L, \tag{13}$$

and set the level of investment equal to the central planner's choice - as given by Proposition 3 -, that is

$$\frac{A}{\Omega + t^* - \overline{f}_H(\rho_0 - \rho_H + s^*)} = \frac{A}{\Omega - \overline{f}_H(\rho_0 - \rho_H)}.$$

This expression implies

$$t^* - \overline{f}_H(\rho_0 - \rho_H + s^*) = -\overline{f}_H(\rho_0 - \rho_H) \Leftrightarrow t^* = \overline{f}_H s^*$$

which implies a zero tax revenue. Replacing t^* in expression (13), yields the value for the optimal contingent subsidy

$$1 = \frac{\left(\Omega + \overline{f}_H s^*\right)(1 - f_{LG})}{f_{LG}\left(\rho_H - \rho_0 - s^*\right)} + \overline{f}_L \Leftrightarrow s^* = \frac{\rho_H - \rho_0 - \frac{\Omega f_{HG}}{\overline{f}_H f_{LG}}}{1 + \frac{1}{f_{LG}}}.$$

Since $s_b = \rho_H - \rho_0 > s^*$, the bailout subsidy (per unit of investment) is higher than the optimal subsidy under commitment. This is because outside investors partially insure entrepreneurs when there is commitment, and the cost of continuation in state ρ_H is partially borne by outside investors. With bailouts, the government bears the whole cost of continuation.[15]

Policies with commitment can replicate the second-best, with the following features:

- Financial markets continue to function, providing insurance in the high liquidity shock state.
- The government uses taxes to reduce incentives to invest.
- Taxes and subsidies are smaller than with ex post bailouts. The net revenue of the policy with commitment is zero.

5.2 Aggregate liquidity shocks

The corporate sector in unable to insure high liquidity shocks, and the best policy is to create outside liquidity. There are two forms of outside liquidity. First, the government provides complete insurance to firms by taxing consumers. In this case, the government implements a bailout policy by transferring income from consumers to firms in the high shock state. Second, the government supplies government bonds, and firms hoard liquid securities that can be resold when needed. Finally, we also consider a strategy which

[15] Entrepreneurs do not want to "resell" their subsidies to consumers as their gain from continuation, $\rho_1 - \rho_0$, is larger than the value of the subsidy (s^* or s_b).

combines the previous two forms of liquidity. The government taxes initial investment and subsidizes liquidity insurance, and firms use government bonds to face liquidity shocks. We consider each of these three alternatives in turn.

5.2.1 Time-consistent policies with ex post bailout

In state $\{\rho_H \rho_H\}$, all firms suffer a high liquidity shock. A bailout policy is equivalent to setting a contingent subsidy $s_b = \rho_H - \rho_0$ in state $\{\rho_H \rho_H\}$. This subsidy implies overinvestment at the initial stage. The bailout policy should be accompanied by taxes which help to fix investment I at the second-best level.

To see how the second-best can be implemented, solve the problem of good entrepreneurs with $s = s_b$. In appendix A.9, we show that good entrepreneurs solve

$$\max_{\{I, i_L, i_H, i_{LH}\}} f_{LB} (\rho_1 - \rho_0) i_L + f_{HG} (\rho_1 - \rho_0) i_H + (1 - f_{LB} - f_{HG}) (\rho_1 - \rho_0) i_{LH} - A$$

$$subject \ \ to$$

$$f_{LB} (\rho_0 - \rho_L) i_L + (1 - f_{LB} - f_{HG}) (\rho_0 - \rho_L) \alpha i_{LH} - tI = I - A$$

$$0 \leq i_L, i_H, i_{LH} \leq I.$$

We obtain $I = \frac{A}{\Omega + t}$. Comparing with the second-best level of investment, we obtain

$$\frac{A}{\Omega + t} = \frac{A}{\Omega - f_{HG} (\rho_0 - \rho_H)} \Leftrightarrow t = f_{HG} (\rho_H - \rho_0)$$

Hence, the expected tax revenue from the bailout programme is zero. The ex post bailouts can replicate the second-best, with the following features:

- The government provides insurance instead of outside investors. The bailout policy prevents insurance markets from working properly.
- The government uses taxes to reduce the incentives to overinvest.

- The net revenue of the bailout policy is zero.

5.2.2 Policies with commitment when $\overline{q} \geq 1$

The financial market functions when $\overline{q} \geq 1$. The government can provide liquidity by promoting the use of government bonds. The larger the supply of government bonds, the lower the price of liquidity q. The optimal provision of government bonds depends on the cost of providing this form of liquidity.

As long as there is no cost in providing outside liquidity beyond the opportunity cost of capital, then the optimal policy prescribes setting the price of liquidity equal to 1. This means that the optimal supply of outside liquidity is larger or equal to $L_D(1)$. An insufficient supply of government bonds would create a positive liquidity premium $q-1$, which would lead to an inefficient level of investment.[16]

5.2.3 Policies with commitment when $\overline{q} < 1$

When $\overline{q} < 1$, raising the supply of government bonds does not solve the liquidity problems. Since $q \geq 1$, it is always the case that entrepreneurs set $i_H = 0$. In this case, one must seek alternative policies. One alternative is to implement an ex post bailout policy, as in Section 5.2.1. The other alternative is to subsidize insurance.

For the sake of simplicity, we assume $L_S \geq L_D(1)$ so that $q = 1$.[17] Appendix A.10

[16] We assumed the good cannot be stored from one period to the next. If there are assets that act as a store of value, then two cases may happen. If these assets provide enough liquidity, then $q = 1$. Otherwise, $q > 1$.

[17] When $q > 1$, injecting liquidity would alleviate the aggregate liquidity problem (as firms need less liquidity than $L_D(1)$), but would not solve the market segmentation problem.

shows that good entrepreneurs solve

$$\max_{\{I,i_L,i_H,i_{LH},\ell\}} f_{LB}(\rho_1-\rho_0)\,i_L+(1-f_{LB}-f_{HG})(\rho_1-\rho_0)\,i_{LH}+f_{HG}(\rho_1-\rho_0)\,i_H-A$$

$$subject\ \ to$$

$$f_{LB}(\rho_0-\rho_L)\,i_L+(1-f_{LB}-f_{HG})(\rho_0-\rho_L)\,i_{LH}\alpha+f_{HG}(\rho_0-\rho_H+s)\,i_H-tI\geq I-A$$

$$(\rho_H-\rho_0-s)\,i_H\leq\ell$$

$$0\leq i_L,i_H,i_{LH}\leq I$$

when there are taxes and subsidies. The participation and liquidity constraints bind, and the optimum prescribes $i_L=i_{LH}=I$. Hence,

$$I(x)=\frac{A}{\Omega+t-f_{HG}(\rho_0-\rho_H+s)\,x}$$

and the profit function

$$\Pi(x)=\left[\frac{(1-f_{HG}+f_{HG}x)(\rho_1-\rho_0)}{\Omega+t-f_{HG}(\rho_0-\rho_H+s)\,x}-1\right]A.$$

Good entrepreneurs set $i_H=I$ when $\Pi(1)\geq\Pi(0)$, that is

$$1\leq\frac{f_{HG}(\Omega+t)}{(1-f_{HG})(\rho_H-\rho_0-s)}+1-f_{HG}\equiv q_a$$

The new cutoff q_a is equivalent to the cutoff $\overline{q}$ when there are taxes and subsidies, and yields a new cutoff value for the market price of liquidity.

Optimal policy with commitment The government computes the optimal values of the contingent subsidy s^* and tax t^* under commitment. These would be the values which set the cutoff value q_a equal to one, so as to induce good entrepreneurs to set

$i_H = I$, that is

$$1 = \frac{f_{HG}\left(\Omega + t^*\right)}{\left(1 - f_{HG}\right)\left(\rho_H - \rho_0 - s^*\right)} + 1 - f_{HG} \tag{14}$$

and set the level of investment equal to the central planner's choice, that is

$$\frac{A}{\Omega + t^* - f_{HG}\left(\rho_0 - \rho_H + s^*\right)} = \frac{A}{\Omega - f_{HG}\left(\rho_0 - \rho_H\right)}.$$

This expression implies

$$t^* = f_{HG} s^*$$

which implies an expected zero tax revenue. Replacing t^* in expression (14) yields the value of the optimal contingent subsidy

$$s^* = \left(\rho_H - \rho_0\right) - \left[f_{HG}\left(\rho_H - \rho_0\right) + \Omega\right]$$

with $s^* < s_b = \rho_H - \rho_0$. Policies with commitment can replicate the second-best with the following features:

- Insurance markets continue to function, providing insurance in the high liquidity shock state.
- The government uses taxes to reduce incentives to invest.
- Taxes and subsidies are smaller than with ex post bailouts. The net revenue of a policy with commitment is zero.

5.3 Discussion of the policy results

With time-consistent policies, authorities bail out firms with bad shocks and must use ex ante policies to alleviate moral hazard problems. When authorities are able to commit not to bail out firms ex post, there are two situations to consider.

In one situation entrepreneurs want insurance in the absence of a liquidity shortage (that is, when obtaining liquidity does not require the payment of a liquidity premium). If there is a liquidity shortage in this situation, then policies that increase government-provided liquidity can be of use and in effect the recommendation is for the government, as low cost liquidity provider, to make liquidity readily available. This situation only arises in the aggregate shocks case; without aggregate shocks, the system generates sufficient liquidity internally.

In the other situation, entrepreneurs do not want liquidity insurance even if it is costless to obtain additional liquidity. In this situation, in the absence of intervention, liquidity has different ex post shadow values for different firms. The optimal policy involves subsidizing the liquidity insurance. However this generates an additional moral hazard problem, for which the solution is a tax on debt. This situation can arise either in the idiosyncratic or the aggregate shocks case.

Since the toolkit of public authorities includes instruments which are economically equivalent to taxes and subsidies, there is a variety of ways in which governments can implement the optimal policy mix. One can relate the optimal government intervention prescribed in our paper with the actual policies implemented in Europe and in the US. Roughly three categories of interventions were pursued in order to provide liquidity and improve the resilience of the financial system.

Policy rates. The use of taxes on investment and insurance subsidies is open to broader interpretations. The taxes and subsidies in the model are used to change intertemporal tradeoffs. The same results can be achieved with interest rate policies. Higher interest rates make current investment relatively more expensive, and can be used to correct entrepreneurs' incentives. Still, policy rates are insufficient to achieve the second-best in the idiosyncratic shocks case since they are non-targeted instruments, and must be complemented with transfer policies.[18][19]

Unconventional monetary policy. The central bank has available a variety of techniques by which it can provide liquidity to banks at subsidized rates. It can accept sub-standard assets as collateral, it can exchange illiquid assets for it own liquid assets, it can lower the haircuts on its lending, or it can provide a variety of debt guarantees to financial institutions. In each case, thanks to the central bank, the financial institution manages to increase its liquidity at a lower cost than it would face on the open market.

Prudential policy. Many non-targeted restrictions on leverage put forward by regulators are equivalent to ex ante taxes on debt. Examples of these restrictions include capital and liquidity requirements.

6 Conclusion

This paper primary aim is to draw attention to the limits to the flow of liquidity under adverse selection. We analyze the allocation of liquidity among firms with heterogeneous liquidity shocks. The efficient allocation of liquidity requires channeling liquidity from liquidity long to liquidity short firms, thus making the ex post shadow value of liquidity equal across projects. Still, the existence of a small set of firms with bad projects may prevent financial markets from performing the efficient allocation of liquidity.

The model shows the limits to aggregate liquidity policies, which are unable to deal with the market segmentation that can arise as a result of adverse selection. We analyze economic policies which rebuild the liquidity channels throughout the economy.

[18] The relation between the interest rate policy and taxes and subsidies has had a revival in recent years, as the financial crisis exposed the limitations of conventional monetary policy. Taxes and subsidies can be used to overcome the zero lower bound, and implement allocations which would require negative nominal policy rates.

[19] Our insights suggest that loose interest rate policies today lead to risk shifting behavior. Low interest rates change the intertemporal rate of substitution, making initial investment cheaper and raising the cost of liquidity insurance. As a result, loose monetary policy today encourages large size projects and increases the likelihood of future liquidity freezes.

The optimal policy mix in effect provides liquidity insurance to institutions; since this insurance can create moral hazard problems, prudential restrictions are also part of the policy mix. The liquidity insurance can come in the form of lower interest rates, reducing the quality of the assets accepted as collateral with low haircuts in loans or repurchase agreements, the purchase of illiquid assets at favorable terms, extending a variety of debt guarantees to financial institutions. The prudential restrictions can come in the form of restrictions on debt, capital and liquidity.

In this paper we have examined both time-consistent and time-inconsistent policies. Recent research has emphasized the importance of time-consistent policies. These policies are likely to be more relevant in the aggregate shocks case. In practical terms, it is easier for public authorities to credibly commit not to rescue firms when the number of failures is small, since welfare losses are small and failed firms may be acquired by the surviving firms. Yet, the lack of commitment in the aggregate shocks case creates implicit bailout guarantees, thus inducing firms to correlate the risk inherent in their investment choices. In this case, firms have incentives to herd ex ante so as to increase the likelihood of being rescued ex post, and public authorities should use *macro*prudential policies which induce firms to differentiate their risks.

Nonetheless, the basic conclusions apply whether or not the regulator can commit to time-inconsistent policies. However, when the regulator is unable to commit, moral hazard is more difficult to contain and prudential activities must become more extreme. When the regulator cannot commit not to bail out banks, the liquidity insurance market unravels.

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A Mathematical Appendix

A.1 Proof of Lemma 1

In our simple setup, firms do not need liquidity in the low shock state. Since only one Arrow-Debreu security is needed, it is enough to have a government bond to complete the markets for state-contingent claims on pledgeable income. In this proof we assume there are government bonds. The firm buys ℓ units of liquidity at date 0. The pledgeable income of good projects is shared in the following way

$$\mathfrak{R}_{OL} + \mathfrak{R}_{EL} = (\rho_0 - \rho_L)\, i_L + \ell \tag{15}$$

$$\mathfrak{R}_{OH} + \mathfrak{R}_{EH} = (\rho_0 - \rho_H)\, i_H + \ell \tag{16}$$

where $\mathfrak{R}_{OL}$ and $\mathfrak{R}_{EL}$ represent the income received by outside investors and the entrepreneur, respectively, in the low liquidity shock case (with $\mathfrak{R}_{EL} \geq 0$, because of limited liability). The values of $\mathfrak{R}_{OH}$ and $\mathfrak{R}_{EH}$ represent the income received by outside investors and the entrepreneur, respectively, in the high liquidity shock case (with $\mathfrak{R}_{EH} \geq 0$, because of limited liability). Without loss of generality, we assume that the funds A are entirely invested at $t = 0$.

In a pooling equilibrium, bad entrepreneurs mimic the good entrepreneurs. Hence, all entrepreneurs choose the same I, i_L and i_H, and the participation constraint of outside investors is

$$\alpha[f_{LG}\mathfrak{R}_{OL} + f_{HG}\mathfrak{R}_{OH}] + (1-\alpha)\,[f_{LB}\mathfrak{R}_{OL} + f_{HB}\mathfrak{R}_{OH}] \geq I - A + q\ell.$$

Using expressions (15) and (16), this constraint can be rewritten as

$$\overline{f}_L\,(\rho_0 - \rho_L)\, i_L + \overline{f}_H\,(\rho_0 - \rho_H)\, i_H - \overline{f}_L\mathfrak{R}_{EL} - \overline{f}_H\mathfrak{R}_{EH} \geq I - A + (q-1)\,\ell.$$

Hence, the good entrepreneur solves

$$\max_{\{I,i_L,i_H,\Re_{EL},\ell\}} f_{LG}(\rho_1-\rho_0)\,i_L + f_{HG}(\rho_1-\rho_0)\,i_H + f_{LG}\Re_{EL} + f_{LH}\Re_{EH} - A.$$

$$subject \quad to$$

$$\overline{f}_L(\rho_0-\rho_L)\,i_L + \overline{f}_H(\rho_0-\rho_H)\,i_H - \overline{f}_L\Re_{EL} - \overline{f}_H\Re_{EH} \geq I - A + (q-1)\,\ell$$

$$(\rho_H-\rho_0)\,i_H \leq \ell$$

$$0 \leq i_L, i_H \leq I$$

$$\Re_{EH}, \Re_{EL} \geq 0$$

where we considered the liquidity constraint. Variable i_L has a positive impact on the objective function, and since high values of i_L have no impact on the participation constraint, then $i_L = I$. Replace i_L with I in the maximization problem, and the objective function indicates that the entrepreneur wants to set I as high as possible. Expression (2) guarantees that the participation constraint of outside investors binds. Replacing this participation constraint into the objective function, one can see that $\Re_{EH} = 0$ since $f_{HG} < \overline{f}_H$. Moreover, liquidity ℓ has a negative impact on profit so that the liquidity constraint holds with equality if $q > 1$. Let $x = \frac{i_H}{I}$, and rewrite the participation constraint of outside investors as

$$I(x,q) = \frac{A - \overline{f}_L\Re_{EL}}{1 - \overline{f}_L(\rho_0-\rho_L) - \left(\overline{f}_H + q - 1\right)(\rho_0-\rho_H)\,x}.$$

Define $\Pi(x,q)$ as the expected profit of the entrepreneur as a function of x, that is,

$$\begin{aligned}\Pi(x,q) &= \left(\frac{(f_{LG}+f_{HG}x)(\rho_1-\rho_0)}{1 - \overline{f}_L(\rho_0-\rho_L) - \left(\overline{f}_H + q - 1\right)(\rho_0-\rho_H)\,x} - 1\right)A + \\ &\quad + \left(f_{LG} - \frac{(f_{LG}+f_{HG}x)(\rho_1-\rho_0)}{1 - \overline{f}_L(\rho_0-\rho_L) - \left(\overline{f}_H + q - 1\right)(\rho_0-\rho_H)\,x}\overline{f}_L\right)\Re_{EL}.\end{aligned}$$

Since the problem is linear, $i_H \in \{0, I\}$ so that $x \in \{0,1\}$. The term multiplying $\Re_{EL}$ is negative when $x = 0$ because of Assumption 1 (expression 1c). When $x = 1$, two cases are possible.

Case 1. If $q \leq \widehat{q} \equiv \frac{(1-f_{LG})\Omega}{f_{LG}(\rho_H-\rho_0)} + \overline{f}_L$, then the term multiplying $\Re_{EL}$ is negative because of Assumption 1 (expression 1c). To see this, write

$$\frac{f_{LG}}{\overline{f}_L} \leq \frac{\rho_1 - \rho_0}{1 - \overline{f}_L(\rho_0 - \rho_L) - \left(\overline{f}_H + q - 1\right)(\rho_0 - \rho_H)} \Leftrightarrow q \leq \frac{\overline{f}_L(\rho_1 - \rho_0) - \Omega}{f_{LG}(\rho_H - \rho_0)} + \widehat{q}$$

with $\overline{f}_L(\rho_1 - \rho_0) - \Omega > 0$ because of Assumption 1 (expression 1c). The term multiplying $\Re_{EL}$ is negative because $q \leq \widehat{q}$.

Case 2. If $q > \widehat{q}$, then $x = 0$ which is a contradiction. To see this, note that the problem is linear, and $i_H \in \{0, I\}$ so that it suffices to compare $\Pi(0,q)$ with $\Pi(1,q)$. The entrepreneur sets $i_H = I$ if and only if

$$\Pi(0,q) \leq \Pi(1,q) \Leftrightarrow q \leq \widehat{q}$$

which is a contradiction. Hence, it is not possible to have $q > \widehat{q}$ and $x = 1$.

To finish the proof, we conclude that the optimal value for $\Re_{EL}$ is zero and we can rewrite the maximization problem of good entrepreneurs as

$$\begin{aligned}
&\max_{\{I, i_L, i_H\}} f_{LG}(\rho_1 - \rho_0)\, i_L + f_{HG}(\rho_1 - \rho_0)\, i_H - A. \\
&\textit{subject to} \\
&\overline{f}_L(\rho_0 - \rho_L)\, i_L + \left(\overline{f}_H + q - 1\right)(\rho_0 - \rho_H)\, i_H \geq I - A \\
&0 \leq i_L, i_H \leq I.
\end{aligned}$$

Bad entrepreneurs pool with good entrepreneurs, and f_{LB} and f_{HB} replace f_{LG} and f_{HG} in their profit function.■

A.2 Proof of Proposition 1

We start by solving Problem 5. Variable i_L has a positive impact on the objective function, and since high values of i_L have no impact on the participation constraint, then $i_L = I$. Replace i_L with I in the maximization problem, and the objective function indicates that the entrepreneur wants to set I as high as possible. Expression (2) guarantees that the participation constraint of outside investors binds, and one obtains the investment function 6. Define $\Pi(x,q)$ as the expected profit of the entrepreneur as a function of x and the market price of liquidity, that is

$$\Pi(x,q) = \left(\frac{(f_{LG} + f_{HG}x)(\rho_1 - \rho_0)}{\Omega - \left(\overline{f}_H + q - 1\right)(\rho_0 - \rho_H)x} - 1 \right) A.$$

Since the problem is linear, $i_H \in \{0, I\}$ and it suffices to compare $\Pi(0,q)$ with $\Pi(1,q)$. Hence, the entrepreneur sets $i_H = I$ if and only if $\Pi(0,q) \leq \Pi(1,q)$ and we obtain the condition for continuation in the high shock state (7). In Proposition 1 we consider the case $\widehat{q} \geq 1$ and in Proposition 2 we consider the case $\widehat{q} < 1$. Consider the case $\widehat{q} \geq 1$.

For the existence of a pooling equilibrium, two conditions must be fulfilled:

1. Both types of entrepreneur want to pool, that is the participation constraints of good and bad entrepreneurs (4) are satisfied.
2. Good entrepreneurs do not want to deviate from the pooling equilibrium. They cannot make a profit with a contract that bad entrepreneurs reject.

We check these conditions in turn.

Participation constraints of both types of entrepreneur With $i_H = i_L = I = \frac{A}{1-\overline{f}_L(\rho_0-\rho_L)-\overline{f}_H(\rho_0-\rho_H)}$, one can write participation constraint of bad entrepreneurs as

$$\pi\left(I, I, I; f_{LB}, A\right) \geq 0 \Leftrightarrow \rho_1 \geq 1 + \overline{f}_L \rho_L + \overline{f}_H \rho_H,$$

which is satisfied due to Assumption 1 (expression 1c). The participation constraint of good entrepreneurs is also satisfied.

Good firms do not want to deviate The following lemma establishes the profit of a good entrepreneur when he deviates from the pooling equilibrium.

Lemma 2 *The maximum profit of a good entrepreneur with a deviation from a pooling equilibrium is equal to* $\Pi_D = \left(\frac{f_{LG}}{f_{LB}} - 1\right) A$.

Proof. Good entrepreneurs can signal who they are, by proposing a contract that bad entrepreneurs do not want. Consider a separating contract with

$$f_{LB}\left(\rho_1 - \rho_0\right) i_L + f_{HB}\left(\rho_1 - \rho_0\right) i_H + f_{LB}\mathfrak{R}_{EL} - A \leq 0$$

where we have already set $\mathfrak{R}_{EH} = 0$, since raising $\mathfrak{R}_{EH}$ above zero benefits more bad entrepreneurs than good ones. A contract satisfying this restriction does not attract bad entrepreneurs, as it violates their participation constraint. An optimal contract for the good entrepreneur, which does not attract bad entrepreneurs, must solve the following problem

$$\begin{aligned}
&\max_{\{I,i_L,i_H\}} f_{LG}(\rho_1-\rho_0)\,i_L + f_{HG}(\rho_1-\rho_0)\,i_H + f_{LG}\mathfrak{R}_{EL} - A \\
&subject \quad to \\
&f_{LG}(\rho_0-\rho_L)\,i_L + (f_{HG}+q-1)(\rho_0-\rho_H)\,i_H + f_{LG}\mathfrak{R}_{EL} \geq I - A \\
&f_{LB}(\rho_1-\rho_0)\,i_L + f_{HB}(\rho_1-\rho_0)\,i_H + f_{LB}\mathfrak{R}_{EL} - A \leq 0 \\
&0 \leq i_L, i_H \leq I \\
&\mathfrak{R}_{EL} \geq 0.
\end{aligned}$$

where we have substituted expressions (15) and (16) into the participation constraint of outside investors. The participation constraint binds, and replace it in the objective function, so as to obtain

$$f_{LG}(\rho_1-\rho_L)\,i_L + f_{HG}(\rho_1-\rho_H)\,i_H - I - (q-1)(\rho_H-\rho_0)\,i_H$$

Since $\mathfrak{R}_{EL}$ has no impact on the objective function, but tightens the participation and the signaling constraints, then $\mathfrak{R}_{EL} = 0$. The problem of good entrepreneurs can be rewritten as

$$\begin{aligned}
&\max_{\{I,i_L,i_H\}} \pi(I, i_L, i_H; f_{LG}, A) \\
&subject \quad to \\
&f_{LG}(\rho_0-\rho_L)\,i_L + (f_{HG}+q-1)(\rho_0-\rho_H)\,i_H = I - A \\
&\pi(I, i_L, i_H; f_{LB}, A) \leq 0 \\
&0 \leq i_L, i_H \leq I
\end{aligned}$$

The signaling constraint $\pi(I, i_L, i_H; f_{LB}, A) \leq 0$ binds. Suppose it did not bind. Then good entrepreneurs would act as if they were alone, and they would be solving the first-best. But bad entrepreneurs want to mimic in the first-best. Hence a contradiction, and the signaling constraint binds.

Since this constraint binds, one can write i_H as a function of i_L. Good entrepreneurs can separate from the bad entrepreneurs by picking the right combination (i_L, i_H). In this case,

$$\begin{aligned} & f_{LB}(\rho_1 - \rho_0)\, i_L + f_{HB}(\rho_1 - \rho_0)\, i_H - A = 0 \\ \Leftrightarrow\quad & i_H = i_H(i_L) = \frac{A}{f_{HB}(\rho_1 - \rho_0)} - \frac{f_{LB}}{f_{HB}} i_L. \end{aligned} \tag{17}$$

Write the maximization problem as

$$\begin{aligned} & \max_{\{I, i_L, i_H\}} \left(f_{LG} - \frac{f_{HG} f_{LB}}{f_{HB}} \right)(\rho_1 - \rho_0)\, i_L + \left(\frac{f_{HG}}{f_{HB}} - 1 \right) A \\ & \textit{subject to} \\ & \left[f_{LG}(\rho_0 - \rho_L) - \frac{(f_{HG} + q - 1) f_{LB}}{f_{HB}} (\rho_0 - \rho_H) \right] i_L + \left[\frac{(f_{HG} + q - 1)(\rho_0 - \rho_H)}{f_{HB}(\rho_1 - \rho_0)} + 1 \right] A = I \\ & 0 \le i_L, i_H(i_L) \le I \end{aligned}$$

Since the objective function and the left-hand side of the participation constraint of outside investors are increasing in i_L, then i_L should be as large as possible. As a result, one of the two constraints, $i_L \le I$ or $0 \le i_H(i_L) \Leftrightarrow i_L \le \frac{A}{f_{LB}(\rho_1 - \rho_0)}$ must bind, and $i_L = \min\left\{ \frac{A}{f_{LB}(\rho_1 - \rho_0)}, I \right\}$. Hence, two cases are possible.

Case 1. Setting $i_L = I < \frac{A}{f_{LB}(\rho_1 - \rho_0)}$ (and $i_H > 0$) is not feasible. Suppose it was feasible, and let $i_L = I$. The participation constraint of outside investors binds and $I = \frac{(f_{HG}+q-1)(\rho_0 - \rho_H) + f_{HB}(\rho_1 - \rho_0)}{1 - f_{LG}(\rho_0 - \rho_L) + \frac{(f_{HG}+q-1) f_{LB}}{f_{HB}} (\rho_0 - \rho_H)} \frac{A}{f_{HB}(\rho_1 - \rho_0)}$, so that one can write $I < \frac{A}{f_{LB}(\rho_1 - \rho_0)} \Leftrightarrow 1 > f_{LG}(\rho_0 - \rho_L) + f_{LB}(\rho_1 - \rho_0)$. Since Assumption 2 states that $f_{LB}(\rho_1 - \rho_0) + \overline{f}_L(\rho_0 - \rho_L) \ge 1$, then $f_{LB}(\rho_1 - \rho_0) + f_{LG}(\rho_0 - \rho_L) \ge 1$ which is a contradiction.

Case 2. Setting $i_L = \frac{A}{f_{LB}(\rho_1 - \rho_0)}$ (and $i_H = 0$) is optimal. When $i_L = \frac{A}{f_{LB}(\rho_1 - \rho_0)}$, the participation constraint of outside investors becomes

$$f_{LG}(\rho_0 - \rho_L) i_L \geq I - A \Leftrightarrow \frac{f_{LB}(\rho_1 - \rho_0) + f_{LG}(\rho_0 - \rho_L)}{f_{LB}(\rho_1 - \rho_0)} A = I.$$

Since $i_L = \min\left\{\frac{A}{f_{LB}(\rho_1 - \rho_0)}, I\right\}$, it must be the case that $\frac{A}{f_{LB}(\rho_1 - \rho_0)} \leq I$, and

$$\frac{A}{f_{LB}(\rho_1 - \rho_0)} \leq \frac{f_{LB}(\rho_1 - \rho_0) + f_{LG}(\rho_0 - \rho_L)}{f_{LB}(\rho_1 - \rho_0)} A \Leftrightarrow 1 \leq f_{LB}(\rho_1 - \rho_0) + f_{LG}(\rho_0 - \rho_L).$$

This condition holds, because $f_{LG} > \overline{f}_L$ in Assumption 2. Hence, the profit of a deviation for the good entrepreneur equals $\Pi_D = \left(\frac{f_{LG}}{f_{LB}} - 1\right) A$. ■

We compare the profit with pooling with insurance $\Pi(1, q)$ with the profit Π_D. First, note that $\Pi(1, q) \geq \Pi(0, q)$ for $\widehat{q} \geq 1$. There is not a profitable deviation when

$$\Pi(0, q) \geq \Pi_D \Leftrightarrow f_{LB}(\rho_1 - \rho_0) + \overline{f}_L(\rho_0 - \rho_L) \geq 1.$$

Due to Assumption 2, the above condition is satisfied and there is no profitable deviation for the good entrepreneurs. We have a pooling equilibrium.■

A.3 Proof of Proposition 2

The proof follows the same steps as the proof of Proposition 1.

Participation constraints of both types of entrepreneur Assumption 2 guarantees that the participation constraint of bad entrepreneurs

$$\pi\left(\frac{A}{1 - \overline{f}_L(\rho_0 - \rho_L)}, \frac{A}{1 - \overline{f}_L(\rho_0 - \rho_L)}, 0; f_{LB}, A\right) \geq 0$$

is satisfied. The participation constraint of good entrepreneurs is satisfied, since it is looser than the participation constraint of bad entrepreneurs.

Good firms do not want to deviate We compare the profit with pooling without insurance

$$\Pi(0,q)=\pi\left(\frac{A}{1-\overline{f}_L(\rho_0-\rho_L)},\frac{A}{1-\overline{f}_L(\rho_0-\rho_L)},0;f_{LG},A\right)=\left(\frac{f_{LG}(\rho_1-\rho_0)}{1-\overline{f}_L(\rho_0-\rho_L)}-1\right)A$$

with the profit Π_D obtained in Lemma 2. There is not a profitable deviation when

$$\Pi(0,q)\geq\Pi_D\Leftrightarrow f_{LB}(\rho_1-\rho_0)+\overline{f}_L(\rho_0-\rho_L)\geq 1.$$

Due to Assumption 2, the above condition is satisfied and there is no profitable deviation for the good entrepreneurs. We have a pooling equilibrium.■

A.4 Proof of Proposition 3

The boundary for α is

$$\alpha>\frac{1-f_{HB}(\rho_0-\rho_H)-f_{LB}(\rho_0-\rho_L)+\frac{f_{LB}(\rho_1-\rho_0)^2}{1-f_{LG}(\rho_1-\rho_L)-f_{LB}(\rho_1-\rho_0)}}{(f_{LG}-f_{LB})(\rho_H-\rho_L)}.$$

Bad entrepreneurs are not socially useful ex ante, and the central planner may want to offer contracts which exclude them. The central planner can fix i_L and i_H such that it is able to separate the good from the bad entrepreneurs. The latter become outside investors. In this case, the central planner solves

$$\begin{aligned}
&\max_{\{I,i_L,i_H\}} f_{LG}(\rho_1-\rho_L)\,i_L + f_{HG}(\rho_1-\rho_H)\,i_H - I \\
&\textit{subject to} \\
&f_{LG}(\rho_0-\rho_L)\,i_L + f_{HG}(\rho_0-\rho_H)\,i_H \geq I - A \\
&f_{LB}(\rho_1-\rho_0)\,i_L + f_{HB}(\rho_1-\rho_0)\,i_H - A \leq 0 \\
&0 \leq i_L, i_H \leq I.
\end{aligned}$$

The signaling constraint binds, and we obtain the same function $i_H(i_L)$ which we obtained as a solution to the problem of the good entrepreneur in expression (17). Hence, we can write

$$\begin{aligned}
&\max_{\{I,i_L,i_H\}} \left[f_{LG}(\rho_1-\rho_L) - \frac{f_{HG}f_{LB}}{f_{HB}}(\rho_1-\rho_H)\right] i_L + \frac{f_{HG}(\rho_1-\rho_H)}{f_{HB}(\rho_1-\rho_0)}A - I \\
&\textit{subject to} \\
&\left[f_{LG}(\rho_0-\rho_L) - \frac{f_{HG}f_{LB}}{f_{HB}}(\rho_0-\rho_H)\right] i_L + \left[1 + \frac{f_{HG}(\rho_0-\rho_H)}{f_{HB}(\rho_1-\rho_0)}\right] A \geq I \\
&0 \leq i_L, i_H(i_L) \leq I.
\end{aligned}$$

Since the participation constraint of outside investors is the same as in the problem of the good entrepreneur with $q = 1$ (see Lemma 2), then setting $i_L = I < \frac{A}{f_{LB}(\rho_1-\rho_0)}$ is not feasible, and the central planner may want to set $i_L = \frac{A}{f_{LB}(\rho_1-\rho_0)}$ (and $i_H = 0$). The central planner compares the social welfare with pooling with the social welfare with separation between good and bad entrepreneurs. Social welfare with separation equals

$$U_S = f_{LG}(\rho_1-\rho_L)\,i_L + f_{HG}(\rho_1-\rho_H)\,i_H(i_L) - I = \frac{f_{LG}(\rho_1-\rho_L)}{f_{LB}(\rho_1-\rho_0)}A - I,$$

and the central planner wants to set the scale of the project equal to $i_L = \frac{A}{f_{LB}(\rho_1-\rho_0)}$, so that $U_S = \frac{f_{LG}(\rho_1-\rho_L)-1}{f_{LB}(\rho_1-\rho_0)}A$.

Under Assumption 1 (expression 1c), social welfare with pooling equals

$$U(1) = \left[\frac{\rho_1 - \rho_0}{1 - \overline{f}_L (\rho_0 - \rho_L) - \overline{f}_H (\rho_0 - \rho_H)} - 1 \right] A,$$

and $U(1) > U_S$ because $\alpha > \frac{1 - f_{HB}(\rho_0 - \rho_H) - f_{LB}(\rho_0 - \rho_L) + \frac{f_{LB}(\rho_1 - \rho_0)(\rho_1 - \rho_0)}{1 - f_{LG}(\rho_1 - \rho_L) - f_{LB}(\rho_1 - \rho_0)}}{(f_{LG} - f_{LB})(\rho_0 - \rho_L) + (f_{HG} - f_{HB})(\rho_0 - \rho_H)}$. ■

A.5 Proof of Proposition 4

The proof has two parts. In the first part, we assume there is a pooling equilibrium and we describe the profit function of entrepreneurs. We then compute the threshold $\overline{q}$, which we use to compute the aggregate demand for liquidity and the equilibrium price of liquidity. In the second part, we compute the pooling equilibrium in financial markets.

Part 1. We start with Lemma 3, which is the analog of Lemma 1 in the idiosyncratic shocks case.

Lemma 3 *Under Assumptions 1 and 3, and pooling, the expected profit of each type of entrepreneur is given by*

$$\pi(I, i_L, i, i_H; A) = f_{LB}(\rho_1 - \rho_0) i_L + (1 - f_{LB} - f_{HG})(\rho_1 - \rho_0) i + f_{HG}(\rho_1 - \rho_0) i_H - A,$$

the participation constraint of outside investors is given by

$$f_{LG}(\rho_0 - \rho_L) i_L + (1 - f_{LG} - f_{HG})(\rho_0 - \rho_L) \alpha i_{LH} + (f_{HG} + q - 1)(\rho_0 - \rho_H) i_H = I - A,$$

and the participation constraints of good and bad entrepreneurs are

$$\pi(I, i_L, i, i_H; A) \geq 0,$$

with $i \in \{0, i_{LH}\}$ depending on the type of entrepreneur. The liquidity constraint (10) holds with equality when $q > 1$.

Proof. In our simple setup, good firms do not need liquidity in states $\{\rho_L \rho_L\}$ and $\{\rho_L \rho_H\}$. Since only one Arrow-Debreu security is needed, it is enough to have a government bond to complete the markets. In this proof we assume there are government bonds. The firm buys ℓ units of liquidity at date 0. The pledgeable income of good projects is shared in the following way

$$\begin{aligned} \Re_{OL} + \Re_{EL} &= (\rho_0 - \rho_L)\, i_L + \ell \\ \Re_{OLH} + \Re_{ELH} &= (\rho_0 - \rho_L)\, i_{LH} + \ell \\ \Re_{OH} + \Re_{EH} &= (\rho_0 - \rho_H)\, i_H + \ell \end{aligned}$$

where $\Re_{OH}$ and $\Re_{EH}$ represent the income received by outside investors and the entrepreneur in state $\{\rho_H \rho_H\}$, respectively. Also, $\Re_{OL}$ and $\Re_{EL}$ represent the income received by outside investors and the entrepreneur in state $\{\rho_L \rho_L\}$, respectively, and $\Re_{OLH}$ and $\Re_{ELH}$ represent the income received by outside investors and the entrepreneur in state $\{\rho_L \rho_H\}$, respectively. Limited liability implies $\Re_{EL} \geq 0$, $\Re_{EH} \geq 0$, and $\Re_{ELH} \geq 0$. The participation constraint of outside investors equals

$$f_{LB} \Re_{OL} + (1 - f_{LB} - f_{HG})\, \Re_{OLH} \alpha + (1 - f_{LB} - f_{HG})\, (1 - \alpha)\, \ell + f_{HG} \Re_{OH} \geq I - A + q\ell$$

and, replacing the shares of outside investors in good projects in the participation constraint, yields

$$\begin{aligned} & f_{LB}\, (\rho_0 - \rho_L)\, i_L + (1 - f_{LB} - f_{HG})\, (\rho_0 - \rho_L)\, i_{LH} \alpha + f_{HG}\, (\rho_0 - \rho_H)\, i_H \\ - \; & f_{LB} \Re_{EL} - (1 - f_{LB} - f_{HG})\, \Re_{ELH} \alpha - f_{HG} \Re_{EH} \geq I - A + (q - 1)\, \ell \end{aligned}$$

The profit of a good entrepreneur equals

$$f_{LB}(\rho_1-\rho_0)\,i_L+(1-f_{LB}-f_{HG})(\rho_1-\rho_0)\,i_{LH}+f_{HG}(\rho_1-\rho_0)\,i_H$$
$$+f_{LB}\Re_{EL}+(1-f_{LB}-f_{HG})\,\Re_{ELH}+f_{HG}\Re_{EH}-A.$$

Finally, there is the liquidity constraint (10).

The entrepreneur wishes to set i_L and i_{LH} as high as possible, so that $i_L=i_{LH}=I$, and the participation constraint of outside investors binds. Replacing this constraint into the profit function, the terms $\Re_{EL}$ and $\Re_{EH}$ vanish. Since these terms tighten the participation constraint of outside investors, then $\Re_{EL}=\Re_{EH}=0$. Moreover, liquidity ℓ has a negative impact on profit so that the liquidity constraint (10) holds with equality if $q>1$.

Using these results in the participation constraint of outside investors, we obtain the investment function

$$I(x,q)=\frac{A-(1-f_{LB}-f_{HG})\,\alpha\Re_{ELH}}{1-f_{LB}(\rho_0-\rho_L)-(1-f_{LB}-f_{HG})(\rho_0-\rho_L)\,\alpha-(f_{HG}+q-1)(\rho_0-\rho_H)\,x}.$$

Replacing investment in the profit function, yields

$$\begin{aligned}\Pi(x,q) &= \left[\frac{(1-f_{HG}+f_{HG}x)(\rho_1-\rho_0)}{\Omega-(f_{HG}+q-1)(\rho_0-\rho_H)\,x}-1\right]A \\ &\quad+\left[1-\frac{(1-f_{HG}+f_{HG}x)(\rho_1-\rho_0)\,\alpha}{\Omega-(f_{HG}+q-1)(\rho_0-\rho_H)\,x}\right](1-f_{LB}-f_{HG})\,\Re_{ELH}.\end{aligned}$$

Since the problem is linear, $i_H\in\{0,I\}$ so that $x\in\{0,1\}$. The term multiplying $\Re_{ELH}$ is negative when $x=0$ because of Assumption 3. When $x=1$, two cases are possible.

Case 1. If $q\leq\overline{q}=\frac{(1-f_{LG})\Omega}{f_{LG}(\rho_H-\rho_0)}+f_{LG}$, then the term multiplying $\Re_{ELH}$ is negative because of Assumption 3. To see this, write

$$1 < \frac{(\rho_1 - \rho_0)\,\alpha}{\Omega - (f_{HG} + q - 1)\,(\rho_0 - \rho_H)} \Leftrightarrow q < \frac{f_{LG}\,(\rho_1 - \rho_0)\,\alpha - \Omega}{f_{LG}\,(\rho_H - \rho_0)} + \overline{q}$$

with $f_{LG}\,(\rho_1 - \rho_0)\,\alpha - \Omega > 0$ because of Assumption 3. The term multiplying $\Re_{ELH}$ is negative because $q \leq \overline{q}$.

Case 2. If $q > \overline{q}$, then $x = 0$ which is a contradiction. To see this, note that the problem is linear, and $i_H \in \{0, I\}$ so that it suffices to compare $\Pi(0, q)$ with $\Pi(1, q)$. The entrepreneur sets $i_H = I$ if and only if

$$\Pi(0, q) \leq \Pi(1, q) \Leftrightarrow q \leq \overline{q}$$

which is a contradiction. Hence, it is not possible to have $q > \overline{q}$ and $x = 1$.

To finish the proof, we conclude that the optimal value for $\Re_{EL}$ is zero and we can rewrite the maximization problem of good entrepreneurs as

$$\begin{aligned}
&\max_{\{I, i_L, i_H, i_{LH}\}} f_{LB}\,(\rho_1 - \rho_0)\,i_L + (1 - f_{LB} - f_{HG})\,(\rho_1 - \rho_0)\,i_{LH} + f_{HG}\,(\rho_1 - \rho_0)\,i_H - A \\
&\textit{subject to} \\
&f_{LB}\,(\rho_0 - \rho_L)\,i_L + (1 - f_{LB} - f_{HG})\,(\rho_0 - \rho_L)\,i_{LH}\alpha + f_{HG}\,(\rho_0 - \rho_H)\,i_H = I - A + (q-1)\,\ell \\
&(\rho_H - \rho_0)\,i_H \leq \ell. \\
&0 \leq i_L, i_H, i_{LH} \leq I.
\end{aligned}$$

Since the liquidity constraint (10) binds when $q > 1$, we obtain the results for good entrepreneurs. Bad entrepreneurs pool with good ones, and set $i_{LH} = 0$. ■

All pledgeable income is given to outside investors, and bad entrepreneurs liquidate their projects in state $\{\rho_L \rho_H\}$. The participation constraint of outside investors shows that considering the existence of liquidity at a price $q > 1$ makes investment

i_H comparatively more expensive. Good entrepreneurs set $i_L = i_{LH} = I$ and, from the participation constraint of outside investors, we obtain the following investment function

$$I(x,q) = \frac{A}{\Omega - (f_{HG} + q - 1)(\rho_0 - \rho_H)x}$$

and the expected profit of good entrepreneurs equals

$$\Pi(x,q) = \left[\frac{(1 - f_{HG} + f_{HG}x)(\rho_1 - \rho_0)}{\Omega - (f_{HG} + q - 1)(\rho_0 - \rho_H)x} - 1\right] A.$$

Since the problem is linear it is optimal to set $x \in \{0,1\}$. Setting $i_H = I$ is optimal when $\Pi(1,q) \geq \Pi(0,q)$. Function $\Pi(1,q)$ is decreasing in q, while $\Pi(0,q)$ is constant. At the threshold $\overline{q}$, we obtain $\Pi(1,\overline{q}) = \Pi(0,\overline{q})$.

Since all firms are identical, the aggregate demand for liquidity by the corporate sector equals

$$L_D(q) = (\rho_H - \rho_0) I(1,q) = \frac{(\rho_H - \rho_0) A}{\Omega - (f_{HG} + q - 1)(\rho_0 - \rho_H)} \qquad \text{if} \qquad 1 \leq q < \overline{q},$$

and $L_D(\overline{q}) \in \left[0, \frac{(1-f_{HG})(\rho_H-\rho_0)A}{\Omega}\right]$. The demand for liquidity by consumers is perfectly elastic at $q = 1$. The equilibrium price of liquidity q_e is found by equating the fixed supply of outside liquidity L_S to the demand. When $L_S \leq \frac{(\rho_H-\rho_0)A}{\Omega-(f_{HG}+\overline{q}-1)(\rho_0-\rho_H)} = \frac{(1-f_{HG})(\rho_H-\rho_0)A}{\Omega}$, we assume without loss of generality that $L_D(\overline{q}) = L_S$ and the proportion $\frac{\alpha}{1-\alpha}$ between good and bad entrepreneurs in the demand for liquidity is maintained for $q = \overline{q}$.[20] The equilibrium price is

$$q_e = \begin{cases} \overline{q} & if \quad 0 < L_S \leq \frac{(1-f_{HG})(\rho_H-\rho_0)A}{\Omega} \\ \frac{A}{L_S} - \frac{\Omega}{\rho_H-\rho_0} - f_{HG} + 1 & if \quad \frac{(1-f_{HG})(\rho_H-\rho_0)A}{\Omega} < L_S < L_D(1) \\ 1 & if \quad L_S \geq L_D(1) \end{cases}$$

[20] Or, alternatively, all entrepreneurs set the same value $i_H < I$.

Part 2. For the existence of a pooling equilibrium, two conditions must be fulfilled. Both types of entrepreneur want to pool, and good entrepreneurs do not want to deviate from the pooling equilibrium. We check these conditions for the two possible cases.

A.5.1 Pooling equilibrium when $\overline{q} > q \geq 1$

Participation constraints of both types of entrepreneur Assumption 2 guarantees that the participation constraint of bad entrepreneurs is satisfied. To see this, use the profit of the bad entrepreneur to write

$$(f_{LB} + f_{HG})(\rho_1 - \rho_0) I(1, q) - A \geq 0 \Leftrightarrow \left[\frac{(f_{LB} + f_{HG})(\rho_1 - \rho_0)}{\Omega - (f_{HG} + q - 1)(\rho_0 - \rho_H)} - 1\right] A \geq 0 \Leftrightarrow$$
$$f_{LB}(\rho_1 - \rho_0) + f_{LB}(\rho_0 - \rho_L) + (1 - f_{LB} - f_{HG}) \alpha (\rho_0 - \rho_L)$$
$$+ f_{HG}(\rho_1 - \rho_H) + (q - 1)(\rho_0 - \rho_H) \geq 1$$

Assumption 2 guarantees that $f_{LB}(\rho_1 - \rho_0) + f_{LB}(\rho_0 - \rho_L) + (1 - f_{LB} - f_{HG}) \alpha (\rho_0 - \rho_L) \geq 1$. Moreover, $f_{HG}(\rho_1 - \rho_H) + (q - 1)(\rho_0 - \rho_H) \geq 0$. To see why this expression is positive, note first that it takes its minimum value for $q = \overline{q}$. Computing

$$f_{HG}(\rho_1 - \rho_H) + (\overline{q} - 1)(\rho_0 - \rho_H) = \frac{f_{HG}}{1 - f_{HG}} [(1 - f_{HG})(\rho_1 - \rho_0) - \Omega] > 0$$

by Assumption 2. The participation constraint of good entrepreneurs is looser than the participation constraint of bad entrepreneurs. Hence, it is satisfied.

Good firms do not want to deviate When good entrepreneurs consider a deviation, they solve

$$\max_{\{I,i_L,i_{LH}.i_H,\ell\}} f_{LB}(\rho_1-\rho_0)\, i_L + (1-f_{LB}-f_{HG})(\rho_1-\rho_0)\, i_{LH} + f_{HG}(\rho_1-\rho_0)\, i_H$$
$$+f_{LB}\mathfrak{R}_{EL} + (1-f_{LB}-f_{HG})\,\mathfrak{R}_{ELH} + f_{HG}\mathfrak{R}_{EH} - A \qquad (18)$$

subject to

$$\mathfrak{R}_{OL} + \mathfrak{R}_{EL} = (\rho_0-\rho_L)\, i_L + \ell$$

$$\mathfrak{R}_{OLH} + \mathfrak{R}_{ELH} = (\rho_0-\rho_L)\, i_{LH} + \ell$$

$$\mathfrak{R}_{OH} + \mathfrak{R}_{EH} = (\rho_0-\rho_H)\, i_H + \ell$$

$$f_{LB}\mathfrak{R}_{OL} + (1-f_{LB}-f_{HG})\,\mathfrak{R}_{OLH} + f_{HG}\mathfrak{R}_{OH} \geq I - A + ql$$

$$f_{LB}(\rho_1-\rho_0)\, i_L + f_{HG}(\rho_1-\rho_0)\, i_H + f_{LB}\mathfrak{R}_{EL} + f_{HG}\mathfrak{R}_{EH} - A \leq 0$$

$$(\rho_H-\rho_0)\, i_H \leq \ell$$

$$0 \leq i_L, i_{LH}, i_H \leq I$$

$$\mathfrak{R}_{EL}, \mathfrak{R}_{ELH}, \mathfrak{R}_{EH} \geq 0$$

knowing that $\overline{q} > q \geq 1$. Replacing the equations regarding the division of pledgeable income into the participation constraint of outside investors, yields

$$f_{LB}(\rho_0-\rho_L)\, i_L + (1-f_{LB}-f_{HG})(\rho_0-\rho_L)\, i_{LH} + f_{HG}(\rho_0-\rho_H)\, i_H$$
$$-f_{LB}\mathfrak{R}_{EL} - (1-f_{LB}-f_{HG})\,\mathfrak{R}_{ELH} - f_{HG}\mathfrak{R}_{EH} \geq I - A + (q-1)\,\ell.$$

This participation constraint binds, and replacing it in the objective function we see that the terms $\mathfrak{R}_{EL}, \mathfrak{R}_{ELH}$ and $\mathfrak{R}_{EH}$ vanish. Since these terms tighten the participation constraint of outside investors and the signaling constraint, then $\mathfrak{R}_{EL} = \mathfrak{R}_{ELH} = \mathfrak{R}_{EH} = 0$. Liquidity ℓ has a negative impact on the objective function, so that $\ell = (\rho_H - \rho_0)\, i_H$ when $q > 1$. The signaling constraint binds, and we can rewrite the maximization problem as

$$\max_{\{I,i_L,i_{LH}.i_H,\ell\}} (1 - f_{LB} - f_{HG}) (\rho_1 - \rho_0) i_{LH}$$
$$subject \ \ to$$
$$f_{LB} (\rho_0 - \rho_L) i_L + (1 - f_{LB} - f_{HG}) (\rho_0 - \rho_L) i_{LH} + (f_{HG} + q - 1) (\rho_0 - \rho_H) i_H = I - A$$
$$f_{LB} (\rho_1 - \rho_0) i_L + f_{HG} (\rho_1 - \rho_0) i_H - A = 0$$
$$0 \leq i_L, i_{LH}, i_H \leq I.$$

The value of i_{LH} should be as high as possible, so that $i_{LH} = I$. Use the participation constraint of outside investors to write investment as

$$I = \frac{f_{LB} (\rho_0 - \rho_L) i_L + (f_{HG} + q - 1) (\rho_0 - \rho_H) i_H + A}{1 - (1 - f_{LB} - f_{HG}) (\rho_0 - \rho_L)}$$

and replacing $i_{LH} = I$ in the objective function, profit equals

$$\frac{(1 - f_{LB} - f_{HG}) (\rho_1 - \rho_0)}{1 - (1 - f_{LB} - f_{HG}) (\rho_0 - \rho_L)} [f_{LB} (\rho_0 - \rho_L) i_L + (f_{HG} + q - 1) (\rho_0 - \rho_H) i_H + A]$$

so that the isoprofit curves have a positive slope equal to $\frac{f_{LB}(\rho_0 - \rho_L)}{(f_{HG}+q-1)(\rho_H - \rho_0)}$ in the plane (i_L, i_H) since $q \geq 1$. The signaling constraint can be represented as

$$i_H = -\frac{f_{LB}}{f_{HG}} i_L + \frac{A}{f_{HG} (\rho_1 - \rho_0)}$$

and the constraint $i_L \leq I$ can be written as

$$i_L \leq \frac{f_{LB} (\rho_0 - \rho_L) i_L + (f_{HG} + q - 1) (\rho_0 - \rho_H) i_H + A}{1 - (1 - f_{LB} - f_{HG}) (\rho_0 - \rho_L)}$$
$$\Leftrightarrow \ \ i_H \leq \frac{1 - f_{LG} (\rho_0 - \rho_L)}{(f_{HG} + q - 1) (\rho_0 - \rho_H)} i_L - \frac{A}{(f_{HG} + q - 1) (\rho_0 - \rho_H)}$$

with $1 - f_{LG} (\rho_0 - \rho_L) > 0 \Leftrightarrow \rho_0 < \frac{1+f_{LG}\rho_L}{f_{LG}}$ by Assumption 1 (expression 1a).

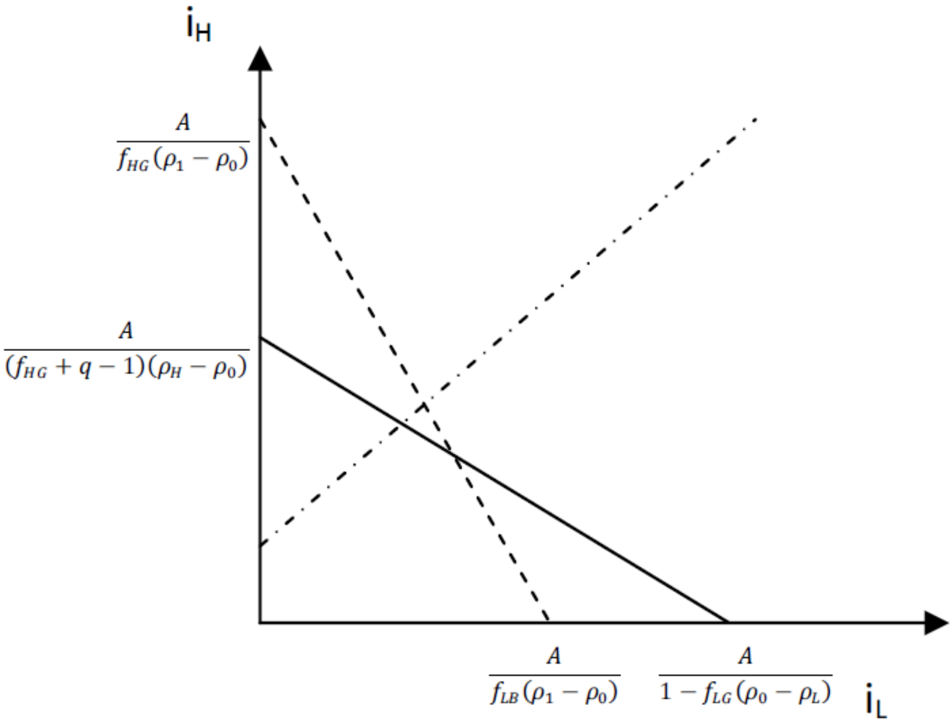

Figure 6: Isoprofit line (dash-dotted line), signaling constraint (dashed line), and $i_L \le I$ line (solid line).

Both constraints have negative slopes in the plane (i_L, i_H). Since the isoprofit curves have positive slopes and profit is increasing with i_L, then $i_H = 0$ and $i_L = \min\left\{\frac{A}{f_{LB}(\rho_1-\rho_0)}, \frac{A}{1-f_{LG}(\rho_0-\rho_L)}\right\}$. See Figure 6.[21] Assumption 2 guarantees that $\frac{A}{f_{LB}(\rho_1-\rho_0)} \le \frac{A}{1-f_{LG}(\rho_0-\rho_L)} \Leftrightarrow f_{LB}(\rho_1-\rho_0) \ge 1 - f_{LG}(\rho_0-\rho_L)$, because $\overline{f}_L < f_{LG}$ and Assumption 2 can be written as $f_{LB}(\rho_1-\rho_0) \ge 1-\overline{f}_L(\rho_0-\rho_L)$. Hence, $i_H = 0$ and $i_L = \frac{A}{f_{LB}(\rho_1-\rho_0)} \le I$ (to see the inequality, replace $i_L = \frac{A}{f_{LB}(\rho_1-\rho_0)}$ and $i_H = 0$ in the expression for investment, and Assumption 2 guarantees that $i_L \le I$).

In this case, the separating contract has no insurance, so that $\ell = 0$. The participation constraint of outside investors yields a level of investment equal to $\frac{\frac{\rho_1-\rho_L}{\rho_1-\rho_0}}{1-(1-f_{LB}-f_{HG})(\rho_0-\rho_L)}A$,

[21] If $i_L = \frac{A}{1-f_{LG}(\rho_0-\rho_L)}$ then $i_L = I$, but the signaling constraint would not bind. This would mean that there are no incentive problems since the bad entrepreneurs would never invest in the project. This case is not interesting.

and profit with the deviation equals

$$\Pi_S = \frac{(1 - f_{LB} - f_{HG})(\rho_1 - \rho_L)}{1 - (1 - f_{LB} - f_{HG})(\rho_0 - \rho_L)} A$$

Recall that $\Pi(1, q) \geq \Pi(0, q)$ for $q < \overline{q}$. Assumption 4 guarantees that $\Pi_S \leq \Pi(0, q)$. Hence, there is no profitable deviation.

A.5.2 Pooling equilibrium when $\overline{q} = q \geq 1$

This is the case when $0 < L_S \leq \frac{(1-f_{HG})(\rho_H - \rho_0)A}{\Omega}$. At $q = \overline{q}$, good firms are indifferent between continuing projects in state $\{\rho_H \rho_H\}$ or not, as they obtain the same profit in both alternatives. The profit of good firms equals $\Pi(1, \overline{q}) = \left[\frac{\rho_1 - \rho_0}{\Omega - (f_{HG} + \overline{q} - 1)(\rho_0 - \rho_H)} - 1\right] A$. Consider the good firms which choose to continue in state $\{\rho_H \rho_H\}$. These firms are in a situation similar to the case $\overline{q} > q \geq 1$, and the proof in Subsection A.5.1 is identical to the proof of the case $\overline{q} = q \geq 1$. As in Subsection A.5.1, Assumption 2 guarantees that those bad entrepreneurs who set $i_H > 0$ have their participation constraints satisfied. Hence the participation constraints of all entrepreneurs are satisfied.

Good firms do not want to deviate. Good entrepreneurs solve program (18) with $q = \overline{q} \geq 1$. These entrepreneurs are in a situation similar to the case $\overline{q} > q \geq 1$, and the proof in Subsection A.5.1 is identical to the proof of the case $\overline{q} = q \geq 1$. The case in which $i_L = \frac{A}{f_{LB}(\rho_1 - \rho_0)}$ and $i_H = 0$ is not optimal, as Assumption 4 guarantees that profit in this case is lower or equal to $\Pi(1, \overline{q}) = \Pi(0, \overline{q})$. Hence there is no profitable deviation.■

A.6 Proof of Proposition 5

We use Lemma 3. Since $\overline{q} < 1 \Leftrightarrow (1 - f_{HG})(\rho_H - \rho_0) > \Omega$, then $\Pi(0, 1) = \left(\frac{(1-f_{HG})(\rho_1 - \rho_0)}{\Omega} - 1\right) A > \Pi(1, 1) = \left(\frac{\rho_1 - \rho_0}{\Omega - f_{HG}(\rho_0 - \rho_H)} - 1\right) A$. Next, we investigate if there is an equilibrium in which firms pool and set $i_H = 0$.

Participation constraints of both types of entrepreneur Since the profit of the bad entrepreneur $\left(\frac{f_{LB}(\rho_1-\rho_0)}{\Omega} - 1\right) A$ is positive because of Assumption 2, then the participation constraints of good and bad entrepreneurs are also satisfied.

Good firms do not want to deviate Good entrepreneurs compare the profit with pooling with $i_H = 0$, with the profit with a separation in which they demand outside liquidity at price $q = 1$ and $\overline{q} < 1$. To find the separating contract, good entrepreneurs must solve program (18) knowing that $q = 1$. The proof in Subsection A.5.1 is identical to the present proof, and $i_L = \frac{A}{f_{LB}(\rho_1-\rho_0)}$ and $i_H = 0$. Assumption 4 guarantees that there is no profitable deviation.■

A.7 Welfare results with aggregate liquidity shocks

This section of the appendix presents the second-best optimum when there are aggregate liquidity shocks. The restriction $\frac{f_{LB}(\rho_1-\rho_L)+(1-f_{LB}-f_{HG})(\rho_1-\rho_L)\alpha+f_{HG}(\rho_1-\rho_H)-1}{\Omega-f_{HG}(\rho_0-\rho_H)} \geq \frac{(1-f_{LB}-f_{HG})(\rho_1-\rho_L)}{1-(1-f_{LB}-f_{HG})(\rho_0-\rho_L)}$ guarantees that the costs of separating good from bad entrepreneurs are too high, and that pooling is indeed the best option.

Proposition 6 *For* $\frac{f_{LB}(\rho_1-\rho_0)+(1-f_{LB}-f_{HG})(\rho_1-\rho_0)\alpha+f_{HG}(\rho_1-\rho_0)}{\Omega-f_{HG}(\rho_0-\rho_H)} - 1 \geq \frac{(1-f_{LB}-f_{HG})(\rho_1-\rho_L)}{1-(1-f_{LB}-f_{HG})(\rho_0-\rho_L)}$, *and under Assumptions 1 and 2, the second-best prescribes setting* $I = \frac{A}{\Omega-f_{HG}(\rho_0-\rho_H)}$. *Good entrepreneurs set* $I = i_L = i_{LH} = i_H$, *and bad entrepreneurs set* $I = i_L = i_H$ *and do not continue in state* $\{\rho_L\rho_H\}$.

Proof. Suppose that the central planner is restricted to pooling contracts. It does not distinguish good from bad firms in states $\{\rho_L\rho_L\}$ and $\{\rho_H\rho_H\}$, so that the second-best solution solves

$$\max_{\{i_L, i_H, i_{LH}\}} f_{LB}(\rho_1 - \rho_L) i_L + (1 - f_{LB} - f_{HG})(\rho_1 - \rho_L)\alpha i_{LH} + f_{HG}(\rho_1 - \rho_H) i_H - I$$

$$\textit{subject to}$$

$$f_{LB}(\rho_0 - \rho_L) i_L + (1 - f_{LB} - f_{HG})(\rho_0 - \rho_L)\alpha i_{LH} + f_{HG}(\rho_0 - \rho_H) i_H \geq I - A$$

$$0 \leq i_L, i_{LH}, i_H \leq I$$

It is easy to check that $i_L = i_{LH} = I$, and that the participation constraint of outside investors binds. Rewrite the participation constraint as

$$I = A + f_{LB}(\rho_0 - \rho_L) i_L + (1 - f_{LB} - f_{HG})(\rho_0 - \rho_L)\alpha i_{LH} + f_{HG}(\rho_0 - \rho_H) i_H$$

and replace it into the objective function, to obtain

$$f_{LB}(\rho_1 - \rho_0) i_L + (1 - f_{LB} - f_{HG})(\rho_1 - \rho_0)\alpha i_{LH} + f_{HG}(\rho_1 - \rho_0) i_H - A. \tag{19}$$

Let $x = \frac{i_H}{I}$, and from the participation constraint of outside investors

$$I = \frac{A}{\Omega - f_{HG}(\rho_0 - \rho_H) x}.$$

If we replace investment i_L, i_{LH} and I in the objective function (19), we obtain social welfare as a function of x

$$U(x) = \left(\frac{f_{LB}(\rho_1 - \rho_0) + (1 - f_{LB} - f_{HG})(\rho_1 - \rho_0)\alpha + f_{HG}(\rho_1 - \rho_0) x}{\Omega - f_{HG}(\rho_0 - \rho_H) x} - 1 \right) A$$

Since the problem is linear, it suffices to compare $U(0)$ with $U(1)$. Since $U(0) < U(1) \Leftrightarrow \rho_H < \frac{1 + [f_{LB} + (1 - f_{LB} - f_{HG})\alpha]\rho_L}{f_{LB} + (1 - f_{LB} - f_{HG})\alpha}$ is equivalent to

$$\frac{1 + [f_{LB} + (1 - f_{LB} - f_{HG})\alpha]\rho_L + f_{HG}\rho_H}{f_{LB} + (1 - f_{LB} - f_{HG})\alpha + f_{HG}} < \frac{1 + [f_{LB} + (1 - f_{LB} - f_{HG})\alpha]\rho_L}{f_{LB} + (1 - f_{HG} - f_{LB})\alpha},$$

Assumption 1 (expression 9) guarantees that $U(0) < U(1)$, so that the second-best prescribes setting $i_H = I$ (since $U(1) > 0$).

The central planner does not use a separating contract Suppose the central planner fixes i_L, i_H, and i_{LH} such that it is able to separate good from bad entrepreneurs using the signaling constraint. Hence, it solves

$$\max_{\{i_L, i_H, i_{LH}\}} f_{LB}(\rho_1 - \rho_L) i_L + (1 - f_{LB} - f_{HG})(\rho_1 - \rho_L) i_{LH} + f_{HG}(\rho_1 - \rho_H) i_H - I$$
$$\textit{subject to}$$
$$f_{LB}(\rho_0 - \rho_L) i_L + (1 - f_{LB} - f_{HG})(\rho_0 - \rho_L) i_{LH} + f_{HG}(\rho_0 - \rho_H) i_H \geq I - A$$
$$f_{LB}(\rho_1 - \rho_0) i_L + f_{HG}(\rho_1 - \rho_0) i_H - A \leq 0$$
$$0 \leq i_L, i_{LH}, i_H \leq I$$

The signaling constraint binds, and we obtain

$$i_H = i_H(i_L) = \frac{A}{f_{HG}(\rho_1 - \rho_0)} - \frac{f_{LB}}{f_{HG}} i_L$$

Replacing i_H in the maximization problem,

$$\max_{\{i_L, i_{LH}\}} f_{LB}(\rho_H - \rho_L) i_L + (1 - f_{LB} - f_{HG})(\rho_1 - \rho_L) i_{LH} + \frac{\rho_1 - \rho_H}{\rho_1 - \rho_0} A - I$$
$$\textit{subject to}$$
$$f_{LB}(\rho_H - \rho_L) i_L + (1 - f_{LB} - f_{HG})(\rho_0 - \rho_L) i_{LH} + \frac{\rho_0 - \rho_H}{\rho_1 - \rho_0} A \geq I - A$$
$$0 \leq i_L, i_{LH}, i_H(i_L) \leq I$$

The central planner wants to set i_{LH} as high as possible, so that $i_{LH} = I$. The central planner wants to set i_L as high as possible and

$$i_H(i_L) \geq 0 \Leftrightarrow i_L \leq \frac{A}{f_{LB}(\rho_1 - \rho_0)},$$

so that $i_L = \min\left\{\frac{A}{f_{LB}(\rho_1-\rho_0)}, I\right\}$. There are two possible cases.

Case 1. Suppose $i_L = I$ (and $i_H > 0$). From the participation constraint of outside investors we obtain

$$I = \frac{\frac{\rho_1-\rho_H}{\rho_1-\rho_0}}{1 - f_{LB}\left(\rho_H - \rho_L\right) - \left(1 - f_{LB} - f_{HG}\right)\left(\rho_0 - \rho_L\right)} A$$

as long as $1 > f_{LB}\left(\rho_H - \rho_L\right) + \left(1 - f_{LB} - f_{HG}\right)\left(\rho_0 - \rho_L\right)$ (otherwise this case cannot happen). Since $i_L = \min\left\{\frac{A}{f_{LB}(\rho_1-\rho_0)}, I\right\}$, it must be the case that

$$\begin{aligned} & \frac{\frac{\rho_1-\rho_H}{\rho_1-\rho_0}}{1 - f_{LB}\left(\rho_H - \rho_L\right) - \left(1 - f_{LB} - f_{HG}\right)\left(\rho_0 - \rho_L\right)} \leq \frac{1}{f_{LB}\left(\rho_1 - \rho_0\right)} \\ \Leftrightarrow \quad & f_{LB}\left(\rho_1 - \rho_L\right) + \left(1 - f_{LB} - f_{HG}\right)\left(\rho_0 - \rho_L\right) \leq 1, \end{aligned} \tag{20}$$

otherwise this case is not possible. Inequality (20) does not hold because of Assumption 2, so that this case does not happen.

Case 2. Suppose $i_L = \frac{A}{f_{LB}(\rho_1-\rho_0)}$ (and $i_H = 0$). The participation constraint of outside investors becomes

$$I = \frac{\frac{\rho_1-\rho_L}{\rho_1-\rho_0}}{1 - \left(1 - f_{LB} - f_{HG}\right)\left(\rho_0 - \rho_L\right)} A$$

as long as $1 > \left(1 - f_{LB} - f_{HG}\right)\left(\rho_0 - \rho_L\right)$ (otherwise this case cannot happen). Since $i_L = \min\left\{\frac{A}{f_{LB}(\rho_1-\rho_0)}, I\right\}$, it must be the case that

$$\begin{aligned} \frac{1}{f_{LB}} &\leq \frac{\rho_1 - \rho_L}{1 - \left(1 - f_{LB} - f_{HG}\right)\left(\rho_0 - \rho_L\right)} \Leftrightarrow \\ 1 &\leq f_{LB}\left(\rho_1 - \rho_L\right) + \left(1 - f_{LB} - f_{HG}\right)\left(\rho_0 - \rho_L\right), \end{aligned} \tag{21}$$

otherwise this case is not possible. When inequality (21) holds, social welfare equals

$$\begin{aligned}
U_{SP} &= f_{LB}\left(\rho_H-\rho_L\right) i_L+\left(1-f_{LB}-f_{HG}\right)\left(\rho_1-\rho_L\right) i_{LH}+\frac{\rho_1-\rho_H}{\rho_1-\rho_0} A-I \\
&= f_{LB}\left(\rho_H-\rho_L\right) \frac{A}{f_{LB}\left(\rho_1-\rho_0\right)}+\left(1-f_{LB}-f_{HG}\right)\left(\rho_1-\rho_L\right) I+\frac{\rho_1-\rho_H}{\rho_1-\rho_0} A-I \\
&= \left[\frac{\left(1-f_{LB}-f_{HG}\right)\left(\rho_1-\rho_L\right)-1}{1-\left(1-f_{LB}-f_{HG}\right)\left(\rho_0-\rho_L\right)}+1\right] \frac{\rho_1-\rho_L}{\rho_1-\rho_0} A \\
&= \frac{\left(1-f_{LB}-f_{HG}\right)\left(\rho_1-\rho_L\right)}{1-\left(1-f_{LB}-f_{HG}\right)\left(\rho_0-\rho_L\right)} A
\end{aligned}$$

Social welfare with pooling $U(1)$ is bigger than U_{SP}, because $\frac{f_{LB}(\rho_1-\rho_0)+(1-f_{LB}-f_{HG})(\rho_1-\rho_0)\alpha+f_{HG}(\rho_1-\rho_0)}{\Omega-f_{HG}(\rho_0-\rho_H)}-1 \geq \frac{(1-f_{LB}-f_{HG})(\rho_1-\rho_L)}{1-(1-f_{LB}-f_{HG})(\rho_0-\rho_L)}$.■

A.8 The profit function of good entrepreneurs with taxes and subsidies in the idiosyncratic shocks case

The objective of this appendix is to derive a reduced form of the profit function of good entrepreneurs, when there are taxes and subsidies. The proof follows the same steps as the proof of Lemma 1. The following assumption strengthens Assumption 1.

Assumption 1' $1+t>\overline{f}_L\left(\rho_0-\rho_L\right)+\overline{f}_H\left(\rho_0-\rho_H+s\right)$ and $\frac{1+t+\overline{f}_L\rho_L}{\overline{f}_L}<\rho_1$.

The entrepreneur must keep the nonpledgeable income. The pledgeable income is divided up so that

$$\Re_{OL}+\Re_{EL} = \left(\rho_0-\rho_L\right) i_L+\ell \tag{22}$$

$$\Re_{OH}+\Re_{EH} = \left(\rho_0-\rho_H+s\right) i_H+\ell \tag{23}$$

where $\Re_{OL}$, $\Re_{OH}$, $\Re_{EL}$, and $\Re_{EH}$ represent the income after taxes and subsidies received by outside investors and entrepreneurs in the low and high shock cases. The participation constraint of outside investors is

$$\overline{f}_L \mathfrak{R}_{OL} + \overline{f}_H \mathfrak{R}_{OH} - tI = I - A + q\ell$$

and the liquidity constraint equals

$$(\rho_0 - \rho_H + s)\, i_H = \ell.$$

The profit of the entrepreneur equals

$$f_{LG}\,(\rho_1 - \rho_0)\, i_L + f_{HG}\,(\rho_1 - \rho_0)\, i_H + f_{LG}\mathfrak{R}_{EL} + f_{HG}\mathfrak{R}_{EH} - A$$

The participation constraint of outside investors binds. Replacing expressions (22) and (23) into the participation constraint of outside investors,

$$\overline{f}_L\,[(\rho_0 - \rho_L)\, i_L - \mathfrak{R}_{EL}] + \left(\overline{f}_H + q - 1\right)(\rho_0 - \rho_H + s)\, i_H - \overline{f}_H \mathfrak{R}_{EH} - (1+t)\, I = -A$$

and replacing this constraint into the profit function, we obtain the following objective function

$$\left[f_{LG}\,(\rho_1 - \rho_0) + \overline{f}_L\,(\rho_0 - \rho_L)\right] i_L + \left[f_{HG}\,(\rho_1 - \rho_0) + \left(\overline{f}_H + q - 1\right)(\rho_0 - \rho_H + s)\right] i_H +$$
$$+ \left(f_{LG} - \overline{f}_L\right)\mathfrak{R}_{EL} + \left(f_{HG} - \overline{f}_H\right)\mathfrak{R}_{EH} - (1+t)\, I$$

It follows that the entrepreneur wants to set $\mathfrak{R}_{EH} = 0$, and $i_L = I$. Let $x = \frac{i_H}{I}$, and rewrite the participation constraint as

$$I\,(x,q) = \frac{A - \overline{f}_L \mathfrak{R}_{EL}}{1 + t - \overline{f}_L\,(\rho_0 - \rho_L) - \left(\overline{f}_H + q - 1\right)(\rho_0 - \rho_H + s)\, x},$$

and replacing in the profit function $f_{LG}\,(\rho_1 - \rho_0)\, i_L + f_{HG}\,(\rho_1 - \rho_0)\, i_H + f_{LG}\mathfrak{R}_{EL} - A$

to obtain

$$\begin{aligned}\Pi(x,q) &= \left[\frac{(f_{LG}+f_{HG}x)(\rho_1-\rho_0)}{1+t-\overline{f}_L(\rho_0-\rho_L)-\left(\overline{f}_H+q-1\right)(\rho_0-\rho_H+s)x}-1\right]A+ \\ &+\left[f_{LG}-\left(\frac{(f_{LG}+f_{HG}x)(\rho_1-\rho_0)}{1+t-\overline{f}_L(\rho_0-\rho_L)-\left(\overline{f}_H+q-1\right)(\rho_0-\rho_H+s)x}\right)\overline{f}_L\right]\Re_{EL}.\end{aligned}$$

Since the problem is linear, $i_H \in \{0, I\}$ so that $x \in \{0,1\}$. The term multiplying $\Re_{EL}$ is negative when $x = 0$ because of Assumption 1'. When $x = 1$, five cases are possible.

Case 1. If $s = \rho_H - \rho_0$, then the term multiplying $\Re_{EL}$ is negative because

$$f_{LG} < \frac{\rho_1-\rho_0}{1+t-\overline{f}_L(\rho_0-\rho_L)}\overline{f}_L \Leftrightarrow 1+t-\overline{f}_L(\rho_0-\rho_L) < \frac{\overline{f}_L}{f_{LG}}(\rho_1-\rho_0)$$

which holds because of Assumption 1'.

Case 2. If $s < \rho_H - \rho_0$ and $q \leq \frac{\overline{f}_L(\rho_1-\rho_0)-f_{LG}\Omega-f_{LG}t}{f_{LG}(\rho_H-\rho_0-s)}+\overline{f}_L$, then the term multiplying $\Re_{EL}$ is negative.

Case 3. If $s < \rho_H - \rho_0$ and $q > \frac{\overline{f}_L(\rho_1-\rho_0)-f_{LG}\Omega-f_{LG}t}{f_{LG}(\rho_H-\rho_0-s)}+\overline{f}_L$, then $x = 0$ which is a contradiction. To see this, compare $\Pi(0,q)$ with $\Pi(1,q)$. The entrepreneur sets $i_H = I$ if and only if

$$\Pi(0,q) \leq \Pi(1,q) \Leftrightarrow q < \frac{(1-f_{LG})(\Omega+t)}{f_{LG}(\rho_H-\rho_0-s)}+\overline{f}_L.$$

Since $\overline{f}_L(\rho_1-\rho_0) > \Omega+t$ by Assumption 1', there is a contradiction. This case is not possible.

Case 4. If $s > \rho_H - \rho_0$ and $q \leq \frac{\overline{f}_L(\rho_1-\rho_0)-f_{LG}\Omega-f_{LG}t}{f_{LG}(\rho_H-\rho_0-s)}+\overline{f}_L$. The term multiplying $\Re_{EL}$ is negative if and only if

$$\begin{aligned}\frac{f_{LG}}{\overline{f}_L} &< \frac{\rho_1-\rho_0}{1+t-\overline{f}_L(\rho_0-\rho_L)-\left(\overline{f}_H+q-1\right)(\rho_0-\rho_H+s)} \\ &\Leftrightarrow\ q > \frac{\overline{f}_L(\rho_1-\rho_0)-f_{LG}(\Omega+t)}{f_{LG}(\rho_0-\rho_H+s)}+\overline{f}_L\end{aligned}$$

which is a contradiction. This case is not possible.

Case 5. If $s > \rho_H - \rho_0$ and $q > \frac{\overline{f}_L(\rho_1-\rho_0)-f_{LG}\Omega-f_{LG}t}{f_{LG}(\rho_H-\rho_0-s)} + \overline{f}_L$, then the term multiplying $\Re_{EL}$ is negative.

We conclude that the optimal value for $\Re_{EL}$ is zero. Since there is plenty of inside liquidity, we set $q = 1$ and we can rewrite the maximization problem of good entrepreneurs as in program (11).

A.9 The profit function of good entrepreneurs with taxes and subsidies in the aggregate shocks case and bailout policy

The objective of this appendix is to derive a reduced form of the profit function of good entrepreneurs, when there are ex post bailouts (that is, $s_b = \rho_H - \rho_0$ in state $\{\rho_H \rho_H\}$). The following assumption is akin to Assumption 3.

Assumption 3' $\Omega + t < f_{LG}(\rho_1 - \rho_0)\,\alpha$.

We use the same notation as in Section 4. Since government bonds are not required ex post, we set $\ell = 0$ without loss of generality. Pledgeable income is divided up so that

$$\begin{aligned}
\Re_{OL} + \Re_{EL} &= (\rho_0 - \rho_L)\, i_L \\
\Re_{OLH} + \Re_{ELH} &= (\rho_0 - \rho_L)\, i_{LH} \\
\Re_{OH} + \Re_{EH} &= (\rho_0 - \rho_H + s_b)\, i_H = 0
\end{aligned}$$

and the participation constraint of outside investors is

$$f_{LB}\Re_{OL} + (1 - f_{LB} - f_{HG})\,\Re_{OLH}\alpha + f_{HG}\Re_{OH} - tI \geq I - A$$

Replacing the equations regarding the division of pledgeable income,

$$\begin{aligned} & f_{LB}(\rho_0-\rho_L)\,i_L+(1-f_{LB}-f_{HG})(\rho_0-\rho_L)\,i_{LH}\alpha \\ - & f_{LB}\Re_{EL}-(1-f_{LB}-f_{HG})\,\alpha\Re_{ELH}-f_{HG}\Re_{EH}-tI\geq I-A. \end{aligned}$$

The profit function equals

$$\begin{aligned} & f_{LB}(\rho_1-\rho_0)\,i_L+(1-f_{LB}-f_{HG})(\rho_1-\rho_0)\,i_{LH}+f_{HG}(\rho_1-\rho_0)\,i_H \\ & +f_{LB}\Re_{EL}+(1-f_{LB}-f_{HG})\,\Re_{ELH}+f_{HG}\Re_{EH}-A. \end{aligned}$$

The participation constraint of outside investors holds with equality, and replacing it in the profit function,

$$\begin{aligned} & f_{LB}(\rho_1-\rho_L)\,i_L+(1-f_{LB}-f_{HG})(\rho_1-\rho_L)\,i_{LH} \\ & -(1-\alpha)(1-f_{LB}-f_{HG})(\rho_0-\rho_L)\,i_{LH}-I-tI+(1-\alpha)(1-f_{LB}-f_{HG})\,\Re_{ELH} \end{aligned}$$

Hence, $\Re_{EL}=\Re_{EH}=0$. From the participation constraint, write the investment function

$$I(x)=\frac{A-(1-f_{LB}-f_{HG})\,\alpha\Re_{ELH}}{\Omega+t}$$

and the profit function

$$\begin{aligned} \Pi(x) &= (1-f_{HG}+f_{HG}x)(\rho_1-\rho_0)\,I(x)+(1-f_{LB}-f_{HG})\,\Re_{ELH}-A \\ &= \left[\frac{(1-f_{HG}+f_{HG}x)(\rho_1-\rho_0)}{\Omega+t}-1\right]A \\ &\quad+\left[1-\frac{\alpha(1-f_{HG}+f_{HG}x)(\rho_1-\rho_0)}{\Omega+t}\right](1-f_{HG}+f_{HG}x)\,\Re_{ELH}. \end{aligned}$$

The term multiplying $\Re_{ELH}$ is negative because of Assumption 3'. Hence, $\Re_{ELH}=0$ and we can rewrite the maximization problem of good entrepreneurs as in the text.

A.10 The profit function of good entrepreneurs with taxes and subsidies in the aggregate shock case with $\overline{q} < 1$

The objective of this appendix is to derive a reduced form of the profit function of good entrepreneurs when there are taxes and subsidies. The following assumptions are akin to Assumptions 1 and 3.

Assumption 1" $\Omega + t > f_{HG}(\rho_0 - \rho_H + s)$.

Assumption 3" $\Omega + t < (\rho_1 - \rho_0)\alpha + f_{HG}(\rho_0 - \rho_H + s)$.

We maintain Assumption 3', and we use the same notation as in Section 4. Pledgeable income is divided up so that

$$\begin{aligned}
\mathfrak{R}_{OL} + \mathfrak{R}_{EL} &= (\rho_0 - \rho_L) i_L + \ell \\
\mathfrak{R}_{OLH} + \mathfrak{R}_{ELH} &= (\rho_0 - \rho_L) i_{LH} + \ell \\
\mathfrak{R}_{OH} + \mathfrak{R}_{EH} &= (\rho_0 - \rho_H + s) i_H + \ell
\end{aligned}$$

and the participation constraint of outside investors is

$$f_{LB}\mathfrak{R}_{OL} + (1 - f_{LB} - f_{HG})\mathfrak{R}_{OLH}\alpha + (1 - f_{LB} - f_{HG})(1-\alpha)\ell + f_{HG}\mathfrak{R}_{OH} - tI \geq I - A + q\ell$$

Replacing the equations regarding the division of pledgeable income,

$$\begin{aligned}
& f_{LB}(\rho_0 - \rho_L) i_L + (1 - f_{LB} - f_{HG})(\rho_0 - \rho_L) i_{LH}\alpha + f_{HG}(\rho_0 - \rho_H + s) i_H \\
- \quad & f_{LB}\mathfrak{R}_{EL} - (1 - f_{LB} - f_{HG})\alpha\mathfrak{R}_{ELH} - f_{HG}\mathfrak{R}_{EH} - tI \geq I - A + (q-1)\ell.
\end{aligned}$$

We have the liquidity constraint

$$(\rho_H - \rho_0 - s) i_H \leq \ell$$

and the profit function

$$f_{LB}(\rho_1-\rho_0)\,i_L+(1-f_{LB}-f_{HG})(\rho_1-\rho_0)\,i_{LH}+f_{HG}(\rho_1-\rho_0)\,i_H$$
$$+f_{LB}\Re_{EL}+(1-f_{LB}-f_{HG})\,\Re_{ELH}+f_{HG}\Re_{EH}-A.$$

The participation constraint of outside investors holds with equality, and replacing it in the profit function,

$$f_{LB}(\rho_1-\rho_L)\,i_L+(1-f_{LB}-f_{HG})(\rho_1-\rho_L)\,i_{LH}+f_{HG}(\rho_1-\rho_H+s)\,i_H$$
$$-(1-\alpha)(1-f_{LB}-f_{HG})(\rho_0-\rho_L)\,i_{LH}-I-tI-(q-1)\,\ell+(1-\alpha)(1-f_{LB}-f_{HG})\,\Re_{ELH}.$$

Hence, $\Re_{EL}=\Re_{EH}=0$. From the participation constraint, write the investment function

$$I(x,q)=\frac{A-(1-f_{LB}-f_{HG})\,\alpha\Re_{ELH}}{\Omega+t-(f_{HG}+q-1)(\rho_0-\rho_H+s)\,x}$$

and the profit function

$$\begin{aligned}\Pi(q,x) &= (1-f_{HG}+f_{HG}x)(\rho_1-\rho_0)\,I(x,q)+(1-f_{LB}-f_{HG})\,\Re_{ELH}-A\\ &= \left[\frac{(1-f_{HG}+f_{HG}x)(\rho_1-\rho_0)}{\Omega+t-(f_{HG}+q-1)(\rho_0-\rho_H+s)\,x}-1\right]A+\\ &\quad+\left[1-\frac{\alpha(1-f_{HG}+f_{HG}x)(\rho_1-\rho_0)}{\Omega+t-(f_{HG}+q-1)(\rho_0-\rho_H+s)\,x}\right](1-f_{HG}+f_{HG}x)\,\Re_{ELH}.\end{aligned}$$

Since $q=1$, we obtain the profit function

$$\begin{aligned}\Pi(1,x) &= (1-f_{HG}+f_{HG}x)(\rho_1-\rho_0)\,I(x,1)+(1-f_{LB}-f_{HG})\,\Re_{ELH}-A\\ &= \left[\frac{(1-f_{HG}+f_{HG}x)(\rho_1-\rho_0)}{\Omega+t+f_{HG}(\rho_0-\rho_H+s)\,x}-1\right]A\\ &\quad+\left[1-\frac{\alpha(1-f_{HG}+f_{HG}x)(\rho_1-\rho_0)}{\Omega+t-f_{HG}(\rho_0-\rho_H+s)\,x}\right](1-f_{HG}+f_{HG}x)\,\Re_{ELH}.\end{aligned}$$

Again, the term multiplying $\Re_{ELH}$ is negative because of Assumptions 3' and 3". Hence, $\Re_{ELH}=0$ and we can rewrite the maximization problem of good entrepreneurs as in the text.