Advance-Purchase Financing of Projects with Few Buyers

Marco Sahm[a,b]

[a] Otto-Friedrich-University
Feldkirchenstraße 21, D–96045 Bamberg, Germany
Tel.: +49–951–863–2728, Fax: +49–951–863–5637
E-mail: Marco.Sahm@uni-bamberg.de.
[b] CESifo, Poschingerstraße 5, D–81679 Munich, Germany

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Abstract

I investigate a simple model of advance-purchase contracts as a mode of financing costly projects. The analysis can easily be reinterpreted as a model of monopolistic provision of excludable public goods under private information. An entrepreneur has to meet some capital requirement in order to start production and sell the related good to a limited number of potential buyers who are privately informed about their willingness to pay. I find that advance-purchase arrangements allow to finance more costly projects than traditional funding sources. The entrepreneur is able to use advance-purchase surcharges as a price discrimination device. However, the discriminatory power is limited by the problem of free-riding which aggravates for an increasing number of potential buyers. I apply the model to research and development activities in the health industry discussing the availability of new drugs and vaccines in poor countries.

Keywords: pre-ordering, price discrimination, excludable public goods, monopolistic provision, innovation and R&D

JEL classification: D42, G32, H41, L12, L26, O31, O32

1 Introduction

In the public health sector, there is a lively political debate about the questions how research and development (R&D) of new drugs and vaccines by private firms can be incentivized properly and how these pharmaceuticals can be made available even in low-income countries. One proposal that has been made in order to jointly address both problems is the use of advance-purchase arrangements:[1] Negotiating with the producer, some national or supra-national health authorities may commit to pre-order the drug and pay in advance (or promise to pay the pre-specified price on delivery). Resolving the producer's uncertainty about the countries' valuations of the drug, the firm can then use these advance payments (or promises) to finance its R&D investments. Moreover, poorer countries might benefit from lower prices once the development of the drug has been financed by the pre-orders of richer countries.

Though the proposal is intuitively appealing, a rigorous analysis based on a theoretical economic model is missing so far. Is it really possible to finance more (costly) R&D activities based on advance-purchase contracts than based on traditional funding like debt or equity? Does the use of advance-purchase financing instead of traditional funding actually improve the availability of pharmaceuticals in low-income countries? (Under which circumstances) does advance-purchase financing Pareto-dominate traditional funding?

In this paper, I examine these questions within a simple model of advance-purchase financing. I consider an entrepreneur who must meet a certain capital requirement in order to start production. Once the fixed costs are covered, the entrepreneur has monopoly power and sells the related product to a limited number of potential buyers. Customers are privately informed about their willingness to pay and buy either one or zero units of the good. Within this framework I compare the allocations resulting from two different funding mechanisms: Under traditional (debt or equity) financing, the entrepreneur relies on standard uniform monopoly pricing, whereas advance-purchase financing can be modeled as a two-stage game: In the first stage, the entrepreneur offers to pre-order the good at some advance-purchase price. If the money collected from pre-orders does not cover the fixed cost, then advance purchasers will be reimbursed. Else the game will move on to the second stage at which the good is produced, delivered to advance-purchasers, and offered to residual customers at a (possibly different) regular price.

[1] Berndt and Hurvitz (2005) provide a comprehensive discussion of this proposal focussing on practical issues.

As will turn out, this enables the entrepreneur to discriminate between customers of different valuations: Agents with a high willingness to pay prefer to pre-order the good at the advance-purchase price because they fear most the non-availability of the product and take into account that their pre-order might be pivotal for its realization. Instead, agents with a lower valuation prefer to wait and possibly purchase the good later on at the regular price. This results in an advance-purchase surcharge, i.e. the entrepreneur optimally sets an advance-purchase price above the regular price. Note that the discriminatory power of the entrepreneur rests upon the threat of the possible non-availability of the product, i.e. on the positive probability that the pre-order of a single agent might be pivotal. Since this probability decreases in the number of potential buyers, the problem that agents tend to free-ride on the advance payments of others limits the discriminatory power. Hence, the differential between advance-purchase price and regular price gets smaller as the market size increases.

Comparing traditional funding and advance-purchase financing, I derive the following results: First, if fixed costs are sufficiently large, the entrepreneur will always prefer advance-purchase financing over traditional funding. Under traditional funding, projects will be realized if and only if they are ex-ante profitable. However, ex-post, they may turn out to create losses. By contrast, advance-purchase financing enables the entrepreneur to price-discriminate and run only projects that are ex-post profitable. If the fixed costs that can be avoided this way are sufficiently large, these expected savings will outweigh the disadvantage that some profitable projects cannot be realized under advance-purchase financing due to the problem of free-riding. Second, as this reasoning implies, under advance-purchase financing indeed more costly projects can be realized than under traditional funding. If the fixed costs exceed a certain threshold, projects will not be profitable ex-ante and, hence, will definitely not be realized under traditional funding, whereas the probability of being ex-post profitable and therefore realized under advance-purchase financing is strictly positive. Third, this shows that advance-purchase financing will actually Pareto-dominate traditional funding if fixed costs are sufficiently large.

Besides the introductory example from health economics, the model captures a series of stylized facts that are characteristic for many markets in which a single seller with increasing returns to scale deals with a limited number of potential buyers. For instance, think of the international defense industry where an arms manufacturer does business with a limited set of countries, or the international airline industry where a producer of jet engines can sell to mainly two aircraft companies only.

Additional to its application to the field of research and development in international health economics (Berndt et al., 2007), this paper is closely related to two further strands of the economic literature. First, it contributes to a series of articles that analyze the role of advance-purchase contracts as a means of price discrimination. Considering markets with a continuum of potential buyers, almost all of the seminal papers on this topic find advance-purchase discounts to characterize the optimal pricing scheme. The optimality of advance-purchase discounts may be due to limited production capacities and uncertainty about the aggregate level of demand (Dana, 1998, 1999, 2001, Gale and Holmes, 1992, 1993) or due to different expected valuations among consumers (Nocke et al., 2011, Möller and Watanabe, 2010). Since in a continuum economy no single pre-order is pivotal for the availability of the respective product, in this part of the literature advance-purchase contracts are irrelevant for the financing decision but solely an instrument of price discrimination. By contrast, taking into account the strategic effects between a finite number of agents, the optimal advance-purchase contract of my model reflects the entrepreneur's simultaneous decision on financing and pricing yielding an advance-purchase surcharge. Belleflamme et al. (2014) and Sahm et al. (2014) derive a similar result considering a model of crowdfunding with a continuum of potential consumers. In their framework, however, the optimality of an advance-purchase surcharge is based on the behavioral assumption that pre-ordering consumers experience community benefits, i.e. additional utility from being part of the funding crowd.

Second, as the advance payments from pre-orders can be understood as contributions to the realization and non-rival availability of the product, this paper also contributes to the literature on monopolistic provision of excludable public goods under private information. Early work in this field has focused on simple pricing mechanisms that put empirically motivated constraints on the class of admissible contracts (Brito and Oakland, 1980). The more recent contributions usually apply a general mechanism design approach in order to specify optimal contracts (Cornelli, 1996, Schmitz, 1997). Since they often find these optimal mechanisms to be rather complex, though, they raise the question how the prevailing use of much simpler contracts in practice can be explained from the viewpoint of contract theory. For example, Schmitz (1997) as well as Norman (2004) show that the monopolist indeed will find it optimal to rely on simple contracts (such as average cost pricing) if the number of potential buyers gets very large. By contrast and more closely related to my paper, Cornelli (1996) focuses on the strategic effects within a small economy emphasizing that the threat of non-production is a useful instrument of price discrimination between customers of different valuations. However, all of these articles assume that the monopolist can commit not to renegotiate with customers once they have been excluded. Though for many instances this might be a reasonable assumption, it seems to be violated for the examples mentioned above. In order to deviate from this assumption in the simplest possible way, I restrict my analysis to simple advance-purchase contracts with posted prices.

The remainder of this paper is organized as follows: In Section 2 I present the model, derive the basic properties of the optimal advance-purchase contract, and compare it to optimal pricing based on traditional funding. Section 3 illustrates the results for the examples with only one and two potential buyers, respectively, and deals with the limit case of a large economy with many potential buyers. In Section 4 I summarize and discuss the main findings.

2 Analysis

2.1 Basic assumptions

A monopolistic entrepreneur seeks to finance a costly project with a commonly known capital requirement K. If the capital requirement is met, the entrepreneur will run the project and produce a related good of a fixed quality normalized to 1. Marginal costs of production are assumed to be zero.

There are $N \in \mathbb{N}$ potential buyers. Depending on their willingness to pay, each of them buys either zero or one unit of the good. The willingness to pay of buyer $i \in \{1, \ldots, N\}$ is his private information and denoted θ_i. It is the realization of a random variable with some commonly known distribution. For the sake of concreteness, assume that all N random variables are uniformly distributed on $[0, 1]$ and independent from each other. If customer i buys one unit of the product at price p, he will realize the surplus $U_i = \theta_i - p$. The surplus from not buying is zero.

The entrepreneur can choose between two mutual exclusive funding mechanisms: either traditional (debt or equity) financing (with opportunity costs of capital normalized to 0) or financing based on advance-purchase commitments. The latter refers to the case in which some customers pre-order the product and pay in advance. The advance payments are used to meet the capital requirement K and realize the project.

I assume that the entrepreneur has the bargaining power to make take-it-or-leave-it price offers. This is common practice for the analysis of monopolies and leads to a tractable screening model with the uninformed party proposing the contract. However, with the number of potential buyers being small, this assumption is debatable and will be discussed in more detail below.

2.2 *Traditional funding*

As a benchmark, consider the standard model of monopoly pricing in which the entrepreneur cannot commit to not running profitable projects.[2] With traditional funding and asymmetric information about customers' preferences, the entrepreneur then relies on uniform pricing in order to maximize expected profits. Once the project is realized, the probability that a certain customer buys at price p_0 is $1 - p_0$. Hence, expected profits equal

$$E(\pi_0) = N(1 - p_0)p_0 - K.$$

The first order condition suggests an optimal price of $p_0 = 1/2$ yielding an expected profit of $E(\pi_0) = N/4 - K$. Accordingly, with traditional funding, the entrepreneur will realize the project if and only if the capital requirement is not exceeding $N/4$.

2.3 *Advance-purchase financing*

2.3.1 *Advance-purchase financing as a two-stage game*

Project funding based on advance-purchase commitments can be described as a sequential game Γ with two periods: In the first period, the entrepreneur offers to all potential buyers the possibility to pre-order the product at price p_c and pay in advance. Individuals then simultaneously decide whether to pre-order at this price. If the money collected from pre-orders falls short of the capital requirement K, advance payments are returned and the game ends. If instead the capital requirement K is met, the project is realized and the game moves to the second period. In the second period, the entrepreneur sets the regular price p_r for buyers who did not pre-order. These residual customers then decide simultaneously whether to buy at this price. I normalize the discount rate to zero, so all pay-offs can be treated as if accruing at the end of period 2. I solve the game by backward induction for its subgame-perfect equilibrium (SPE).

The structure of the game reflects the implicit assumptions about the entrepreneur's bargaining power. Though being able to make take-it-or-leave-it price offers, it is limited in two ways. First, I assume that an advance-purchase contract specifies only the advance-purchase price p_c. In particular, the minimum number of pre-orders that is necessary to run the project cannot be contracted upon explicitly. The idea is that the entrepreneur cannot commit to not running the project if the money collected from pre-orders meets the capital requirement, because then expected profits from realizing

[2] Cornelli (1996) characterizes optimal selling procedures in a model with commitment.

the project are positive. Second, the above timing corresponds to the implicit assumption that the entrepreneur cannot commit to a regular price p_r ex ante. This lack of commitment is due to a problem of time-inconsistency similar to the one for durable goods: As long as the market is not entirely covered from pre-orders in period 1, the entrepreneur has always an incentive to adjust the price in period 2 in order to address additional buyers and make additional profits. As a consequence, in any SPE, anyone who pre-orders must have a higher willingness to pay for the good than any regular customer.

Lemma 1 *For any given prices p_c and p_r there is some $\theta_c \in \mathbb{R} \bigcup \{\pm\infty\}$ such that customer $i \in \{1, \dots, N\}$ will pre-order the product if and only if $\theta_i \geq \theta_c$.*

Proof. Denote by $m_c \in \{1, \dots, N\}$ the minimum number of pre-orders that is necessary to finance the project for the given advance-purchase price p_c, i.e. $m_c p_c \geq K > (m_c - 1)p_c$. Let $\sigma(m)$ be the probability that the number of pre-orders among $N-1$ potential buyers is at least $m \in \mathbb{N}$. Trivially, $\sigma(m-1) \geq \sigma(m)$. Some customer with willingness to pay θ will weakly prefer to pre-order the product if and only if his expected utility from an advance-purchase, $\sigma(m_c - 1)(\theta - p_c)$, is at least as high as the one from a regular purchase, $\sigma(m_c)(\theta - p_r)$, i.e. if and only if

$$(\sigma(m_c - 1) - \sigma(m_c))\theta \geq \sigma(m_c - 1)p_c - \sigma(m_c)p_r. \tag{1}$$

For $\sigma(m_c-1) = \sigma(m_c)$ nobody (everybody) will pre-order if $p_c > p_r$ ($p_c \leq p_r$), and $\theta_c := \infty$ ($\theta_c := -\infty$) has the stated property. For $\sigma(m_c - 1) > \sigma(m_c)$, set $\theta_c := \frac{\sigma(m_c-1)p_c - \sigma(m_c)p_r}{\sigma(m_c-1)-\sigma(m_c)}$.

□

In what follows, of course, only the cases for which $\theta_c \in [0, 1]$ are of further interest. I refer to θ_c as the marginal willingness to pay since it expresses the willingness to pay of any buyer being indifferent between advance-purchasing and not.

2.3.2 Stage 2

Let n_c denote the actual number of pre-orders and suppose that the capital requirement is met, i.e. $n_c p_c \geq K$. Then, the residual potential buyers in period 2 are those for which $\theta \in [0, \theta_c)$. Since they actually purchase the good at price p_r if and only if $\theta - p_r \geq 0$, the conditional probability that such a customer actually buys in period 2 equals

$$\text{Prob}(\theta \geq p_r \mid \theta \leq \theta_c) = \frac{\text{Prob}(p_r \leq \theta \leq \theta_c)}{\text{Prob}(\theta \leq \theta_c)} = \frac{\theta_c - p_r}{\theta_c}.$$

Hence, the entrepreneur maximizes his additional conditional expected profits from period 2

$$E(\pi_{2nd} \mid n_c) = (N - n_c) \cdot \frac{\theta_c - p_r}{\theta_c} \cdot p_r$$

by the choice of p_r. The solution to this problem is given by

$$p_r = \theta_c/2 \quad \text{and} \quad E(\pi_{2nd} \mid n_c) = (N - n_c) \cdot \theta_c/4. \tag{2}$$

2.3.3 Stage 1

Note that by setting the advance-purchase price, the entrepreneur implicitly determines the marginal willingness to pay as well as the minimum number of pre-orders that is necessary to finance the project. To see this, let p_n be the advance-purchase price. Then, $n \in \{1, \ldots, N\}$ with $np_n \geq K > (n-1)p_n$ is the corresponding minimum number of necessary pre-orders. By definition, for any customer with marginal willingness to pay θ_n, the expected payoff from advance-purchase

$$\left[\sum_{i=0}^{N-n} \binom{N-1}{n-1+i} (1-\theta_n)^{n-1+i} \theta_n^{N-n-i}\right] (\theta_n - p_n) \tag{3}$$

equals the expected payoff from regular purchase

$$\left[\sum_{i=1}^{N-n} \binom{N-1}{n-1+i} (1-\theta_n)^{n-1+i} \theta_n^{N-n-i}\right] (\theta_n - p_r). \tag{4}$$

Note that the probability that the project will be realized if the customer pre-orders in (3) and the probability that the project will be realized if he does not pre-order in (4) differ only by the probability that his own pre-order is pivotal for meeting the capital requirement $\binom{N-1}{n-1}(1-\theta_n)^{n-1}\theta_n^{N-n}$. Applying (2), the equality of (3) and (4) characterizes the relation between the marginal willingness to pay θ_n and the advance-purchase price

$$p_n = \theta_n \left(1 - \frac{1}{2} \cdot \frac{\sum_{i=1}^{N-n} \binom{N-1}{n-1+i} (1-\theta_n)^{n-1+i} \theta_n^{N-n-i}}{\sum_{i=0}^{N-n} \binom{N-1}{n-1+i} (1-\theta_n)^{n-1+i} \theta_n^{N-n-i}}\right). \tag{5}$$

Now suppose $n \in \{1, \ldots, N\}$ to be the minimum number of necessary pre-orders. If the realized number of pre-orders is $i \in \{0, \ldots, N\}$, the entrepreneur's profit will be 0 for $i < n$ and $ip_n - K + E(\pi_{2nd} \mid i)$ for $i \geq n$. Applying (2), the entrepreneur's expected profit equals

$$E(\pi_n) = \sum_{i=n}^{N} \binom{N}{i} (1-\theta_n)^i \theta_n^{N-i} \left[ip_n - K + (N-i) \cdot \theta_c/4\right], \tag{6}$$

where p_n is given by (5). Therefore, the problem of finding the advance-purchase price p_c that maximizes expected overall profits can be solved in two steps:

Step 1: For any $n \in \{1, \ldots, N\}$ choose the marginal willingness to pay θ_n that maximizes expected overall profits $E(\pi_n)$ given by (6) subject to the constraint that n indeed is the minimum number of pre-orders that is necessary to finance the project for the corresponding advance-purchase price p_n given by (5), i.e. such that $np_n \geq K > (n-1)p_n$.

Step 2: Choose $p_c := p_{n^*}$ (or equivalently $\theta_c := \theta_{n^*}$) with $n^* \in \arg\max_{n \in \{1,\ldots,N\}} E(\pi_n)$.

2.3.4 General properties of the optimal pricing scheme

Before I take these steps in order to solve the entrepreneur's problem explicitly for $N = 1$ and $N = 2$ in Section 3, I record some general properties of the optimal pricing scheme under advance-purchase financing.

Proposition 1 *In any SPE of the game Γ with $N \in \mathbb{N}$ potential buyers,*

(a) $p_c > p_r$, i.e. the advance-purchase price exceeds the regular price.

(b) If $n^* = N$ then $p_c = \theta_c$ else $p_c < \theta_c$, i.e. the advance-purchase price falls short of the marginal willingness to pay unless it is optimal to realize the project only if all potential buyers pre-order.

Proof. Let $p_c = p_{n^*}$ and $\theta_c = \theta_{n^*}$ for some $n^* \in \arg\max_{n \in \{1,\ldots,N\}} E(\pi_n)$.

(a) Using equations (2) and (5) one computes

$$p_c - p_r = \frac{\theta_{n^*}}{2} \cdot \frac{\binom{N-1}{n^*-1}(1-\theta_{n^*})^{n^*-1}\theta_{n^*}^{N-n^*}}{\sum_{i=0}^{N-n^*}\binom{N-1}{n^*-1+i}(1-\theta_{n^*})^{n^*-1+i}\theta_{n^*}^{N-n^*-i}} > 0$$

(b) Consider equation (5). If $n = N$ then the term in brackets will be equal to 1 else it will be smaller than 1.

□

To get some intuition for these results, note that the product will be available under advance-purchase financing if and only if the entrepreneur collects enough money from pre-orders to run the project. Since agents with a high willingness to pay suffer most from the possible non-availability of the product, the offered advance-purchase contract serves as a price discrimination device attracting only the customers with the highest valuations. In order to reach additional customers after the project has been realized, the entrepreneur must lower the price and choose $p_r < p_c$. However, unless all of the N potential buyers have to purchase in advance to meet the capital requirement, any agent's probability of being pivotal for the availability of the product is smaller than 1. Put differently, from the perspective of any single agent, there is a positive probability that the product will be available later on at some reduced price p_r. Therefore, the agent's willingness to pay must exceed the advance-purchase price p_c by some strictly positive amount for pre-ordering to be attractive to him. The difference $\theta_c - p_c > 0$ can be interpreted as the minimum information rent the entrepreneur has to leave to agents with high valuations in order to make them reveal their willingness to pay.

2.4 Comparison: traditional funding vs. advance-purchase financing

Under traditional funding, only projects with fixed costs $K \leq N/4$ will be realized. Under advance-purchase financing, in contrast, the probability of realization is positive for all projects with fixed costs $K < N$. Put differently, more costly projects can be realized based on advance-purchase financing than based on traditional funding.

To see this, consider the entrepreneur's strategy to choose an advance-purchase price p_N that makes each potential buyer pivotal for running the project. Obviously, the corresponding expected profit

$$E(\pi_N) = (1 - p_N)^N (Np_N - K) \tag{7}$$

is a lower bound for the entrepreneur's optimal profit under advance-purchase financing. The optimal p_N maximizes (7) subject to the constraints $Np_N \geq K > (N-1)p_N$. The unconstrained solution to this problem can be derived from the first-order condition and is given by $p_N = \frac{K+1}{N+1}$. It will satisfy the first constraint $Np_N \geq K$ if and only if $K \leq N$. Moreover, it will meet the second constraint $K > (N-1)p_N$ as well if and only if $\frac{N-1}{2} < K$. Hence, for $\frac{N-1}{2} < K \leq N$, the expected profit equals $E(\pi_N) = \left(\frac{N-K}{N+1}\right)^{N+1} > 0$. If instead $K \leq \frac{N-1}{2}$, the entrepreneur can set p_N arbitrarily close to $\frac{K}{N-1}$. The corresponding expected profit is then given by

$$E(\pi_N) = \left(\frac{N-1-K}{N-1}\right)^N \left(\frac{K}{N-1}\right) > 0.$$

These considerations imply the following

Proposition 2 *For any $N \in \mathbb{N}$ and $K \in \mathbb{R}$ the following statements hold:*

(a) *For any $K \in (N/4, N)$, the project is not realized under traditional funding but has a strictly positive probability of realization under advance-purchase financing.*

(b) *There is some $K_0 \in [0, N/4]$ such that the entrepreneur strictly prefers advance-purchase financing over traditional funding for all $K \in (K_0, N)$.*

(c) *There is some $K_1 \in [K_0, N/4]$ such that the allocation under advance-purchase financing Pareto-dominates the allocation under traditional funding for all $K \in (K_1, N)$.*

As stated in Proposition 2, advance-purchase financing leads to a welfare improvement in at least all those cases in which the capital requirement cannot be met under traditional funding.

3 Examples

In this section I will explicitly solve the game for the cases with one and two potential buyers and derive limit results for the case in which the number of potential buyers goes to infinity. These three cases illustrate all relevant aspects: The example with only one potential buyer highlights how advance-purchase contracts can be used as a price discrimination device. The example with two potential buyers shows that this use is limited by free-riding arising from the public goods character of buyer's advance payments contributing to the realization of the project. As the limit case demonstrates, with an increasing number of potential buyers the free-rider problem becomes more and more severe and, finally, inhibits price discrimination by means of advance-purchase contracts.

3.1 Example: $N = 1$

As a benchmark, consider the case with one potential buyer only.

3.1.1 Traditional Funding

With traditional funding, the project can be realized if and only if $K \leq 1/4$. In this case, the customer buys the product if and only if he has a willingness to pay of at least $p_0 = 1/2$ yielding an expected profit of $E(\pi_0) = 1/4 - K$.

3.1.2 Funding based on advance-purchase contracts

With funding based on an advance-purchase contract, the project is realized if and only if the potential buyer pre-orders at the advance-purchase price

p_c. Accordingly, the entrepreneur maximizes his expected profit

$$E(\pi_c) = (1 - p_c)(p_c - K)$$

by the choice of p_c subject to the constraint that $p_c \geq K$. The unconstrained solution to this problem is given by $p_c = \frac{1+K}{2}$. It is feasible for all $K \leq 1$.

3.1.3 Comparison

The expected profits related to the two alternative financing schemes are depicted in figure 1. A comparison shows, that with funding based on an

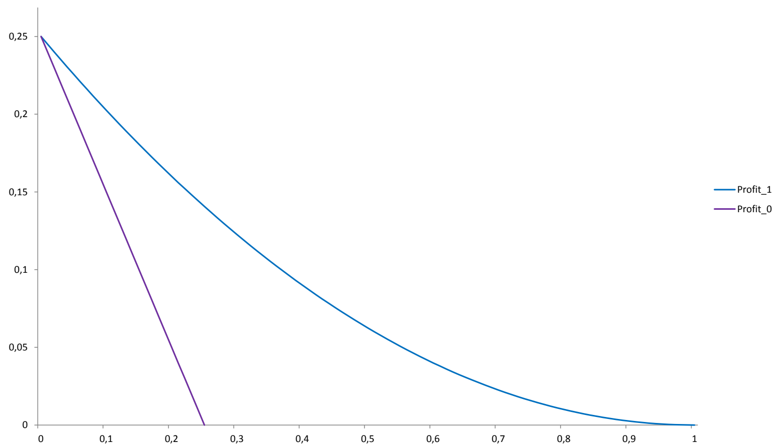

Figure 1: Graphs of $E(\pi_0)$ and $E(\pi_1)$ as functions of K

advance-purchase contract more costly projects can be realized than with traditional funding. Under the requirement of pre-ordering, the entrepreneur will incur the fixed costs only if the purchase actually takes place. This eliminates the possibility of making losses and leads to a stricter policy of exclusion: The entrepreneur raises the advance-purchase price above the monopoly price under traditional funding. Though this decreases the probability of realizing the project and selling the product, the effect of increased profits in case of realization dominates. For $1/4 < K < 1$, the probability of realization is zero under traditional funding but strictly positive under advance-purchase financing. Hence, the latter Pareto-dominates the former for sufficiently large fixed costs.

3.1.4 Discussion

For $0 \leq K \leq 1/4$, any type of customer would (weakly) prefer traditional funding and uniform pricing over financing based on advance-purchase contracts. This raises the question whether the entrepreneur can commit to exclusively rely on the latter funding method. If the entrepreneur lacks such commitment power, only projects with $1/4 < K \leq 1$ can be realized based on advance-purchase contracts whereas all projects with $0 \leq K \leq 1/4$ have to be financed traditionally.

3.2 Example: $N = 2$

To illustrate the strategic effects among customers resulting in the problem of free-riding, consider the case with two potential buyers.

3.2.1 Traditional Funding

With traditional funding, the project can be realized if and only if $K \leq 1/2$. In this case, some customer buys the product if and only if he has a willingness to pay of at least $p_0 = 1/2$ yielding an expected profit of $E(\pi_0) = 1/2 - K$.

To finance the project based on advance-purchase contracts, the entrepreneur can address either one or both potential buyers. Which of the two strategies depicted below yields higher expected profits depends on the capital requirement K.

3.2.2 Financing by at least one pre-order

Suppose the entrepreneur sets the advance purchase price to p_1. The customer who is indifferent between pre-ordering or not is characterized by the marginal willingness to pay θ_1 for which the utility from pre-ordering $\theta_1 - p_1$ equals the expected utility from a possible regular purchase $(1-\theta_1)(\theta_1 - p_r)$. Remember that the regular price is anticipated to equal $p_r = \theta_c/2$ by (2). Hence, the marginal willingness to pay is characterized by (5), i.e.

$$p_1 = \theta_1 \left(1 - \frac{1}{2}(1-\theta_1)\right) = \frac{1}{2}\theta_1(1-\theta_1).$$

Therefore, the entrepreneur's problem can be stated as follows: Choose θ_1 in order to maximize the expected profit

$$\begin{aligned} E(\pi_1) &= (1-\theta_1)^2 \cdot (2p_1 - K) + 2(1-\theta_1)\theta_1 \cdot \left(p_1 + \frac{\theta_1 - p_r}{\theta_1} p_r - K\right) \\ &= -\frac{3}{2}\theta_1^3 + \left(\frac{1}{2} + K\right)\theta_1^2 + \theta_1 - K \end{aligned}$$

subject to the constraint that one pre-order is sufficient to finance the project, i.e. $p_1 = \frac{1}{2}\theta_1(1-\theta_1) \geq K$. The unconstrained solution to this problem is derived from the necessary condition $\partial E(\pi_1)/\partial\theta_1 = 0$ yielding

$$\theta_1 = \frac{1}{9}\left(\sqrt{18 + (1+2K)^2} + 1 + 2K\right).$$

The unconstrained optimum will satisfy the constraint $p_1 \geq K$ if and only if K is sufficiently small.[3] Otherwise, the solution is given by $p_1 = K$ and

$$\theta_1 = \sqrt{2K + \frac{1}{4}} - \frac{1}{2},$$

which is feasible for all $K \leq 1$. The entrepreneur's resulting expected profit from financing the project by at least one pre-order is depicted in Figure 2.

3.2.3 Financing by at least two pre-orders

With a minimum number of two pre-orders the project can be realized if and only if both potential buyers purchase in advance. Therefore, the entrepreneur's problem can be stated as follows: Choose p_2 in order to maximize the expected profit

$$\begin{aligned} E(\pi_2) &= (1-p_2)^2 \cdot (2p_2 - K) \\ &= 2p_2^3 - (4+K)p_2^2 + (2+2K)p_2 - K \end{aligned}$$

subject to the constraints that two pre-orders are sufficient but one pre-order is insufficient to finance the project, i.e. $2p_2 \geq K > p_2$. The unconstrained solution to this problem is derived from the necessary condition $\partial E(\pi_2)/\partial p_2 = 0$ yielding

$$p_2 = \frac{K+1}{3}.$$

The unconstrained optimum will satisfy the constraints $2p_2 \geq K > p_2$ if and only if $1/2 < K \leq 2$. It then yields the expected profit $E(\pi_2) = \left(\frac{2-K}{3}\right)^3$. For $K \leq 1/2$, no solution exists unless there is a smallest monetary unit μ. As $\mu \to 0$, the optimal price p_2 converges to K yielding the asymptotic expected profit $E(\pi_2) = (1-K)^2 K$. The entrepreneur's resulting expected profit from financing the project by at least two pre-orders is also depicted in Figure 2.

3.2.4 Comparison

The optimum expected profit from advance-purchase financing is given by the upper envelope of $E(\pi_1)$ and $E(\pi_2)$ as functions of K. As Figure 2 illustrates, it will be better to use contracts based on at least one (two) pre-order(s) if

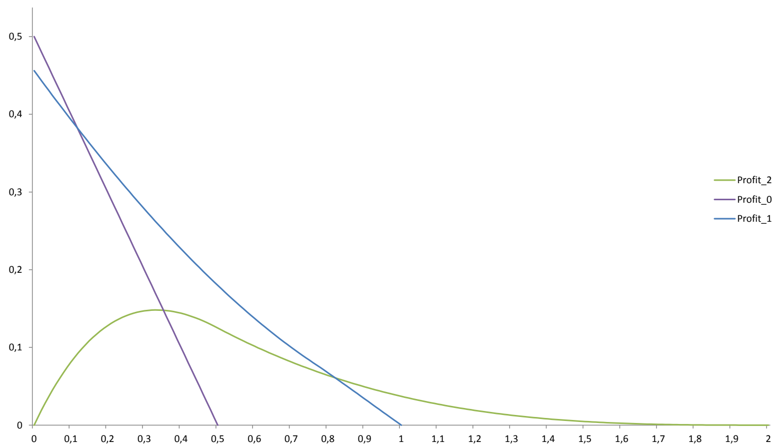

Figure 2: Graphs of $E(\pi_0)$, $E(\pi_1)$, and $E(\pi_2)$ as functions of K

K is sufficiently small (large). However, with two potential buyers, the use of advance-purchase contracts as a device of price discrimination is limited compared to the case with only one potential buyer. While advance-purchase financing is still a profitable measure for sufficiently high fixed costs K,[4] traditional funding is superior for small capital requirements. The reason for this limitation is a problem of free-riding arising from the public goods character of the customers' advance payments as contributions to the fixed costs. First, if financing is based on at least one pre-order, the agents' probability of being pivotal for the realization of the project will be smaller than one. This induces an advance-purchase price p_1 below the marginal willingness to pay θ_1, i.e. a positive information rent even for the marginal pre-order. Second, if financing is based on at least two pre-orders, the advance-purchase price p_2 will be limited by the incentive compatibility constraint that one single pre-order must not be sufficient to cover the fixed costs. Both effects undermine the discriminatory power of advance-purchase contracts reducing the gap between the advance-purchase price p_c and the regular price p_r.

3.2.5 Discussion

For fixed costs $K \leq 1/2$, for which the entrepreneur prefers advance-purchase financing over traditional funding, customers who pre-order would actually prefer traditional funding and uniform pricing over financing based on

[3]The critical capital requirement is about $K \approx 0.76$.

[4]Note once more that for $1/2 < K < 2$, the probability of realization is zero under traditional funding but strictly positive under advance-purchase financing. Hence, the latter also Pareto-dominates the former for sufficiently large fixed costs.

advance-purchase contracts. Similar to the case of one potential buyer only, this again raises the question whether the entrepreneur can commit to exclusively rely on advance-purchase financing. If the entrepreneur lacks such commitment power, only projects with $1/2 < K \leq 2$ can be realized based on advance-purchase contracts whereas all projects with $0 \leq K \leq 1/2$ have to be financed traditionally.

3.3 The limit case: $N \to \infty$

With an increasing number of potential buyers N, the probability that the pre-order of a single agent is pivotal for the availability of the product decreases. Put differently, the problem that the agents free-ride on the contributions of others is intensified. In order to maintain the agent's incentive to purchase in advance, the entrepreneur has to increase the information rent $\theta_c - p_c$. This difference will become larger if the marginal willingness to pay θ_c increases or the advance-purchase price p_c decreases. The former implies that the fraction of pre-orders gets smaller and smaller, the latter means that the difference between the advance-purchase price p_c and the regular price $p_r = \theta_c/2$ shrinks. Both effects make price discrimination based on advance-purchase contracts less profitable. In the limit, the entrepreneur cannot do better than by traditional funding and uniform pricing.

Corollary 1 *For any fixed cost $K > 0$ there exists some $N_0 \in \mathbb{N}$ such that for all $N \in \mathbb{N}$ with $N \geq N_0$ the entrepreneur's profit from traditional funding exceeds the profit from advance-purchase financing.*

Proof. To be done.

Irrespective of this result, the problem of committing to advance-purchase financing discussed above will arise if the fixed cost K is sufficiently small compared to the number of potential buyers N. Suppose $K \leq N/4$ which will always be fulfilled if N is sufficiently large. Even if there was a range of parameters for which the entrepreneur would prefer advance-purchase financing over traditional funding, pre-ordering customers would actually prefer traditional funding and uniform pricing over financing based on advance-purchase contracts. This reemphasizes the result that, in a large economy, advance-purchase surcharges cannot be enforced.

4 Conclusion

I have studied a simple model of advance-purchase financing in which a monopolist has to meet some capital requirement in order to start production. He then sells the related good to a limited number of potential buyers who are privately informed about their willingness to pay. In contrast to most part of the previous literature, I have shown that advance-purchase surcharges may arise as an optimal strategy of price discrimination. The discriminatory power, though, is limited by the problem of free-riding which gets worse as the number of potential buyers increases.

The setting considered in this paper can easily be reinterpreted as a model of monopolistic provision of excludable public goods under private information. Indeed, the introductory example of $R\&D$ in the public health sector allows for this reinterpretation. I have shown that advance-purchase arrangements allow to finance more costly projects than traditional funding sources. Thus, the former mode of financing will Pareto-dominate the latter if the capital requirement is sufficiently large. Besides efficiency concerns, following the *ability-to-pay-principle*, advance-purchase financing may also improve on the fairness of the allocation under traditional funding and uniform pricing: If the buyers' different valuations stem from differences in income and the good under consideration is normal, it will be the richer customers who pay the advance-purchase surcharges. In the context of $R\&D$ in the international health sector this means that, in many instances, advance purchase contracts could indeed improve the availability of new drugs and vaccines in poorer countries.

The analysis presented here assumes that the bargaining power is on the seller-side, though it is limited by the lack of commitment to not renegotiate with initially excluded customers. These limitations are captured by the specific structure of the sequential game I consider. The corresponding constraints on the set of admissible contracts are motivated by their relevance in practise. With the uninformed side of the market proposing the contract, this leads to a tractable screening model. However, further limitations of the entrepreneur's bargaining power could be considered. For example, think of commitment problems with respect of the exclusive use of advance-purchase contracts for funding the project as already discussed above. With more and more restrictions on the entrepreneur's bargaining power and the number of buyers being small, one might prefer to switch to a signaling model in which the informed side of the market makes proposals.

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