Charity as a Tool to Maximize Donations to Congestible Club Good

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1. Introduction

People often engage in numerous voluntary and free-entry socio-economic movements and groups [Tajfel, 1974], [Polletta and Jasper, 2001].

Some groups can produce benefits in the form of a club good for their members. It can take the form of a local public good, lobbying for the policy preferred by the members or collective action, for example, a success of political or social movement [Hindriks et al., 2006], [Olson, 2009], [Cornes, 1996]. A club good is a form of an impure public good, which differs from the pure public good in the way that it has an exclusion mechanism and thus can be enjoyed only by the members of the club. Non-members are excluded from consuming the club good[1].

Standard public good game implies that there is a number of agents with certain endowments that can be donated to produce a public good. The public good is non-competitive in consumption and non-exclusive, i.e. everyone has access to its full amount and each agent can consume this full amount without decreasing the amount of public good consumed by another agent. A typical public good model with linear production function and linear utilities leads to the equilibrium where each agent decides to free-ride [Fehr and Gächter, 2000]. Making a donation produces less utility for the agent than no donation when the marginal per capita return (MPCR) for the good is small enough. However, in many cases it is socially optimal for the agents to donate the whole endowment. A club good is essentially a public good produced and consumed within a certain group. Hence, the free-rider problem holds in the case of a club good as well: with linear production function and linear utilities, provided there is a free entry to the group, the Nash equilibrium implies zero donation. As a result, club goods suffer from underprovision.

This paper shows that maximizing donations to such a club good can sometimes be reached if, in addition to the club good, the group produces a public good that can be consumed by all members of the society. The public good can come in the form of charity, volunteer work, social projects or any kind of positive social activity. This external orientation of the group activity helps to overcome the free-rider problem by making the outgroup benefits large enough to drive the free riders out of the club.

The paper considers an economy with only one group that produces

[1] According to [Cornes, 1996, p. 159], club good is excludable because it emerges in clubs which by their nature are excludable.

both the club and the public good. The former can be enjoyed only by group members, while the latter is available to all agents in the economy. When a club member makes a donation to the club fund it is then distributed in some proportion between the club and the public good.

The club good is assumed to be congestible. It means that with greater number of club participants its benefits for each participant decrease [Cornes, 1996, p. 159]. Congestion takes place in such club goods as swimming pools, recreations facilities and roads, where the increase in the number of users creates problems such as pollution and overcrowding. Public good is not considered as congestible because it is a secondary good for the club and by nature is similar to charity and social activity, where there is no congestion.

There are mechanisms which can be applied to mitigate the free-rider problem and to make more agents donate. These include introduction of punishment of free-riders by other agents [Fehr and Gächter, 2000], [Carpenter et al., 2009] and rewards for active donors [Sefton et al., 2007], organizing public good donation process as an auction [Clarke, 1971], [Groves, 1973], as well as providing external competition with other groups (for club goods) [Erev et al., 1993]. The free-rider problem can be also overcome naturally when agents have social preferences that include utilities of other agents [Bowles, 2009].

This paper proposes another way of solving the free-rider problem for the club good. This mechanism implies that the club should provide benefits for the society as a whole, not only for the club members.

There are two important features of the club goods that are taken into account in the present paper. First, the club is assumed to be mixed, i.e. its members are heterogeneous in their personal value of the club good.

Second, the club good is assumed to be congestible. Congestion makes the club good less attractive with the increase in the number of its users (club members). In the paper I assume that the degree of congestion depends on the group size. It is also assumed that in small groups congestion term $f(S)$ is positive, but in groups which are too large it is negative. This assumption reflects the fact that congestion may become so large that it starts harming people. Also, sociological research suggests that people often seek balance between engagement in social groups and individuality, or assimilation and differentiation [Brewer, 1991]. As a result, when the group is overcrowded, this balance cannot be reached, and the utility of belonging to the group starts to decrease.

Generally, there are two main points of view on the relationship between the degree of congestion and the group size. Some authors claim that due to congestion, outcome of the group monotonously decreases with larger group size [Cornes, 1996], [Brewer and Kramer, 1986]. An alternative point of view suggests that there is parabolic relationship between the two: participation in the group may be characterized by the network effects, the benefits of which are decreased when the group size is too large or too small [Brewer, 1991], and there may exist the group size that minimizes congestion. Both cases are considered in the present paper, although main conclusions do not differ significantly between the two.

The model conclusions can be applied to maximize donations in social and political movements. In these movements, donations can take forms of both tangible and intangible resources, and this covers a large variety of cases [Cornes, 1996]. Participants of these movements experience collective identity and attachment to some of them [Cornes, 1996][p. 1], which corresponds well to model assumptions. Importantly, the present paper predicts that social and political movements can maximize their resources and power through sacrificing some of these resources to external activities that could benefit the population as a whole. For example, social change movements may use charity as their secondary goal or do volunteer work with the help of their members.

Various online communities that unite people for a specific purpose (such as discussing and solving technical computer problems or maintaining a common online resource, e.g. Wikipedia) may benefit from a similar strategy[2]. The model predicts that for the closed online groups (especially in social networks) with useful content it may be optimal to open the content for everyone to see in order to boost donations from members in the form of new content, which eventually will lead to greater total donations and better performance of the group.

The paper is structured as follows: section 2 provides general setting of the model, section 2.2 solves the model for a congestible club good, section **??** presents the case of a congestible club good with network effect as a robustness check, section 3 concludes.

[2]Attachment to online communities promotes participation, brings positive feelings and makes members help the community more, however, it is typically reduced with greater group size [Kraut and Kiesler, 2006].

2. The Model

I consider the economy consisting of heterogeneous agents with an initial endowment. The agents may choose to participate in a group which produces both the public and the congestible club good. Entrance to the group is voluntary and costless. The group produces the goods from donations of its participants. The whole donated amount is distributed between production of the public and the club good in some proportion.

An important feature of clubs is that they are voluntary, [Cornes, 1996], which means that nobody can be forced to enter the club against their will. Additionally, nobody can exclude agents from the club against their will. In other words, I assume that there is a natural exclusion mechanism for the club that prevents agents from having an obligation to participate in it, so that each agent can freely decide whether to enter it or not.

A. Structure

The economy consists of n agents who can decide whether to enter a club[3] or not. There is only one free-entrance group in the economy. Each agent is endowed with some amount of a private good $y = 1/n$ so that the total amount of good in the economy equals 1. Agents who are the members of a group can donate part (or all) of the private good to a group fund. The fund then distributes the donation by contributing fixed share $1-q$ to the club good and share q to the public good. The public good is produced with constant marginal per capita return (MPCR) b and the club good is produced with constant marginal per capita return a. Public good can be consumed by all individuals whereas club good can be consumed by group members only.

Denote by I the set of all group members, and by O the set of agents who are not members of the group.

B. Actions

Each agent in the economy can decide whether to enter the club or not. If one participates, he or she can choose the amount to donate to the group fund out of their endowment.

The game description is the following: in the first stage each agent decides whether to become a member of the club. In the second stage,

[3]The terms *club* and *group* in this section are used interchangeably.

each club member announces the amount of money he wants to donate. Agents who are not members of the club have no action to choose in the second stage.

C. Preferences

Preferences of agents are represented by the linear utility function which includes the amount of the good consumed privately, and the benefits of the public and the club good consumed. Congestion of the club good affects only its users, i.e. club members.

Since there is a finite number of agents in the economy, the total amount donated will be $\sum_{j\in I} g_j$ — the sum of donations g_j of ingroup members (set I) willing to donate.

Utility function of an outgroup member is:

$$\bar{u}_i = y + bq\sum_{j\in I} g_j$$

And utility of an ingroup member is as follows:

$$u_i(g_i) = y - g_i + a(1-q)\sum_{j\in I} g_j + bq\sum_{j\in I} g_j + \gamma_i\sum_{j\in I} g_j + f_i(S)\left(1-(1-q)\sum_{j\in I} g_j\right),$$

where g_j is a donation of j-th agent and $f_i(S)$ is the congestion term.

The last term in this utility function means that the agents dislike the participation of others (see below). Yet they also feel better when the total pie is bigger, so that $\sum_{j\in I} g_j$ is larger, and it cannot exceed 1 — total amount of good in the economy.

D. Types

Each agent experiences the club good congestion if he or she decides to participate in the group. Congestion is decreasing in absolute terms with the amount of the total group donation $\sum_{j\in I} g_j$. Congestion depends on S which is a share of the ingroup members in the population. On the other hand, agents are heterogenous in the parameter γ_i which corresponds to the (additional) individual value of the club good. Individual parameter of the club good value γ_i is assumed to be uniformly distributed in $[0, h]$ and reflects heterogeneity of preferences of agents for the club good. The congestion term depends on S negatively[4].

[4] Congestion may depend negatively on the size of the group because large groups typically discourage the members' participation in the activities of the group

2.1. Benchmark Cases

Consider the case when $f_i(S) = 0$, all $\gamma_i = 0$ and $q = 0$. In this case a typical public good game emerges. If $a > 1$ everyone decides to donate to the public good, if $a < \frac{1}{n}$ everyone decides to free-ride. In the case of $\frac{1}{n} < a < 1$ there are no donations, and every agents decides to free-ride again. Here, the social dilemma emerges. Whereas everyone donates nothing, it would be better off for everyone to donate their whole endowment to the public good.

Consider the case when there is the public good and the club good, and $f_i(S) = 0$, all $\gamma_i = 0$. Club membership does not bring any disutility, and everyone participates and has the opportunity to donate. Then the utility of each member equals:

$$u_i(g_i) = y - g_i + a(1-q)\sum_{j\in I} g_j + bq\sum_{j\in I} g_j,$$

and under assumptions $a < 1$, $b < 1$ everyone free-rides again. It is also assumed that MPCR of the public good b is less than that of the club good a, or club good should be abandoned due to its inefficiency. These assumptions together with $a(1-q) + bq > \frac{1}{n}$ create the free-rider problem, because it is not profitable to donate to the fund alone, but it is Pareto-efficient to donate jointly with other agents. When nobody donates, the utility of each agent equals y, but when everyone donates jointly, then utilities are $(a(1-q) + bq)ny > \frac{1}{n} \cdot ny = y$, and the case where everyone donates Pareto-dominates free-riding.

2.2. Economy with the congestible club good

Suppose $f_i(S) = -f(S)$, and $f(S)$ is an increasing function of the group size S. This form of preferences reflects symmetric congestion which increases when a greater share of the population participates in the group. In addition, we assume that $f(0) = 0$ and $f(1) > 0$. The first condition reflects the fact that (almost) all agents benefit from being part of the group if this group is small ($\gamma_i - f(0) > 0$), and have negative feeling towards the group if it is too large due to congestion.

The utility function of an ingroup member includes part of the endowment y being consumed privately, a donation (g_i), the club good utility ($(\gamma_i + a(1-q))\sum_{j\in I} g_j$), the public good utility ($a(1-q)\sum_{j\in I} g_j$) and

[Olson, 2009]. Additionally, when network effects are present, congestion may increase up to some point, and then starts to decrease, because less people may benefit from participating in a group that is too large.

congestion $(-f(S))\left(1-(1-q)\sum_{j\in I} g_j\right)$. The agent i maximizes the following utility function:

$$u_{\text{ingroup}} = y - g_i + (a(1-q)+\gamma_i)\sum_{j\in I} g_j + bq\sum_{j\in I} g_j - f(S)\left(1-(1-q)\sum_{j\in I} g_j\right) \to \max_{0\le g_i\le y}$$

where a and b are the MPCRs of the club and the public good respectively, q is a share of the donation that finances the production of the public good, and $\sum_{j\in I} g_j$ is the total donation of the ingroup agents.

If an agent stays out of the club, his utility depends only on the endowment and the amount of the public good is not affected by his decisions because outgroup members cannot donate to the fund. His utility equals:

$$\bar{u}_i = y + bq \sum_{j\in I, j\neq i} g_j$$

It is now possible to solve the game described in Part B using backward induction. In the second stage, the agent decides on the donation that he wants to make to the group fund. In other words, the agent maximizes the following utility function with respect to the donation g_i:

$$u_{i,\text{ingroup}} = y - g_i + (\gamma_i + a(1-q))\sum_{j\in I} g_j + bq\sum_{j\in I} g_j - f(S)\left(1-(1-q)\sum_{j\in I} g_j\right) \to \max_{0\le g_i\le y}$$

For the agent to make a donation the following must be true, call it provision condition:

$$a(1-q) + bq + \gamma_i + f(S)(1-q) \geq 1$$

To derive the optimal amount of the donation for a group member we solve the maximization problem of a club member.

Lemma 2.1. *If an agent is the member of the club with $\gamma \in [0,h]$, he will donate only y or 0 to the fund, and $\exists \bar{\gamma} : \forall i$ if $\gamma_i > \bar{\gamma}$, the agent donates y, and if $\gamma_i < \bar{\gamma}$ he donates 0.*

Proof. The club member solves the following problem:

$$u_{\text{ingroup}} = y - g_i + (\gamma_i + a(1-q))\sum_{j\in I} g_j + bq\sum_{j\in I} g_j - f(S)\left(1-(1-q)\sum_{j\in I} g_j\right) \to \max_{0\le g_i\le y}$$

As the problem is linear with respect to g_i, the solution of i'th member is determined by the sign of the coefficient of g_i.

The condition on coefficient turns out to be the following:

$$a(1-q)+bq+\gamma_i+f(S)(1-q)-1 \gtreqless 0$$

$$\gamma_i \gtreqless 1-a(1-q)-bq-f(S)(1-q)=\bar{\gamma}$$

If the left hand side is greater than the right one, then the coefficient before g_i turns out to be positive, and the agent donates full endowment (y). If the coefficient is negative, the agent donates 0. If it turns out to be exactly zero, then one is indifferent and can donate any amount between 0 and y.

Because γ_i is uniformly distributed in $[0,h]$, the boundary $\bar{\gamma}$ turns out to be equal to $a(1-q)+bq+f(S)-1$, if $0<a(1-q)+bq+f(S)(1-q)-1<h$, zero if $0 \geq a(1-q)+bq+f(S)(1-q)-1$, and h if $a(1-q)+bq+f(S)(1-q)-1 \geq h$. Because the subgame is considered, parameters a, b, q and S are fixed here. Since there is a large number of agents in the economy, the agents who happen to be indifferent for any amount of donation are assumed to not influence the solution of the game, as they provide an insufficient amount of total donation. Further, the actions of such agents will not be taken into account. □

Among the club members, the agents with high enough value of the club good $\gamma_i > \gamma$ decide to donate, and the agents with low values of the club good, i.e. with $\gamma_i < \bar{\gamma}$, decide to free ride. In addition, the members of the group consider only two donation strategies — to donate everything (y) or nothing.

Consider now the first stage of the game. In the first stage, the agent decides whether to enter the group or not. Here, one compares his utility as an ingroup member taking into account his possible donation, and the utility as an outgroup member. The agent decides to enroll in the group if:

$$u_{i,\text{ingroup}}-\bar{u}_i=(\gamma_i+a(1-q))\sum_{j\in I} g_j - f(S)\left(1-(1-q)\sum_{j\in I} g_j\right)+(bq-1)g_i \geq 0$$

$$a(1-q)+\gamma_i+f(S)(1-q) \geq \frac{f(S)+(1-bq)g_i}{\sum_{j\in I} g_j}$$

call it a membership condition.

Remember that provision condition is

$$\gamma_i+f(S)(1-q) \geq 1-bq-a(1-q)$$

.

Case 1. $1-bq>0$**,** $1-bq-a(1-q)\leq 0$

This means that either $a>1$ or $b>1$, or both.

If $1-bq-a(1-q)\leq 0$ then provision condition is always satisfied, all agents donate y if they enter the club, and we have to derive the share of agents entering the club (S). To do that we rewrite membership condition as

$$\gamma_i \geq \frac{f(S)+(1-bq)g_i}{\sum_{j\in I} g_j} - a(1-q) - f(S)(1-q) =$$

$$= \frac{f(S)+(1-bq)y}{nyS} - a(1-q) - f(S)(1-q) = \gamma_{enter}.$$

Remember that $ny=1$. Because of uniform distribution the share S equals to

$$S = 1 - \frac{1}{h}\gamma_{enter},$$

$$hS - a(1-q) - f(S)(1-q) + \frac{f(S)+(1-bq)y}{S} = h.$$

Clearly $S=0$ makes LHS higher than RHS. At the same time at $S=1$ we get

$$LHS - RHS = -a(1-q) + f(1)q + \frac{1-bq}{n}.$$

If this is less than 0 then there exists a solution. However, it's not clear if the solution is unique.

We will consider specific $f(S)=\beta S$**.**

This choice of the function allows us to characterize all equilibria. We get

$$hS^2 - a(1-q)S - \beta S^2(1-q) + \beta S + \frac{1-bq}{n} = hS,$$

$$(h-\beta(1-q))S^2 + (\beta - a(1-q) - h)S + \frac{1-bq}{n} = 0.$$

In this case there could be no more than two solutions for the following reason. Consider again

$$hS - a(1-q) - f(S)(1-q) + \frac{f(S)+(1-bq)y}{S} = h.$$

For linear $f(S)=\beta S$ we get

$$z(S) = hS - a(1-q) - \beta(1-q)S + \beta + \frac{(1-bq)y}{S} = h. \quad (1)$$

Take first derivative of $z(S)$:

$$z'(S) = h - \beta(1-q) - \frac{(1-bq)y}{S^2}.$$

This function is negative around $S = 0$. As $S \in [0,1]$ this means that the derivative may change sign at most once (switching from being negative to being positive). Thus $z(S)$ either decreases in the interval, or first decreases and then increases. For that reason, equation $z(S) = h$ could have zero, one or two solutions.

Note that the point where $z(S)$ changes sign is the solution to

$$z'(S) = h - \beta(1-q) - \frac{(1-bq)y}{S^2} = 0, \quad S_0 = \sqrt{\frac{1-bq}{n(h-\beta(1-q))}},$$

and we assume here that $h - \beta(1-q) > 0$ (congestion is not too high; we keep assumption $1 - bq > 0$). This point is the point of (unrestricted) minimum of $z(S)$ in the whole interval $[0,\infty]$. If this is greater than 0 then $z(S)$ decreases in $[0,1]$ and thus there exists unique solution iff

$$\frac{1-bq}{n(h-\beta(1-q))} \geq 1, \quad z(1) = h - \alpha(1-q) - \beta(1-q) + \beta + \frac{1-bq}{n} \leq h.$$

We could rewrite all conditions as

$$\frac{1-bq}{n} - (h-\beta(1-q)) \geq 0, \beta q - \alpha(1-q) + \frac{1-bq}{n} \leq 0, 1-bq > 0, 1-\alpha(1-q)-bq \leq 0. \tag{2}$$

Condition $h - \beta(1-q) > 0$ is not necessary for uniqueness: if $h - \beta(1-q) \leq 0$ then $z'(S)$ is always negative and so we could not have more than one solution.

We can now invert the equation (1) to find $S(q)$ that solves the equation (this function $S(q)$ also depends on other parameters, however we will only change q in order to maximize S and hence will keep other parameters constant).

Now we can prove the following

Proposition 1.

If (2) holds then $z(S)$ is decreasing in the interval [0,1] and

1) $S(q)$ is unique;

2) $S(q)$ is increasing;

3) $S(q)$ reaches maximum when one or more conditions (2) are binding.

Proof.

1) As the left hand side of the membership condition is a decreasing function of S and the right hand side does not depend on S, then there is only one solution of the equation with respect to all parameters including q and, hence, $S(q)$ is unique.

2) Consider $z'(S) = h - \beta - \frac{y}{S^2} + \beta q + \frac{bqy}{S^2}$. With greater values of q this function increases which means $z(S)$ decreases with a slower pace. So the whole function $z(S)$ moves up. Thus the solution $S(q)$ increases in q.

3) As $S(q)$ is increasing, we only need to stay in the area of unique solution when we increase q. The area is determined by conditions (2) and maximum is reached at the boundary of this area.

□

Note that in Case 1 we get $D = S$. This proposition provides clear result: if we consider area with unique solution and congestion linear in the share of people participating, then the maximum donations increase with q — the share spent on public good — and the only restriction is to be within the bounds determined by say $bq < 1$ or $1 - a(1-q) - bq \le 0$. The share spent on public good should not be too high, otherwise conditions (2) will not hold.

The idea behind the results is that when $1 \le a(1-q) + bq$, utility of donations is high enough. If at the same time congestion is not too serious ($\beta q - a(1-q) + \frac{1-bq}{n} \le 0$), then the trade-off between higher utility of club/public goods consumption and congestion always leads to participation.

Note that $\beta q - a(1-q) + \frac{1-bq}{n} \le 0$ and $bq < 1$ are harder and harder to satisfy when q increases. The first one is the condition of existence of solution ($z(1) \le h$). If it does not hold then we may think in terms of γ_{enter}:

$$\gamma_{enter} = \frac{f(S) + (1-bq)y}{nyS} - a(1-q) - f(S)(1-q) = \beta + \frac{(1-bq)y}{S} - a(1-q) - \beta(1-q)S.$$

This function is decreasing in S. This means that the cutoff level for entering the group is very high at low S. So nobody participates if S is low — if one participates then due to participation condition she invests whole endowment y, but then due to low participation of others the gain is too low and it is better to consume privately. But then there is no equilibrium with $S \approx 1$: when $S = 1$ we get

$$\gamma_{enter}(S = 1) = \beta q - a(1 - q) + \frac{1 - bq}{n} > 0,$$

so the cutoff level is greater than 0 even if $S = 1$ — so there is no equilibrium.

Case 2. $1-bq<0$

In this case provision condition is always satisfied (every participant of the club contributes y). Similarly to the case 1 we derive (1) — equation for S:

$$z(S) = hS - \alpha(1-q) - \beta(1-q)S + \beta + \frac{(1-bq)y}{S} = h.$$

However, now we have different case as around $S=0$, $z(S)$ is negative (high increase in utility from small increases in S). This means that

$$z'(S) = h - \beta(1-q) - \frac{(1-bq)y}{S^2}$$

is now decreasing function of S and is positive around $S=0$. So $z(S)$ is increasing around 0. If $h-\beta(1-q) \geq 0$ then $z(S)$ is always increasing and the condition of existence of equilibrium is

$$z(1) = h - \alpha(1-q) + \beta q + \frac{1-bq}{n} \geq h.$$

This condition is symmetric to case 1.

If $h-\beta(1-q)<0$ but $S_0 = \sqrt{\frac{1-bq}{n(h-\beta(1-q))}} \geq 1$ we still have unique solution when $z(1) \geq h$. Moreover, as

$$z'(S) = h - \beta - \frac{y}{S^2} + \frac{bqy}{S^2} + \beta q,$$

the solution $S(q)$ now decreases in q: higher q means more positive $z'(S)$ and thus higher $z(S)$. But now $z(S)$ is an increasing function — so moving it up means lower solution to $z(S)=h$.

We can rewrite the conditions as

$$\frac{1-bq}{n} - (h-\beta(1-q)) \geq 0, \beta q - \alpha(1-q) + \frac{1-bq}{n} \geq 0, 1-bq<0. \tag{3}$$

Proposition 2. *If (3) holds then $z(S)$ is increasing in the interval [0,1] and*

1) *$S(q)$ is unique;*

2) *$S(q)$ is decreasing;*

3) *$S(q)$ reaches maximum when one or more conditions (3) are binding.*

Case 3. $1-bq-a(1-q)>0$. Then provision condition is binding. Let D be the share of donors in the population. Consider the boundary agent being indifferent between the membership with and without donation. Denote his club good value as $\hat{\gamma}$. For such agent it holds that $\frac{h-\hat{\gamma}}{h}=D$ or $\hat{\gamma}=h(1-D)$.

The provision condition equality then becomes:

$$a(1-q)+bq+f(S)(1-q)+h(1-D)=1 \tag{4}$$

The membership condition equality similarly becomes:

$$a(1-q)+h(1-S)+f(S)(1-q)=\frac{f(S)}{Dny}. \tag{5}$$

So we have a system of two equations:

$$f(S)(1-q)+h(1-D)=1-a(1-q)-bq,\ aD(1-q)+hD(1-S)+Df(S)(1-q)=f(S). \tag{6}$$

Note that if in equilibrium $D(1-bq)<f(S)$ then given $D\leq S$

$$(1-bq)D=(f(S)(1-q)+h(1-D)+a(1-q))D\geq(f(S)(1-q)+h(1-S)+a(1-q))D=f(S).$$

Thus we should have in equilibrium $D(1-bq)\geq f(S)$; we call it "function condition".

Using still $f(S)=\beta S$ we get

$$\beta S(1-q)+h(1-D)=1-a(1-q)-bq,\ aD(1-q)+hD(1-S)+\beta(1-q)DS=\beta S.$$

Due to function condition and $D\leq S$ we get

$$\beta S\leq D(1-bq)\leq S(1-bq),$$

so we should have

$$\beta\leq 1-bq. \tag{7}$$

This condition is really restrictive in comparison to cases 1 and 2: first, $\beta<1$ which means congestion is low; second, without (7) there is no equilibrium.

We can express D in terms of S from the second equation in (6):

$$D=\frac{\beta S}{a(1-q)+h(1-S)+\beta(1-q)S}$$

and plug it into the first equation:

$$z_1(S) = \beta S(1-q) + h - h\frac{\beta S}{a(1-q) + h(1-S) + \beta(1-q)S},$$

then the following equation defines equilibrium:

$$z_1(S) = 1 - a(1-q) - bq.$$

We can slightly rewrite

$$z_1(S) = \beta S(1-q) + h - h\frac{\beta S}{a(1-q) + h + (\beta(1-q) - h)S}.$$

If $\beta(1-q) - h = 0$ then $z_1(S)$ is linear in S and we only need $(z_1(1) - (1 - a(1-q) - bq))(z_1(0) - 1 - a(1-q) - bq) \leq 0$ to obtain equilibrium.

If $\beta(1-q) - h \neq 0$ then

$$z_1(S) = \beta S(1-q)+h-h\frac{\beta S}{a(1-q)+h+(\beta(1-q)-h)S} = \beta S(1-q)+h-h\frac{\frac{\beta S}{\beta(1-q)-h}}{\frac{a(1-q)+h}{\beta(1-q)-h}+S} =$$

$$= \beta S(1-q) + h - h\frac{\beta}{\beta(1-q)-h} + h\frac{\frac{\beta}{\beta(1-q)-h}\frac{a(1-q)+h}{\beta(1-q)-h}}{\frac{a(1-q)+h}{\beta(1-q)-h}+S} =$$

$$= \beta S(1-q) + h - h\frac{\beta}{\beta(1-q)-h} + h\frac{\beta(a(1-q)+h)}{(a(1-q)+h)(\beta(1-q)-h) + S(\beta(1-q)-h)^2}.$$

Assume now that $h \geq 1$ (personal preference over club good is dispersed). Then, given $\beta \leq 1 - bq < 1$ and $S \leq 1$ we get

$$h > \beta(1-q),\ S(h - \beta(1-q)) < 1 \times h \leq a(1-q) + h.$$

So denominator in the ratio

$$\frac{\beta(a(1-q)+h)}{(a(1-q)+h)(\beta(1-q)-h) + S(\beta(1-q)-h)^2}$$

is negative. We can write

$$z_1(S) = \beta S(1-q)+h+h\frac{\beta}{h-\beta(1-q)} - h\frac{\beta(a(1-q)+h)}{h-\beta(1-q)}\frac{1}{a(1-q)+h-S(h-\beta(1-q))}.$$

Now all the summands are positive except the last one. Note that first term increases in S and the last term decreases in S.

Consider

$$z_1'(S) = \beta(1-q) - h\frac{\beta(a(1-q)+h)}{h-\beta(1-q)}\frac{h-\beta(1-q)}{(a(1-q)+h-S(h-\beta(1-q)))^2} =$$

$$= \beta(1-q) - h\frac{\beta(a(1-q)+h)}{(a(1-q)+h-S(h-\beta(1-q)))^2}.$$

This function decreases with S,

$$z_1'(0) = \beta(1-q) - h\frac{\beta(a(1-q)+h)}{(a(1-q)+h)^2} = \beta(1-q) - h\frac{1}{a(1-q)+h}.$$

Note that $z_1(0) = h > 1-a(1-q)-bq$. If $z_1'(0) < 0$ (say when h is large enough) then $z_1(S)$ is a decreasing function, so condition of existence of equilibrium is

$$z_1(1) \le 1-a(1-q)-bq.$$

Now we would like to study D as a function of q. To do that we could express from the second equation in (6):

$$aD(1-q)+hD(1-S)+\beta(1-q)DS = \beta S,\ S = \frac{(a(1-q)+h)D}{\beta-\beta(1-q)D+hD},$$

and then plug in the first one:

$$\beta S(1-q)+h(1-D) = 1-a(1-q)-bq,\ -\beta(1-q)\frac{(a(1-q)+h)D}{\beta-\beta(1-q)D+hD}+hD = h+a(1-q)+bq-1.$$

Denote

$$z_2(D) = -\beta(1-q)\frac{(a(1-q)+h)D}{\beta-\beta(1-q)D+hD}+hD = hD-\beta(1-q)\frac{\frac{a(1-q)+h}{h-\beta(1-q)}D}{\frac{\beta}{h-\beta(1-q)}+D} =$$

$$= hD-\beta(1-q)\frac{a(1-q)+h}{h-\beta(1-q)}+\beta(1-q)\frac{\frac{a(1-q)+h}{h-\beta(1-q)}\frac{\beta}{h-\beta(1-q)}}{\frac{\beta}{h-\beta(1-q)}+D} =$$

$$= hD-\beta(1-q)\frac{a(1-q)+h}{h-\beta(1-q)}+\frac{\beta^2(a(1-q)+h)(1-q)}{\beta(h-\beta(1-q))+(h-\beta(1-q))^2D}.$$

Then $z_2(0) = 0 < h+a(1-q)+bq-1$,

$$z_2'(D) = h-\frac{\beta^2(a(1-q)+h)(1-q)(h-\beta(1-q))^2}{(\beta(h-\beta(1-q))+(h-\beta(1-q))^2D)^2} =$$

$$= h - \frac{\beta^2(a(1-q)+h)(1-q)}{(\beta+(h-\beta(1-q))D)^2}.$$

This function increases in D. We see that $z_2'(0) = h-(a(1-q)+h)(1-q) = hq-a(1-q)^2$, and if this is greater than 0 then $z_2(D)$ is an increasing function of D — which means that there could be no more than one solution. To have an equilibrium we have to demand

$$z_2(1) = h-\beta(1-q)\frac{a(1-q)+h}{h-\beta(1-q)} + \frac{\beta^2(a(1-q)+h)(1-q)}{\beta(h-\beta(1-q))+(h-\beta(1-q))^2} =$$

$$= h-\beta(1-q)\frac{a(1-q)+h}{h-\beta(1-q)} + \frac{\beta^2(a(1-q)+h)(1-q)}{h-\beta(1-q)}\frac{1}{\beta+(h-\beta(1-q))} =$$

$$= h - \frac{\beta(1-q)(a(1-q)+h)}{h-\beta(1-q)}\left[1-\frac{\beta}{\beta+(h-\beta(1-q))}\right] =$$

$$= h - \frac{\beta(1-q)(a(1-q)+h)}{h-\beta(1-q)}\frac{h-\beta(1-q)}{\beta+(h-\beta(1-q))} = h - \frac{\beta(1-q)(a(1-q)+h)}{\beta+(h-\beta(1-q))}.$$

If $z_2(1) \geq h+a(1-q)+bq-1$ then there exists unique equilibrium.

Now let's see how D depends on q:

$$z_2'(D) = h - \frac{\beta^2(a(1-q)+h)(1-q)}{(\beta+(h-\beta(1-q))D)^2}.$$

When q increases, numerator of the ratio decreases and denominator increases which means that $z_2'(D)$ increases with q. So if we consider the case with $hq-a(1-q)^2 \geq 0$ then increasing q means faster increasing $z_2(D)$ and lower D as a solution. Here increasing q decreases agents' desire to invest in goods.

The very last case is if $hq-a(1-q)^2 < 0$. However, this only means that $z_2(D)$ first decreases and then increases as $z_2'(D)$ has one point of sign change (from negative to positive). As $z_2(0) = 0 < h+a(1-q)+bq-1$, this means that $z_2(D)$ lies below $h+a(1-q)+bq-1$ until $z_2'(D) \leq 0$ (and in some interval after that). If we assume $z_2(1) \geq h+a(1-q)+bq-1$ we still have unique solution, and that solution should satisfy $z_2' > 0$ — so our analysis of dependence of $z_2'(D)$ on q above applies.

Let's summarize the results: we assume that

$$z_2(1) \geq h+a(1-q)+bq-1, \beta \leq 1-bq, a(1-q)+bq \leq 1, h \geq 1. \tag{8}$$

Proposition 3. *If (8) holds then $z_2(D)$ is either increasing or first decreasing and then increasing in the interval [0,1] and*

1) $D(q)$ is unique;

2) $D(q)$ is decreasing;

3) $D(q)$ reaches maximum when one or more conditions (8) *are binding.*

3. Conclusion

Introducing charity motives to the mixed clubs is proposed to be one of the mechanisms which can help to overcome the free-rider problem in case when the club good is congestible. This mechanism partially copes with the free-rider problem, reducing the number of free-riders in the group and boosting donations.

The main result of the paper is the following: it may be optimal for the clubs to spend more resources on social activities and charities, not only for club projects even if the latter can be more profitable in terms of outcome per unit of resources spent. It happens because more social activities encourages more people to stay out of the group reducing club good congestion, and it leads to greater donations of group members. Fine-tuning of the share of external activities may lead to greater amounts of donation and to prosperity of some groups. It may also explain why some groups have become more successful in terms of donation rate than others.

In the presence of a congestible club good, higher levels of external activities can increase total donations. Because of heterogeneity all agents of the economy choose one of the three following strategies. The first is to enter the club and donate, the second is to enter the club and not donate and, finally, not to enter the club. Greater external activity (production of a public good) leads to the reduction in the number of free-riders (those who enter the club, but not donate) and helps to maximize the share of donors. The point with maximum share of donors is also the point where the maximum amount of donations is attained.

Since sponsoring the public good may help reduce the number of free-riders and increase the number of donors, the described mechanism can serve as an alternative to monitoring within the group. Differently from monitoring, the proposed mechanism does not impose costs on the participating agents. The only loss that occurs under this mechanism is related to the fact that donations used for the club good are generally more productive than those used for the public good. However, potential benefits of reducing the number of free-riders may outweigh these costs and become a welfare improving alternative to various monitoring and punishment schemes.

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