

Self-Fulfilling Debt Crises: Can Monetary Policy Really Help?¹

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Abstract

This paper examines quantitatively the potential for monetary policy to avoid self-fulfilling sovereign debt crises. We combine a version of the slow-moving debt crisis model proposed by Lorenzoni and Werning (2014) with a standard New Keynesian model. We consider both conventional and unconventional monetary policy. With price rigidity, the real cost of debt can be reduced through lower real interest rates. On the other hand, deflation of long-term debt is less effective and requires higher inflation rates. In general, we show that crisis equilibria can only be avoided with steep inflation rates for a sustained period of time, the cost of which is likely to be much larger than government default.

1 Introduction

A popular explanation for the sovereign debt crisis that has impacted European periphery countries since 2010 is self-fulfilling sentiments. If market participants believe that sovereign default of a country is more likely, they demand higher spreads, which over time raises the debt level and therefore indeed makes eventual default more likely.¹ This view of self-fulfilling beliefs is consistent with the evidence that the surge in sovereign bond spreads in Europe during 2010-2011 was disconnected from debt ratios and other macroeconomic fundamentals (e.g., de Grauwe and Ji, 2013). However, countries with comparable debt and deficits outside the Eurozone (e.g., the US, Japan or the UK) were not impacted. This difference in experience has often been attributed to the fact that the highly indebted non-Eurozone countries have their own currency.² The central bank has additional tools to support the fiscal authority, either in the form of standard inflation policy or by providing liquidity, which can avoid self-fulfilling debt crises. In fact, the decline in European spreads since mid 2012 is widely attributed to a change in ECB policy towards explicit backing of periphery government debt.

The question that we address in this paper is whether central banks can credibly avert self-fulfilling debt crises. This is a quantitative question that requires a reasonably realistic model. Existing models of self-fulfilling sovereign debt crises either take the form of liquidity or rollover crises, such as Cole and Kehoe (2000), or models in the spirit of Calvo (1988), where default becomes self-fulfilling by raising the spread on sovereign debt. In this paper we are interested in the second type of self-fulfilling crises, which fits more closely with the experience in Europe. However, while the contribution by Calvo was important in highlighting the mechanism, it uses a two-period setup that quantitatively is of limited interest.

¹This view was held by the ECB President Draghi himself: "... the assessment of the Governing Council is that we are in a situation now where you have large parts of the euro area in what we call a "bad equilibrium", namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios." (press conference, September 6, 2012). In the academic literature, versions of this argument can be found, among others, in Aguiar et al. (2013), Camous and Cooper (2014), Cohen and Villemot (2011), Corsetti and Dedola (2014), de Grauwe (2011), de Grauwe and Ji (2013), Gros (2012), Jeanne (2012), Jeanne and Wang (2013), Krugman (2013), Lorenzoni and Werning (2014) and Miller and Zhang (2012).

²See for example de Grauwe (2011), de Grauwe and Jin (2013), Jeanne (2012) and Krugman (2013).

We therefore analyze the role that the central bank can play in the context of a framework developed by Lorenzoni and Werning (2014), which extends the mechanism of Calvo (1988) to a more dynamic setting. The model exhibits "slow moving" debt crisis. The anticipation of a possible future default on long term bonds leads interest rates and debt to gradually rise over time, justifying the belief of ultimate default. This framework has two advantages. First, while the mechanism is in the spirit of Calvo (1988), the presence of long-term debt and more realistic dynamics provides a better framework for quantitatively evaluating the role of monetary policy. The slow-moving nature of the crisis also gives the central bank more time to act to support the fiscal authority. Second, the model connects closely to the recent experience in Europe, where sovereign default spreads rose over several years without setting off immediate default events.

While the LW model is real and does not have a monetary authority, we analyze the role of monetary policy by incorporating the LW framework into a standard New Keynesian model. We follow the literature and consider a specification that yields empirically consistent responses of output and inflation to monetary shocks. We then first analyze the role of conventional monetary policy. Expansionary policy that lowers interest rates, raises inflation and raises output slows down government debt accumulation in three ways. First, lower real interest rates reduce the real cost of new borrowing. Second, inflation erodes the value of the outstanding debt. Third, higher output raises government tax revenue.

Most of the paper considers the case, also analyzed in LW, where the decision to default or not takes place at a known future date T . At that time uncertainty about future fiscal surpluses is resolved. At an initial date 0 a self-fulfilling expectation shock can lead to beliefs of default at time T . Investors then demand a higher yield on new debt, which leads to a more rapid accumulation of debt between the initial period 0 and the default period T . If debt is large enough, default may occur due to insolvency. There is a range of initial debt levels at time 0 for which self-fulfilling crises may occur. Monetary policy can be used to relax the solvency constraint both ex ante, before T , and ex post, after T . We will also consider an extension in which there is uncertainty about T .

Sufficiently aggressive monetary policy can in principle preclude a self-fulfilling debt crisis. However, the policy needs to be credible and therefore not too costly, especially in terms of inflation. Assuming reasonable parameters of the model and the debt maturity structure, we find that avoiding a crisis equilibrium is typically

very costly. For example, with an initial debt level in the middle of the multiplicity range (112% of GDP), optimal policy that avoids a self-fulfilling crisis implies that prices ultimately increase by a factor of 5 and the peak annual inflation rate is 24%. Avoiding self-fulfilling equilibria requires very steep inflation rates for a sustained period of time, the cost of which is likely to be much larger than that of allowing the government to default. We find that this result is robust to significant changes in the assumed parameters of both the LW and NK components of the model.

Apart from conventional interest rate policy, which can be conducted in a cashless economy, we also analyze monetary backstops. These rely on the resources that the central bank can bring to bear through its balance sheet. Specifically, the central bank can buy government debt in exchange for monetary liabilities. Outside of the zero lower bound (ZLB) we find that this is of little help. It generates some seigniorage, but this is typically small.³ Once the ZLB is reached, there is no limit to the central bank's ability to exchange bonds for monetary liabilities. However, we will argue that this can only help avert a self-fulfilling debt crisis if the ZLB is structural and long-lasting.

There is one way in which a monetary backstop would work, even outside a structural ZLB. This applies to a monetary union, where the central bank supports a periphery government rather than the central government. The ECB could for example sell German bonds and buy Spanish bonds at low interest rates. This explains why the bond purchasing program announced by the ECB in the summer of 2012 was successful. But this does not explain why highly indebted non-Eurozone countries have not experienced a sovereign debt crisis.

The impact of monetary policy in a self-fulfilling debt crisis environment was first analyzed by Calvo (1988), who examined the trade-off between outright default and debt deflation. Corsetti and Dedola (2014) extend the Calvo model to allow for both fundamental and self-fulfilling default. They show that with optimal monetary policy debt crises can still happen, but for larger levels of debt. They also analyze monetary backstops, showing that a crisis can be avoided if the central bank buys government debt in exchange for riskfree interest-paying central bank liabilities. Reis (2013) and Jeanne (2012) both develop stylized two-period

³This is consistent with Reis (2013) and Hilscher et al. (2014), who also find that monetary backstops have little effect when interest rates are positive. As Reis (2013) puts it, "In spite of the mystique behind a central bank's balance sheet, its resource constraint bounds the dividends it can distribute by the present value of seigniorage, which is a modest share of GDP."

models with multiple equilibria to illustrate ways in which the central bank can act to avoid the bad equilibrium.

Some papers consider more dynamic models. Camous and Cooper (2014) use a dynamic overlapping-generation model with strategic default. They show that the central bank can avoid self-fulfilling default if they commit to a policy where inflation depends on the state (productivity, interest rate, sunspot). Aguiar et al. (2013) consider a dynamic model with self-fulfilling roll-over crises and analyze the vulnerability to self-fulfilling rollover crises, depending on the aversion of the central bank to inflation. Although a rollover crisis occurs suddenly, it is assumed that there is a grace period to repay the debt, allowing the central bank time to reduce the real value of the debt through inflation. They find that only for intermediate levels of the cost of inflation do debt crises occur under a narrower range of debt values.

All these papers derive analytical conditions under which central bank policy would avoid a self-fulfilling debt crisis. While this delivers interesting insights, it does not answer the more quantitative question whether realistically the central bank can be expected to adopt a policy that prevents a self-fulfilling crisis. In order to do so we relax the assumptions of one-period bonds, flexible prices, and instantaneous crises that are adopted in the literature above for tractability reasons.⁴

The rest of the paper is organized as follows. Section 2 presents the slow-moving debt crisis model based on LW. It starts with a real version of the model and then presents its extension to a monetary environment. Subsequently, it analyzes the various channels of monetary policy in this framework. Section 3 describes the New Keynesian part of the model and its calibration. Section 4 analyzes the quantitative impact of conventional monetary policy and provides extensive sensitivity analysis. Section 5 discusses the monetary backstop options and Section 6 concludes. Some of the technical details are left to the Appendix, while additional algebraic details can be found in a separate Technical Appendix.

⁴There are recent models that examine the impact of monetary policy in the presence of long-term government bonds. Leeper and Zhou (2013) analyze optimal monetary (and fiscal) policy with flexible prices, while Bhattarai et al. (2013) consider a New Keynesian environment at ZLB. These papers, however, do not allow for the possibility of sovereign default. Sheedy (2014) and Gomes et al. (2014) examine monetary policy with long-term private sector bonds.

2 A Model of Slow-Moving Self-Fulfilling Debt Crisis

In this section we present a dynamic sovereign debt crisis model based on LW. We first describe the basic structure of the model in a real environment. We then extend the model to a monetary environment and discuss the impact of monetary policy on the existence of self-fulfilling debt crises. We finally extend this to an economy with positive money demand by considering the resources that a central bank can bring to bear through its balance sheet. We focus on the dynamics of asset prices and debt for given interest rates and goods prices. The latter will be determined in a New Keynesian model that we describe in Section 3.

2.1 A Real Model

We consider a simplified version of the LW model. As in the applications considered by LW, there is a key date T at which uncertainty about future primary surpluses is resolved and the government makes a decision to default or not.⁵ Default occurs at time T when the present value of future primary surpluses is insufficient to repay the debt. We assume that default does not happen prior to date T as there is always a possibility of large primary surpluses from T onward. We also follow LW by abstracting from the possibility of default after date T . In one version of their model LW assume that T is known to all agents, while in another they assume that it is unknown and arrives each period with a certain probability. In most of the analysis in this paper we will adopt the former assumption, but in section 4.5 we consider an extension where T is uncertain and can take two possible values.

The only simplification we adopt relative to LW concerns the process of the primary surplus. For now we assume that the primary surplus s_t is constant at \bar{s} between periods 0 and $T - 1$. Below we extend this by allowing for a procyclical primary surplus.⁶ A second assumption concerns the primary surplus value starting at date T . Let \tilde{s} denote the maximum potential primary surplus that the

⁵One can for example think of countries that have been hit by a shock that adversely affected their primary surpluses, which is followed by a period of uncertainty about whether and how much the government is able to restore primary surpluses through higher taxation or reduced spending.

⁶LW assume a fiscal rule whereby the surplus is a function of debt.

government is able to achieve, which becomes known at time T and is constant from thereon. LW assume that it is drawn from a log normal distribution. Instead we assume that it is drawn from a binary distribution, which simplifies the algebra and the presentation. It can take on only two values: s_{low} with probability ψ and s_{high} with probability $1 - \psi$. When the present discounted value of \tilde{s} is at least as large as what the government owes on the debt, there is no default at time T and the actual surplus is just sufficient to satisfy the budget constraint (generally below \tilde{s}). We assume that s_{high} is big enough such that this is always the case when $\tilde{s} = s_{high}$.⁷ When $\tilde{s} = s_{low}$ and its present value is insufficient to repay the debt, the government defaults and the primary surpluses from T onwards is s_{low} .

A key feature of the model is the presence of long-term debt. As usual in the literature, assume that bonds pay coupons (measured in goods) that depreciate at a rate of $1 - \delta$ over time: κ , $(1 - \delta)\kappa$, $(1 - \delta)^2\kappa$, and so on.⁸ A smaller δ therefore implies a longer maturity of debt. This facilitates aggregation as a bond issued at $t - s$ corresponds to $(1 - \delta)^s$ bonds issued at time t . We can then define all outstanding bonds in terms of the equivalent of newly issued bonds. We define b_t as debt measured in terms of the equivalent of newly issued bonds at $t - 1$ on which the first coupon is due at time t .

Let Q_t be the price of a government bond. At time t the value of government debt is $Q_t b_{t+1}$. In the absence of default the return on the government bond from t to $t + 1$ is

$$R_t^g = \frac{(1 - \delta)Q_{t+1} + \kappa}{Q_t} \quad (1)$$

If there is default at time T , bond holders are able to recover a proportion $\zeta < 1$ of the present discounted value s^{pdv} of the primary surpluses s_{low} . In that case the return on the government bond is

$$R_{T-1}^g = \frac{\zeta s^{pdv}}{Q_{T-1} b_T} \quad (2)$$

Government debt evolves according to

$$Q_t b_{t+1} = R_{t-1}^g Q_{t-1} b_t - s_t \quad (3)$$

In the absence of default this may also be written as $Q_t b_{t+1} = ((1 - \delta)Q_t + \kappa)b_t - s_t$. The initial stock of debt b_0 is given.

⁷See Appendix A for details.

⁸See for example Hatchondo and Martinez (2009).

We assume that investors also have access to a short-term bond with a gross real interest rate R_t . The only shocks in the model occur at time 0 (self-fulfilling shock to expectations) and time T (value of \tilde{s}). In other periods the following risk-free arbitrage condition holds (for $t \geq 0$ and $t \neq T - 1$):

$$R_t = \frac{(1 - \delta)Q_{t+1} + \kappa}{Q_t} \quad (4)$$

For now we assume, as in LW, a constant interest rate, $R_t = R$. In that case $s^{pdv} = Rs_{low}/(R - 1)$ is the present discounted value of s_{low} . There is no default at time T if s^{pdv} covers current and future debt service at T , which is $((1 - \delta)Q_T + \kappa)b_T$. For convenience it is assumed that $\kappa = R - 1 + \delta$, so that (4) implies that $Q_T = 1$. This means that there is no default as long as $s^{pdv} \geq Rb_T$, or if

$$b_T \leq \frac{1}{R - 1}s_{low} \equiv \tilde{b} \quad (5)$$

When $b_T > \tilde{b}$, the government partially defaults on the debt, with investors seizing a fraction ζ of the present value s^{pdv} of surpluses.

This framework may lead to multiple equilibria and to a slow moving debt crisis, as described in LW. The existence of multiple equilibria can be seen graphically from the intersection of two schedules, as illustrated in Figure 1. The first schedule, labeled "pricing schedule" is given by:

$$Q_{T-1} = 1 \quad \text{if } b_T \leq \tilde{b} \quad (6)$$

$$= \psi \frac{\zeta s^{pdv}}{Rb_T} + (1 - \psi) \quad \text{if } b_T > \tilde{b} \quad (7)$$

When $b_T \leq \tilde{b}$, the arbitrage condition (4) also applies to $t = T - 1$, implying $Q_{T-1} = 1$. When b_T is just above \tilde{b} , there is a discrete drop of the price because only a fraction ζ of primary surpluses can be recovered by bond holders in case of default. For larger values of the debt, Q_{T-1} will be even lower as the government will default on a larger portion of the debt in case $\tilde{s} = s_{low}$.

The second schedule is the "debt accumulation schedule". It is found by first integrating (4) backwards from $T - 1$ to 0, which gives

$$Q_t - 1 = \left(\frac{1 - \delta}{R} \right)^{T-1-t} (Q_{T-1} - 1) \quad (8)$$

Substituting in (3) and integrating the government budget constraint forward from 0 to $T - 1$, we get (see Appendix B):

$$b_T = (1 - \delta)^T b_0 + \frac{\chi^\kappa \kappa b_0 - \chi^s \bar{s}}{Q_{T-1}} \quad (9)$$

where

$$\begin{aligned} \chi^\kappa &= R^{T-1} + (1 - \delta)R^{T-2} + (1 - \delta)^2 R^{T-3} + \dots + (1 - \delta)^{T-1} \\ \chi^s &= 1 + R + R^2 + \dots + R^{T-1} \end{aligned}$$

The numerator $\chi^\kappa \kappa b_0 - \chi^s \bar{s}$ in (9) corresponds to the accumulated new borrowing between 0 and T . We assume that it is positive, which happens when the primary surplus is insufficient to pay the coupons on the initial debt. A sufficient, but not necessary, condition is that the primary surplus itself is negative during this time. The debt accumulation schedule then gives a negative relationship between b_T and Q_{T-1} . When Q_{T-1} is lower, asset prices from 0 to $T - 2$ are also lower. This implies a higher yield as coupons remain the same, reflecting a premium for possible default at time T . These default premia imply a more rapid accumulation of debt and therefore a higher debt b_T at $T - 1$.

Figure 1 shows these two schedules and illustrates the multiplicity of equilibria. There are two stable equilibria, represented by points A and B. At point A, $Q_{T-1} = 1$. The bond price is then equal to 1 at all times. This is the "good" equilibrium in which there is no default. At point B, $Q_{T-1} < 1$. This is the "bad" equilibrium. Asset prices starting at time 0 are less than 1 in anticipation of possible default at time T . Intuitively, when agents believe that default is likely, they demand default premia (implying lower asset prices), leading to a more rapid accumulation of debt, which in a self-fulfilling way indeed makes default more likely.

In the bad equilibrium there is a slow-moving debt crisis. As can be seen from (8), using $Q_{T-1} < 1$, the asset price instantaneously drops at time 0 and then continues to drop all the way to $T - 1$. Correspondingly, default premia gradually rise over time. Such a slow-moving crisis occurs only for intermediate levels of debt. When b_0 is sufficiently low, the debt accumulation schedule is further to the left, crossing below point C, and only the good equilibrium exists. When b_0 is sufficiently high, the debt accumulation schedule is further to the right, crossing above point D, and only a bad equilibrium exists with s_{low} . In that case default is unavoidable.

2.2 A Monetary Model

We now extend the model to a monetary economy. The goods price level is P_t . R_t is now the gross nominal interest rate and $r_t = R_t P_t / P_{t+1}$ the gross real interest rate. The central bank can set the interest rate R_t and affect P_t . The nominal debt level at time $t - 1$ is B_t and the initial level of nominal debt is B_0 . We define $b_t = B_t / P_t$. The arbitrage equation with no default remains (4), while the government budget constraint for $t \neq T - 1$ becomes

$$Q_t B_{t+1} = ((1 - \delta)Q_t + \kappa)B_t - s_t P_t \quad (10)$$

s_t is now the real primary surplus and $s_t P_t$ the nominal surplus.

At time T the real obligation of the government to bond holders is $[(1 - \delta)Q_T + \kappa]b_T$. The no default condition remains $b_T \leq \tilde{b}$, with the latter now defined as

$$\tilde{b} = \frac{s^{pdv}}{(1 - \delta)Q_T + \kappa}. \quad (11)$$

where

$$s^{pdv} = \left[1 + \frac{1}{r_T} + \frac{1}{r_T r_{T+1}} + \dots \right] s_{low} \quad (12)$$

and Q_T is equal to the present discounted value of coupons:

$$Q_T = \frac{\kappa}{R_T} + \frac{(1 - \delta)\kappa}{R_T R_{T+1}} + \frac{(1 - \delta)^2 \kappa}{R_T R_{T+1} R_{T+2}} + \dots \quad (13)$$

In analogy to the real model, the new pricing schedule becomes

$$Q_{T-1} = \frac{(1 - \delta)Q_T + \kappa}{R_{T-1}} \quad \text{if } b_T \leq \tilde{b} \quad (14)$$

$$= \psi \frac{\zeta s^{pdv}}{R_{T-1} b_T} + (1 - \psi) \frac{(1 - \delta)Q_T + \kappa}{R_{T-1}} \quad \text{if } b_T > \tilde{b} \quad (15)$$

This implies a relationship between Q_{T-1} and b_T that has the same shape as in the real model, but is now impacted by monetary policy through real and nominal interest rates and inflation.

The debt accumulation schedule is again derived by first integrating the arbitrage condition backwards from $T - 1$ to 0 and using the result to integrate the government budget constraint forward from 0 to $T - 1$. This gives (see Appendix B):

$$b_T = (1 - \delta)^T \frac{B_0}{P_T} + \frac{P_{T-1}}{P_T} \frac{\chi^\kappa \kappa B_0 / P_0 - \chi^{s\bar{s}}}{Q_{T-1}} \quad (16)$$

where

$$\begin{aligned}\chi^\kappa &= \left[r_{T-2} \dots r_1 r_0 + (1 - \delta) r_{T-2} \dots r_1 \frac{P_0}{P_1} + (1 - \delta)^2 r_{T-2} \dots r_2 \frac{P_0}{P_2} + \dots + (1 - \delta)^{T-1} \frac{P_0}{P_{T-1}} \right] \\ \chi^s &= 1 + r_{T-2} + r_{T-2} r_{T-3} + \dots + r_{T-2} \dots r_1 r_0\end{aligned}$$

The schedule again implies a negative relationship between Q_{T-1} and b_T . Monetary policy shifts the schedule through its impact on interest rates and inflation.

2.3 The Impact of Monetary Policy

Monetary policy affects the path of interest rates and prices, which in turn shift the two schedules and therefore can affect the existence of self-fulfilling debt crises. The idea is to implement a monetary policy strategy conditional on expectations of sovereign default, which only happens in the crisis equilibrium. If this strategy is successful and credible, the crisis equilibrium is avoided altogether and the policy does not need to be implemented. It is therefore the threat of such a policy that may preclude the crisis equilibrium.

In terms of Figure 1, the crisis equilibrium is avoided when the debt accumulation schedule crosses below point C. This is the case when

$$\frac{\chi^\kappa \kappa B_0 / P_0 - \chi^s \bar{s}}{s^{pdv} - ((1 - \delta) Q_T + \kappa) (1 - \delta)^T B_0 / P_T} r_{T-1} < \psi \zeta + 1 - \psi \quad (17)$$

The central bank can impact this condition through both *ex ante* policies, taking place between 0 and $T - 1$, and *ex post* policies, taking place in period T and afterwards.

First consider *ex ante* policies. Both inflation and lower real interest rates between 0 and T can help to avert a self-fulfilling debt crisis. Inflation before time T reduces the real value of the coupons before T on the initial debt B_0 . This is captured through χ^κ in the numerator of (17). It also erodes the real value of the coupons on the initial debt after time T , which is captured by the term B_0 / P_T in the denominator in (17). In terms of Figure 1, both have the effect of shifting the debt accumulation schedule downward.

Reducing real interest rates lowers the cost of new borrowing. This is captured through both χ^κ and χ^s in the numerator of (17), which represents the accumulated new borrowing from 0 to T . Similar to *ex-ante* inflation policy, it has the effect of shifting the debt accumulation schedule downward. There is one additional real

interest rate effect, which is specific to the assumption that the central bank knows exactly when the default decision is made. By reducing the real interest rate r_{T-1} the central bank can offset the negative impact of expected default on Q_{T-1} . This is captured through the last term on the left hand side of (17). This effect implies an upward shift of the pricing schedule.

Finally, there are two ways that ex-post monetary policy can help to avoid a crisis. First, lower real interest rates starting at time T raise the present discounted value of the primary surpluses. This makes it easier to repay the debt at time T . This is captured through s^{pdv} in the denominator of (17).⁹ Finally, inflation after time T reduces the real value of the coupons after time T on the original debt B_0 . This is reflected in a lower value of Q_T in the denominator. In terms of Figure 1, both of these ex-post policies lead to a rise in \tilde{b} , shifting the vertical section of the pricing schedule to the right.

There is one additional impact of monetary policy that we have not explicitly modeled yet, but will introduce at the end of the next section. We will allow the primary surpluses to be pro-cyclical. In that case expansionary monetary policy, by increasing output, will raise primary surpluses. Ex-ante policy then implies a rise in \bar{s} , lowering the numerator in (17), while ex-post policy raises s^{pdv} , raising the denominator in (17). Similar to the other ex-ante and ex-post policies, these have the effects of respectively lowering the debt accumulation schedule and shifting the vertical portion of the pricing schedule to the right.

2.4 Central Bank Resources

So far we have considered the role of standard monetary policy aimed at affecting inflation, interest rates and output. We now consider additional ways that monetary policy can help in avoiding the crisis equilibrium by considering resources that a central bank can bring to bear through its balance sheet. It is useful to start from the budget constraint of the central bank:

$$Q_t B_{t+1}^c + D_{t+1}^c = ((1 - \delta)Q_t + \kappa)B_t^c + R_{t-1}D_t^c + [M_t - M_{t-1}] - Z_t \quad (18)$$

A superscript c refers to assets held by the central bank. We assume that the central bank holds both government bonds B_t^c and one-period bonds D_t^c . The

⁹This is offset to some extent by a higher present discounted value of the coupons after time T , captured by a rise in Q_T in the denominator.

value of central bank assets decreases due to the depreciation of government bonds and payments Z_t to the treasury. It increases due to the coupon and interest payments and an expansion $M_t - M_{t-1}$ of monetary liabilities. The government budget constraint remains (10), except that Z_t is now subtracted on the right hand side.

The balance sheets of the central bank and government are interconnected as most central banks pay a measure of net income (including seigniorage) to the Treasury as a dividend.¹⁰ We will therefore consider the consolidated government budget constraint by substituting the central bank constraint into the government budget constraint:

$$Q_t B_{t+1}^p = ((1 - \delta)Q_t + \kappa)B_t^p + [D_{t+1}^c - R_{t-1}D_t^c] - [M_t - M_{t-1}] - s_t P_t \quad (19)$$

where $B_t^p = B_t - B_t^c$ is government debt held by the general public. There are now two additional ways to reduce the debt issued to the private sector: selling other assets held by the central bank and earning positive seigniorage $M_t - M_{t-1}$.

Let \tilde{m} represent accumulated seigniorage between 0 and $T - 1$:

$$\tilde{m} = \frac{M_{T-1} - M_{T-2}}{P_{T-1}} + r_{T-2} \frac{M_{T-2} - M_{T-3}}{P_{T-2}} + \dots + r_0 r_1 \dots r_{T-2} \frac{M_0 - M_{-1}}{P_0} \quad (20)$$

This is affected by ex-ante policies. Similarly, let m^{pdv} denote the present discounted value of seigniorage revenues starting at date T :

$$m^{pdv} = \frac{M_T - M_{T-1}}{P_T} + \frac{1}{r_T} \frac{M_{T+1} - M_T}{P_{T+1}} + \frac{1}{r_T r_{T+1}} \frac{M_{T+2} - M_{T+1}}{P_{T+2}} + \dots \quad (21)$$

This is affected by ex-post policies.

In terms of Figure 1, with the new consolidated government budget constraint (19) the pricing schedule becomes

$$Q_{T-1} = \frac{(1 - \delta)Q_T + \kappa}{R_{T-1}} \quad \text{if } b_T^p \leq \tilde{b} \quad (22)$$

$$= \psi \frac{\zeta s^{pdv} + m^{pdv} + r_{T-1} d_T^c}{R_{T-1} b_T^p} + (1 - \psi) \frac{(1 - \delta)Q_T + \kappa}{R_{T-1}} \quad \text{if } b_T^p > \tilde{b} \quad (23)$$

with

$$\tilde{b} = \frac{s^{pdv} + m^{pdv} + r_{T-1} d_T^c}{(1 - \delta)Q_T + \kappa}. \quad (24)$$

¹⁰See Hall and Reis (2013) for a discussion.

where $d_T^c = D_T^c/P_{T-1}$ is the real value at time $T - 1$ of one-period bonds held by the central bank. The debt accumulation schedule now becomes

$$b_T^p = (1 - \delta)^T \frac{B_0^p}{P_T} + \frac{P_{T-1}}{P_T} \frac{\chi^\kappa \kappa B_0^p / P_0 - \chi^s \bar{s} - \tilde{m} + d_T^c - r_{-1} r_0 r_1 \dots r_{T-2} d_0^c}{Q_{T-1}} \quad (25)$$

When the central bank sells all its short-term bonds, so that $d_T^c = 0$, the pricing schedule remains unchanged but the debt accumulation schedule shifts down. Ex-ante seigniorage \tilde{m} shifts the debt accumulation schedule down also, while ex-post seigniorage $m^{p dv}$ shifts the vertical section of the price schedule to the right (\tilde{b} rises), similar to the other ex-post policies discussed above. We discuss these policies in section 5. In the next two sections we focus on conventional monetary policy that operates through interest rates.

3 A Basic New Keynesian Model

We consider a standard cashless New Keynesian model based on Galí (2008, ch.3), with three extensions suggested by Woodford (2003): i) habit formation; ii) price indexation; iii) lagged response in price adjustment. These extensions are standard in the monetary DSGE literature and are introduced to generate more realistic responses to monetary shocks. The main effect of these extensions is to generate a delayed impact of a monetary policy shock on output and inflation, leading to the humped-shaped response seen in the data.

3.1 Households

With habit formation, households maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - \eta C_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} - z \iota_t \right) \quad (26)$$

where total consumption C_t is

$$C_t = \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (27)$$

and N_t is labor and z is a default cost. We have $\iota_t = 0$ if there is no default at time t and $\iota_t = 1$ if there is default. The default cost does not affect households'

decisions, but provides an incentive for authorities to avoid default. Habit persistence, measured by η , is a common feature in NK models to generate a delayed response of expenditure and output.

The budget constraint is

$$P_t C_t + D_{t+1}^p + Q_t B_{t+1}^p = W_t N_t + \Pi_t + R_{t-1} D_t^p + R_{t-1}^g Q_{t-1} B_t^p - T_t \quad (28)$$

where P_t is the standard aggregate price level and W_t is the wage level. Π_t are firms profits distributed to households and T_t are lump-sum taxes. We will abstract from government consumption, so that the primary surplus is $P_t s_t = T_t$. For now we assume that $B_t^p = B_t$ since we abstract from the central bank balance sheet. Moreover, we set the supply of short-term bonds to zero so that in equilibrium $D_t^p = 0$.

The first-order conditions with respect to asset holdings are

$$\tilde{C}_t = \beta E_t R_t \frac{P_t}{P_{t+1}} \tilde{C}_{t+1} \quad (29)$$

$$\tilde{C}_t = \beta E_t R_t^g \frac{P_t}{P_{t+1}} \tilde{C}_{t+1} \quad (30)$$

where

$$\tilde{C}_t \equiv (C_t - \eta C_{t-1})^{-\sigma} - \eta \beta E_t (C_{t+1} - \eta C_t)^{-\sigma}$$

The combination of (29) and (30) gives the arbitrage equations (4), (14), and (15). This is because government default, which lowers the return on government bonds, does not affect consumption due to Ricardian equivalence.¹¹

Let Y_t denote real output and c_t , y_t and y_t^n denote logs of consumption, output and the natural rate of output. Using $c_t = y_t$, and defining $x_t = y_t - y_t^n$ as the output gap, log-linearization of the Euler equation (29) gives the dynamic IS equation

$$\tilde{x}_t = E_t \tilde{x}_{t+1} - \frac{1 - \beta \eta}{\sigma} (i_t - E_t \pi_{t+1} - r^n) \quad (31)$$

where

$$\tilde{x}_t = x_t - \eta x_{t-1} - \beta \eta E_t (x_{t+1} - \eta x_t) \quad (32)$$

Here $i_t = \ln(R_t)$ will be referred to as the nominal interest rate and $r^n = -\ln(\beta)$ is the natural rate of interest. The latter uses our assumption below of constant productivity, which implies a constant natural rate of output.

¹¹When substituting the government budget constraint $Q_t B_{t+1} = R_t^g Q_{t-1} B_t - T_t$ into the household budget constraint (28), and imposing asset market equilibrium, we get $C_t = Y_t$, which is unaffected by default.

3.2 Firms

There is a continuum of firms on the interval $[0, 1]$, producing differentiated goods. The production function of firm i is

$$Y_t(i) = AN_t(i)^{1-\alpha} \quad (33)$$

We follow Woodford (2003) by assuming firm-specific labor.

Calvo price setting is assumed, with a fraction $1 - \theta$ of firms re-optimizing their price each period. In addition, it is assumed that re-optimization at time t is based on information from date $t - d$. This feature, adopted by Woodford (2003), is in the spirit of the model of information delays of Mankiw and Reis (2001). It has the effect of a delayed impact of a monetary policy shock on inflation, consistent with the data.¹² Analogous to Christiano et al. (2005), Smets and Wouters (2003) and many others, we also adopt an inflation indexation feature in order to generate more persistence of inflation. Firms that do not re-optimize follow the simple indexation rule

$$\ln(P_t(i)) = \ln(P_{t-1}(i)) + \gamma\pi_{t-1} \quad (34)$$

where $\pi_{t-1} = \ln P_{t-1} - \ln P_{t-2}$ is aggregate inflation one period ago.

Leaving the algebra to the Technical Appendix, these features give the following Phillips curve (after linearization):

$$\pi_t = \gamma\pi_{t-1} + \beta E_{t-d}(\pi_{t+1} - \gamma\pi_t) + E_{t-d}(\omega_1 x_t + \omega_2 \tilde{x}_t) \quad (35)$$

where

$$\omega_1 = \frac{1-\theta}{\theta}(1-\theta\beta)\frac{\phi+\alpha}{1-\alpha+(\alpha+\phi)\varepsilon}$$

$$\omega_2 = \frac{1-\theta}{\theta}(1-\theta\beta)\frac{1-\alpha}{1-\alpha+(\alpha+\phi)\varepsilon}\frac{\sigma}{1-\eta\beta}$$

3.3 Monetary Policy

The question is not whether monetary policy can avoid a self-fulfilling crisis, but whether it can credibly do so. This will be the case when the cost of such a policy is less than the cost of default. The latter is hard to measure with any precision

¹²This feature can also be justified in terms of a delay by which newly chosen prices go into effect.

and we will not attempt to do so. Instead we will focus on what we can measure, which is the cost associated with the monetary policy that avoids a self-fulfilling crisis. If this cost is excessively high, we consider it to be implausible and therefore not credible.

We follow most of the literature by using a quadratic approximation of utility. The central bank then minimizes the following objective function:

$$E_0 \sum_{t=0}^{\infty} \beta^t \{ \mu_x (x_t - \nu x_{t-1})^2 + \mu_\pi (\pi_t - \gamma \pi_{t-1})^2 \} \quad (36)$$

where ν , μ_x and μ_π a function of model parameters (see the Technical Appendix for the derivation). The central bank chooses the optimal path of nominal interest rates over $H > T$ periods. After that, we assume an interest rate rule as in Clarida et al. (1999):

$$i_t - \bar{i} = \rho(i_{t-1} - \bar{i}) + (1 - \rho)(\psi_\pi E_t \pi_{t+1} + \psi_y x_t) + \varepsilon_t \quad (37)$$

where $\bar{i} = -\ln(\beta)$ is the steady state nominal interest rate. We will choose H to be large. Interest rates between time T and H involve ex-post-policy.¹³

Optimal policy is chosen conditional on two types of constraints. The first is the ZLB constraint that $i_t \geq 0$ for all periods. In the good equilibrium that is the only constraint and the optimal policy implies $i_t = \bar{i}$ each period, delivering zero inflation and a zero output gap. However, conditional on expectations of default that raise default premia, the central bank will engage in expansionary policy that is just sufficient to avoid the self-fulfilling bad equilibrium so that (17) is satisfied as an equality. Graphically, this means that the two schedules meet at point C in Figure 1. Technically the self-fulfilling equilibrium still exists at point C. But infinitesimally stronger monetary policy pushes the debt accumulation schedule below point C, so that the two schedules only meet in the good equilibrium.

Using the NK Phillips curve (35), the dynamic IS equation (31), and the policy rule (37) after time H , we solve for the path of inflation and output gap conditional on the set of H interest rates chosen. We then minimize the welfare cost (36) over the H interest rates subject to $i_t \geq 0$ and (17) as an equality.

¹³Since H will be large, the precise policy rule after H does not have much effect on the results.

3.4 Calibration

We consider one period to be a quarter and we normalize the constant productivity A such that the natural rate of output is equal to 1 annually (0.25 per quarter). The other parameters are listed in Table 1. The left panel shows the parameters from the LW model, while the right panel lists the parameters that pertain to the New Keynesian part of the model.

Consider first the LW parameters. We set $\beta = 0.99$, implying a 4% annualized interest rate. A key parameter, which we will see has an impact on the results, is δ . In the benchmark parameterization we set it equal to 0.05, which implies a government debt duration of 4.2 years. This is typical in the data. For example, OECD estimates of the Macauley duration in 2010 are 4.0 in the US and 4.4 for the average of the five European countries that experienced a sovereign debt crisis (Greece, Italy, Spain, Portugal and Ireland). The coupon is determined such that $\kappa = 1/\beta - 1 + \delta$.

The other parameters, T and the fiscal surplus parameters, do not have a direct empirical counterpart, but are chosen so that there is a broad range of self-fulfilling equilibria. If the range of initial debt B_0 for which multiple equilibria are feasible is very narrow, the entire problem would be a non-issue.

The range of B_0 for which there are multiple equilibria under passive monetary policy ($i_t = \bar{i}$) is $[B_{low}, B_{high}]$, where¹⁴

$$B_{low} = \frac{\beta}{1 - \beta} \frac{(\psi\zeta + 1 - \psi)\beta^T s_{low} + (1 - \beta^T)\bar{s}}{1 - (1 - \zeta)(1 - \delta)^T \beta^T \psi} \quad (38)$$

$$B_{high} = \frac{\beta}{1 - \beta} (\beta^T s_{low} + (1 - \beta^T)\bar{s}) \quad (39)$$

Under the parameters in Table 1 this range is $[0.79, 1.46]$. This means that debt is between 76% and 146% of GDP. This is not unlike debt of the European periphery hit by the 2010 crisis, where debt ranged from 62% in Spain to 148% in Greece. Note that the assumption $\bar{s} = -0.01$, corresponding to a 4% annual primary deficit, also corresponds closely to Europe, where the five periphery crisis countries had an average primary deficit of 4.4% in 2010. We set $T = 20$ for the benchmark, corresponding to 5 years. We will see in section 4.3 that there are other parameter choices that lead to a similar range for B_0 with multiple equilibria without much effect on results.

¹⁴These values lead to equilibria at points C and D in Figure 1.

The New Keynesian parameters are standard in the literature. The first 5 parameters correspond exactly to those in the Galí (2008) textbook. The habit formation parameter, the indexation parameter and the parameters in the interest rate rule are all the same as in Christiano et al. (2005). We take $d = 2$ from Woodford (2003, p. 218-219), which also corresponds closely to Rotemberg and Woodford (1997). This set of parameters implies a response to a small monetary policy shock under the Taylor rule that is similar to the empirical VAR results reported by Christiano et al. (2005). The level of output and inflation at their peak correspond exactly to that in the data. Both the output and inflation response is humped shaped like the data, although the peak response (quarter 6 and 3 respectively for inflation and output) occurs a bit earlier than in the data.

4 How Can Monetary Policy Avoid a Debt Crisis?

4.1 Flexible Prices

It is useful to start with flexible prices, where only inflation can affect the government budget constraint and the no-default condition. In that case the output gap is zero and the real interest rate is constant at $1/\beta$. In the NK model inflation is only costly because infrequent price adjustments lead to changes in relative prices that generate an inefficient equilibrium. When all firms adjust their prices simultaneously, there is no inflation cost even if inflation is very high. This is obviously unrealistic. We therefore focus on the inflation rate needed to avoid default rather than the welfare cost, which is zero under flexible prices.

Assume that the central bank increases inflation to a constant level during H periods, after which inflation goes back to zero. The left panel of Figure 2 shows what inflation rate would be needed to avoid a self-fulfilling crisis. It is shown as a function of B_0 for the full range of initial debt $[B_{low}, B_{high}]$ where multiple equilibria exist in the absence of monetary policy, indicated by the thick segment on the horizontal axis. The right panel shows the cumulative price increase. This is the price level at the end of the H quarters, starting with $P_0 = 1$. The results are shown both for $H = T = 20$, so that there is inflation for 5 years, and for $H = 40$, implying 10 years of constant inflation. With $H = 20$ there is only ex-ante policy,

while $H = 40$ implies also ex-post policy.

The level of inflation needed to avoid default is very high. Let $B_0 = B_{middle} = 1.12$ be the level of initial debt in the middle of the range giving rise to multiple equilibria. At that initial debt level either a constant 16% inflation rate is needed for 5 years or a constant 14% inflation rate for 10 years, which respectively more than doubles and quadruples the price level. This is simply implausible as a policy to avoid an equilibrium with sovereign default. Even much higher inflation rates are needed for higher initial debt levels, closer to B_{high} , the upper bound for multiple equilibria. Only when the level of debt is quite close to the lower bound B_{low} would a relatively modest inflation rate be needed.

Most of the impact comes from ex ante policy. Having the inflation last beyond T periods does not help much in reducing the level of inflation needed to avoid self-fulfilling crises. The reason is that inflation reduces the real value of coupons on outstanding debt at time 0. For the assumed maturity of debt, most of this effect comes within the first 5 years. If in theory we could generate all of the inflation in the first quarter, when it is most effective, we would only need to raise the price level by 42% when $B_0 = B_{middle}$. However, during that quarter the annualized inflation rate would be 168%.

The results are critically dependent on the maturity of the debt. We have calibrated it to fit the data. However, inflation would be more effective when the duration of government bonds is longer. The slower the debt depreciates, the larger the impact of inflation that reduces the real value of the coupons on the debt. For example, if we set $\delta = 0.025$ instead of 0.05, implying a longer duration of 7.2 years, the required inflation would be either 10% for 5 years or 7% for 10 years for debt in the middle of the multiplicity range. Similarly, a shorter debt maturity would imply even higher inflation rates.

With flexible prices debt deflation can be more effective than with price rigidity because prices can increase immediately at time 0. With price rigidity, initial inflation is lower. It is this initial inflation that is most effective in deflating the debt because of the depreciation of the debt. As a result, higher levels of inflation later on are needed to avoid a crisis. On the other hand, with price rigidity expansionary monetary policy has the additional effect of reducing real interest rates, which also contributes to reducing the debt.

4.2 Optimal Policy in the New Keynesian Model

We now determine the inflation rate required to avoid a crisis in the New Keynesian model. We consider the optimal path of the interest rate for $H = 40$ quarters. Figure 3 shows the dynamics of inflation under optimal policy under the benchmark parameterization. The results are shown for various levels of B_0 . The optimal path for inflation is hump shaped. Optimal inflation gradually rises, both due to rigidities and because the welfare cost (36) depends on the change in inflation. Eventually optimal inflation decreases as it becomes less effective over time when the original debt depreciates and is replaced by new debt that incorporates inflation expectations. When $B_0 = B_{middle} = 1.12$, the maximum inflation rate reaches 23.7%, while inflation ultimately leads the price level to more than quintuple. It increases by a factor 5.2.

Such high inflation is implausible. Inflation needed to avoid default gets even much higher for higher debt levels. When B_0 reaches the upper bound B_{high} for multiple equilibria, the maximum inflation is close to 45% and ultimately the price level increases by a factor 21! Only when B_0 is very close to the lower bound for multiplicity, as illustrated for $B_0 = 0.8$ is little inflation needed.

Inflation is now even higher than under flexible prices. Even though it would be best to generate as much inflation as possible right away in order to reduce the real value of the outstanding debt, with price rigidities it is costly to do so. This leads to a delay that ultimately requires even more inflation to avoid self-fulfilling equilibria.

While nominal rigidities allow the central bank to control real interest rates, and thereby reduce the cost of new borrowing, in practice the benefit from this turns out to be limited. Under the benchmark parameterization the real interest rate goes to zero for two quarters, since we reach the ZLB and inflation is initially zero, but after that it soon goes back to its steady state. In order to understand why this result is more general than the specific parameterization here, consider the consumption Euler equation, which in linearized form implies (31). It is well known that without habit formation ($\eta = 0$) this can be solved as

$$x_0 = -\frac{1}{\sigma} \sum_{t=0}^{\infty} E_0 r_t \quad (40)$$

This precludes a large and sustained drop in the real interest rate as it would imply an enormous and unrealistic immediate change in output at time zero, especially

with $\sigma = 1$ as often assumed.

For the benchmark parameterization, where $\sigma = 1$ and $\eta = 0.65$, we derive an analogous expression in the Technical Appendix (removing the expectation operator):

$$x_0 = -0.36r_0 - 0.59r_1 - 0.73r_2 - 0.83r_3 - 0.89r_4 - 0.93r_5 - 0.95r_6 - 0.97r_7 - \dots \quad (41)$$

Subsequent coefficients are very close to -1. For the path of real interest rates under optimal policy this implies $x_0 = 0.0235$. This implies an immediate increase in output of 9.4% on an annualized basis, which is already pushing the boundaries of what is plausible.¹⁵

4.3 Sensitivity Analysis

We now consider changes to both the LW and NK parameters. An issue arises when changing the LW parameters as they affect the region $[B_{low}, B_{high}]$ for B_0 under which multiple equilibria arise. For example, when $T = 10$, there is less time for a debt crisis to develop and a higher level of initial debt is needed to have a self-fulfilling crisis. Naturally the question that we address here has little content when this region $[B_{low}, B_{high}]$ is very narrow. This issue does not arise for the NK parameters, which leave this region unchanged.

We should first point out that the same region $[B_{low}, B_{high}]$ under which there are multiple equilibria under the benchmark parameterization applies to many other reasonable combinations of LW parameters. The left panel of Figure 4 shows combinations of T , \bar{s} and s_{low} that generate the same B_{low} and B_{high} . The panel on the right shows that this has little effect on the path of optimal inflation. Varying T from 10 to 30, while adjusting \bar{s} and s_{low} to keep B_{low} and B_{high} unchanged, gives very similar paths for optimal inflation.

In Figure 5 and Table 5 we present results when varying one parameter at a time, but keeping B_0/B_{low} the same as under the benchmark parameterization. Table 5 shows that B_{low} and B_{high} can be significantly affected by the LW parameters. But the inflation and welfare results control for this by keeping $B_0/B_{low} = 1.42$ as

¹⁵Note that the coefficients for the first 8 quarters are less than 1 in absolute value, reflecting the smaller response under habit formation. While this gives more leeway to larger changes in the real interest rate, it is still severely constrained by plausible levels of x_0 .

under the benchmark. For the LW parameters this implies values of B_0 that can be relatively closer to B_{low} or B_{high} , dependent on their values for that parameter.¹⁶

Each panel of Figure 5 reports optimal inflation for two values of a parameter, one higher and the other lower than in the benchmark. The last two columns of Table 5 report the price level after inflation and a measure of welfare. The latter is the percentage drop in consumption or output that generates the same drop in welfare. It makes little difference whether the drop in consumption or output happens all in one year or is spread over several years, as long as it is measured as a percentage of one year's GDP.

Figure 5 shows that for most parameters the optimal inflation path is remarkably little affected by the level of parameters. For example, optimal inflation is only slightly higher for $T = 10$ than $T = 30$. When T is low, ex-post policies will be much more important than for higher values of T , but the overall impact on inflation is similar. Also notice that setting the probability ψ of the bad state equal to 1 has little effect on the results.

There are three parameters, δ , γ and d , for which there are more significant differences. As already discussed in the context of flexible prices, lower debt depreciation δ , which implies a longer maturity of debt, implies lower inflation. But even when $\delta = 0.025$, so that the duration is 7.2 years, optimal inflation is still above 10% for 6.5 years and the price level ultimately triples. A lower value for the lag in price adjustment, d , also allows for a lower inflation rate. With $d = 0$ it is possible to increase inflation from the start, when debt deflation is the most powerful. Note though that even with $d = 0$, optimal inflation still peaks close to 20% and the price level still more than quadruples as a result of years of inflation. No matter what the parameter values, an implausibly high level of inflation is needed to avert a self-fulfilling debt crisis.

Finally, we also see a clear difference when we lower the inflation indexation parameter γ . Lower indexation reduces inflation persistence. But more importantly, it directly affects optimal policy through (36). With $\gamma = 1$, only changes in inflation matter, while with $\gamma < 1$ the level of inflation is also undesirable. To avoid higher inflation levels, the central bank takes advantage of the real interest rate channel to avoid the bad equilibrium. But the sharp drop in the real interest rate leads to an unrealistic output response: with $\gamma = 0.8$, output increases

¹⁶Only for $\zeta = 0.7$ is B_0 now slightly above B_{high} . For all other parameters the B_0 is within the interval for B_0 generating multiple equilibria.

at an annual rate of 24% in the first quarter. If we introduced other features that restrict such unrealistic changes in output, inflation would be closer to the benchmark again.

So far we have focused on inflation rather than on welfare. The reason for this is that the welfare criterion has many limitations. While the NK model gives us an explicit welfare criterion, it is very sensitive to parameters that otherwise have little effect on optimal inflation. This can be seen in the last two columns of Table 5. Changes in θ , ϵ and ϕ lead dramatically different welfare numbers, while having little effect on inflation. To a lesser extent this is also the case for d . This makes looking at optimal inflation more appealing. Related to this, note that when $\gamma = 0.8$, so that not just the change in inflation, but also its level matters, the welfare cost of 44% is four times as high as under the benchmark. This happens even though inflation is now a bit lower than under the benchmark.¹⁷

In addition the welfare criterion only captures one aspect of the cost of inflation, associated with inefficiencies due to changes in relative prices. This abstracts from other costs of inflation, a problem we already noticed when discussing flexible prices. It is also well known that this welfare metric significantly understates the welfare cost of a non-zero output gap because of its representative agent nature. The weight on the output gap in the welfare metric is therefore very low relative to that of inflation. For all these reasons the welfare cost of 11.1% under the benchmark parameterization is likely to significantly understate the true cost of the monetary policy.

It is finally useful to report the results for a less realistic set of parameters that are nonetheless consistent with the textbook NK model, as in Gali (2008) for example. To this end we set $\gamma = d = \eta = 0$. This removes inflation indexation, habit formation and delayed price changes, all features that have been introduced to generate more realistic output and inflation responses to monetary policy shocks. In this case optimal inflation peaks right away and then gradually declines. It starts very high at a 23% annualized rate in the first quarter, but after two years drops below 10%. Overall the price index now rises much less, by 66%. But the welfare cost is a staggering 341%. This is because now the absolute level of inflation drives the welfare cost rather than the change in the inflation rate. This case is highly unrealistic though. In order to avoid inflation while at the same time avoiding

¹⁷It is also well known that welfare is sensitive to the exact form of price setting. Taylor pricing leads to lower welfare costs than Calvo pricing. See Ambler (2007) for a discussion.

an equilibrium with default, there is a very steep drop in real interest rates that implies a 25% increase in output in the first quarter, which is a 100% annualized growth rate. This obviously makes little sense. Introducing additional features that limit such unrealistic changes in the level of output would again generate significantly higher inflation rates.

4.4 Procyclical Primary Surplus

Nominal rigidities also give the central bank control over the accumulation of debt through the level of output that affects the primary surplus. So far we have abstracted from this channel, but we now introduce a pro-cyclical primary surplus. From 0 through $T - 1$ we have

$$s_t = \bar{s} + \lambda(y_t - \bar{y}) \tag{42}$$

where \bar{y} is steady state output. We similarly assume that s_{low} is pro-cyclical: $s_{low} = \bar{s}_{low} + \lambda(y_t - \bar{y})$. We set the value of the cyclical parameter of the fiscal surplus to $\lambda = 0.1$, in line with empirical estimates.¹⁸

With this additional effect from an output increase, the required inflation decreases. For $B_0 = B_{middle}$, the maximum inflation rate is reduced from 23.7% in the benchmark to 18.9%. The increase in the price level after inflation is reduced from 5.2 under the benchmark to 3.7. This is still an excessive amount of inflation. Moreover, if anything it understates the needed inflation and overstates the gain from higher tax revenue due to increased output. In order to avoid inflation, the optimal policy now gives more emphasis to raising output. It implies an output increase in the first quarter of 13% at annualized rate (compared to 9% in the benchmark), which is unrealistic. Higher inflation would be needed if, for example, we introduced reasonable limits to hours worked or the rate of change in hours worked.

4.5 Uncertainty about the Date of Default Decision

So far we have assumed that the only uncertainty in the model is about the level of primary surpluses that can be generated from T onward. In other words, there is

¹⁸Note that since $\bar{Y} = 0.25$ for quarterly GDP, the specification implies that $\Delta s = 0.4\Delta Y$. This is consistent for example with estimates by Girouard and André (2005) for the OECD.

uncertainty about whether the government is able to enact reforms that raise the primary surplus. But this uncertainty is resolved at a known date and the default decision is then made at that time. We will now briefly discuss an extension whereby there is uncertainty about T itself. Since this significantly complicates the model, all details of this case are left to the Technical Appendix. We only discuss the setup and results.

In general there is uncertainty about both the date that we find out if reforms will be enacted and about the reforms themselves. We now abstract from the latter by setting $\psi = 1$. In this case the agents know that there will be no reform that raises primary surpluses, but they do not know at what time a decision will be made to default or not. We further simplify by considering only two possible dates for the default decision. The default decision will take place at T_1 with probability p and at T_2 with probability $1 - p$, with $T_1 < T_2$. The asset price prior to T_1 now takes into account the possibility of default at either T_1 or T_2 .

Monetary policy again involves setting interest rates for the first $H = 40$ periods, after which the Taylor rule applies again. The interest rates starting at T_1 will be contingent on whether there was a default decision at time T_1 . Figure 6 shows the impact of optimal monetary policy on inflation when $T_1 = 10$ and $T_2 = 20$. Except for $\psi = 1$, all other parameters are the same as in the benchmark parameterization. The chart on the left shows the maximum inflation rate under optimal policy for different values of B_0 , while the chart on the right shows the ultimate price level as a result of the optimal policy. The charts also show results for the case where $T = 10$ and $T = 20$ without uncertainty, and $\psi = 1$. The thick section on the horizontal axis represents the range of B_0 for which there are multiple equilibria under uncertainty about T in the absence of monetary policy, which is 0.84 to 1.47.

The case of uncertainty lies right between the two cases without uncertainty. The range of B_0 for which there are multiple equilibria is shifted to somewhere in between the two cases without uncertainty, but otherwise the results remain very similar to those without uncertainty. Unless B_0 is very close to the lowest value for which there are multiple equilibria, it remains the case that very significant inflation is needed to avoid multiple equilibria. For example, when B_0 is 1.42 times B_{low} , which is 1.19 and again near the middle of the multiplicity range, the maximum inflation rate is 20%, while ultimately the price level will increase by a factor 3.9.

5 Monetary Backstop

So far we have only considered what the central bank can do through its interest rate policy in a cashless economy. In this section we discuss whether the central bank can do more to help avert a self-fulfilling debt crisis by using the resources on its balance sheet. As discussed in section 2.4, the central bank has two potential resources, which come from its ability to issue monetary liabilities and to sell assets other than government bonds.

5.1 Selling other Assets on Central Bank Balance Sheet

First consider the assets other than government bonds held by the central bank, which we have denoted D_t^c . It is not surprising that having such additional assets on hand will help in avoiding a default on government debt if the central bank is willing to use them to support the government. To give a specific example, consider the benchmark parameterization, under which there are multiple equilibria when B_0 is in the range $[0.79, 1.46]$ and monetary policy is passive. Now assume that the other assets are equal to 10% of GDP ($D_t^c = 0.1$). Using the results from section 2.4, we find that only for B_0 in the range of 0.79 to 0.90 is self-fulfilling default now avoided.¹⁹ Most likely though, even this is an overstatement of what is feasible. For example, in 2000, prior to their recent balance sheet expansion, these assets amounted to 1% of GDP in the US and 4% of GDP in the Bank of Japan.

5.2 Standard Seigniorage

Next consider the role of issuing monetary liabilities. To discuss this, we first need to introduce money demand to the model. We assume a standard specification for money demand when $i_t > 0$ ($m_t = \ln(M_t)$):

$$m_t = \alpha_m + p_t + y_t - \alpha_i i_t \quad (43)$$

This can be derived by introducing a transactions cost $f(M_t, Y_t)$ to the budget constraint of the agents.²⁰ When i_t is close to zero, money demand reaches the

¹⁹This is the case both when the central bank sells these assets prior to time T , so that $D_T^c = 0$, and when it does not. In the latter case $D_T^c = R^T D_0^c$. The difference is non-zero but tiny.

²⁰The transaction cost $f(M_t, Y_t) = \alpha_0 + M_t \left(\ln \left(\frac{M_t}{P_t Y_t} \right) - 1 - \alpha_m \right)$ gives rise to money demand (43). This function applies for values of M_t where as $\partial f / \partial M > 0$. Once the derivative becomes

satiation level $\alpha_m + p_t + y_t$. At the ZLB money and bonds are indistinguishable and the money supply is not limited by the satiation level.

We first abstract from the ZLB by assuming that the money supply does not go beyond the satiation level. The condition to avoid self-fulfilling equilibria, previously (17) with an equality, now becomes

$$\frac{\chi^\kappa \kappa B_0 / P_0 - \chi^s \bar{s} - \tilde{m}}{s^{pdv} + m^{pdv} - ((1 - \delta)Q_T + \kappa)(1 - \delta)^T B_0 / P_T} r_{T-1} = \psi \zeta + 1 - \psi + \frac{(1 - \zeta)\psi m^{pdv}}{s^{pdv} + m^{pdv}}$$

We maximize utility as before, subject to this constraint (with money equal to (43)) and the non-negativity constraints for interest rates.

The impact of seigniorage revenue is larger for lower values of α_i . A higher semi-elasticity α_i of money demand implies a larger drop in real money demand when inflation rises. Estimates of α_i vary a lot, from as low as 6 in Ireland (2009) to as high as 60 in Bilson (1978).²¹ The biggest effect from seigniorage therefore comes from the lowest value $\alpha_i = 6$. But even in that case the effect is limited. When $B_0 = B_{middle}$, the maximum inflation rate is reduced from 23.7% to 19.8% and the price level ultimately increases by a factor 4.1 instead of 5.2.²² There is clearly some benefit from seigniorage, but quantitatively it is small and does not change our conclusion that an excessive amount of inflation is needed to avoid the crisis equilibrium. This result is consistent with Reis (2013), who also points out that outside of the ZLB seigniorage is significantly constrained by real money demand.

5.3 Monetary Backstop at the ZLB

Next consider the case where we are at the ZLB. The central bank can then increase money supply beyond the satiation level as bonds and money become perfect zero, we reach a satiation level and we assume that the transaction cost remains constant for larger M_t . If this cost is paid to intermediaries that do not require real resources and return their profits to households, it will remain the case that $C_t = Y_t$.

²¹Lucas (2000) finds a value of 28 when translated to a quarterly frequency. Engel and West (2005) review many estimates that also fall in this range.

²²We calibrate α_m to the U.S., such that the satiation level of money corresponds to the monetary base just prior to its sharp rise in the Fall of 2008 when interest rates approached the ZLB. At that time the velocity of the monetary base was 17. This gives $\alpha_m = -1.45$. The velocity is $4P_t Y_t / M_t$ as output needs to be annualized, which is equal to $4e^{-\alpha_m}$ at the satiation level.

substitutes. We can write the accumulated seigniorage between 0 and $T - 1$ as

$$\tilde{m} = \frac{M_{T-1}}{P_{T-1}} - r_0 \dots r_{T-2} \frac{M_{-1}}{P_0} + r_1 \dots r_{T-2} (R_0 - 1) \frac{M_0}{P_1} + \dots + (R_{T-2} - 1) \frac{M_{T-2}}{P_{T-1}} \quad (44)$$

It follows that being at the ZLB is not helpful if it comes to an end at $T - 1$ or earlier. If any of the money balances from time 0 through $T - 2$ are beyond the satiation level, they are multiplied by a zero nominal interest rate $R - 1$ in the expression for \tilde{m} . Intuitively, any seigniorage earned from increasing the money supply beyond the satiation level will be of no help if it needs to be unwound prior to the default decision.

However, if we are still at the ZLB at $T - 1$, and the central bank issues sufficient monetary liabilities beyond the satiation point at $T - 1$ or before, a self-fulfilling crisis can be avoided even if this monetary expansion is unwound at time T or later. This is illustrated in Appendix C for the case where we are structurally at the ZLB in that β is temporarily equal to 1 for a period that lasts at least through $T - 1$. Unwinding the monetary expansion at time T or later would imply a negative ex-post seigniorage m^{pdv} , which may be of similar magnitude as the positive ex-ante seigniorage \tilde{m} . Nonetheless, for a sufficient increase in the money supply beyond the satiation level prior to time T such a policy will avoid the bad equilibrium.

To understand this, assume that there is a bad equilibrium and agents anticipate default at time T . The central bank could then buy a lot of the government debt at time $T - 1$ or earlier at a depressed price in exchange for monetary liabilities, and sell the debt again at time T or later at a high price as there is no further default risk. If this transaction is large enough, the profit is sufficient to avoid default. This means that the bad equilibrium cannot happen in the first place.

In principle the central bank could do this even when $\beta < 1$ and the natural real interest rate is positive. A sufficient monetary expansion at $T - 1$ or before will cause the nominal interest rate to go to zero and the money supply can be raised well beyond the satiation level. This can then again be undone at time T or later. However, if we are not structurally at the ZLB such a policy has little practical interest as it depends crucially on the assumption that the central bank knows exactly the date T at which the default decision is made. In section 4.5 we considered uncertainty about T . There we assumed for simplicity that T can take on only two values. More generally though, the default decision may happen any time between say T_1 and T_2 . In that case this scheme would only work to avoid

the bad equilibrium if the central bank keeps the interest rate at zero for the entire period from T_1 to T_2 , which may be many years. But forcing the nominal interest rate to zero when $\beta < 1$ would quickly lead to an explosion of inflation.

5.4 Discussion

These results show that unless the economy is at a structural ZLB for an extended time, there is not much that can be done in the form of a monetary backstop. Reis (2013) draws the same conclusion when discussing how the central bank can avoid sovereign default: “The central bank’s main lever over fundamentals is to raise inflation, but otherwise the balance sheet gives it little leeway.” Outside of the ZLB, the ability of the central bank to issue monetary liabilities is only special to the extent that money is valued for transaction purposes. But the corresponding seigniorage revenue is small in present value.

Corsetti and Dedola (2014) argue that the central bank can avoid self-fulfilling debt crises by issuing risk-free liabilities that are convertible into cash. Risky government liabilities are then replaced by risk-free central bank liabilities. But when consolidating the accounts of the central government and central bank, there is in principle little difference between debt issued by the government and the central bank.²³ There is not an inelastic demand for non-monetary central bank liabilities and nothing stops investors (and banks) from demanding a default premium on these assets.²⁴

Two questions remain. First, if our conclusion that in most cases the central bank cannot avoid self-fulfilling debt crises is correct, why is it the case that many highly indebted non-Eurozone countries with their own currencies have escaped such crises recently? Second, why is it the case that a change in ECB policy in

²³While the model in Corsetti and Dedola (2014) has only two periods, in a dynamic model with long term debt their proposal to exchange government debt for central bank reserves would also significantly shorten the maturity of the debt. We have not explicitly considered a change in debt maturity to avoid self-fulfilling crises. Two points are worth making though. First, a move to short term debt may be undesirable for reasons outside of the model. In particular, it leads to more exposure to rollover crises. Second, as also emphasized by LW, multiple equilibria are present in the model even for short term (one period) debt.

²⁴If the accounts of the central bank and government were separate, then institutions that hold non-monetary central bank liabilities should also be concerned about central bank losses when the government defaults.

the summer of 2012 was successful in significantly lowering sovereign debt spread of periphery countries?

The answer to the first question may partly be that since the end of 2008 many countries have been at a structural ZLB, which is the only case we identified where central bank policy can be effective. The threat of such policy alone is sufficient. But the answer can also be that these countries are less exposed to self-fulfilling debt crises even with passive monetary policy. In particular, as shown by LW, if the government responds to an increase in debt by significantly increasing the primary surplus, a self-fulfilling crisis is avoided. We have abstracted from such a policy here as our focus has been on what the central bank can accomplish on its own.

Regarding the second question, the analysis in this paper applies to a central bank that aims to avoid a self-fulfilling default by the central government. The situation where the central bank of a currency union aims to avoid sovereign default in periphery countries of the union is quite different. Specifically, the ECB could buy government bonds of the periphery countries that experience high default premia and sell government bonds of countries that are not subject to a sovereign debt crisis. No monetary liabilities need to be issued in the process, generating no inflation.

The ECB could keep interest rates on new debt of the periphery governments equal to their no-default levels and buy all new bonds that would otherwise be sold to the private sector at that low interest rate. The threat alone of doing so is sufficient, which is exactly what happened under the OMT policy in the summer of 2012 and the famous Draghi statement “to do whatever it takes”. Such a threat was credible as such an intervention would not overwhelm the ECB. For example, in 2010 the sum of all the periphery country government deficits together (Greece, Ireland, Portugal, Spain, Italy) amounted to 13% of the ECB balance sheet. And a self-fulfilling default can be avoided even if only a portion of these financing needs are covered by the ECB. This explains why sovereign spreads quickly fell due to the change in policy in the summer of 2012.

6 Conclusion

Several recent contributions have derived analytical conditions under which the central bank can avoid a self-fulfilling sovereign debt crisis. Extreme central bank intervention, generating extraordinary inflation, would surely avoid a sovereign debt crisis. But the cost would be excessive, making such actions not credible. The aim of this paper has been to quantify this cost in order to better assess whether countries with their own currency (and therefore central bank) are less likely to be subject to such self-fulfilling debt crises.

To address this question, we have adopted a dynamic model with many realistic elements that make a quantitative assessment more meaningful. We introduced a NK model with nominal rigidities in which monetary policy has realistic effects on output and inflation. We introduced long-term bonds and calibrated the maturity to what is observed in many industrialized countries. We allowed for slow-moving debt crises that are a good representation of the recent European sovereign debt crisis. We have considered both conventional monetary policy that impacts inflation, real interest rates and output, and less conventional monetary backstop policies.

Overall our conclusion is that the ability to avert self-fulfilling crises is limited. Unless debt is close to the bottom of an interval where multiple equilibria occur, conventional policies involve very high inflation for a sustained period of time. Monetary backstop policies are generally only useful when the economy is at the structural ZLB for a sustained length of time.

Several extensions are worthwhile considering for future work. We have focused on a closed economy. In an open economy monetary policy also affects the exchange rate, which affects relative prices and output. While we made some brief comments at the end, it would also be of interest to more explicitly consider a monetary union, where sovereign default may be limited to only a segment of the union. Finally, we have only considered one type of self-fulfilling debt crises, associated with the interaction between sovereign spreads and debt. It would be of interest to also consider rollover crises or even a combination of both types of crises. This also provides an opportunity to consider the optimal maturity of sovereign debt, which we have taken as given.

Appendix

A. Minimum Level of s_{high}

As mentioned in section 2.1, we assume that the primary surplus s_{high} in the good state is sufficiently high such that default never happens in that state. We derive a condition for s_{high} under which this is the case. We only do so in the non-monetary LW model of section 2.1. If $b_T > s_{high}/(R-1)$, there is default even when $\tilde{s} = s_{high}$. When $b_T = s_{high}/(R-1)$ the price schedule then drops down a second time, to

$$Q_{T-1} = \frac{\zeta}{(R-1)b_T} (\psi s_{low} + (1-\psi)s_{high}) \quad (45)$$

We need to show that there can be a level of s_{high} such that is no equilibrium with $b_T > s_{high}/(R-1)$. This is the case if the pricing schedule is always above the debt accumulation schedule. For a given $b_T > s_{high}/(R-1)$, the Q_{T-1} from the pricing schedule must be higher than from the debt accumulation schedule. This is the case when

$$\frac{\zeta}{(R-1)b_T} (\psi s_{low} + (1-\psi)s_{high}) > \frac{\chi^\kappa \kappa b_0 - \chi^{\bar{s}}}{b_T - (1-\delta)^T b_0} \quad (46)$$

It is sufficient that this is the case for $b_T = s_{high}/(R-1)$, which gives the condition

$$\frac{\zeta}{(R-1)} (\psi s_{low} + (1-\psi)s_{high}) > \frac{\chi^\kappa \kappa b_0 - \chi^{\bar{s}}}{1 - (R-1)(1-\delta)^T b_0/s_{high}} \quad (47)$$

Since the left hand side of this expression depends positively on s_{high} and goes to infinity when $s_{high} \rightarrow \infty$, while the right hand side depends negatively on s_{high} and goes to a constant when $s_{high} \rightarrow \infty$, it follows that for s_{high} above some cutoff level this condition is always satisfied.

B. Derivation of the Debt Accumulation Schedule.

We derive the debt accumulation schedule in the general case of section 2.4. The debt accumulation schedule in the cashless economy (section 2.2) is a special case of this where $M_t = D_t^c = 0$ at all times. We first derive a relationship between Q_0 and Q_{T-1} . Integrating forward the one-period arbitrage equation (4) from $t = 1$ to $t = T-1$, we have:

$$Q_0 = A^\kappa \kappa + A^Q Q_{T-1} \quad (48)$$

where

$$A^\kappa = \frac{1}{R_0} + \frac{1-\delta}{R_0R_1} + \frac{(1-\delta)^2}{R_0R_1R_2} + \dots + \frac{(1-\delta)^{T-2}}{R_0R_1R_2\dots R_{T-2}} \quad (49)$$

$$A^Q = \frac{(1-\delta)^{T-1}}{R_0R_1R_2\dots R_{T-2}} \quad (50)$$

Next consider the consolidated budget constraint (19):

$$Q_t B_{t+1}^p = ((1-\delta)Q_t + \kappa)B_t^p - v_t - s_t P_t \quad (51)$$

where $v_t = [M_t - M_{t-1}] - [D_{t+1} - R_{t-1}D_t^c]$. The government budget constraint at $t = 0$ is:

$$\frac{Q_0 B_1^p}{P_0} = ((1-\delta)Q_0 + \kappa)b_0^p - \bar{s} - \frac{v_0}{P_0} \quad (52)$$

For $1 < t < T$

$$\frac{Q_t B_{t+1}^p}{P_t} = r_{t-1} \frac{Q_{t-1} B_t^p}{P_{t-1}} - \bar{s} - \frac{v_t}{P_t} \quad (53)$$

Using equations (53) and (52) and integrating forward, we obtain

$$\begin{aligned} \frac{Q_{T-1} B_T^p}{P_{T-1}} &= r_{T-2} \dots r_1 r_0 \frac{Q_0 B_1^p}{P_0} - \bar{s}(1 + r_{T-2} + r_{T-2}r_{T-1} + \dots + r_{T-2}r_{T-1} \dots r_1) \\ &- \left[r_1 \dots r_{T-2} \frac{v_1}{P_1} + r_2 \dots r_{T-2} \frac{v_2}{P_2} + \dots + \frac{v_{T-1}}{P_{T-1}} \right] \end{aligned} \quad (54)$$

Combining equation (54) with (52) and (48), we obtain:

$$\begin{aligned} \frac{Q_{T-1} B_T^p}{P_{T-1}} &= r_{T-2} \dots r_1 r_0 (1-\delta) b_0^p Q_0 + r_{T-2} \dots r_1 r_0 \kappa b_0^p \\ &- \bar{s}(1 + r_{T-2} + r_{T-2}r_{T-3} + \dots + r_{T-2} \dots r_1 r_0) \\ &- \left[r_0 \dots r_{T-2} \frac{v_0}{P_0} + r_1 \dots r_{T-2} \frac{v_1}{P_1} + \dots + \frac{v_{T-1}}{P_{T-1}} \right] \end{aligned} \quad (55)$$

Using equations (48)-(50), we can rewrite equation (55) as

$$\begin{aligned} \frac{Q_{T-1} B_T^p}{P_{T-1}} &= \frac{P_0}{P_{T-1}} (1-\delta)^T b_0^p Q_{T-1} \\ &+ r_{T-2} \dots r_1 r_0 \left[1 + \frac{1-\delta}{R_0} + \frac{(1-\delta)^2}{R_0R_1} + \dots + \frac{(1-\delta)^{T-1}}{R_0R_1R_2 \dots R_{T-2}} \right] \kappa b_0^p \\ &- \bar{s}(1 + r_{T-2} + r_{T-2}r_{T-3} + \dots + r_{T-2} \dots r_1 r_0) \\ &- \left[r_0 \dots r_{T-2} \frac{v_0}{P_0} + r_1 \dots r_{T-2} \frac{v_1}{P_1} + \dots + \frac{v_{T-1}}{P_{T-1}} \right] \end{aligned} \quad (56)$$

Using the expression for v_t , the last term in brackets is equal to $\tilde{m} + r_{-1}r_0r_1\dots r_{T-1}d_0^c - d_T^c$ with \tilde{m} as defined in (20). This yields

$$b_T^p = (1 - \delta)^T \frac{B_0^p}{P_T} + \frac{P_{T-1} \chi^\kappa \kappa b_0^p - \chi^s \bar{s} - \tilde{m} + d_T^c - r_{-1}r_0r_1\dots r_{T-1}d_0^c}{Q_{T-1}} \quad (57)$$

C. Avoiding Self-fulfilling Equilibria at a Structural ZLB

Assume that the discount rate β is 1 for $t = 0, \dots, \bar{T}$ with $\bar{T} \geq T - 1$ and it is a constant $\beta < 1$ for $t > \bar{T}$. Under this assumption the central bank can keep the interest rate zero through time \bar{T} , and raise it to $1/\beta$ after time \bar{T} , while keeping inflation and the output gap at zero all along. So we have $R_t = 1$ for $t = 0, \dots, \bar{T}$ and $R_t = 1/\beta$ for $t > \bar{T}$. The price level is always 1. We then have $Q_T = \kappa \sum_{i=0}^{\bar{T}-T} (1 - \delta)^i + (1 - \delta)^{\bar{T}-T+1}$.

Using (44) we have $\tilde{m} = M_{T-1} - M_{-1}$. Let \bar{M} be the level of money demand starting at $\bar{T}+1$, when we are no longer at the ZLB. Then $m^{pdv} = \bar{M} - M_{T-1}$. Define $\Delta M = M_{T-1} - M_{-1}$ and $dm = \bar{M} - M_{-1}$. Then $\tilde{m} = \Delta M$ and $m^{pdv} = dm - \Delta M$. Assuming that we are already at the ZLB at time -1, dm is negative.

Using the results from section 2.4, the pricing schedule is then

$$Q_{T-1} = (1 - \delta)Q_T + \kappa \quad \text{if } B_T^p \leq \tilde{b} \quad (58)$$

$$= \psi h(\Delta M) + (1 - \psi)((1 - \delta)Q_T + \kappa) \quad \text{if } B_T^p > \tilde{b} \quad (59)$$

where $h(\Delta M) = 0$ if $\zeta s^{pdv} + dm - \Delta M \leq 0$ and otherwise $h(\Delta M) = \frac{\zeta s^{pdv} + dm - \Delta M}{B_T^p}$, and

$$\tilde{b} = \frac{s^{pdv} + dm - \Delta M}{(1 - \delta)Q_T + \kappa} \quad (60)$$

The debt accumulation schedule is

$$B_T^p = (1 - \delta)^T B_0^p + \frac{\chi^\kappa \kappa B_0^p - T\bar{s} - \Delta M}{Q_{T-1}} \quad (61)$$

Passive monetary policy takes the form $\Delta M = 0$. The central bank then does not expand the money supply between -1 and $T - 1$. Under passive monetary policy there are multiple equilibria when B_0 is within a range that we have called $[B_{low}, B_{high}]$. The condition $B_0 < B_{high}$ implies that the debt accumulation schedule crosses below $(1 - \delta)Q_T + \kappa$ when $B_T^p = \tilde{b}$ (point D in Figure 1) and $\Delta M = 0$. This can be written as

$$\frac{\chi^\kappa \kappa B_0^p - T\bar{s}}{\tilde{b} - (1 - \delta)^T B_0^p} < (1 - \delta)Q_T + \kappa \quad (62)$$

Substituting the expression for \tilde{b} with $\Delta M = 0$, this becomes

$$s^{pdv} + dm - (1 - \delta)^T B_0^p ((1 - \delta)Q_T + \kappa) > \chi^\kappa \kappa B_0^p - T\bar{s} \quad (63)$$

Now assume that $\Delta M > \chi^\kappa \kappa B_0^p - T\bar{s}$ conditional on a sunspot shock. We will show that this is a sufficient condition to avoid the bad equilibrium. If there were a bad equilibrium with default, we know from the pricing schedule that $Q_{T-1} < (1 - \delta)Q_T + \kappa$. In that case the debt accumulation schedule, together with the assumption $\Delta M > \chi^\kappa \kappa B_0^p - T\bar{s}$, implies

$$B_T^p < (1 - \delta)^T B_0^p + \frac{\chi^\kappa \kappa B_0^p - T\bar{s} - \Delta M}{(1 - \delta)Q_T + \kappa} \quad (64)$$

It can be shown that the right hand side is less than \tilde{b} , so that $B_T^p < \tilde{b}$ and there cannot be a default equilibrium. The condition that the right hand side is less than \tilde{b} is

$$(1 - \delta)^T B_0^p + \frac{\chi^\kappa \kappa B_0^p - T\bar{s} - \Delta M}{(1 - \delta)Q_T + \kappa} < \frac{s^{pdv} + dm - \Delta M}{(1 - \delta)Q_T + \kappa} \quad (65)$$

Multiplying both sides by $(1 - \delta)Q_T + \kappa$, we can rewrite this as (63), which holds if there are multiple equilibria under passive monetary policy.

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Table 1 Calibration

Lorenzoni-Werning parameters		New Keynesian parameters	
Parameter	Description	Parameter	Description
$\beta = 0.99$	discount rate	$\sigma = 1$	elasticity of intertemporal substitution
$\delta = 0.05$	coupon depreciation rate	$\phi = 1$	Frisch elasticity
$\kappa = 0.06$	coupon	$\varepsilon = 6$	demand elasticity
$T = 20$	quarters before default decision	$\alpha = 0.33$	capital share
$\zeta = 0.5$	recovery rate	$\theta = 0.66$	Calvo pricing parameter
$\psi = 0.95$	probability low surplus state	$\eta = 0.65$	habit parameter
$s_{low} = 0.02$	low state surplus	$\gamma = 1$	indexation parameter
$\bar{s} = -0.01$	surplus before T	$d = 2$	lag in price adjustment
		$\rho = 0.8$	persistence in interest rate rule
		$\psi_\pi = 1.5$	inflation parameter in interest rule
		$\psi_y = 0.1$	output parameter in interest rule

Table 2 Sensitivity Analysis				
Parameters	B_{low}	B_{high}	price level after inflation	welfare (% of GDP)
Benchmark	0.79	1.46	5.2	11.1
Lorenzoni-Werning Parameters				
$T = 10$	1.15	1.71	6.1	13.6
$T = 30$	0.56	1.22	4.4	8.4
$\delta = 1/40$	0.89	1.46	3.0	3.8
$\delta = 1/10$	0.71	1.46	10.7	38.4
$\bar{s} = -0.02$	0.58	1.28	4.9	10.6
$\bar{s} = 0$	1.00	1.64	5.3	11.2
$s_{low} = 0.01$	0.28	0.63	4.9	10.6
$s_{low} = 0.03$	1.27	2.25	5.2	11.1
$\zeta = 0.3$	0.46	1.46	5.9	14.8
$\zeta = 0.7$	1.08	1.46	4.5	8.4
$\psi = 0.7$	0.98	1.46	4.7	9.2
$\psi = 1$	0.75	1.46	5.3	11.4
$\beta = 0.98$	0.61	1.17	6.7	13.4
$\beta = 0.995$	0.90	1.62	4.5	9.8
New Keynesian Parameters				
$\theta = 0.5$	0.79	1.46	4.9	4.0
$\theta = 0.8$	0.79	1.46	5.2	33.9
$\gamma = 0.8$	0.79	1.46	2.6	43.8
$\gamma = 1$	0.79	1.46	5.2	11.1
$\eta = 0$	0.79	1.46	4.7	10.2
$\eta = 0.9$	0.79	1.46	5.1	10.5
$\epsilon = 4$	0.79	1.46	5.1	5.3
$\epsilon = 8$	0.79	1.46	5.1	18.3
$d = 0$	0.79	1.46	4.2	7.1
$d = 4$	0.79	1.46	6.9	18.4
$\sigma = 2$	0.79	1.46	4.7	9.3
$\sigma = 0.5$	0.79	1.46	5.3	11.7
$\phi = 2$	0.79	1.46	5.2	18.5
$\phi = 0.5$	0.79	1.46	5.1	7.2

Table 1 Calibration

Lorenzoni-Werning parameters		New Keynesian parameters	
Parameter	Description	Parameter	Description
$\beta = 0.99$	discount rate	$\sigma = 1$	elasticity of intertemporal substitution
$\delta = 0.05$	coupon depreciation rate	$\phi = 1$	Frisch elasticity
$\kappa = 0.06$	coupon	$\varepsilon = 6$	demand elasticity
$T = 20$	quarters before default decision	$\alpha = 0.33$	capital share
$\zeta = 0.5$	recovery rate	$\theta = 0.66$	Calvo pricing parameter
$\psi = 0.95$	probability of low state	$\eta = 0.65$	habit parameter
$\bar{s}_{low} = 0.02$	constant in low state surplus	$\gamma = 1$	indexation parameter
$\bar{s} = -0.01$	constant in fiscal rule before T	$d = 2$	lag in price adjustment
		$\rho = 0.8$	persistence in interest rate rule
		$\psi_\pi = 1.5$	inflation parameter in interest rule
		$\psi_y = 0.1$	output parameter in interest rule

Table 2 Sensitivity Analysis				
Parameters	B_{low}	B_{high}	price level after inflation	welfare (% of GDP)
Benchmark	0.79	1.46	5.2	11.1
Lorenzoni-Werning Parameters				
$T = 10$	1.15	1.71	6.1	13.6
$T = 30$	0.56	1.22	4.4	8.4
$\delta = 1/40$	0.89	1.46	3.0	3.8
$\delta = 1/10$	0.71	1.46	10.7	38.4
$\bar{s} = -0.02$	0.58	1.28	4.9	10.6
$\bar{s} = 0$	1.00	1.64	5.3	11.2
$s_{low} = 0.01$	0.28	0.63	4.9	10.6
$s_{low} = 0.03$	1.27	2.25	5.2	11.1
$\zeta = 0.3$	0.46	1.46	5.9	14.8
$\zeta = 0.7$	1.08	1.46	4.5	8.4
$\psi = 0.7$	0.98	1.46	4.7	9.2
$\psi = 1$	0.75	1.46	5.3	11.4
$\beta = 0.98$	0.61	1.17	6.7	13.4
$\beta = 0.995$	0.90	1.62	4.5	9.8
New Keynesian Parameters				
$\theta = 0.5$	0.79	1.46	4.9	4.0
$\theta = 0.8$	0.79	1.46	5.2	33.9
$\gamma = 0.8$	0.79	1.46	2.6	43.8
$\gamma = 1$	0.79	1.46	5.2	11.1
$\eta = 0$	0.79	1.46	4.7	10.2
$\eta = 0.9$	0.79	1.46	5.1	10.5
$\epsilon = 4$	0.79	1.46	5.1	5.3
$\epsilon = 8$	0.79	1.46	5.1	18.3
$d = 0$	0.79	1.46	4.2	7.1
$d = 4$	0.79	1.46	6.9	18.4
$\sigma = 2$	0.79	1.46	4.7	9.3
$\sigma = 0.5$	0.79	1.46	5.3	11.7
$\phi = 2$	0.79	1.46	5.2	18.5
$\phi = 0.5$	0.79	1.46	5.1	7.2

Figure 1 Multiple Equilibria Lorenzoni-Werning Model

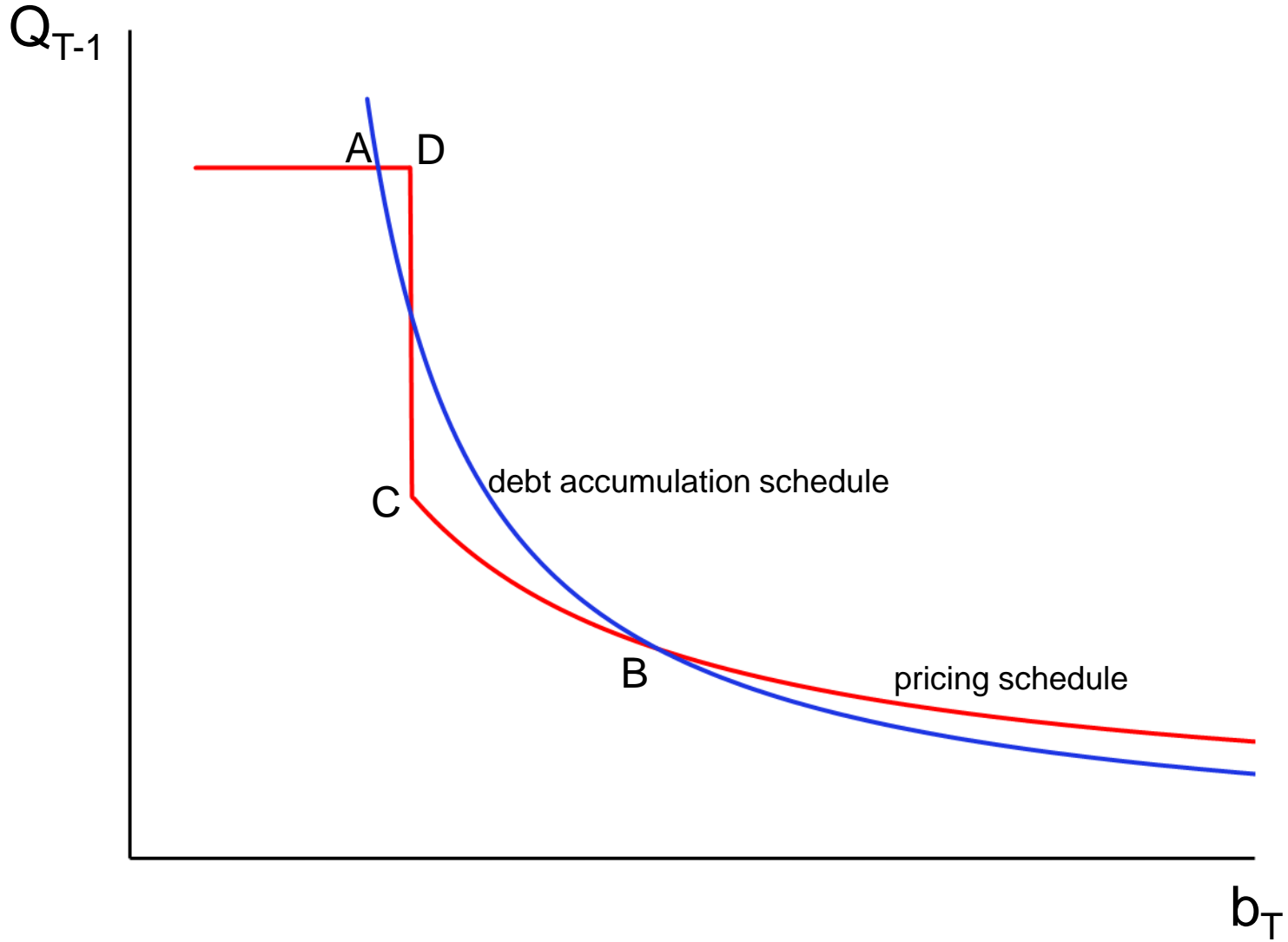
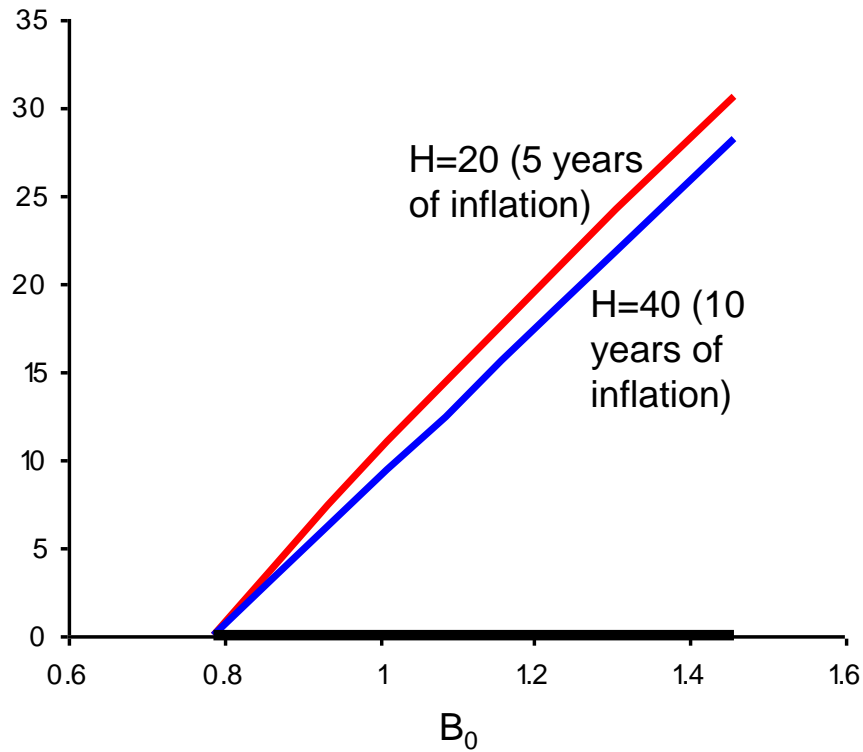


Figure 2 Flexible Prices: Inflation Needed to Avoid Default

Constant inflation rate (APR)



Price Level After Inflation

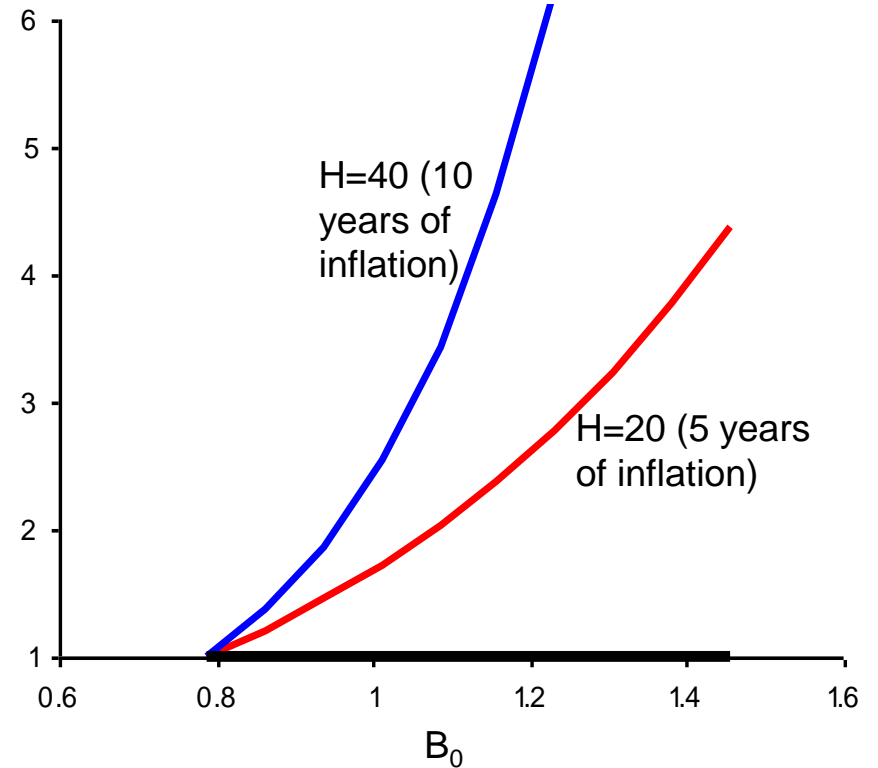
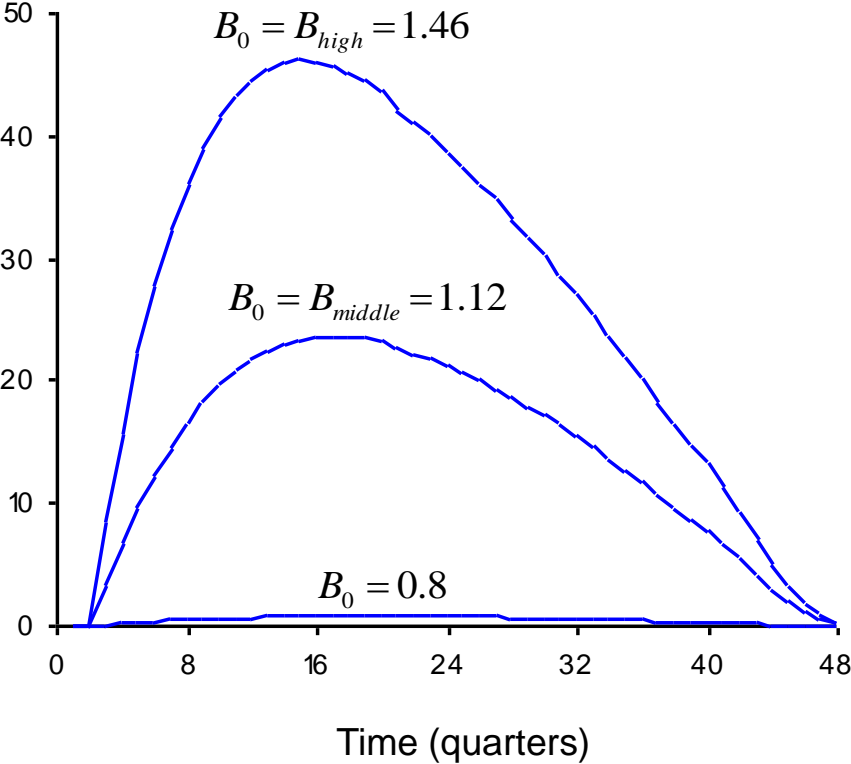


Figure 3 Benchmark NK Model: Inflation Needed to Avoid Default

Inflation (APR)



Price Level After Inflation

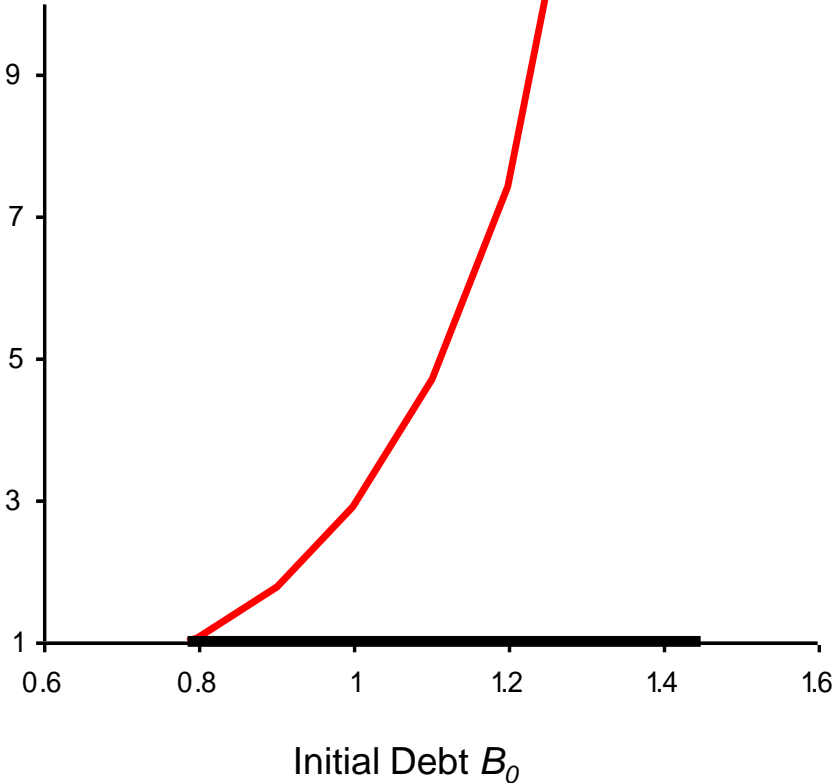
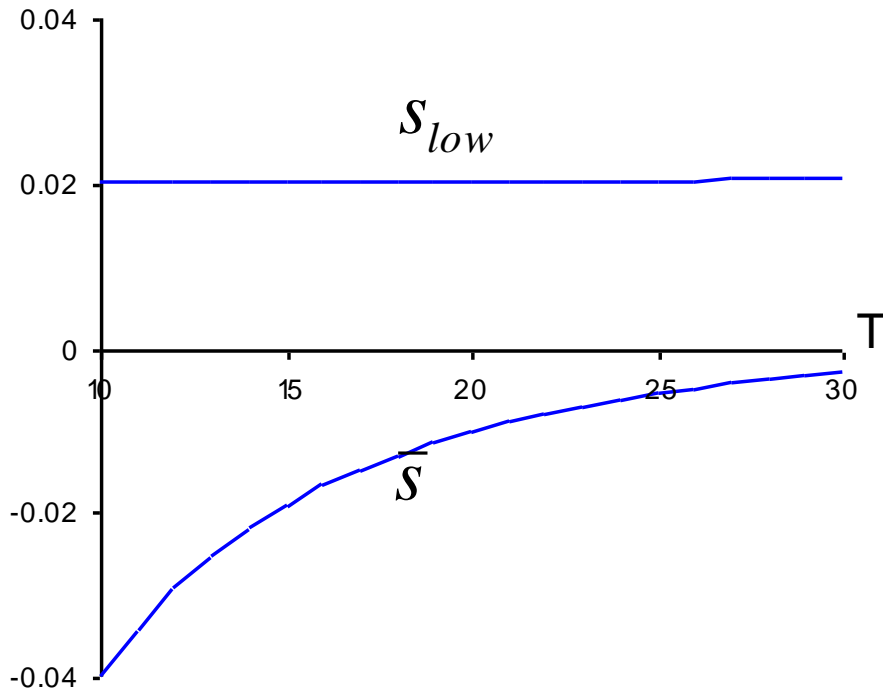


Figure 4 Sensitivity Analysis LW Parameters

T, \bar{s} and s_{low} for same $[B_{low}, B_{high}]$



Inflation when $B_0=B_{middle}=1.12$

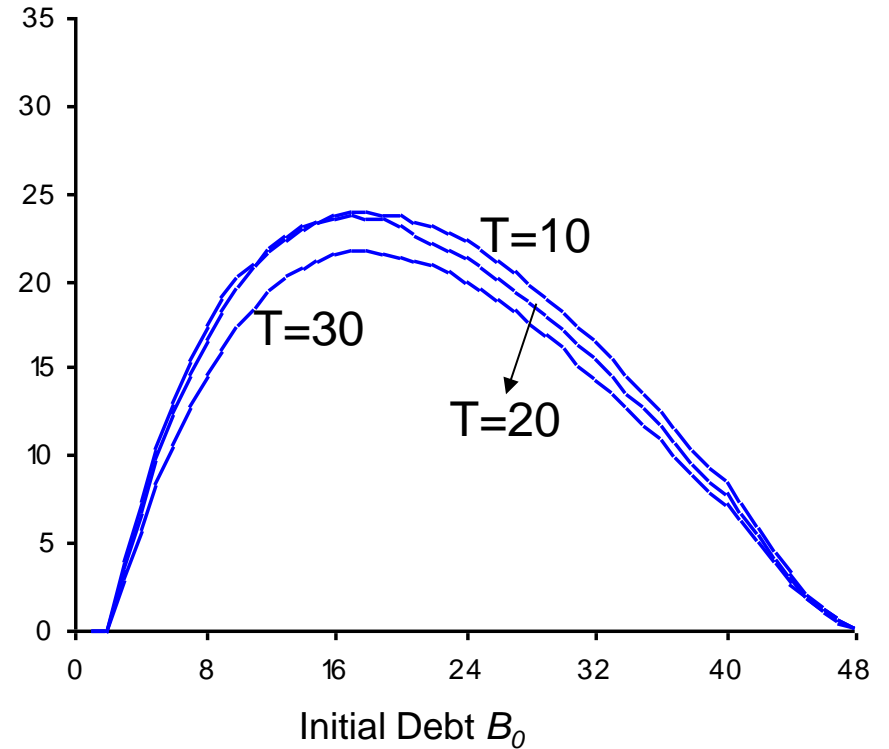


Figure 5 Sensitivity Analysis Optimal Inflation ($B_0/B_{low}=1.42$)

