Fiscal Multipliers and Sovereign Risk *

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Abstract

We investigate the impact of fiscal expansions on corporate loans by exploiting highly granular Colombian data. Our study identifies the crowding out impact of government spending, and closely traces firms which have multiple banking relationships. As such, we monitor banks that take on high shares of government securities and which of those pass on their shortage of funds to firms. Additionally, we study how increased government spending raises sovereign risk, which in turn translates into higher costs of credit. Finally, we construct a quantitative endogenous sovereign default model featuring these stylized facts. In brief, we believe that our paper brings important insights on the transmission mechanism of government spending.

Keywords: fiscal multipliers, sovereign risk
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*The views expressed herein are those of the authors and should not be attributed to the the Central Bank of Colombia, its Executive Board, or its management.

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1 Introduction

Monetary policy has now gone to its limits characterized by interest rates that now fall in the negative territory. Monetary policy makers were left out with a smaller number of policy options and had to rely on unorthodox central bank policies such as macroprudential policies or forward guidance with which central bankers try to influence the current interest rates by making projections on the long-term path of the interest rates. Given these limitations of monetary policy, one of the natural remaining candidates is to utilize fiscal policy. This paper, to this end, explores the channel in which government spending operates with a highly granular loan level Colombian data for the period after 2002.

Our preliminary results suggest that banks pass their shortage of funds to firms when they hold higher share of government securities. Firms, particularly small ones are unable obtain the credit they are seeking for, which leads large drops in borrowing and investment.

Our estimation and identification strategy relies on firms' borrowing from multiple banks, where the banks differ in their exposure to the valuation shocks on their government security holdings. Using fixed effects, we analyze how the same firm's loan growth shifts from one bank to another more affected bank. This within firm comparison controls firm-specific changes in the demand for credit, leaving the estimation differences to be attributed to differences in bank's exposure to the valuation in the changes for government security holdings. As part of our robustness checks, we focus our attention to the primary dealer banks as the group of banks who are most affected bank from these shocks as these banks are obliged to be primary holders of sovereign securities. We show that the affect is more pronounces because these primary dealer banks are particularly hit from the government's spending decision and cannot easily adjust their portfolio. So they simply pass their shortage of funds to firms by loaning less.

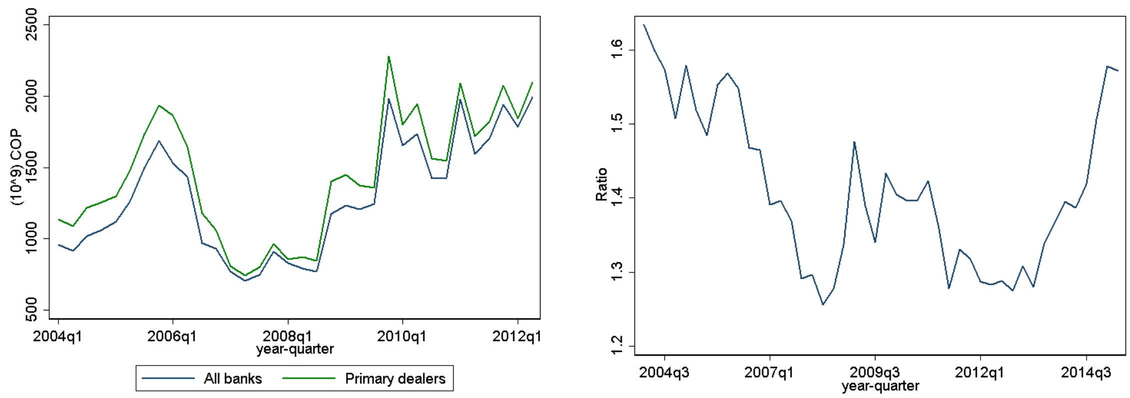
2 Data

In order to carry out the estimates, the 341 format was used from 2004-Q4 to 2012-Q3, which has quarterly data on the firm-bank credit relation for each, according to its credit rating. This database has 5,633,054 observations and includes all the information concerning the loans of a firm with a bank, such as the rate, bank's provisions, start and expiration, approved amount, among others. In order to incorporate the due firm controls such as size of its assets, indebtedness, and patrimony, a merge was made with the Superintendencia de Sociedades databases. This allows for non-contaminated data on each firmâĂŹs characteristics. Subsequent to this step, the remainder data consists of 1,501,685 observations representing the relation of 30 banks with 32,479 firms. This exercise seeks to analyze the impact of TES bonds holdings on the acquisition of new loans by firms, which is why the base was limited to new credit relationships between firms and banks, which from now on will be referred to as "new loans". After applying this filter, data consists of 731,141 new loans. As a fundamental part of the study, the total balance of public debt bonds of Colombia (TES) in the hands of banks and also the stock of the economy at the end of each quarter was included. In turn, in order to analyze substitution effects among banks by firms, we consider only firms that have already had a credit relationship with that bank in the past. Finally, macroeconomic controls such as the industrial production index, rates of new loans, exchange rate, among others are included. Once the pertinent depuration was made, the base consists of 134,522 observations.

Graph 2 illustrates the relationship between the total debt of the government in relation to the GDP is between 20% and 60% above GDP. Furthermore, figure 3 shows that as the percentage of bank bonds over its assets increases, their portfolio decreases (i.e amount of loans offered is lower). Moreover, small banks have a higher proportion of bank bonds over its assets compared to large banks. The lack of clarity of this relationship in figure 3a was, as a consequence, of considerable motivation for the research question posed on this paper.

On average, a bank has 24 thousand new loans with different firms per quarter, however,

Figure 1



large banks, with assets greater than the 75th percentile, account for 44% of the relationships with firms. Only one bank concentrates 25% of the relationships. Additionally, a large bank has, on average, 2.40 trillion pesos in TES while the general average is 1.78 trillion pesos. The percentage of TES over assets held by a large bank is approximately 5% while that of a small bank is 13%. However, as shown in graph 1, this ratio has been decreasing for all banks over time, on average. As for the entities that trade in the primary market, it is observed, on average, they have a greater amount of TES than the other banks (figure 2). Finally, the total debt that a large firm has over its total assets is approximately 35% compared to a small firm whose value is 48%.

Figure 2

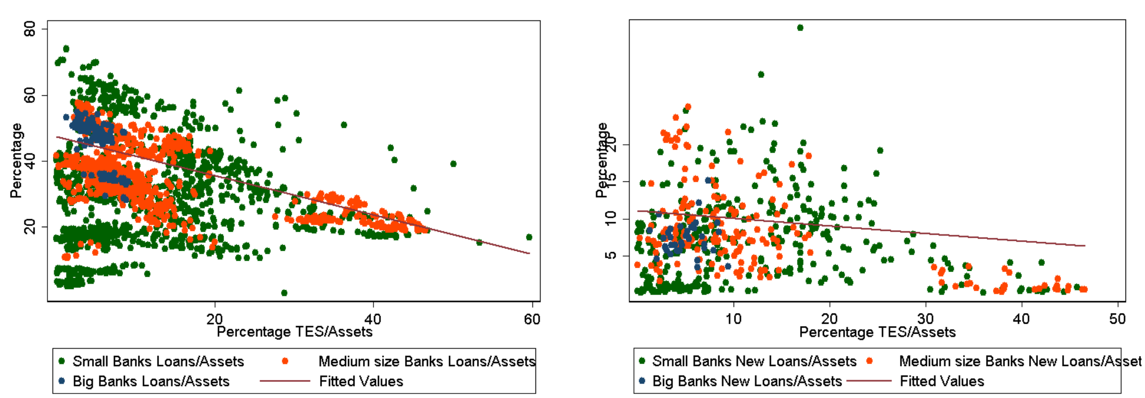


Table 1: Descriptives

Variable	Observations	Mean	St. dev.
Main Variables			
Logarithm of credit amount	246,866	18.98296	2.736915
Logarithm of government bonds	246,474	27.6204	0.954065
Bonds over assets	240,032	0.083699	0.063839
Debt over GDP	246,866	1.399239	0.097517
Firm controls			
Total assets	246,866	41.05141	244.1302
Securities	246,866	1.012422	12.81384
ROA	246,850	4.018454	172.413
Solvency	246,842	2.337085	39.85075
Indebtedness ratio	246,850	0.607874	3.096291
Acid test ratio	246,825	1.238304	5.689338
Bank controls			
Bonds of other banks	246,865	1372.634	1691.981
Provisions	246,866	34.80029	27.59406
Yields of ordinary credit	236,263	43.19154	7.507011
Risky portfolio	240,034	544.2078	441.9362
Past due portfolio	240,034	443.7186	330.027
Operational income	240,034	115.2032	176.4539
Others available	240,034	178.9975	234.6305
Total assets	240,034	21386.59	15352.07
Macroeconomic controls			
Policy rate	246,866	6.131776	2.237724
Net inflows	246,866	11.75228	36.02999
IPI (Industrial production index)	246,866	3.861118	6.023086

FACT 1. *Higher levels of sovereign debt crowd out the available credit to private firms.*

FACT 2. *Reduction in the availability of credit is associated with lower investment.*

FACT 3. *Increased government spending exacerbates the decline in investment through the transmission of sovereign risk on the cost of credit for firms..*

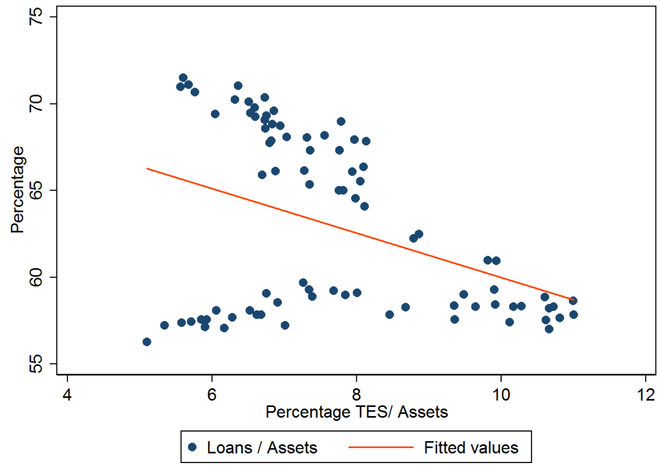


Figure 3: This figure shows that as the share of sovereign securities with respect to total assets increase, amount of new credit provided to firms tend to decline. Data: Own Calculations.

3 The Model[1]

3.1 Households

Households choose how much to consume and supply labor to maximize expected discounted utility streams, $E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, L_t)$, where $0 < \beta < 1$ is the subjective discount factor, and c_t and L_t denote consumption and labor, respectively. $u(.)$ is the period utility function which is continuous, strictly increasing in consumption, strictly decreasing in labor, and strictly concave in both arguments. E_t is the expectation operator conditional on the information set available at t.

Household receive real wage per labor hour, w_t, profits paid by the final-goods producers, π_t, and transfers from the government (T_t). Formally, the household solves

$$\max_{c_t, L_t} E_t \left[\sum_{t=0}^{\infty} \beta^t u(c_t, L_t) \right] \tag{3.1}$$

[1]Our quantitative model relies on the framework provided by **?**.

subject to the period budget constraint

$$c_t = w_t L_t + \pi_t + T_t \tag{3.2}$$

Household preferences are governed by a **?**-type utility function, which ensures no wealth effect on labor supply.[2] In particular, we use the utility function of the form

$$u(c_t, L_t) = \frac{\left(c_t - \frac{L_t^{1+v}}{1+v}\right)^{1-\sigma} - 1}{1-\sigma} \tag{3.3}$$

where $\sigma > 0$ is the constant relative risk aversion, and $v > 0$ governs the (inverse) Frisch elasticity of labor supply. The optimal labor supply is given by $L_t^v = w_t$.

3.2 Final-Goods Producers

We formulate the final goods producers problem following **?**. In particular, these firms maximize profits, π_t, net of depreciation and discounted at the country borrowing rate:

$$E_0 \left[\sum_t \left(\frac{1}{1+r_t^*} \right) \left(\pi_t^f - p_t^I \left(K_t - (1-\delta) K_{t-1} \right) \right) \right] \tag{3.4}$$

where p_t^I is the price of investment good, $I_t \equiv (K_t - (1-\delta)K_{t-1})$ denotes investment, and π_t^f is the output net of labor costs ($\pi_t^f = A_t K_t^{\alpha^K} L_t^{\alpha^L} - w_t L_t$). The production technology is governed by a Cobb-Douglas function given by

$$Y_t = A_t K_t^{\alpha^K} L_t^{\alpha^L} \tag{3.5}$$

α^K is the output elasticity of capital, α^L the output elasticity of labor, and A_t the shock to level of total factor productivity. They operate under decreasing returns to scale, i.e.

[2] This formulation for preferences removes wealth effect on labor supply and helps explain key business cycle facts for small open economies. See, e.g., **?**.

$0 < \alpha^K + \alpha^L < 1$.[3] Final-goods producers take factor prices, w_t and p_t^I, and the country borrowing rate, r_t^*, as given, and maximize their profits given by equation (3.4).

We assume that firms perceive the TFP shock as i.i.d., which then renders an optimal level of capital as a function of country borrowing rate (**?**). That is, for a given r^*, there is a unique and constant level of optimal K_t. The simplification, without loss of generality, is that final goods producers are myopic, and therefore, their maximization problem satisfies $K \equiv K(r^*)$ and $I(r^*) = \delta K(r^*)$. This assumption is not as restrictive as it might appear, since in the sovereign's dynamic problem that we are going to postulate below, r^* fluctuates endogenously due to fluctuations in sovereign external borrowing and fluctuations in TFP, which then results in endogenously time-varying K_t and I_t.[4]

Profit maximization yields optimal factor demand schedules:

$$\alpha^K A_t K_t^{\alpha^K - 1} L_t^{\alpha^L} = \delta P_t^I \tag{3.6}$$

and

$$\alpha^L A_t K_t^{\alpha^K} L_t^{\alpha^L - 1} = w_t \tag{3.7}$$

3.3 Capital Accumulation and Investment Goods Producers

The importance of foreign investment goods in accumulating capital is well recognized in the literature (e.g. **?** and **?**). Along with this literature, we assume that investment good, I_t, is produced via a CES-aggregate of domestic and imported foreign investment goods, I_t^d

[3]Under decreasing-returns-to-scale production technology, the firm makes positive economic profits (which are then transferred to households). Such transfer of profit helps match consumption dynamics better for default episodes.

[4]Our assumption also boils down to the *non-contractibility of investment* as laid out by **?** that the firm cannot commit to an exact investment level when signing the contract, and, afterwards, it is free to choose investment to maximize its own objective function. Or, similar in notion to **?**, firms, in the face of realization of shock in the next period, moves quickly to bring investment to the new level, given the new endogenous borrowing constraint due to a given (r_{t+1}, A_{t+1}).

and I_t^f, respectively. The aggregator satisfies

$$I_t = \left[\lambda(I_t^d)^\epsilon + (1-\lambda)(I_t^f)^\epsilon\right]^{\frac{1}{\epsilon}} \tag{3.8}$$

where $\lambda \in [0,1)$ is the weight of I_t^d, and $\epsilon < 1$ governs the (inverse) elasticity of substitution.

Moreover, I_t^f is composed of imperfectly substitutable varieties, for reasons that we discuss below. In particular, I_t^f is a CES-aggregate of foreign investment goods varieties,

$$I_t^f = \left(\int_{j\in[0,1]} (I_{i,t}^f)^\nu\right)^{\frac{1}{\nu}} \tag{3.9}$$

where $I_{i,t}^f$ is the foreign investment good variety i, and the elasticity of substitution across the varieties is given by $|\frac{1}{\nu-1}|$.

Investment goods producers minimize the cost of producing a certain level of investment, $\overline{I_t}$, given by

$$\min_{I_t^d, I_t^f} I_t^d + P_t^f I_t^f \tag{3.10}$$

subject to $\overline{I_t} = \left[\lambda(I_t^d)^\epsilon + (1-\lambda)(I_t^f)^\epsilon\right]^{\frac{1}{\epsilon}}$. At the second stage, given their optimal demand for aggregate foreign investment good, they choose their demand for the each variety.

The optimality conditions for the first-stage are

$$I_t^d = \left(\frac{1-\lambda}{\lambda}\frac{1}{P_t^f}\right)^{\frac{1}{\epsilon-1}} I_t^f \tag{3.11}$$

$$I_t^f = \left(\lambda\left(\frac{1-\lambda}{\lambda}\frac{1}{P_t^f}\right)^{\frac{\epsilon}{\epsilon-1}} + (1-\lambda)\right)^{-\frac{1}{\epsilon}} I_t \tag{3.12}$$

and given the zero profit condition $P_t^I I_t = I_t^d + P_t^f I_t^f$, the price of investment goods satisfies

$$P_t^I = \left(\lambda^{\frac{1}{1-\epsilon}} + (P_t^f)^{\frac{\epsilon}{\epsilon-1}}(1-\lambda)^{\frac{1}{1-\epsilon}}\right)^{\frac{\epsilon-1}{\epsilon}} \tag{3.13}$$

At the second stage, investment goods producers choose optimal demand for each variety i, subject to a working capital constraint. In particular, they need to finance a θ-fraction of their purchase of foreign investment good. One can show that the demand for each variety i satisfies

$$I_{it}^f = \begin{cases} \left(\frac{p_i^f(1+r_t^*)}{P^f(r_t^*)}\right)^{-\frac{1}{1-\nu}} I_t^f, & \text{for } i \in [0,\theta]; \\ \left(\frac{p_i^f}{P^f(r_t^*)}\right)^{-\frac{1}{1-\nu}} I_t^f, & \text{for } i \in [\theta,1]. \end{cases} \tag{3.14}$$

Moreover, when the country is in default, the investment goods producing firms cannot access to working capital loans, and therefore

$$I_{it}^f = \begin{cases} 0, & \text{for } i \in [0,\theta]; \\ \left(\frac{P_{aut}^f}{p_i^f}\right)^{-\frac{1}{1-\nu}} I_t^f, & \text{for } i \in [\theta,1]. \end{cases} \tag{3.15}$$

where P_{aut}^f is the rental rate of foreign investment during autarky. Given the CES aggregator for foreign capital (equation (3.9)), and given that a random θ fraction of varieties needs to be financed by working capital loans, then $P^f(r_t^*)$ satisfies the following:

$$P_t^f(r_t^*) = \left[\int_0^\theta (p_i^f(1+r_t^*))^{\frac{\nu}{\nu-1}} di + \int_\theta^1 (p_i^f)^{\frac{\nu}{\nu-1}} di\right]^{\frac{\nu-1}{\nu}} \tag{3.16}$$

and where, in autarky,

$$P_{aut}^f = \left[\int_\theta^1 (p_i^f)^{\frac{\nu}{\nu-1}}\right]^{\frac{\nu-1}{\nu}}. \tag{3.17}$$

When a sovereign defaults, the investment goods producers cannot have access to working capital financing and the demand function system is computed with the limit of that system as $r_t^* \to \infty$.

3.4 Sovereign's problem

This section describes the benevolent government's optimization problem. The government lacks a commitment technology, thus cannot commit to its future default and borrowing decisions, and decides how much non-state contingent long-term debt to issue each period after repayment.[5] The timing of our paper is as in **?** to rule out the multiplicity dynamics that is present in **?** and **?** where they assume that the government makes the default decision after borrowing.[6]

We introduce an endogenous link between sovereign default and private economic activity. This endogenous link hinges on two assumptions: First, there is one-to-one mapping between the government's implied one-period borrowing rate and the firm's short-term working capital loan rate; and (ii) both the government and domestic firms are excluded from the financial markets after a default decision. The first is instrumental to capture the debt overhang: a rise in government debt holdings increases the likelihood of government default and thus amplifies the bond spreads as well as the loan rates for domestic firms. The second facilitates the significant investment drop during financial exclusion since lack of access to credit markets inhibits firms to find necessary funds to finance investment expenditures and leads to an efficiency loss.

Long-term debt contracts. The government issues long-term non-state contingent debt in real goods which are traded at a price q_t by infinitely many risk neutral international lenders. Similar to **?** and **?**, a debt issued at time t promises a stream of geometrically decreasing coupons κ which depreciates at a rate $\delta_b \in (0, 1]$. Thus, a government promises

[5] Allowing for long-term maturity for sovereign debt helps obtain more realistic sovereign bond spreads and default frequencies. Note also that, under certain conditions, long-term debt incorporates short-term debt as a special case (**?**, **?**).

[6] **?** documents that the models and the extensions of Eaton-Gersovitz type of sovereign default models are not prone to multiple equilibria.

to pay $\kappa(1-\delta_b)^{n-1}$ units of consumption good in period $t+n$ for $n \geq 1$. Coupon payment κ is computed such that in the absence of default risk, price of non-state contingent long-term debt is equal to the price of average one-period debt and it reads $\frac{r+\delta_b}{1+r}$. This is a common formulation for long-term debt contracts to avoid keeping track of the entire maturity distribution. Hence, the dynamics of the long-term can be represented as follows:

$$b_{t+1} = (1-\delta_b)b_t + m_t \tag{3.18}$$

where b_t and b_{t+1} are total outstanding debt obligations at the beginning of period t and $t+1$ and m_t is the amount of debt that is issued or purchased back by the government in period t. Note that one-period debt is a special case of long-term debt where $\delta_b = 1$. The budget constraint of a government conditional on its repayment decision reads as

$$c_t = A_t f(K_t, L_t) - I_t^f P_t^f - \kappa b_t + q(b_{t+1}, A_t) m_t. \tag{3.19}$$

Defaults. It is common in the literature to assume that the government is excluded from the credit markets for a stochastic period of time upon its default decision and gains access to the credit markets next period with constant probability $\psi \in [0,1]$. Bond contracts typically include acceleration and collective action clauses, thus all debt obligations become due when a government defaults.

In order to regain access to the credit markets, the government needs to repay α fraction of its defaulted debt. This is a simple way of introducing recovery to the model. However, in a model with long-term debt the government issues infinite amount of debt prior to its default decision which may distort the moments in the simulations. To account for that it is assumed the government cannot issue bonds with a price lower than $\underline{q}$ as in **?**. We confirm that the constraint never binds in model simulations. In default, total output is used for consumption and imports of foreign investment goods at the price P_{aut}^f. Therefore, the budget constraint of an economy in default reads as

$$c_t = A_t f(K_t, L_t) - I_t^f P_{aut}^f. \tag{3.20}$$

International lenders. The international risk-neutral lenders invest in non-state contingent long-term debt and within-period private working capital loans in a perfectly competitive market, taking decision and borrowing rules given. Thus, lenders' supply schedule reflects a no-arbitrage condition with an opportunity cost of funds equals to the world's risk-free rate r. We denote the pricing schedule of bonds that are repaid with $q_t(b_{t+1}, A_t)$ and for bonds that are in arrears with $q_t^D(b_t, A_t)$.

3.5 Recursive representation

We now formulate the government's optimization problem recursively. Let V be the value function of the government that has the option of defaulting. The government chooses to repay if value of repayment V^R is greater than the value of default V^D. The function V satisfies the following functional equation:

$$V(b, A) = \max \left\{ V^R(b, A), V^D(b, A) \right\}, \tag{3.21}$$

where the government's continuation value for repayment reads

$$\begin{aligned} V^R(b, A) &= \underset{b' \geq 0, c \geq 0, L \geq 0, I^f \geq 0, I^d \geq 0}{Max} \left\{ u(c, L) + \beta E_A \left[V(b', A') \right] \right\}, \qquad (3.22) \\ s.t. \quad & c = Af(K, L) - I^f P^f(r^*) - b\kappa + q(b', A) \left[b' - (1 - \delta_b) b \right], \\ q(b', A) &\geq \underline{q} \text{ if } b' > (1 - \delta_b) b, \end{aligned}$$

where $f(K, L) = K_t^{\alpha^K} L_t^{\alpha^L}$. I^f and $P^f(r^*)$ denote the imported foreign investment goods and the price of these goods, respectively. The term r^* is the government's implied interest rate on one-period bonds. It is worth noting that the benevolent government faces the same allocations of output and factors of production as the agents in the private economy. To be specific, for a given TFP shock and implied short-term borrowing rate r^* –which will be obtained below under the price function–, the optimal factor allocations (I^f, I^d, L) chosen by the benevolent government characterize the private sector competitive equilibrium.

The government stays in default at least for one period and regains access to the credit markets with constant probability ψ. The continuation value of default takes into account the positive repayment when the government regains access to the credit markets. We assume that debt in arrears grows at the international risk free rate r during the sovereign's exclusion from the credit markets. Along these lines, the value of default is:

$$\begin{aligned} V^D(b, A) &= \underset{c \geq 0, L \geq 0, I^f \geq 0, I^d \geq 0}{Max} \left\{ u(c, L) + \beta E_A \left[\psi V(\alpha b(1+r), A) + (1-\psi) V^D(b(1+r), A) \right] \right\} \\ s.t. \quad & c = Af(K, L) - I^f P^f_{aut}. \end{aligned} \tag{3.23}$$

The solution to the government's optimization problem yields decision rules for default $\hat{d}(b, A)$ and borrowing $\hat{b}(b, A)$. A default decision rule is denoted by the binary variable $\hat{d} \in \{0, 1\}$; $\hat{d} = 1$ if the government finds value of defaulting higher than value repayment and 0 otherwise. In equilibrium, defined in Section 3.6, lenders use these decision rules for contract pricing. The price of a bond satisfies

$$\begin{aligned} q(b', A) &= \frac{1}{1+r} E_A \left[\left(1 - \hat{d}\left(b', A'\right)\right) \left(\kappa + (1-\delta_b)\, q\left(b'', A'\right)\right) + \right. \\ & \qquad \left. \hat{d}\left(b', A'\right) q^D \left((1+r)\, b', A'\right) \right], \\ b'' &= \hat{b}(b', A'). \end{aligned} \tag{3.24}$$

Equation (3.24) indicates that with no-arbitrage condition, selling a bond and investing it in a risk-free asset today at an international risk-free rate r (left hand-side of the equation) is equal to the value of keeping the bond (right hand-side of the equation). If the lender keeps the bond, he will receive one unit of coupon κ and can sell the unamortized portion of its assets $(1-\delta_b)$ at tomorrow's prevailing price $q\left(b'', A'\right)$ conditional on government's repayment decision ($\hat{d}\left(b', A'\right)=0$). If the government defaults, then lenders can still trade their bonds at the secondary market at a price q^D. The value of a bond in default satisfies

$$q^D(b', A) = \frac{1}{1+r} E_A \left[\begin{array}{l} (1-\psi)\,(1+r) q^D\left((1+r)b', A'\right) + \\ \alpha\psi \hat{d}(b''^{DD}, A') q^D\left(b''^{DD}, A'\right) + \\ \alpha\psi\left(1-\hat{d}(\alpha(1+r)b', A')\right)\left[\kappa + (1-\delta_b) q\left(b''^{DR}, A'\right)\right] \end{array} \right],$$

where $b''^{DD} = \alpha(1+r)b'$ denotes the next-period end-of-period total debt if the government defaults after exiting the current default, $b''^{DR} = \hat{b}(\alpha b')$ denotes the next-period end-of-period debt obligations chosen by a government that repays after exiting the default.

The short-term interest rate r^* that is used by the investment good producers in the economy is computed by setting $\delta_b = 1$. Notice that when $\delta_b = 1$, equation (3.24) boils down to the price of one-period debt which is determined by tomorrow's default probability and the recovery rate.

3.6 Definition of equilibrium

This paper focuses on Markov Perfect Equilibrium. The government cannot commit to any future (repayment and borrowing) decisions. Hence, government's strategies depend only on the payoff relevant state variables.

Definition 1 (Markov perfect equilibrium) *A Markov perfect equilibrium is characterized by value functions $V(b, A), V^D(b, A), V^R(b, A)$, bond pricing function $q(b', A)$, default rule $\hat{d}$ and borrowing rule $\hat{b}$ such that*

1. *Given the bond pricing function, government policy rules $\left\{\hat{d}, \hat{b}\right\}$ solve the utility maximization problem defined in equations (3.21), (3.22) and (3.23).*

2. *Given government policy rules $\left\{\hat{d}, \hat{b}\right\}$, the pricing function q satisfies condition (3.24).*

A solution to the Markov Perfect Equilibrium consists of the solutions for the optimality conditions of the private sector including the factor allocations as well as production with and without having an access to the credit markets. Solutions for equilibrium wages and profits follow from the optimality conditions characterized earlier.

3.7 Numerical Solution

The solution to this problem is obtained using global solution methods. This section briefly sketches the main numerical algorithm, relegating the details of the implementation to the Appendix. Solving the model relies on iterating the value functions V^R and V^D and price function q as well as an approximation scheme to the private sector's allocation problem. To avoid a potential multiplicity problem outlined in **?**, we first solve the equilibrium of the finite-horizon economy. We start with an initial guess of the terminal value and iterate backwards until the differences in value and price functions for two subsequent periods are less than 10^{-5}. We then use obtained values as the equilibrium of the infinite horizon economy.

4 Econometric model specification

The econometric model employed for this study is a data panel containing information on firms and banks in a period of time between 2004 Q4 to 2012 Q3. The exercise consists on running several regressions taking into account fixed effects for variables that are constant over time by bank and firm. In order to estimate the implied impulse response functions (IRF's) we follow Jorda's (2005) method of local projections. Essentially, we estimate sequential regressions in which the outcome variable log loans is shifted forward at each forecasting period.

$$\arg\min_{\theta} \sum_{i=1}^{I} \sum_{j=1}^{J} \sum_{t=2}^{T-J} \left(log(L_{t+j}) - a_j - \theta_j log(B_t) - b_j(WA_t) - \tau_j(DG_t) - \psi_j log(B_t)^*(WA_t)^*(DG_t) \right)^2 \quad (4.1)$$

$L_{i,j,t}$ is the loan (the credit relationship that one specific firm has with an specific bank), B_t are the government bonds (TES) that the bank owns, WA_t is the total won amount in auction over assets by bank and $DG_{j,t-1}$ is total debt over Colombia's GDP.

Time, bank, firm and firm-bank fixed effects were used as in **?** to take advantage of the variation of the variable of interest. Time fixed effects are used to tamper the possible macroeconomic effects which are invariant over time. Banks and firms fixed effects, on the other hand, isolate the effect of invariant variables in their balance sheets that may have an effect on the decisions they make.

Finally, as robustness checks, the proportion of the TES holdings on the assets of each bank is taken as a variable of interest to control for the amount of assets of each one of them, the decisions that they make based on their balance sheet and to rescale Government bonds in proportion to assets. Additional exercises were carried out using clusters per firm to control for the heterogeneity that exists between firms not only by sector but by size.

5 Results

Figure 4

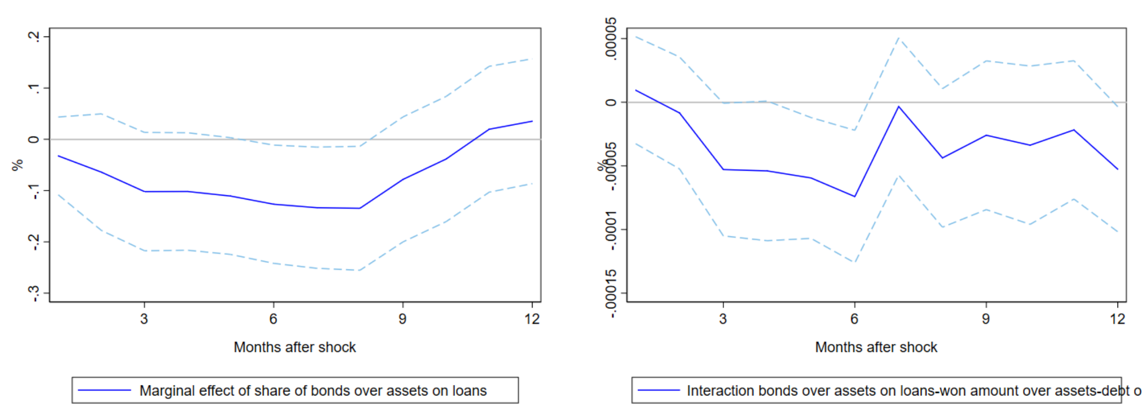
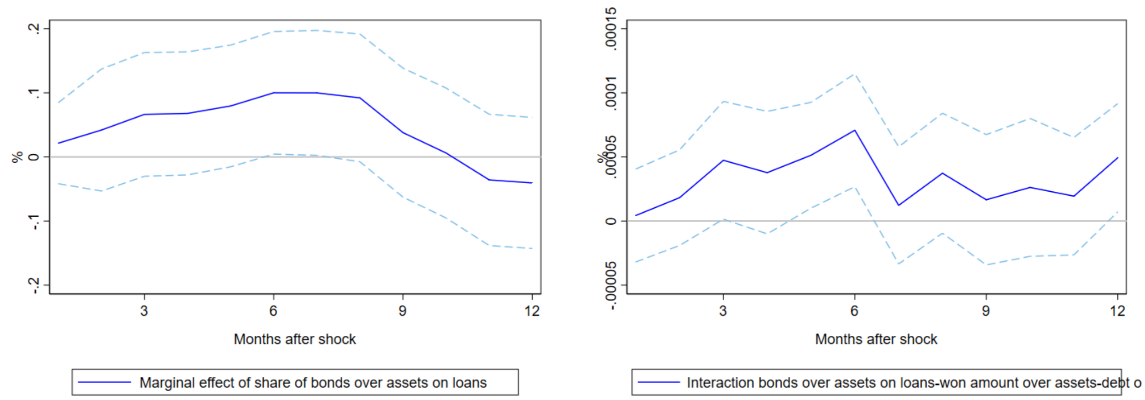


Figure 5



As it was previously stated in the methodology section, we use a panel with time, bank, firm and bank-firm as fixed effects as our main specification to analyze the impact of an increase in TES holdings by a bank over new loans to the firms, at the micro level. Figure 4.a shows the marginal effect of the amount of TES held by a bank on new loan to a firm with that bank. The impact is negative and in month 7 it starts to be significant (i.e. an increase of 1% in bonds over assets, the credits decrease by 0.1%). Figure 4.b is the coefficient of the triple interaction between bonds over assets, amount won in auction over assets, and debt over GDP. It shows the additional effect over loans of shocks on those variables. Each regression was run with cluster by firm and fixed effects of firm, bank and time. On the other hand, figures 5.a and 5.b show the impact of the amount of TES held by a bank on the amount of credits that a firm futherly starts having with other banks (i.e. substitution effect)

We ran 3 types of regressions with a difference in one of the explanatory variables. Table 2 shows the results of the main specification. It is evident that the different lags of the Bonds over Assets variable are significant negative, which supports the initial hypothesis. For example, in column 1 its observed that by a 1% increase in Bonds over Assets decreases the new loan amount of a firm with that same bank in 1.978%. On the other hand, the coefficients of the different lags of the triple interaction between bonds over assets, won amount in auction over assets and debt over GDP are also negative. This shows the additional effect of the shock in some of these variables. An increase of the debt and/or the amount won in the auction,strenghtens the impact of bonds over assets, and reduces even more the amount of new loans a firm has with that specific bank. Same interpretaation applies for table 3 and 4. As for the interaction, the total amount won in the auction over the approved net amount, and the primary dealers, is taken into account instead of won amount over assets. Results show that these variables also have a negative sign and causes the same effect.

6 Conclusions

Our results show the impact of bond holdings on the amount of loans lent by firms in Colombia. If a country rises its public debt, banks (especially primary dealers) buy more bonds and, as a consequence, borrow less to firms. Using firm-bank data from new loans in Colombia from Q4 2004 to Q3 2012, we study the impact of the logarithm of the total amount held by a bank as our main explanatory variable. Our methodology for estimating the bank lending channel focuses on firms' borrowing from multiple banks, where the banks differ in their exposure to liquidity shocks. Using firm fixed effects (FEs) in first-differenced data, we compare how the same firmâĂŹs loans grow from one bank to a more affected bank. The firm-specific changes in credit demand are subsequently absorbed since there are comparisons made within each firm. The estimated difference in loan growth can be plausibly attributed to differences in bank liquidity shocks. Our findings suggest that an increase of 1 percentage point in the amount of bonds of a bank decreases 1.678 percentage points the new loan amount of a firm with that same bank. These results hold when using the total amount of bonds over the total assets of the bank as explanatory variable, which strengthens our results.

Appendices

A Appendix -Tables

Table 2: Impact of bonds over assets on new loans (interacted with won amount in auction/assets)

VARIABLES	(1) Log(loans)	(2) Log(loans)	(3) Log(loans)	(4) Log(loans)	(5) Log(loans)	(6) Log(loans)	(7) Log(loans)	(8) Log(loans)
Bonds/Assets	-1.978***					-0.307		
	(0.635)					(1.004)		
Won Amount in Auction/Assets	-0.00986***					0.00397		
	(0.00260)					(0.00522)		
Bonds/Assets*Won Amount in Auction/Assets	0.0353***					-0.0296		
	(0.0114)					(0.0219)		
Bonds/Assets_t-1		-2.013***				0.747	1.412	1.315
		(0.612)				(1.092)	(0.898)	(0.949)
Bonds/Assets_t-2			-2.418***			-0.630	-1.997**	-0.781
			(0.603)			(1.116)	(0.913)	(1.100)
Bonds/Assets_t-3				-2.547***		-1.825*		-1.914**
				(0.571)		(0.964)		(0.951)
Bonds/Assets*Debt/GDP	1.397***					0.151		
	(0.447)					(0.705)		
Won Amount in Auction/Assets*Debt/GDP	0.00709***					-0.00285		
	(0.00178)					(0.00357)		
Bonds/Assets*Won Amount in Auction/Assets*Debt/GDP	-0.0254***					0.0208		
	(0.00810)					(0.0152)		
Won Amount in Auction/Assets_t-1		-0.0185***				-0.0207***	-0.0214***	-0.0187***
		(0.00308)				(0.00626)	(0.00485)	(0.00499)
Bonds/Assets_t-1*Won Amount in Auction/Assets_t-1		0.0761***				0.0606**	0.0453**	0.0347*
		(0.0149)				(0.0258)	(0.0199)	(0.0210)
Bonds/Assets_t-1*Debt/GDP_t-1		1.427***				-0.566	-1.064*	-0.987
		(0.427)				(0.749)	(0.622)	(0.656)
Won Amount in Auction/Assets_t-1*Debt/GDP_t-1		0.0132***				0.0132***	0.0138***	0.0115***
		(0.00215)				(0.00430)	(0.00336)	(0.00344)
Bonds/Assets_t-1*Won Amount in Auction/Assets_t-1*Debt/GDP_t-1		-0.0546***				-0.0416**	-0.0320**	-0.0233
		(0.0107)				(0.0184)	(0.0143)	(0.0149)
Won Amount in Auction/Assets_t-2			-0.0160***			0.0142**	0.00318	0.0162***
			(0.00296)			(0.00586)	(0.00432)	(0.00571)
Bonds/Assets_t-2*Won Amount in Auction/Assets_t-2			0.0748***			-0.0306	0.00452	-0.0343
			(0.0150)			(0.0223)	(0.0189)	(0.0219)
Bonds/Assets_t-2*Debt/GDP_t-2			1.720***			0.264	1.201*	0.361
			(0.424)			(0.763)	(0.627)	(0.754)
Won Amount in Auction/Assets_t-2*Debt/GDP_t-2			0.0114***			-0.00845**	-0.00133	-0.00953**
			(0.00205)			(0.00380)	(0.00288)	(0.00372)
Bonds/Assets_t-2*Won Amount in Auction/Assets_t-2*Debt/GDP_t-2			-0.0535***			0.0225	-0.00260	0.0242
			(0.0106)			(0.0158)	(0.0134)	(0.0156)
Won Amount in Auction/Assets_t-3				-0.0162***		-0.0185***		-0.0190***
				(0.00316)		(0.00591)		(0.00538)
Bonds/Assets_t-3*Won Amount in Auction/Assets_t-3				0.0671***		0.0598**		0.0576**
				(0.0158)		(0.0250)		(0.0227)
Bonds/Assets_t-3*Debt/GDP_t-3				1.899***		1.312**		1.376**
				(0.402)		(0.662)		(0.656)
Won Amount in Auction/Assets_t-3*Debt/GDP_t-3				0.0115***		0.0124***		0.0126***
				(0.00217)		(0.00403)		(0.00363)
Bonds/Assets_t-3*Won Amount in Auction/Assets_t-3*Debt/GDP_t-3				-0.0481***		-0.0429**		-0.0412**
				(0.0112)		(0.0180)		(0.0163)
Bonds/Assets_t-4					-2.303***			
					(0.548)			
Won Amount in Auction/Assets_t-4					-0.0149***			
					(0.00311)			
Bonds/Assets_t-4*Won Amount in Auction/Assets_t-4					0.0477***			
					(0.0143)			
Bonds/Assets_t-4*Debt/GDP_t-4					1.777***			
					(0.387)			
Won Amount in Auction/Assets_t-4*Debt/GDP_t-4					0.0106***			
					(0.00212)			
Bonds/Assets_t-4*Won Amount in Auction/Assets_t-4*Debt/GDP_t-4					-0.0345***			
					(0.00999)			
Clustered by firm	X	X	X	X	X	X	X	X
Firm FE	X	X	X	X	X	X	X	X
Bank FE	X	X	X	X	X	X	X	X
Time FE	X	X	X	X	X	X	X	X
Firm - Bank FE						X	X	X
Observations	162,491	163,564	164,996	165,918	166,699	128,118	135,222	132,088
R-squared	0.752	0.753	0.754	0.756	0.757	0.871	0.869	0.871

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 3: Impact of bonds over assets on new loans (interacted with won amount in auction/approved amount in auction)

VARIABLES	(1) Log(loans)	(2) Log(loans)	(3) Log(loans)	(4) Log(loans)	(5) Log(loans)	(6) Log(loans)	(7) Log(loans)	(8) Log(loans)
Bonds/Assets		-2.110**					2.074	2.206
		(1.048)					(2.739)	(2.780)
Won amount/Approved		0.301					0.353	0.191
		(1.405)					(2.292)	(2.313)
Bonds/Assets*Won amount/Approved	-0.647***	-2.120					1.339	2.721
	(0.205)	(9.440)					(15.59)	(15.78)
Bonds/Assets_t-1			-2.267**				-3.238	-3.964
			(0.997)				(2.810)	(2.905)
Bonds/Assets_t-2				-3.069***			0.283	1.019
				(1.044)			(3.191)	(3.250)
Bonds/Assets_t-3					-2.249**		-1.288	1.016
					(0.955)		(2.538)	(3.080)
Bonds/Assets*Debt/GDP		0.00143*					-0.00166	-0.00177
		(0.000746)					(0.00193)	(0.00196)
Won amount/Approved*Debt/GDP		-0.000261					-0.000401	-0.000277
		(0.00104)					(0.00169)	(0.00171)
Bonds/Assets*Won amount/Approved*Debt/GDP		0.00178					-0.000225	-0.00123
		(0.00686)					(0.0113)	(0.0115)
Won amount/Approved_t-1			-1.286				0.716	1.297
			(1.367)				(2.367)	(2.385)
Bonds/Assets_t-1*Won amount/Approved_t-1			3.024				-13.81	-16.77
			(8.490)				(15.07)	(15.24)
Bonds/Assets_t-1*Debt/GDP_t-1			0.00160**				0.00198	0.00253
			(0.000714)				(0.00200)	(0.00206)
Won amount/Approved_t-1*Debt/GDP_t-1			0.000931				-0.000662	-0.00111
			(0.00102)				(0.00178)	(0.00179)
Bonds/Assets_t-1*Won amount/Approved_t-1*Debt/GDP_t-1			-0.00240				0.0105	0.0127
			(0.00629)				(0.0111)	(0.0113)
Won amount/Approved_t-2				-1.738			-1.120	-0.976
				(1.361)			(2.540)	(2.552)
Bonds/Assets_t-2*Won amount/Approved_t-2				9.972			0.428	-4.175
				(8.772)			(16.55)	(16.61)
Bonds/Assets_t-2*Debt/GDP_t-2				0.00206***			-9.38e-05	-0.000635
				(0.000738)			(0.00226)	(0.00230)
Won amount/Approved_t-2*Debt/GDP_t-2				0.00136			0.00102	0.000912
				(0.00101)			(0.00189)	(0.00190)
Bonds/Assets_t-2*Won amount/Approved_t-2*Debt/GDP_t-2				-0.00734			-0.00129	0.00206
				(0.00646)			(0.0121)	(0.0122)
Won amount/Approved_t-3					-1.313		0.311	1.673
					(1.221)		(2.222)	(2.501)
Bonds/Assets_t-3*Won amount/Approved_t-3					9.264		9.172	-3.494
					(8.229)		(14.00)	(16.27)
Bonds/Assets_t-3*Debt/GDP_t-3					0.00152**		0.000797	-0.000786
					(0.000678)		(0.00179)	(0.00219)
Won amount/Approved_t-3*Debt/GDP_t-3					0.00114		-0.000183	-0.00123
					(0.000903)		(0.00165)	(0.00185)
Bonds/Assets_t-3*Won amount/Approved_t-3*Debt/GDP_t-3					-0.00713		-0.00652	0.00288
					(0.00598)		(0.0103)	(0.0119)
Bonds/Assets_t-4						-2.619***		-3.365
						(1.002)		(2.608)
Won amount/Approved_t-4						0.0874		-2.763
						(1.437)		(2.432)
Bonds/Assets_t-4*Won amount/Approved_t-4						8.821		26.05*
						(9.343)		(14.99)
Bonds/Assets_t-4*Debt/GDP_t-4						0.00173**		0.00236
						(0.000714)		(0.00185)
Won amount/Approved_t-4*Debt/GDP_t-4						3.62e-05		0.00215
						(0.00107)		(0.00181)
Bonds/Assets_t-4*Won amount/Approved_t-4*Debt/GDP_t-4						-0.00635		-0.0194*
						(0.00690)		(0.0111)
Constant	21.35***							
	(0.00317)							
Clustered by firm		X	X	X	X	X	X	X
Firm FE		X	X	X	X	X	X	X
Bank FE		X	X	X	X	X	X	X
Time FE		X	X	X	X	X	X	X
Firm - Bank FE							X	X
Observations	84,458	58,912	59,076	59,160	58,660	58,578	38,139	37,979
R-squared	0.000	0.802	0.802	0.802	0.803	0.803	0.910	0.910

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 4: Impact of bonds over assets on new loans (interacted with primary dealers)

VARIABLES	(1) Log(loans)	(2) Log(loans)	(3) Log(loans)	(4) Log(loans)	(5) Log(loans)	(6) Log(loans)	(7) Log(loans)	(8) Log(loans)
Bonds/Assets							-1.999	
							(2.717)	
Primary Dealer							0.0121	
							(0.224)	
Bonds/Assets*Primary Dealer	-0.137***						0.242	
	(0.0249)						(2.798)	
Bonds/Assets_t-1		-4.936***					-3.886	-5.015*
		(1.597)					(3.369)	(2.606)
Bonds/Assets_t-2			-2.204				2.106	0.931
			(1.606)				(3.441)	(3.437)
Bonds/Assets_t-3				-2.990*			-1.712	-0.553
				(1.672)			(2.726)	(3.466)
Bonds/Assets*Debt/GDP							1.334	
							(1.808)	
Primary Dealer*Debt/GDP							-0.0161	
							(0.157)	
Bonds/Assets*Primary Dealer*Debt/GDP							-0.190	
							(1.863)	
Primary Dealer_t-1		-0.0114					-0.698***	-0.766***
		(0.136)					(0.267)	(0.216)
Bonds/Assets_t-1*Primary Dealer_t-1		3.412**					5.632*	6.433**
		(1.611)					(3.418)	(2.672)
Bonds/Assets_t-1*Debt/GDP_t-1		3.225***					2.593	3.362*
		(1.070)					(2.215)	(1.732)
Primary Dealer_t-1*Debt/GDP_t-1		-0.0300					0.500***	0.544***
		(0.0967)					(0.184)	(0.151)
Bonds/Assets_t-1*Primary Dealer_t-1*Debt/GDP_t-1		-2.106*					-3.865*	-4.459**
		(1.080)					(2.255)	(1.781)
Primary Dealer_t-2			0.335**				0.625**	0.549**
			(0.134)				(0.271)	(0.268)
Bonds/Assets_t-2*Primary Dealer_t-2			0.269				-3.534	-2.900
			(1.603)				(3.515)	(3.502)
Bonds/Assets_t-2*Debt/GDP_t-2			1.499			0.0772	-1.518	-0.732
			(1.086)			(0.0777)	(2.266)	(2.274)
Primary Dealer_t-2*Debt/GDP_t-2			-0.274***			-0.00791	-0.454**	-0.401**
			(0.0960)			(0.00700)	(0.187)	(0.185)
Bonds/Assets_t-2*Primary Dealer_t-2*Debt/GDP_t-2			-0.0684			-0.247***	2.394	1.956
			(1.080)			(0.0775)	(2.317)	(2.318)
Primary Dealer_t-3				0.310**			0.103	0.241
				(0.132)			(0.208)	(0.272)
Bonds/Assets_t-3*Primary Dealer_t-3				0.513			0.0625	0.582
				(1.659)			(2.784)	(3.537)
Bonds/Assets_t-3*Debt/GDP_t-3				2.062*			1.155	0.328
				(1.131)			(1.792)	(2.274)
Primary Dealer_t-3*Debt/GDP_t-3				-0.259***			-0.0891	-0.174
				(0.0947)			(0.144)	(0.186)
Bonds/Assets_t-3*Primary Dealer_t-3*Debt/GDP_t-3				-0.172			0.0179	-0.364
				(1.117)			(1.827)	(2.318)
Bonds/Assets_t-4					-0.617			0.348
					(1.617)			(2.524)
Primary Dealer_t-4					0.516***			0.0848
					(0.129)			(0.203)
Bonds/Assets_t-4*Primary Dealer_t-4					-2.332			-3.326
					(1.631)			(2.587)
Bonds/Assets_t-4*Debt/GDP_t-4					0.481			-0.225
					(1.090)			(1.657)
Primary Dealer_t-4*Debt/GDP_t-4					-0.397***			-0.0788
					(0.0920)			(0.140)
Bonds/Assets_t-4*Primary Dealer_t-4*Debt/GDP_t-4					1.764			2.395
					(1.099)			(1.699)
Constant	21.31***							
	(0.00261)							
Clustered by firm		X	X	X	X	X	X	X
Firm FE		X	X	X	X		X	X
Bank FE		X	X	X	X	X	X	X
Time FE		X	X	X	X	X	X	X
Firm - Bank FE						X	X	X
Observations	165,096	163,564	164,996	165,918	166,699	149,665	128,118	129,218
R-squared	0.000	0.753	0.754	0.756	0.757	0.747	0.871	0.872

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 5: Weighted average of bonds over groth of specific variables of the balance sheet of firms

VARIABLES	(1) Growth of assets	(2) Growth of liabilities	(3) Growth of equity	(4) Growth of investments	(5) Growth of profits
Weighted average of bonds	-0.0633	-0.0706	0.0298	-0.684**	-0.114*
	(0.0504)	(0.0503)	(0.0476)	(0.279)	(0.0661)
Clustered by firm	X	X	X	X	X
Time FE	X	X	X	X	X
Observations	17,071	17,070	16,973	6,582	16,922
R-squared	0.021	0.021	0.005	0.006	0.014

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1