Optimal Monetary Policy Regime Switches[*]

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January 28, 2019

Abstract

An economy that switches between high and low growth regimes creates incentives for the monetary authority to change its rule. As lower growth tends to produce lower real interest rates, the monetary authority has an incentive to increase the inflation target and increase the degree of inertia in setting rates in an attempt to keep the nominal rate positive. An optimizing monetary authority therefore responds to permanently lower growth by slightly increasing both the inflation target and inertia; focusing solely on the inflation target ignores a key margin of adjustment. With repeated growth rate regime switches, an optimal monetary rule that switches at the same time internalizes both the direct effects of growth regime change and the indirect expectation effects generated by switching in policy. The switching rule improves economic outcomes relative to a constant rule and one that does not consider the impact of regime changes; this result is robust to the case when the monetary authority misidentifies the growth regime with relatively high frequency.

Keywords: growth rate; optimal policy; regime switching; Taylor rule; inflation target

JEL Codes: C63, E31, E52

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1 Introduction

The slow growth after the financial crisis prompted a renewed debate about objectives and conduct of monetary policy due to structural changes that have taken place. In particular, slower growth has contributed to lowering the real interest rate, leading to questions about how, if at all, the systematic conduct of monetary policy should respond to this shift. Much of the debate has called for a reassessment of inflation targets in light of a lower real interest rate.

The emphasis on changing fundamental policy parameters such as the inflation target reflects the widespread view that lower growth is in fact due to a persistent structural change rather than temporary factors that can be addressed by monetary policy without an overhaul of the policy framework. At the same time, while the period of low growth is persistent, the US economy has experienced repeated and lengthy periods of high or low growth (Fernald (2012)). This fact suggests that at some point in the future, the economy may return to a period of higher growth, only to switch back to a period of lower growth again at a later date.

Periods of higher or lower growth affect the conduct of monetary policy though direct effects on output and inflation, but also indirectly through the connection between growth and the real interest rate. If growth is persistently low, then an associated low real interest rate provides some justification for increasing the inflation target: a higher inflation target is intended to increase the nominal interest rate in order to avoid hitting the zero lower bound, at which time central banks may have to use less conventional means of providing monetary accommodation such as forward guidance or quantitative easing. However, adjusting the inflation target due to structural change requires two important considerations.

First, moving the inflation target is only one of possibly multiple adjustments to the conduct of monetary policy that can be made. Rather than focusing solely on the inflation target, it makes sense to also consider how strongly the nominal rate should react to current conditions versus have inertia in policy, as this systematic feature may have trade-offs relative to moving the target.

A second issue for changing monetary policy due to structural change is that the economy tends to shift between periods of high and low growth. These repeated regime changes, while not

commonplace, are not rare either, and so a monetary authority should account for the possibility of future shifts in the growth rate when setting policy. In addition, if the monetary authority is changing how it reacts as the economy undergoes structural shifts, then households and firms will respond as well, leading to feedback effects.

This paper studies optimal simple rules for monetary policy when the structural economy experiences regime shifts. Using a standard New Keynesian model modified so that growth rate switches between high and low growth regimes, it assesses the implications for setting the inflation target and degree of interest rate inertia when monetary policy follows a Taylor rule. The switching in the growth rate process tends to generate a low real interest rate in one regime and affects inflation and output dynamics, which causes the monetary authority to need to adjust its policy parameters.

The focus on the inflation target and inertia is because these two options present a trade-off for the monetary authority. A higher inflation target can be used to raise the level of nominal rates when real rates are low, which tends to keep the nominal rate positive but comes at the cost of higher inflation. On the other hand, making interest rates more inertial and hence dampening their responsiveness to current conditions can keep interest rates stable at positive values at the cost of injecting more volatility into inflation and output. An optimizing monetary authority that follows a rule will set both of these policy parameters in tandem depending on growth. Focusing on only one, for instance the inflation target, ignores a key degree of freedom available in setting monetary policy. More specifically, comparing optimal rules between a high and low growth rate economy, the lower growth rule has a slightly higher inflation target and more inertia; ignoring the possibility of an increase in inertia causes the monetary authority to set a substantially higher inflation target.

In addition, the results show that taking into account repeated switches between high and low growth is a key consideration for setting policy. The monetary authority can generate better welfare by internalizing several expectational effects, not only the direct effects that switching between high and low growth regimes has on monetary policy through household and firm behavior, but also the fact that if the monetary rule changes across regimes, households and

firms will internalize the possibility of future policy changes. In other words, communication of a switching policy has effects beyond the current rule set in place. As a result, the optimal inflation target and degree of inertia are somewhat between the optimal parameters if each regime occurred in isolation without switching behavior.

One possible caveat to switching monetary policy rules due to structural change is that clearly identifying changes in the growth regime may be difficult. If the monetary authority misidentifies the regime and puts the wrong rule into place, such an outcome could be costly from a welfare perspective. Under these circumstances, setting an optimal constant rule that is not susceptible to the error of misidentifying the growth regime could be preferred. However, the paper shows that an optimal rule that switches can outperform a constant rule, even when the incidence of misidentification is relatively high. This result then suggests that providing flexibility for a monetary policy rule can generate gains, even if flexibility sometimes generates errors.

Much of the literature on optimal monetary policy with simple rules assumes constant rules over time. For example, Schmitt-Grohe and Uribe (2007) characterize optimal simple rules in an economy without instability in the structural economy, and show these rules nearly replicate welfare achieved by a Ramsey planner. In the context of regime switches in the structural economy, this paper shows simple rules that switch alongside the structural economy welfare dominate fixed rules.

More recently, there has been considerable discussion about whether to change inflation targets in light of lower growth and real interest rates. For example, Rogoff (2008), Blanchard et al. (2010), and Ball (2013) all call for higher inflation targets in the United States. Billi (2011), Coibion et al. (2012), Dordal-i-Carreras et al. (2016) and Blanco (2018) consider the optimal inflation target in the presence of the zero lower bound on the nominal interest rate. In these cases, the monetary authority sets a constant inflation target that weighs the costs of higher inflation against the chance of hitting the zero lower bound. The optimal simple rules considered in this paper allow this trade-off to be regime-dependent, which enables the authority to set a higher inflation target in regimes where hitting the zero bound is relatively more likely.

Papers studying switching in monetary policy rules often have non-optimal switches that are independent of any underlying changes in the economy. For example, Davig and Leeper (2007) and Bianchi (2013) consider switches in the coefficients dictating how the monetary authority responds to deviations from its targets. Schorfheide (2005) and Liu et al. (2011) allow for switches in the inflation target, and Foerster (2016) considers both types. However, in each of these frameworks, changes in the monetary policy rule occur randomly and without regard to the state of the private economy. In contrast, this paper motivates regime switching in the policy rule as an optimal response to switches in the private economy.

In the case where papers consider optimal policy with regime switching in the private economy, the monetary authority may face a reduced-form representation of the structural economy as in Blake and Zampolli (2011). On the other hand, Debortoli and Nunes (2014) interpret regime switching in monetary policy as coming from explicit changes in the authority's loss function. Davig (2016) shows how regime switches in price-setting behavior in the structural economy map into switches in the loss function when the authority operates with discretion. In contrast to these frameworks, this paper considers optimal simple rules, and how changes in the structural economy affect the optimal choice of policy parameters.

The remainder of the paper proceeds as follows: Section 2 presents the model, Section 3 discusses computation of the optimal policy rules, Section 4 shows results for optimal policy rules without regime switching, Section 5 considers optimal policy rules in the presence of regime switches, Section 6 studies the effects of monetary policy errors, and Section 7 concludes.

2   Model

This section describes a prototypical New Keynesian model. The six key features of the model are: (i) nominal rigidities without indexation of prices that create a role for inflation stabilization near price stability, (ii) a markup shock that generates a trade-off between output and inflation stabilization, (iii) a preference shock that affects the inter-temporal decisions of the household, (iv) regime switching that affects the growth rate of the economy, (v) government spending

financed solely by lump-sum taxes, and (vi) a monetary authority with only access to a Taylor rule for setting nominal interest rates.

The following presents the model's several parts: households, final and intermediate goods firms, fiscal policy and the aggregate resource constraint, the monetary authority, and how regimes switch.

2.1 Households

Households maximize lifetime expected discounted utility of the form

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t d_t \left(\log\left(C_t/A_t\right) - \frac{\phi}{1+\phi} H_t^{\frac{1+\phi}{\phi}} \right) \tag{1}$$

where $\mathbb{E}_0$ is the expectations operator conditional on information at time 0, $\beta \in (0,1)$ is the discount factor, C_t is consumption, A_t the level of technology, H_t is hours worked, and ϕ denotes the Frisch elasticity. The inter-temporal preference shifter d_t follows

$$\log d_t = \rho_d \log d_{t-1} + \sigma_d \varepsilon_{d,t}. \tag{2}$$

Households face the budget constraint

$$C_t + \frac{B_t}{P_t} + T_t = W_t H_t + R_{t-1} \frac{B_{t-1}}{P_t} + D_t, \tag{3}$$

where B_t denotes bonds purchased at time t that pay out a gross nominal interest rate R_t at $t+1$, T_t is real lump-sum taxes paid to the government, W_t is the real wage rate, and D_t is real dividend payments from firms.

Standard optimality conditions for the household produce an Euler equation of the form

$$\beta \mathbb{E}_t \left(\frac{C_t}{C_{t+1}} \right) \left(\frac{d_{t+1}}{d_t} \right) \frac{R_t}{\Pi_{t+1}} = 1, \tag{4}$$

which highlights how, all else equal, shocks or regime shifts that decrease the growth rate of consumption will increase the term C_t/C_{t+1} and hence will tend to lower the nominal interest rate. Likewise, given the autoregressive process in equation (2), negative realizations of $\varepsilon_{d,t}$ will tend to increase d_{t+1}/d_t, which, all else equal, will then tend to lower R_t.

2.2 Firms

There are two types of firms: intermediate goods firms that produce with labor, and final good firms that bundle intermediate goods into a final output to be consumed by households and the government.

2.2.1 Final Good Firms

A competitive final good producer combines a continuum of intermediate goods $Y_{j,t}$, $j \in [0,1]$, by a constant elasticity of substitution technology to produce a final good

$$Y_t = \left(\int_0^1 Y_{j,t}^{\frac{1}{1+m_t}} dj \right)^{1+m_t}, \tag{5}$$

where m_t denotes the time-varying net markup. This specification implies the demand for a good $Y_{j,t}$ depends on its relative price, the markup, and aggregate demand by

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\frac{1+m_t}{m_t}} Y_t. \tag{6}$$

The net markup m_t follows an autoregressive process

$$\log m_t = (1 - \rho_m) \log m_{ss} + \rho_m \log m_{t-1} + \sigma_m \varepsilon_{m,t}. \tag{7}$$

Positive markup shocks $\varepsilon_{m,t}$ produce opposite movements in inflation and the output gap, generating a trade-off for their stabilization.

2.2.2 Intermediate Goods Firms

Intermediate goods producers are indexed by j and have production functions

$$Y_{j,t} = A_t H_{j,t}, \tag{8}$$

where total factor productivity nests stationary and unit root components:

$$\log A_t = \log \omega \left(s_t \right) + \log A_{t-1} + a_t, \tag{9}$$

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_{a,t}. \tag{10}$$

In this case, the mean growth rate of TFP, $\omega(s_t)$, switches with the regime variable s_t to be discussed in detail shortly.

Intermediate goods firms adjust prices according to Rotemberg pricing without indexation of prices to inflation. Consequently, the firm's maximization problem is to choose $H_{j,t}$ and $P_{j,t}$ to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0 P_t} \left(\left(\frac{P_{j,t}}{P_t} \right)^{-\frac{1}{m_t}} Y_t - W_t H_{j,t} - \frac{\gamma}{2} \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 Y_t \right) \tag{11}$$

where λ_t denotes the marginal utility of consumption for the household, and γ determines the cost of adjusting prices. Firms are also subject to the constraint that supply (8) must meet demand (6) at the posted price. The price adjustment generates a cost of inflation in output terms, as deviations from price stability generate progressively higher losses in output.

2.3 Fiscal Policy and Aggregate Resource Constraint

The government purchases a fraction ζ_t of aggregate output Y_t,

$$G_t = \zeta_t Y_t, \tag{12}$$

where the fraction of goods purchased satisfies $g_t = \frac{1}{1-\zeta_t}$, and g_t follows an autoregressive process

$$\log g_t = \left(1 - \rho_g \right) \log g_{ss} + \rho_g \log g_{t-1} + \sigma_g \varepsilon_{g,t}. \tag{13}$$

The government collects lump-sum taxes to cover spending, and nominal bonds are in zero net supply. Importantly, fiscal policy does not have access to a production subsidy to firms that eliminates the distortions associated with monopolistic competition; the inefficiency from imperfect competition must be taken into account by the monetary authority when setting policy (Woodford (2003)).

The aggregate resource constraint is given by

$$Y_t = C_t + G_t + \frac{\gamma}{2} \left(\Pi_t - 1 \right)^2 Y_t. \tag{14}$$

This resource constraint again highlights the cost of inflation, as deviations from price stability produce losses in output that cannot go to consumption or the government. Moreover, this cost

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is high when output is high, so changes in the growth rate of technology have implications for the costliness of price changes.

2.4   Monetary Policy

The monetary authority sets the nominal interest rate using a Taylor rule of the form

$$\frac{R_t}{R_t^*} = \left(\frac{R_{t-1}}{R_t^*}\right)^{\rho_r(s_t)} \left(\left(\frac{\Pi_t}{\Pi^*(s_t)}\right)^{\psi_\pi} \left(\frac{\tilde{Y}_t}{\tilde{Y}_{ss}}\right)^{\psi_y}\right)^{1-\rho_r(s_t)}. \tag{15}$$

This form allows for regime switches in the inflation target $\Pi^*(s_t)$ and the degree of interest rate inertia $\rho_r(s_t)$, which change according to the regime s_t. The rule has a constant responsiveness to the inflation and output deviations from steady state, ψ_π and ψ_y, respectively. The output deviation is in terms of de-trended output $\tilde{Y}_t = Y_t/A_t$ from its steady state value $\tilde{Y}_{ss}$. The reasons to require ψ_π and ψ_y to remain fixed across regimes is that optimal policy typically places significant weight on inflation stabilization around target, and a low weight on the output gap, and these incentives are similar across regimes.[1]

Note that the above has a time-varying neutral nominal rate R_t^*, which follows

$$R_t^* = \Pi^*(s_t)\frac{\omega(s_t)}{\beta}, \tag{16}$$

and is made up of the inflation target, $\Pi^*(s_t)$, and the steady state real rate that would prevail if each regime occurred in isolation, which is in turn a function of the growth rate of the economy.

2.5   Regime Switching

As noted, the structural economy experiences switches in the growth rate of TFP, $\omega(s_t)$. The variable s_t denotes the growth regime, and takes one of two values $s_t \in \{L, H\}$, where L denotes low growth and H denotes high growth. The regime follows a Markov processes governed by a

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[1]Analysis of the model that allowed switching inflation and output responses found the inflation response went to the maximal permitted value and the output response was low, and these did not change across regimes. Thus, this paper focuses on the policy parameters that have meaningful differences across regimes.

transition matrix with elements $p_{ij} = \Pr(s_t = j | s_{t-1} = i)$:

$$P = \begin{bmatrix} p_{LL} & 1 - p_{LL} \\ 1 - p_{HH} & p_{HH} \end{bmatrix}.$$

At the same time, the monetary authority switches the parameters $\rho_r(s_t)$ and $\Pi^*(s_t)$ to reflect the new growth regime.

3 Computation and Welfare

Given the model previously described, this section turns to the calibration and solution method, how the monetary authority sets optimal implementable rules, and the welfare calculations.

3.1 Calibration and Solution

The set of parameters shown in Table 1 describe preferences and production assuming the unit of time is a quarter. These parameters largely follow the estimates in a similar model by Schorfheide (2005).

For the growth processes, the parameterization is motivated by Fernald (2012), who notes that productivity growth has switched between periods of high average growth rates of around 2 percent per year, and periods of low average growth rates of around 0.5 percent per year. The transition probabilities are picked so that each regime has expected duration of 12.5 years, which is roughly in line with the durations of the subsamples considered by Fernald (2012).

More broadly, the growth regimes capture concerns about switches to slower growth regimes (for example, Gordon (2012)), as well as a lower real interest rate (Summers (2014)). The presence of growth rate switches affects the household's Euler equation (4) and the intermediate goods firms' problem via technology (8). In addition, lower growth, by affecting the real interest rate, also changes the neutral nominal rate in the monetary policy rule (16). In the low growth regime, the real rate tends to be low, which corresponds to a lower–and hence a greater chance of negative–nominal rate.

Table 1: Fixed Parameters

Description	Parameter Value
Discount Factor	$\beta = 0.9987$
Steady State Fraction of Government Purchases	$\zeta_{ss} = 0.2$
Steady State Net Markup	$m_{ss} = 0.10$
Cost of Price Adjustment	$\gamma = 100$
Technology Shock Persistence	$\rho_a = 0.8$
Government Spending Shock Persistence	$\rho_g = 0.9$
Inter-temporal Preference Shock Persistence	$\rho_d = 0.9$
Markup Shock Persistence	$\rho_m = 0.9$
Technology Shock Std Dev	$\sigma_a = 0.003$
Government Spending Shock Std Dev	$\sigma_g = 0.01$
Inter-temporal Preference Shock Std Dev	$\sigma_d = 0.01$
Markup Shock Std Dev	$\sigma_m = 0.01$
Growth Rate, High Growth Regime	$\omega(1) = 1.020^{1/4}$
Growth Rate, Low Growth Regime	$\omega(2) = 1.005^{1/4}$
Transition Probabilities	$p_{LL} = p_{HH} = 0.98$
Monetary Policy Inflation Response	$\psi_\pi = 3$
Monetary Policy Output Response	$\psi_y = 0.25$

Given the switching in the structural economy and the monetary policy rule, as well as a need to perform welfare calculations, the results in Section 4-6 use the perturbation method for Markov-switching DSGE models from Foerster et al. (2016). This methodology allows for second-order approximations to the decision rules, which enable accurate welfare calculations and capture the certainty non-equivalence generated by Markov-switching, especially in the monetary policy rule (see Foerster (2016)). In addition, perturbation allows for checking existence and uniqueness of the equilibrium generated by the monetary policy rule, which is key for policy to be implementable.

3.2 Optimal Implementable Rules

Given the economic structure, the monetary policy sets policy parameters for the Taylor rule given by equation (15). First, they set the inflation response to $\psi_\pi = 3.0$ and the output gap response to $\psi_y = 0.25$, and does not change these across regimes. This parameterization reflects the fact that optimal policy rules typically place high weight on inflation stabilization and low weight on output stabilization, and this incentive is largely unaffected by regime change.[2] More substantively, the monetary authority can set the degree of inertia $\rho_r(s_t)$ and the inflation target $\Pi^*(s_t)$ across regimes. These two parameters have important implications across regimes, since the inflation target affects the level of inflation and hence the level of the nominal rate, whereas the degree of inertia affects the volatility of the nominal rate and hence of inflation and output.

When setting the policy parameters for each regime, $\rho_r(s_t)$ and $\Pi^*(s_t)$, the monetary authority sets them to be optimal within the class of implementable rules.

Definition 1 (Implementable) *For a policy rule to be implementable, it must meet the following two conditions:*

1. *Existence and Uniqueness of Equilibrium: The policy parameters generate a unique equilibrium when considering mean square stability (MSS) of minimum state variable (MSV) solutions.*

2. *Non-Negative Rate Dynamics: The stochastic steady state admits non-negative dynamics for the net nominal interest rate. If $\boldsymbol{\mu}_R$ and $\boldsymbol{\sigma}_R$ denote the mean and standard deviations for the log of the gross interest rate, respectively, then this condition requires $\boldsymbol{\mu}_R - 2\boldsymbol{\sigma}_R \geq 0$.*

This definition of implementable policy largely follows Schmitt-Grohe and Uribe (2007), but adapted to the framework with regime switching.

Condition (1) requires that the policy parameters across regimes produce a unique equilibrium when considered as a whole. The MSS concept allows temporarily explosive regimes

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[2]As previously noted, prior analysis of the model that also allowed these parameters to change across regimes found little incentive to do so.

as long as the entire system has finite first and second moments in expectation. Under these circumstances, satisfying equilibrium existence and uniqueness regime-by-regime is neither necessary nor sufficient for achieving existence and uniqueness overall, and the associated parameter regions can be complex functions of all the policy parameters (see Davig and Leeper (2007) and Foerster (2016)).

Condition (2) imposes that, in the longer-run distribution of the economy, hitting a negative net nominal interest rate should be at least a two standard deviation event, which ensures a sufficiently low probability of that occurring. Note that the perturbation solution approach allows negative dynamics, and this condition requires a low volatility relative to the average value. From a technical standpoint, perturbation does not easily handle the occasionally binding zero lower bound constraint; an alternative would be to solve the model globally (for example, Coibion et al. (2012)), but given switching in the monetary policy rule, this procedure would not handle uniqueness well (see Richter et al. (2014)).

From an economic standpoint, recent experience in Europe and Japan shows that policy rates can indeed go below zero, and there may be an effective lower bound that is negative. Perturbation has the benefit of not explicitly taking a stand on a hard bound for the nominal rate, by simply seeking to limit the probability that the nominal rate goes below zero. In addition, the model abstracts from policy strategies at the zero lower bound such as forward guidance and asset purchases that may lessen the direct impact of zero lower bound episodes. These additional tools can be viewed as providing additional accommodation that pushes rates below zero (Krippner (2013), and Wu and Xia (2016)). Rather than model these features, Condition (2) implies that these additional tools might be costly in some way so that the monetary authority wishes to avoid them.

Among the set of policy parameters that generate an implementable rule, the monetary authority chooses those that are optimal in the sense that they maximize the household's expected lifetime utility.

Definition 2 (Optimal) *For an implementable policy to be optimal, it must maximize the*

household's value function

$$\mathbb{E}\left[V_0\right] = \mathbb{E}\sum_{t=0}^{\infty} \beta^t d_t \left(\log\left(C_t/A_t\right) - \frac{\phi}{1+\phi} H_t^{\frac{1+\phi}{\phi}} \right). \tag{17}$$

By using the household's preferences, the monetary authority is making optimal policy from the household's view, rather than holding its own objective function.[3] To compare welfare across rules, the results below compute consumption unit equivalent losses relative to the zero-inflation steady state.

4   Optimal Monetary Policy without Regime Switches

Before analyzing the economy with switches between growth regimes, this Section first analyzes optimal monetary policy rules without switching. The model therefore has two separate versions: one in which the average growth rate is always high, and one in which the average growth rate is always low. Comparisons between the optimal policy and its economic effects illustrate the trade-offs the monetary authority faces when setting its policy parameters.

Given the framework discussed in Section 2, the monetary authority faces a number of trade-offs when setting policy. First, the presence of nominal rigidities without price indexation implies a need to keep inflation low and stable; in particular, exact price stability minimizes the resource cost from inflation in equation (14). However, given that the policy instruments are only lump-sum taxes and a nominal interest rate rule and not a production subsidy to eliminate the distortion from monopolistic competition, the first-best inflation rate will not necessarily be zero (Schmitt-Grohe and Uribe (2007)). In the case of non-zero inflation targets, the model does not contain features such as quality improvements or heterogeneity among individuals that might warrant higher inflation targets, so these inflation targets will tend to be low in

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[3]The monetary authority uses the unconditional expectation of welfare rather than using a single point and regime combination as an initial condition. An alternative would be to condition the starting welfare on each of the two growth regimes, and some point for the state variables. The specific parameters chosen tend to be numerically different–which highlights possible time-inconsistencies in choosing optimal rules–but gives qualitatively similar results.

Figure 1: Implementable Rules and Welfare without Regime Switching
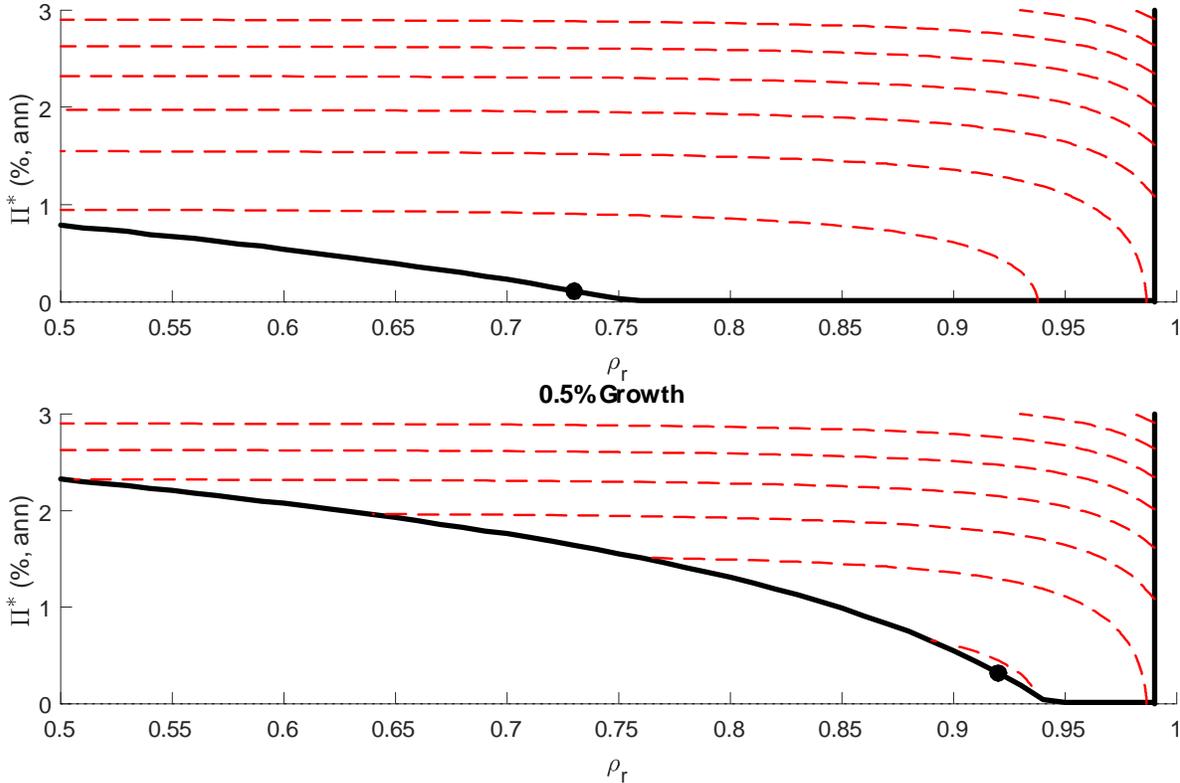
Note: Black solid line shows boundary of implementable rules, red dashed lines show contours of welfare for implementable rules that are decreasing in both parameters, dot shows optimal implementable rule.

absolute terms (Schmitt-Grohe and Uribe (2010)). Second, the presence of a markup shock generates opposing co-movement between inflation and the flexible-price output gap, which in turn creates a trade-off for stabilization policy. Third, the constraint for non-negative dynamics for the nominal interest rate imposed by the implementability condition creates incentives to increase the nominal rate and limit how much it fluctuates; the monetary authority will tend to want to raise the inflation target or make the nominal rate more inertial. These incentives work in contrast to the ability to keep inflation low and stable.

Figure 1 shows the trade-off the monetary authority faces in setting the policy parameters in each growth regime. First, considering the case of 2% growth, the top panel of Figure 1 shows a negative relationship between the inflation target and the degree of interest rate inertia that

allow for implementable rules. In this case, the real interest rate is relatively high due to the high growth rate, which tends to push up the nominal interest rate. At a zero inflation target, if the interest rate inertia is too small (less than around 0.75), then the interest rate fluctuates strongly to current conditions, which in turn makes nominal rates exhibit negative rate dynamics too frequently. In order to diminish negative rate dynamics, the monetary authority can either increase the inflation target, which increases the level of the nominal rate directly, or raise the degree of interest rate inertia, which lessens the volatility of interest rates. On the other hand, an interest rate inertia of 1 implies an interest rate that is pegged, and hence generates an indeterminate equilibrium and the rule is not implementable.

In addition to the consideration of whether a parameter configuration produces an implementable rule or not, an optimizing monetary authority must take into account the welfare of possible implementable rules. The contour lines in Figure 1 show that, at moderate levels of inertia, interest rate inertia tends to not have an impact on welfare, whereas changes in the inflation target have more drastic effects. This result is due to the fact that changes in inertia mainly impact the volatility of the nominal rate and hence inflation and output, which have relatively minor effects on welfare; on the other hand, changes in the inflation target mainly impact the level of inflation and the distortion in output that it causes, which have a relatively larger effect on welfare. As ρ_r increases towards unity, the interest rate rule becomes more like a peg, and increases in volatility become larger and eventually produce a more substantial welfare loss than changes in the inflation target.

The above discussion therefore illustrates why the monetary authority will end up picking an optimal rule with parameters, shown in by the dot in Figure 1 and listed in Table 2, of $\rho_r = 0.73$ and $\Pi^* = 0.11$. In order to have an implementable rule, it must have either a positive inflation target or a high degree of inertia. Changes in inertia have relatively low welfare impact at moderate levels, leading the monetary authority to pick a slightly inflationary target and a moderate degree of inertia.

Now turning to the case with lower growth, the bottom panel of Figure 1 shows that the boundary for what constitutes an implementable rule shifts upwards. The lower growth produces

Table 2: Optimal Policy Rules and Outcomes without Regime Switching

	Parameters		Economic Performance (Mean / Std)		Welfare Loss
Regime and Rule	ρ_r	Π^*	R_t	Π_t	(Normalized)
High Growth Regime					
Optimized High Rule	0.73	0.11	3.2634 (1.6269)	0.0924 (0.7767)	1.00
Optimized Low Rule	0.92	0.32	3.4675 (0.9896)	0.3036 (1.0547)	1.48
Low Growth Regime					
Optimized Low Rule	0.92	0.32	1.9814 (0.9896)	0.3036 (1.0547)	1.00
Re-Optimized Target	0.73	1.64	3.2958 (1.6446)	1.6104 (0.7730)	1.90

Note: Welfare Loss is relative to the zero inflation steady state, and normalized in each regime so the optimized rule is 1.

a lower real interest rate, which, all else equal, in turn lowers the nominal rate and increases the incidence of negative rate dynamics. As a result, the monetary authority needs to either raise the inflation target or increase the degree of interest rate inertia in order to produce an implementable rule. The welfare contours in this case are similar in shape to the higher growth case, and as a result the optimal rule sets $\rho_r = 0.92$ and $\Pi^* = 0.32$, parameters that are quite different from the high growth case.

A comparison of the high and low growth cases in Figure 1, along with a comparison of economic performance of the associated rules in Table 2, yields important implications for monetary policy. If the economy moves from the high to the low growth case, it must change its rule in some dimension, as the high growth rule is not implementable in the low growth regime. From an optimal policy perspective, it is important for the monetary authority to re-optimize both the inflation target and the degree of inertia. The trade-off between inertia and the inflation target implies that only re-optimizing on the inflation target dimension, which has been the subject of much study and discussion, can produce inferior economic outcomes.

Table 2 shows that, if the economy moves from the high to the low growth regime, while keeping in place the optimal inertia from the high growth rule and only changing the inflation

target–as many policy makers tend to advocate–the inflation target would have to be dramatically increased by 1.32 percentage points over the optimal value, to 1.64 percent. This increase in the inflation target would move the nominal rate up in order to decrease the incidence of negative rate dynamics and keep the rule implementable. Such a change would lead to sub-optimally high inflation with a too-low volatility. As a result, the welfare losses–relative to the zero inflation steady state–would be nearly double the losses of the optimal rule. In the event of a change from high to low growth, therefore, the monetary authority would need to consider the trade-off between inertia in its rule and the inflation target, rather than focusing solely on a re-optimization of the inflation target.

A change in the other direction, from low growth to high growth, likewise requires a re-optimization of the policy parameters. However, in this case, the need to re-optimize is less apparent because the low growth rule is implementable in the high growth regime. Instead, a monetary authority that did not re-optimize would simply find a higher real interest rate pushing up the nominal rate without any difference in inflation dynamics from the low growth regime. These dynamics would have sub-optimally high inflation that was too volatile, producing a welfare loss about 50 percent higher than if the inflation target and the inertia were at their optimal values.

Lastly, in the presence of monetary policy re-optimization, a change in the growth regime naturally produces different dynamics for inflation and the nominal rate. In the low growth regime, optimal parameters lead to higher, more volatile inflation than in the high growth regime. The nominal rate should also tend to be lower and less volatile under the optimal policy rule when growth is low.

While the above discussion shows how optimal policy rules differ when growth is either low or high, it considers changes in the growth rate, and hence changes in monetary policy, as complete surprises. Given that regime switches in growth do occur and repeat themselves, it is natural for households and firms to build expectations about regime changes into their decision-making process and hence alter macroeconomic dynamics. Setting policy coefficients must then take these changes into account; the next section turns to addressing this issue.

5   Optimal Monetary Policy Regime Switches

This Section now turns to an analysis of setting optimal monetary policy rules in the presence of regime switching between high and low growth regimes. In this full version of the economic model, the growth rate changes between high and low average growth rates, which in turn affects macroeconomic dynamics directly but also through expectational effects. That is, when the economy is in the high growth regime, households and firms will internalize the possibility of lower growth in the future, which affects their desire to consume, supply labor, and set prices; similarly, in the low growth regime, the expectation of a possible shift to higher growth will affect decision making.

In addition to the trade-offs of low and stable inflation, stabilizing the output gap, and having non-negative nominal rate dynamics as discussed in the previous section, the monetary authority faces an additional trade-off with regime switches. Specifically, the presence of regime switches–in both the private economy and possibly the monetary policy rule–generate equilibrium existence and uniqueness considerations, as well as expectational effects (Davig and Leeper (2007), Liu et al. (2009), and Foerster (2016)) that the monetary authority must internalize when setting optimal policy. The interaction between regimes can imply that the parameters in one regime could possibly imply non-uniqueness if that regime occurred in isolation, but switches between regimes mean the overall equilibrium is unique. High values of interest rate inertia in one regime produce peg-like behavior that can possibly produce indeterminacy in economies without regime switching; the regime-switching setup implies that it is possible for parameter configurations that produce indeterminacy in one regime when considered in isolation can produce determinacy when considered across regimes. In addition, expectational effects of regime switching alter the equilibrium outcomes based on how policy switches, and the monetary authority must take these effects into account. For example, in the case when the inflation target may switch between regimes, firms and households will internalize this switching, leading to differences in behavior and hence realized inflation relative to the case when regime changes do not exist. Under these circumstances, realized inflation in each regime may differ from that regime's inflation target, and the authority sets policy knowing this result will occur.

5.1 Optimal Constant Rule

First, consider the case where the growth rate changes between high and low growth regimes, but the monetary authority sets a constant rule. In this case, they set the nominal rate according to a fixed-parameter version of equation (15):

$$\frac{R_t}{R_{ss}^*} = \left(\frac{R_{t-1}}{R_{ss}^*}\right)^{\rho_r} \left(\left(\frac{\Pi_t}{\Pi^*}\right)^{\psi_\pi} \left(\frac{\tilde{Y}_t}{\tilde{Y}_{ss}}\right)^{\psi_y}\right)^{1-\rho_r}. \tag{18}$$

For this rule, the interest rate inertia and inflation target are unchanged when the growth regime changes, and the monetary authority sets the nominal rate according to deviations from the steady state nominal rate $R_{ss}^* = \Pi^* \bar{\omega}/\beta$, where $\bar{\omega}$ is the ergodic mean of $\omega(s_t)$. In other words, the monetary authority does not adjust the real interest rate implicit in its neutral nominal rate as the regime changes, but instead targets an average real rate plus its inflation target.

Figure 2 shows the trade-off between setting an inflation target and the interest rate inertia in the regime-switching case with a constant monetary policy rule. Similar to the case without regime switching, the monetary authority faces a choice between increasing the inflation target and raising the degree of inertia. The exact nature of the trade-off in this case depends on dynamics in both regimes. In particular, the boundary for the implementability region is between the boundaries for the fixed regime cases. Intuitively, for a fixed inertia choice, the monetary authority needs to have a higher inflation target than if the high growth regime occurred in isolation, but can support a lower target than if the low growth regime only occurred. This result is due to the fact that switching between high and low real interest rates will tend to push the nominal rate down during the low regime, but this regime only occurs half the time. The welfare trade-off between inertia and the inflation target are similarly shaped to the no switching case, but the set of implementable rules restricts the parameter space to the high end of the inertia interval, where the trade off is strongest. The optimal policy parameters in this case, shown in Table 3 as well, are $\rho_r = 0.85$ and $\Pi^* = 0.23$.

Figure 2: Implementable Constant Rules and Welfare with Regime Switching
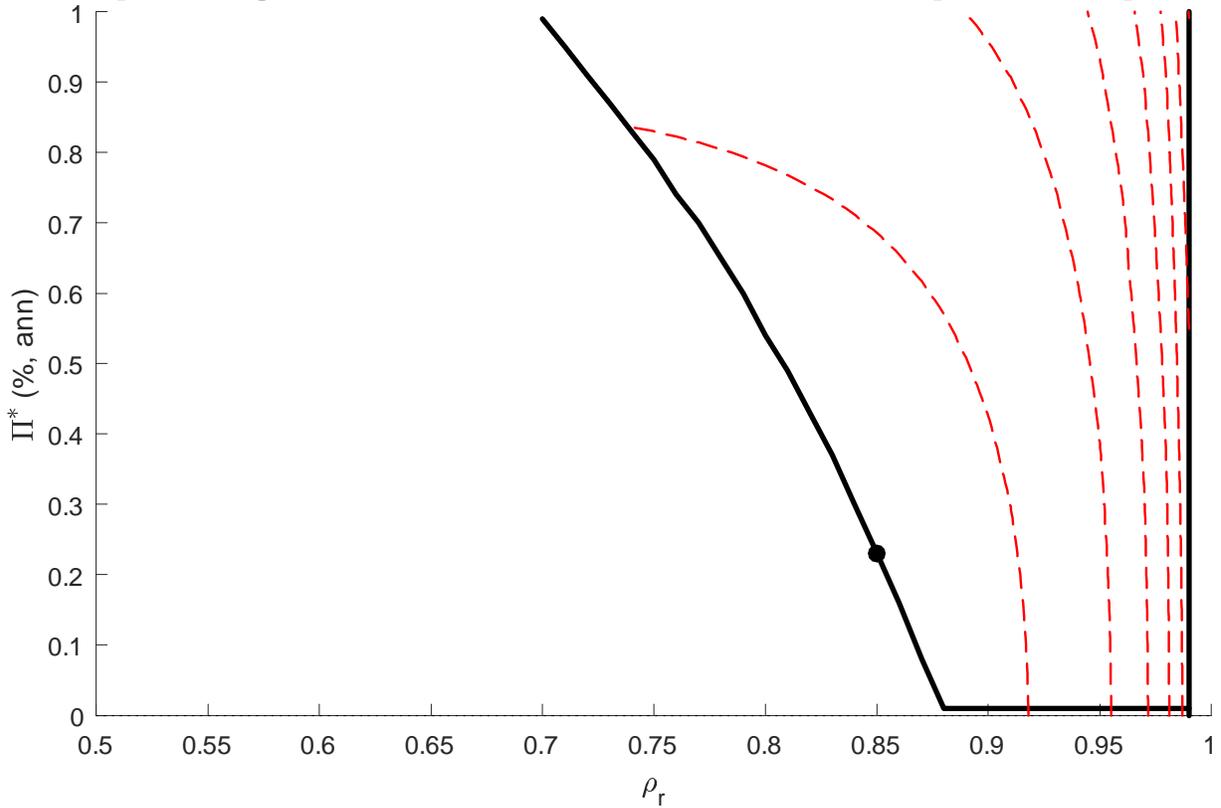
Note: Black solid line shows boundary of implementable rules, red dashed lines show contours of welfare for implementable rules, dot shows optimal implementable rule.

5.2   Optimal Switching Rule

Table 3 also shows the optimal parameters when the monetary policy rule switches across regimes and the real interest rate used to determine the neutral nominal rate changes across regimes as well. The optimal parameters are quite close to the constant rule parameters: the degree of inertia does not change across regimes, while there is slight movement in the inflation target. With four parameters to optimize over, both the set of implementable rules and the welfare of these rules become multi-dimensional objects. Figure 3 shows slices of these objects, keeping two parameters fixed at their optimal values.

The top two panels show that the trade-off between inertia and inflation targets within a regime remains similar to the no regime switching and constant rule cases. The lower panels show

Table 3: Optimal Policy Rules and Outcomes with Regime Switching

| Rule Specification | Parameters | | Economic Performance (Mean / Std) | | Welfare Loss |
	$\rho_r(s_t)$	$\Pi^*(s_t)$	$R_t\|s_t$	$\Pi_t\|s_t$	(Normalized)
Naive Switching Rule					0.98
Low Growth $s_t = L$	0.92	0.32	2.0590 (1.0091)	0.3291 (1.0691)	
High Growth $s_t = H$	0.73	0.11	3.0857 (1.5607)	0.0256 (0.7474)	
Constant Rule					1.00
Low Growth $s_t = L$	0.85	0.23	1.7373 (1.3149)	-0.0721 (0.8754)	
High Growth $s_t = H$	0.85	0.23	3.5320 (1.3149)	0.4950 (0.8754)	
Switching Rule					0.92
Low Growth $s_t = L$	0.85	0.24	2.0375 (1.3149)	0.2722 (0.8754)	
High Growth $s_t = H$	0.85	0.21	3.2272 (1.3149)	0.1433 (0.8754)	

Note: Welfare Loss is relative to the zero inflation steady state, and normalized so the constant rule is 1.

trade-offs between inertia and inflation targets across regimes. The lower left panel, which shows the interaction between $\rho_r(L)$ and $\rho_r(H)$, shows a curved boundary for implementable rules. Changes in one inertia parameter, say $\rho_r(L)$, have competing effects: lower inertia can makes inflation more volatile within the low growth regime, which can spill over and make inflation in the high growth regime more volatile as well; alternatively, the relatively stronger response to current conditions that comes with lower inertia can dampen the volatility of inflation in the high growth regime. As a result, there is a nonlinear trade-off depending on the strength of these forces, with decreases in $\rho_r(L)$ requiring large increases in $\rho_r(H)$ when $\rho_r(L)$ is high in order to preserve implementability, but smaller ones when $\rho_r(L)$ is lower. Further, with welfare relatively steep at high values of inertia as in the fixed regime case, the incentive for the monetary authority is to push inertia to equal values across regimes

The bottom right panel shows a linear trade-off between the inflation target in both regimes. In the low growth regime, as discussed, the low real interest rate generates an incentive for a higher inflation target than the high growth regime. However, due to expectational effects, the

Figure 3: Implementable Switching Rules and Welfare with Regime Switching
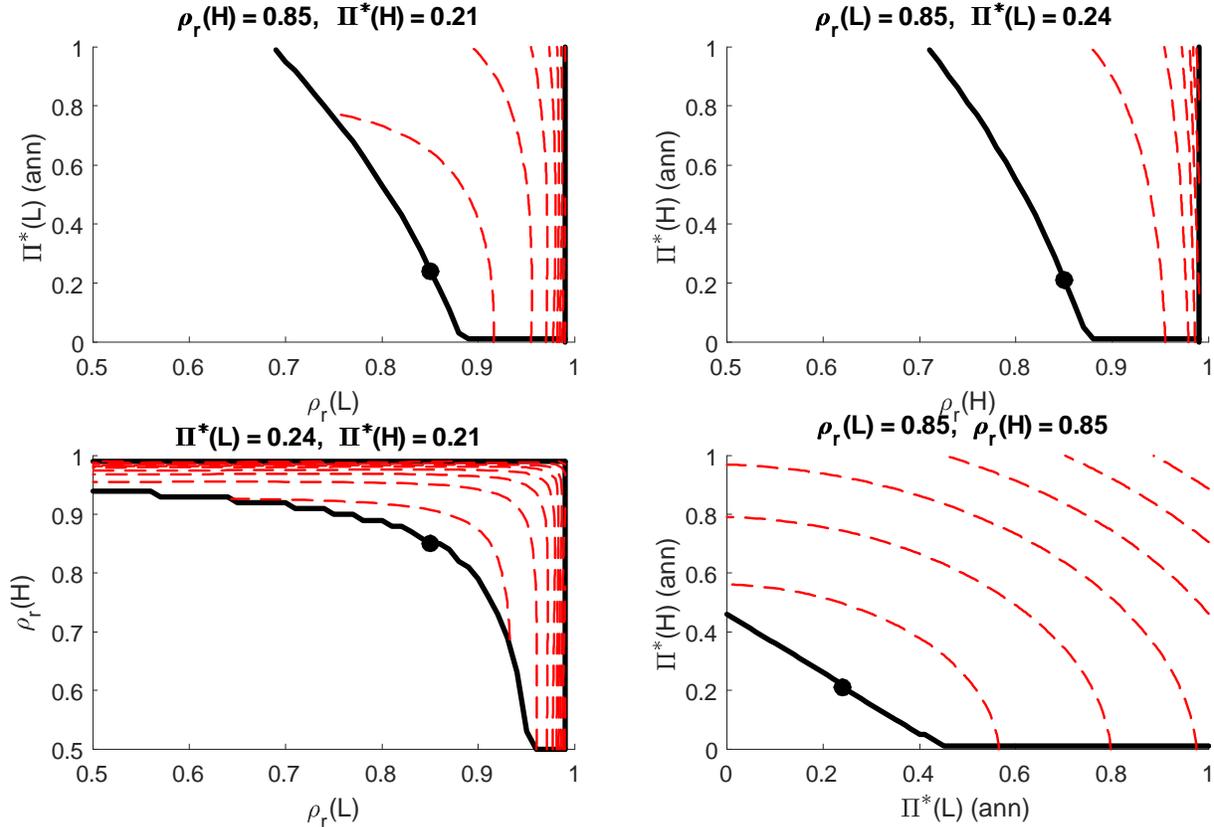
Note: Black solid line shows boundary of implementable rules, red dashed lines show contours of welfare for implementable rules, dot shows optimal implementable rule.

differences are not as large as when regime changes do not occur. Since higher inflation targets produce higher inflation and hence have a cost in terms of lost output, the optimizing monetary authority effectively tries to avoid negative rate dynamics in the low growth regime somewhat indirectly, by changing the inflation target in the high growth regime. In other words, welfare considerations make it optimal to have moderate output losses in both regimes rather than larger losses in the low growth regime.

Figure 3 also implicitly shows the importance of exploiting the trade-off between inertia and the inflation target in both regimes. A monetary authority that was forced to have lower inertia, for example around 0.73 as in the fixed regime, high growth case, would require much larger inflation targets in order to avoid negative rate dynamics too often.

Given the similarities in parameter values in the constant and switching rule specifications, it would be natural to think these rules produce similar dynamics and welfare implications, but Table 3 shows this conclusion is not the case. The reason for the differences in dynamics and welfare comes from the constant versus switching real interest rate portion of the rule, which moves the neutral nominal rate accordingly, affecting inflation and output dynamics. Comparing the two different rule specifications, the changes in the means of the nominal rate and inflation are much larger across regimes in the constant rule case. When the monetary authority uses a constant real rate rather than adjusting it with the regime, the nominal rate will tend to be too high in the low growth regime, which will push down inflation; likewise the nominal rate will tend to be too low in the high growth regime, which will boost inflation. The switching rule, by contrast, will have the neutral nominal rate more aligned across regimes, dampening the level shifts.

It is interesting to note that the mean inflation rates in each regime shown in Table 3 differ from the stated inflation targets, even in the case when the inflation target switches. The monetary authority chooses the inflation target, but expectational effects and precautionary behavior yield average inflation rates that are either above or below target depending on the regime and rule. In the constant rule case, the average inflation rate across regimes is quite close to the inflation target, but in each regime the realized inflation misses the target, with inflation high in the high growth regime and low in the low growth regime due to the effects of the neutral nominal rate. The optimal switching rule, by changing the neutral nominal rate, adjusts the nominal rate by a larger degree when regimes change, which reverses the misses of the inflation target: the lower nominal rate in the low growth regime pushes inflation above target slightly, and the higher nominal rate in the high growth regime pushes inflation below target slightly. From the standpoint of an optimizing monetary authority, these misses are expected and internalized when setting policy.

These considerations produce an implication for the dynamics of inflation and the nominal rate across regimes. Specifically, while both optimal constant and optimal switching rules will have lower nominal rates in the low growth regime than the high growth regime, the implications

for inflation differ. The constant rule produces lower inflation in the low growth regime, whereas the opposite is true for the switching rule. These differences generated by changes in the neutral nominal rate therefore produce different welfare implications. Using the zero inflation steady state as a benchmark, the switching rule produces less welfare loss than the constant rule by around 8%. In fact, even a switching rule with policy parameters optimized in the non-regime switching context welfare dominate the constant rule.

To summarize, the results in this section point to the need to consider cross-regime expectation effects when setting policy parameters. The trade-off between inertia and inflation targets is an important piece of setting optimal policy rules, as higher levels of inertia can substitute for high inflation targets. Further, there are welfare gains from having a rule that switches along with the regime, with the real rate being used in the neutral nominal rate an important factor. A key assumption in this analysis, however, is that it relies on synchronized switching for the monetary policy rule and the growth regime. The next section consider the implications when the monetary policy may misidentify the regime and put the wrong rule into place.

6   The Cost of Monetary Policy Errors

The results in the previous section show that a switching rule can have welfare gains over a constant one, in particular by adjusting the real interest rate used to determine the neutral nominal rate. However, identifying regime changes in technology growth in real time may be difficult, possibly casting doubt on the assumption that the monetary authority switches its policy rule exactly in sync with changes in the growth rate. To address this issue, consider an environment where the economy switches between high and low growth regimes as before, but now the monetary authority correctly identifies the regime and puts the relevant rule in place with probability q. Then, with probability $1 - q$, they make a policy error, either putting the low growth rule into place during the high growth regime, or the high growth rule into place during the low growth regime.[4] When $q = 1$, the monetary policy authority never makes an

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[4]The probabilty q can be thought of as a shorthand for a learning process which allows for incorrectly realized rules in a framework close to the original specification that doesn't resort to requiring a filtering problem by the

Figure 4: Monetary Policy Errors and Relative Welfare Loss
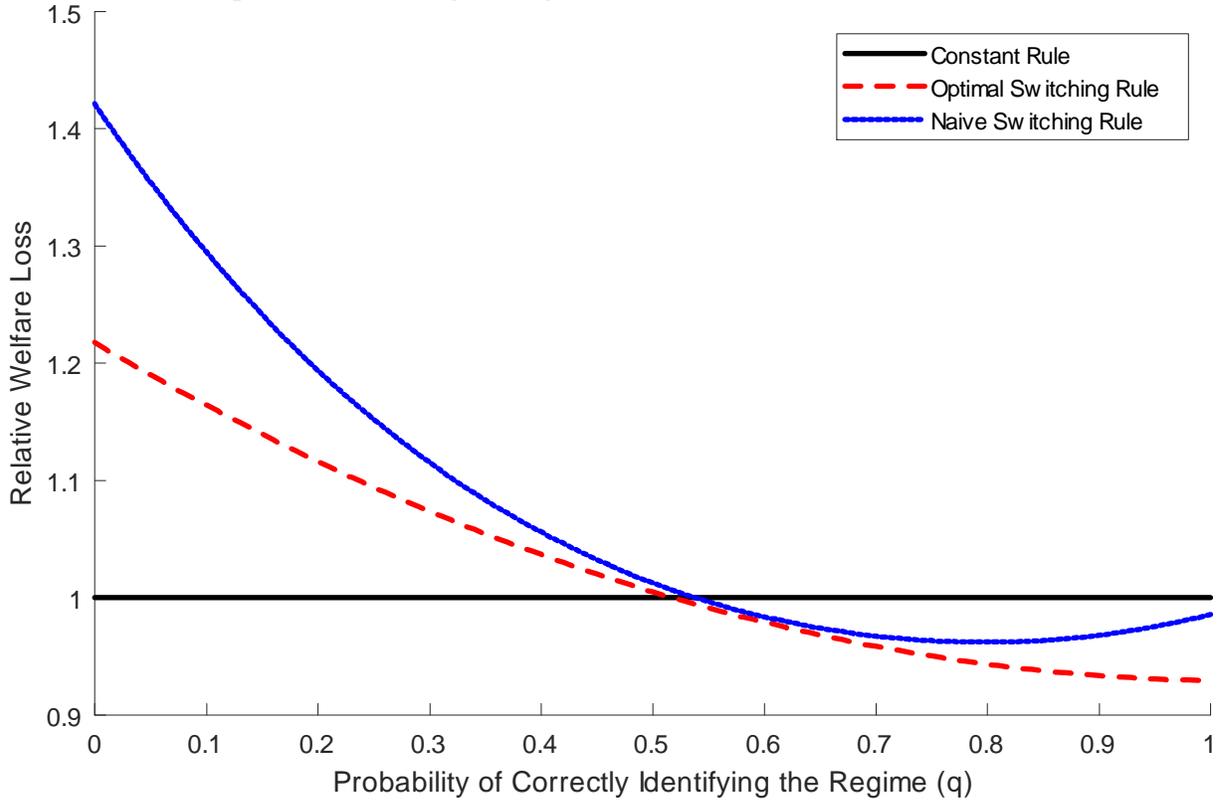
Note: Welfare Loss is relative to the zero inflation steady state, and normalized so the constant rule is 1.

error and the model collapses back to the version studied in the previous section.

Figure 4 shows how the probability of correctly identifying the regime q affects the welfare loss, relative to the zero inflation steady state and normalized by the constant rule loss, for the constant rule, the optimal switching rule, and the naive switching rule shown in Table 3. The constant rule is unaffected by the probability of misidentifying the regime, since it is unresponsive to regime switches. The optimal switching rule, as noted above, achieves about an 8% lower relative welfare when $q = 1$. As the probability of misidentifying the regime increases, the switching rule becomes increasingly less beneficial. This result is due to the feature that the switching rule changes the real interest rate used to determine the neutral nominal rate, so misidentifying the regime throws off the level of the nominal rate. For values of q under 0.5, the

monetary authority.

regime is misidentified with such a high frequency that the constant rule welfare dominates the switching rule. The naive switching rule follows a similar pattern, although due to the fact that it is not optimized when $q = 1$, it actually shows marginal welfare gain as q declines.

The results in Figure 4 therefore imply that the switching rule is relatively robust to regime misidentification. Even though it had been optimized in model where no monetary policy errors were made, it provides sufficient flexibility across regimes to be welfare-preferred relative to a constant rule. The switching rule that responds to changes in the real interest rate thus generates better outcomes than the constant rule, even taking into account that the monetary authority may make mistakes. Of course, if the probability of an error is exceedingly high, the constant rule eliminates the issue of making errors and produces superior outcomes.

7 Conclusion

Motivated by recent arguments for changing the conduct of monetary policy in the presence of regime shifts in the growth rate of the economy, this paper has studied optimal monetary policy rules that switch parameters. These rules differ from the optimal choice under fixed regimes, and setting parameters in a way that considers the trade-off between inertia and the inflation target, as well as the expectational effects of regime switching, is an important aspect of monetary policy rules. The rule that switches whenever the growth rate regime switches outperforms a constant rule from a welfare standpoint, and this outperformance is robust to a relatively high incidence of misidentifying the regime.

The results in this paper therefore have several implications for the conduct of monetary policy when the economy experiences growth rate switches. First, setting a monetary policy rule should be done as a whole rather than focusing on a single aspect of that rule such as the inflation target. Second, monetary policy parameters set during times of high growth or low growth may not be optimal in the presence of regime switching, taking these changes into account and the feedback effects from expectations is crucial in setting optimal rules. Finally, having a flexible rule that switches whenever the growth regime changes can yield superior

macroeconomic outcomes from a constant rule, and the benefits from such flexibility outweigh the costs associated with even moderately frequent errors.

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