Dispersion of Asset Prices – an Axiomatic Definition

The extent to which a set of asset prices tends to diverge over time is of considerable interest to academics and practitioners. But there is no standard definition of dispersion. A definition is offered here which derives from four simple axioms, and its properties are discussed. Viewed in the cross-section of assets, dispersion is closely related to the variance of excess returns against the average return; viewed in the time series it is the difference between the average variance of the assets, and the variance of a portfolio of the assets. A closely related correlation statistic is also derived that provides a simple measure of the degree to which the assets comove.

Keywords: Price dispersion, diversification, correlation risk, axiomatic definition, return comovement

Dispersion of Asset Prices – an Axiomatic Definition

Dispersion – the extent to which a set of asset prices tends to diverge over time – has long been of interest to academic researchers. It has been used to measure the scope for diversification, and to detect the presence of herding behaviour. More recently, there has been mounting evidence that dispersion risk – time variation in the degree of dispersion – matters, and it is priced by investors. It is called by various names - correlation risk, idiosyncratic risk or time varying diversification opportunities – but the concepts are closely related. The way dispersion is measured varies from paper to paper, and the methodology chosen generally lacks a clear theoretical basis.

The purpose of this paper is to develop a definition of dispersion that has a firm theoretical basis. The definition is derived from a set of four primitive axioms. The axioms determine the definition of dispersion uniquely (up to a scaling factor). The axiomatic foundation brings with it simplicity, precision and a set of desirable properties.

With the axiomatic definition, dispersion is independent of the numeraire; the dispersion of a set of currencies is identical whether currency returns are measured against the dollar or some other currency. Dispersion aggregates nicely across portfolios; the dispersion of the stocks in the world market portfolio can be decomposed into the dispersion between national stock market indices and the average dispersion of stocks within each country. The decomposition is exact and requires only that the global market is the aggregate of national markets. The relationship between realized and implied dispersion is simple. With a suitably rich set of European options traded, a dispersion swap can be synthesized. This has important asset pricing implications. It means that implied dispersion can be measured precisely and without specific modelling assumptions; a pure dispersion trade can be synthesized, allowing dispersion risk premia to be measured and analysed. There is a natural decomposition of dispersion into the sum of the contributions of the individual assets so one can readily identify the relative contribution of each asset to the overall dispersion of the market.

An axiomatic definition is only as good as the axioms that determine it. As primitives, the acceptability of the axioms is a subjective matter. I assume a set of assets whose prices are observed at discrete times, say daily. The assets may be equally or unequally weighted. The first axiom is that dispersion over a period is an analytic function of the daily returns over that period and of the vector of weights. The second axiom is that dispersion is additive over time. The third axiom concerns the way dispersion aggregates across assets. The dispersion of a portfolio of portfolios is the sum of two components: the dispersion across the portfolios, and the average dispersion within the portfolios. The final axiom specifies that the dispersion of prices of a set of assets is independent of the currency or numeraire in which the returns are measured.

These axioms lead to the definition of dispersion as the difference between the average log return of the individual assets and the log return of the portfolio of the assets (multiplied by 2). Apart from a multiplicative constant, the definition is unique – no other definition of dispersion satisfies all the four axioms. Dispersion over a single period is twice the log of the ratio of the arithmetic average return to the geometric average return, where the averages are across the individual assets. Using the relationship of the geometric and arithmetic averages as a measure of dispersion is not novel; Norris (1938) argues that it is an efficient estimator of dispersion for a wide class of non-normal distributions. In the continuous time limit, and under the assumption that the price processes are diffusions, our definition converges to the average variance of asset excess returns, where excess returns are measured relative to the portfolio of assets, a measure of dispersion used for example (in very different contexts) by Christie and Huang (1995), and Heston and Rouwenhorst (1994). Interpreting the definition in the time series, dispersion is seen to be the difference between the average variance of the individual assets and the variance of the portfolio of the assets.

To complement the definition of dispersion, I seek an axiomatic definition of correlation. Three axioms are proposed for correlation: a regularity axiom that requires that correlation be a function of weights and returns, with an absolute value less than 1; a time aggregation requirement that the correlation over a period is spanned by the correlations in the sub-periods of which it is composed; and a cross-sectional requirement that if assets are merged or combined into a single asset, correlation rises. None of the definitions of correlation that have been used in the academic literature or by practitioners satisfies all three axioms. An alternative definition is proposed – in effect, the ratio of the variance of the aggregate index to the average variance of the individual assets - that does satisfy them; the uniqueness of the proposed definition is not proven.

The paper is set out as follows. Section 1 contains a brief overview of the prior literature, focusing on the various interpretations of dispersion in research into financial markets, and the different ways in which it has been measured. The second section contains the main contribution of the paper, the axioms and the derivation of the definition of dispersion that satisfies them. Section 3 documents some of the properties of dispersion. Section 4 is devoted to a discussion of a companion definition of correlation, while section 5 concludes.

1. Dispersion in the Academic Literature

The following review seeks to make two points: that the degree to which asset prices move together or apart has been the focus of much research in many different contexts; and that the way that dispersion or correlation has been measured has varied widely, and generally not for strong theoretical reasons. The review is not intended as a comprehensive survey of dispersion in the finance literature, but rather to illustrate these two points.

Pollet and Wilson (2010) show that correlation risk is priced in the equity market. An increase in the average pairwise correlation between stocks positively predicts future stock market returns 3 months and more ahead. The pairwise correlation is a negative measure of dispersion – the lower the correlation, the more dispersed the performance of individual stocks. They see the result through the lens of the Roll Critique; they interpret the changing levels of correlation as reflecting changing volatility in the true market portfolio that is orthogonal to the stock market.

Driessen, Maenhout and Vilkov (2009) argue that dispersion should be priced in equity markets for a rather different reason. Dispersion is time varying, and when dispersion is low, correlation is high, and opportunities to diversify are reduced. This is a bad state of the world, and they show that stocks that do well when dispersion is low have higher expected returns. They measure dispersion risk using options. They construct a portfolio that is short at the money index straddles and long straddles on the individual components; the portfolio is designed to be vega and delta neutral (that is hedged against changes in the individual stock prices and in the individual stock volatilities) and thus be a pure play on the average correlation between the stocks. Buraschi, Kosowski and Trojani (2013) who also use correlation as a measure time-varying diversification opportunities, measure it using the price of over-the-counter correlation swaps which investors can use to speculate explicitly on the average pairwise correlation between stocks in the market index.

Dispersion has been used as a measure of herding behaviour, following Christie and Huang (1995). They compute dispersion as the equally weighted standard deviation of daily returns, and argue that the smaller the dispersion, the greater the herding. While the use of dispersion to measure herding has been criticised (eg by Bikhchandani and Sharma, 2000) it continues to be widely used, though the standard deviation is often replaced by the mean absolute deviation, following Chang, Cheng and Khorana (2000).

Another area where dispersion figures is in the debate over the relative importance of country and industry factors for diversification of risk. Heston and Rouwenhorst (1994) regress monthly stock returns on industry and country dummies cross-sectionally. The coefficients can be interpreted as the returns on single industry and single country portfolios. They use the cross-sectional variance of these country and industry returns as a measure of the relative importance of diversification across the two dimensions.

Interest in dispersion is not confined to the equity market. Menkhoff, Sarno, Schmeling and Schrimpf (2012) show that the profitability of the carry strategy (the strategy of borrowing in low interest rate currencies and lending in high interest rate currencies) can be explained, at least in part, as a risk premium compensating investors for global foreign exchange volatility. They measure global foreign exchange volatility as the equally weighted average absolute daily log return on a set of currencies against the dollar, aggregated over time. This can be seen as a measure of the degree of dispersion amongst currencies.

Mueller, Stathopoulos and Vedolin (2017) also find that correlation risk is priced in currency markets. They construct an FX correlation factor (FXC) and find a negative relation between a currency's loading on FXC and its subsequent return. FXC is computed by taking all possible pairs of currencies from the G10 currencies, ranking them by their correlation, and taking the difference between the average correlation of the top and bottom decile.

Dispersion, or the lack of it, has also been the subject of much debate in the commodity market since Pindyck and Rotemberg (1990) put forward the Excess Comovement (ECM) hypothesis. The ECM states that commodity prices move together far more than is implied by fundamentals. Pindyck and Rotemberg measure the comovement by regressing price returns of seven commodities on a number of macro-economic variables, and analysing the correlations between the residuals. They focus attention on the statistic $|R|^{N/2}$, where $|R|$ is the determinant of the correlation matrix between the residuals and N is the number of observations.

Dispersion and correlation of returns across a set of assets has been the object of study in the academic literature for many years, and they have been measured in a variety of ways. For our purposes it is useful to distinguish between dispersion and correlation. Dispersion we define as a rate, like a variance; like variance, it can be annualized, to produce something akin to a volatility. By contrast, correlation is a pure number with an absolute value bounded by 1. If all assets become more volatile but the relationship between them stays the same, one would expect dispersion to increase while correlation is unchanged.

It turns out to that dispersion is a simpler concept than correlation, so we deal with it first, in section 2. We arrive at an axiomatic definition and proceed to examine its properties in section 3. In section 4 an axiomatic basis for the definition of correlation is proposed, and a candidate definition is put forward.

2. The Axiomatic Definition of Dispersion

The object is to find the dispersion d of a set of S assets over some period $[0, T]$. Asset prices are observed at discrete points in time $0, 1, \ldots, T$, which are not necessarily equally spaced. On occasion, it is convenient to refer to the whole period as a year, and the sub-periods as days, but obviously nothing hangs on this. The returns of the assets are contained in a SxT matrix $\mathbf{R}$ where the element $R_{s,t}$ is the price relative of asset s at time t (ie the price at time t divided by its price at t-1). The vector $\mathbf{r}_t$ is the vector of asset returns at time t, so $\mathbf{r}_t$ is the t'th column of $\mathbf{R}$.

The assets may or may not be of equal importance, so dispersion takes account of the weight of each asset in the whole. The weights are represented by the 1xS vector $\mathbf{q}$ whose element q_s is the weight of asset s. A pair $(\mathbf{q}, \mathbf{R})$ is *admissible* if

1. There exist integers S and T such that the dimensions of $\mathbf{q}$ is 1xS and of $\mathbf{R}$ SxT;
2. The elements of $\mathbf{R}$ are strictly positive;
3. $\mathbf{q}$ is a *weighting vector* in the sense that the elements of $\mathbf{q}$ are positive and sum to 1.

The four axioms are:

AXIOM 1 (*Regularity*): dispersion d is an analytic function of $\mathbf{q}$ and $\mathbf{R}$, so $d = d(\mathbf{q}, \mathbf{R})$ is an analytic function for any admissible $(\mathbf{q}, \mathbf{R})$.

AXIOM 2 (*Time Aggregation*): dispersion d is additive over time, so

$$d(\mathbf{q},\mathbf{R}) = \sum_{t=1}^{T} d(\mathbf{q},\mathbf{r}_t). \qquad (1)$$

AXIOM 3 (*Cross-sectional Aggregation*): dispersion d is additive across portfolios, so

$$d(\mathbf{pQ},\mathbf{r}) = d(\mathbf{p},\mathbf{Qr}) + \mathbf{p}d(\mathbf{Q},\mathbf{r}), \tag{2}$$

for any UxS matrix $\mathbf{Q}$, and 1xU vector $\mathbf{p}$, where $\mathbf{p}$ and the rows of $\mathbf{Q}$ are weighting vectors, and $\mathbf{r}$ is a strictly positive 1xS matrix.

AXIOM 4 (*Numeraire Invariance*): dispersion d is invariant to a change of numeraire, so

$$d(\mathbf{q},k\mathbf{r}) = d(\mathbf{q},\mathbf{r}) \text{ for any scalar } k > 0, \text{ and admissible } \mathbf{q} \text{ and } \mathbf{r}. \tag{3}$$

Axiom 2 stipulates that dispersion cumulates over time. This allows us to focus on the one period case. We now drop the time subscript and look at the properties of $d(\mathbf{q}, \mathbf{r})$, where $\mathbf{q}$ is the weighting vector and $\mathbf{r}$ is the vector of one-period returns.

The third axiom concerns the way dispersion behaves when portfolios of assets are combined. It formalises the intuition that when portfolios of assets are combined, the dispersion of the aggregate portfolio comes from two sources: the intra-portfolio dispersion and the inter-portfolio dispersion.

A concrete example may be helpful. Think of a global equity index. It is typically the weighted average of national indices. The axiom requires that the dispersion of stocks within the global index be the sum of two elements:

- the dispersion between national indices;
- the average dispersion of stocks within national indices.

With this interpretation, axiom 3 envisages a world with U countries and S stocks. The matrix $\mathbf{Q}$ has a typical element $Q_{u,s}$ which represents the weight of stock s in country u. The vector $\mathbf{p}$ has typical element p_u which represents the weight of country u in the world portfolio. The expression $d(\mathbf{Q}, \mathbf{r})$ is interpreted as the Ux1 vector whose u'th element is $d(\mathbf{q}_u, \mathbf{r})$ where $\mathbf{q}_u$ is the u'th row of $\mathbf{Q}$. There is no presumption here that stocks cannot appear in more than one country index.

The final axiom states that the dispersion measure should be independent of the choice of numeraire. The extent to which the world's currencies are moving apart from each other should not depend on whether currency moves are measured relative to the dollar or the euro.

THEOREM: the only definition of dispersion that satisfies the four axioms is

$$d(\mathbf{q},\mathbf{r}) = const \times \{\log \mathbf{qr} - \mathbf{q}\log \mathbf{r}\} \tag{4}$$

where the log function is applied element by element.

PROOF: in Appendix.

For reasons that will shortly become apparent, we fix the constant in equation (4) to be 2, so we now define dispersion as

$$d(\mathbf{q},\mathbf{r}) := 2(\log \mathbf{qr} - \mathbf{q}\log \mathbf{r}). \tag{5}$$

qr is the weighted average return on the assets; we refer to it as the index return. The concavity of the log function ensures that dispersion is non-negative.

3 Properties of Dispersion

Dispersion as defined is a function of the price paths followed by a set of assets over a time period [0, *T*] under a partition (0, 1, …, *T*) with a set of weights **q**. It is not a statistical property of the distribution, but rather it is a realization, and for that reason can properly be called *realized dispersion*.

Since the time period, partition and weights remain fixed in the analysis that follows, we suppress them and, using the time additivity axiom, write the realized dispersion over the year as

$$\begin{aligned} RD &:= 2\sum_{t=1}^{T}\left(\log R_t^A - \sum_{s=1}^{S} q_s \log R_{s,t}\right) \\ \text{where } R_t^A &:= \sum_{s=1}^{S} q_s R_{s,t}. \end{aligned} \tag{6}$$

R_t^A is the index return on day *t*. The superscript *A* highlights the fact that the index is an arithmetic weighted index; the geometric index will also play a role in the analysis.

Dispersion has a number of nice properties on top of those coming directly from the axioms. In particular, we show that

- the realized dispersion of a set of assets is equal to the difference between the average realized variance of the assets, and the realized variance of the index;

- the same holds true when "realized" is replaced by "model free implied";
- realized dispersion is the average of the realized variance of the excess returns of the individual assets relative to the index;
- realized dispersion is approximately equal to one half of the weighted average variance of all the cross-rates between pairs of assets;
- realized dispersion is the sum over time of the cross-sectional variance of asset returns;
- realized dispersion over a period is twice the log of the ratio of the performance of the arithmetic and geometric indices.

3.1 Realized Dispersion and Realized Variance

There is a close relation between realized variance and realized dispersion. While the definition of realized variance in the case of a continuously sampled diffusion process is unambiguous – it is the integrated squared instantaneous volatility – there are a number of alternative definitions when there are jumps, or the path is observed discretely. Following Bondarenko (2014), we define the realized variance of an asset over the given period as[1]

$$
\begin{aligned}
&RV(R) := \sum_{t=1}^{T} v(R_t), \\
&\text{where } v(x) := 2(x - 1 - \log x),
\end{aligned} \tag{7}
$$

and R_t is the gross return on the asset. Realized dispersion can then be written as the difference between the average realized variance of the individual assets and the realized variance of the index

$$
RD = \sum_{s=1}^{S} q_s RV(R_s) - RV(R^A). \tag{8}
$$

3.2 Implied Dispersion

The key feature of Bondarenko's definition of realized variance is that the fair price of a variance swap whose floating leg is given by equation (7) is the model-free implied variance. To put the point another way, within the standard probability space framework, if $\mathbb{Q}$ is a pricing measure then

[1] This differs from the traditional definition, used for example in variance swap contracts, which is the sum of squared log returns. They converge to the same limit in the case of a continuously sampled diffusion, but the definition we use has superior mathematical properties.

$$\mathbb{E}_0^{\mathbb{Q}}\left[RV(R)\right] = IV(R), \tag{9}$$

where $IV(R)$ is the model-free implied variance of the asset at time 0.

By taking expectations of both sides of equation (8) under the pricing measure, this naturally leads to a definition of the model free implied dispersion of the assets, *ID*, as the difference between the average implied variance of the individual assets and that of the index

$$ID := \sum_{s=1}^{S} q_s IV(R_s) - IV(R^A). \tag{10}$$

The feasibility of extracting estimates of implied dispersion from options markets depends on the set of options that are traded. If there are variance swaps both on the individual assets and on the index, estimation is straightforward. If no variance swaps are traded, they can be synthesized if a sufficiently rich set of European calls and puts with appropriate maturity are traded (Jiang and Tian, 2005). There are some practical issues. Even in the case where we are interested in the dispersion of precisely those stocks that make up an index on which options are traded, the way the index is constructed matters. Most equity indexes on which options are traded are capitalization weighted. The weight of an asset in the index is not constant (as in our index) but increases as it outperforms other assets. For short horizons and diversified indexes, the implied volatility of the index and its regularly rebalanced cousin are likely to be similar (Deelstra, Liinev and Vanmaele, 2004).

It is worth highlighting the fact that implied dispersion, unlike realized dispersion, is numeraire dependent. The payout of a dispersion swap, like the payout of a variance swap, is the difference between the realized value and the fixed leg, multiplied by the notional value. The swap rate depends on the currency of the notional value. A dispersion contract where the notional is in a currency whose value is positively correlated with dispersion is more valuable than one which is negatively correlated.

3.3 Attribution to Individual Assets

Further insight into the definition of dispersion can be obtained by exploiting its numeraire independence. Define the gross *excess* return of asset *s* against the index as

$$R_{s,t}^e := R_{s,t} / R_t^A . \tag{11}$$

Realized dispersion is simply the weighted average of the realized variance of the excess returns of the individual assets

$$RD = \sum_{s=1}^{S} q_s RV\left(R_s^e\right). \tag{12}$$

This suggests that the dispersion of a set of assets can be decomposed into the sum of contributions from each individual asset – an asset will contribute more or less to the dispersion of the market than its weight in the market index implies according to the volatility of its excess return against the index.

3.4 Dispersion and the Volatility of Cross-rates

Realized dispersion is also related to the volatility of what in a currency setting would be called the cross-exchange rates – that is the volatility of the rates of exchange between the individual assets. To see this, using the fact that the weighted average excess gross return is 1, we can derive from (12) the relationship

$$\begin{gathered} RD = \frac{1}{2}\sum_{s,u=1}^{S} q_s q_u \sum_{t=1}^{T} w\left(R_{s,t}^e, R_{u,t}^e\right), \\ \text{where } w(x,y) := 2\left(R_{s,t}^e R_{u,t}^e - 1 - R_{s,t}^e \ln R_{u,t}^e - R_{u,t}^e \ln R_{s,t}^e\right). \end{gathered} \tag{13}$$

The significance of this is that, ignoring terms of third order in log returns and higher, $w(x,y) = v\left(\frac{x}{y}\right) = \left(\ln\frac{x}{y}\right)^2$. So $\sum_{t=1}^{T} w\left(R_{s,t}^e, R_{u,t}^e\right)$ is approximately equal to the realized variance of the exchange rate between asset s and asset u. Realized dispersion is approximately equal to half the weighted average cross-rate variance.

3.5 Dispersion in the Cross-section

There is a debate in the literature (Solnik and Roulet, 2000) about the relative merits of measuring dispersion in the cross-section, by looking at the variance of excess returns on all assets in each time period, or in the time series, by measuring correlations and covariances of returns on pairs of assets over time. So far we have summed first in the time series and then across assets. But we could reverse the order of summation and show that with our axiomatic measure, the two approaches are identical. From equation (12), and using the definition of realized variance in (7),

$$RD = \sum_{t=1}^{T} RD_t \text{ where } RD_t := \sum_{s=1}^{S} q_s v\left(R_{s,t}^e\right) \tag{14}$$

This says that realized dispersion in each period is the variance of excess returns in that period where the term "variance" is used in the same loose sense in the cross-section that Bondarenko (2014) and other use it in the time series.

3.6 Dispersion and Price Indices

There is one more property of dispersion that is worth highlighting. Going back to equation (6) and using the fact that the return on a geometrically weighted index is

$$R_t^G = \prod_{s=1}^{S} R_{s,t}^{q_s}, \tag{15}$$

we can write

$$RD = 2\sum_{t=1}^{T}\left(\log R_t^A - \log R_t^G\right). \tag{16}$$

Denote the arithmetic and geometric weighted indexes by A_t and G_t respectively, and normalizing them to 1 at time 0, this becomes

$$RD = 2\log\left(A_T / G_T\right). \tag{17}$$

The fact that the difference between the arithmetic index and the geometric index, and the difference depends on the volatility of the index components has long been recognized (see Brennan and Schwartz 1985).

The relationship between dispersion and geometric and arithmetic indices raises the prospect of drawing direct conclusions about implied dispersion from the price of index forward contracts. Suppose there is a forward contract on the geometric index with maturity T that has price F_0 at time 0. The strategy of going short the forward contract, and investing the money F_0 in the market portfolio, creates a synthetic dispersion forward contract. The terminal cash flow is

$$F_0 A_T - G_T = \left(F_0 - e^{-RD/2}\right) A_T. \tag{18}$$

The implied dispersion – the level at which the contract breaks even – is $-(\log F_0)/2$. The practical usefulness of this result depends on the existence of futures contracts on geometric indices. The first stock index futures contract, the Value Line Composite Stock Index (VLCI), was traded from its inception in 1982 as a geometric index. It was overtaken in importance by other indices which were all arithmetically weighted, and in 1988 its computation was changed to an arithmetic basis in a vain attempt to recover market share. For most of the period when the VLCI was traded as a geometric index, Thomas (2002) argues that it was actually priced as an arithmetic index, so its price would not have been very informative about the market view of dispersion even in that period.

The currency markets do not offer a much better laboratory. Currency indices are often geometric; it is the standard methodology for computing effective exchange rates (see Klau and Fung, 2006). But the only futures contract traded on a geometric currency index of which I am aware is the USDX contract on the Intercontinental Exchange (ICE). The index reflects the value of a basket of six currencies against the dollar, but it is the wrong way round for our purposes; although the contract is denominated in dollars, the exchange rates are foreign currency per dollar rather than dollars per foreign currency (see Eytan, Harpaz and Krull 1988 for a discussion of the pricing of USDX index contracts).

Equation (17) shows that the realized dispersion over the year depends only on the closing arithmetic and geometric index levels (when the opening index levels are normalized to unity). It is tempting to conclude that the observation interval used for the calculation of dispersion (in our case 1 day) is immaterial. That is not the case; while the geometric index does indeed depend only on opening and closing prices, the terminal level of the arithmetic index depends on the frequency with which it is rebalanced. The more frequently the arithmetic index is rebalanced, the more closely does it track the geometric index, and the less the noise in measuring the realized dispersion. To the extent that past returns on an asset are correlated with future returns, there will also be a systematic difference in the expected level of realized dispersion if the rebalancing interval is changed. If there is momentum in asset prices, with an asset's past excess returns against the index being positively correlated with its future excess returns, rebalancing less frequently will tend to raise the arithmetic index and increase the realized dispersion; if there is mean reversion, the converse is true.

3.7 Continuous Time Diffusions

In this subsection the dispersion measure is taken from the discrete time framework to the standard continuously observed diffusion. The properties of dispersion that were identified above have their natural counterparts in a diffusion world.

Assume that the process driving returns is a pure diffusion. At time t, the covariance matrix is $\mathbf{V_t}$ where

$$\frac{dP_{s,t}}{P_{s,t}}\frac{dP_{u,t}}{P_{u,t}} = v_{s,u}dt, \tag{19}$$

where $P_{s,t}$ is the price of asset s at time t and $v_{s,u}$ is an element of $\mathbf{V_t}$. To avoid convergence issues, assume that the elements of $\mathbf{V}$ are bounded above and below.

The natural extension of the definition of dispersion in equation (5) to this setting is to define a random process RD_t which satisfies the differential equation

$$dRD_t = 2\left(\log\left(\sum_{s=1}^{S} q_s\left(1+\frac{dP_{s,t}}{P_{s,t}}\right)\right) - \sum_{s=1}^{S} q_s \log\left(1+\frac{dP_{s,t}}{P_{s,t}}\right)\right). \tag{20}$$

with initial condition $RD_0 = 0$. Using Ito calculus and equation (19)

$$\begin{aligned} dRD_t &= \left(\sum_{s=1}^{S} q_s v_{s,s} - \sum_{s,u=1}^{S} q_s q_u v_{s,u}\right) dt \\ &= \left(\mathbf{q}.\text{diag}\left(\mathbf{V}_t\right)' - \mathbf{q}.\mathbf{V}_t.\mathbf{q}'\right) dt, \end{aligned} \tag{21}$$

where diag(.) is the diagonal of the matrix.

The index A_t satisfies the differential equation

$$\frac{dA_t}{A_t} = \sum_{s=1}^{S} q_s \frac{dP_{s,t}}{P_{s,t}}, \tag{22}$$

with initial condition $A_0 = 1$. The definition of realized variance of the different assets is also straightforward

$$dRV\left(R_s\right)=\left(\frac{dP_{s,t}}{P_{s,t}}\right)^2=v_{s,s}dt;$$
$$dRV\left(R^A\right)=\left(\frac{dA_t}{A_t}\right)^2=\sum_{s,u=1}^{S}q_sq_uv_{s,u}dt. \tag{23}$$

We then have the counterpart to equation (8)

$$RD_t=\sum_{s=1}^{S}q_sRV_t\left(R_s\right)-RV_t\left(R^A\right). \tag{24}$$

The implied dispersion result also goes through directly by taking expectations of both sides under the risk-adjusted measure.

The realized variance of excess returns follows the process

$$dRV\left(R_s^e\right)=\left(\frac{dP_{s,t}}{P_{s,t}}-\frac{dA_t}{A_t}\right)^2=dRV\left(R_s\right)+dRV\left(R^A\right)-2q_s\sum_{u=1}^{S}q_uv_{s,u}dt. \tag{25}$$

Substituting for $RV(R_s)$ from (25) into (24) gives the result that realized dispersion is the weighted average of the realized variance of excess returns on the assets

$$RD_t=\sum_{s=1}^{S}q_sRV_t\left(R_s^e\right). \tag{26}$$

Let $\mathbf{W}=\{w_{s,u}\}$ be the variance covariance matrix of cross-rate variances, so

$$\left(\frac{d\left(P_{s,t}/P_{u,t}\right)}{P_{s,t}/P_{u,t}}\right)^2=w_{s,u}dt. \tag{27}$$

It follows that

$$w_{s,u}=v_{s,s}+v_{u,u}-2v_{s,u}. \tag{28}$$

Putting (21) and (28) together

$$dRD_t=\frac{1}{2}\sum_{s,u=1}^{S}q_sq_uw_{s,u}dt. \tag{29}$$

Realized dispersion is half the average realized variance of the cross-rates

$$RD_t = \frac{1}{2}\sum_{s,u=1}^{S} q_s q_u RV\left(R_s/R_u\right). \tag{30}$$

Finally, the variance in the cross section at time t is

$$\sum_{s=1}^{S} q_s \left(\frac{dP_s}{P_s}\right)^2 - \left(\sum_{s=1}^{S} q_s \frac{dP_s}{P_s}\right)^2 = dRD_t, \tag{31}$$

showing that realized dispersion is the sum of the cross-sectional variance over time.

4. Correlation

In the literature looking at the collective behaviour of the prices of a set of assets, there has been as much focus on correlation as on dispersion. Correlation has been widely used in the literature as a measure of the scope the market offers for diversification – the lower the correlation, the more potential there is for diversification (see for example Driessen, Maenhout and Vilkov, 2009, and Buraschi, Kosowski and Trojani, 2013). Dispersion and correlation are clearly related. Holding the average volatility of the assets constant, the degree of dispersion depends on the correlation between them – the higher the correlation, the lower the dispersion. But the two concepts differ dimensionally; dispersion (like variance) is cumulative over time, and is quoted as a rate. Correlation is a pure number, one whose absolute value is bounded by unity.

In this section we look for a definition of correlation for multiple assets which is a natural complement to our definition of dispersion. A set of three axioms is proposed. They roughly correspond to three of the four axioms of dispersion (for reasons we discuss, numeraire independence is not imposed). We show that the definitions of correlation currently in use fail to satisfy one or other of the axioms, and propose a definition which is closely related to the definition of dispersion, and show it does satisfy the axioms. We do not claim uniqueness.

The three axioms are:

AXIOM 1 (*Regularity*): correlation ρ is an analytic function of $\mathbf{q}$ and $\mathbf{R}$; $\rho = \rho(\mathbf{q}, \mathbf{R})$ lies in the range [-1, +1], and is defined almost everywhere on the domain of admissible ($\mathbf{q}, \mathbf{R}$).

AXIOM 2 (*Time Aggregation*): correlation ρ is cumulative over time, so for any admissible pair ($\mathbf{q}$, $\mathbf{R}_1$) and ($\mathbf{q}$, $\mathbf{R}_2$)

$$\rho\left(\mathbf{q},\left[\mathbf{R}_1,\mathbf{R}_2\right]\right)\in\left[\rho\left(\mathbf{q},\mathbf{R}_1\right),\rho\left(\mathbf{q},\mathbf{R}_2\right)\right], \tag{32}$$

where [$\mathbf{R}_1$, $\mathbf{R}_2$] is the $S \times (T_1 + T_2)$ matrix obtained by concatenating $\mathbf{R}_1$ and $\mathbf{R}_2$.

AXIOM 3 (*Cross-sectional Aggregation*): if ($\mathbf{q}$, $\mathbf{R}$) and ($\mathbf{q}^*$, $\mathbf{R}^*$) are admissible and identical apart from two assets s and u where

$$q_s^* = q_s + q_u; \quad q_u^* = 0; \quad \text{and } R_s^* = \frac{q_s R_s + q_u R_u}{q_s + q_u}. \tag{33}$$

then

$$\rho\left(\mathbf{q}^*,\mathbf{R}^*\right)\geq\rho\left(\mathbf{q},\mathbf{R}\right). \tag{34}$$

The first axiom allows correlation to be undefined for some pathological cases (such as when $R_{st} = 1$ for all s and t). The second requires that the correlation over a period must be some kind of average of the correlations observed in each of the sub-periods. The third axiom embodies the intuition that if two assets get stapled together, so they can only be traded in fixed proportion (say as the result of a merger), the possibilities for diversification decline and correlation increases.

The choice of numeraire is central to the magnitude of correlation. To see this, consider a symmetrical diffusion model on the lines of equation (19), but with the covariance between assets s and u taking the symmetric form

$$\begin{aligned} &v_{su} = v \text{ if } s = u, \text{ and } \rho v \text{ otherwise,} \\ &\text{and with } q_s = 1/S \text{ for all } s. \end{aligned} \tag{35}$$

ρ is the obvious measure of correlation in this case. By taking as the numeraire the equally weighted portfolio of assets together with another asset uncorrelated with any of the existing assets, the symmetric structure is retained, but the correlation changed. Specifically, take as numeraire the asset P with

$$\frac{dP_t}{P_t} = \frac{1}{S}\sum\frac{dP_{s,t}}{P_{s,t}} + adz_t, \tag{36}$$

where dz is standard Brownian and uncorrelated with any other prices, and a is a constant. With this numeraire, the symmetry between assets is maintained. All assets have the same volatility as each other and the same correlation with all other assets. But the level of correlation can be set anywhere in the range [-1/S, +1) by varying the level of idiosyncratic volatility in the numeraire, a.

This argument shows why there is no point in requiring the definition of correlation to be numeraire independent. It also follows that correlation – its level, and the way it changes over time - is only economically meaningful in cases where there is a natural choice of numeraire. In the context of the US equity market, the use of the US dollar as the numeraire may be rather uncontroversial. But in looking at international portfolio choice with agents who have different home currencies, the correlation measure is likely to be investor specific[2].

The simplest portmanteau measure of portfolio correlation is the average Pearson correlation of pairs of assets:

$$\rho^{PW} = \frac{\sum_{s,u=1,u\neq s}^{S} q_s q_u \rho_{s,u}}{1-\sum_{s=1}^{S} q_s^2}, \tag{37}$$

$$\text{where } \rho_{s,u} = correl\left(R_s, R_u\right).$$

This formulation is used for example by Pollet and Wilson (2010).

The average is formed without regard to the level of volatility of the assets. In a market where some assets are much more volatile than others, the scope for diversification depends much more heavily on the correlations between the volatile assets than it does on the correlations between less volatile assets. This provides a justification for weighting correlations by the volatility of the assets, as the Chicago Board Option Exchange's Implied Correlation Index (CBOE, 2009) where the index is defined as

[2] For example, looking at the eight most widely traded currencies (USD, EUR, JPY, GBP, AUD, CAD, CHF,CNY) over the period 2011-2015, the average pairwise correlation in daily returns ranges from 0.30, when the US Dollar is used as numeraire to 0.71 using the Swiss Franc.

$$\rho^{CBOE} = \frac{\sum_{s=1}^{S}\sum_{u=1,u\neq s}^{S} q_s q_u \sigma_s \sigma_u \rho_{s,u}}{\sum_{s=1}^{S}\sum_{u=1,u\neq s}^{S} q_s q_u \sigma_s \sigma_u}, \quad (38)$$

where σ_s is the implied volatility of asset s. This measure of average correlation is also used by Buss, Schonleber and Vilkov (2017) for estimating a model in which all pairwise correlations are assumed equal.

Neither of the measures of average pairwise correlation (in (37) and (38)) satisfy either Axiom 2 or Axiom 3. To see this, consider two examples, in both of which the assets follow a diffusion process with variance-covariance matrix **V**.

In the first example, there are two assets, equally weighted, and two sub-periods of equal length, with covariance matrices $\mathbf{V}_1$ and $\mathbf{V}_2$ where

$$\mathbf{V}_1 = \begin{bmatrix} h^2 & \rho hl \\ \rho hl & l^2 \end{bmatrix}; \quad \mathbf{V}_2 = \begin{bmatrix} l^2 & \rho hl \\ \rho hl & h^2 \end{bmatrix} \quad (39)$$
with $h > l > 0$ and $1 > \rho > 0$.

The correlation between the two assets is the same in both sub-periods, and is equal to ρ on both definitions. But taking the period as a whole, the correlation (on both definitions) is $\rho\left(2hl/\left(h^2+l^2\right)\right)$, which is strictly lower than its level in either sub-period, thus violating Axiom 2.

The second example has three equally weighted assets with covariance matrix **V** where

$$\mathbf{V} = \begin{bmatrix} h & \rho h & 0 \\ \rho h & h & 0 \\ 0 & 0 & h \end{bmatrix} \quad (40)$$
with $h > 0$ and $1 > \rho > 0$.

$\rho^{PW} = \rho^{CBOE} = \rho/3 > 0.$ Now suppose that the first two assets are merged. The new covariance matrix is

$$\mathbf{V}^* = \begin{bmatrix} l & 0 \\ 0 & h \end{bmatrix} \quad (41)$$
with $l = h(1+\rho)/2$.

The average correlation on both definitions is now 0, so merging assets has led to a fall in the correlation measure, contrary to Axiom 3.

There is a definition of correlation which is consistent with the axioms. Equation (8) shows that the realized dispersion over the period [0, T] is the difference between the average realized variance of the assets, and the realized variance of the index. The ratio between the two is the most straightforward way of calculating a dimensionless measure of the extent to which a set of assets comove, giving a definition of correlation

$$\rho^{NEW} = \frac{RV\left(R^A\right)}{\sum_{t=1}^{T} q_s RV\left(R_s\right)} \quad . \tag{42}$$

ρ^{NEW} is an analytic function for all admissible (**q**, **R**), except when the denominator is zero, which only occurs when R_{st} = 1for all s and t. ρ^{NEW} lies in the range [0, 1], so Axiom 1 is satisfied. In the case of a period divided into two sub-periods, ρ^{NEW} over the period as a whole is the weighted average of the correlations in each of the sub-periods, with the weights being the average realized variance in each sub-period, which means that Axiom 2 is satisfied.

To see that Axiom 3 is satisfied, note that when assets s and u are merged, the numerator of equation (42) does not change, but the denominator falls by

$$\left(q_s + q_u\right)\left(RV\left(R_s\right) + RV\left(R_u\right) - RV\left(\frac{q_s R_s + q_u R_u}{q_s + q_u}\right)\right). \tag{43}$$

It is straightforward to demonstrate from the definition of realized variance in equation (7) that this expression is always positive – diversification reduces risk - so the merger of two assets necessarily causes ρ^{NEW} to rise, or at least not to fall.

To gain further insight into this definition, note that in the diffusion world equation (42) becomes

$$\rho^{NEW} = \frac{\sum_{s=1}^{S}\sum_{u=1}^{S} q_s q_u \sigma_s \sigma_u \rho_{s,u}}{\sum_{s=1}^{S} q_s \sigma_s^2}. \tag{44}$$

This measure is quite similar to the CBOE definition, but with two differences. The CBOE excludes the diagonal elements. The diagonals of a correlation matrix carry no information, since they consist of 1's, but their exclusion leads to the failure to satisfy Axiom 2. The other difference is that the denominator of ρ^{NEW} is the mean variance of the assets, while the denominator of ρ^{CBOE} is (ignoring the diagonal elements) the square of the average volatility of the assets. Since it is variance that averages over time, not volatility, this is what prevents ρ^{CBOE} satisfying Axiom 3.

It should be noted that ρ^{NEW} is not an average of the pairwise correlations – the weights in the numerator only sum to the denominator when all assets have the same volatility – so it does not converge to the Pearson correlation coefficient. But it does inherit many of the properties of the dispersion measure from which it is derived. With ρ^{NEW} as defined, it has the desirable properties of aggregation over time and across assets that are embodied in Axioms 2 and 3. Realized correlation is straightforward to calculate from discretely observed data – it is the ratio of the realized variance of the index to the average realized variance of its components. There is no need to compute covariances of returns across all possible pairs of assets. Its implied counterpart is equally straightforward to calculate, being the difference between the model free implied variance of the index and the average model free implied variance of its constituents. By going long variance swaps on the index, and short variance swaps on the individual assets, it is possible to construct a correlation swap whose payoff π is

$$\pi = \left(\rho^{real} - \rho^{imp}\right) \times \frac{\sum_s q_s RV\left(R_s\right)}{\sum_s q_s IV\left(R_s\right)}, \tag{45}$$

where ρ^{real} and ρ^{imp} are the realized and implied versions of ρ^{NEW}. The expected profit on such a swap can be interpreted as a correlation risk premium; the numeraire of the payoff is the currency in which the variance swaps are denominated scaled by the average realized variance of the assets.

5. Conclusions

The level of dispersion in asset returns in many financial markets, and the degree to which the returns on assets comove have been the subject of much academic study and practitioner interest. There is no widely accepted method of measuring dispersion or correlation among multiple assets, and practice varies widely. There are substantial advantages in having a standardized definition, accepted throughout the academic community. Having terms that are precisely defined and widely understood makes debate and discussion easier and avoids confusion. The connection between different concepts become clearer, and empirical investigation can become more precise. It is easier to understand and compare empirical studies when researchers measure quantities in a standard fashion. Stylized facts, such as that correlations increase in a bear market, or that dispersion between national equity markets has reduced over time, are easier to test and verify, and can be established or refuted more easily if the objects of the propositions have a precise and well-understood meaning.

Without guiding principles, it is difficult to see how one can decide whether one formulation of a broad concept such as dispersion or correlation is better than another. So we have followed an axiomatic approach. While axiomatic definitions are not necessarily unique – different axioms produce different definitions – they are not arbitrary, and they tend to be simple. They are also likely to have useful properties. Artzner *et al* (1999) follow a somewhat similar approach in characterizing coherent risk measures by specifying a set of four axioms that they should satisfy, though in their case there are multiple measures of risk that satisfy the axioms. In the case of dispersion, the axiomatic approach produces a unique definition that is simple to formulate and to calculate, and which has many attractive properties.

ANNEX

Proof of Theorem

Assume for the moment that the set of non-trivial functions (that is, apart from $d = 0$) that satisfy the axioms is non-empty. Take a member of the set, d. Let

$$\begin{aligned} &\mathbf{p} = \left(\tfrac{1}{2}, \tfrac{1}{2}\right), \\ &\mathbf{q} \text{ be a } 1\times S \text{ strictly positive weighting vector,} \\ &\mathbf{e} \text{ be a } 1\times S \text{ vector of infinitesimals,} \\ &\mathbf{r} \text{ be a } S\times 1 \text{ vector of returns,} \\ &\mathbf{Q} \text{ be the } 2\times S \text{ matrix } \begin{pmatrix} \mathbf{q}+\mathbf{e} \\ \mathbf{q}-\mathbf{e} \end{pmatrix}. \end{aligned} \tag{46}$$

Axiom 3 requires that

$$\begin{aligned} &d(\mathbf{q},\mathbf{r}) = d(\mathbf{p},\mathbf{x}) + \tfrac{1}{2} d(\mathbf{q}+\mathbf{e},\mathbf{r}) + \tfrac{1}{2} d(\mathbf{q}-\mathbf{e},\mathbf{r}), \\ &\text{where } \mathbf{x} = \left((\mathbf{q}+\mathbf{e})\mathbf{r}, (\mathbf{q}-\mathbf{e})\mathbf{r}\right). \end{aligned} \tag{47}$$

Rewrite this as

$$\tfrac{1}{2} d(\mathbf{q}+\mathbf{e},\mathbf{r}) + \tfrac{1}{2} d(\mathbf{q}-\mathbf{e},\mathbf{r}) - d(\mathbf{q},\mathbf{r}) = -d(\mathbf{p},\mathbf{x}). \tag{48}$$

Since $\mathbf{e}$ is infinitesimal and d is analytic, the left hand side simplifies to

$$\begin{aligned} &\tfrac{1}{2} d(\mathbf{q}+\mathbf{e},\mathbf{r}) + \tfrac{1}{2} d(\mathbf{q}-\mathbf{e},\mathbf{r}) - d(\mathbf{q},\mathbf{r}) = \tfrac{1}{2} \mathbf{e} \frac{\partial^2 d}{\partial \mathbf{q}^2} \mathbf{e}' \\ &\text{where } \frac{\partial^2 d}{\partial \mathbf{q}^2} \text{ is the Hessian of dispersion with respect to weights.} \end{aligned} \tag{49}$$

Turning to the right hand side of (48), use the second element of $\mathbf{x}$ as the numeraire. Axiom 4 requires that

$$d(\mathbf{p},\mathbf{x}) = d(\mathbf{p},\mathbf{x}^*), \text{ where } \mathbf{x}^* = \left(1 + \frac{2\mathbf{er}/\mathbf{qr}}{1-\mathbf{er}/\mathbf{qr}}, 1\right). \tag{50}$$

With $\mathbf{e}$ being infinitesimal

$$d(\mathbf{p},\mathbf{x}) = a_0 + 2\frac{\mathbf{er}}{\mathbf{qr}}a_1 + 2\left(\frac{\mathbf{er}}{\mathbf{qr}}\right)^2(a_1 + a_2),$$
$$\text{where } a_n = \frac{\partial^n}{\partial x^n} d\left(\left(\tfrac{1}{2},\tfrac{1}{2}\right),(1+x,1)'\right) \text{ evaluated at } x = 0. \tag{51}$$

Equating (49) and (51) gives

$$a_0 = a_1 = 0;$$
$$\frac{\partial^2 d}{\partial q_i \partial q_j} = -4a_2 \frac{r_i r_j}{(\mathbf{qr})^2}. \tag{52}$$

The general solution to this partial differential equation takes the form

$$d(\mathbf{q},\mathbf{r}) = const \times \left\{\log \mathbf{qr} + \mathbf{q}f(\mathbf{r}) + g(\mathbf{r})\right\}, \tag{53}$$

for some function f and g where $f(\mathbf{r})$ is a S x 1 vector and $g(\mathbf{r})$ a scalar). Substituting this form for d back into (2) for general **p**, **Q** and **r** requires that

$$\begin{aligned}&\log \mathbf{pQr} + \mathbf{pQ}f(\mathbf{r}) + g(\mathbf{r})\\ &= \log \mathbf{pQr} + \mathbf{p}f(\mathbf{Qr}) + g(\mathbf{Qr}) + \mathbf{p}\left(\log \mathbf{Qr} + \mathbf{Q}f(\mathbf{r}) + g(\mathbf{r})\right).\end{aligned} \tag{54}$$

Writing **y** = **Qr**, this simplifies to

$$\mathbf{p}\left(\log \mathbf{y} + f(\mathbf{y})\right) + g(\mathbf{y}) = 0, \tag{55}$$

for all admissible (**p**, **y**). This requires that

$$f(\mathbf{y}) = -\log \mathbf{y} \text{ and } g(\mathbf{y}) = 0. \tag{56}$$

Any function d that satisfies the four axioms must therefore take the form

$$d(\mathbf{q},\mathbf{r}) = const \times \{\log \mathbf{qr} - \mathbf{q}\log \mathbf{r}\} \tag{57}$$

It remains to demonstrate that all functions of this form do satisfy the four axioms. It is obvious that they satisfy the first two axioms. Axiom 3 is satisfied because

$$
\begin{aligned}
d\left(\mathbf{pQ},\mathbf{r}\right) &= const \times \left\{\log \mathbf{pQr} - \mathbf{pQ}\log \mathbf{r}\right\};\\
d\left(\mathbf{p},\mathbf{Qr}\right) &= const \times \left\{\log \mathbf{pQr} - \mathbf{p}\log \mathbf{Qr}\right\};\\
\mathbf{p}d\left(\mathbf{Q},\mathbf{r}\right) &= const \times \left\{\mathbf{p}\log \mathbf{Qr} - \mathbf{pQ}\log \mathbf{r}\right\};\\
\text{so } d\left(\mathbf{pQ},\mathbf{r}\right) - d\left(\mathbf{p},\mathbf{Qr}\right) - \mathbf{p}d\left(\mathbf{Q},\mathbf{r}\right) &= 0.
\end{aligned}
\tag{58}
$$

Numeraire invariance (axiom 4) holds since

$$
\begin{aligned}
d\left(\mathbf{q},k\mathbf{r}\right) &= const \times \left\{\log k\mathbf{qr} - \mathbf{q}\log k\mathbf{r}\right\}\\
&= const \times \left\{\log \mathbf{qr} - \mathbf{q}\log \mathbf{r}\right\}\\
&= d\left(\mathbf{q},\mathbf{r}\right).
\end{aligned}
\tag{59}
$$

REFERENCES

Artzner, P., Delbaen, F., Eber, J.M. and Heath, D., 1999. Coherent measures of risk. Mathematical finance, 9(3), pp.203-228.

Bikhchandani, S. and Sharma, S., 2000. Herd behavior in financial markets. IMF Staff papers, 47(3), pp.279-310.

Bondarenko, O., 2014. Variance trading and market price of variance risk. Journal of Econometrics, 180(1), pp.81-97.

Brennan, M.J. and Schwartz, E.S., 1985. On the geometric mean index: a note. Journal of Financial and Quantitative Analysis, 20(1), pp.119-122.

Buraschi, A., Kosowski, R. and Trojani, F., 2013. When there is no place to hide: Correlation risk and the cross-section of hedge fund returns. The Review of Financial Studies, 27(2), pp.581-616.

Buss, A., Schönleber, L. and Vilkov, G., 2017. Option-implied correlations, factor models, and market risk. Instead working paper series 2017/20/FIN.

Chang, E.C., Cheng, J.W. and Khorana, A., 2000. An examination of herd behavior in equity markets: An international perspective. Journal of Banking & Finance, 24(10), pp.1651-1679.

Chicago Board Options Exchange, 2009. CBOE S&P500 Implied Correlation Index White Paper, http://www.cboe.com/micro/impliedcorrelation/impliedcorrelationindicator.pdf.

Christie, W.G. and Huang, R.D., 1995. Following the pied piper: Do individual returns herd around the market? Financial Analysts Journal, 51(4), pp.31-37.

Deelstra, G., Liinev, J. and Vanmaele, M., 2004. Pricing of arithmetic basket options by conditioning. Insurance: Mathematics and Economics, 34(1), pp.55-77.

Driessen, J., Maenhout, P.J. and Vilkov, G., 2009. The price of correlation risk: Evidence from equity options. The Journal of Finance, 64(3), pp.1377-1406.Heston and Rouwenhorst (1994).

Eytan, T.H., Harpaz, G. and Krull, S., 1988. The pricing of dollar index futures contracts. Journal of Futures Markets, 8(2), pp.127-139.

Heston, S.L. and Rouwenhorst, K.G., 1994. Does industrial structure explain the benefits of international diversification?. Journal of Financial Economics, 36(1), pp.3-27.

Jiang, G.J. and Tian, Y.S., 2005. The model-free implied volatility and its information content. The Review of Financial Studies, 18(4), pp.1305-1342.

Klau, M. and Fung, S.S., 2006. The new BIS effective exchange rate indices.

Menkhoff, L., Sarno, L., Schmeling, M. and Schrimpf, A., 2012. Carry trades and global foreign exchange volatility. The Journal of Finance, 67(2), pp.681-718.

Mueller, P., Stathopoulos, A. and Vedolin, A., 2017. International correlation risk. Journal of Financial Economics, 126(2), pp.270-299.

Norris, N., 1938. Some efficient measures of relative dispersion. The Annals of Mathematical Statistics, 9(3), pp.214-220.

Pindyck, R., and Rotemberg, J., 1990, The Excess Co-movement of Commodity Prices, Economic Journal, 100(403), pp.1173-89.

Pollet, J.M. and Wilson, M., 2010. Average correlation and stock market returns. Journal of Financial Economics, 96(3), pp.364-380.

Solnik, B. and Roulet, J., 2000. Dispersion as cross-sectional correlation. Financial Analysts Journal, 56(1), pp.54-61.

Thomas, S., 2002. The saga of the first stock index futures contract: benchmarks, models, and learning. Journal of Money, Credit and Banking, pp.767-808.