

Money Illusion: A Rationale for the TIPS Puzzle

Abraham Lioui¹ and Andrea Tarelli²

¹EDHEC Business School

²Catholic University of Milan

Outline

Background and contribution

Model

Empirical findings

Conclusions

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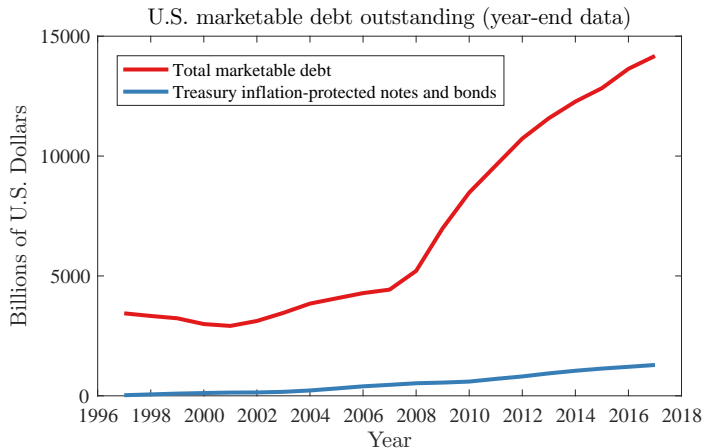
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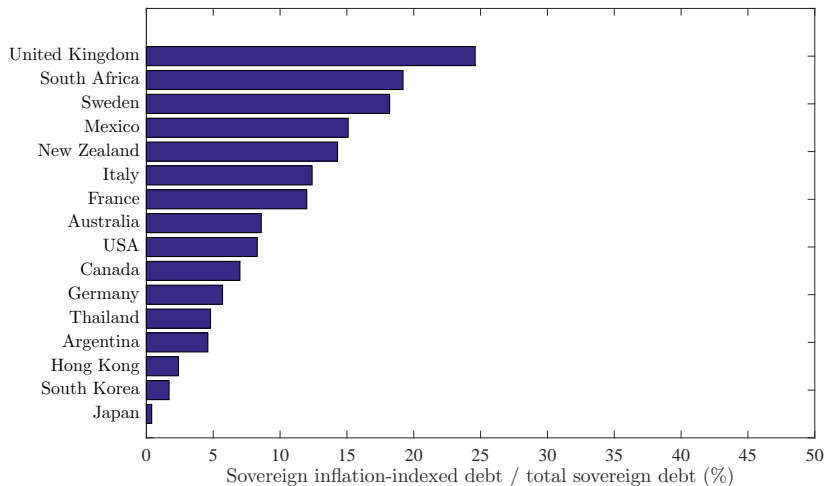
The TIPS puzzle



Treasury Inflation-Protected Securities

- ▶ Treasury bonds which notional is adjusted w.r.t. CPI
- ▶ First auctioned in the U.S. in January 1997

The TIPS puzzle



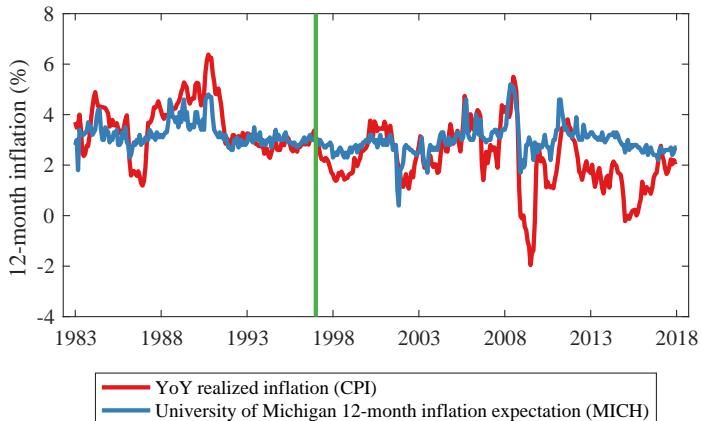
As of 2014, small fractions of outstanding inflation-indexed debt

The TIPS puzzle

Incentives for development of TIPS market

- ▶ Supply-side (government)
 - ▶ Hedging: tax revenues correlated with inflation
 - ▶ Signaling a credible monetary policy
 - ▶ Break-even inflation rates as measure for inflation expectations
 - ▶ If demand is high, reduce cost of funding
- ▶ Demand-side
 - ▶ Risk-less asset in real terms (inflation hedging)
 - ▶ Diversification

The TIPS puzzle



- ▶ Realized and expected inflation far from being constant
- ▶ Since 1997, annualized volatility of realized inflation: 1.21%
- ▶ Significant inflation risk even in low-inflation (Illeditsch, 2017)

The TIPS puzzle

Fleckenstein, Longstaff and Lustig (JF, 2014)

- ▶ Replicate nominal bonds with TIPS + inflation swaps
- ▶ Find a significant mispricing of TIPS
- ▶ 2004–2009: TIPS cheaper than nominal Treasuries by 54.5bps on average (up to 200bps)

Andreasen, Christensen and Riddell (2018)

- ▶ 1997–2013: positive average TIPS liquidity premium (38bps), making TIPS comparatively cheap for buyers

The TIPS puzzle

In our view

- ▶ TIPS liquidity premium compatible with a lack of demand
- ▶ The scarce demand for TIPS is the true puzzle
 - ▶ We show that a rational agent has tangible welfare incentives to hold large amounts of TIPS
- ▶ We provide a behavioral explanation for this allocation puzzle

Money illusion

Money illusion

« The failure to perceive that the dollar, or any other unit of money, expands or shrinks in value »

Irving Fisher (1928)

- ▶ Tendency of evaluating investment outcomes in nominal terms
- ▶ In our model, money illusion can also be a rational choice
 - ▶ Fund manager compensated according to nominal returns
 - ▶ Pension fund which liabilities are not inflation-linked
 - ▶ ...

Money illusion: experimental evidence

Experimental evidence

- ▶ Shafir, Diamond and Tversky (QJE, 1997)
- ▶ Fehr and Tyran (AER, 2001; GEB, 2007; AER, 2014)
- ▶ Stephens and Tyran (2016)
- ▶ Cavallo, Cruces and Perez-Truglia (AEJ, 2017)
 - ▶ Individuals in low inflation environment have weaker priors about the inflation rate (rational inattention)
 - ▶ Money illusion can be even stronger in low inflation

Money illusion: asset pricing literature

Housing market

- ▶ Brunnermeier and Julliard (RFS, 2008)
 - ▶ Stronger impact of money illusion when inflation is low
- ▶ Piazzesi and Schneider (2008)

Valuation of corporate claims

- ▶ Modigliani and Cohn (FAJ, 1979)
- ▶ Campbell and Vuolteenaho (AER, 2004)
- ▶ Cohen, Polk and Vuolteenaho (QJE, 2005)
- ▶ Schmeling and Schrimpf (EER, 2011)
- ▶ Bhamra, Dorion, Jeanneret and Weber (2017)

Asset prices at the equilibrium

- ▶ Basak and Yan (RES, 2010)
- ▶ Miao and Xie (JEDC, 2013)
- ▶ David and Veronesi (JPE, 2013)

Contribution: dynamic asset allocation problem

- ▶ Investor featuring a given degree of money illusion
- ▶ Finite investment horizon: 0 – 30 years
- ▶ **Impact of money illusion** on optimal portfolio positions
 - ▶ 10-year nominal bond: **increase**
 - ▶ 10-year inflation-indexed bond: **strong decrease**
 - ▶ Stock index: **almost unaffected**
 - ▶ Money market account: **almost unaffected**
- ▶ Opportunity cost of money illusion
 - ▶ **Substantial**
- ▶ Opportunity cost of not having access to TIPS
 - ▶ As perceived by a non-illusioned investor: **substantial**
 - ▶ As perceived by a money-illusioned investor: **negligible**

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Utility over real (w_T) and nominal (W_T) terminal wealth

$$U(w_T, W_T) = \frac{(w_T^{1-\alpha} W_T^\alpha)^{1-\gamma}}{1-\gamma} = \frac{W_T^{1-\gamma} P_T^{-(1-\alpha)(1-\gamma)}}{1-\gamma}$$

P_T is the terminal level of the price index: $W_T = w_T P_T$

Degree of money illusion $\alpha \in [0, 1]$

- ▶ $\alpha = 0$: no money illusion
 - ▶ standard CRRA utility over real terminal wealth
- ▶ $\alpha = 1$: complete money illusion
 - ▶ CRRA utility over nominal terminal wealth

The financial market

- ▶ Nominal stochastic discount factor (SDF)

$$\frac{d\Phi_t}{\Phi_t} = -R_t dt - \boldsymbol{\Lambda}'_t d\mathbf{z}_t$$

- ▶ Nominal instantaneous interest rate

$$R_t = R_0 + \mathbf{R}'_1 \mathbf{X}_t$$

- ▶ Market prices of risk

$$\boldsymbol{\Lambda}_t = \boldsymbol{\Lambda}_0 + \boldsymbol{\Lambda}_1 \mathbf{X}_t$$

- ▶ 3 state variables

$$d\mathbf{X}_t = \boldsymbol{\Theta} (\bar{\mathbf{X}} - \mathbf{X}_t) dt + \boldsymbol{\Sigma}'_{\mathbf{X}} d\mathbf{z}_t$$

- ▶ Price level

$$\frac{dP_t}{P_t} = \pi_t dt + \boldsymbol{\sigma}'_P d\mathbf{z}_t$$

- ▶ Expected inflation

$$\pi_t = \pi_0 + \boldsymbol{\pi}'_1 \mathbf{X}_t$$

The financial market: risky assets

No-arbitrage affine nominal and real term structures

- ▶ Nominal zero-coupon bond

$$B_t(\mathbf{X}_t, \tau) = e^{A_0^N(\tau) + \mathbf{A}_1^N(\tau)\mathbf{X}_t}$$

$$\frac{dB_t}{B_t} = \left(R_t + \mathbf{A}_1^N(\tau) \boldsymbol{\Sigma}'_{\mathbf{X}} \boldsymbol{\Lambda}_t \right) dt + \mathbf{A}_1^N(\tau) \boldsymbol{\Sigma}'_{\mathbf{X}} d\mathbf{z}_t$$

- ▶ Inflation-indexed zero-coupon bond

$$I_t(\mathbf{X}_t, P_t, \tau) = P_t e^{A_0^I(\tau) + \mathbf{A}_1^I(\tau)\mathbf{X}_t}$$

$$\frac{dI_t}{I_t} = \left(R_t + \left(\mathbf{A}_1^I(\tau) \boldsymbol{\Sigma}'_{\mathbf{X}} + \boldsymbol{\sigma}'_P \right) \boldsymbol{\Lambda}_t \right) dt + \left(\mathbf{A}_1^I(\tau) \boldsymbol{\Sigma}'_{\mathbf{X}} + \boldsymbol{\sigma}'_P \right) d\mathbf{z}_t$$

- ▶ Stock

$$\frac{dS_t}{S_t} = \left(R_t + \boldsymbol{\sigma}'_S \boldsymbol{\Lambda}_t \right) dt + \boldsymbol{\sigma}'_S d\mathbf{z}_t$$

$A_0^N(\tau)$, $\mathbf{A}_1^N(\tau)$, $A_0^I(\tau)$ and $\mathbf{A}_1^I(\tau)$ depend on model parameters

Optimal portfolio

Maximum expected terminal utility

$$\max_{[\omega_s]_{s=t}^T} E_t \left[\frac{W_T^{1-\gamma} P_T^{-(1-\alpha)(1-\gamma)}}{1-\gamma} \right]$$

Optimal strategy à la Merton ($\tau = T - t$)

$$\begin{aligned} \omega_t = & \underbrace{\frac{1}{\gamma} (\boldsymbol{\Sigma}'\boldsymbol{\Sigma})^{-1} \boldsymbol{\Sigma}'\boldsymbol{\Lambda}_t}_{\text{mean-variance}} + \underbrace{(1-\alpha) \left(1 - \frac{1}{\gamma}\right) (\boldsymbol{\Sigma}'\boldsymbol{\Sigma})^{-1} \boldsymbol{\Sigma}'\boldsymbol{\sigma}_P}_{\text{unexpected inflation hedging } (\alpha\text{-dependent})} \\ & + \underbrace{(\boldsymbol{\Sigma}'\boldsymbol{\Sigma})^{-1} \boldsymbol{\Sigma}'\boldsymbol{\Sigma}_X \left(\tilde{\mathbf{B}}_3(\tau) \mathbf{X}_t + \mathbf{B}'_2(\tau) \right)}_{\text{intertemporal hedging } (\alpha\text{-dependent})} \end{aligned}$$

- ▶ $\boldsymbol{\Sigma}$: volatility matrix of available risky assets
- ▶ $\mathbf{B}_2(\tau)$ and $\tilde{\mathbf{B}}_3(\tau)$ depend on model parameters (including α)

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Data and estimation

MLE, similar to Joslin, Singleton and Zhu (RFS, 2011)

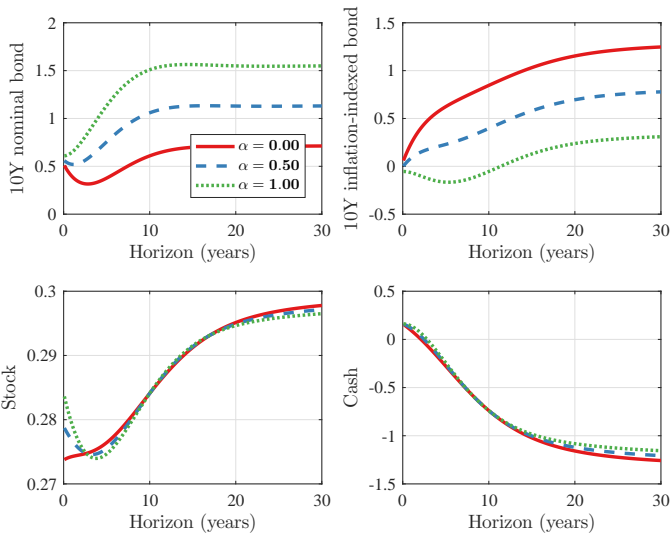
- ▶ U.S. monthly data from 1999 to 2016
- ▶ Nominal zero-coupon yields: 3M, 6M, 1Y, 2Y, 3Y, 5Y, 7Y, 10Y
- ▶ Real zero-coupon yields: 5Y, 7Y, 10Y
- ▶ Stock: NYSE/Amex/NASDAQ/ARCA Value-weighted Index
- ▶ Consumer Price Index for All Urban Consumers

Two estimation settings

- ▶ Constant risk premia
- ▶ Time-varying risk premia

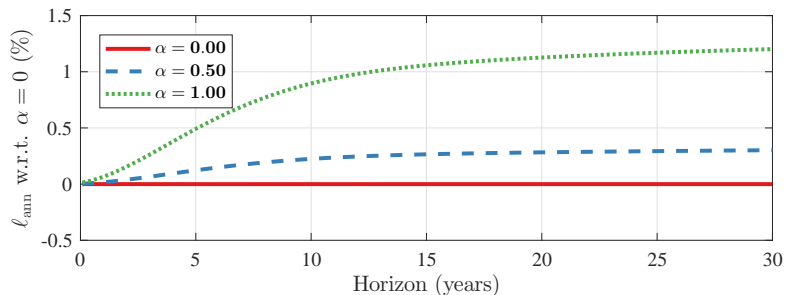
Constant risk premia, moderate investor ($\gamma = 10$)

Optimal portfolio strategy



Constant risk premia, moderate investor ($\gamma = 10$)

Opportunity cost of money illusion

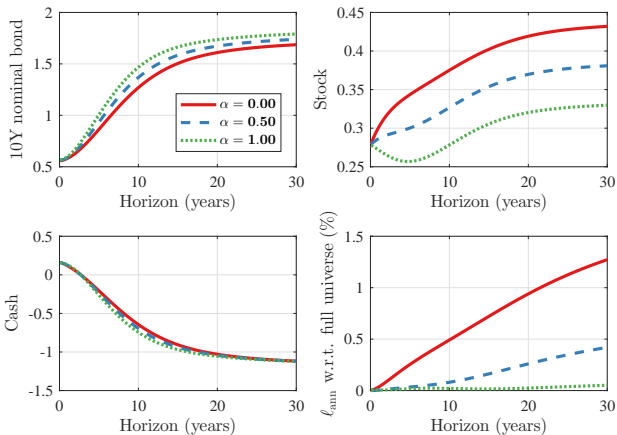


l_{ann} : certainty equivalent annualized loss

- ▶ suffered by a rational non-illusioned investor
- ▶ following the optimal strategy for an agent with a degree α of money illusion

Constant risk premia, moderate investor ($\gamma = 10$)

Opportunity cost of not having access to TIPS

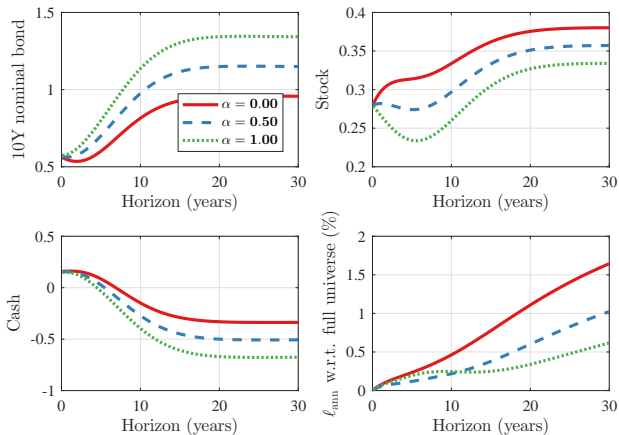


ℓ_{ann} : certainty equivalent annualized loss

- ▶ as perceived by an investor with a degree of money illusion α
- ▶ when the inflation-indexed bond is not accessible

Time-varying risk premia, moderate investor ($\gamma = 10$)

Opportunity cost of not having access to TIPS

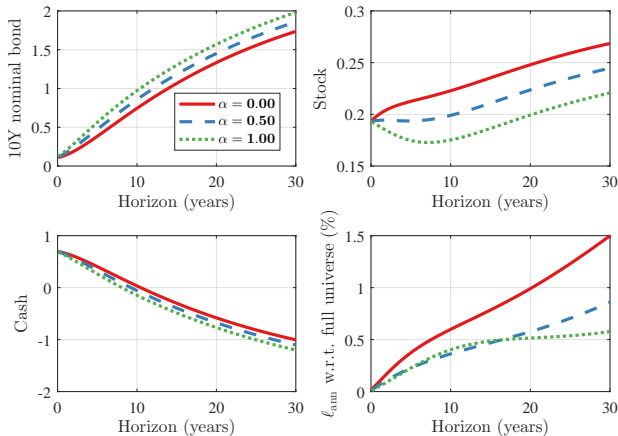


Robust estimation of time-varying risk premia

- ▶ All assets (including TIPS) hedge variations of risk premia
- ▶ Opportunity cost for money-illusioned investor still far smaller

UK, time-varying risk premia, moderate investor ($\gamma = 10$)

Opportunity cost of not having access to TIPS



UK data: longer sample (1985 – 2018) than for U.S. market

- ▶ Negative inflation risk premium (-0.07% vs 1.18% in U.S.)
- ▶ Higher realized inflation volatility (1.46% vs 1.07% in U.S.)

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- ▶ TIPS puzzle hard to reconcile with rational finance
- ▶ Behavioral explanation: money illusion
- ▶ Impact of money illusion on portfolio strategy and welfare
 - ▶ Shift from inflation-indexed to nominal bonds
 - ▶ Negligible effect on stock allocation
 - ▶ Sizable certainty equivalent loss for long horizons
- ▶ Economic incentive to enter the TIPS market
 - ▶ High for non-illusioned long-term investors
 - ▶ Much smaller for money-illusioned investors

Additional material

Estimation (constant risk premia): parameters

R_0	R_1	π_0	π_1	Θ			σ_ϵ^B	σ_ϵ^I
0.0185 (0.0001)	0.3626 (0.0013)	0.0210 (0.0022)	0.0716 (0.0054)	0.0639 (0.0033)	-0.5484 (0.0149)	1.0786 (0.0396)	0.0012 (0.0000)	0.0007 (0.0000)
	-0.4174 (0.0053)		-0.8305 (0.0301)	0.0215 (0.0019)	0.2526 (0.0085)	-0.3158 (0.0213)		
	0.3323 (0.0138)		-1.1252 (0.0771)	-0.0228 (0.0019)	0.0953 (0.0093)	0.2829 (0.0254)		
Λ_0	Λ_1			Σ_X			σ_P	σ_S
-0.6746 (0.0332)	0	0	0	0.0209 (0.0010)	0.0066 (0.0009)	-0.0043 (0.0006)	0.0001 (0.0007)	0.0380 (0.0107)
0.4464 (0.0517)	0	0	0	0	0.0123 (0.0006)	0.0014 (0.0006)	-0.0009 (0.0007)	-0.0325 (0.0108)
-0.1015 (0.0572)	0	0	0	0	0	0.0087 (0.0004)	-0.0040 (0.0007)	-0.0427 (0.0101)
1.3054 (0.2718)	0	0	0	0	0	0	0.0090 (0.0005)	-0.0181 (0.0100)
0.7694 (0.2765)	0	0	0	0	0	0	0	0.1413 (0.0070)

Estimation (constant risk premia): moments

Time series	Mean value		Volatility	
	Estimation	Data	Estimation	Data
3M nominal yield	1.91%	1.87%	0.63%	0.64%
6M nominal yield	1.97%	1.97%	0.62%	0.61%
1Y nominal yield	2.10%	2.15%	0.63%	0.74%
2Y nominal yield	2.37%	2.38%	0.71%	0.85%
3Y nominal yield	2.64%	2.63%	0.80%	0.91%
5Y nominal yield	3.14%	3.12%	0.93%	0.96%
7Y nominal yield	3.55%	3.54%	0.96%	0.96%
10Y nominal yield	3.99%	4.01%	0.91%	0.95%
5Y real yield	1.29%	1.28%	0.92%	0.97%
7Y real yield	1.51%	1.54%	0.88%	0.86%
10Y real yield	1.81%	1.80%	0.84%	0.77%
Log realized inflation	2.09%	2.17%	0.99%	1.07%
Equity log returns	5.55%	4.98%	15.69%	15.72%
3M nominal risk premium	0.12%		0%	
6M nominal risk premium	0.25%		0%	
1Y nominal risk premium	0.51%		0%	
2Y nominal risk premium	1.07%		0%	
3Y nominal risk premium	1.62%		0%	
5Y nominal risk premium	2.55%		0%	
7Y nominal risk premium	3.20%		0%	
10Y nominal risk premium	3.71%		0%	
5Y real risk premium	2.22%		0%	
7Y real risk premium	2.74%		0%	
10Y real risk premium	3.28%		0%	
Realized inflation risk premium	1.18%		0%	
Equity risk premium	4.93%		0%	
Nominal risk-free rate	1.85%		0.65%	
Expected inflation	2.10%		1.54%	

Estimation: pairwise return correlations (data)

Data

	3M nom	1Y nom	2Y nom	5Y nom	10Y nom	5Y real	10Y real	Equity	CPI
3M nom	1.000								
1Y nom	0.658	1.000							
2Y nom	0.471	0.924	1.000						
5Y nom	0.257	0.708	0.889	1.000					
10Y nom	0.108	0.499	0.669	0.904	1.000				
5Y real	-0.000	0.311	0.409	0.501	0.476	1.000			
10Y real	-0.009	0.322	0.440	0.623	0.682	0.910	1.000		
Equity	-0.160	-0.299	-0.352	-0.323	-0.253	0.038	0.000	1.000	
CPI	-0.124	-0.150	-0.139	-0.190	-0.234	0.350	0.156	0.070	1.000

Estimation (constant risk premia): correlations

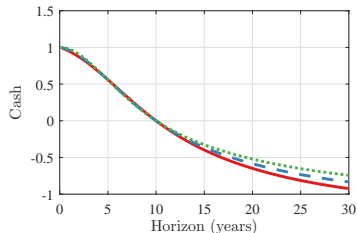
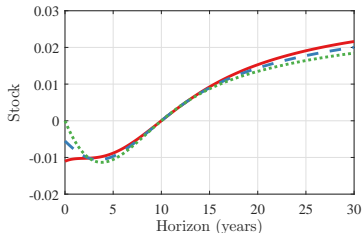
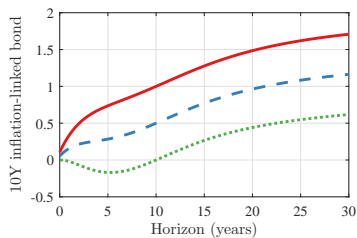
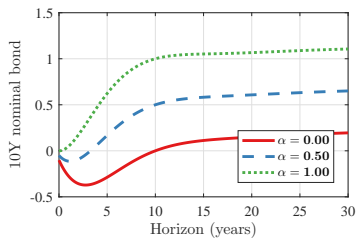
Model-implied pairwise correlations of asset returns and economic variables

	3M nom	1Y nom	2Y nom	5Y nom	10Y nom	5Y real	10Y real	Equity	CPI	R	π	r
3M nom	1.000											
1Y nom	0.926	1.000										
2Y nom	0.744	0.939	1.000									
5Y nom	0.398	0.705	0.904	1.000								
10Y nom	0.209	0.537	0.782	0.969	1.000							
5Y real	0.166	0.274	0.366	0.494	0.618	1.000						
10Y real	0.114	0.319	0.492	0.686	0.807	0.958	1.000					
Equity	-0.203	-0.295	-0.335	-0.306	-0.237	0.089	-0.002	1.000				
CPI	0.050	-0.048	-0.122	-0.158	-0.115	0.270	0.156	0.046	1.000			
R	-0.978	-0.860	-0.634	-0.254	-0.064	-0.120	-0.028	0.156	-0.091	1.000		
π	-0.340	-0.404	-0.385	-0.216	-0.013	0.715	0.561	0.346	0.380	0.296	1.000	
r	-0.073	0.046	0.124	0.114	-0.015	-0.795	-0.595	-0.292	-0.434	0.128	-0.910	1.000

- ▶ R : nominal short-term rate
- ▶ π : instantaneous expected inflation
- ▶ r : real short-term rate

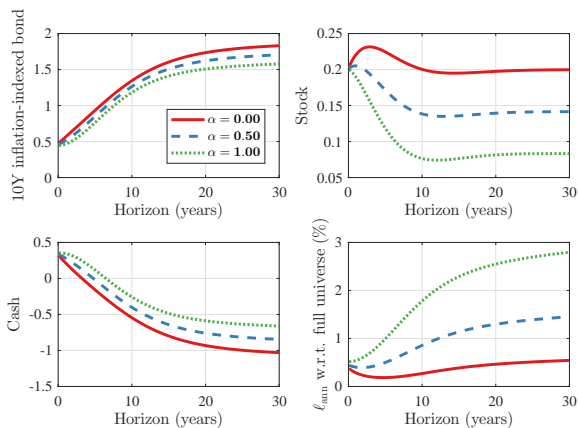
Constant risk premia, conservative investor ($\gamma \rightarrow \infty$)

Optimal portfolio strategy



Constant risk premia, moderate investor ($\gamma = 10$)

Opportunity cost of not having access to nominal bonds



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- ▶ as perceived by an investor with a degree of money illusion α
- ▶ when the nominal bond is not accessible

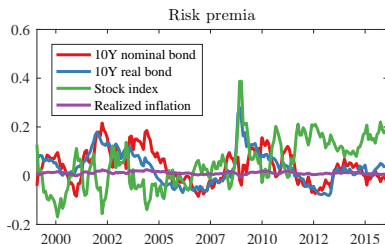
Estimation (time-varying risk premia)

No-arbitrage term structure models with time-varying risk premia seriously suffer from **overfitting issues**

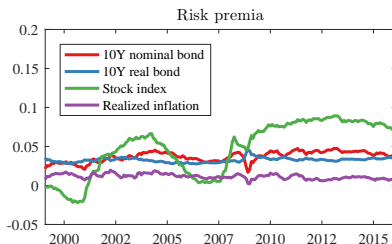
- ▶ Duffee (2011): “Flexibility and overfitting go hand-in hand”
 - ▶ A 5-factor unconstrained term structure model leads to a maximum in-sample Sharpe ratio on the order of 10^{30} !
 - ▶ This is obtained through huge portfolio positions
 - ▶ Proposes to set a constraint on maximum Sharpe ratio
- ▶ Feldhütter, Heyerdahl-Larsen and Illieditsch (2012)
 - ▶ Utility losses due to parameter uncertainty in portfolio strategies based on essentially affine term structure models
 - ▶ These increase with number of factors and model complexity
 - ▶ Better using a parsimonious model with constant risk premia
- ▶ Sarno, Schneider and Wagner (JEF, 2016)
 - ▶ Robust model forecasts with time-varying risk premia lead to portfolio returns lower than in the case of constant risk premia

Estimation (time-varying risk premia)

Time-varying risk premia (unconstrained)



Time-varying risk premia (volatility constrained)

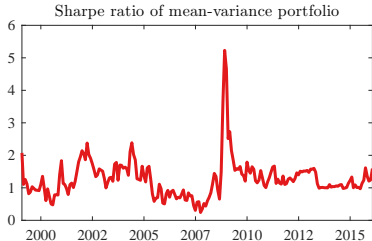
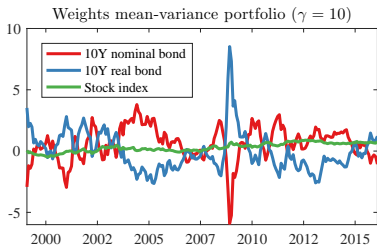


To avoid overfitting, in the estimation we impose reasonable **bounds** to the **volatility of the risk premia**

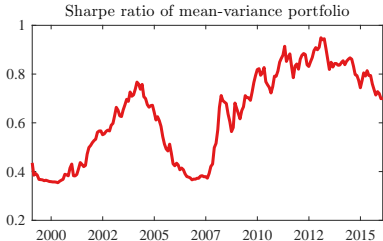
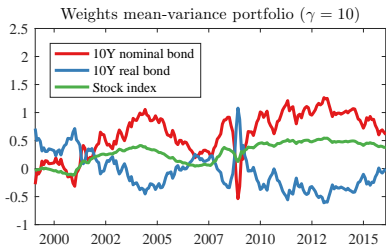
- ▶ Bond premia: 0.64% p.a. (same as short-term rate volatility)
- ▶ Realized inflation risk premium: 0.5% p.a.
- ▶ Equity premium: 1% p.a.

Estimation (time-varying risk premia)

Time-varying risk premia (unconstrained)

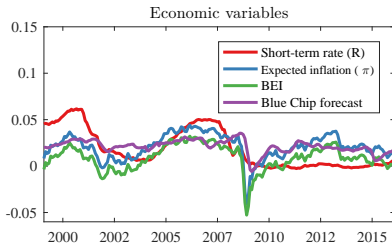


Time-varying risk premia (volatility constrained)

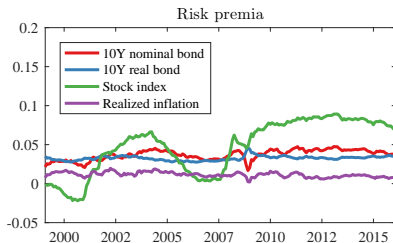
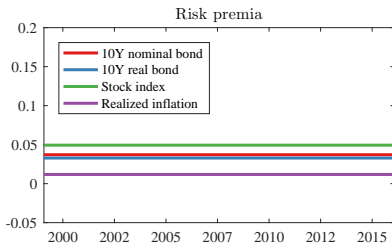
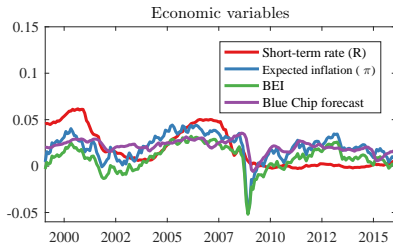


Estimation (constant vs. time-varying risk premia)

Constant risk premia

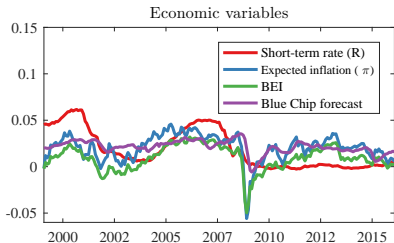


Time-varying risk premia

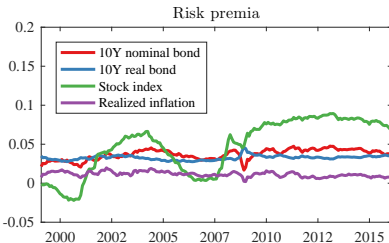
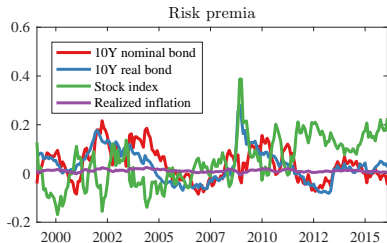
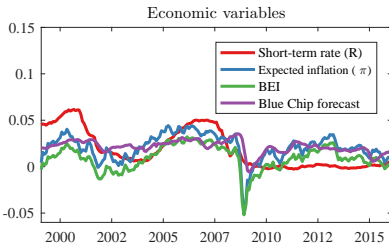


Estimation (time-varying risk premia)

Time-varying risk premia (unconstrained)



Time-varying risk premia (volatility constrained)



Estimation (time-varying risk premia): parameters

Time-varying risk premia with volatility restrictions

R_0	R_1	π_0	π_1	Θ			σ_ϵ^B	σ_ϵ^I
0.0185 (0.0001)	0.3626 (0.0013)	0.0208 (0.0022)	0.0949 (0.0395)	0.0642 (0.0563)	-0.2302 (0.1384)	1.8305 (0.2640)	0.0012 (0.0000)	0.0007 (0.0000)
	-0.4181 (0.0053)		-0.7257 (0.1119)	0.0031 (0.0426)	0.0620 (0.1067)	-0.6949 (0.1989)		
	0.3304 (0.0137)		-1.5538 (0.2107)	-0.0113 (0.0327)	0.1566 (0.0708)	0.6364 (0.1220)		
Λ_0	Λ_1			Σ_X			σ_P	σ_S
-0.6922 (0.0340)	-0.0048 (2.7657)	-15.7462 (6.9130)	-37.4191 (13.1314)	0.0204 (0.0010)	0.0069 (0.0009)	-0.0044 (0.0006)	-0.0001 (0.0007)	0.0365 (0.0105)
0.4902 (0.0547)	1.5536 (3.9027)	25.2406 (12.1339)	53.6283 (23.5016)	0	0.0119 (0.0006)	0.0015 (0.0006)	-0.0008 (0.0007)	-0.0302 (0.0106)
-0.1367 (0.0595)	-1.6353 (3.8535)	-20.0336 (10.7760)	-71.5528 (20.6652)	0	0	0.0085 (0.0004)	-0.0040 (0.0006)	-0.0431 (0.0099)
1.2851 (0.2752)	2.1164 (4.2971)	4.5350 (14.0612)	-79.6940 (28.0800)	0	0	0	0.0088 (0.0004)	-0.0190 (0.0099)
0.7610 (0.2772)	-4.2405 (3.2702)	5.6160 (7.0408)	-16.8258 (14.0275)	0	0	0	0	0.1401 (0.0070)

Estimation (time-varying risk premia): moments

Time-varying risk premia with volatility restrictions

Time series	Mean value		Volatility	
	Estimation	Data	Estimation	Data
3M nominal yield	1.91%	1.87%	0.59%	0.64%
6M nominal yield	1.97%	1.97%	0.59%	0.61%
1Y nominal yield	2.10%	2.15%	0.60%	0.74%
2Y nominal yield	2.37%	2.38%	0.69%	0.85%
3Y nominal yield	2.64%	2.63%	0.79%	0.91%
5Y nominal yield	3.14%	3.12%	0.92%	0.96%
7Y nominal yield	3.55%	3.54%	0.95%	0.96%
10Y nominal yield	3.99%	4.01%	0.90%	0.95%
5Y real yield	1.29%	1.28%	0.90%	0.97%
7Y real yield	1.51%	1.54%	0.87%	0.86%
10Y real yield	1.81%	1.80%	0.83%	0.77%
Log realized inflation	2.08%	2.17%	0.97%	1.07%
Equity log returns	5.46%	4.98%	15.52%	15.72%
3M nominal risk premium	0.12%		0.14%	
6M nominal risk premium	0.25%		0.25%	
1Y nominal risk premium	0.51%		0.44%	
2Y nominal risk premium	1.07%		0.63%	
3Y nominal risk premium	1.62%		0.64%	
5Y nominal risk premium	2.55%		0.38%	
7Y nominal risk premium	3.20%		0.21%	
10Y nominal risk premium	3.70%		0.64%	
5Y real risk premium	2.20%		0.76%	
7Y real risk premium	2.71%		0.64%	
10Y real risk premium	3.26%		0.34%	
Realized inflation risk premium	1.15%		0.50%	
Equity risk premium	4.81%		1.00%	
Nominal risk-free rate	1.85%		0.61%	
Expected inflation	2.08%		1.76%	

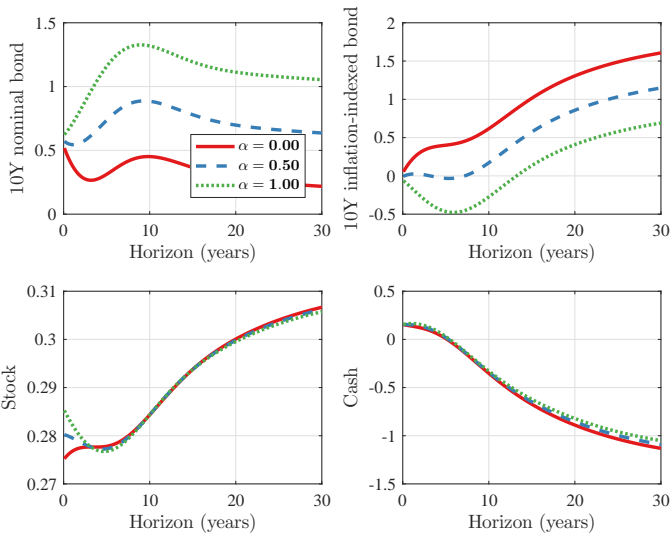
Estimation (time-varying risk premia): correlations

Time-varying risk premia with volatility restrictions

	3M nom	1Y nom	2Y nom	5Y nom	10Y nom	5Y real	10Y real	Equity	CPI	R	π	r
3M nom	1.000											
1Y nom	0.924	1.000										
2Y nom	0.741	0.939	1.000									
5Y nom	0.399	0.710	0.907	1.000								
10Y nom	0.209	0.544	0.787	0.970	1.000							
5Y real	0.127	0.255	0.362	0.507	0.633	1.000						
10Y real	0.087	0.308	0.491	0.693	0.813	0.961	1.000					
Equity	-0.195	-0.287	-0.328	-0.302	-0.238	0.084	-0.006	1.000				
CPI	0.050	-0.052	-0.128	-0.165	-0.123	0.270	0.153	0.046	1.000			
R	-0.974	-0.844	-0.613	-0.238	-0.052	-0.100	-0.012	0.133	-0.113	1.000		
π	-0.341	-0.480	-0.523	-0.417	-0.237	0.567	0.365	0.381	0.423	0.224	1.000	
r	-0.022	0.080	0.139	0.105	-0.029	-0.792	-0.598	-0.293	-0.449	0.113	-0.911	1.000

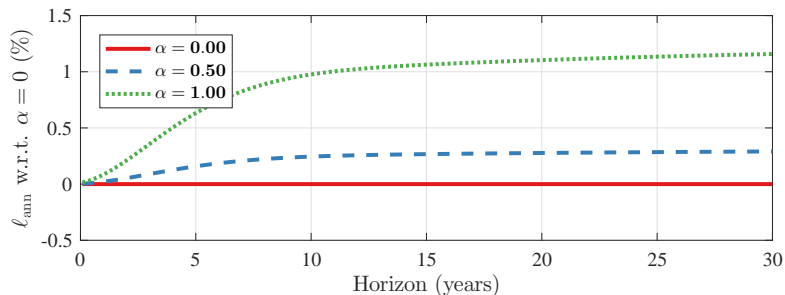
Time-varying risk premia, moderate investor ($\gamma = 10$)

Optimal portfolio strategy



Time-varying risk premia, moderate investor ($\gamma = 10$)

Opportunity cost of money illusion

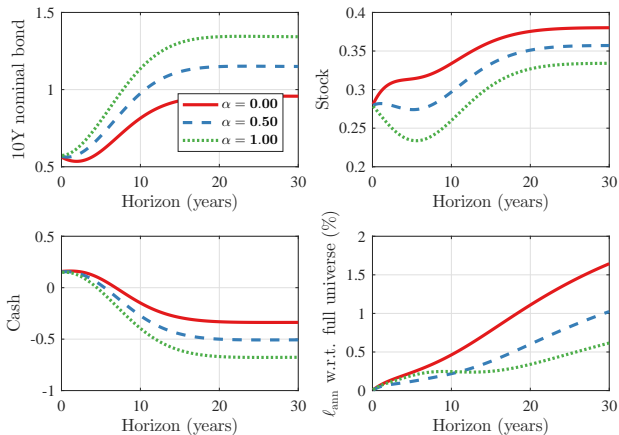


l_{ann} : certainty equivalent annualized loss

- ▶ suffered by a rational non-illusioned investor
- ▶ following the optimal strategy for an agent with a degree α of money illusion

Time-varying risk premia, moderate investor ($\gamma = 10$)

Opportunity cost of not having access to TIPS

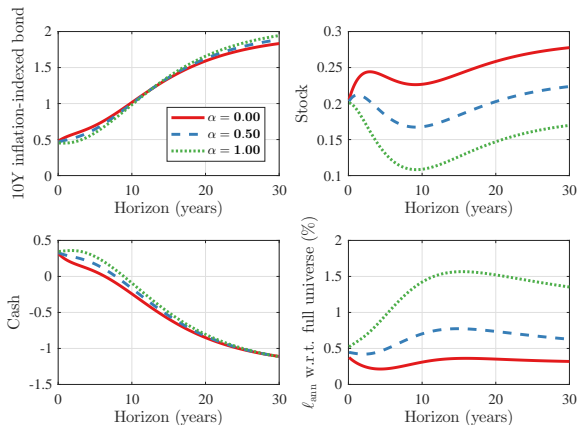


ℓ_{ann} : certainty equivalent annualized loss

- ▶ as perceived by an investor with a degree of money illusion α
- ▶ when the inflation-indexed bond is not accessible

Time-varying risk premia, moderate investor ($\gamma = 10$)

Opportunity cost of not having access to nominal bonds

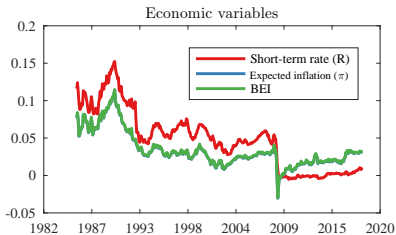


ℓ_{ann} : certainty equivalent annualized loss

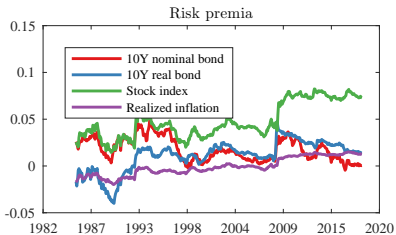
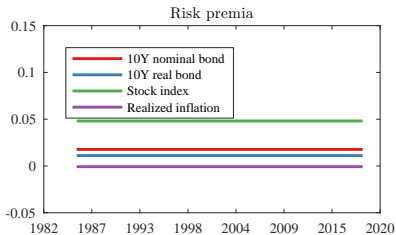
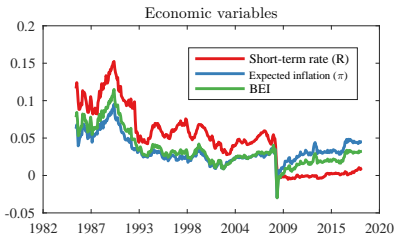
- ▶ as perceived by an investor with a degree of money illusion α
- ▶ when the nominal bond is not accessible

UK, estimation (constant vs. time-varying risk premia)

Constant risk premia

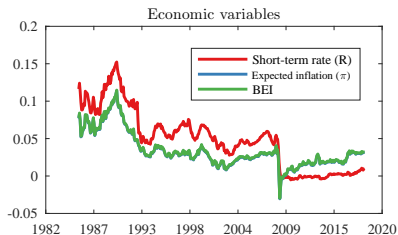


Time-varying risk premia

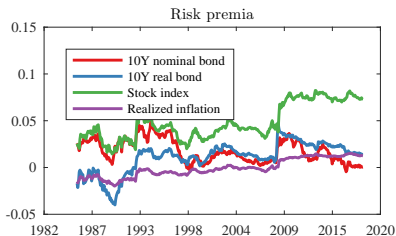
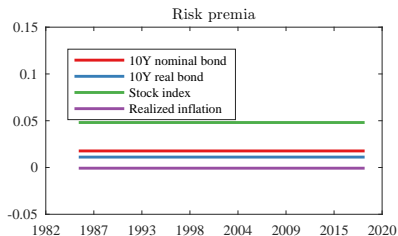
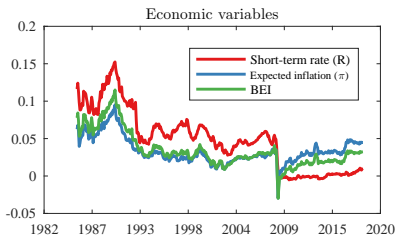


UK, estimation (time-varying risk premia)

Time-varying risk premia (unconstrained)



Time-varying risk premia (volatility constrained)



UK, estimation (time-varying risk premia): parameters

Time-varying risk premia with volatility restrictions

R_0	R_1	π_0	π_1	Θ			σ_ϵ^B	σ_ϵ^I
0.0482 (0.0001)	0.4425 (0.0009)	0.0338 (0.0024)	0.1078 (0.0283)	0.0674 (0.0538)	0.1204 (0.3656)	-0.8229 (0.5294)	0.0009 (0.0000)	0.0009 (0.0000)
	-0.6442 (0.0079)		-1.1201 (0.1903)	-0.0297 (0.0237)	0.1696 (0.1596)	0.0712 (0.2320)		
	-0.7438 (0.0099)		0.3843 (0.2764)	0.0054 (0.0172)	-0.0939 (0.1158)	0.3181 (0.1684)		
Λ_0	Λ_1			Σ_X			σ_P	σ_S
-0.2172 (0.0075)	-0.0513 (2.0332)	-24.5926 (13.9069)	-4.1829 (20.0000)	0.0265 (0.0010)	-0.0005 (0.0006)	0.0009 (0.0004)	0.0007 (0.0007)	-0.0242 (0.0079)
-0.0143 (0.0103)	0.9121 (2.0439)	10.7036 (14.0087)	2.5268 (20.0193)	0	0.0116 (0.0004)	-0.0012 (0.0004)	-0.0002 (0.0007)	-0.0218 (0.0077)
0.0452 (0.0115)	-0.9774 (2.0448)	38.3922 (14.0594)	-14.0737 (19.9555)	0	0	0.0083 (0.0003)	0.0007 (0.0007)	-0.0075 (0.0077)
-0.0410 (0.1775)	-7.7586 (2.0690)	-2.3811 (14.2380)	15.2210 (20.0102)	0	0	0	0.0137 (0.0005)	0.0001 (0.0076)
0.2870 (0.1769)	-1.2047 (2.0720)	-1.4300 (14.2063)	5.9979 (19.9233)	0	0	0	0	0.1493 (0.0053)

UK, Estimation (time-varying risk premia): moments

Time-varying risk premia with volatility restrictions

Time series	Mean value		Volatility	
	Estimation	Data	Estimation	Data
1Y nominal yield	4.94%	4.96%	1.24%	1.18%
2Y nominal yield	5.05%	5.03%	1.11%	1.15%
3Y nominal yield	5.16%	5.14%	1.05%	1.12%
5Y nominal yield	5.33%	5.34%	1.02%	1.07%
7Y nominal yield	5.48%	5.49%	1.03%	1.03%
10Y nominal yield	5.63%	5.62%	1.05%	0.97%
5Y real yield	1.68%	1.67%	0.89%	0.91%
7Y real yield	1.77%	1.79%	0.81%	0.79%
10Y real yield	1.90%	1.89%	0.72%	0.68%
Log realized inflation	3.37%	3.34%	1.38%	1.46%
Equity log returns	8.45%	9.28%	15.30%	15.31%
1Y nominal risk premium	0.25%		0.53%	
2Y nominal risk premium	0.47%		0.84%	
3Y nominal risk premium	0.68%		1.00%	
5Y nominal risk premium	1.05%		1.05%	
7Y nominal risk premium	1.36%		0.97%	
10Y nominal risk premium	1.77%		1.00%	
5Y real risk premium	0.59%		0.97%	
7Y real risk premium	0.81%		1.00%	
10Y real risk premium	1.11%		0.90%	
Realized inflation risk premium	-0.07%		0.32%	
Equity risk premium	4.81%		1.00%	
Nominal risk-free rate	4.82%		1.45%	
Expected inflation	3.38%		1.43%	

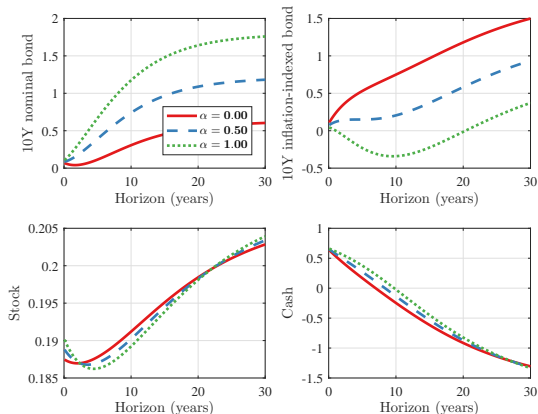
UK, estimation (time-varying risk premia): correlations

Time-varying risk premia with volatility restrictions

	1Y nom	2Y nom	5Y nom	10Y nom	5Y real	10Y real	Equity	CPI	R	π	r
1Y nom	1.000										
2Y nom	0.983	1.000									
5Y nom	0.834	0.920	1.000								
10Y nom	0.651	0.768	0.949	1.000							
5Y real	0.333	0.418	0.471	0.313	1.000						
10Y real	0.385	0.513	0.666	0.585	0.942	1.000					
Equity	0.074	0.107	0.156	0.152	0.185	0.213	1.000				
CPI	-0.055	-0.062	-0.076	-0.088	0.023	-0.008	-0.004	1.000			
R	-0.981	-0.929	-0.716	-0.509	-0.227	-0.235	-0.037	0.047	1.000		
π	-0.491	-0.434	-0.341	-0.388	0.635	0.474	0.084	0.080	0.538	1.000	
r	-0.409	-0.413	-0.289	-0.029	-0.897	-0.714	-0.119	-0.045	0.385	-0.569	1.000

UK, time-varying risk premia, moderate investor ($\gamma = 10$)

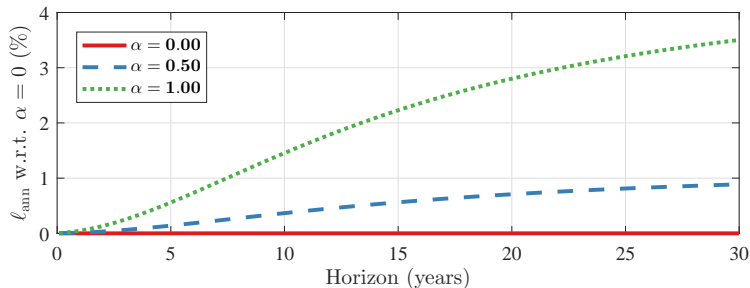
Optimal portfolio strategy



- ▶ UK sample longer than U.S. sample (from 1985 to 2018)
- ▶ Negative inflation risk premium (-0.07% vs 1.18% in U.S.)
- ▶ Higher realized inflation volatility (1.46% vs 1.07% in U.S.)

UK, time-varying risk premia, moderate investor ($\gamma = 10$)

Opportunity cost of money illusion

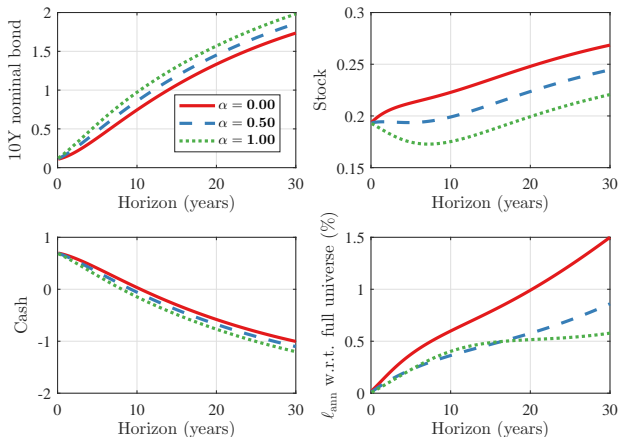


ℓ_{ann} : certainty equivalent annualized loss

- ▶ higher cost of money illusion than for U.S. sample
- ▶ due to high inflation volatility and negative inflation premium

UK, time-varying risk premia, moderate investor ($\gamma = 10$)

Opportunity cost of not having access to TIPS

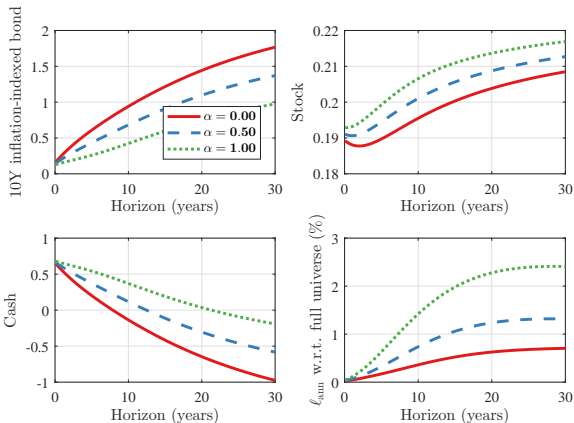


ℓ_{ann} : certainty equivalent annualized loss

- ▶ opportunity cost similar to U.S. analysis for long horizons

UK, time-varying risk premia, moderate investor ($\gamma = 10$)

Opportunity cost of not having access to nominal bonds



ℓ_{ann} : certainty equivalent annualized loss

- ▶ as perceived by an investor with a degree of money illusion α
- ▶ when the nominal bond is not accessible