Welfare-enhancing properties of countercyclical bank capital requirements*

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Abstract

The paper studies optimal macroprudential policy in a New Keynesian model with risk-averse bankers, entrepreneurs and households—an extension of Iacoviello (2015). Bankers manage banks that are subject to bank capital requirements, while entrepreneurs are collateral constrained. There is no systemic risk and thus no standard rationale for macroprudential policy. The Ramsey optimal policy, however, prescribes countercyclical capital requirements, is Pareto improving (relative to the economy with constant capital requirements) and extremely effective in alleviating the inefficiency of the decentralized equilibrium arising due to the binding collateral constraint. Entrepreneurs reap the benefits of the optimal policy, while bankers, and especially households, gain significantly less. Nominal rigidities and capital adjustment costs are crucial for the optimal policy to matter. Suboptimal practical policy rules are not Pareto improving, but still lead to sizable welfare gains for entrepreneurs and are countercyclical. Such rules are consistent with the countercyclical capital buffer of Basel III; however, the credit-to-GDP ratio proves to be a worse cyclical indicator than credit or asset prices.

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1 Introduction

The importance of macroprudential dimension of financial regulation was emphasized as early as by Crockett (2000)[1], but it was not until the recent global financial crisis that the problem started receiving substantial attention from both policymakers and academic economists. The main objective of macroprudential policy is to battle systemic risk. Several measures have been proposed, such as the countercyclical capital buffer of Basel III.[2] The buffer must be built up by banks in "good times" and can be used as a safety net in "bad times".[3] Since an increase in the capital requirement depresses credit and economic growth, such a buffer is countercyclical. Is there a role for such policy when there is no default risk, and thus no systemic risk whatsoever? This paper argues that the answer is yes.

I build on the framework of Iacoviello (2015), augmenting it with nominal rigidities. The model features three risk-averse welfare-relevant agents: bankers, entrepreneurs and households. Bankers manage banks which operate subject to a regulatory requirement on the minimum amount of bank capital. Entrepreneurs produce the wholesale good. They can borrow from banks to consume, invest or pay labor costs. Borrowing is limited by a collateral constraint as in Kiyotaki and Moore (1997). Specifically, entrepreneurs pledge the expected future value of their capital stock as collateral. Both bankers and entrepreneurs have finite expected lifetimes, and thus are more impatient than households. As a result, borrowing constraints bind in the neighborhood of the steady state. First, this leads to the inability of bankers and entrepreneurs to smooth consumption across time and states. Second, the binding collateral constraint generates a loop between the price of capital and the value of collateral, resulting in the financial amplification/accelerator, similar to Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999). Under the assumption of log utility, in equilibrium, all choice variables of bankers and entrepreneurs are proportional to their net worth.

Optimal state-contingent bank capital requirements address the inefficiency of the decentralized equilibrium arising due to the binding collateral constraint. An increase in capital requirements in the time of expansion puts an upward pressure on the loan interest rate, makes entrepreneurs to borrow and invest less, which through the financial amplification loop makes borrowing even harder and stabilizes entrepreneurs' consumption and the price of capital. The same mechanism works in recessions to relax the collateral constraint and prevent sharp drops in consumption and the asset price. Quantitatively, I find that under the baseline calibration, the Ramsey optimal policy is Pareto improving relative to the decentralized equilibrium with the same steady state level of capital requirements, leading to a 0.88% welfare gain in consumption equivalents: 0.32% gain for bankers, 8.69% gain for entrepreneurs, and 0.02% gain for households. Both bankers and households benefit from the increased macroeconomic stability, but their benefits are limited for the following reasons. Dynamic bank capital requirements make bankers' net worth and consumption

[1]See also Borio (2003).

[2]It is currently in the process of implementation: `https://www.bis.org/bcbs/ccyb/`.

[3]Whether the time is good or bad is determined by the deviation of the credit-to-GDP ratio from trend. The usefulness of the latter as an indicator of systemic risk has been questioned by Repullo and Saurina (2012).

more volatile. Households are not directly subject to financial frictions and lose from the increased inflation volatility after TFP shocks. The latter happens because the optimal macroprudential policy is always countercyclical, while monetary policy is procyclical after TFP shocks.

Nominal rigidities and capital adjustment costs are crucial for the countercyclical optimal policy to arise and to matter. If there are no capital adjustment costs, the price of capital is constant, and the financial accelerator mechanism is shut down. If capital adjustment costs do exist, but there are no nominal rigidities, financial accelerator does not reach its full extent. The reason is that the increasing amplification loop increases the expected inflation rate, which relaxes the collateral constraint even more, with the opposite effect in recessions. Quantitatively, I find that in a model with flexible prices and zero capital adjustment costs, the overall welfare gain from the optimal policy is 0.001%: 0.052% for bankers, -0.006% for entrepreneurs, and -0.005% for households. In the model with flexible prices, but with capital adjustment costs, under the baseline calibration, the overall welfare gain from the optimal policy is 0.03%: 0.14% for bankers, 0.39% for entrepreneurs, and -0.03% for households. The overall gain is not practically sensitive to the magnitude of adjustment costs, changing from 0.00% to 0.05% for a wide range of values of the adjustment cost parameter (the higher are the adjustment costs, the higher are the gains).

I also consider suboptimal policy rules in the spirit of the countercyclical capital buffer of Basel III. As possible cyclical indicators I choose output, credit, credit-to-output ratio and the price of capital, following Drehmann et al. (2010) and (BCBS, 2010b). I consider both current values of these variables and lags up to four (quarters). I search for the policy rule parameters that maximize social welfare. The optimized policy rules are not Pareto improving, but the overall welfare gains are almost as high as in the case of optimal policy. The price of capital and credit prove to be the best cyclical indicators among those considered. For example, the optimized rule based on credit achieves the overall welfare gain of 0.82%: -0.16% for bankers, 8.32% for entrepreneurs, and 0.05% for households.

Importantly, and this is a limitation of the current version of this study, monetary policy is postulated in the form of a Taylor rule with "conventional" response parameters, and is taken as a constraint by the Ramsey planner. The overall results, however, are qualitatively unchanged under strict or flexible inflation targeting and different magnitudes of response parameters. The key findings are also robust to different values of the steady state loan-to-value ratio and different magnitudes of capital adjustment costs.

This paper is mainly related to the literature studying bank capital requirements from a macroprudential perspective. Angeloni and Faia (2013) and Angelini, Neri and Panetta (2014) use New Keynesian models with and without default risk, respectively, to compare different monetary and macroprudential (suboptimal) policy rules. They find that the rules that lead to higher welfare feature countercyclical macroprudential policy. They do not study the Ramsey optimal policy. On the contrary, Martinez-Miera and Suarez (2014) and Collard et al. (2017) find that procyclically adjusted capital requirements are preferable to prevent excessive risk-taking in models with default risk. Martinez-Miera and Suarez (2014) only focus on policy rules, while Collard et al. (2017) study the Ramsey optimal policy. The latter paper is the most relevant to my analysis. Collard et al. (2017), however, have an entirely different model, where the credit market equilibrium is determined through static optimization problems that allow for tractable modeling of limited liability and default risk. In their model, capital good "furbishers" need debt funding to cover the full cost of "unfurbished" capital, do not choose the optimal amount of borrowing, and are not subject to a collateral constraint of Kiyotaki and Moore (1997). The conventional financial accelerator mechanism is thus absent from their framework, hence macroprudential policy does not have a specific role that I investigate in this paper. Finally, in their model the representative household is the only welfare-relevant agent, unlike in my framework. Our analyses can thus be seen as complementary to each other. A related paper is also by Mendicino et al. (2018) who build on Bernanke, Gertler and Gilchrist (1999) and feature default risk both on the borrower and lender sides. They restrict attention to policy rules and find that optimized rules are either constant or procyclical, which depends on particular welfare weighting schemes. They abstract from nominal rigidities and monetary policy.

Another strand of literature that is also related is about macroprudential policy as a tool to prevent overborrowing in the decentralized equilibrium due to pecuniary externalities arising from particular types of borrowing constraints. For example, the work of Lorenzoni (2008), Bianchi (2011), Dávila and Korinek (2018), Bianchi and Mendoza (2018), Jeanne and Korinek (2018). Since my framework features a collateral constraint and a similar pecuniary externality, their insights are relevant to my analysis. However, the collateral constraint is binding in equilibrium, hence the mechanism is different. A slightly related fact is that the Ramsey optimal policy features bank capital requirements that are around 17% in the steady state, which implies lower equilibrium borrowing amounts than in the baseline calibration with a 10% capital requirement reflecting the Basel accords.

The paper is structured as follows. Section 2 describes the model and its calibration. Section 6 presents the welfare analysis. Section 7 contains sensitivity checks. Section 8 concludes.

2 Model

There are three types of risk-averse agents: bankers, entrepreneurs and households. There is a continuum of measure one of identical agents of each type. The time horizon is infinite. Following Blanchard (1985), there is a constant probability $\delta_i \in [0, 1)$ that an agent of type $i \in \{b, e, h\}$ dies at the end of any period.[4] The probabilities are the same for agents of the same type, but different across types. Each period a measure δ_i of agents of type i dies, while the same measure of new agents is born, so that the measure of agents remains constant over time. Agents discount period utilities using the discount factor $\beta \in (0, 1)$, while the adjusted discount factors are $\beta_i \equiv \beta(1 - \delta_i)$.

Bankers and entrepreneurs are borrowing constrained. Bankers manage banks that intermediate funds between households and entrepreneurs. They operate subject to a regulatory constraint on

[4] Andrés, Arce and Thomas (2013) and Gertler and Kiyotaki (2015) use similar assumptions.

the minimum amount of bank capital. Entrepreneurs manage firms producing the wholesale good. They can take a loan from banks, but the amount of a loan is limited such that the expected repayment does not exceed a share of the expected future value of entrepreneurs' capital stock. Households save through bank deposits.

There are also retailers who purchase the wholesale good from entrepreneurs and transform it into a specific variety of a differentiated good which they sell to final good producers. Retailers can set a new price for their varieties only with a probability $1-\theta$ each period. Final good producers bundle the continuum of varieties into a final good and sell it to bankers, entrepreneurs, households and capital good producers. A central bank sets the deposit rate in the economy. A macroprudential authority specifies the prudential requirement for bank capital.

3 Bankers

Following Iacoviello (2015), bankers consume the final good and manage banks, taking deposits from households and providing loans to entrepreneurs. A bank has loans on the asset side, deposits and capital on the liability side. There is a regulatory constraint on the minimum amount of bank capital as a share of loans. The existence of such a constraint is taken as a fact, reflecting the real world practice, but it can be rationalized in various ways.[5] A banker chooses $\left\{C_t^b, D_t, L_t\right\}_{t=0}^{\infty}$—where C_t^b is consumption[6], D_t are deposits, L_t are loans—to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} {\beta_b}^t U_b\left(C_t^b\right)$$

subject to

$$C_t^b + L_t \leq NW_t^b + D_t, \tag{1}$$

$$L_t - D_t \geq k_t^m L_t, \tag{2}$$

where

$$NW_t^b \equiv \frac{R_t^l}{\Pi_t} L_{t-1} - \frac{R_{t-1}}{\Pi_t} D_{t-1}, \tag{3}$$

$\mathbb{E}_t(\cdot) \equiv \mathbb{E}\left(\cdot \mid \mathbb{I}_t\right)$ and $\mathbb{I}_t$ is the information set at t, $U_b(\cdot)$ is strictly concave and strictly increasing (hence, (1) binds), NW_t^b is net worth (loan repayments minus deposit repayments), k_t^m denotes the regulatory capital requirement, R_t^l is the gross nominal loan rate, Π_t is the gross inflation rate, R_t is gross nominal deposit rate. The bank capital, $L_t - D_t$, is a part of net worth left after consumption.

[5]In the model of Holmstrom and Tirole (1997), a certain amount of intermediary capital is required for credible monitoring of entrepreneurs. In the models of Gertler and Kiyotaki (2010) and Gertler and Karadi (2011), a banker can divert a fraction of their assets. For lenders to be willing to lend, a banker must abide by an incentive constraint that creates an endogenous capital requirement.

[6]All variables are in real terms (in units of the final good), unless stated otherwise.

The demand for deposits and supply of loans are described by the Euler equations

$$\frac{\gamma_t^b}{U_b'\left(C_t^b\right)} = 1 - \mathbb{E}_t\left\{\beta_b \frac{U_b'\left(C_{t+1}^b\right)}{U_b'\left(C_t^b\right)}\frac{R_t}{\Pi_{t+1}}\right\}, \tag{4}$$

$$\frac{\gamma_t^b}{U_b'\left(C_t^b\right)}(1-k_t^m) = 1 - \mathbb{E}_t\left\{\beta_b \frac{U_b'\left(C_{t+1}^b\right)}{U_b'\left(C_t^b\right)}\frac{R_{t+1}^l}{\Pi_{t+1}}\right\}, \tag{5}$$

where $\gamma_t^b \geq 0$ is the Lagrange multiplier on the regulatory constraint (2). If the capital requirement constraint is binding, bankers' ability to smooth consumption is distorted.

A detailed characterization of the bankers' problem is presented in section 4.

3.1 Entrepreneurs

As in Bernanke, Gertler and Gilchrist (1999), entrepreneurs manage firms that use physical capital and labor to produce the wholesale good which is sold to the continuum of retailers. Entrepreneurs consume the final good as in Iacoviello (2005) and can borrow from banks subject to a collateral constraint, following Kiyotaki and Moore (1997). An entrepreneur chooses $\{C_t^e, K_t, L_t, N_t\}_{t=0}^{\infty}$—where C_t^e is consumption, K_t is capital, N_t is labor—to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} {\beta_e}^t U_e(C_t^e)$$

subject to

$$C_t^e + Q_t K_t \leq NW_t^e + L_t, \tag{6}$$

$$\mathbb{E}_t\left\{R_{t+1}^l L_t\right\} \leq m_t \mathbb{E}_t\left\{Q_{t+1}\Pi_{t+1}K_t\right\}, \tag{7}$$

where

$$NW_t^e \equiv \frac{Y_t^w}{X_t} - W_t N_t + Q_t(1-\delta)K_{t-1} - \frac{R_t^l}{\Pi_t}L_{t-1}, \tag{8}$$

$$Y_t^w \equiv A_t {K_{t-1}}^{\alpha} {N_t}^{1-\alpha}, \tag{9}$$

$U_e(\cdot)$ is strictly concave and strictly increasing (hence, (6) binds), Q_t is the price of capital, NW_t^e is net worth (the revenue from selling the wholesale good net wage payments plus the value of the underpreciated capital minus loan repayments), m_t is the LTV ratio, Y_t^w is wholesale output, X_t is the gross markup of the final good price over the wholesale good price, W_t is wage, $\delta \in [0,1]$ is the depreciation rate, A_t is TFP, $\alpha \in (0,1)$ is the capital share.

The optimality conditions are

$$W_t = (1-\alpha)\frac{Y_t^w}{N_t X_t}, \tag{10}$$

$$Q_t = \mathbb{E}_t\left\{\beta_e \frac{U_e'(C_{t+1}^e)}{U_e'(C_t^e)}\left(\alpha\frac{Y_{t+1}^w}{K_t X_{t+1}} + (1-\delta)Q_{t+1}\right)\right\} + \frac{\gamma_t^e}{U_e'(C_t^e)} m_t \mathbb{E}_t\left\{Q_{t+1}\Pi_{t+1}\right\}, \tag{11}$$

$$\frac{\gamma_t^e}{U_e'(C_t^e)}\mathbb{E}_t\{R_{t+1}^l\} = 1 - \mathbb{E}_t\left\{\beta_e \frac{U_e'(C_{t+1}^e)}{U_e'(C_t^e)}\frac{R_{t+1}^l}{\Pi_{t+1}}\right\}, \tag{12}$$

where $\gamma_t^e \geq 0$ is the Lagrange multiplier on the collateral constraint (7).

Equation (10) represents labor demand which, given X_t, is a decreasing convex function of the real wage. Similarly, other things equal, demand for capital, described by (11), is a decreasing convex function of the price of capital. If $\gamma_t^e > 0$, so that the collateral constraint is binding, then the price of capital reflects not only its expected marginal product, but also its marginal value as collateral. Equation (12) describes the demand for loans. If $\gamma_t^e > 0$, the intertemporal consumption smoothing is distorted and the financial accelerator makes entrepreneurs' consumption very volatile.

As with bankers, the detailed characterization of entrepreneurs' problem is left for section 4.

3.2 Households

Households are not directly affected by financial frictions, and their problem is standard. They consume the final good, supply labor to entrepreneurs and save through bank deposits. A household chooses $\left\{C_t^h, D_t, N_t\right\}_{t=0}^{\infty}$, where C_t^h is consumption, to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} {\beta_h}^t U_h\left(C_t^h, N_t\right)$$

subject to

$$C_t^h + D_t \leq W_t N_t + \frac{R_{t-1}}{\Pi_t} D_{t-1} + \Xi_t - T_t, \tag{13}$$

where $U_h(\cdot)$ is strictly concave, strictly increasing in the first argument (hence, (13) binds) and strictly decreasing in the second argument, Ξ_t are profits of capital good producers and retailers, T_t are lump-sum taxes.

The optimality conditions are

$$\frac{\partial U_h}{\partial N_t} + W_t \frac{\partial U_h}{\partial C_t^h} = 0, \tag{14}$$

$$\mathbb{E}_t\left\{\beta_h \frac{\partial U_h/\partial C_{t+1}^h}{\partial U_h/\partial C_t^h}\frac{R_t}{\Pi_{t+1}}\right\} = 1, \tag{15}$$

where $\partial U_h/\partial C_{t+j}^h$ should be understood as $\partial U_h\left(C_{t+j}^h, N_{t+j}\right)/\partial C_{t+j}^h$.

Equation (14) describes labor supply, (15)—intertemporal consumption choice.

3.3 Capital good producers

Capital good producers make physical capital using the final good as an input and are owned by households. They maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left\{ Q_t I_t \left[1 - f\left(\frac{I_t}{I_{t-1}} \right) \right] - I_t \right\}$$

where I_t is investment, $f(1) = f'(1) = 0$, $f'' > 0$, and

$$\Lambda_{t,s} \equiv \beta^{s-t} \frac{\partial U_h / \partial C_s^h}{\partial U_h / \partial C_t^h}$$

for any $s \geq t$. The adjustment cost is modeled as in Andrés, Arce and Thomas (2017).

The supply of capital is described by

$$1 = Q_t \left[1 - f\left(\frac{I_t}{I_{t-1}} \right) - f'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + \mathbb{E}_t \left\{ \Lambda_{t,t+1} Q_{t+1} f'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right\}. \quad (16)$$

3.4 Final good producers

Final good producers purchase retail good varieties to produce the final good using the production function $Y_t \equiv \left(\int_0^1 Y_t(i)^{(\epsilon-1)/\epsilon} di \right)^{\epsilon/(\epsilon-1)}$, where $Y_t(i)$ is the quantity of variety i and $\epsilon > 1$ is the elasticity of substitution between varieties. Minimizing costs $\int_0^1 P_t(i) Y_t(i) di$, where $P_t(i)$ is the price of variety i, subject to the production technology leads to the demand for variety i of the form $Y_t(i) = Y_t (P_t(i)/P_t)^{-\epsilon}$, where $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{1/(1-\epsilon)}$ is the aggregate price level.

3.5 Retailers

There is a measure one continuum of retailers owned by households. A retailer buys the wholesale good, differentiates it and sells its variety to final good producers. The price of each variety "survives" to the next period with a probability $\theta \in [0, 1]$ as in Calvo (1983). The profits in a period $t + j$ under a price P_t^* set in a period t are $\Pi_{t+j|t} \equiv \left(P_t^* / P_{t+j} - 1/X_{t+j} \right) Y_{t+j} \left(P_t^* / P_{t+j} \right)^{-\epsilon}$. Let $V_{j|k}$, where $j \geq k$, denote the value of a retailer in a period j under a price set in a period k. If able to do so, a retailer chooses P_t^* to maximize $V_{t|t} = \Pi_{t|t} + \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left(\theta V_{t+1|t} + (1-\theta) V_{t+1|t+1} \right) \right\}$. The optimality condition is $\partial V_{t|t} / \partial P_t^* = \partial \Pi_{t|t} / \partial P_t^* + \mathbb{E}_t \left\{ \Lambda_{t,t+1} \theta \partial V_{t+1|t} / \partial P_t^* \right\} = 0$ or

$$\mathbb{E}_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \theta^j \frac{\partial \Pi_{t+j|t}}{\partial P_t^*} = 0,$$

which can be represented as a system

$$\Pi_t^* = \frac{\epsilon}{\epsilon - 1}\frac{\Omega_{1,t}}{\Omega_{2,t}}, \tag{17}$$

$$\Omega_{1,t} = {\Pi_t}^{\epsilon}\left(\frac{\partial U_h}{\partial C_t^h}\frac{Y_t}{X_t} + \beta_h\theta\mathbb{E}_t\left\{\Omega_{1,t+1}\right\}\right), \tag{18}$$

$$\Omega_{2,t} = {\Pi_t}^{\epsilon-1}\left(\frac{\partial U_h}{\partial C_t^h}Y_t + \beta_h\theta\mathbb{E}_t\left\{\Omega_{2,t+1}\right\}\right), \tag{19}$$

where $\Pi_t^* \equiv P_t^*/P_{t-1}$.

As the price survival probability is constant across retailers and across time and there is a continuum of retailers, $P_t = \left(\theta {P_{t-1}}^{1-\epsilon} + (1-\theta){P_t^*}^{1-\epsilon}\right)^{1/(1-\epsilon)}$ or

$${\Pi_t}^{1-\epsilon} = \theta + (1-\theta){\Pi_t^*}^{1-\epsilon}. \tag{20}$$

3.6 Government

The central bank is assumed to be following a simple interest rate rule of the form

$$R_t = \bar{R}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi}, \tag{21}$$

where R and $\bar{\Pi}$ are the steady state values of R_t and Π_t, $\phi_\pi \geq 0$, but in practice the restriction is stronger for the model to have a unique solution. Strict inflation targeting is assumed for simplicity and to isolate the effect of macroprudential policy as much as possible, while still preserving nominal rigidities. In section 7.1, I explore how flexible inflation targeting affects the results.

The budget of the fiscal authority is

$$g_t Y_t = T_t, \tag{22}$$

where g_t is an exogenous process determining the share of government expenditures to GDP.

3.7 Market clearing

Market clearing in deposits, loans, labor and retail varieties markets is already imposed in the text.

Market clearing in the capital goods market requires the supply of capital to be equal to the entrepreneurs' investment demand:

$$I_t\left[1 - f\left(\frac{I_t}{I_{t-1}}\right)\right] = K_t - (1-\delta)K_{t-1}. \tag{23}$$

Market clearing in the wholesale goods market equates the supply of wholesale goods to the aggregate demand from retailers : $Y_t^w = \int_0^1 Y_t(i)di$. Substituting the demand for variety i, we get

$$Y_t^w = Y_t\Delta_t, \tag{24}$$

where $\Delta_t \equiv \int_0^1 (P_t(i)/P_t)^{-\epsilon}\, di$. Taking into account the same considerations as for deriving equation (20), Δ_t can be expressed recursively:

$$\Delta_t = \theta {\Pi_t}^{\epsilon} \Delta_{t-1} + (1-\theta)\left(\frac{\Pi_t}{\Pi_t^*}\right)^{\epsilon}. \tag{25}$$

By Walras' law, the final goods market clears automatically:

$$(1-g_t)Y_t = C_t^b + C_t^e + C_t^h + I_t. \tag{26}$$

4 Decentralized equilibrium

The definition of a competitive equilibrium for a given macroprudential policy $\{\{k_t^m(s^t)\}_{s^t \in S^t}\}_{t=0}^{\infty}$ is standard. Let me instead focus on its properties.

First, consider the steady state equilibrium: all variables are constant, and there are no shocks. Let $\bar{k}^m$ be the value of k_t^m in such a steady state, and let

$$\tilde{\beta}_e \equiv \frac{\beta_h}{1 + \bar{k}^m \left(\frac{\beta_h}{\beta_b} - 1\right)}.$$

Then the following results hold.

Proposition 1. *There exists a steady state decentralized equilibrium with* $k_t^m = \bar{k}^m \ \forall t \iff \left(\beta_b \leq \beta_h \wedge \beta_e \leq \tilde{\beta}_e\right)$.

Proof. Appendix. □

Proposition 2. *There exists a* ***unique*** *steady state decentralized equilibrium with* $k_t^m = \bar{k}^m \ \forall t \iff \left(\beta_b < \beta_h \wedge \beta_e < \tilde{\beta}_e\right)$.

Proof. Appendix. □

Now, let me come back to the stochastic environment. Consider, first, the bankers' problem.

Proposition 3. *Suppose* $U_b(\cdot) \equiv \ln(\cdot)$ *and* $\beta_b < \beta_h$. *Then in a "neighborhood" of the steady state,*

$$C_t^b = (1-\beta_b) NW_t^b, \tag{27}$$

$$D_t = \beta_b \frac{1-k_t^m}{k_t^m} NW_t^b, \tag{28}$$

$$L_t = \frac{\beta_b}{k_t^m} NW_t^b, \tag{29}$$

$$NW_t^b = \frac{\beta_b}{\Pi_t}\left(\frac{R_t^l - R_{t-1}}{k_{t-1}^m} + R_{t-1}\right) NW_{t-1}^b. \tag{30}$$

Proof. Appendix C. □

Consumption is a constant share of net worth. A higher discount factor makes a banker more patient and willing to consume more tomorrow instead of today which requires supplying more loans and demanding more deposits. Deposits and loans are proportional to net worth too, and the coefficient of proportionality is decreasing in capital requirements because bankers are required to hold more bank capital. For a given level of inflation, an increase in the equilibrium loan rate increases net worth. Then for a given level of capital requirements, supply of loans increases. Therefore, keeping inflation and capital requirements constant, supply of loans is an increasing linear function of the loan rate. Similarly, demand for deposits is a decreasing linear function of the deposit rate, keeping inflation, loan rate and capital requirements constant. Equation (30), describing the evolution of net worth, is sufficient to characterize bankers' optimal decisions.

An analogous result holds for entrepreneurs.

Proposition 4. *Suppose $U_e(\cdot) \equiv \ln(\cdot)$ and $\beta_e < \widetilde{\beta}_e$. Then in a "neighborhood" of the steady state,*

$$C_t^e = (1-\beta_e) NW_t^e, \tag{31}$$

$$K_t = \eta_t NW_t^e, \tag{32}$$

$$L_t = (Q_t \eta_t - \beta_e) NW_t^e, \tag{33}$$

$$N_t = \left[\frac{(1-\alpha)A_t}{W_t X_t}\right]^{\frac{1}{\alpha}} \eta_{t-1} NW_{t-1}^e, \tag{34}$$

$$NW_t^e = \left\{\alpha \frac{A_t}{X_t}\left[\frac{(1-\alpha)A_t}{W_t X_t}\right]^{\frac{1-\alpha}{\alpha}} \eta_{t-1} + Q_t(1-\delta)\eta_{t-1} - \frac{R_t^l}{\Pi_t}(Q_{t-1}\eta_{t-1} - \beta_e)\right\} NW_{t-1}^e, \tag{35}$$

where

$$\eta_t \equiv \beta_e \left(Q_t - \frac{m_t \mathbb{E}_t\{Q_{t+1}\Pi_{t+1}\}}{\mathbb{E}_t\left\{R_{t+1}^l\right\}} \right)^{-1}.$$

Proof. Appendix D. □

Consumption is a constant share of net worth. When the collateral constraint is binding, the movements in the price of capital directly affect net worth and consumption making them more volatile. The demands for capital, labor and loans are proportional to net worth too. The coefficients of proportionality are positively related to η_t which is increasing in $\beta_e, m_t, \mathbb{E}_t\{Q_{t+1}\Pi_{t+1}\}$ and decreasing in Q_t and $\mathbb{E}_t\left\{R_{t+1}^l\right\}$. A higher discount factor means that an entrepreneur is more patient and prefers to consume more in the future, thus needing to borrow and produce more today. A higher LTV ratio or expected future price of capital directly relaxes the collateral constraint, allowing to borrow and produce more. The higher price of capital decreases the demand for factors of production. As for loans, there is also an opposite effect because the amount paid when purchasing capital is $Q_t K_t$, hence there is a direct effect through an increased price of capital and an indirect through a decreased quantity. The higher expected loan rate makes borrowing less attractive and decreases factor demands. As in the case of bankers, the recursive dynamics of net worth (35) is sufficient to summarize the optimal conditions.

Finally, market clearing in the loan market—the equality of (29) and (33)—implies that bankers' consumption is proportional to entrepreneurs' consumption:

$$C_t^b = \frac{1-\beta_b}{\beta_b(1-\beta_e)}(Q_t\eta_t - \beta_e)k_t^m C_t^e. \tag{36}$$

The relative consumption share is increasing in capital requirements, price of capital and η_t. The higher capital requirements make bankers supply less loans as a share of net worth and consume more today which makes the consumption share increase. The higher $Q_t\eta_t$ increases the demand for loans from entrepreneurs, and they tend to consume less today, so that bankers tend to consume relatively more.

5 Quantitative properties

The deterministic steady state of the model is described in Appendix B. I assume that $U_h(\cdot)$ has a constant relative risk aversion (RRA) property for consumption, while disutility from supplying labor is additively separable: $U_h\left(C_t^h, N_t\right) \equiv \left({C_t^h}^{1-\sigma} - 1\right)/(1-\sigma) - {N_t}^{1+\phi}/(1+\phi)$, where $\sigma > 0$ is the RRA coefficient, $\phi \geq 0$ is the inverse Frisch elasticity of labor supply, and it is understood that the period utility from consumption is equal to $\ln C_t^h$ if $\sigma = 1$.

$$f\left(\frac{I_t}{I_{t-1}}\right) = \frac{\phi_k}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2,$$

where $\phi_k \geq 0$.

$$\begin{aligned}
\ln(A_t) &= \rho_a \ln(A_{t-1}) + (1-\rho_a)\ln(A) + \epsilon_t^a, \\
\ln(m_t) &= \rho_m \ln(m_{t-1}) + (1-\rho_m)\ln(m) + \epsilon_t^m,
\end{aligned}$$

where for $i \in \{a, m\}$, $\rho_i \in [0,1)$, $\epsilon_t^i \sim \mathcal{N}(0, {\tau_i}^2)$, $\tau_i > 0$, where ϵ_t^i are independent random variables in i and t. A and m are the steady state values of A_t and m_t, as well as their approximate expected values.[7]

5.1 Parameter values

The parameter values that I set are outlined in Table 1. The capital share (α), discount factor (β) and capital depreciation rate (δ) take standard values when the analysis is applied to the US economy using quarterly data.

Bankers' adjusted discount factor (β_b) is set to a value similar to Gertler and Kiyotaki (2015) and Iacoviello (2015). It implies a steady state credit spread of about 1.8%. As for the adjusted discount

[7] For $x \in \{A, m\}$, x_t is log-normal with mean $x \exp\left\{{\tau_i}^2/\left[2\left(1-{\rho_i}^2\right)\right]\right\}$, which is very close to x for reasonable parameter values.

factor of entrepreneurs (β_e), it is based on Andrés, Arce and Thomas (2013) and Iacoviello (2015). The implied exit probabilities (δ_b and δ_e) are approximately 0.04, so that banks and wholesale good firms on average exist for 6 years and 1 quarter. As in Andrés, Arce and Thomas (2013) and Gertler and Kiyotaki (2015), I assume that $\delta_h = 0$, so that $\beta_h = \beta$. The values for adjusted discount factors imply that borrowing constraints of bankers and entrepreneurs bind in the neighborhood of the steady state.

The steady state minimum bank capital requirement (k^m) is close to the minimum total capital requirement recommended in the Basel III framework—8% plus 2.5% conservation buffer (BCBS, 2010a, p. 12 and 55). The steady state LTV ratio (m) takes different values in the literature: 0.89 in Iacoviello (2005), 0.85 in Andrés, Arce and Thomas (2013), 0.35 in Angelini, Neri and Panetta (2014), 0.9 in Iacoviello (2015), 0.64 in Andrés, Arce and Thomas (2017).[8] I set m to 0.9 because it implies the steady state value of k^m in the Ramsey optimal equilibrium that is closer to the calibrated value. The effect of alternative values of m on the results is explored in section 7.2.

The price survival probability (θ) implies that on average prices of retail good varieties remain constant for a year as in Iacoviello (2005) and Fernández-Villaverde et al. (2015). The value of the elasticity of substitution between retail good varieties (ϵ) implies a steady state markup of the final good price over the wholesale good price (X) of approximately 1.2 as in (Galí, 2008, p. 52) and Fernández-Villaverde et al. (2015). The inverse Frisch elasticity of labor supply (ϕ) varies substantially in the literature, depending on the type of data used (Chetty et al., 2011). I set the parameter to a neutral value, following (Galí, 2008, p. 52) and Fernández-Villaverde et al. (2015). The capital good adjustment cost parameter (ϕ_k) is set as in Andrés, Arce and Thomas (2017). The central bank's inflation response parameter (ϕ_π) takes a standard value, satisfying the Taylor principle, as in (Galí, 2008, p. 52) and Fernández-Villaverde et al. (2015). The steady state inflation rate (Π) in annualized terms is about 2% as in Fernández-Villaverde et al. (2015) which is consistent with the dynamics of advanced economies in non-crisis times.[9] Log utility from consumption is assumed for households ($\sigma = 1$) as common in the literature.

The TFP process autocorrelation coefficient (ρ_a) is set as in Fernández-Villaverde et al. (2015). As for the LTV ratio process, as Andrés, Arce and Thomas (2013) mention, it is difficult to recover it from the data. I set ρ_m to 0.9 which is consistent with findings in Gerali et al. (2010). The standard deviation of the TFP shock (τ_a) is based on the estimates in Gerali et al. (2010), Andrés, Arce and Thomas (2013) and Iacoviello (2015), taking into account a lower value in Fernández-Villaverde et al. (2015). The standard deviation of the LTV ratio process (τ_m) is again set to the same value as for the TFP process, being close to the estimate in Gerali et al. (2010).

[8] In these papers, the "value" is the value of a stock of housing, while in my case it is the value of the capital stock. The general idea is the same—a durable asset used as collateral, following Kiyotaki and Moore (1997).

[9] It also corresponds to the current IMF projections for advanced economies for 2018 and beyond.

6 Welfare analysis

6.1 Welfare criterion

Period t welfare for an agent of type $i \in \{b, e, h\}$ is the corresponding value function $V_t^i = U_i(\cdot) + \beta_i \mathbb{E}_t \left\{ V_{t+1}^i \right\}$. A social planner cares about currently living agents and agents that will be born in the future periods, so that aggregate welfare is $\mathcal{W}_t^i = V_t^i + \delta_i \sum_{s=1}^{\infty} \beta^s V_{t+s}^i$. It can be shown that in such a setting $\mathcal{W}_t^i = U_i(\cdot) + \beta \mathbb{E}_t \left\{ \mathcal{W}_{t+1}^i \right\}$.[10] Then period t social welfare is

$$\mathcal{W}_t \equiv \omega_b \mathcal{W}_t^b + \omega_e \mathcal{W}_t^e + \omega_h \mathcal{W}_t^h,$$

where $\omega_b, \omega_e, \omega_h \geq 0$ are Pareto weights, $\omega_b + \omega_e + \omega_h = 1$. Expected social welfare is $\mathcal{W} = \omega_b \mathcal{W}^b + \omega_e \mathcal{W}^e + \omega_h \mathcal{W}^h$, where $\mathcal{W} \equiv \mathbb{E}(\mathcal{W}_t)$, $\mathcal{W}^i \equiv \mathbb{E}\left(\mathcal{W}_t^i\right)$ for $i \in \{b, e, h\}$.

Following Lucas (1987), it is convenient to report welfare changes in consumption units. Consider households, for example. $\mathcal{W}^h = \mathbb{E}\left(\sum_{s=0}^{\infty} \beta^s U_h(C_{t+s}^h, N_{t+s})\right)$ by the law of iterated expectations. Let $\widetilde{\mathcal{W}^h}$ be expected welfare under some alternative policy. We are looking for λ_h that satisfies $\widetilde{\mathcal{W}^h} = \mathbb{E}\left(\sum_{s=0}^{\infty} \beta^s U_h\left([1+\lambda_h] C_{t+s}^h, N_{t+s}\right)\right)$. Similarly, for $i \in \{b, e\}$, $\widetilde{\mathcal{W}^i} = \mathbb{E}\left(\sum_{s=0}^{\infty} \beta^s U_i\left([1+\lambda_i] C_{t+s}^i\right)\right)$. Under the made assumptions on utility functions,

$$\lambda_i = \exp\left[(1-\beta)\left(\widetilde{\mathcal{W}^i} - \mathcal{W}^i\right)\right] - 1.$$

Applying the same idea to social welfare, we are looking for λ that satisfies

$$\widetilde{\mathcal{W}} = \mathbb{E}\left(\sum_{s=0}^{\infty} \beta^s \left[\omega_b U_b\left([1+\lambda] C_{t+s}^b\right) + \omega_e U_e([1+\lambda] C_{t+s}^e) + \omega_h U_h\left([1+\lambda] C_{t+s}^h, N_{t+s}\right)\right]\right).$$

Under the assumed period utility functions,

$$\lambda = \exp\left[(1-\beta)\left(\widetilde{\mathcal{W}} - \mathcal{W}\right)\right] - 1.$$

To perform quantitative analysis, we have to assign values to Pareto weights which is not an obvious task. Andrés, Arce and Thomas (2013) analyze optimal monetary policy in a model with risk-averse households and entrepreneurs. They set the population share of households, which serves as their Pareto weight, to 0.979 to equate the tax on entrepreneurs' profits that implements the efficient steady state in the decentralized equilibrium to zero. They mention that the population share of entrepreneurs should be small in reality. Mendicino et al. (2018) study optimal implementable rules for bank capital requirements in a model with risk-averse dynasties of savers and borrowers. Savers include workers, bankers and entrepreneurs. They calibrate the share of borrowers to 0.437, but do not use it as a unique Pareto weight, exploring different weighting schemes instead.

In our model there are three welfare relevant agents and it is hard to find an exact counterpart in the data to calibrate Pareto weights. In the analysis of practical macroprudential rules, I

[10] The proof is analogous to Andrés, Arce and Thomas (2013), Appendix A.2.

consider what happens under alternative weights similarly to Mendicino et al. (2018). When it is required to choose one particular weighting scheme, I set $\omega_b = 0.1$, $\omega_e = 0.1$, $\omega_h = 0.8$. I agree with Andrés, Arce and Thomas (2013) that entrepreneurs constitute a smaller share of population than households-workers.[11] Moreover, the steady state household consumption in the model is significantly higher than that of bankers and entrepreneurs.

6.2 Efficient allocation

A social planner chooses $\left\{C_t^b, C_t^e, C_t^h, I_t, K_t, N_t\right\}_{t=0}^{\infty}$ to maximize $\mathcal{W}_0$ subject to the resource constraints (23) and (26). The Lagrangian is

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg[\omega_b \ln C_t^b + \omega_e \ln C_t^e + \omega_h \left(\ln C_t^h - \frac{{N_t}^{1+\phi}}{1+\phi} \right) + \lambda_t \Big(A_t {K_{t-1}}^{\alpha} {N_t}^{1-\alpha} - C_t^b - C_t^e - C_t^h - I_t \Big) + \gamma_t \left(I_t \left(1 - \frac{\phi_k}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right) - K_t + (1-\delta) K_{t-1} \right) \Bigg].$$

The efficient allocation is described by the resource constraints and the following equations.

$$\lambda_t = \frac{\omega_i}{C_t^i} \quad \forall i \in \{b, e, h\}, \tag{37}$$

$$\frac{C_t^i}{C_t^j} = \frac{\omega_i}{\omega_j} \quad \forall i, j \in \{b, e, h\}, \tag{38}$$

$$C_t^h {N_t}^{\phi} = (1-\alpha) A_t \left(\frac{K_{t-1}}{N_t} \right)^{\alpha}, \tag{39}$$

$$\frac{\gamma_t}{\lambda_t} = \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \left(\alpha A_{t+1} \left(\frac{K_t}{N_{t+1}} \right)^{\alpha-1} + (1-\delta) \frac{\gamma_{t+1}}{\lambda_{t+1}} \right) \right\}, \tag{40}$$

$$1 = \frac{\gamma_t}{\lambda_t} \left[1 - \frac{\phi_k}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \phi_k \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right] + \beta \phi_k \mathbb{E}_t \left\{ \frac{\lambda_{t+1}}{\lambda_t} \frac{\gamma_{t+1}}{\lambda_{t+1}} \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right\}. \tag{41}$$

The perfect consumption insurance condition (38) postulates that relative consumption allocations correspond to relative Pareto weights. This implies that relative variances of consumption correspond to squared relative Pareto weights. As discussed earlier, a reasonable Pareto weighting scheme specifies a relatively higher weight for households. However, in the decentralized equilibrium with constant bank capital requirements, the variance of entrepreneurs' consumption is significantly higher than that of households due to the financial accelerator. Binding borrowing constraints are the first source of inefficiency of the competitive equilibrium.

[11] According to the Global Entrepreneurship Monitor data, there were more than 25 million entrepreneurs in the US in 2016—12.6% of 18–64 age population. According to the U.S. Bureau of Labor Statistics, the number of employed persons was 151.4 million.

As for other optimality conditions, equation (39)—the equality between the marginal rate of substitution between consumption and leisure and the marginal product of labor—would hold in the decentralized equilibrium provided that $X_t = 1$ (no monopolistic competition of retailers). By setting the price of capital to $Q_t = \gamma_t/\lambda_t$, for (40) to hold, we additionally need $\delta_e = 0$, so that $\beta_e = \beta$ and as a result $\gamma_t^e = 0$ in the neighborhood of the steady state. If, moreover, $\delta_h = 0$, so that $\beta_h = \beta$ (which is true in the baseline calibration), (41) would hold too. Of course, for the final goods resource constraint to hold in the first place, we also need flexible prices.

6.3 Ramsey optimal policy

A relevant problem is finding the optimal competitive equilibrium. We would like to find sequences of allocations and prices that maximize $\mathcal{W}_0$ subject to all the conditions describing the decentralized equilibrium. This problem makes sense if there are policy instruments that are or can be made exogenous. The minimum bank capital requirement (k_t^m) satisfies this condition.

6.3.1 Optimal policy in the simplified model

Before analyzing the complete model of section 2, let us consider its flexible price, perfect competition, no adjustment cost version. In this model, entrepreneurs produce the final good, there are no retailers and central bank (the deposit rate is determined endogenously). The equilibrium conditions can be obtained by imposing $X_t = 1$, $\Delta_t = 1$, $\phi_k = 0$ and redefining interest rates as real rates: R_{t-1} now corresponds to R_{t-1}/Π_t before, R_t^l corresponds to R_t^l/Π_t.

The competitive equilibrium can be summarized by the following equations.[12]

$$
\begin{aligned}
g_{1t} &\equiv \beta_b \left(R_t^l - \frac{1-k_{t-1}^m}{\beta_h C_{t-1}^h \mathbb{E}_{t-1}\left\{ {C_t^h}^{-1} \right\}} \right)(\eta_{t-1} - \beta_e) C_{t-1}^e - (\eta_t - \beta_e) k_t^m C_t^e = 0, \\
g_{2t} &\equiv \left(\eta_{t-1} \left[\alpha {A_t}^{\frac{1}{\alpha}} \left(\frac{C_t^h {N_t}^{\phi}}{1-\alpha} \right)^{\frac{\alpha-1}{\alpha}} + 1 - \delta - R_t^l \right] + \beta_e R_t^l \right) C_{t-1}^e - C_t^e = 0, \\
g_{3t} &\equiv \left[1 + \frac{\eta_t}{1-\beta_e} + \frac{1-\beta_b}{\beta_b(1-\beta_e)}(\eta_t - \beta_e) k_t^m \right] C_t^e + C_t^h - (1-\delta)\frac{\eta_{t-1} C_{t-1}^e}{1-\beta_e} \\
&\quad - A_t \left(\frac{\eta_{t-1} C_{t-1}^e}{1-\beta_e} \right)^{\alpha} {N_t}^{1-\alpha} = 0, \\
g_{4t} &\equiv ((1-\alpha)A_t)^{\frac{1}{\alpha}} \frac{\eta_{t-1} C_{t-1}^e}{1-\beta_e} - {C_t^h}^{\frac{1}{\alpha}} {N_t}^{1+\frac{\phi}{\alpha}} = 0, \\
g_{5t} &\equiv \left(\mathbb{E}_t\{R_{t+1}^l\} - m_t \right) \eta_t - \beta_e \mathbb{E}_t\{R_{t+1}^l\} = 0.
\end{aligned}
$$

A Ramsey planner chooses $\left\{ C_t^e, C_t^h, N_t, R_t^l, k_t^m, \eta_t \right\}_{t=0}^{\infty}$ to maximize $\mathcal{W}_0$ subject to the equilib-

[12] In principle, one can solve for N_t and η_t using $g_{4t} = 0$ and $g_{5t} = 0$, and then substitute in the remaining conditions to get a system of three equations, though more cumbersome.

rium conditions $\{g_{it} = 0\}_{i \in \{1,\dots,5\}}$. The Lagrangian is

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\omega_b \left(\ln(\eta_t - \beta_e) + \ln k_t^m \right) + (\omega_b + \omega_e) \ln C_t^e + \omega_h \left(\ln C_t^h - \frac{{N_t}^{1+\phi}}{1+\phi} \right) + \sum_{i=1}^{5} \lambda_{it} g_{it} \right].$$

The optimal conditions are described in Appendix E.

Figure 1 shows impulse responses to an approximately 1% TFP shock ($\epsilon_1^a = 0.01$) in the decentralized equilibrium with constant capital requirements and the Ramsey optimal equilibrium.[13] The shock directly increases output, marginal products of labor and capital. Wages and household income increase, households consume more. Entrepreneurs sell more output, their net worth increases, they consume and invest more. The collateral constraint is relaxed, entrepreneurs borrow more. Interest rates rise, bankers' net worth and consumption increases. The Ramsey planner increases capital requirements on impact, but then the optimal policy is procyclical, although the change in capital requirements is very small. The optimal policy decreases the variance of entrepreneurs' consumption a little. Mostly, the differences between two economies are insignificant.

Figure 2 reports impulse responses to an approximately 0.9 percentage point LTV ratio shock ($\epsilon_1^m = 0.01$). The shock directly relaxes the collateral constraint. Entrepreneurs invest and borrow more, although borrowing peaks later. A higher interest rate induces households to save more and decrease consumption, but only on impact. Required loan repayments rise which has a negative effect on entrepreneurs' net worth, consumption, investment and output. Bankers' net worth and consumption increases. The optimal policy is mostly countercyclical. It reduces the volatility of output and household consumption to some extent, but the effect is small.

The simplified model does not contain many important mechanisms present in the complete model, therefore now we compare what happens in the latter.

6.3.2 Optimal policy in the complete model

The decentralized equilibrium in the complete model is more complicated, but in principle, it is possible to characterize the optimal policy explicitly. This would take several pages of equations, so I transition directly to the impulse response analysis.

Figure 3 shows impulse responses to a TFP shock.[14] In the complete model, the price of capital is no longer constant. The increase in the expected marginal product of capital and investment leads to a large increase in the price of capital and thus to a much more significant relaxation of the collateral constraint. Entrepreneurs supply more wholesale output, the wholesale price and marginal cost for retailers falls, inflation decreases, and the central bank decreases the deposit rate. Entrepreneurs' consumption jumps by almost 30%, and there is a higher increase in output. The optimal policy has a huge effect. A countercyclical increase in capital requirements shuts

[13] The steady state value of capital requirements in the Ramsey optimal equilibrium is 16.8%. I impose the same steady state value in the decentralized equilibrium with constant capital requirements.

[14] I impose the same steady state in two economies. The steady state capital requirements in the Ramsey optimal equilibrium are 17%.

down the increase in entrepreneurs' consumption almost completely. A tighter collateral constraint leads to smaller investment and a much smaller increase in the price of capital. Output increases significantly less too, and there is a larger initial decrease in inflation and deposit rate.

Figure 4 reports impulse responses to an LTV ratio shock. In the complete model, the effect of the direct relaxation of the collateral constraint is much more pronounced. An increase in investment and output is more persistent. There is a significant jump in the price of capital, which relaxes the constraint even more. The unfolding of the financial accelerator results in an almost 80% increase in entrepreneurs' consumption. Inflation and deposit rate increase. The increase in loan repayments outweighs deposit repayments, and bankers' net worth and consumption increases. The optimal policy hugely restricts the variance of the price of capital and entrepreneurs' consumption. The policy is again countercyclical, and its impact on the economy is even more substantial than after the TFP shock.

The effectiveness of optimal macroprudential policy is significantly higher in the complete model with nominal rigidities and capital adjustment costs. The optimal policy is countercyclical after both real and financial shocks. It is extremely successful in containing the financial accelerator—reducing the volatility of the price of capital and entrepreneurs' consumption. It also has a general stabilization effect. At the same time, bankers' net worth and consumption, being inherently linked to capital requirements, becomes more volatile. The social welfare gain from the optimal policy, comparing to the decentralized equilibrium with constant capital requirements, is equal to an approximately 1% increase in consumption units (measured by λ).[15]

The Ramsey optimal policy can provide insights into the design of practical macroprudential rules. Table 2 reports correlations of k_t^m with lags and leads of selected variables that serve as indicators of the business cycle and aggregate risk and could be used as target variables in macroprudential rules: loans, loans-to-output ratio, price of capital and output.[16] In terms of comovement with optimal capital requirements, loans and loans-to-output ratio seem to be better indicators. The correlation with the price of capital is a little lower, being the highest with the first lag, and it is significantly lower with output. As for the latter, the highest correlation is with future output, which is less practical for rule-based capital requirements. Of course, in the case of contemporaneous correlations it is less clear whether optimal policy is mainly a response to or a cause of movements in the considered variables.

[15] The computational algorithm of the program Dynare used for quantitative analysis does not allow to define agents' welfare functions when computing the Ramsey optimal policy because the social welfare function is a linear combination of individual welfare functions, and thus model variables would be linearly dependent, creating singularity problems. For this reason, only the aggregate welfare gain is reported.

[16] The set of variables is chosen based on considerations by policy institutions such as the Basel Committee on Banking Supervision. In particular, see Drehmann et al. (2010) and (BCBS, 2010b).

6.4 Suboptimal policy rules

6.4.1 One-variable rules

I consider policy rules of the form

$$k_t^m = k^m + \nu \left(\frac{x_t}{x} - 1 \right) \tag{42}$$

or

$$k_t^m = k^m + \nu \left(x_t - x \right), \tag{43}$$

where $\nu \in \mathbb{R}$ is a response parameter, x_t is an indicator of aggregate risk and x is its steady state value. I consider current values of x_t and lags up to four. Loans, output and price of capital are introduced in the rules of type (42), while loans-to-output ratio is more suitable for (43).

The value of ν that maximizes $\mathcal{W}$ is searched in the grid $[-5, 5]$ with a step 0.01. In practice, the model admits a unique solution only in a subset of the grid, different for each particular x_t. The values on the boundary of such subsets are discarded due to near singularity and precision problems. The model is solved approximately using a second order Taylor expansion of equilibrium conditions about a steady state.

I consider Pareto weights for households in an interval $[0, 1]$ with a step 0.1, and for simplicity bankers' and entrepreneurs' weights are set to be equal. For each Pareto weighting scheme I compare the optimized rules for lag lengths of x_t from 0 to 4 and choose the welfare maximizing rule. Tables 3–6 summarize quantitative results.

Optimal parameters are very robust to alternative Pareto weighting schemes, except when only one agent matters. This is due to a highly heterogeneous impact of macroprudential policy: there is a significant effect on entrepreneurs, a smaller effect on bankers and a negligible effect on households. Except under extreme Pareto weightings, all the optimized rules are countercyclical as could be expected based on the Ramsey optimal policy analysis. Entrepreneurs benefit a lot from optimized rules, gaining up to 10% in consumption equivalents.[17] The ability of countercyclical macroprudential policy to contain the financial accelerator is the key. On the other hand, bankers lose from optimized rules, although their losses are smaller. This is the result of the higher volatility of their net worth and consumption tightly linked to capital requirements. Households are not directly affected by financial frictions. They benefit from the overall stabilization effect of countercyclical policy, but suffer from the higher variance of inflation after TFP shocks, and in total gain negligibly.

The optimized rules that achieve the highest gains in aggregate (represented by λ) are based on loans and lagged price of capital. Their welfare implications are similar due to the link through the collateral constraint: when the price of capital is higher, other things equal, the value of collateral is higher, and entrepreneurs can borrow more. The optimized rules based on the loans-to-output

[17] The magnitude of gains depends on standard deviations of shocks hitting the economy. The bigger are the shocks, the bigger are the gains.

ratio and output perform worse. The latter is consistent with the Ramsey optimal policy finding. The problem with the former is that the loan dynamics lags output dynamics, hence the dynamics of the loans-to-output ratio does not always correspond to the business cycle, especially after TFP shocks. This is consistent with the empirical findings of Repullo and Saurina (2012) who criticize the use of the credit-to-GDP gap in the countercyclical capital buffer of Basel III.

Figures 5–6 report impulse responses under the optimized rule based on loans comparing to constant capital requirements.[18] Table 7 reports standard deviations of consumption for bankers, entrepreneurs and households under constant and optimized rules. Under constant capital requirements, the variance of bankers' consumption is much smaller than of household consumption, while the latter is significantly smaller than the variance of entrepreneurs' consumption. Recall that in the efficient equilibrium relative variances of consumption correspond to relative Pareto weights. Optimized rules based on loans and price of capital succeed in preserving the efficient ranking, and variances for bankers and entrepreneurs are close, which agrees with the fact that their Pareto weights are set to be equal in this exercise. The impulse responses reflect this effect on the variances and the notions already discussed.

Practical rules are not as effective as the Ramsey optimal policy, but still are successful in alleviating the inefficiencies of the decentralized equilibrium. In terms of gains in social welfare comparing to the economy with constant capital requirements, there is a 1% increase in consumption units (measured by λ) in the Ramsey optimal equilibrium, 0.84% under the optimized rule based on the price of capital, 0.83% under the loans rule, 0.56% under the loans-to-output ratio rule and 0.51% under the output rule.[19]

6.4.2 Two-variable rules

One-variable rules can be generalized to rules responding to several variables. An optimized rule of this kind obviously achieves at least as high welfare as one-variable rules based on its components. A disadvantage of several-variable rules is that it might be harder to explain them to the general public, for their interpretation may be less clear. Consider two-variable rules of the form

$$k_t^m = k^m + \nu_1 \left(\frac{x_{1t}}{x_1} - 1 \right) + \nu_2 \left(\frac{x_{2t}}{x_2} - 1 \right),$$

where $\nu_1, \nu_2 \in \mathbb{R}$ and x_{1t}, x_{2t} are indicators of aggregate risk.

I performed welfare analysis of rules based on all possible combinations of target variables considered in the previous section, using both their current values and optimal lag lengths found in the one-variable case. I used the $[-5, 5] \times [-5, 5]$ grid with a step 0.01. The rule that achieves the highest welfare among the optimized rules is based on the lagged loans-to-output ratio and lagged price of capital. The welfare gains are reported in Table 8.[20] The welfare effects are similar

[18] Similar figures for other optimized rules are available upon request.

[19] After setting $\omega_b = 0.1$, $\omega_e = 0.1$, $\omega_h = 0.8$, imposing the steady state value of capital requirements from the Ramsey optimal equilibrium (approximately 17%) and recomputing optimal parameters for this steady state.

[20] Results for other optimized rules are available upon request.

to the optimal one-variable rules based on loans and price of capital. Entrepreneurs gain a lot, households gain insignificantly, bankers lose. As before, the optimized rule is robust to alternative Pareto weighting schemes, except the extreme weightings. The gain in welfare comparing to optimal one-variable rules is small, suggesting that simpler rules may be preferable.

7 Sensitivity analysis

7.1 Flexible inflation targeting

To isolate the effect of macroprudential policy on the economy as much as possible, the baseline analysis was conducted under strict inflation targeting. Now suppose the central bank cares about output too. Similar to Angelini, Neri and Panetta (2014), who conduct the joint analysis of monetary and macroprudential policies, I assume that the central bank follows[21]

$$R_t = \bar{R}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi}\left(\frac{Y_t}{Y_{t-1}}\right)^{\phi_y}, \tag{44}$$

where $\phi_y > 0$. Specifically, I set $\phi_y = 0.25$.

First, I consider the Ramsey optimal policy. Figures 7 and 8 show impulse responses to the TFP and LTV ratio shocks. The effects of the optimal policy are similar to the case with strict inflation targeting. The difference is that flexible inflation targeting reduces the fluctuations of the economy by itself, although being particularly successful only after TFP shocks. After both shocks, optimal macroprudential policy is countercyclical and limits substantially the volatility of entrepreneurs' consumption, price of capital and other variables. The especially striking impact of the optimal policy after LTV ratio shocks remains.

Tables 9–12 parallel 3–6. As in the case of strict inflation targeting, all the optimized macroprudential rules are countercyclical, except under the extreme Pareto weighting schemes. As before, the welfare maximizing rules are based on loans and price of capital. The optimized rules are less aggressive, the magnitudes of welfare gains are smaller (including smaller welfare losses for bankers), but the qualitative effects are exactly the same: entrepreneurs gain significantly, households gain negligibly, bankers lose.

Table 13 reports the statistics for the optimized rule based on the lagged loans-to-output ratio and price of capital which is the welfare maximizing rule as in the case of strict inflation targeting. Again, we observe smaller magnitudes, but the same qualitative effects as in the baseline case.

[21] An alternative would be to replace Y_{t-1} by Y, but this specification creates stability problems and lack of interior solutions in some cases. The reason is that monetary policy and macroprudential policy become more correlated. The Ramsey optimal policy is not affected by this issue, and the impact of the optimal policy is similar to the one under (44), especially after LTV ratio shocks. The results are available upon request.

7.2 Steady state LTV ratio

There is substantial variability in the literature regarding the calibration of the steady state LTV ratio. Table 14 reports the optimal response parameters under alternative calibrations. Optimized rules remain countercyclical in almost all cases. On average, the higher is the steady state LTV ratio, the higher are the optimal parameters. The reason is that entrepreneurs can borrow more, which increases the magnitude of the financial accelerator and requires a stronger response from the macroprudential authority.

8 Conclusion

The paper explored optimal macroprudential policy in a New Keynesian model with risk-averse bankers, entrepreneurs and households. The model does not feature default and thus systemic risk whatsoever, but has a clear rationale for macroprudential policy as a tool to dampen the financial accelerator mechanism and alleviate the inefficiency of the decentralized equilibrium due to the binding collateral constraint. The Ramsey optimal policy is Pareto improving and prescribes countercyclical bank capital requirements, being quite effective in containing the financial accelerator. Nominal rigidities and capital adjustment costs are important for the optimal macroprudential policy to be countercyclical. In the simplified flexible-price model, the optimal policy is almost indistinguishable from constant capital requirements. Suboptimal policy rules are not Pareto improving, but still lead to sizable welfare gains, especially for entrepreneurs. The most effective rules are based on credit and asset prices as cyclical indicators, while the credit-to-GDP ratio, employed in the countercyclical capital buffer of Basel III, is less desirable to use.

There are several limitations in the current analysis. For example, monetary policy is taken as given by the Ramsey planner in the form of a conventional Taylor rule. It is important to generalize the analysis and consider the joint determination of the optimal macroprudential and monetary policy. Moreover, the results are based on applying perturbation methods, restricting to the local analysis in a neighborhood of the steady state that features binding borrowing constraints. In the future research, it is important to take into account the nonlinearities resulting from the occasionally binding constraints.

References

Andrés, Javier, Óscar Arce, and Carlos Thomas (2013). "Banking Competition, Collateral Constraints, and Optimal Monetary Policy". *Journal of Money, Credit and Banking*, 45(s2), 87–125.

Andrés, Javier, Óscar Arce, and Carlos Thomas (2017). "Structural reforms in a debt overhang". *Journal of Monetary Economics*, 88, 15–34.

Angelini, Paolo, Stefano Neri, and Fabio Panetta (2014). "The Interaction between Capital Requirements and Monetary Policy". *Journal of Money, Credit and Banking*, 46(6), 1073–1112.

Angeloni, Ignazio, and Ester Faia (2013). "Capital Regulation and Monetary Policy with Fragile Banks". *Journal of Monetary Economics*, 60(3), 311–324.

Basel Committee on Banking Supervision (2010*a*). *Basel III: A global regulatory framework for more resilient banks and banking systems.* Bank for International Settlements.

Basel Committee on Banking Supervision (2010*b*). *Guidance for national authorities operating the countercyclical capital buffer.* Bank for International Settlements.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist (1999). "The Financial Accelerator in a Quantitative Business Cycle Framework". In *Handbook of Macroeconomics.* Vol. 1, ed. John B. Taylor and Michael Woodford, 1341–1393. Elsevier Science B.V.

Bianchi, Javier (2011). "Overborrowing and Systemic Externalities in the Business Cycle". *American Economic Review*, 101(7), 3400–3426.

Bianchi, Javier, and Enrique G. Mendoza (2018). "Optimal Time-Consistent Macroprudential Policy". *Journal of Political Economy*, 126(2), 588–634.

Blanchard, Olivier J. (1985). "Debt, Deficits, and Finite Horizons". *The Journal of Political Economy*, 93(2), 223–247.

Borio, Claudio (2003). "Towards a macroprudential framework for financial supervision and regulation?". Bank for International Settlements Working Paper No 128.

Calvo, Guillermo A. (1983). "Staggered prices in a utility-maximizing framework". *Journal of Monetary Economics*, 12(3), 383–398.

Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber (2011). "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins". *American Economic Review Papers and Proceedings*, 101(2), 471–475.

Collard, Fabrice, Harris Dellas, Behzad Diba, and Olivier Loisel (2017). "Optimal Monetary and Prudential Policies". *American Economic Journal: Macroeconomics*, 9(1), 40–87.

Crockett, Andrew D. (2000). "Marrying the micro- and macro-prudential dimensions of financial stability". Conference remarks, Bank for International Settlements.

Dávila, Eduardo, and Anton Korinek (2018). "Pecuniary Externalities in Economies with Financial Frictions". *The Review of Economic Studies*, 85(1), 352–395.

Drehmann, Mathias, Claudio Borio, Leonardo Gambacorta, Gabriel Jiménez, and Carlos Trucharte (2010). "Countercyclical capital buffers: exploring options". Bank for International Settlements Working Paper No 317.

Fernández-Villaverde, Jesús, Grey Gordon, Pablo Guerrón-Quintana, and Juan F. Rubio-Ramírez (2015). "Nonlinear adventures at the zero lower bound". *Journal of Economic Dynamics and Control*, 57(Supplement C), 182–204.

Galí, Jordi (2008). *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework.* Princeton University Press.

Gerali, Andrea, Stefano Neri, Luca Sessa, and Federico M. Signoretti (2010). "Credit and Banking in a DSGE Model of the Euro Area". *Journal of Money, Credit and Banking*, 42, supplement(6), 107–141.

Gertler, Mark, and Nobuhiro Kiyotaki (2010). "Financial Intermediation and Credit Policy in Business Cycle Analysis". In . Vol. 3 of *Handbook of Monetary Economics*, ed. Benjamin M. Friedman and Michael Woodford, 547–599. Elsevier.

Gertler, Mark, and Nobuhiro Kiyotaki (2015). "Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy". *American Economic Review*, 105(7), 2011–43.

Gertler, Mark, and Peter Karadi (2011). "A model of unconventional monetary policy". *Journal of Monetary Economics*, 58(1), 17–34.

Holmstrom, Bengt, and Jean Tirole (1997). "Financial Intermediation, Loanable Funds, and The Real Sector". *The Quarterly Journal of Economics*, 112(3), 663–691.

Iacoviello, Matteo (2005). "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle". *The American Economic Review*, 95(3), 739–764.

Iacoviello, Matteo (2015). "Financial Business Cycles". *Review of Economic Dynamics*, 18(1), 140–164.

Jeanne, Olivier, and Anton Korinek (2018). "Managing credit booms and busts: A Pigouvian taxation approach". *Journal of Monetary Economics.*

Kiyotaki, Nobuhiro, and John Moore (1997). "Credit Cycles". *The Journal of Political Economy*, 105(2), 211–248.

Lorenzoni, Guido (2008). "Inefficient Credit Booms". *The Review of Economic Studies*, 75(3), 809–833.

Lucas, Robert E. Jr. (1987). *Models of Business Cycles.* Basil Blackwell.

Martinez-Miera, David, and Javier Suarez (2014). "Banks Endogenous Systemic Risk Taking". CEMFI, mimeo.

Mendicino, Caterina, Kalin Nikolov, Javier Suarez, and Dominik Supera (2018). "Optimal Dynamic Capital Requirements". *Journal of Money, Credit and Banking*, Forthcoming.

Repullo, Rafael, and Jesús Saurina (2012). "The Countercyclical Capital Buffer of Basel III: A Critical Assessment". In *The Crisis Aftermath: New Regulatory Paradigms*. ed. Mathias Dewatripont and Xavier Freixas, 45–67. CEPR, London.

Appendix

A Equilibrium conditions

Functional forms

$$U_b(C_t^b) = \ln C_t^b,$$
$$U_e(C_t^e) = \ln C_t^e,$$
$$U_h(C_t^h, N_t) = \ln C_t^h - \chi \frac{{N_t}^{1+\phi}}{1+\phi},$$
$$f\left(\frac{I_t}{I_{t-1}}\right) = \frac{\phi_k}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2.$$

Derivatives

$$U_b'(C_t^b) = \frac{1}{C_t^b},$$
$$U_e'(C_t^e) = \frac{1}{C_t^e},$$
$$\frac{\partial U_h(C_t^h, N_t)}{\partial C_t^h} = \frac{1}{C_t^h},$$
$$\frac{\partial U_h(C_t^h, N_t)}{\partial N_t} = -\chi {N_t}^{\phi}$$
$$f'\left(\frac{I_t}{I_{t-1}}\right) = \phi_k\left(\frac{I_t}{I_{t-1}} - 1\right).$$

Bankers

$$C_t^b + L_t = NW_t^b + D_t, \tag{45}$$
$$0 = \gamma_t^b[(1-k_t^m)L_t - D_t], \tag{46}$$
$$NW_t^b \equiv \frac{R_t^l}{\Pi_t}L_{t-1} - \frac{R_{t-1}}{\Pi_t}D_{t-1}, \tag{47}$$
$$\frac{\gamma_t^b}{U_b'\left(C_t^b\right)} = 1 - \mathbb{E}_t\left\{\beta_b \frac{U_b'\left(C_{t+1}^b\right)}{U_b'\left(C_t^b\right)}\frac{R_t}{\Pi_{t+1}}\right\}, \tag{48}$$
$$\frac{\gamma_t^b}{U_b'\left(C_t^b\right)}(1-k_t^m) = 1 - \mathbb{E}_t\left\{\beta_b \frac{U_b'\left(C_{t+1}^b\right)}{U_b'\left(C_t^b\right)}\frac{R_{t+1}^l}{\Pi_{t+1}}\right\}. \tag{49}$$

Entrepreneurs

$$
\begin{aligned}
C_t^e + Q_t K_t &= NW_t^e + L_t, && (50)\\
0 &= \gamma_t^e \left[m_t \mathbb{E}_t \left\{ Q_{t+1} \Pi_{t+1} K_t \right\} - \mathbb{E}_t \left\{ R_{t+1}^l L_t \right\} \right], && (51)\\
NW_t^e &\equiv \frac{Y_t^w}{X_t} - W_t N_t + Q_t (1-\delta) K_{t-1} - \frac{R_t^l}{\Pi_t} L_{t-1}, && (52)\\
Y_t^w &\equiv A_t {K_{t-1}}^{\alpha} {N_t}^{1-\alpha}, && (53)\\
W_t &= (1-\alpha) \frac{Y_t^w}{N_t X_t}, && (54)\\
Q_t &= \mathbb{E}_t \left\{ \beta_e \frac{U_e'(C_{t+1}^e)}{U_e'(C_t^e)} \left(\alpha \frac{Y_{t+1}^w}{K_t X_{t+1}} + (1-\delta) Q_{t+1} \right) \right\} \\
&\quad + \frac{\gamma_t^e}{U_e'(C_t^e)} m_t \mathbb{E}_t \left\{ Q_{t+1} \Pi_{t+1} \right\}, && (55)\\
\frac{\gamma_t^e}{U_e'(C_t^e)} \mathbb{E}_t \{ R_{t+1}^l \} &= 1 - \mathbb{E}_t \left\{ \beta_e \frac{U_e'(C_{t+1}^e)}{U_e'(C_t^e)} \frac{R_{t+1}^l}{\Pi_{t+1}} \right\}, && (56)
\end{aligned}
$$

Households

$$
\begin{aligned}
\frac{\partial U_h}{\partial N_t} + W_t \frac{\partial U_h}{\partial C_t^h} &= 0, && (57)\\
\mathbb{E}_t \left\{ \beta_h \frac{\partial U_h / \partial C_{t+1}^h}{\partial U_h / \partial C_t^h} \frac{R_t}{\Pi_{t+1}} \right\} &= 1, && (58)
\end{aligned}
$$

Capital good producers

$$
1 = Q_t \left[1 - f\left(\frac{I_t}{I_{t-1}} \right) - f'\left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + \mathbb{E}_t \left\{ \Lambda_{t,t+1} Q_{t+1} f'\left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right\}. \quad (59)
$$

Retailers

$$
\begin{aligned}
\Pi_t^* &= \frac{\epsilon}{\epsilon - 1} \frac{\Omega_{1,t}}{\Omega_{2,t}}, && (60)\\
\Omega_{1,t} &= {\Pi_t}^{\epsilon} \left(\frac{\partial U_h}{\partial C_t^h} \frac{Y_t}{X_t} + \beta_h \theta \mathbb{E}_t \left\{ \Omega_{1,t+1} \right\} \right), && (61)\\
\Omega_{2,t} &= {\Pi_t}^{\epsilon-1} \left(\frac{\partial U_h}{\partial C_t^h} Y_t + \beta_h \theta \mathbb{E}_t \left\{ \Omega_{2,t+1} \right\} \right), && (62)\\
{\Pi_t}^{1-\epsilon} &= \theta + (1-\theta)(\Pi_t^*)^{1-\epsilon}. && (63)
\end{aligned}
$$

Government

$$R_t = \bar{R}\left(\frac{\Pi_t}{\bar{\Pi}}\right)^{\phi_\pi}, \tag{64}$$

$$g_t Y_t = T_t. \tag{65}$$

Market clearing

$$I_t\left[1 - f\left(\frac{I_t}{I_{t-1}}\right)\right] = K_t - (1-\delta)K_{t-1}, \tag{66}$$

$$Y_t^w = Y_t \Delta_t, \tag{67}$$

$$\Delta_t = \theta {\Pi_t}^{\epsilon}\Delta_{t-1} + (1-\theta)\left(\frac{\Pi_t}{\Pi_t^*}\right)^{\epsilon}, \tag{68}$$

$$(1-g_t)Y_t = C_t^b + C_t^e + C_t^h + I_t. \tag{69}$$

B Deterministic steady state

The exogenous variables take values $A = \bar{A}$, $g = \bar{g}$ and $m = \bar{m}$. The macroprudential policy is $k^m = \bar{k}^m$.

First set of variables

$$\Pi = \bar{\Pi}, \tag{70}$$

$$\Pi^* = \left[\frac{\Pi^{1-\epsilon} - \theta}{1-\theta}\right]^{\frac{1}{1-\epsilon}}, \tag{71}$$

$$\Delta = \frac{(1-\theta)\left(\frac{\Pi}{\Pi^*}\right)^{\epsilon}}{1-\theta\Pi^{\epsilon}}, \tag{72}$$

$$X = \frac{\epsilon}{\epsilon - 1}\frac{\Pi^{1-\epsilon} - \beta_h\theta}{\Pi^*(\Pi^{-\epsilon} - \beta_h\theta)}, \tag{73}$$

$$Q = 1, \tag{74}$$

$$R = \frac{\Pi}{\beta_h}, \tag{75}$$

$$R^l = R\left[1 + k^m\left(\frac{\beta_h}{\beta_b} - 1\right)\right]. \tag{76}$$

Auxiliary elements

Choose some $\epsilon^b \geq 0$ and $\epsilon^e \geq 0$, and let

$$\Psi_1 \equiv \left[\frac{1}{\beta_e} - (1-\delta) - \left(\frac{\Pi}{\beta_e R^l} - 1\right) m\right] \frac{QX}{\alpha},$$

$$\Psi_2 \equiv \left(\frac{1-g}{\Delta} - \frac{\alpha}{X}\right) \Psi_1 + (Q-1)\delta + \left(\frac{R}{\Pi} - 1\right) \left[\frac{1-k^m}{R^l} \left(mQ\Pi - \mathbb{1}\{\beta_e = \tilde{\beta}_e\}\epsilon^e\right) - \mathbb{1}\{\beta_b = \beta_h\}\epsilon^b\right]$$

Second set of variables

$$N = \left(\frac{1-\alpha}{\chi} \frac{\Psi_1}{\Psi_2 X}\right)^{\frac{1}{1+\phi}}, \tag{77}$$

$$K = \left(\frac{A}{\Psi_1}\right)^{\frac{1}{1-\alpha}} N, \tag{78}$$

$$Y^w \equiv AK^\alpha N^{1-\alpha}, \tag{79}$$

$$L = \frac{1}{R^l} \left(mQ\Pi K - \mathbb{1}\{\beta_e = \tilde{\beta}_e\}\epsilon^e K\right), \quad \epsilon^e \geq 0, \tag{80}$$

$$D = (1-k^m)L - \mathbb{1}\{\beta_b = \beta_h\}\epsilon^b K, \quad \epsilon^b \geq 0, \tag{81}$$

$$C^b = \frac{R^l - R}{\Pi} L + \left(\frac{R}{\Pi} - 1\right)(L - D), \tag{82}$$

$$\gamma^b = \frac{1}{C^b}\left(1 - \frac{\beta_b}{\beta_h}\right), \tag{83}$$

$$C^e = \alpha \frac{Y^w}{X} - \delta QK - \left(\frac{R^l}{\Pi} - 1\right) L, \tag{84}$$

$$\gamma^e = \frac{1}{R^l C^e}\left(1 - \beta_e \frac{R^l}{\Pi}\right), \tag{85}$$

$$NW^b \equiv \frac{R^l}{\Pi} L - \frac{1}{\beta_h} D, \tag{86}$$

$$NW^e \equiv \alpha \frac{Y^w}{X} + Q(1-\delta)K - \frac{R^l}{\Pi} L, \tag{87}$$

$$Y = \frac{Y^w}{\Delta}, \tag{88}$$

$$C^h = (1-g)Y - C^b - C^e - \delta K, \tag{89}$$

$$I = \delta K, \tag{90}$$

$$\Omega_1 = \frac{Y}{C^h X(\Pi^{-\epsilon} - \beta_h \theta)}, \tag{91}$$

$$\Omega_2 = \frac{Y}{C^h(\Pi^{1-\epsilon} - \beta_h \theta)}, \tag{92}$$

$$T = gY, \tag{93}$$

$$W = \chi C^h N^\phi. \tag{94}$$

C Proof of proposition 3

As $\beta_b < \beta_h$, the regulatory constraint is binding in the neighborhood of the steady state. Therefore, $1 - k_t^m = D_t/L_t$. Using this result, the definition of NW_t^b, substituting (4) into (5), multiplying both sides by L_t and rearranging,

$$\mathbb{E}_t \left\{ \beta_b \frac{U_b'\left(C_{t+1}^b\right)}{U_b'\left(C_t^b\right)} NW_{t+1}^b \right\} = NW_t^b - C_t^b.$$

Using the assumption of log utility and dividing both sides by C_t^b,

$$\frac{NW_t^b}{C_t^b} = 1 + \beta_b \mathbb{E}_t \left\{ \frac{NW_{t+1}^b}{C_{t+1}^b} \right\}. \tag{95}$$

Solving (95) forward, we obtain the first result:

$$C_t^b = (1 - \beta_b) NW_t^b. \tag{96}$$

Substituting (96) into the budget constraint and using the regulatory constraint,

$$L_t = \frac{\beta_b}{k_t^m} NW_t^b. \tag{97}$$

Using the regulatory constraint,

$$D_t = \beta_b \frac{1 - k_t^m}{k_t^m} NW_t^b. \tag{98}$$

Substituting (97) and (98) into the definition of net worth,

$$NW_t^b = \frac{\beta_b}{\Pi_t} \left(\frac{R_t^l - R_{t-1}}{k_{t-1}^m} + R_{t-1} \right) NW_{t-1}^b,$$

which completes the proof.

D Proof of proposition 4

Substituting the binding collateral constraint into (11), then substituting (12) into (11), multiplying both sides by K_t, rearranging, using the budget constraint and the definition of net worth,

$$\mathbb{E}_t \left\{ \beta_e \frac{U_e'(C_{t+1}^e)}{U_e'(C_t^e)} NW_{t+1}^e \right\} = NW_t^e - C_t^e.$$

Dividing by C_t^e and using assumption of log utility,

$$\frac{NW_t^e}{C_t^e} = 1 + \beta_e \mathbb{E}_t \left\{ \frac{NW_{t+1}^e}{C_{t+1}^e} \right\}. \tag{99}$$

Solving (99) forward, we get the first result:

$$C_t^e = (1-\beta_e)NW_t^e. \tag{100}$$

Substituting (100) into the budget constraint, we obtain $L_t = Q_t K_t - \beta_e NW_t^e$. Substituting this into the collateral constraint and rearranging,

$$K_t = \eta_t NW_t^e, \tag{101}$$

where

$$\eta_t \equiv \beta_e \left(Q_t - \frac{m_t \mathbb{E}_t\{Q_{t+1}\Pi_{t+1}\}}{\mathbb{E}_t\left\{R_{t+1}^l\right\}} \right)^{-1}.$$

Therefore,

$$L_t = (Q_t\eta_t - \beta_e)NW_t^e. \tag{102}$$

Using (10) and (101),

$$N_t = \left[\frac{(1-\alpha)A_t}{W_t X_t}\right]^{\frac{1}{\alpha}} \eta_{t-1} NW_{t-1}^e. \tag{103}$$

Substituting (101), (102) and (103) into the definition of net worth,

$$NW_t^e = \left\{ \alpha \frac{A_t}{X_t}\left[\frac{(1-\alpha)A_t}{W_t X_t}\right]^{\frac{1-\alpha}{\alpha}} \eta_{t-1} + Q_t(1-\delta)\eta_{t-1} - \frac{R_t^l}{\Pi_t}(Q_{t-1}\eta_{t-1} - \beta_e) \right\} NW_{t-1}^e,$$

which completes the proof.

E Ramsey optimal policy in the simplified model

The optimal policy is described by $\{g_{it}=0\}_{i\in\{1,\ldots,5\}}$, complementary slackness conditions and the following equations:

$$
\begin{aligned}
0 =\ & \omega_b + \omega_e - \lambda_{1t}(\eta_t - \beta_e)C_t^e k_t^m - \lambda_{2t}C_t^e + \lambda_{3t}(C_t^b + C_t^e + K_t) \\
& + \beta\mathbb{E}_t\Bigg\{\lambda_{1,t+1}\beta_b\left(R_{t+1}^l - R_t(1-k_t^m)\right)(\eta_t - \beta_e)C_t^e + \lambda_{2,t+1}C_t^e\Bigg(\eta_t\Bigg[\alpha A_{t+1}{}^{\frac{1}{\alpha}}\left(\frac{W_{t+1}}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} \\
& + 1 - \delta - R_{t+1}^l\Bigg] + \beta_e R_{t+1}^l\Bigg) - \lambda_{3,t+1}\left[(1-\delta)K_t + \alpha Y_{t+1}\right] + \lambda_{4,t+1}\left[(1-\alpha)A_{t+1}\right]^{\frac{1}{\alpha}} K_t\Bigg\}, \\
0 =\ & \omega_h - \lambda_{2t}C_{t-1}^e\eta_{t-1}(1-\alpha)A_t{}^{\frac{1}{\alpha}}\left(\frac{W_t}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} + \lambda_{3t}C_t^h - \lambda_{4t}\frac{1}{\alpha}W_t{}^{\frac{1}{\alpha}}N_t \\
& - \frac{\beta_b(\eta_{t-1}-\beta_e)(1-k_{t-1}^m)C_{t-1}^e R_{t-1}\mathbb{E}_{t-1}\{\lambda_{1t}\}}{C_t^h\mathbb{E}_{t-1}\left\{C_t^{h\,-1}\right\}} + \beta\beta_b(\eta_t-\beta_e)(1-k_t^m)C_t^e R_t\mathbb{E}_t\{\lambda_{1,t+1}\}, \\
0 =\ & \omega_h N_t{}^{1+\phi} + \phi\lambda_{2t}C_{t-1}^e\eta_{t-1}(1-\alpha)A_t{}^{\frac{1}{\alpha}}\left(\frac{W_t}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} + \lambda_{3t}(1-\alpha)Y_t + \lambda_{4t}\left(1+\frac{\phi}{\alpha}\right)W_t{}^{\frac{1}{\alpha}}N_t, \\
0 =\ & \beta(\lambda_{1t}\beta_b - \lambda_{2t})C_{t-1}^e + \lambda_{5,t-1}, \\
0 =\ & \frac{1-\beta_b}{\beta_b(1-\beta_e)}\left(\frac{\omega_b}{C_t^b} + \lambda_{3t}\right) - \lambda_{1t} + \beta\beta_b R_t\mathbb{E}_t\{\lambda_{1,t+1}\}, \\
0 =\ & \frac{\omega_b + \lambda_{3t}C_t^b}{(\eta_t-\beta_e)C_t^e} - \lambda_{1t}k_t^m + \frac{\lambda_{3t}}{1-\beta_e} + \frac{\lambda_{5t}}{C_t^e}\left(\mathbb{E}_t\left\{R_{t+1}^l\right\} - m_t\right) \\
& + \beta\mathbb{E}_t\Bigg\{\lambda_{1,t+1}\beta_b\left(R_{t+1}^l - R_t(1-k_t^m)\right) + \lambda_{2,t+1}\left[\alpha A_{t+1}{}^{\frac{1}{\alpha}}\left(\frac{W_{t+1}}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} + 1 - \delta - R_{t+1}^l\right] \\
& - \frac{\lambda_{3,t+1}}{1-\beta_e}\left[1-\delta+\alpha\frac{Y_{t+1}}{K_t}\right] + \frac{\lambda_{4,t+1}}{1-\beta_e}\left[(1-\alpha)A_{t+1}\right]^{\frac{1}{\alpha}}\Bigg\},
\end{aligned}
$$

where $C_t^b \equiv (1-\beta_b)(\eta_t-\beta_e)k_t^m C_t^e/\left[\beta_b(1-\beta_e)\right]$, $K_t \equiv \eta_t C_t^e/(1-\beta_e)$, $R_t \equiv \left[\beta_h C_t^h \mathbb{E}_t\left\{C_{t+1}^h{}^{-1}\right\}\right]^{-1}$, $W_t \equiv C_t^h N_t{}^{\phi}$, $Y_t \equiv A_t K_{t-1}{}^{\alpha} N_t{}^{1-\alpha}$.

Working with the above equations, it is possible to express $\lambda_{2t}, \lambda_{3t}, \lambda_{5t}$ in terms of $\{\lambda_{1t}\}, \{\lambda_{4t}\}$ and other variables. After solving for $\{\lambda_{1t}\}$ and $\{\lambda_{4t}\}$ and eliminating all the Lagrange multipliers, we are left with one equation which together with $\{g_{it}=0\}_{i\in\{1,\ldots,5\}}$ describes the optimal competitive equilibrium and in particular optimal bank capital requirements.

Table 1: Parameter values

Parameter	Value	Definition
α	1/3	Capital share in the wholesale good production
β	0.99	Discount factor
β_b	0.95	Bankers' adjusted discount factor
β_e	0.95	Entrepreneurs' adjusted discount factor
β_h	0.99	Household adjusted discount factor
δ	0.025	Capital depreciation rate
ϵ	6	Elasticity of substitution between retail good varieties
k^m	0.1	Steady state minimum bank capital requirement
m	0.9	Steady state LTV ratio
ϕ	1	Inverse Frisch elasticity of labor supply
ϕ_k	2.4	Capital good adjustment cost parameter
ϕ_π	1.5	Central bank's response to inflation parameter
Π	1.005	Steady state inflation rate
ρ_a	0.9	TFP AR(1) autocorrelation coefficient
ρ_m	0.9	LTV ratio AR(1) autocorrelation coefficient
σ	1	Household RRA coefficient
τ_a	0.005	Standard deviation of the TFP shock
τ_m	0.005	Standard deviation of the LTV ratio shock
θ	0.75	Retail price survival probability

Table 2: Correlation of k_t^m with selected variables in the Ramsey optimal equilibrium

x	x_{t-4}	x_{t-3}	x_{t-2}	x_{t-1}	x_t	x_{t+1}	x_{t+2}	x_{t+3}	x_{t+4}
L	0.25	0.33	0.42	0.51	0.56	0.55	0.52	0.47	0.43
L/Y	0.40	0.46	0.52	0.57	0.60	0.57	0.51	0.45	0.39
Q	0.15	0.24	0.34	0.42	0.41	0.34	0.32	0.25	0.18
Y	-0.04	0.01	0.08	0.15	0.20	0.21	0.22	0.22	0.21

Notes: $\omega_b = 0.1$, $\omega_e = 0.1$, $\omega_h = 0.8$. L denotes loans, L/Y—loans-to-output ratio, Q—price of capital, Y—output. The value in row L and column x_{t-4}, for example, is $Corr(k_t^m, L_{t-4})$.

Table 3: Welfare gains under optimized rule based on loans

ω_h	ν	λ_b, %	λ_e, %	λ_h, %	λ, %	Lag
0.0	0.74	-0.60	9.46	0.06	4.31	0
0.1	0.74	-0.60	9.46	0.06	3.88	0
0.2	0.74	-0.60	9.46	0.06	3.45	0
0.3	0.74	-0.60	9.46	0.06	3.02	0
0.4	0.74	-0.60	9.46	0.06	2.59	0
0.5	0.74	-0.60	9.46	0.06	2.16	0
0.6	0.74	-0.60	9.46	0.06	1.74	0
0.7	0.74	-0.60	9.46	0.06	1.32	0
0.8	0.74	-0.60	9.46	0.06	0.90	0
0.9	0.74	-0.60	9.46	0.06	0.48	0
1.0	-1.32	-36.56	6.23	0.14	0.14	1

Notes: $\omega_b = \omega_e = 0.5(1 - \omega_h)$. λ-s represent welfare gains in permanent consumption units relative to the economy with constant capital requirements.

Table 4: Welfare gains under optimized rule based on loans-to-output ratio

ω_h	ν	λ_b, %	λ_e, %	λ_h, %	λ, %	Lag
0.0	0.44	-3.75	9.16	-0.19	2.51	1
0.1	0.44	-3.75	9.16	-0.19	2.23	1
0.2	0.44	-3.75	9.16	-0.19	1.96	1
0.3	0.43	-3.72	9.12	-0.19	1.69	1
0.4	0.43	-3.72	9.12	-0.19	1.42	1
0.5	0.43	-3.72	9.12	-0.19	1.15	1
0.6	0.42	-3.69	9.08	-0.18	0.88	1
0.7	0.41	-3.66	9.02	-0.18	0.62	1
0.8	0.39	-3.61	8.88	-0.16	0.35	1
0.9	0.35	-3.55	8.37	-0.13	0.10	1
1.0	0.06	-0.38	-0.96	0.02	0.02	0

Notes: $\omega_b = \omega_e = 0.5(1 - \omega_h)$. λ-s represent welfare gains in permanent consumption units relative to the economy with constant capital requirements.

Table 5: Welfare gains under optimized rule based on price of capital

ω_h	ν	$\lambda_b, \%$	$\lambda_e, \%$	$\lambda_h, \%$	$\lambda, \%$	Lag
0.0	2.39	-0.69	9.93	0.03	4.49	1
0.1	2.38	-0.69	9.93	0.03	4.03	1
0.2	2.37	-0.69	9.93	0.03	3.58	1
0.3	2.37	-0.69	9.93	0.03	3.13	1
0.4	2.35	-0.69	9.93	0.03	2.68	1
0.5	2.34	-0.69	9.93	0.03	2.24	1
0.6	2.32	-0.69	9.93	0.03	1.79	1
0.7	2.28	-0.68	9.93	0.03	1.35	1
0.8	2.22	-0.68	9.93	0.03	0.91	1
0.9	2.08	-0.68	9.92	0.03	0.47	1
1.0	0.34	-0.59	6.34	0.04	0.04	4

Notes: $\omega_b = \omega_e = 0.5(1 - \omega_h)$. λ-s represent welfare gains in permanent consumption units relative to the economy with constant capital requirements.

Table 6: Welfare gains under optimized rule based on output

ω_h	ν	$\lambda_b, \%$	$\lambda_e, \%$	$\lambda_h, \%$	$\lambda, \%$	Lag
0.0	0.77	-1.09	6.48	0.02	2.62	3
0.1	0.77	-1.09	6.48	0.02	2.36	3
0.2	0.77	-1.09	6.48	0.02	2.10	3
0.3	0.77	-1.09	6.48	0.02	1.84	3
0.4	0.77	-1.09	6.48	0.02	1.58	3
0.5	0.76	-1.07	6.46	0.02	1.31	3
0.6	0.76	-1.07	6.46	0.02	1.05	3
0.7	0.76	-1.07	6.46	0.02	0.80	3
0.8	0.75	-1.05	6.44	0.02	0.54	3
0.9	0.74	-1.03	6.41	0.02	0.28	3
1.0	-3.63	-86.32	13.59	2.50	2.50	1

Notes: $\omega_b = \omega_e = 0.5(1 - \omega_h)$. λ-s represent welfare gains in permanent consumption units relative to the economy with constant capital requirements.

Table 7: Standard deviations of consumption under optimized macroprudential rules

	Constant	L	L/Y	Q	Y
Bankers	0.001	0.008	0.016	0.008	0.008
Entrepreneurs	0.036	0.011	0.013	0.007	0.022
Households	0.020	0.017	0.028	0.019	0.018

Notes: $\omega_b = 0.1$, $\omega_e = 0.1$, $\omega_h = 0.8$.

Table 8: Welfare gains under optimized rule based on loans-to-output ratio and price of capital

ω_h	ν_1	ν_2	$\lambda_b, \%$	$\lambda_e, \%$	$\lambda_h, \%$	$\lambda, \%$
0.0	0.10	1.71	-0.41	9.96	0.06	4.65
0.1	0.10	1.70	-0.41	9.96	0.06	4.18
0.2	0.10	1.68	-0.41	9.96	0.06	3.71
0.3	0.10	1.66	-0.41	9.96	0.06	3.25
0.4	0.10	1.63	-0.41	9.96	0.06	2.79
0.5	0.11	1.59	-0.41	9.96	0.06	2.33
0.6	0.11	1.55	-0.41	9.95	0.06	1.87
0.7	0.11	1.49	-0.40	9.95	0.06	1.42
0.8	0.11	1.39	-0.40	9.94	0.07	0.96
0.9	0.12	1.23	-0.40	9.91	0.07	0.51
1.0	-0.20	-0.40	-22.04	-16.23	0.62	0.62

Notes: $\omega_b = \omega_e = 0.5(1 - \omega_h)$, $x_{1t} = L_{t-1}/Y_{t-1}, x_{2t} = Q_{t-1}$, λ-s represent welfare gains in permanent consumption units relative to the economy with constant capital requirements.

Table 9: Flexible inflation targeting: welfare gains under optimized rule based on loans

ω_h	ν	$\lambda_b, \%$	$\lambda_e, \%$	$\lambda_h, \%$	$\lambda, \%$	Lag
0.0	0.64	-0.49	5.58	0.05	2.50	0
0.1	0.64	-0.49	5.58	0.05	2.25	0
0.2	0.64	-0.49	5.58	0.05	2.00	0
0.3	0.64	-0.49	5.58	0.05	1.76	0
0.4	0.64	-0.49	5.58	0.05	1.51	0
0.5	0.64	-0.49	5.58	0.05	1.27	0
0.6	0.64	-0.49	5.58	0.05	1.02	0
0.7	0.64	-0.49	5.58	0.05	0.78	0
0.8	0.64	-0.49	5.58	0.05	0.53	0
0.9	0.65	-0.51	5.60	0.05	0.29	0
1.0	-1.37	-32.52	5.73	0.12	0.12	1

Notes: $\omega_b = \omega_e = 0.5(1 - \omega_h)$. λ-s represent welfare gains in permanent consumption units relative to the economy with constant capital requirements.

Table 10: Flexible inflation targeting: welfare gains under optimized rule based on loans-to-output ratio

ω_h	ν	$\lambda_b, \%$	$\lambda_e, \%$	$\lambda_h, \%$	$\lambda, \%$	Lag
0.0	0.34	-2.36	5.86	-0.04	1.67	1
0.1	0.34	-2.36	5.86	-0.04	1.49	1
0.2	0.34	-2.36	5.86	-0.04	1.32	1
0.3	0.34	-2.36	5.86	-0.04	1.15	1
0.4	0.34	-2.36	5.86	-0.04	0.98	1
0.5	0.34	-2.36	5.86	-0.04	0.81	1
0.6	0.33	-2.31	5.79	-0.04	0.64	1
0.7	0.33	-2.31	5.79	-0.04	0.47	1
0.8	0.32	-2.27	5.71	-0.03	0.30	1
0.9	0.31	-2.22	5.60	-0.03	0.13	1
1.0	0.08	-0.54	0.52	0.05	0.05	0

Notes: $\omega_b = \omega_e = 0.5(1 - \omega_h)$. λ-s represent welfare gains in permanent consumption units relative to the economy with constant capital requirements.

Table 11: Flexible inflation targeting: welfare gains under optimized rule based on price of capital

ω_h	ν	$\lambda_b, \%$	$\lambda_e, \%$	$\lambda_h, \%$	$\lambda, \%$	Lag
0.0	2.32	-0.62	6.48	0.02	2.87	1
0.1	2.32	-0.62	6.48	0.02	2.58	1
0.2	2.32	-0.62	6.48	0.02	2.30	1
0.3	2.32	-0.62	6.48	0.02	2.01	1
0.4	2.31	-0.62	6.48	0.02	1.72	1
0.5	2.31	-0.62	6.48	0.02	1.44	1
0.6	2.3	-0.61	6.48	0.02	1.15	1
0.7	2.29	-0.61	6.48	0.02	0.87	1
0.8	2.27	-0.61	6.48	0.02	0.59	1
0.9	2.21	-0.61	6.48	0.02	0.30	1
1.0	0.37	-0.50	4.04	0.03	0.03	4

Notes: $\omega_b = \omega_e = 0.5(1 - \omega_h)$. λ-s represent welfare gains in permanent consumption units relative to the economy with constant capital requirements.

Table 12: Flexible inflation targeting: welfare gains under optimized rule based on output

ω_h	ν	$\lambda_b, \%$	$\lambda_e, \%$	$\lambda_h, \%$	$\lambda, \%$	Lag
0.0	0.63	-0.77	3.33	0.01	1.26	3
0.1	0.63	-0.77	3.33	0.01	1.13	3
0.2	0.63	-0.77	3.33	0.01	1.01	3
0.3	0.63	-0.77	3.33	0.01	0.88	3
0.4	0.63	-0.77	3.33	0.01	0.76	3
0.5	0.63	-0.77	3.33	0.01	0.63	3
0.6	0.63	-0.77	3.33	0.01	0.51	3
0.7	0.63	-0.77	3.33	0.01	0.38	3
0.8	0.62	-0.76	3.31	0.01	0.26	3
0.9	0.61	-0.74	3.29	0.01	0.14	3
1.0	-3.82	-82.42	9.22	2.00	2.00	1

Notes: $\omega_b = \omega_e = 0.5(1 - \omega_h)$. λ-s represent welfare gains in permanent consumption units relative to the economy with constant capital requirements.

Table 13: Flexible inflation targeting: welfare gains under optimized rule based on loans-to-output ratio and price of capital

ω_h	ν_1	ν_2	$\lambda_b, \%$	$\lambda_e, \%$	$\lambda_h, \%$	$\lambda, \%$
0.0	0.10	1.44	-0.33	6.51	0.05	3.03
0.1	0.10	1.43	-0.33	6.51	0.05	2.73
0.2	0.11	1.39	-0.33	6.51	0.06	2.43
0.3	0.11	1.36	-0.33	6.51	0.06	2.13
0.4	0.11	1.34	-0.32	6.50	0.06	1.83
0.5	0.11	1.30	-0.32	6.50	0.06	1.53
0.6	0.11	1.26	-0.32	6.50	0.06	1.24
0.7	0.12	1.19	-0.32	6.49	0.06	0.94
0.8	0.12	1.09	-0.31	6.48	0.06	0.65
0.9	0.13	0.94	-0.32	6.44	0.06	0.35
1.0	-0.20	-0.42	-19.43	-10.89	0.61	0.61

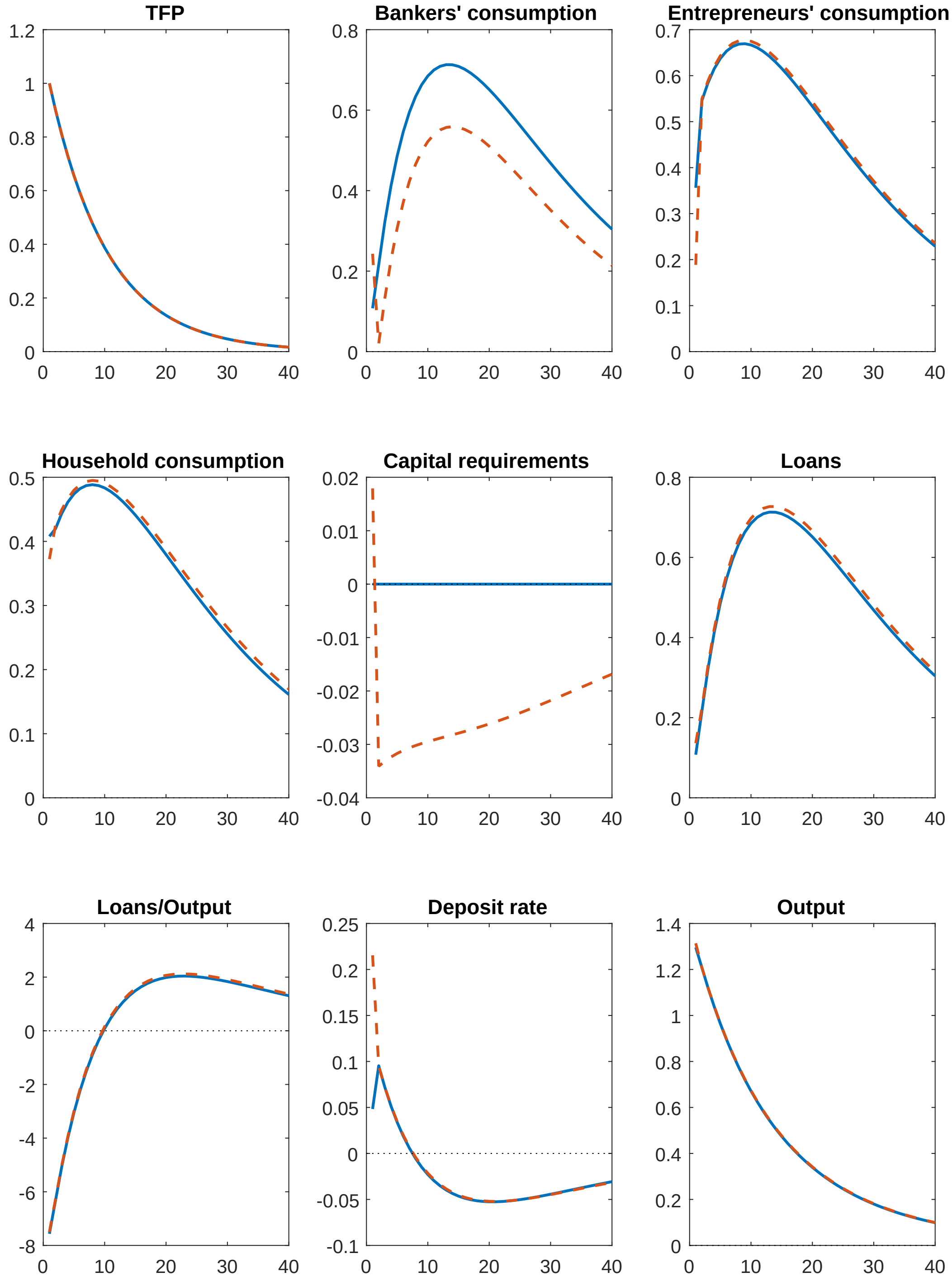
Notes: $\omega_b = \omega_e = 0.5(1 - \omega_h)$, $x_{1t} = L_{t-1}/Y_{t-1}, x_{2t} = Q_{t-1}$, λ-s represent welfare gains in permanent consumption units relative to the economy with constant capital requirements.

Table 14: Optimized rules under alternative calibration of the steady state LTV ratio

	One-variable				Two-variable	
m	L_t	L_{t-1}/Y_{t-1}	Q_{t-1}	Y_{t-3}	L_{t-1}/Y_{t-1}	Q_{t-1}
0.50	0.26	0.06	0.06	-3.72	0.19	0.27
0.55	0.34	0.05	0.08	0.08	0.19	0.35
0.60	0.41	0.04	0.11	0.12	0.18	0.44
0.65	0.48	0.03	1.97	0.17	0.17	0.57
0.70	0.55	0.02	2.01	0.24	0.16	0.70
0.75	0.61	0.02	2.04	0.31	0.15	0.84
0.80	0.66	0.01	2.07	0.41	0.14	0.98
0.85	0.71	0.00	2.11	0.55	0.12	1.15
0.90	0.74	0.39	2.22	0.75	0.11	1.39
0.95	0.76	0.47	2.55	1.10	0.11	1.85

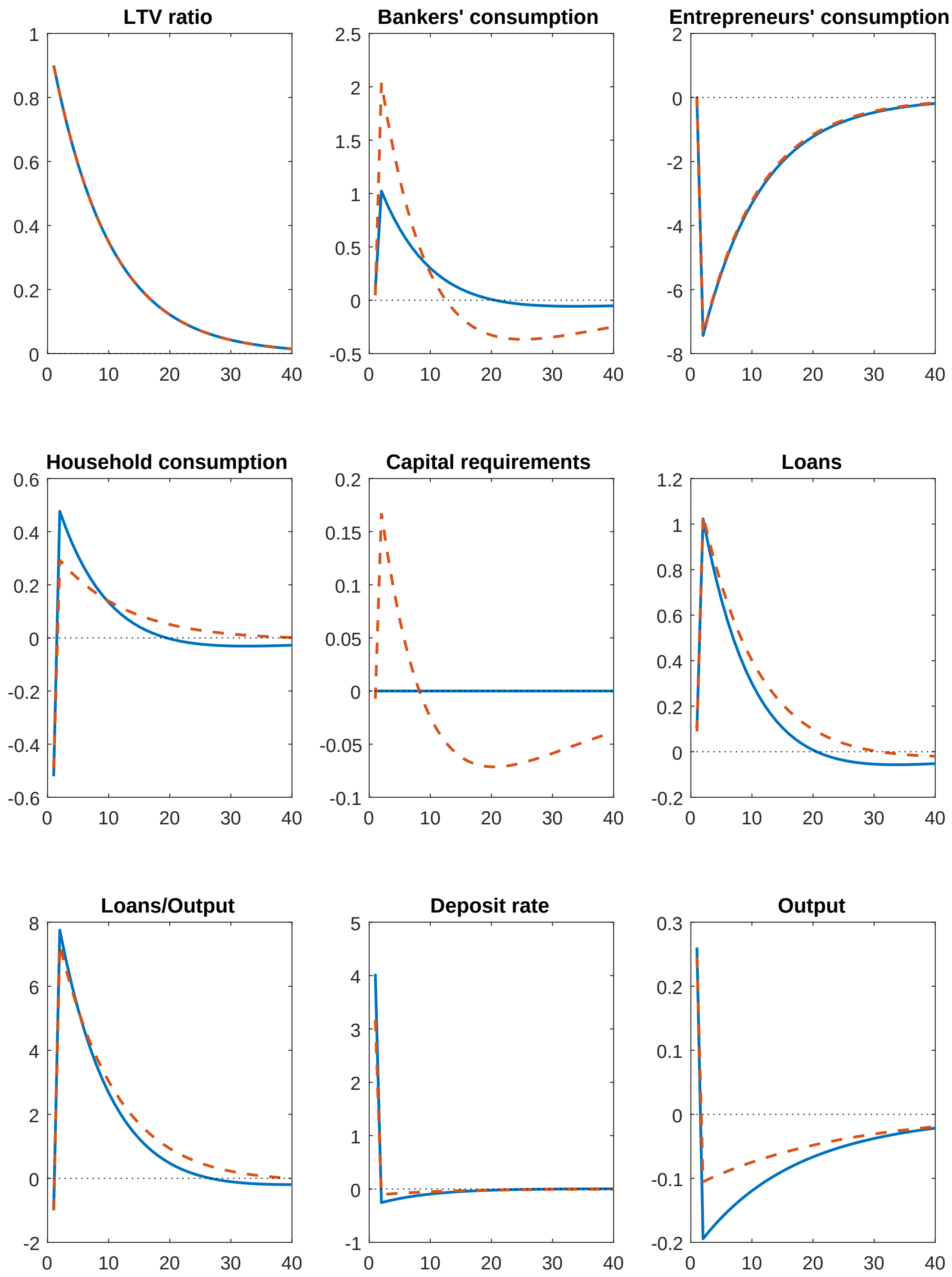
Notes: $\omega_b = 0.1$, $\omega_e = 0.1$, $\omega_h = 0.8$. The numbers in the table are optimal values of response parameters.

Figure 1: Ramsey optimal policy in the simplified model: TFP shock



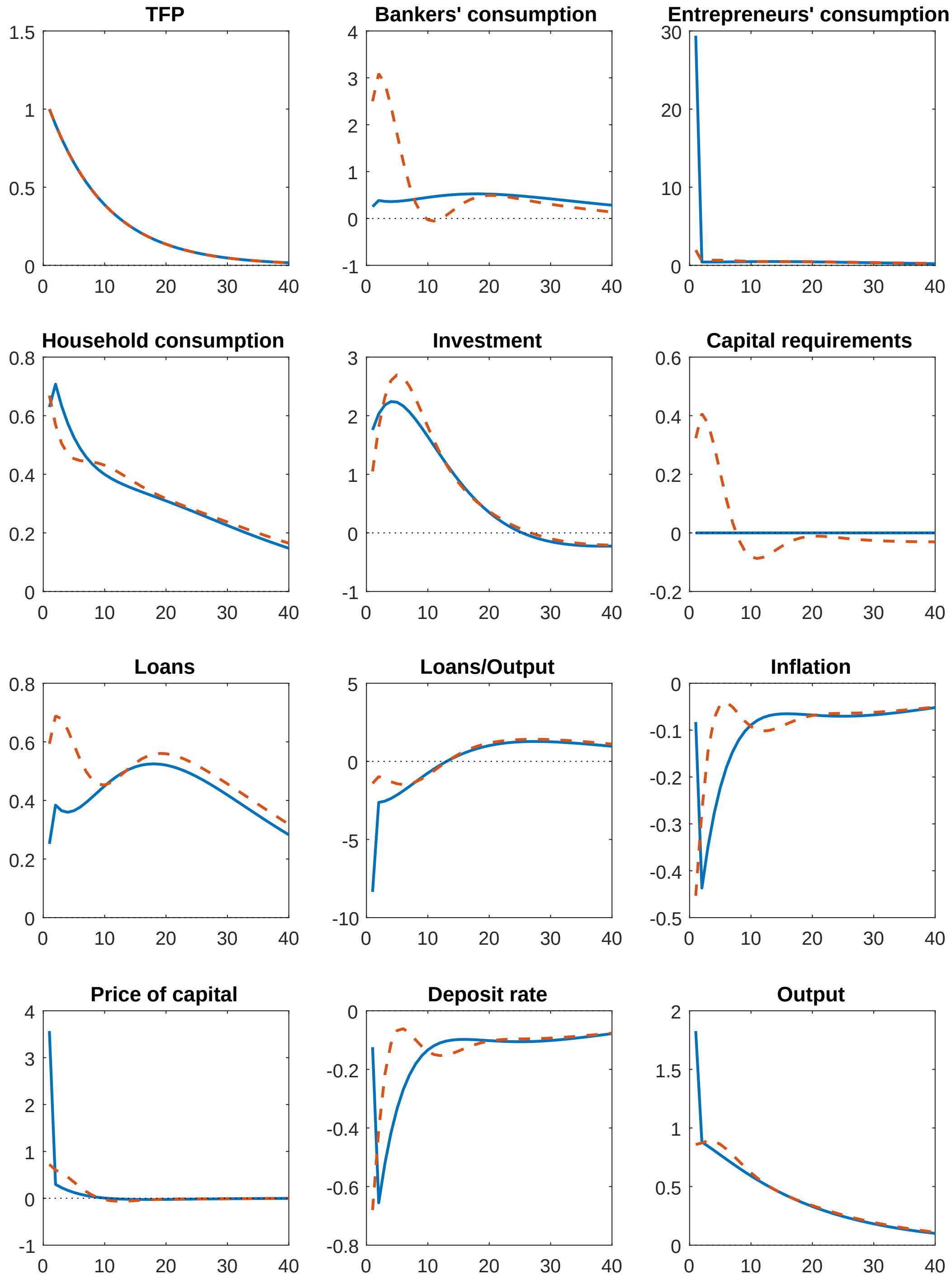
Notes: $\omega_b = 0.1$, $\omega_e = 0.1$, $\omega_h = 0.8$. Solid lines correspond to the decentralized equilibrium with constant capital requirements, dashed lines—to the Ramsey optimal equilibrium. Horizontal axes are quarters, the shock hits in quarter 1. Capital requirements, loans-to-output ratio and deposit rate are in percentage point deviations from the steady state values. The deposit rate is annualized. Other variables are in percent deviations from the steady state.

Figure 2: Ramsey optimal policy in the simplified model: LTV ratio shock



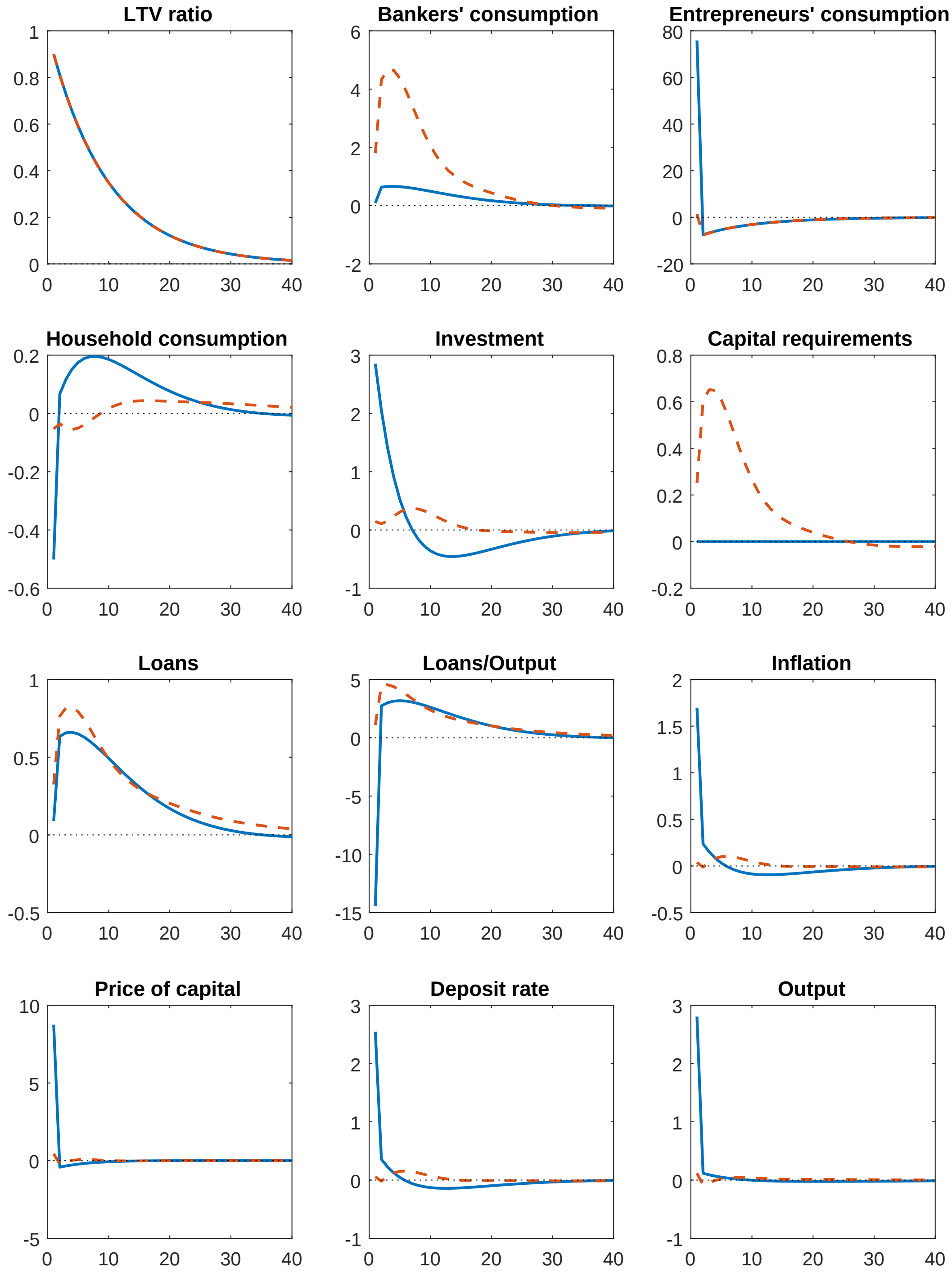
Notes: $\omega_b = 0.1$, $\omega_e = 0.1$, $\omega_h = 0.8$. Solid lines correspond to the decentralized equilibrium with constant capital requirements, dashed lines—to the Ramsey optimal equilibrium. Horizontal axes are quarters, the shock hits in quarter 1. Capital requirements, loans-to-output ratio, LTV ratio and deposit rate are in percentage point deviations from the steady state values. The deposit rate is annualized. Other variables are in percent deviations from the steady state.

Figure 3: Ramsey optimal policy in the complete model: TFP shock



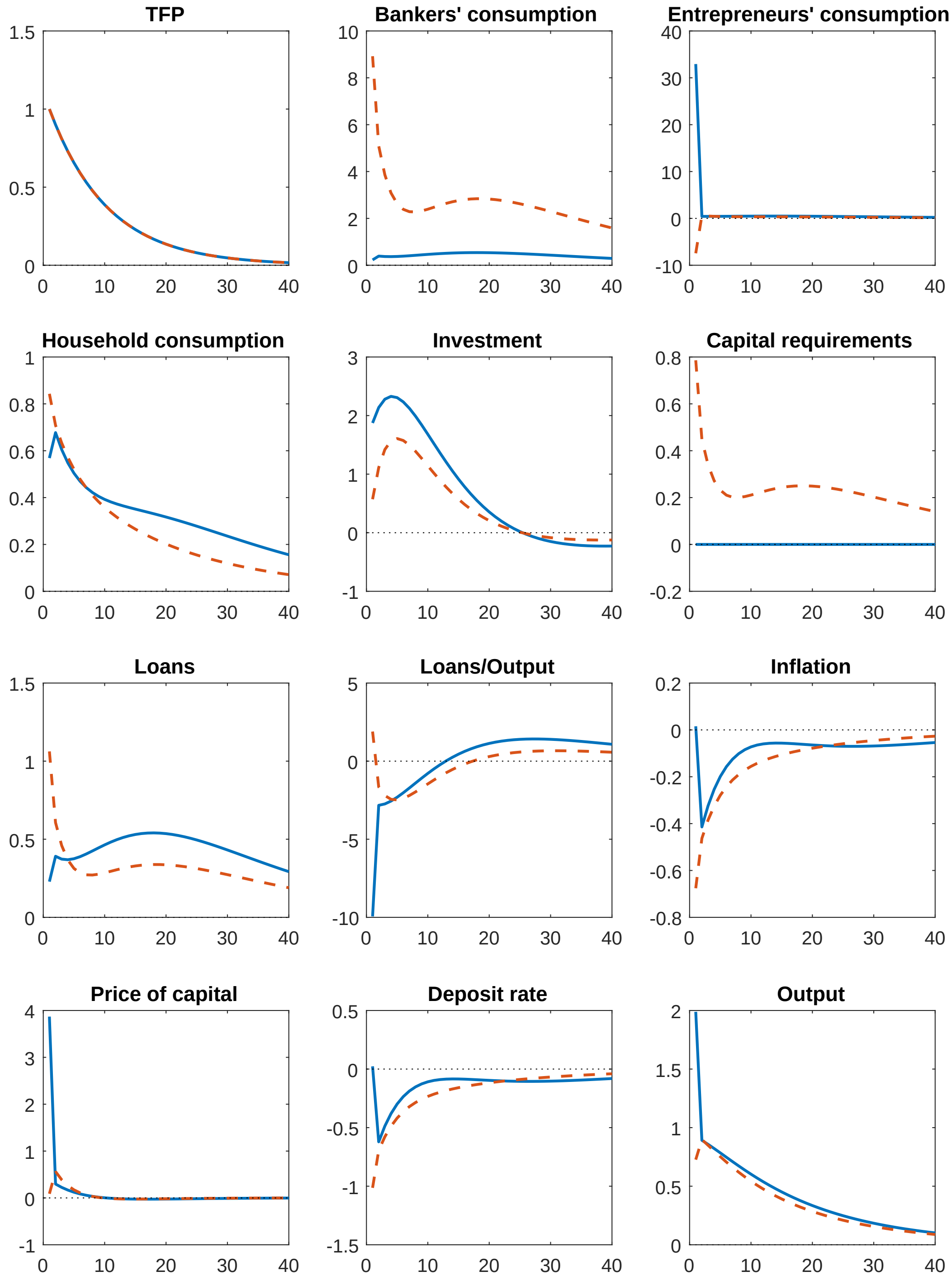
Notes: $\omega_b = 0.1$, $\omega_e = 0.1$, $\omega_h = 0.8$. Solid lines correspond to the decentralized equilibrium with constant capital requirements, dashed lines—to the Ramsey optimal equilibrium. Horizontal axes are quarters, the shock hits in quarter 1. Capital requirements, loans-to-output ratio, inflation and deposit rate are in percentage point deviations from the steady state values. Inflation and deposit rate are annualized. Other variables are in percent deviations from the steady state.

Figure 4: Ramsey optimal policy in the complete model: LTV ratio shock



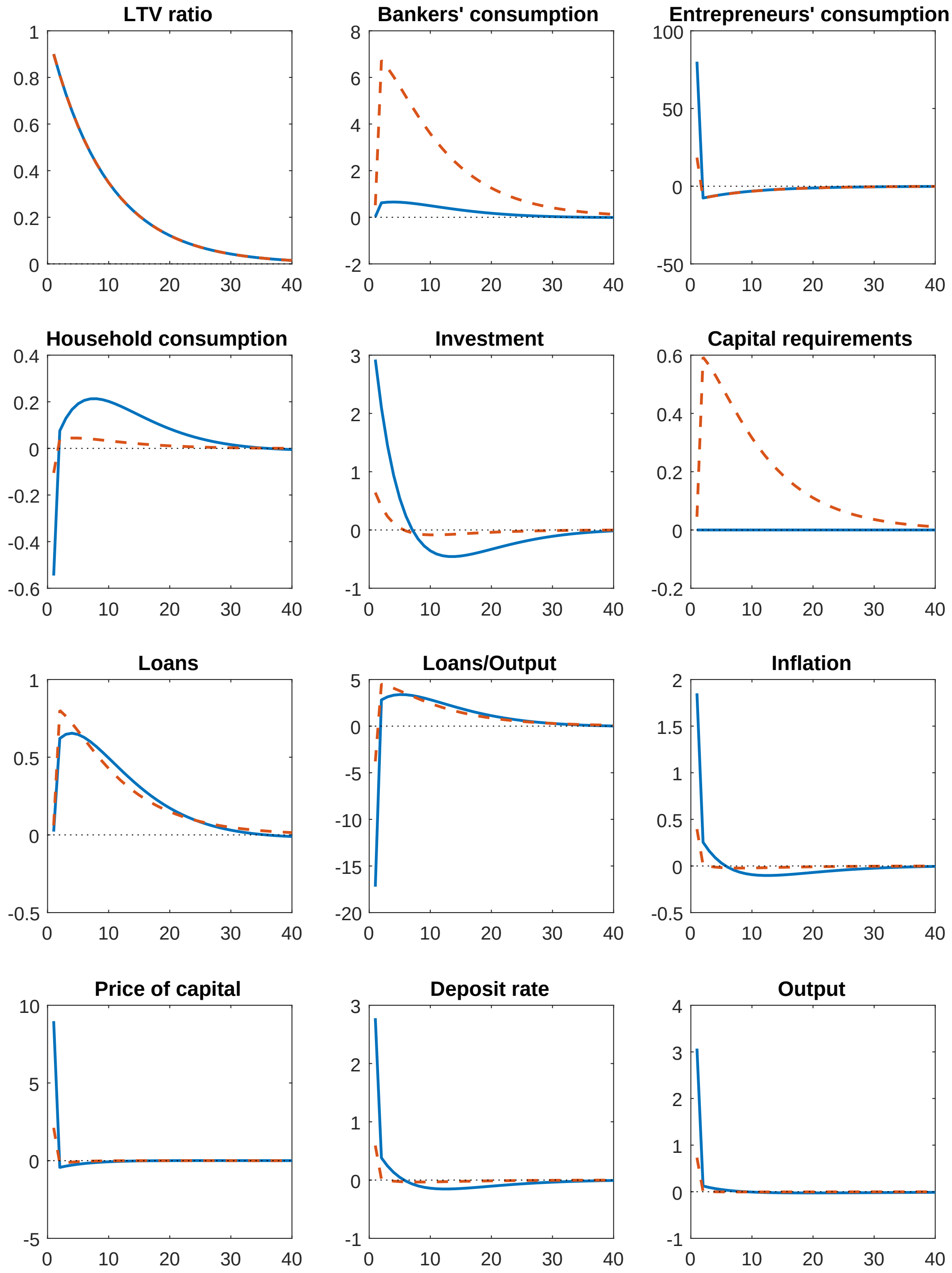
Notes: $\omega_b = 0.1$, $\omega_e = 0.1$, $\omega_h = 0.8$. Solid lines correspond to the decentralized equilibrium with constant capital requirements, dashed lines—to the Ramsey optimal equilibrium. Horizontal axes are quarters, the shock hits in quarter 1. Capital requirements, loans-to-output ratio, LTV ratio, inflation and deposit rate are in percentage point deviations from the steady state values. Inflation and deposit rate are annualized. Other variables are in percent deviations from the steady state.

Figure 5: Optimized macroprudential rule based on loans: TFP shock



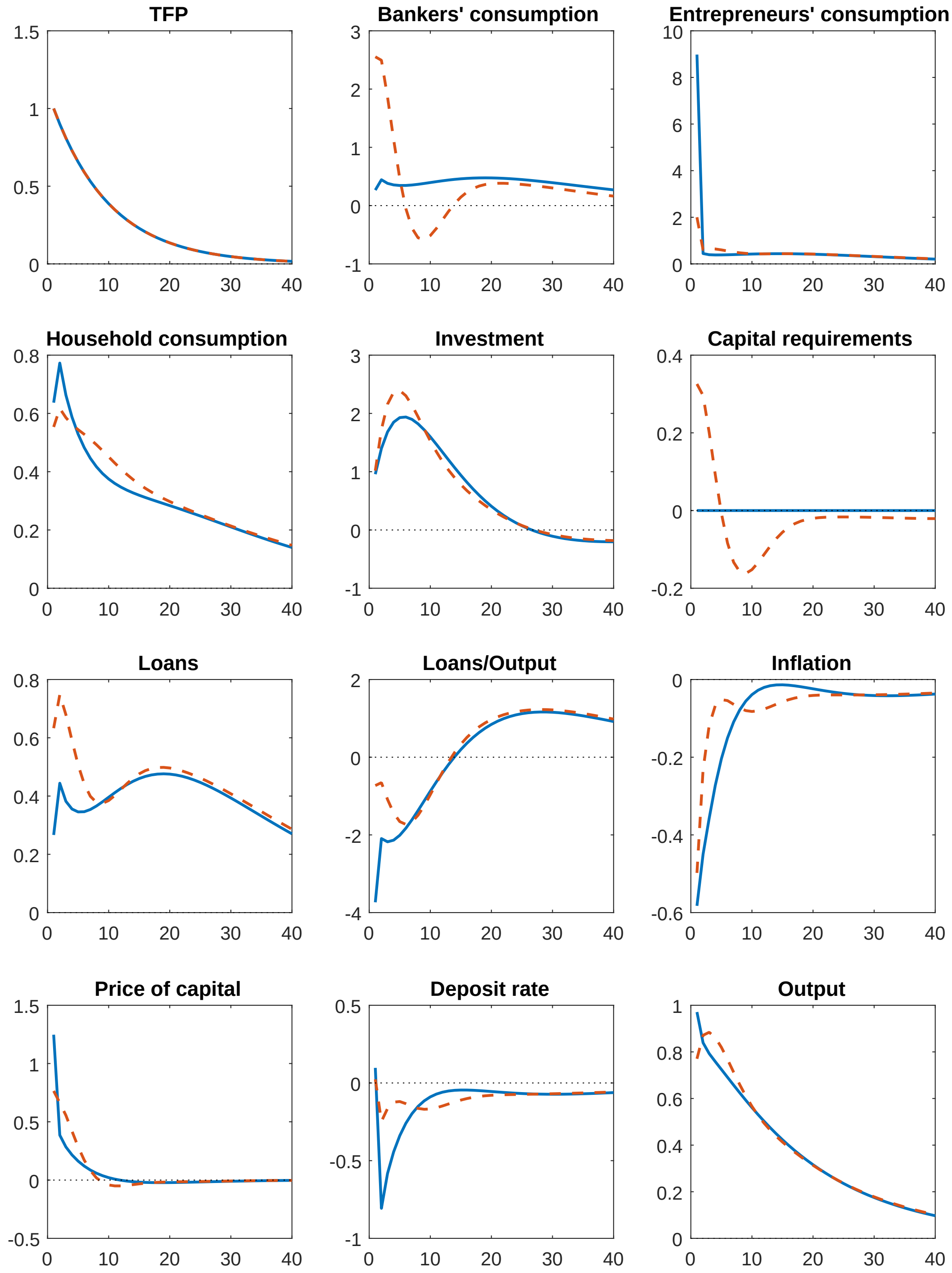
Notes: $\omega_b = 0.1$, $\omega_e = 0.1$, $\omega_h = 0.8$. Solid lines correspond to the decentralized equilibrium with constant capital requirements, dashed lines—with the optimized macroprudential rule. Horizontal axes are quarters, the shock hits in quarter 1. Capital requirements, loans-to-output ratio, inflation and deposit rate are in percentage point deviations from the steady state values. Inflation and deposit rate are annualized. Other variables are in percent deviations from the steady state.

Figure 6: Optimized macroprudential rule based on loans: LTV ratio shock



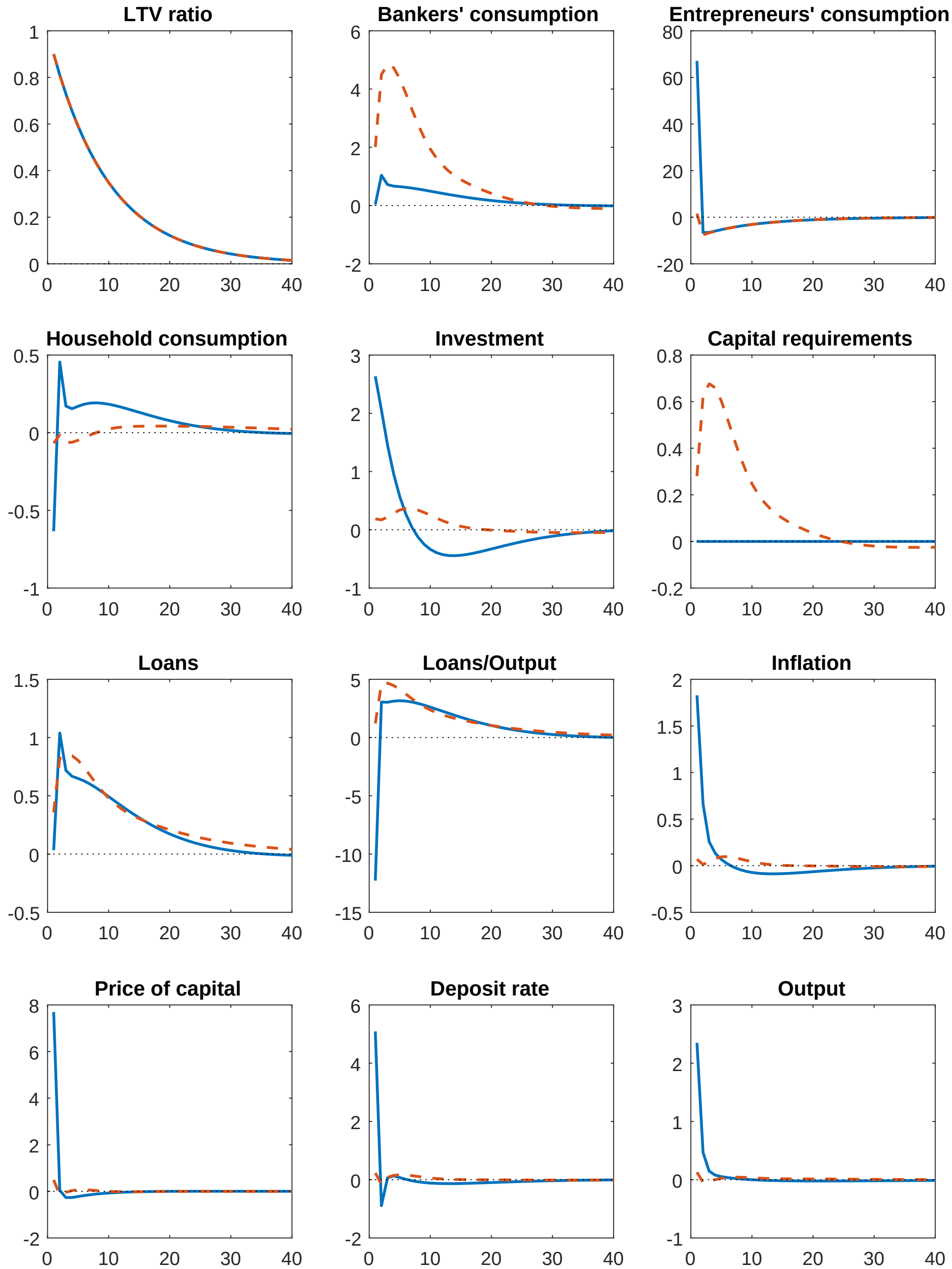
Notes: $\omega_b = 0.1$, $\omega_e = 0.1$, $\omega_h = 0.8$. Solid lines correspond to the decentralized equilibrium with constant capital requirements, dashed lines—with the optimized macroprudential rule. Horizontal axes are quarters, the shock hits in quarter 1. Capital requirements, loans-to-output ratio, LTV ratio, inflation and deposit rate are in percentage point deviations from the steady state values. Inflation and deposit rate are annualized. Other variables are in percent deviations from the steady state.

Figure 7: Ramsey optimal policy under flexible inflation targeting: TFP shock



Notes: $\omega_b = 0.1$, $\omega_e = 0.1$, $\omega_h = 0.8$. Solid lines correspond to the decentralized equilibrium with constant capital requirements, dashed lines—to the Ramsey optimal equilibrium. Horizontal axes are quarters, the shock hits in quarter 1. Capital requirements, loans-to-output ratio, inflation and deposit rate are in percentage point deviations from the steady state values. Inflation and deposit rate are annualized. Other variables are in percent deviations from the steady state.

Figure 8: Ramsey optimal policy under flexible inflation targeting: LTV ratio shock



Notes: $\omega_b = 0.1$, $\omega_e = 0.1$, $\omega_h = 0.8$. Solid lines correspond to the decentralized equilibrium with constant capital requirements, dashed lines—to the Ramsey optimal equilibrium. Horizontal axes are quarters, the shock hits in quarter 1. Capital requirements, loans-to-output ratio, LTV ratio, inflation and deposit rate are in percentage point deviations from the steady state values. Inflation and deposit rate are annualized. Other variables are in percent deviations from the steady state.