Explanation of US Interest Rates based on Inflation Tax Avoidance

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Abstract

This paper provides a general equilibrium quantity-theoretic explanation of interest rates by employing the eloquent Canzoneri, Cumby and Diba (2007) empirical method to compute our model's respective Euler equation. The agent uses exchange credit supplied through the financial intermediary production function. Our model generated interest rate Euler equation has a 86% correlation with the actual 3-month Treasury bill for the $1959Q1 - 2018Q2$ quarterly sample. We also focus on the post- Great Recession subsample explanation in terms of the backed out bank productivity shock.

JEL Classification: E13, E31, E43, E52

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1 Introduction

Relative to the well-known Fisher relation of interest rates, the Gibson paradox is that nominal interests follow the inflation rate, with high positive correlation, during some periods as expected but then depart from this in others, such as in the US after 1995 (Cogley, Sargent and Surico, 2012). Casares and Vazquez (2018) explain such interest rate "inversions" as an aspect of the Gibson paradox, relative to the Fisherian "norm", based on a monetary extended New Keynesian Taylor (1993) rule based on Canova and Ferroni (2012). This paper provides a general equilibrium quantity-theoretic explanation of interest rates more broadly by employing the eloquent Canzoneri et al. (2007) empirical method to compute our model's Euler equation based on velocity which, in contrast to Canzoneri et al. (2007) establishes a high 86% correlation of the model's estimated interest rate with actual US 3-month Treasury bills for the full quarterly sample of $1959Q1 - 2018Q2$. short term interest rate data.

This paper uses an exchange based economy in which, instead of transaction costs of consumer banking as tightly calibrated to end products of financial intermediation services, as in Alvarez and Lippi (2009), Silva (2012), Lucas and Nicolini (2015), Benati et al (2016), or with related transaction costs plus limited commitment that gives rise to exchange credit, as in Berentsen et al (2015, 2016), and Gu et al. (2016), the cost of financial intermediation is in terms of the cost of inputs into the general financial intermediation services output as in Sealey and Sealey and Lindley (1977), Clark (1984), Hancock (1985), Berk and Greene (2004), Benk et al (2005, 2008, 2010), Berger and Humphrey (1997), Degryse et al. (2009), Wheelock and Wilson (2006), Antunes et al. (2013) and Csabafi et al (2019). Using the bank production of exchange credit as based on inputs of labor and deposited funds, a unique equilibrium velocity arises for all nominal interest rates greater than or equal to zero, thereby avoiding important determinicacy concerns raised in alternative frameworks such as studied in Barthélemy and Marx (2019). The theory implies an Euler equation of interest rates, which depends in a graphically demonstrated way upon the degree of fiscal inflation tax avoidance, that is successfully estimated in the sense of the high correlation using the well-known Canzoneri et al. (2007) estimation of the Euler equation by using VAR estimation of variables that enter the theoretical model of interest rates, as compared to the reported 0.20 in Canzoneri et al. (2007).

For a prominent alternative approach, Del Negro et al. (2015) explain interest rates after the Great Recession using time varying inflation rate targets and Bernanke et al (1999) (BGG) finance premium shocks within as an extension of Smets and Wouters (2007) to focus on why inflation remained high relative to the nominal interest rate. Our approach instead assumes a stationary money supply growth process that is shocked because of fiscal needs, with inflation being a fiscal tax phenomenon which the agent optimally avoids, rather than a change in marginal cost unrelated to a fiscal tax as in the New Keynesian framework well exposited by Gali (2008) and also in Del Negro et al. (2015). Rather than intertemporal credit with shocks, as in BGG, Nolan and Theonissen (2009), Del Negro et al. (2015), and Csabafi et al. (2019), we also differ by having exchange credit as a costly way to avoid the inflation tax, as for example in Prescott (1987), and shocks to the bank sector producing this exchange credit that in turn facilitates the inflation tax avoidance that drives our model's Euler equation of interest rates; in addition, rather than a forecast focus, ours uses Canzoneri et al. (2007) methods to show performance of the model in terms of correlation with the interest rate and then we explain the Great Recession interest rate and inflation rate in terms of the shocks backed out from the model.

Our results also include study of pre-Great Recession and post- Great Recession sample periods, with the bank shock backed out and shown as the basis for our model's explanation of the post- Great Recession sample. For the pre- Great Recession period of $1959Q1 - 2007Q4$, the correlation of the model's interest rate and the actual 3-month Treasury bill rate is 89%; for the post- Great Recession period of $2008Q1 - 2018Q2$, this correlation is 93%. These results are with the same calibration and with differences only in that the Canzoneri et al. (2007) method produces somewhat different results for the variables entering the Euler equation, as dependent on the sample period, resulting for example in the 86% correlation for the full sample being less than in each of the above sub-periods.

Figure 1 shows the actual US CPI inflation rate, US Treasury 3-month and Federal

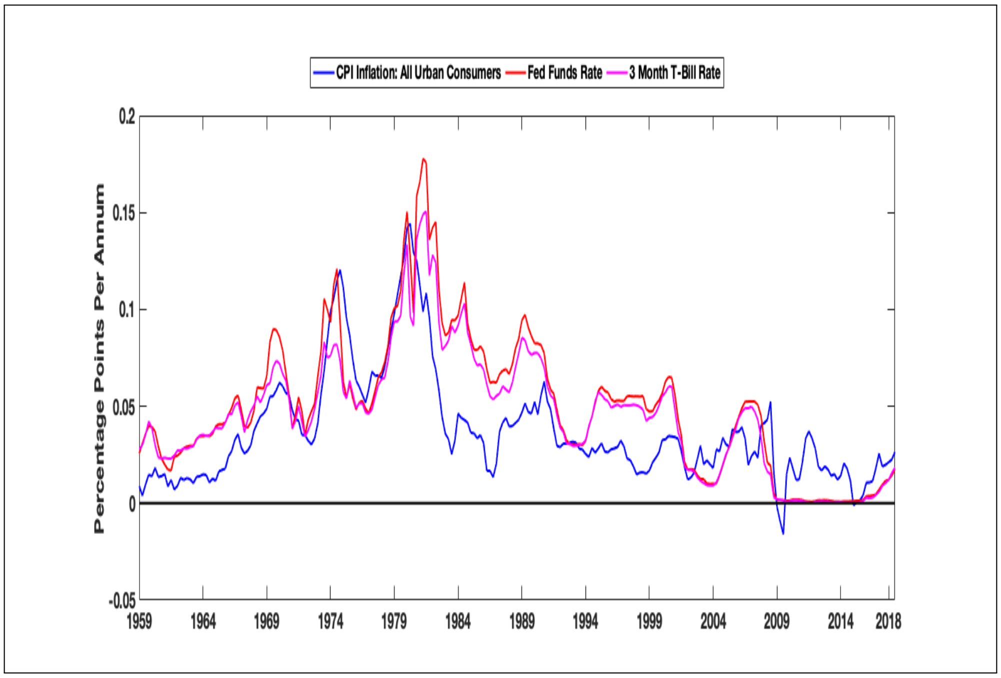

Figure 1: US CPI Inflation Rate, US 3-month Treasury Bill, and Federal Funds Rate: $1959Q1 - 2018Q2$.

Funds Rate, for $1959Q1 - 2018Q2$. This period includes the somewhat anomalous, relative to "Fisherian" norms, post 1995 period which has given rise to analysis of a re-emergence of the Gibson paradox. We explain the whole post- 1959 interest rate series, with a focus on this post- Great Recession period data.

Section 2 sets out related literature, Section 3 presents the model and Section 4 the Euler equation focus. Section 5 computes the model's Euler equation of interest rates using the Canzoneri et al. (2007) method; Section 6 presents the bank shock as backed out from data for the post Great Recession period; Section 7 and 8 provide Discussion and a Conclusion.

2 Related Literature

The determination of nominal interest rates in asset markets, rather than by central banks per se, is suggested by Fama (2013). In contrast, a large literature either derives or posits general theoretical properties of the Taylor (1993) principle coefficient within a central bank

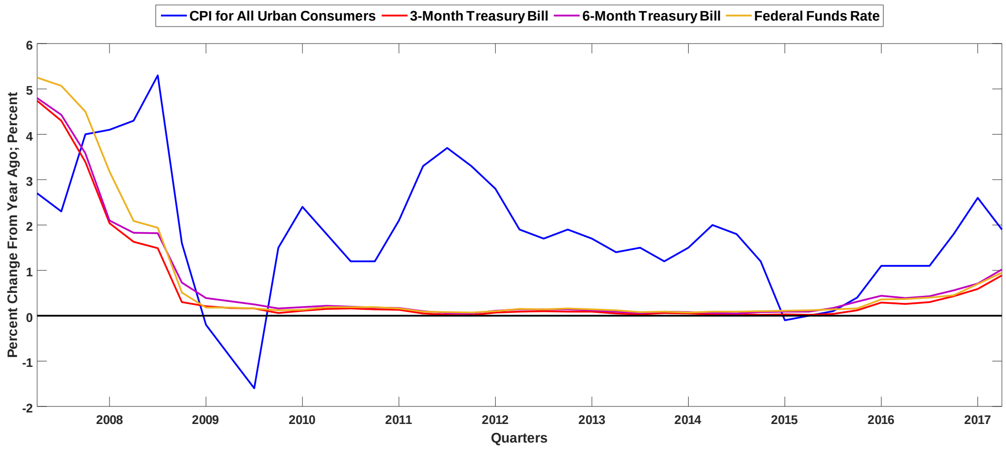

Figure 2: US CPI Inflation Rate and US Treasury 3-month, 6-month, and Federal Funds Rate, June 2007 to April 2017.

reaction literature, and a similar sized literature estimates a wide variety of versions of the Taylor (1993) rule. A smaller literature focuses on the nominal interest rate Euler equations in monetary models and the implied inflation coefficient, such as Coleman (1995, 1996), Bansal and Coleman (1996), and Canzoneri et al. (2007). In the latter, the expected inflation coefficient is one (as in standard nominal asset pricing models including Lucas, 1980), and using a sequence of extended models they show ways to improve on the empirical performance of the Euler equation. They conclude that having liquidity functions of assets, which Coleman (1995, 1996) and Bansal and Coleman (1996), include, may be useful; this liquidity function is also the focus of Gu et al. (2016) who derive an equilibrium with exchange credit and money as based on intertemporal debt limitations. This paper's use of productivity shocks allows a link back to this coefficient of the Euler equation and provides a real business cycle (RBC) approach with its use of productivity shocks to explain nominal variables, as do for example Gavin and Kydland (1999).[1]

The approach of adding an exchange credit substitute to money goes back to Prescott (1987), with this mix made endogenous for example in Gillman (1993), Ireland (1994), Marquis and Reffett (1994), Coleman (1995, 1996), Bansal and Coleman (1996), Kydland

[1] Gavin and Kydland (1999) add a money feedback rule, which we do not employ, in a model with productivity shocks to help explain nominal volatilities.

and Freeman (2000), and Lucas and Nicolini (2015). These latter endogenous velocity papers use a transaction cost approach to specify a cost of exchange credit that is not pinned down through the technology of bank production, although Coleman (1996) interacts the transaction cost with a bank sector assumed to have a fixed cost per unit. Another general transaction cost approach without explicit exchange credit is found in the literature of McCallum (1983), Brock (1989), Gavin and Kydland (1999), Lucas (2000), Schmitt-Grohe and Uribe (2004), Kimbrough (2006), and Teles and Uhlig (2016) in which exchange costs in the form of (shopping) time or of goods are lowered by using more money in exchange. All of these but McCallum (1983) backwards engineer the time or goods transaction cost to yield either a Baumol (1956)-Tobin (1965) or Cagan (1956) money demand function, rather than being based on any microeconomic provision of a substitute to money such as exchange credit produced by banks.

This paper has exchange credit supplied by a bank production function, as suggested by King and Plosser (1984), instead of using an general transaction cost. This enables an RBC style productivity shock to banking, thereby extracting from the recent literature in modeling the bank balance sheet, the role of leverage, and the onset of crises as in BGG, Kiyotaki and Moore (1997), Nolan and Thoenissen (2009) or Del Negro et al. (2015); see Gunther and Nolan (2018) for a review of this literature. This financial intermediation approach to banking also results in the asset pricing equation having a coefficient on its expected inflation rate term that is always greater than one (for positive nominal interest rates). And it provides the price-theoretic graphical interpretation of this inflation coefficient along the dynamic transition during shocks to the economy. It results in a central role for banking in money demand theory that resonates with Reynard (2004), while allowing the paper to exploit the Euler equation to help in explaining its results, as in Bansal and Coleman (1996). The money supply is stochastic as in a fiscal policy inflation such as documented in De Graeve and von Heideken (2015). [2]

[2] An alternative rich literature builds upon Kiyotaki and Wright (1989, 1993), as reviewed for example in Lagos et al. (2016), and extends monetary results in many directions, including constructing equilibria using credit, without reliance on nominal rigidities or Taylor interest rate rules in many cases, as is similar to the approach of this paper which like their work also avoids such indeterminacy issues as highlighted in Cochrane (2011).

3 Stochastic Exchange Economy with Banking

The economy is a special case of Benk et al. (2010), without capital accumulation. While in Coleman (1996) the bank has a fixed cost per unit in supplying checkable deposits plus a transactions cost for the household, the model here also has an implicit convex cost of producing exchange credit per unit of goods as a result of its specification of this Cobb-Douglas production function. Shocks occurs at the beginning of the period, and then asset and goods trading and production occur after the shock is revealed. Given the random transfer of money from the government each period to the household, of an amount denoted by T_t, the household is choosing M_{t+1}, as well as choosing nominal bond purchases, B_{t+1}, given the asset carryover of $M_t + T_t$, plus the net interest income and principle on those bonds, $R_t B_t$, with R_t the gross nominal interest rate. The government finances any payment of interest on bonds by a lump sum real tax each period on the household, τ_t.[3]

The exchange process is that the representative household uses exchange credit to avoid some of the inflation tax as long as the net nominal interest rate is above the Friedman (1969) optimum of zero. The household as goods producer agrees to accept both money and exchange credit for goods purchasing given the deposit of wage earnings at the bank. The household as banker charges a competitive price per unit of exchange credit, which in nominal terms is denoted by P_{Qt}, while the credit itself is denoted in real terms by q_t. It results that the credit service price in real terms in equilibrium is set to the net government bond risk-free interest rate $\bar{R}_t$. Since the household owns the bank, the banker gives back the profit to the household in the form of dividends per unit of deposits that are made in the bank, where the deposits equal the total wage income of the household, net of the banking expense. With the deposits in the bank denoted by d_t, and the net dividend rate per deposit denoted as $\bar{R}_t^d$, the bank gives back $\bar{R}_t^d d_t$, while the deposits are used up by the end of the period from either cash taken out in advance or credit purchases paid off. There is no reserve requirement and no uncertainty in the bank's operation, and it can create credit backed by the household's income in the sense that it is fully collateralized.

[3]Bansal and Coleman (1996) and Coleman (1996) use separate credit purchases constraints backed by bonds to assume a liquidity provision of such bonds, through their transaction cost specification, which is not explored here.

As a candidate simplest model in order to model the inflation rate and interest rate relation found in the data, there is no physical capital or leisure. All time is employed within a linear production function of goods and a Cobb-Douglas banking production function. There is no trend up in productivity, or human capital investment as in Benk et al. (2010), so the growth rate is zero along the stationary equilibrium balanced growth path (BGP). There are AR(1) shocks to the goods sector productivity, the bank sector productivity, and the money supply growth rate [delete: , but the latter shock is not employed in the simulation].

The representative household consumer has expected lifetime utility from consumption of goods, c_t; with $\beta \in (0,1)$, and $\theta > 0$, this is given by

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \frac{{c_t}^{1-\theta}}{1-\theta}. \tag{1}$$

The agent allocates a time endowment of one between labor in goods production, l_{Gt}, and time spent working in the bank sector, denoted by l_{Qt} (Q subscript for Credit):

$$l_{Gt} + l_{Qt} = 1. \tag{2}$$

With w denoting the real wage, the consumer receives nominal income of wages, $P_t w_t (l_{Gt} + l_{Qt}) = P_t w_t$, dividends from the bank $P_t \bar{R}_t^d d_t$ and a nominal random transfer of money from the government, T_t, as well as paying a lump sum tax of τ_t in real terms. The consumer pays the bank a fee for credit services, whereby one unit of credit service is required for each unit of credit that the bank supplies the consumer for use in buying goods. With P_{Qt} the nominal price of each unit of credit and q_t the real quantity of credit, the consumer pays $P_{Qt} q_t$ in credit fees. With other expenditures on goods, of $P_t c_t$, and on investment in cash for purchases, of $M_{t+1} - M_t$, and in nominal bonds $B_{t+1} - B_t(1+\bar{R}_t)$, the consumer's budget constraint is:

$$
\begin{aligned}
& P_t w_t + P_t \bar{R}_t^d d_t + T_t \\
\geq \quad & P_{Qt} q_t + P_t c_t + M_{t+1} - M_t + B_{t+1} - B_t(1 + \bar{R}_t) + \tau_t.
\end{aligned} \tag{3}
$$

The consumer can purchase goods by using either money after-transfer money balances $M_t + T_t$ or exchange credit, which in nominal terms is $P_t q_t$ so that the consumer's exchange technology is:

$$
M_t + T_t + P_t q_t \geq P_t c_t. \tag{4}
$$

Since all cash comes out of deposits at the bank and credit purchases are paid off after production takes place out of the same deposits, total deposits are equal to consumption at the end of the period, such that

$$
d_t = c_t; \tag{5}
$$

please see Appendix A for the full consumer optimization problem, equilibrium conditions, BGP equations, and log-linearized model.

The bank produces exchange credit via a functional form used extensively in the financial intermediation microeconomics literature starting with Clark (1984) and promulgated by Berger and Humphrey (1997), Wheelock and Wilson (2006), Degryse et al. (2009) and Inklaar and Wang (2013), for example. The bank produces credit that is available for exchange at the point of purchase. The bank determines the amount of such credit by maximizing profit subject to the labor and deposit costs of producing the credit. The *competitive* production of credit uses a constant returns to scale technology with effective labor and deposited funds as inputs. In particular, with $A_Q > 0$ and $\gamma \in (0,1)$:[4]

$$q_t = A_Q \left(l_{Qt}\right)^{\gamma} d_t^{1-\gamma}, \tag{6}$$

where A_{Qt} is the banking productivity with an AR(1) process around a BGP value of $\bar{A}_Q > 0:$

$$A_{Qt} - \overline{A}_Q = \rho_Q(A_{Qt-1} - \overline{A}_Q) + \epsilon_{Qt}; \tag{7}$$

where ρ_Q is the autocorrelation parameter and ϵ_{Qt} is a sequence of i.i.d. innovations with zero mean and a constant variance. Subject to the production function in equation (6), the bank maximizes profit Π_{Qt} with respect to the labor l_{Ft} and deposits d_t, in a current period problem [5]

$$\frac{\Pi_{Qt}}{P_t} = \frac{P_{Qt}}{P_t} q_t - w l_{Qt} - \bar{R}_t^d d_t. \tag{8}$$

Equilibrium implies that:

$$\left(\frac{P_{Qt}}{P_t}\right) \gamma A_Q \left(\frac{l_{Qt}}{d_t}\right)^{\gamma-1} = w_t; \tag{9}$$

$$\left(\frac{P_{Qt}}{P_t}\right) (1-\gamma)\, A_Q \left(\frac{l_{Qt}}{d_t}\right)^{\gamma} = \bar{R}_t^d. \tag{10}$$

[4] For example, in support of this functional form, Degryse et al. (2009), p. 39, writes that " Shaffer (1993) follows the so-called financial intermediation approach to banking. According to this view, bank use labor and deposits to originate loans." And he finds empirical support for perfect competition, which is assumed in our model.

[5] The problem is a current period one similar to Coleman (1996) with the main exception being that he instead uses a fixed cost of banking services.

Equation (9) indicates that the marginal cost of credit, $\left(\frac{P_{Qt}}{P_t}\right)$, is equal to the marginal factor price divided by the marginal factor product, or $\frac{w}{\gamma A_F e^{v_t}\left(\frac{l_{Ft}h_t}{d_t}\right)^{\gamma-1}}$, and equation (10) that the zero profit dividend yield paid on deposits is equal to the fraction of the marginal cost given by

$$\left(\frac{P_{Qt}}{P_t}\right)(1-\gamma)\left(\frac{q_t}{d_t}\right)=\bar{R}_{dt}.$$

In turn the consumer problem, from equations (42), (44), and (45), implies that

$$\frac{P_{Qt}}{P_t}=\bar{R}_t, \tag{11}$$

and so

$$\bar{R}_t \geq \bar{R}_t(1-\gamma)\frac{q_t}{d_t}=\bar{R}_{dt}. \tag{12}$$

Define the after-cash-transfer of money at time t by M_t' and the real quantity of this by $m_t \equiv \frac{M_t'}{P_t}$. Then using the notation of the share of money in consumption expenditure, as denoted by $a_t \equiv \frac{m_t}{c_t}$, where $c_t = d_t$ from equation (5), and similarly from equation (4) that $(1-a_t)=\frac{q_t}{c_t}$, the rate of return to depositors $\bar{R}_{dt}$ can be expressed as

$$\bar{R}_t \geq \bar{R}_t(1-\gamma)(1-a_t)=\bar{R}_{dt}. \tag{13}$$

The firm maximizes profit given by $y_t - wl_{Gt}$ (G subscript for goods sector) subject to a linear production function in effective labor and capital:

$$y_t = A_{Gt}l_{Gt}, \tag{14}$$

where A_{Gt} is the stochastic goods sector productivity with an AR(1) shock process around its BGP value of $\bar{A}_G > 0$:

$$A_{Gt} - \overline{A}_G = \rho_G(A_{Gt-1} - \overline{A}_G) + \epsilon_{Gt}; \tag{15}$$

ρ_G is the autocorrelation parameter and ϵ_{Gt} is a sequence of i.i.d. innovations with zero mean and a constant variance. The first order condition for the firm's problem yields that

$$w_t = A_{Gt}. \tag{16}$$

It is assumed that government policy includes sequences of nominal cash transfers to the consumer as given by:

$$T_t = \sigma_t M_t, \tag{17}$$

where σ_t is the net growth rate of money, an AR(1) process of the form:

$$\sigma_t - \overline{\sigma} = \rho_\sigma(\sigma_{t-1} - \bar{\sigma}) + \epsilon_{\sigma t}; \tag{18}$$

where ρ_σ is the autocorrelation parameter, $\bar{\sigma} > 0$ the long-run stationary money supply net growth rate, and $\epsilon_{\sigma t}$ is a sequence of i.i.d. innovations with zero mean and a constant variance. Including bond sales and redemptions, the government budget constraint is

$$M_{t+1} - M_t - T_t + B_{t+1} - B_t(1 + \bar{R}_t) + \tau_t = 0. \tag{19}$$

Given M_0 and the evolution of M_t ($t > 0$) as given by the exogenous monetary policy in equation (17) below, the consumer maximizes utility in equation (1) subject to the budget, exchange and deposit constraints (3)-(5), with the definition of equilibrium and equilibrium conditions given in Appendix A. Bond holdings are zero in equilibrium, but provide the equilibrium asset price value of R_t.

4 General Equilibrium Asset Pricing

From equations (41)-(45), the Euler equation of interest rates follows as

$$1 = \beta E_t \left\{ \frac{c_{t+1}^{-\theta}}{c_t^{-\theta}} \frac{\tilde{R}_t}{\tilde{R}_{t+1}} \frac{R_{t+1}}{\Pi_{t+1}} \right\}, \tag{20}$$

where $\bar{R}$ and $\bar{\pi}$ are net rates of nominal interest and inflation, such that $R \equiv 1+\bar{R}$, $\Pi = 1+\bar{\pi}$. The term $\tilde{R}_t$ is a gross rate equal to one plus a 'weighted average cost of exchange' defined as follows:

$$\tilde{R}_t \equiv 1 + \frac{m_t}{c_t}(R_t - 1) + \gamma \left(1 - \frac{m_t}{c_t}\right)(R_t - 1), \tag{21}$$

where a weight of $\frac{m}{c}$ is attached to the opportunity cost of money $(R_t - 1)$ and a weight of $(1 - \frac{m}{c})$ is attached to the average cost of credit, $\gamma (R_t - 1)$, and $\frac{m_t}{c_t}$ is the real consumption normalized demand for money (i.e. the inverse of the consumption velocity of money). In effect, equation (20) augments a standard consumption Euler equation with the (growth rate of) the weighted average cost of exchange. If all goods purchases are restricted to using money $(m_t/c_t = 1)$ then equation (20) reverts back to the familiar consumption Euler equation which would constitute an equilibrium condition of a standard, unit velocity cash-in-advance model without a money alternative: $1 = \beta E_t \left\{ \frac{c_{t+1}^{-\theta}}{c_t^{-\theta}} \frac{R_t}{\Pi_{t+1}} \right\}$,

With $a \equiv \frac{m}{c} \leq$, a is the fraction of goods bought with money and the exchange cost can be written as

$$\tilde{R}_t = 1 + a_t \bar{R}_t + \gamma (1 - a_t) \bar{R}_t. \tag{22}$$

For some intuition on the role played by equation (22), consider a log-approximation of the Euler equation (20), with the interest rate terms brought to the lefthandside:

$$\begin{aligned} & \bar{R}_t\left[a_t+\gamma\left(1-a_t\right)\right]+E_t\left(\bar{R}_{t+1}\left[1-\left[a_{t+1}+\gamma\left(1-a_{t+1}\right)\right]\right]\right)-\bar{R} \\ = \quad & E_t\left(\overline{\pi}_{t+1}-\overline{\pi}\right)+\theta E_t\left(g_{c,t+1}-g\right). \end{aligned} \tag{23}$$

Note that $\bar{R}_t\left[a_t+\gamma\left(1-a_t\right)\right]$ is exactly the average exchange cost of buying goods, using both money with a cost of $a_t\bar{R}_t$, and credit with an average cost of $\gamma\left(1-a_t\right)\bar{R}_t$ (proved below). Then the interest rate terms on the lefthandside are a weighted average, where the weights on $\bar{R}_t$ is the average exchange cost per unit of consumption using both money and credit, or $\left[a_t+\gamma\left(1-a_t\right)\right]$. The weight on the next period interest rate is one minus the expected exchange cost using both money and credit, or $1-\left[a_{t+1}+\gamma\left(1-a_{t+1}\right)\right]$. In the BGP equilibrium, the weights add to one since $\left[a+\gamma\left(1-a\right)\right]+1-\left[a+\gamma\left(1-a\right)\right]=1$, but do not add to one during the transition dynamics when the economy reacts to sequences of monetary shocks.

Since the future period interest rate factor of $1-\left[a_{t+1}+\gamma\left(1-a_{t+1}\right)\right]$ reduces to $\left(1-\gamma\right)\left(1-a_{t+1}\right)$, then when multiplied by the expected nominal interest rate $E_t\left(\bar{R}_{t+1}\right)$, this gives $E_t\left(\bar{R}_{t+1}\right)\left(1-\gamma\right)\left(1-a_{t+1}\right)$, which in turn is equal to $E_t\left(\bar{R}^d_{t+1}\right)$, by equation (13) (with equality). The return per unit of deposits in the bank is

$$E_t\left(\bar{R}_{t+1}\left[1-\left[a_{t+1}+\gamma\left(1-a_{t+1}\right)\right]\right]\right)=E_t\bar{R}^d_{t+1}.$$

Each of these above weights on $E_t\left(\bar{R}_{t+1}\right)$ and on R_t are important since their BGP forms make up the numerator and denominator respectively of the expected inflation coefficient as solved in the log-linearization of the Euler equation (20) around the steady state. To see this, start with standard definitions such that for any variable z_t, define $\widehat{z}_t\equiv\ln z_t-\ln z$, where the absence of a time subscript denotes a BGP stationary value, and define $\widehat{g}_{z,t+1}\equiv$

$\ln z_{t+1} - \ln z_t$. Then the log-linear approximation of (20) evaluated around the BGP is

$$0 = E_t \left\{ \theta \widehat{g}_{c,t+1} + \widehat{g}_{\tilde{R},t+1} - \widehat{R}_{t+1} + \widehat{\Pi}_{t+1} \right\}. \tag{24}$$

After considerable algebra, as presented in the Appendix B, it results that the Taylor condition can be expressed in log-deviations from the BGP equilibrium as:

$$\widehat{R}_t = (1+\omega) E_t \left\{ \widehat{\Pi}_{t+1} + \theta \widehat{g}_{c,t+1} \right\} + \omega E_t \left\{ \left(\frac{\bar{R}\frac{m}{c}}{1-\frac{m}{c}} \right) \widehat{g}_{\frac{m}{c},t+1} - \widehat{R}_{t+1} \right\} \tag{25}$$

where

$$\omega \equiv \frac{1}{1+\bar{R}} \frac{(1-\gamma)\left(1-\frac{m}{c}\right)}{\left[1-(1-\gamma)\left(1-\frac{m}{c}\right)\right]} \geq 0. \tag{26}$$

Using that $a \equiv \frac{m}{c}$, the key inflation coefficient can be expressed as

$$1+\omega \equiv 1 + \frac{1}{1+\bar{R}} \frac{(1-\gamma)(1-a)}{[a(1-\gamma)+\gamma]}. \tag{27}$$

Since $a \equiv \frac{m}{c}$, $\gamma \in [0,1)$, and the BGP money demand is given from equations (6), (9) and (11) by

$$a \equiv \frac{m}{c} = 1 - q_t = 1 - A_Q \left(\frac{\bar{R}\gamma A_Q}{w} \right)^{\frac{\gamma}{1-\gamma}}. \tag{28}$$

it is clear in equation (27) that $\omega \geq 0$, and $1+\omega \geq 1$.

The money demand interest elasticity, denoted by $\eta^{R}_{m/c}$, is a type similar to the Cagan (1956) model with a rising elasticity in magnitude as inflation increases and exchange credit is substituted for money at an increasing rate; $\eta^{\bar{R}}_{m/c} = -\frac{1-a}{a}\frac{\gamma}{1-\gamma}$ is proportional to the ratio of credit to money, since $\frac{1-a}{a} = q/m$. At the Friedman optimum, $\bar{R} = 0$, $a = 1$, and $\omega = 0$.

The net inflation coefficient ω here is the degree to which the coefficient of the inflation rate exceeds one. A simple description of ω results from the above discussion and equation (13) (with equality). In particular the net inflation coefficient ω indicates the ratio of the one period discounted value amount of the inflation tax that is avoided to the total exchange cost per unit of consumption. This is because the inflation tax avoided per unit of consumption equals the bank dividend return per unit of credit, $\bar{R}^d$, while the inflation tax paid equals $a_t\bar{R}_t$ (that paid on money used) plus the total cost of producing credit $(\bar{R}\gamma(1-a))$, per unit of consumption, which in turn can be expressed as $\bar{R}-\bar{R}^d$:

$$\begin{aligned}\omega &= \frac{1}{1+\bar{R}}\frac{\bar{R}(1-\gamma)(1-a)}{\bar{R}[a+\gamma(1-a)]} = \frac{1}{1+\bar{R}}\frac{\bar{R}^d}{\bar{R}-\bar{R}^d}, \\ &= \frac{1}{1+\bar{R}}\frac{\textit{Dividend return Per Unit}}{\textit{Total Exchange Cost Per Unit}}.\end{aligned}$$

For example if $\gamma = 0.2$ and $\bar{R} = 0.1$ with $m/c \equiv a = 0.44$, then velocity is 2.27 and ω would be equal to

$$\omega = \frac{1}{1.1}\frac{(1-0.2)(1-0.44)}{0.44+0.2(1-0.44)} = 0.74.$$

The gross value of the inflation rate coefficient would be $1+\omega = 1.74$.

If for the same γ and R, but different A_G and/or A_Q such that the BGP velocity was higher at say $a = (1/12)$, so that velocity equals 12, then

$$\omega = \frac{1}{1.1}\frac{(1-0.2)(1-0.0833)}{0.0833+0.2(1-0.0833)} = 2.5,$$

and the inflation rate coefficient would be $1+\omega = 3.5$. So in higher velocity eras, the inflation coefficient may be higher, in this case doubled from 1.74 to 3.5. A higher velocity would result from pivoting down the marginal cost curve for banking, or by a higher stationary money supply growth rate that raises R, and moves the equilibrium up along the marginal cost curve to a higher point.

4.1 Banking Marginal Cost and Money Demand

To graph the BGP one-period future value of the parameter ω, which is $\omega\left(1+\bar{R}\right)$, consider the marginal cost of credit per unit of deposits, as graphed in the same units against the marginal cost of money, which is the nominal interest rate $\bar{R}$. To derive the marginal cost of credit function, first take the bank's equilibrium condition with respect to its choice of banking time labor l_{Qt}, as given by equation (9), use the households first order conditions that equates the real market price of credit to $\bar{R}$ ($\frac{P_{Qt}}{P_t}=\bar{R}$, in equation (11)) and solve for $\bar{R}$, as the ratio of the marginal factor cost (w) to the marginal factor product of labor in banking ($\gamma\left(\frac{l_{Ft}h_t}{d_t}\right)^{\gamma-1}$). This later ratio is equal to the marginal cost of banking output in price theory.

$$\begin{aligned}\left(\frac{P_{Qt}}{P_t}\right)\gamma A_Q\left(\frac{l_{Ft}}{d_t}\right)^{\gamma-1} &= w_t,\\ \bar{R} &= \frac{w_t}{\gamma\left(\frac{l_{Qt}}{d_t}\right)^{\gamma-1}}.\end{aligned} \tag{29}$$

To graph the marginal cost in terms of the output space of credit per unit of deposits, or q/d, normalize the bank production function of equation (6) by dividing by deposits d, which gives $q/d=A_Q\left(\frac{l_{Ft}}{d_t}\right)^{\gamma}$. Then solve for the input ratio $\frac{l_{Qt}}{d_t}$ as

$$\frac{l_{Qt}}{d_t}=\left(\frac{q/d}{A_Q}\right)^{1/\gamma}, \tag{30}$$

and substitute for $\frac{l_{Qt}}{d_t}$ in equation (29) to get the marginal cost of credit per unit of deposits. Denote the latter by $MC_{q/d}$, express it in terms of the output q/d and view it in terms of the equalization of the marginal cost of money to that of credit:

$$\bar{R}=\frac{w}{\gamma\left(A_Q\right)^{(1-\gamma)/\gamma}}\left(q/d\right)^{(1-\gamma)/\gamma}=\frac{P_{Qt}}{P_t}\equiv MC_{q/d}. \tag{31}$$

Figure 2 graphs the marginal cost of credit per unit of deposits (denoted as MC) as against the example nominal interest rate $\bar{R}=0.10$. In this illustration $\gamma=0.2$ and $w/\left[\gamma\left(A_Q\right)^{(1-\gamma)/\gamma}\right]=1$, so that $MC_{q/d}=\left(q/d\right)^4$ is graphed. This gives an equilibrium

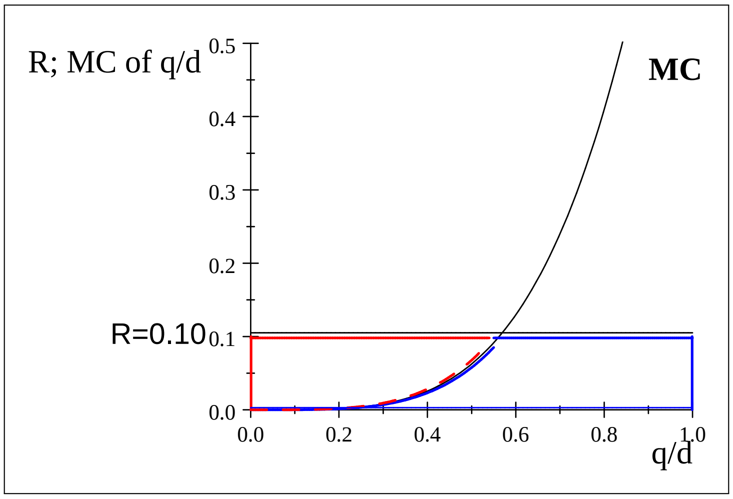

Figure 3: Equilibrium Endogenous Velocity and Baumol-Tobin Margin

$q^*/d = 0.1^{0.25} = 0.56$. Equilibrium consumption velocity follows as $1/\left[1-(q^*/d)\right] = c/m = 1/\left(1-0.56\right) = 2.27$.

The total producer surplus is given by the Red-outlined producer surplus area in the graph. It is the integral of the difference between the nominal interest rate and the marginal cost curve from $q/d = 0$ up to the (Baumol-Tobin type) equilibrium q^*/d where the intersection occurs of the nominal interest rate with the marginal cost curve. The marginal cost of money, per unit of goods, equals that of credit, per unit of deposits, which also is in the same units of "goods" since $c = d$ is an equilibrium condition.

The total cost of exchange, per unit of deposits, is comprised of two areas that are represented by the Blue-outlined area of Figure 3. This consists of the cost of exchange using money, which is the square area $\bar{R}\cdot(m^*/d)$, the traditional inflation tax revenue per unit of deposits, plus the total cost of producing the credit per unit of deposits, q^*/d, as given by the integral of the area under the marginal cost curve from $q/d = 0$ up to the equilibrium credit q^*/d. This latter area under the MC curve also equals wl_{Qt}/d_t, which is analogous to the normalized value of shopping time in the Lucas (2000) formulation of the welfare cost of inflation.

It is clear that $\bar{R}\cdot 1$ is the total area of the two color-outlined areas, the Red and the Blue. And since the Red area is the producer surplus that is returned to the consumer, as given above mathematically by equation (12), along the BGP it results that this area is $\bar{R}(1-\gamma)\frac{q}{d}=R^d$. Then the Blue area is $(R\cdot 1)$ minus the producer surplus, or simply $\bar{R}-\bar{R}^d$. And the ratio of the Red to the Blue area is the one-period forward value of the net inflation coefficient ω, which equals

$$\left(1+\bar{R}\right)\omega=\frac{\bar{R}^d}{\bar{R}-\bar{R}^d}.$$

Therefore the inflation coefficient ω is given by the one-period forward value of the balanced growth path (BGP) ratio of the per-unit total inflation tax that is avoided to the value of the per-unit total cost of exchange, the latter of which is comprised of the per-unit inflation tax paid plus the per-unit banking cost of avoiding the inflation tax.

The average cost of exchange credit per unit of deposits, as denoted by $AC_{q/d}$ (and in Figure 2 by AC), is given as the total bank cost wl_Q per unit of deposits d divided by the total equilibrium output of exchange credit q per unit of deposits d, or as

$$AC_{q/d}\equiv\frac{wl_Q}{q}=\frac{\frac{wl_Q}{d}}{q/d}.$$

Substituting in from the bank production function for $\frac{l_Q}{d}$ from equation (30), the average cost can be written as a function of q/d :

$$AC\equiv w\left(\frac{l_Q}{d}\right)\frac{1}{(q/d)}=w\left(\frac{q/d}{A_Q}\right)^{(1/\gamma)-1}=\frac{w}{A_Q^{(1-\gamma)/\gamma}}\left(q/d\right)^{(1-\gamma)/\gamma}.$$

And since the marginal cost is given in equation (31) as $MC_{q/d}\equiv\frac{1}{\gamma}\frac{w}{(A_Q)^{(1-\gamma)/\gamma}}\left(q/d\right)^{(1-\gamma)/\gamma}$, it results as in microeconomic price theory that

$$AC_{q/d}=\gamma MC_{q/d}.$$

And the equilibrium equivalence of $\bar{R}$ to the $MC_{q/d}$ means that the average cost can be expressed simply as $\gamma\bar{R}$, which is the value of the AC at the quantity $q*/d=0.56$ in Figure 2.

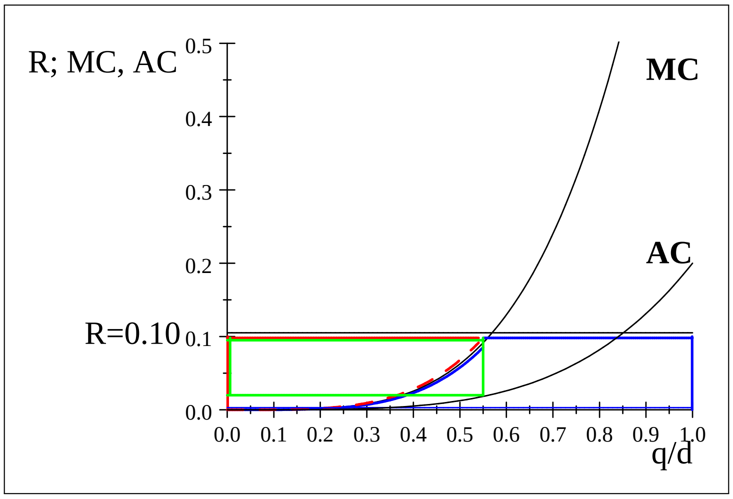

Figure 4: Equilibrium Endogenous Velocity and Baumol-Tobin Margin

In Figure 4, graph both the marginal cost of Figure 3 plus the average cost function $AC_{q/d} = \gamma MC_{q/d}$, as well as the equivalent expression for average cost as $\gamma \bar{R}$. With $\gamma = 0.2$, and $\gamma \bar{R} = 0.02$, the profit box, instead of the producer surplus equivalent, is given by the Green box as the area of $\bar{R}(1-\gamma)\frac{q}{d}$, which indeed is the definition of $\bar{R}^d$ as the profit dividend given back to the consumer. This proves that the producer surplus area in the Green box equals $\bar{R}^d$, which also equals the area given by the Red outline in Figures 3 and 4. This means that alternatively the inflation coefficient can be expressed in Figure 4 as the one-period forward value of the ratio of the Green box to the Blue area.

Both the forward values of the ratio of the Red producer surplus to the Blue cost of exchange and the ratio of the Green profit box to the Blue cost of exchange are equivalent graphical measures of $(1+R)\omega$.

Conversely, the graph of money demand in this economy, using equation (28) and the same assumptions that $\frac{w}{\gamma(A_Q)^{(1-\gamma)/\gamma}} = 1$ and $\gamma = 0.2$, can be presented by solving for $\bar{R}$:

$$\bar{R} = \left(1 - \frac{m}{c}\right)^{(1-0.2)/0.2},$$

Figure 5 graphs this money demand function. It is a simple corollary using geometry (by superimposing the marginal cost of Figure 3 onto the money demand of Figure 5) that the

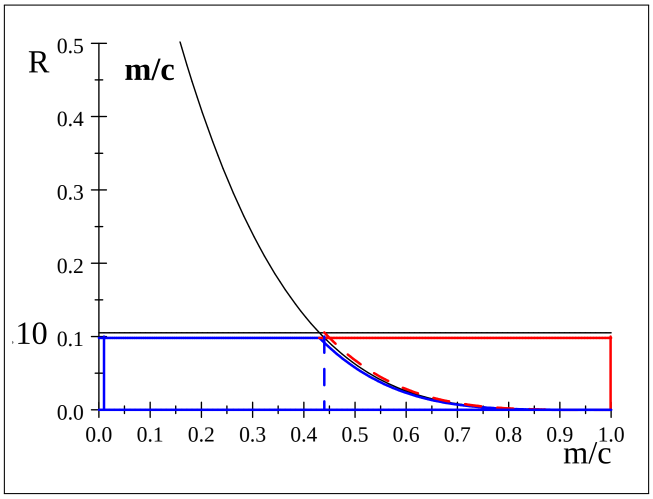

Figure 5: Equilibrium Endogenous Velocity and Baumol-Tobin Margin

lost consumer surplus from going from a zero nominal interest rate up to $\bar{R} = 0.10$ is equal to the total cost of producing exchange credit, per unit of goods, which comprises the integral of the area under the bank marginal cost curve from $\bar{R} = 0$ up to $\bar{R} = 0.10$.

The lost consumer surplus is seen in the Figure 5 using the Blue dashed line and the money demand function, making a Blue "curved" Harberger (1964) triangle. Here, the welfare cost is the value of time spent in banking to avoid the inflation tax optimally, while in Lucas (2000) and Alvarez and Lippi (2009), it is the value of the time spent shopping to avoid the inflation tax optimally.[6]

Another corollary is the graphical derivation of the one-period future value of the ω coefficient within this money demand function instead of marginal cost function. In particular $\left(1 + \bar{R}\right) \omega$ is again equal to the ratio of the bank's producer surplus as outlined in Red, to the total exchange cost, as given by the sum of the standard inflation tax revenue box in Blue and the standard welfare loss (of consumer surplus) triangle in Blue. The Red and Blue areas are exactly the same area as given in Figure 3. As the ratio of the Red to Blue area gets larger, so does $\left(1 + \bar{R}\right) \omega$, and so does the BGP inflation tax avoidance.

[6] The money demand can be viewed exactly as a special case of the shopping time function by solving for $l_Q = g(\underset{-}{m}, \underset{+}{c})$, from equations (5) and (30), with $g(\cdot)$ a function with the derivatives with respect to m and c of the same sign as found in the shopping time literature: $l_{Qt} = c_t \left(\frac{1 - \frac{m_t}{c_t}}{A_Q} \right)^{1/\gamma}$.

4.2 Conservation of "Total" Inflation Tax

By using units in terms of per unit of consumption, the normalization is such that one can think of how the inflation tax applies on a per unit basis. The "total" inflation tax then can be viewed as the given nominal interest rate $\bar{R}$ multiplied by one, in the rectangles in Figures 2, 3, and 4 given by the sum of Blue and Red areas (where $q = 1$ in Figures 2 and 3, and $a = 1$ in Figure 4). This total inflation tax of $\bar{R} \cdot 1$ is divided amongst what is paid to the government outright, $a\bar{R}$, plus what is paid to the bank and returned as dividends to the consumer, $(1 - \gamma) q\bar{R}$, plus the what is paid for work done in banking, the latter being the deadweight loss Harberger (1964) triangle that equals the smaller rectangle given by $\gamma q\bar{R}$. The sum of $a\bar{R} + (1 - \gamma) q\bar{R} + \gamma q\bar{R} = a\bar{R} + (1 - \gamma)(1 - a)\bar{R} + \gamma(1 - a)\bar{R} = \bar{R} \cdot 1$, as stated. Therefore no matter how the box is divided by the nature of the marginal cost of banking function, the result is that the total returned inflation tax revenue, or $a\bar{R}+ (1 - \gamma) q\bar{R}$, plus the value of the work done in the banking sector is constant, given $\bar{R}$. Whether the work done in banking is a "deadweight loss" in economic terms or a "friction" from having to use banking, in either case the value of banking time (as work or a friction) plus the returned inflation tax revenue is always constant, at $\bar{R} \cdot 1$.

For example, given $\bar{R}$, if the marginal cost of banking shifts down, the $\bar{R} \cdot 1$ total inflation tax box stays the same but its division changes amongst its three components. The Red producer surplus would rise in Figures 2 and 3, and so ω would rise as well. Bailey (1956) emphasized this collection of the inflation tax by banks, and it plays a key role in determining ω.

4.3 Alternative Versions

The ω factor is a part of the magnitude of every coefficient of the log-linearized Euler equation (25). Consider the first two variables which are factored by one plus ω, in particular: $(1 + \omega) E_t \left\{\widehat{\Pi}_{t+1} + \theta\widehat{g}_{c,t+1}\right\}$. Since consumption equals output in this economy, this term could equivalently be expressed as $(1 + \omega) E_t \left\{\widehat{\Pi}_{t+1} + \theta\widehat{g}_{y,t+1}\right\}$. If the rest of equation (25) were ad hoc eliminated, then the result would be that

$$\widehat{R}_t = (1+\omega)\, E_t \widehat{\Pi}_{t+1} + E_t\,(1+\omega)\,\theta \widehat{g}_{y,t+1}. \tag{32}$$

Except for the forward looking nature of the growth in output term, this would seem to be similar to various versions of the Taylor rule, such as the so-called "speed limit" version that uses the growth rate of output. But it is clearly not at all the same as a standard Taylor rule equation, despite its coefficient $1+\omega$ on inflation seeming to obey the Taylor principle quality of being greater than one (for $\bar{R} > 0$). That is because the equation is forward looking in output growth, it has the additional forward terms also added, these being $\omega\left[\left(\frac{\bar{R}\frac{m}{c}}{1-\frac{m}{c}}\right)\widehat{g}_{\frac{m}{c},t+1} - \widehat{R}_{t+1}\right]$, and it is a market based determination of the interest rate rather than a policy rule for the interest rate.

While the paper has established that the inflation coefficient has characteristics of the Taylor principle coefficient in the Taylor rule, in the strict sense only that $\omega > 0$ for $R > 0$, the reason that the coefficient $1+\omega$ is greater than one for $R > 0$ is not related to the central bank reaction interpretation given in Taylor rule literature. The economy includes only a stochastic money supply growth rule with inflation tax avoidance being possible. Should $\omega \longrightarrow 0$, a case in which there would be zero inflation tax avoidance, then the equation (25) reduces to a Fisher equation.

With this qualification on the nature of the Euler equation, then taking the second term $E_t\,(1+\omega)\,\theta \widehat{g}_{y,t+1}$ by itself adds a positive effect to the nominal interest rate determination, and thereby does not offset in any way the positive effect of the expected inflation rate on the nominal interest rate. So considering only this second term alone, it would be safe to set the interest rate at least as high as $(1+\omega)\, E_t \widehat{\Pi}_{t+1}$, unless the output growth were negative, if setting interest rates was a desired directive even as inflation was targeted by the long run money supply growth rate of the model economy.

The rest of the terms add a significant qualification. These terms can be gathered as $+\omega\left[\left(\frac{\bar{R}\frac{m}{c}}{1-\frac{m}{c}}\right)\widehat{g}_{\frac{m}{c},t+1} - \widehat{R}_{t+1}\right]$. The money demand growth term, $\omega\left(\frac{\bar{R}\frac{m}{c}}{1-\frac{m}{c}}\right)\widehat{g}_{\frac{m}{c},t+1}$, can be written equivalently using the growth in terms of velocity, c/m, rather than the growth in normalized money m/c. Using $g_{V,t+1}$ to denote the gross growth rate in c/m, and substituting it in $\widehat{g}_{V,t+1}$ for $\widehat{g}_{\frac{m}{c},t+1}$ the only other change is that the sign on the term becomes negative. Now the log-linearized Euler equation can be expressed as

$$\widehat{R}_t = (1+\omega)\, E_t \left(\widehat{\Pi}_{t+1} + \theta \widehat{g}_{y,t+1}\right) - \omega E_t \left(\bar{R}\frac{a}{1-a}\widehat{g}_{V,t+1} + \widehat{R}_{t+1}\right). \tag{33}$$

In general, it can be proved that the magnitude of the velocity coefficient term is strictly less than one, and so less than the inflation coefficient term, since $0 \leq \omega \bar{R}\frac{a}{1-a} = \frac{\bar{R}}{1+\bar{R}}\frac{(1-\gamma)a}{\gamma+(1-\gamma)a} \leq 1 \leq (1+\omega)$. This velocity growth term is a factor that can offset some of the effect of the first two, inflation and output growth, if velocity is increasing.

The forward looking interest rate term is $\omega E_t \widehat{R}_{t+1}$. Consider the special case of $E_t \widehat{R}_{t+1} = \widehat{R}_t$. Since ω factors the expected interest rate term, then dividing through by $1+\omega$ to solve for $\widehat{R}_t$ causes the equation (25) to reduce down to

$$\widehat{R}_t = E_t \widehat{\Pi}_{t+1} + \theta E_t \widehat{g}_{y,t+1} - \left(\frac{\omega}{1+\omega}\right) \bar{R}\frac{a}{1-a} E_t g_{V,t+1}. \tag{34}$$

The expected inflation coefficient is now exactly one. In general though, only along the balanced growth path (BGP) does $E_t \widehat{R}_{t+1} = \widehat{R}_t$, and then also velocity is stationary, the velocity growth term disappears, and the equation is back to a Fisher one.

Along the dynamic path, both the velocity growth term and the expected interest rate term effect the nominal interest rate with a negative sign, and typical interest rate rules do not include such terms. Dropping the expected interest rate term alone, with its coefficient of $-\omega$, could force an estimation relation based on one similar to equation (32) to find empirically that the inflation coefficient is close to or even below one, as the estimation combines the expected interest rate effect into the expected inflation rate effect (see Davies et al., 2016). With the expected nominal interest rate term being in a near neighborhood of the current period nominal interest rate, its coefficient of $-\omega$ would in itself act to offset the $+\omega$ part of the effect of the expected inflation rate $(1+\omega)$ on nominal interest rates. An exact offset would mean, as in equation (34), that the nominal interest rate should be set, if the policy involves such setting of the nominal interest rate, near to the expected inflation rate, rather than $1 + \omega$ times the expected inflation rate, as qualified by the effect of the velocity growth term. This case shows how the nominal interest rates could move in what is closer to one to one comovement with the inflation rate, even though the inflation term as taken by itself suggests a greater than one relation of $1 + \omega$.

5 Euler Equation Computation by Canzoneri et al. (2007) Method

The methodology of Canzoneri et al. (2007) is used to evaluate the ability of the Euler equation to explain the nominal interest rate. Here the variables entering the Euler equation (25) are each estimated by a vector autoregression (VAR). Following Fuhrer (2000) and Canzoneri et al. (2007) the dynamics of consumption, inflation, the nominal interest rate, and the velocity of money are assumed to be described by the following first order difference equation in general form with the vector $Y_t \equiv \left\{ \log c_t, \log \frac{P_t}{P_{t-1}}, \log \frac{m_t}{c_t}, R_t \right\}$:

$$Y_t = A_0 + A_1 Y_{t-1} + \nu_t; \tag{35}$$

the error term, ν_t, is assumed to be iid $N(0, \Sigma)$.[7]

Given the calibration described in the next Section 6, the consequent steady state of the model the nominal interest rates are computed by using the VAR based expectations of variables as inputed at each point in time within the log-linearized Euler condition in equation (25). Data is described in Appendix For data please see Appendix D.

Figure 6 displays the Model implied nominal interest rate (Dashed Red) as computed from the Euler equation (25) using Canzoneri et al. (2007) methods. The correlation with the US 3-month Treasury rate (Solid Red) is 86% for the whole $1959Q1 - 2018Q2$ period,

[7] Unlike in Canzoneri et al. (2007) log normality is not assumed here. As pointed out in their paper, the Euler condition under log normality and under log linear approximation only differ by a constant. As a robustness check, the same exercise has been carried out under the log normality assumption with nearly identical results.

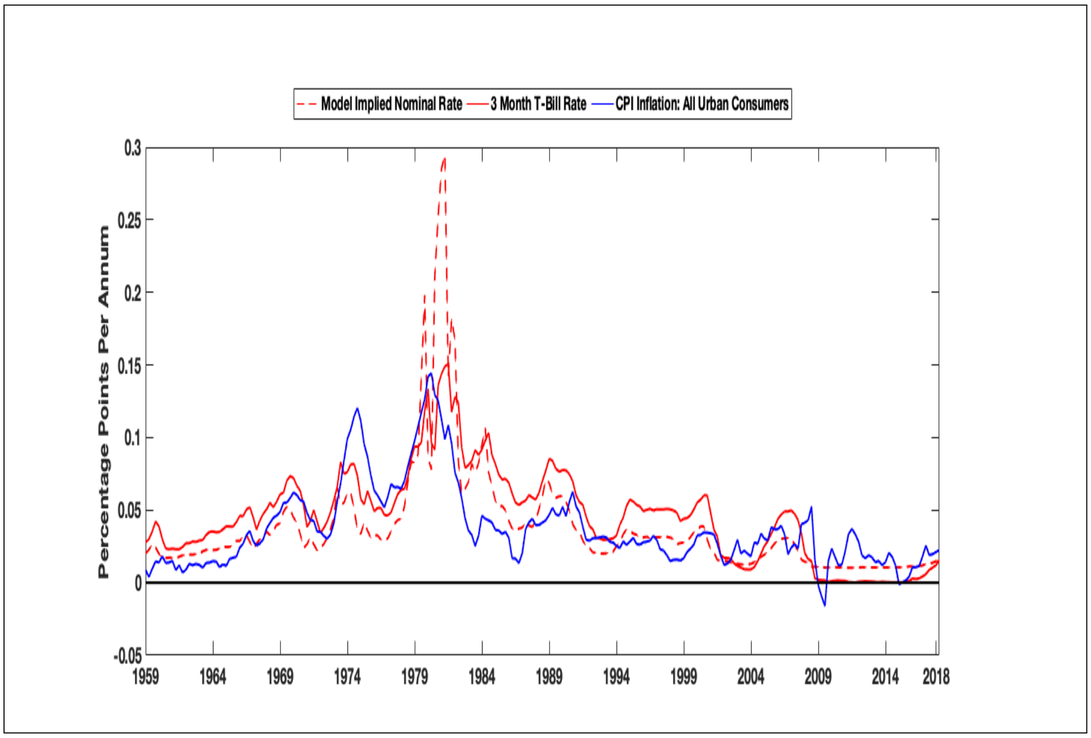

Figure 6: Euler Equation Nominal Interest Rate as Computed by Canzoneri et al. (2007) methods, versus the Actual 3 Month T-Bill Rate and CPI Inflation Rate, $1959Q1-2018Q2$.

89% for the $1959Q1 - 2007Q4$ sub-period, and 93% for $2008Q1 - 2018Q2$ sub-period. The inflation rate data (Blue Solid) is also fit by the model

For the post- Great Recession period, $2008Q1-2018Q2$, Figure 7 focuses in on the model explanation of the nominal interest (Dashed Red), computed using the same Canzoneri et al. (2007) method. The other two lines are the actual inflation rate (Solid Blue) and 3-month Treasury Bill rate (Solid Red) as above. This shows how the correlation between the model R and the actual R is rather high, but off by some magnitude.

6 Calibration and Simulation

The calibration is used for the Canzoneri et al. (2007) Euler equation computation and for backing out the shocks for the post- Great Recession period. The BGP quarterly money supply growth rate is set to $\sigma = 0.0044$, which gives a 1.79% annual CPI inflation rate.

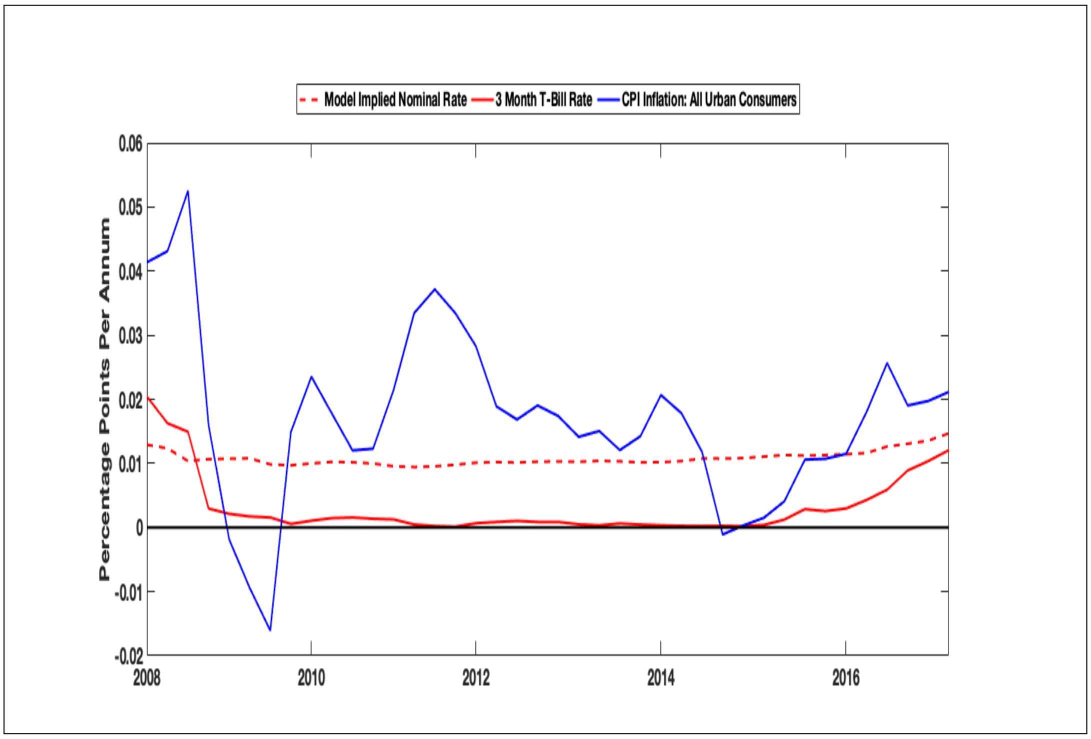

Figure 7: Euler Equation Implied Nominal Interest Rate versus the 3 Month T-Bill and CPI Inflation Rates, January 2008 to July 2018.

The productivity parameters are set at $A_Q = 0.5421$ and $A_G = 1$, which give a quarterly consumption velocity of money equal to 1.2795, or 5.12 annually. In addition $\beta = 0.995$ and $\gamma = 0.11$ as in Benk et al. (2008). The constant elasticity parameter is set to $\theta = 2$, at the upper limit of the Mehra and Prescott (1985) range. For the shocks, the autocorrelations are set to $\rho_Q = \rho_G = 0.9$, and $\rho_\sigma = 0.8$, as in Benk et al. (2008), who use a data period of 1972-2003. These shock settings in a related model can be viewed as a pre-Great Recession sample period, allowing us to view the simulation for the period after 2007 as a type of "out-of-sample" experiment.

The model is simulated for a shock sequence that is chosen through the model's solution so as to match the interest rate and inflation rate profile of the quarterly data in Figure 1, for the $2008Q1 - 2018Q2$ period. To do this, the actual "policy functions" for the nominal interest rate and the inflation rate, using the concepts of Ljungqvist and Sargent (2012), are solved using the method of undetermined coefficients. Using these solutions and a model calibration, sequences of exogenous shocks are then uniquely imputed for this data period by using data series for the nominal interest rate, the inflation rate, and the money supply; please see Appendix C for the computation of the policy function determination of the interest rate.

Figure 8 presents a less than perfect fit of the inflation rate (Dashed Blue) by the model using the backed out shocks, as compared to the data (Solid Blue), and a near perfect fit of the US 3-month Treasury rate. Figure 9 shows the backed out bank shock using the Benk et al. (2005, 2010) and Nolan and Thoenissen (2009) methods. The latter shows a negative bank productivity shock hitting during this period. Figure 10 shows the bank productivity as estimated using Solow residual methods from US data, and it also shows a negative productivity shock after 2007, although it turns positive before that computed in Figure 9.

The Figure 10 shock is broader than only an exchange credit output shock, in that it uses for output "Loans and Leases in Bank Credit, All Commercial Banks". For the banking time, it uses " Average Weekly Hours of Production and Nonsupervisory Employees: Financial Activities". The production function then assumes only banking time (without

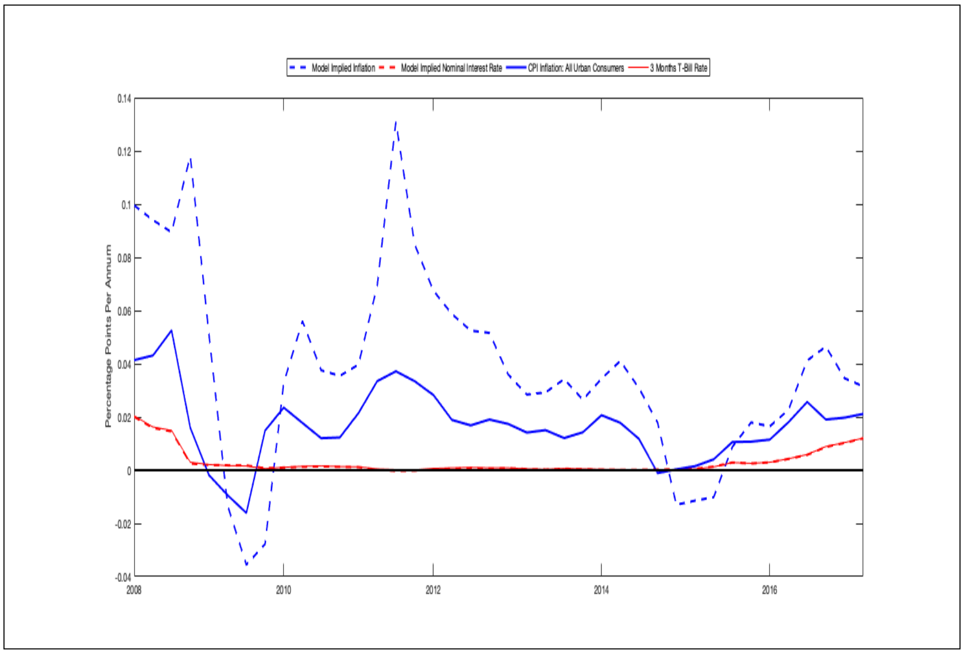

Figure 8: Nominal Interest Rate and Inflation Rate Constructed Using Model Implied Shocks versus US Data, $2008Q1 - 2018Q2$.

physical capital) and the loan output, and estimates the "Solow-residual" for this bank sector, normalized around zero.[8]

The estimation uses the bank production function of the model in Section 3 to find a Solow-type residual from the bank sector's output of loans as the measure of bank productivity. This is a standard productivity measure for a sector. This bank productivity sectoral shock is likely to be well correlated with any one financial intermediary sub-output such as exchange credit services; alternatively, Nolan and Theonissen (2009) estimate a BGG shock for their credit shock, while also having a goods sector productivity shock and money supply growth rate shock as in our economy.

[8]Loans and Leases in Bank Credit, All Commercial Banks (Series ID: LOANS), from the Board of Governors of the Federal Reserve System; release H.8: Assets and Liabilities of Commercial Banks in the United States (Seasonally adjusted; in billions of US Dollars). Average Weekly Hours of Production and Nonsupervisory Employees: Financial Activities (Series: CES5500000007), the US Bureau of Labor Statistics; release Employment Situation (Seasonally Adjusted,Hours).

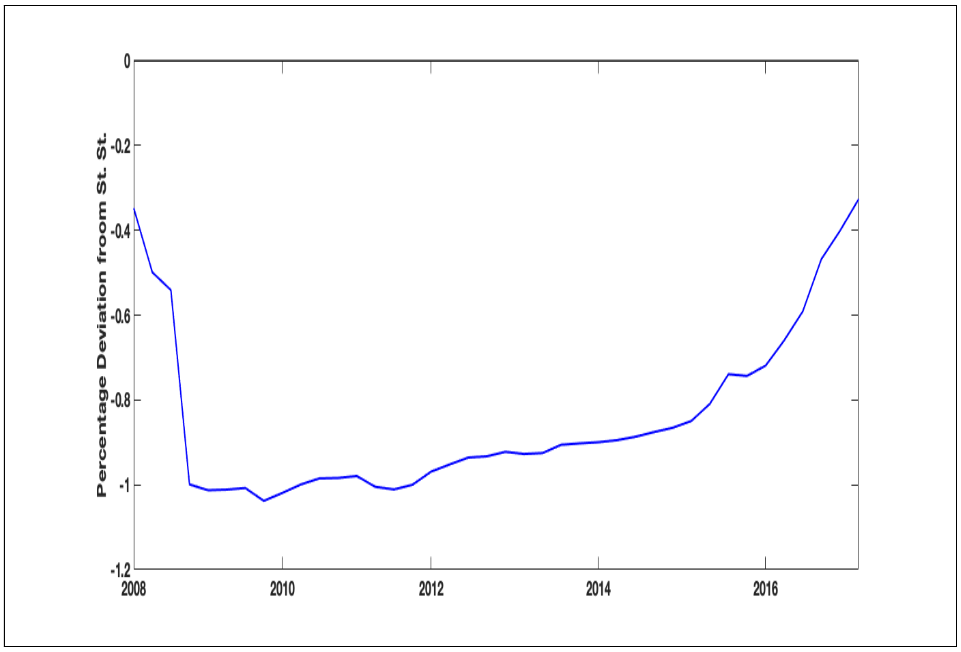

Figure 9: Model Solution Based Bank Productivity Shock Series, $2008Q1 - 2018Q2$.

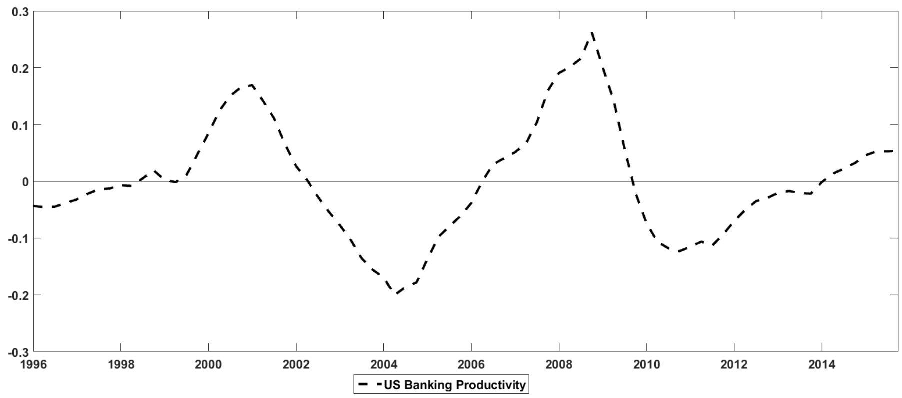

Figure 10: Bank Productivity Measured as a Solow Residual for a Banking Time Production funcion, 1996-2016.

7 Discussion

Explaining how a nominal rate below its normal level could leave the actual inflation rate above the nominal interest rate means that equation (25), or in the form of (33), is a viable, non-trivial, explanation. The main coefficient in that equation, relative to utility and technology parameters, is the ω. This is why the analysis of ω in the above sections of the paper helps complete the model's explanation of the data, in that ω is proportional to the relative fraction of inflation tax that is avoided; the more the avoidance of the fiscal inflation tax, the higher is ω.

The bank productivity decrease measured variously in Figures 9 and 10 could be motivated from the run on investment bank mutual funds during the Great Recession and the subsequent Dodd-Frank 2010 Banking Act that tightened capital reserve requirements among other restrictions. At the same time, the interest on excess reserves (IOER) could also have acted as a decrease in bank productivity by keeping the part of the increased monetary base as excess reserves, rather than as profitable loans, in the sense of inducing moral hazard by paying banks to withhold some amount of new loans that would be made under a zero excess reserves regime as held pre- 2008.

The equation (33) offers a straightforward way to can explain how the short term government debt interest rates remained below the inflation rate since August 2009. It is as if every period the nominal interest was expected to go up, even slightly, because the nominal interest rate fixing was seen to be temporary but long-lived, as is consistent with the forward guidance offered by the Federal Reserve Bank which involved many minuted discussions over time on raising interest rates soon, even though in fact the nominal interest rate only began rising more somewhat recently. De Graeve and von Heideken (2015) confirm that inflation expectations did increase during this period even though standard measures did not reflect this.

According to the model, this episode would end once the deviations from the BGP values of the expected nominal interest rate and of the expected inflation rate coincide in terms of their magnitude and in terms of being of the same sign. In that case the inflation rate and the nominal interest rate in this economy would again tend to move together, since the negative $1+\omega$ expected inflation multiplier would be approximately offset by the opposite-signed ω expected nominal interest rate multiplier, with a net effect of 1 rather than $1+\omega$.

The model's basis in the production of banking services in a competitive equilibrium fashion is consistent with the estimation methods used extensively in the microeconometrics of banking. An excellent review of the foundations of such estimation based in part on the financial intermediation approach is found in Degryse et al. (2009). For example, these authors discuss estimation "of a translog cost function with total costs due to labor, capital, and borrowed funds" (p.33), with a related discussion of input prices based on similar factors.[9] They also cite the role of banking marginal costs (p. 36-37), findings of perfect competition in banking (p. 39), and how a higher product quality of banks holds for certain banks, such as a result of nationwide branching (p. 42), which in our framework would be reflected as a higher bank productivity. Further the authors cite literature showing how bank runs, "banking distress", or a loan supply shock can be associated with or causal of lower GDP growth (p.154), which is not inconsistent with our Figures 9 and 10.

8 Conclusion

The paper presents a model based generated nominal interest rate with a 86% correlation with the actual US 3-month Treasury rate for $1959Q1-2018Q2$. Microfoundations for the interpretation of the Euler equation are in the price-theoretic tradition in a simple monetary economy with an endogenous income velocity of money demand. A graphical representation is made whereby the net expected inflation rate coefficient is the ratio of the amount of the inflation tax avoided per unit of consumption divided by the amount of the exchange cost per unit of consumption.

The exchange cost includes both the payment of the inflation tax on the purchases made

[9] "In most studies, three different input prices are considered: (1) the deposit rate, measured by the ratio of annual interest expense to total assets; (2) wages, measured by the ratio of personnel expense to total assets; and (3) price of equipment or fixed capital, measured by the ratio of capital expenditures and other expenses to total assets (Degryse et al., 2009; p. 34)". This is consistent with the equilibrium upward sloping marginal cost function per unit of deposits in our Figure 3.

with money and the value of resources used up in banking to avoid the inflation tax. The graphical representation provides a simple way to understand the effect of expected inflation on nominal interest rates, along the dynamic path of the economy in response to shocks. And it can be used to graph how changes in parameters such as bank productivity change the inflation coefficient.

The model is then applied to explain the nominal interest rates and the inflation rate, as seen in the post- Great Recession US data. The negative bank productivity shock scenario, from the backed-out shock, is consistent with a Solow residual estimation of the bank productivity in the financial intermedation sector. The model is the simplest possible setting with this type of endogenous velocity, as based on financial intermediation production, while excluding capital.

The model's Euler equation of the government bond one-period, nominal, risk-free, interest rate has a coefficient on the expected inflation rate is always greater than one for a positive nominal interest rate. This coefficient equals one plus the ratio of the BGP bank's producer surplus, in providing exchange credit that avoids the inflation tax, divided by the actual exchange costs on a per unit of consumption basis. This ties the Euler equation to the graphs of marginal cost producer surplus and money demand welfare loss.

The expected inflation coefficient in a nominal bond asset pricing condition being established as greater than one, as well as being based on the microfoundations of the degree of avoidance of inflation tax, is novel to our knowledge. This dual approach of graphing the expected inflation coefficient using each the banking cost function and the money demand function are also apparently novel here, making it both price-theoretic in bank production and "quantity theoretic" in a modern money demand sense with endogenous velocity. Extension to inclusion of physical and capital accumulation is under investigation to see if improvement upon the 86% to 93% correlations found here with respect to the 3-month Treasury rate is possible; similar correlation results are found for the Federal Funds rate.

A Competitive Equilibrium

The representative agent's optimization problem can be written recursively as:

$$V(s) = \max_{c,q,d,M',B'} \{u(c) + \beta EV(s')\} \tag{36}$$

subject to the conditions (3) to (5), where the state of the economy is denoted by $s = (M, B; u)$ and a prime (') indicates next-period values. A competitive equilibrium consists of a set of policy functions $c(s)$, $q(s)$, $d(s)$, $M'(s)$, $B'(s)$ pricing functions $P(s)$, $w(s)$, $R^d(s)$, $P_Q(s)$ and a value function $V(s)$, such that:

i. the consumer maximizes utility, given the pricing functions and the policy functions, so that $V(s)$ solves the functional equation (36);

ii. the goods producer maximizes profit similarly, with the resulting functions for w being given by equation (16);

iii. the bank firm maximizes profit similarly in equation (8) subject to the technology of equation (6)

iv. the goods, money and credit markets clear, in equations (3) and (14), and in (4), (6) and (17).

The consumer's first-order conditions of the following problem

$$\max_{c_t,q_t,d_t,M_{t+1},B_{t+1}} \mathcal{V}(M_0, B_0; u_0) = E_0 \sum_{t=0}^{\infty} \beta^t \frac{(c_t)^{1-\theta}}{1-\theta} \tag{37}$$

subject to the following constraints that have the Lagrangian multipliers of λ, μ, and ε,

$$
\begin{aligned}
\lambda \quad &: \quad w_t + \bar{R}_t^d d_t + \frac{T_t}{P_t} \qquad (38)\\
&\geq \quad \frac{P_{Qt}}{P_t} q_t + c_t + \frac{M_{t+1}}{P_t} - \frac{M_t}{P_t} + \frac{B_{t+1} - B_t(1+\bar{R}_t)}{P_t} + \tau_t \\
\mu \quad &: \quad \frac{M_t}{P_t} + \frac{T_t}{P_t} + q_t \geq c_t \qquad (39)\\
\varepsilon \quad &: \quad c_t = d_t, \qquad (40)
\end{aligned}
$$

are

$$
\begin{aligned}
0 &= (c_t)^{-\theta} - \lambda_t - \mu_t + \varepsilon_t, \qquad (41)\\
0 &= -\lambda_t \left(\frac{P_{Qt}}{P_t} \right) + \mu_t, \qquad (42)\\
0 &= \lambda_t \bar{R}_t^d - \varepsilon_t, \qquad (43)\\
0 &= -\frac{\lambda_t}{P_t} + \beta E_t \left\{ \frac{\lambda_{t+1} + \mu_{t+1}}{P_{t+1}} \right\}, \qquad (44)\\
0 &= -\frac{\lambda_t}{P_t} + \beta E_t \left\{ \frac{\lambda_{t+1} \left(1 + \bar{R}_{t+1}\right)}{P_{t+1}} \right\}. \qquad (45)
\end{aligned}
$$

A.1 The Dynamic Equilibrium Conditions

Using the first order conditions and the constraints in (2), (4), and (5), the underlying model can be fully described by the following system of 14 equations in 14 unknowns $\{c_t,\ y_t,\ m_t,\ a_t,\ q_t,\ l_{Gt},\ l_{Qt},\ d_t,\ \bar{\pi}_t,\ p_{Qt},\ \overline{R}_t,\ \bar{R}_t^d,\ w_t,\ \tau_t\}$:

$$
1 = \beta E_t \left(\frac{c_t}{c_{t+1}} \right)^{\theta} \left[\frac{1 + \overline{R}_t - R_t^d}{1 + \overline{R}_{t+1} - R_{t+1}^d} \right] \left[\frac{1 + \overline{R}_{t+1}}{1 + \bar{\pi}_{t+1}} \right]; \qquad (46)
$$

$$
y_t = c_t; \qquad (47)
$$

$$d_t = c_t; \tag{48}$$

$$p_{Qt} = \overline{R}_t; \tag{49}$$

$$m_t + \tau_t + q_t = c_t; \tag{50}$$

$$1 = l_{Gt} + l_{Qt}; \tag{51}$$

$$a_t = \frac{m_t}{c_t} \tag{52}$$

$$\tau_t = \sigma_t m_t; \tag{53}$$

$$(1 + \bar{\pi}_{t+1})m_{t+1} = (1 + \sigma_t)m_t; \tag{54}$$

$$y_t = A_{Gt} l_{Gt} \tag{55}$$

$$w_t = A_{Gt}; \tag{56}$$

$$q_t = A_{Qt} l_{Qt}^{\gamma} d_t^{1-\gamma}; \tag{57}$$

$$\bar{R}_t^d = p_{Qt}(1-\gamma) A_{Qt} \left[\frac{l_{Qt}}{d_t}\right]^{\gamma}; \tag{58}$$

$$w_t = p_{Qt} \gamma A_{Qt} \left[\frac{l_{Qt}}{d_t}\right]^{\gamma-1}, \tag{59}$$

and three forcing processes for goods TFP, A_{Gt}, the banking productivity, A_{Qt}, and the money supply, σ_t:

$$A_{Gt} - \overline{A}_G = \rho_G (A_{Gt-1} - \overline{A}_G) + \epsilon_{Gt}; \tag{60}$$

$$A_{Qt} - \overline{A}_Q = \rho_Q (A_{Qt-1} - \overline{A}_Q) + \epsilon_{Gt}; \tag{61}$$

$$\sigma_t - \overline{\sigma} = \rho_\sigma (\sigma_{t-1} - \overline{\sigma}) + \epsilon_{\sigma t}, \tag{62}$$

where ρ_G, ρ_Q, and ρ_σ are the persistence parameters of the respective processes; $\overline{A}_G$,

$\overline{A}_Q$, and $\overline{\sigma}$ are the long-run (BGP) values; and ϵ_{Gt}, ϵ_{Gt}, and $\epsilon_{\sigma t}$ are the respective i.i.d. innovations of the shock processes with zero means and constant variances.

A.2 The Balanced Growth Path Solution

The structural equations of the model in (46) - (59) along the zero balanced growth path can be written without time subscripts as:

$$1 = \beta \left[\frac{1 + \overline{R}}{1 + \bar{\pi}} \right]; \tag{63}$$

$$y = c; \tag{64}$$

$$d = c; \tag{65}$$

$$p_Q = \bar{R}; \tag{66}$$

$$m + \tau + q = c; \tag{67}$$

$$1 = l_G + l_Q; \tag{68}$$

$$a = \frac{m}{c} \tag{69}$$

$$\tau = \overline{\sigma} m; \tag{70}$$

$$(1 + \bar{\pi}) m = (1 + \overline{\sigma}) m; \tag{71}$$

$$y = \overline{A}_G l_G \tag{72}$$

$$w = \overline{A}_G; \tag{73}$$

$$q = \overline{A}_Q l_Q^{\gamma} d^{1-\gamma}; \tag{74}$$

$$\bar{R}^d = p_Q (1 - \gamma) \overline{A}_Q \left[\frac{l_Q}{d} \right]^{\gamma}; \tag{75}$$

$$w = p_Q \gamma \overline{A}_Q \left[\frac{l_Q}{d} \right]^{\gamma - 1}. \tag{76}$$

To solve for the long-run values of variables in terms of parameters use equation (71) to get an expression for the net inflation rate in terms of the long-run money supply:

$$\bar{\pi} = \overline{\sigma}. \tag{77}$$

Next, substitute (77) into equation(63) and rearrange it to get an expression for the net nominal interest rate:

$$\bar{R} = \frac{1 + \overline{\sigma}}{\beta} - 1 = \frac{1 + \overline{\sigma} - \beta}{\beta}. \tag{78}$$

It follows from (66) that the price of exchange credit along the BGP is:

$$p_Q = \bar{R} = \frac{1 + \overline{\sigma} - \beta}{\beta}. \tag{79}$$

The BGP wage rate directly follows from equation (73) as it is equal to the long-run value of the goods TFP:

$$w = \overline{A}_G. \tag{80}$$

Then using the results in equations (78), (79), and (80) in equation (76) one can express the BGP factor input ratio $\frac{l_Q}{d}$ in terms of parameters only as:

$$\frac{l_Q}{d} = \left[\frac{\beta \overline{A}_G}{(1 + \overline{\sigma} - \beta)\gamma \overline{A}_Q} \right]^{\frac{1}{\gamma - 1}}. \tag{81}$$

Substituting (78), (79), and (81) into (75) yields an expression for the bank dividend rate, $\bar{R}^d$:

$$\bar{R}^d = \left[\frac{(1+\overline{\sigma}-\beta)(1-\gamma)\overline{A}_Q}{\beta}\right]\left[\frac{\beta\overline{A}_G}{(1+\overline{\sigma}-\beta)\gamma\overline{A}_Q}\right]^{\frac{\gamma}{\gamma-1}}. \tag{82}$$

Using (81) the labor time in the banking sector can be expressed in terms of consumption by substituting in (65):

$$l_Q = \left[\frac{\beta\overline{A}_G}{(1+\overline{\sigma}-\beta)\gamma\overline{A}_Q}\right]^{\frac{1}{\gamma-1}} c. \tag{83}$$

Then labor in the goods sector can also be expressed in terms consumption by combining (64) and (72):

$$l_G = \frac{c}{\overline{A}_G}. \tag{84}$$

After substituting (83) and (84) into the time constraint in (68) one can obtain the BGP consumption in terms of parameters only:

$$c = \frac{1}{\left[\frac{1}{\overline{A}_g} + \left(\frac{\beta\overline{A}_g}{(1+\overline{\sigma}-\beta)\gamma\overline{A}_q}\right)^{\frac{1}{\gamma-1}}\right]}, \tag{85}$$

from which the BGP output and deposits follow because $c = y = d$. The remaining variables directly follow from this result.

A.3 The Log-linearized Model

Define $\Pi_t \equiv 1 + \bar{\pi}_t$ and $R_t = 1 + \overline{R}_t$ as the gross inflation and nominal interest rates respectively. Then, after log-linearizing the equations in (46) - (59) one obtains to following system:

$$\hat{R}_t = (1+\omega)E_t\left\{\hat{\Pi}_{t+1} + \theta[\hat{c}_{t+1} - \hat{c}_t]\right\} + \omega E_t\left\{\left(\frac{\overline{R}a}{1-a}\right)[\hat{a}_{t+1} - \hat{a}_t] - \hat{R}_{t+1}\right\}; \quad (86)$$

$$\hat{y}_t = \hat{c}_t; \quad (87)$$

$$\hat{d}_t = \hat{c}_t; \quad (88)$$

$$\hat{p}_{Qt} = \hat{\overline{\overline{R}}}_t; \quad (89)$$

$$m\hat{m}_t + \tau_t\hat{\tau}_t + q\hat{q}_t = c\hat{c}_t; \quad (90)$$

$$0 = l_G\hat{l}_{Gt} + l_Q\hat{l}_{Qt}; \quad (91)$$

$$\hat{a}_t = \hat{m}_t - \hat{c}_t \quad (92)$$

$$\hat{t}_t = \hat{\sigma}_t + \hat{m}_t; \tag{93}$$

$$\left(\frac{\bar{\pi}}{1+\bar{\pi}}\right)\hat{\pi}_{t+1} + \hat{m}_{t+1} = \left(\frac{\overline{\sigma}}{1+\bar{\sigma}}\right)\hat{\sigma}_t + \hat{m}_t; \tag{94}$$

$$\hat{y}_t = \hat{A}_{Gt} + \hat{l}_{Gt} \tag{95}$$

$$\hat{w}_t = \hat{A}_{Gt}; \tag{96}$$

$$\hat{q}_t = \hat{A}_{Qt} + \gamma\hat{l}_{Qt} + (1-\gamma)\hat{d}_t; \tag{97}$$

$$\widehat{\bar{R}}_t^d = \hat{p}_{Qt} + \hat{A}_{Qt} + \gamma\hat{l}_{Qt} - \gamma\hat{d}_t; \tag{98}$$

$$\hat{w}_t = \hat{p}_{Qt} + \hat{A}_{Qt} + (\gamma-1)\hat{l}_{Qt} + (1-\gamma)\hat{d}_t, \tag{99}$$

$$\hat{A}_{Gt} = \rho_G\hat{A}_{Gt-1} + \epsilon_{Gt}; \tag{100}$$

$$\hat{A}_{Qt} = \rho_Q \hat{A}_{Qt-1} + \epsilon_{Qt}; \tag{101}$$

$$\hat{\sigma}_t = \rho_\sigma \hat{\sigma}_{t-1} + \epsilon_{\sigma t}, \tag{102}$$

where variables with tilde are in terms of percentage deviations from their respective steady state values.

B Log-Linearization of Euler Equation

Log-linearization of the Euler Condition will make use of both the gross rate R_t and the net rate $\bar{R}_t$, where $R_t = 1 + \bar{R}_t$. The Euler equation is,

$$1 = \beta E_t \left[\left(\frac{c_t}{c_{t+1}} \right)^\theta \frac{\tilde{R}_t}{\tilde{R}_{t+1}} \frac{R_{t+1}}{\Pi_{t+1}} \right]; \tag{103}$$

where $\Pi_{t+1} \equiv 1 + \pi_{t+1}$ are gross rates of nominal interest and inflation, respectively, and $\beta \equiv 1/(1+\rho)$. The term $\tilde{R}_t$ represents one plus a 'weighted average cost of exchange' as follows:

$$\tilde{R}_t = R_t - \bar{R}_t^d = 1 + \bar{R}_t - \bar{R}_t^d, \tag{104}$$

where $\bar{R}^d$ is the net dividend rate of the exchange credit producer. Define the gross growth rates of consumption, and consumption velocity of money as $g_{c,t+1} \equiv \frac{c_{t+1}}{c_t}$ and

$g_{a,t+1} \equiv \frac{a_{t+1}}{a_t}$. where $a \equiv \frac{m_t}{c_t}$, and then (103) can be written as

$$1 = \beta E_t \left[(g_{c,t+1})^{-\theta} \left(\frac{R_t - \bar{R}_t^d}{R_{t+1} - \bar{R}_{t+1}^d} \right) \frac{R_{t+1}}{\Pi_{t+1}} \right]. \tag{105}$$

Log-linearization of Euler condition gives that

$$0 = E_t \left[-\hat{\Pi}_{t+1} - \theta \hat{g}_{c,t+1} - \left(\frac{R}{R - \bar{R}^d} \right) [\hat{R}_{t+1} - \hat{R}_t] + \left(\frac{\bar{R}^d}{R - \bar{R}^d} \right) \left(\hat{R}_{t+1}^d - \hat{R}_t^d \right) + \hat{R}_{t+1} \right]. \tag{106}$$

We know from the exchange credit producer's problem that $\bar{R}_t^d = (1-\gamma)(1-a_t)(R_t-1)$, so that log-linearizing $\bar{R}_t^d$, where $\hat{R}_t^d$ is that log-linearization notationally (of the net rate $\bar{R}_t^d$), yields

$$\begin{aligned} \bar{R}_t^d &= (1-\gamma)(1-a_t)(R_t-1) \qquad (107) \\ \ln \bar{R}_t^d &= \ln(1-\gamma) + \ln(1-a_t) + \ln(R_t-1) \\ \hat{R}_t^d &= -\left(\frac{a}{1-a} \right) \hat{a}_t + \left(\frac{R}{R-1} \right) \hat{R}_t. \end{aligned}$$

Substituting in $\hat{R}_t^d$ to $\left(\frac{\bar{R}^d}{R - \bar{R}^d} \right) \left(\hat{R}_{t+1}^d - \hat{R}_t^d \right)$ yields that

$$\left(\frac{\bar{R}^d}{R - \bar{R}^d} \right) \left(\hat{R}_{t+1}^d - \hat{R}_t^d \right) = \left(\frac{\bar{R}^d}{R - \bar{R}^d} \right) \left[\left(\frac{R}{R-1} \right) [\hat{R}_{t+1} - \hat{R}_t] - \left(\frac{a}{1-a} \right) [\hat{a}_{t+1} - \hat{a}_t] \right]$$

Substituting this result into (106) gives an equation without R_t^d or R_{t+1}^d such that

$$0 = E_t \left[\begin{array}{c} -\hat{\Pi}_{t+1} - \theta \hat{g}_{c,t+1} + \psi(1-\theta)\hat{g}_{x,t+1} - \left(\frac{R}{R - \bar{R}^d} \right) [\hat{R}_{t+1} - \hat{R}_t] \\ + \left(\frac{\bar{R}^d}{R - \bar{R}^d} \right) \left[\left(\frac{R}{R-1} \right) [\hat{R}_{t+1} - \hat{R}_t] - \left(\frac{a}{1-a} \right) [\hat{a}_{t+1} - \hat{a}_t] \right] + \hat{R}_{t+1} \end{array} \right]. \tag{108}$$

Rearranging to express the gross nominal interest rate, collecting terms, defining $\hat{g}_{a,t+1} \equiv \hat{a}_{t+1} - \hat{a}_t$ yields:

$$\Gamma \hat{R}_t = E_t \left\{ \hat{\Pi}_{t+1} + \theta \hat{g}_{c,t+1} + \left(\frac{\bar{R}^d}{R - \bar{R}^d} \right) \left(\frac{a}{1-a} \right) g_{a,t+1} + (\Gamma - 1) \hat{R}_{t+1} \right\}, \tag{109}$$

where

$$\Gamma = \frac{R - \bar{R}^d - 1}{R - \bar{R}^d} \frac{R}{R-1}.$$

Substituting the definition of R and R^d into the coefficient of $\hat{R}_t$:

$$\Gamma = \frac{R - \bar{R}^d - 1}{R - \bar{R}^d} \frac{R}{R-1} = \frac{[1 - (1-\gamma)(1-a)](1+\bar{R})}{1 + [a + \gamma(1-a)]\bar{R}}.$$

From this it results that the coefficient of the gross inflation Π_t rate is

$$1 + \omega \equiv \frac{1}{\Gamma} = \frac{1 + [a + \gamma(1-a)]\bar{R}}{[1 - (1-\gamma)(1-a)]} \frac{1}{1+\bar{R}} = \frac{1 + [a + \gamma(1-a)](R-1)}{[1 - (1-\gamma)(1-a)]} \frac{1}{R}$$

From this the formula for ω follows:

$$1 + \omega = 1 + \frac{1 + [a + \gamma(1-a)]\bar{R}}{[1 - (1-\gamma)(1-a)]} \frac{1}{1+\bar{R}} - 1$$

$$\begin{aligned} 1 + \omega &= 1 + \frac{1 + [a + \gamma(1-a)]\bar{R} - [1 - (1-\gamma)(1-a)](1+\bar{R})}{(1+\bar{R})[1 - (1-\gamma)(1-a)]} \\ &= 1 + \frac{1 + [1 - (1-\gamma)(1-a)]\bar{R} - [1 - (1-\gamma)(1-a)](1+\bar{R})}{(1+\bar{R})[1 - (1-\gamma)(1-a)]} \end{aligned}$$

$$\omega = \frac{(1-\gamma)(1-a)]}{[1-(1-\gamma)(1-a)]} \frac{1}{(1+\bar{R})} = \frac{\bar{R}^d}{\bar{R}-\bar{R}^d} \frac{1}{1+\bar{R}},$$

with a final form of

$$\hat{R}_t = E_t \left\{ (1+\omega)\,\hat{\Pi}_{t+1} + \theta\,(1+\omega)\,\hat{g}_{c,t+1} + \omega \left[\left(\frac{\bar{R}a}{1-a} \right) \hat{g}_{a,t+1} - \hat{R}_{t+1} \right] \right\}. \tag{110}$$

C Policy Functions

The policy functions depend on the shocks $\tilde{A}_{Qt}$, $\tilde{A}_{Gt}$, $\tilde{\sigma}_t$ and the state variable ($M_t/P_t \equiv m_t$). In log-deviations, the three following equations express the solution form for the general equilibrium, in terms of the net nominal interest rate, the net inflation rate, and the state variable m_t :

$$\tilde{\bar{R}}_t = \Phi_{RQ}\tilde{A}_{Qt} + \Phi_{RG}\tilde{A}_{Gt} + \Phi_{R\sigma}\tilde{\sigma}_t + \Phi_{Rm}\tilde{m}_t; \tag{111}$$

$$\tilde{\pi}_{t+1} = \Phi_{\pi Q}\tilde{A}_{Qt} + \Phi_{\pi G}\tilde{A}_{Gt} + \Phi_{\pi\sigma}\tilde{\sigma}_t + \Phi_{\pi m}\tilde{m}_t; \tag{112}$$

$$\tilde{m}_{t+1} = \Phi_{mQ}\tilde{A}_{Qt} + \Phi_{mG}\tilde{A}_{Gt} + \Phi_{m\sigma}\tilde{\sigma}_t + \Phi_{mm}\tilde{m}_t. \tag{113}$$

The BGP quarterly money supply growth rate is set to $\sigma = 0.0044$, which gives a 1.79% annual CPI inflation rate, as in the data. The productivity parameters are set at $A_Q = 0.5421$ and $A_G = 1$, which give a quarterly consumption velocity of money equal to 1.2795, or 5.12 annually, as in the data. In addition $\beta = 0.995$, and as in Benk et al. (2008), $\gamma = 0.11$. The constant elasticity parameter is set to $\theta = 2$, as in the range of Mehra and Prescott (1985). For the shocks, the autocorrelations are set to $\rho_Q = \rho_G = 0.9$, and $\rho_\sigma = 0.8$, as in Benk et al. (2008), who use a data period of 1972-2003. These shock settings in a related model can be viewed as a pre-Great Recession sample period, allowing us to view the simulation for the period after 2007 as a type of "out-of-sample" experiment.

Table 1 shows the implied Φ coefficients. This shows for example, that a positive bank productivity shock moves the nominal interest rate downwards, but it lowers the inflation

	Φ_{kQ}	Φ_{kG}	$\Phi_{k\sigma}$	Φ_{km}
R_t	-1.46	1.84	-0.007	-1.68
π_t	-5.79	-6.33	1.99	4.03
m_{t+1}	0.054	0.028	-0.004	0.98

Table 1: Coefficients Φ of Policy Solutions.

rate by more than triple the magnitude. A positive goods sector productivity shock in contrast raises the nominal interest rate and lowers the inflation rate by three times the magnitude of the nominal interest rate effect. Together this suggests that the bank and goods sector productivity shocks can be combined to explain the inversion of nominal interest rates and inflation rates from their usual pattern of a higher nominal interest rate than inflation rate.

The estimation of shocks is made using equations (111) to (113) and the Φ values in Table 1, along with the data series for the 3-month Treasury Bill interest rate, the CPI inflation rate, and the M1 money stock as normalized by the CPI price level. Given the model's structure, allowing only goods sector and bank sector productivity shocks enables a unique set of shocks to [delete: can] fit the data exactly. This is not automatic because while there are only two shocks and two variables in such a case of the model, there is still the state variable m_t which is model-determined as the sequence of shocks unfold and which requires the use of equation (113) as well.

As a further detailing of the shock processes in terms of the model's dynamics, consider how ω of the Euler equation (33) itself affects the impact of the shocks, while equation (114) expands out the component contributions for the effect of each shock on the nominal interest rate. For example, positive bank shocks affect R through the two terms $(1+\omega)\left(\frac{\bar{\pi}}{1+\bar{\pi}}\right)\Phi_{\bar{\pi}Q}$ and $-\omega\left(\frac{\bar{R}}{1+\bar{R}}\right)\Phi_{\bar{R}Q}$. Since both $\Phi_{\bar{\pi}Q}$ and $\Phi_{\bar{R}Q}$ are negative, the first term from inflation acts to lower R and the second from the expected nominal interest rate *acts* to raise R. If they were of equal sign, then their effects would be such that, taken alone, R would approximately rise by a factor of one, since $1+\omega$ would get approximately offset by ω. But since $\Phi_{\bar{\pi}Q} = -5.8$ and $\Phi_{\bar{R}Q} = -1.46$ the first effect dominates the second. This means *a* [delete: the] fall proportional to $1+\omega$ is only somewhat offset by the rise proportional to

ω, so the former effect drives down R in net as a result. For a positive goods productivity shock, from the terms $(1+\omega)\left(\frac{\bar{\pi}}{1+\bar{\pi}}\right)\Phi_{\bar{\pi}G}$ and $-\omega\left(\frac{\bar{R}}{1+\bar{R}}\right)\Phi_{\bar{R}G}$, the $\Phi_{\bar{\pi}G}$ equals 1.84 and the $\Phi_{\bar{R}G}$ equals -6.33, so both of these act to raise R. The upshot is that a positive bank productivity shock decreases R and a positive goods sector productivity shock increases R.

For a higher ω, coming for example from a higher steady state bank productivity, the positive bank shock would more negatively impact R, and the positive goods sector productivity shock would more positively impact R. The magnitudes of the effect of both shocks would be greater, so that smaller shocks would be needed to generate the same data changes in R.

$$
\begin{aligned}
\widehat{\bar{R}}_t \quad = \quad & \rho_Q \hat{A}_{Qt} \left\{ \begin{array}{c} (1+\omega)\left[\left(\frac{\bar{\pi}}{1+\bar{\pi}}\right)\Phi_{\bar{\pi}Q} + \theta\Phi_{g_Y Q}\right] \\ -\omega\left[\bar{R}\left(\frac{a}{1-a}\right)\Phi_{g_V Q} + \left(\frac{\bar{R}}{1+\bar{R}}\right)\Phi_{\bar{R}Q}\right] \end{array} \right\} \\
& + \rho_G \hat{A}_{Gt} \left\{ \begin{array}{c} (1+\omega)\left[\left(\frac{\bar{\pi}}{1+\bar{\pi}}\right)\Phi_{\bar{\pi}G} + \theta\Phi_{g_Y G}\right] \\ -\omega\left[\bar{R}\left(\frac{a}{1-a}\right)\Phi_{g_V G} + \left(\frac{\bar{R}}{1+\bar{R}}\right)\Phi_{\bar{R}G}\right] \end{array} \right\} \\
& + \rho_\sigma \hat{\sigma}_t \left\{ \begin{array}{c} (1+\omega)\left[\left(\frac{\bar{\pi}}{1+\bar{\pi}}\right)\Phi_{\bar{\pi}\sigma} + \theta\Phi_{g_Y \sigma}\right] \\ -\omega\left[\bar{R}\left(\frac{a}{1-a}\right)\Phi_{g_V \sigma} + \left(\frac{\bar{R}}{1+\bar{R}}\right)\Phi_{\bar{R}\sigma}\right] \end{array} \right\} \\
& + \hat{m}_t \Phi_{mm} \left\{ \begin{array}{c} (1+\omega)\left[\left(\frac{\bar{\pi}}{1+\bar{\pi}}\right)\Phi_{\bar{\pi}m} + \theta\Phi_{g_Y m}\right] \\ -\omega\left[\bar{R}\left(\frac{a}{1-a}\right)\Phi_{g_V m} + \left(\frac{\bar{R}}{1+\bar{R}}\right)\Phi_{\bar{R}m}\right] \end{array} \right\}.
\end{aligned} \tag{114}
$$

D Data Description

All data series for the VAR estimation and shock construction are from 1959Q1 until 2018Q2 and of quarterly frequency. All data series are seasonally adjusted except for interest rate series.

M1 Money Stock (M1SL)

M2 Money Stock (M2SL)

MZM Money Stock (MZM)

Monetary Base (BOGMBASE)

CPI: All Urban Areas (CPIAUCSL)

Personal Consumption Expenditures: Nondurable Goods (implicit price deflator) (DNDGRD3Q086SBEA)

Personal Consumption Expenditures: Services (implicit price deflator) (DSERRD3Q086SBEA)

Gross Domestic Product (GDP)

Effective Federal Funds Rate (FEDFUNDS)

3-Month Treasury Bill: Secondary Market Rate (TB3MS)

Noninstitutional Population of the United States: Ages 16 to 65 (CNP16OV)

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