Conditional Choice Probability Estimation of Continuous-Time Job Search Models

Preliminary and Incomplete

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Abstract

In this paper we propose a new method to estimate continuous-time job search models. Our approach is based on an adaptation of the conditional choice probability estimation methods to a continuous time job search environment. To do so, the proposed framework incorporates preference shocks into the search framework, resulting in a tight connection between value functions and conditional choice probabilities. A key advantage of the proposed approach relative to standard estimation methods for continuous-time job search models is computational. In particular, this makes it possible to estimate rich nonstationary job search models without having to solve any differential equation, and in some cases even avoiding any optimization. We apply our method to analyze the effect of unemployment benefit expiration on the duration of unemployment using rich longitudinal data from Hungarian administrative records.

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1 Introduction

In this paper we propose a new method to estimate continuous-time job search models. The idea of this approach is to adapt the conditional choice probability (CCP) estimation methods that have found many applications in the (discrete-time) dynamic discrete choice literature since the seminal work of Hotz and Miller (1993), to a continuous-time job search environment. To do so, our framework incorporates preference shocks into the search framework, resulting in a tight connection between value functions and conditional choice probabilities. A key advantage of the proposed approach relative to standard estimation methods for continuous-time job search models is computational. While the empirical labor search literature has been rapidly growing over the last few years (see Eckstein and van den Berg, 2007 for a survey), structural estimation of these models often remains challenging. This is particularly true for nonstationary environments, which tend to be the norm rather than the exception in the context of job search (van den Berg, 2001, van den Berg, 1990, Cahuc et al., 2014). The first and main contribution of this paper is to provide a novel empirical framework that makes it possible to estimate job search models in a simple, tractable and transparent way.

We apply our methods to analyze the effect of unemployment benefit expiration on duration of unemployment spells and on accepted wages, using rich longitudinal administrative data from Hungary. Beyond illustrating how these methods can be used to estimate nonstationary job search models at a very limited computational cost, this application contributes to the vast and growing literature on the relationship between unemployment benefits and job search behavior (see, e.g., Johnston and Mas, 2018, Nekoei and Weber, 2017, Lollivier and Rioux, 2010, Card et al., 2007, van den Berg, 1990, and Schmieder and von Wachter, 2016 for a recent survey).

This paper fits into three different literatures. First, it contributes to the literature on the estimation of dynamic discrete choice models using conditional choice probabilities. Since the seminal work of Hotz and Miller (1993), CCP methods have been increasingly used as a way to estimate complex dynamic discrete choice models at a limited computational cost (see Arcidiacono and Ellickson, 2011, and Aguirregabiria and Mira, 2010 for recent surveys). While CCP methods have been used a variety of settings, they have been mostly used in a discrete time environment. A recent exception is Arcidiacono et al. (2016), who apply CCP methods to estimate dynamic equilibrium models of market competition. However, to the best of our knowledge, none of these papers have explored the use of CCP methods to estimate job search models in continuous time.

This paper also contributes to the empirical job search literature. Since the seminal work of Flinn and Heckman (1982), a large number of papers have structurally estimated various types of job search models (see Eckstein and van den Berg, 2007 for a survey). In this literature, structural parameters are generally estimated via maximum likelihood or indirect inference methods, where the model needs to be solved *within* the estimation procedure. As a result, estimation tends to be computationally demanding, especially so in complex environments that are needed to capture important features of the labor market. Nonstationarity in job search, which arises in particular when the level of unemployment benefits varies over the unemployment spell, is an important example. Since the important work of van den Berg (1990) who structurally estimates a continuous-time nonstationary search model,[1] examples of structural estimates of nonstationary job search models remain scarce, likely in part because of the computational burden involved. Important references include Cockx et al. (2018), Launov and Walde (2013), Lollivier and Rioux (2010), Paserman (2008), and Frijters and van der Klaauw (2006).

Finally, our application fits into the vast and growing empirical literature that investigates the impact of unemployment benefit levels and duration on labor supply (see, e.g., Johnston and Mas, 2018, Nekoei and Weber, 2017, Le Barbanchon et al., 2017, Lollivier and Rioux, 2010, Card et al., 2007, van den Berg, 1990, and Schmieder and von Wachter, 2016 and Krueger and Meyer, 2002 for overviews of this literature).

The rest of the paper is structured as follows. In Section 2, we extend a stationary continuous-time job search model to allow for preference shocks, and explain how the model changes when UI expires. In Section 3, we show how the value of unemployment can be expressed in terms of probability to accept a job offer. In Section 4,

[1]See also Wolpin (1987), which is the first study to estimate a (discrete time) nonstationary search model.

we demonstrate our method: after describing the data, we outline a two-stage estimation procedure and discuss its extension to feature unobserved heterogeneity. Finally, Section 5 concludes.

2 Model

Our baseline framework is a continuous-time nonstationary job search model with wage posting. While this model shares many of the features of nonstationary job search models that have been considered in the literature (van den Berg, 1990, Lollivier and Rioux, 2010), a key distinction is that it incorporates preference shocks into the search framework. This feature is instrumental to our approach as it makes it possible to connect the value functions of unemployment and employment to the conditional choice probabilities. We first discuss the particular case of a stationary environment, then we turn to the nonstationary case where the value of unemployment is allowed to vary over time.

2.1 Stationary environment

Consider an economy in continuous time with infinitely lived workers, who discount the future at a rate $\rho > 0$. In our model, both employed and unemployed workers are looking for jobs. They can receive job offers which are characterized by wages drawn from a common discrete wage distribution with finite support $\Omega_w = \{w_1, \ldots, w_{N_w}\}$ and with the probability mass function $f(\cdot)$. Each time a worker receives an offer, she has to decide whether to accept it or turn it down based on the expected value which she can get if she continues to search. We model job offer arrival as a Poisson process, and allow employed and unemployed workers to sample jobs at different frequencies.

Workers have heterogeneous valuations of the job offers they receive. Specifically, we model these differences through the preference shock ε which is drawn independently whenever a new job offer arrives. The preference shock ε represents the relative attractiveness of a new job compared to the current state of the individual (employment at the current job for the employed or unemployment for the unemployed) and is supposed to affect the instantaneous utility. Thus, a job offer with the wage w can be accepted or rejected depending on the realization of the preference shock ε. We denote the *ex ante* probability of accepting a job offer w – before the realization of the shock – by $p_1(w_0, w)$ for employed workers, where w_0 is the wage at the current job, and by $p_0(w)$ for unemployed individuals. Finally, workers get laid off at the exogenous Poisson rate $\delta > 0$.

We now write the problem of the unemployed individuals. The flow utility of unemployment is given by b and include the value of unemployment insurance and leisure. Job offers arrive at a rate λ_0. Upon arrival of a job offer w, and denoting by V_0 the value of unemployment and by $V_1(w)$ the value of employment (at a wage w), the individual decides to accept the offer if and only if $V_1(w) + \varepsilon > V_0$. The Bellman equation for V_0 writes:

$$(\lambda_0 + \rho)\, V_0 = b + \lambda_0 \mathbb{E}_{\varepsilon,w} \max\left\{V_1(w) + \varepsilon, V_0\right\}$$

The Emax term is the expected value resulting from the optimal choice conditional on the job offer arrival. The expectation is taken both with the respect to the possible offered wages and realizations of the preference shocks. Assuming that ε is drawn from a logistic distribution, along with the independence between preference shocks and wage offers, we obtain the following expression:

$$\rho V_0 = b - \lambda_0 \mathbb{E}_w \log\left(1 - p_0(w)\right) \tag{2.1}$$

where $p_0(w)$ is the *ex ante* probability of accepting a job offer w, given by:

$$p_0(w) = \frac{1}{1 + \exp\left\{V_0 - V_1(w)\right\}}.$$

Equation (2.1) yields a simple expression of the value of unemployment as a function of the structural parameters $(\rho, b, \lambda_0)'$ and the conditional choice probabilities $p_0(w)$.

We now turn to the value of employment $V_1(w)$. The Bellman equation in this case writes:

$$(\lambda_1 + \delta + \rho) V_1(w) = u(w) + \delta V_0 + \lambda_1 \mathbb{E}_{\varepsilon, w'} \max \{V_1(w') - c_1 + \varepsilon, V_1(w)\}$$

where $u(w)$ is the flow utility associated with wage w, and c_1 is a job-switching cost. Assuming that the shocks ε are drawn from a logistic distribution, we can rewrite this equality as:

$$(\lambda_1 + \delta + \rho) V_1(w) = u(w) + \delta V_0 + \lambda_1 V_1(w) + \lambda_1 \log [1 + \exp (V_1(w') - V_1(w) - c_1)]$$

which, in turn, can be rewritten as:

$$(\delta + \rho) V_1(w) = u(w) + \delta V_0 - \lambda_1 \mathbb{E}_{w'} \log (1 - p_1(w, w')) \tag{2.2}$$

where $p_1(w, w')$ denotes the probability of accepting a new job offer w' given the current wage rate w, given by:

$$p_1(w, w') = \frac{1}{1 + \exp \{V_1(w) - V_1(w') + c_1\}}.$$

Combined with Equation (2.1) above, it follows from Equation (2.2) that the value of employment at a wage rate w is a function of the flow utility of employment $u(w)$, the structural parameters $(\rho, b, \lambda_0, \delta, \lambda_1)$ and the conditional choice probabilities $p_1(w, w')$ and $p_0(w')$.

2.2 Nonstationary environment

We now extend the model discussed in the previous section by relaxing the assumption that the value of unemployment is constant over time. Specifically, we introduce two sources of nonstationarity: we allow the flow utility of unemployment $b(t)$ and the job offer arrival rate $\lambda_0(t)$ to vary as a function of time from the start of the unemployment spell. We assume that getting a job resets the unemployment duration, which implies that $V_1(w)$ remains stationary. Indexing time spent unemployed by t, it is useful to first write the Bellman equation for the unemployment value function $V_0(t)$ in discrete time.

Denoting by Δt the discrete time unit, it follows from Bellman's optimality principle that:

$$V_0(t) = b(t)\Delta t + \frac{\lambda_0(t)\Delta t}{1+\rho\Delta t}\mathbb{E}_{\varepsilon,w}\max\{V_1(w)+\varepsilon, V_0(t+\Delta t)\} + \frac{1-\lambda_0(t)\Delta t}{1+\rho\Delta t}V_0(t+\Delta t).$$

which can be rewritten as:

$$\rho V_0(t) = b(t)(1+\rho\Delta t) + \lambda_0(t)\mathbb{E}_{\varepsilon,w}\max\{V_1(w) - V_0(t+\Delta t) + \varepsilon, 0\} + \frac{V_0(t+\Delta t) - V_0(t)}{\Delta t}$$

Next, letting $\Delta t \to 0$ and denoting by $\dot{V}_0(t)$ the derivative of the $V_0(t)$ with respect to unemployment duration, we obtain the following (continuous-time) differential equation in $V_0(\cdot)$:

$$\rho V_0(t) = b(t) + \lambda_0(t)\mathbb{E}_{\varepsilon,w}\max\{V_1(w) - V_0(t) + \varepsilon, 0\} + \dot{V}_0(t)$$

Finally, denoting by $p_0(w,t)$ the *ex ante* probability of accepting a job offer w at time t, this expression can be rewritten as a function of $p_0(w,t)$:

$$\rho V_0(t) = b(t) - \lambda_0(t)\mathbb{E}_w \log(1 - p_0(w,t)) + \dot{V}_0(t) \tag{2.3}$$

A couple of remarks are in order. First, an important difference relative to the stationary environment is that Equation (2.3) now involves the derivative of the value of unemployment with respect to duration of unemployment ($\dot{V}_0(t)$). This term represents the change in the option value of job search due to variation over time in the value of unemployment. In the particular case where nonstationarity arises because of over-time changes in the level of unemployment benefits, the option value of searching for a job will decrease as job seekers get closer to the unemployment insurance expiration date.

Second, Equation (2.3) is a simple linear first-order differential equation in $V_0(\cdot)$, which admits an exact analytical solution as a function of the structural parameters and the conditional choice probabilities $p_0(w,t)$. This solution is given by:

$$V_0(t) = \exp(\rho t)\left(V_0(t_0)\exp(-\rho t_0) - \int_{t_0}^{t}\exp(-\rho u)\phi(u)du\right)$$

for any given $t_0 \in \mathbb{R}^+$, with:

$$\phi(u) = -b(u) + \lambda_0(u)\mathbb{E}_w \log\left(1 - p_0(w,t)\right)$$

Note that the existence of preference shocks ε is key to this derivation. Absent these shocks, $V_0(t)$ would satisfy instead the following nonlinear second order differential equation:

$$\rho V_0(t) = b(t) + \lambda_0(t)\mathbb{E}_w \max\left\{V_1(w) - V_0(t), 0\right\} + \dot{V}_0(t)$$

This equation needs to solved numerically as is done in particular in the seminal work of van den Berg (1990).

Finally, following similar arguments to the stationary case, the value of employment $V_1(w)$ is given by:

$$\left(\delta + \rho\right) V_1(w) = u(w) + \delta V_0(0) - \lambda_1 \mathbb{E}_{w'} \log\left(1 - p_1(w, w')\right) \tag{2.4}$$

where $V_0(0)$ is the value of unemployment at the beginning of an unemployment spell, and $p_1(w, w')$ denotes the probability of accepting a new job offer w' given the current wage rate w.

3 Identification

We have shown in the previous section that the unemployment and employment value functions can be simply expressed as a function of the structural parameters of the model, the wage offer distributions, as well as the conditional job acceptance probabilities. There are two fundamental differences compared to Hotz-Miller type approach for dynamic discrete choice models. First, in a search environment, choices (i.e., job offer acceptance or rejection) are generally not observed by the analyst. Second, wage offers are generally unobserved as well. Nonetheless, we provide in the following a simple constructive identification strategy for the structural parameters, wage offer distributions as well as (conditional) job acceptance probabilities. These results hold in a standard empirical setting where one has access to longitudinal data on i) accepted wages, along with ii) transitions from unemployment to employment, iii) transitions from employment to unemployment, and iv) job-to-job transitions. Note that we assume throughout this identification sketch that wages are drawn from a discrete distribution with finite support. This distribution can be thought of as a discrete approximation to the underlying (continuous) wage distribution.[2]

3.1 Restrictions implied by job-to-job transitions

3.1.1 Additional notation and assumptions

Given the structural model of the previous section, we now turn to identification. We assume the following are directly identified from the data:

1. h_{ij}, the hazard rate of moving from a job with wage w_i to a job with wage w_j;
2. $h_i(t)$, the hazard rate out of unemployment at time t to a job that pays w_i;
3. h_0, the hazard rate of moving from employment to unemployment.

Note that the job destruction rate δ is directly identified from h_0. Finally, as is standard in this class of models, we assume throughout that the discount rate ρ is known.

We first show that under the assumptions outlined above, the offered wage distribution, on-the-job offer arrival rate λ_1, job switching cost c_1, conditional choice probabilities of accepting a new job offer given the current wage rate, and the flow utility of employment (up to an additive constant) can be identified from the hazard rates associated with the various job-to-job transitions from w to w', with $(w, w') \in \Omega_w^2$. We then show how to recover the parameters associated with unemployment, namely the offer arrival rate $\lambda_0(t)$, the flow utility of unemployment $b(t)$, and the conditional choice probabilities of accepting a job offer while unemployed.

[2] We will investigate this issue further in our project. In particular, we plan on borrowing tools from machine learning and discretize the wage distribution via k-means clustering (see, e.g., Pollard, 1981, Dougherty, Kohavi, and Sahami, 1995, and Bonhomme, Lamadon, and Manresa, 2017 for recent use of k-means clustering in the context of unobserved heterogeneity discretization).

3.1.2 Wage offer distribution

By definition, h_{ij} is given by:

$$h_{ij} = \lambda_1 f(w_j) p_1(w_i, w_j)$$

where $p_1(w_i, w_j)$ is the probability of accepting a new job offering w_j given the individual's current job pays w_i.

Note that $p_1(w_i, w_i) = p_1(w_j, w_j)$ for all $\{j, i\}$ as the level of the wage does not affect the probability of switching given the assumptions made earlier. It follows that:

$$\frac{h_{ii}}{h_{jj}} = \frac{f(w_i)}{f(w_j)}$$

Summing over w_i in the support Ω_w yields:

$$f(w_j) = \frac{h_{jj}}{\sum_i h_{ii}} \tag{3.1}$$

3.1.3 Identification of λ_1, c_1, and $p_1(w_i, w_j)$

The distributional assumption on the preference shocks ε yields a simple relationship between probabilities of accepting a new job offer, the employment value functions and the switching cost:

$$\ln\left(\frac{p_1(w_i, w_j)}{1 - p_1(w_i, w_j)}\right) = V_1(w_j) - c_1 - V_1(w_i)$$

Thus implying:

$$\ln\left(\frac{p_1(w_i, w_j)}{1 - p_1(w_i, w_j)}\right) + \ln\left(\frac{p_1(w_j, w_i)}{1 - p_1(w_j, w_i)}\right) = -2c_1$$

Using the fact that:

$$p_1(w_i, w_j) = \frac{h_{ij}}{\lambda_1 f(w_j)} \tag{3.2}$$

we obtain:

$$\ln\left(\frac{h_{ij}}{\lambda_1 f(w_j) - h_{ij}}\right) + \ln\left(\frac{h_{ji}}{\lambda_1 f(w_i) - h_{ji}}\right) = -2c_1 \tag{3.3}$$

It follows that the following equality holds for any given pair $\{w_{i'}, w_{j'}\} \in \Omega_w^2$:

$$\ln\left(\frac{h_{ij}}{\lambda_1 f(w_j) - h_{ij}}\right) + \ln\left(\frac{h_{ji}}{\lambda_1 f(w_i) - h_{ji}}\right) = \ln\left(\frac{h_{i'j'}}{\lambda_1 f(w_{j'}) - h_{i'j'}}\right) + \ln\left(\frac{h_{j'i'}}{\lambda_1 f(w_{i'}) - h_{j'i'}}\right)$$

For any $\{w_{i'}, w_{j'}\} \in \Omega_w^2$ such that $p_1(w_i, w_j)p_1(w_j, w_i) \neq p_1(w_{i'}, w_{j'})p_1(w_{j'}, w_{i'})$ this equality identifies λ_1, which is given by:

$$\lambda_1 = \frac{(f(w_{j'})h_{j'i'} + f(w_{i'})h_{i'j'})h_{ij}h_{ji} - (f(w_j)h_{ji} + f(w_i)h_{ij})h_{i'j'}h_{j'i'}}{f(w_{j'})f(w_{i'})h_{ij}h_{ji} - f(w_j)f(w_i)h_{i'j'}h_{j'i'}} \tag{3.4}$$

Finally, having identified λ_1, the switching cost c_1 and the conditional choice probabilities are directly identified from Equation (3.3) and Equation (3.2), respectively.

3.1.4 Flow utility of wages $u(\cdot)$

Consider the log odds of choosing to accept a job offering w_j when the current job pays w_i. Expressing the Emax term with respect to the value of the new job, we can write:

$$\ln\left(\frac{p_1(w_i, w_j)}{1 - p_1(w_i, w_j)}\right) = \frac{u(w_j) - u(w_i) + \lambda_1 \sum_w \left[\ln\left(p_1(w_i, w)\right) - \ln\left(p_1(w_j, w)\right)\right] f(w)}{\rho + \delta + \lambda_1} - c_1 \tag{3.5}$$

Since the job destruction rate δ is directly identified from the data on job-to-unemployment transitions, it follows from the previous steps that the flow utility $u(w)$ is identified up to a constant. Assuming that the flow utility of employment is linear in the wage rate, $u(w) = \alpha w$, this in turn identifies $u(\cdot)$.

3.2 Restrictions implied by unemployment-to-job transitions

3.2.1 Identification of $\lambda_0(t)$

Denote the probability of accepting an offer of w out of unemployment at time t as $p_0(w,t)$. The hazard rate from unemployment to employment at wage w_i in period t is then:

$$h_i(t) = \lambda_0(t) f(w_i) p_0(w_i, t)$$

implying that we can write $p_0(w_i, t)$ as a function of one unknown parameter, $\lambda_0(t)$:

$$p_0(w_i, t) = \frac{h_i(t)}{\lambda_0(t) f(w_i)} \tag{3.6}$$

Taking the difference in log odds ratios of accepting jobs that offer w_i and w_j out of unemployment at time t, we obtain:

$$\ln\left(\frac{p_0(w_i,t)}{1-p_0(w_i,t)}\right) - \ln\left(\frac{p_0(w_j,t)}{1-p_0(w_j,t)}\right) = \frac{u(w_i) - u(w_j) + \lambda_1 \sum_w \left[\ln\left(p_1(w_j,w)\right) - \ln\left(p_1(w_i,w)\right)\right] f(w)}{\rho + \lambda_1 + \delta}$$

Note that everything on the right hand side is known. Further, everything on the left hand side can be expressed as an (identified) function of $\lambda_0(t)$ using Equation (3.6):

$$\ln\left(\frac{h_i(t)}{\lambda_0(t) f(w_i) - h_i(t)}\right) - \ln\left(\frac{h_j(t)}{\lambda_0(t) f(w_j) - h_j(t)}\right) = \frac{u(w_i) - u(w_j) + \lambda_1 \sum_w \left[\ln\left(p_1(w_j,w)\right) - \ln\left(p_1(w_i,w)\right)\right] f(w)}{\rho + \lambda_1 + \delta}$$

Denoting the right hand side as κ_{ij}, this equation yields a closed-form expression for $\lambda_0(t)$:

$$\lambda_0(t) = \frac{h_i(t) h_j(t) (e^{\kappa_{ij}} - 1)}{f(w_i) h_j(t) e^{\kappa_{ij}} - f(w_j) h_i(t)} \tag{3.7}$$

3.2.2 Identification of $p_0(w,t)$

Having identified $\lambda_0(t)$, the conditional choice probabilities $p_0(w,t)$ are identified from Equation (3.6).

3.2.3 Identification of $b(t)$

We express the following log odds ratio by normalizing the future value of working relative to staying at the same job:

$$\ln\left(\frac{p_0(w_i,t)}{1-p_0(w_i,t)}\right) = \frac{u(w_i) - \lambda_1 \sum_w \left[\ln\left(1-p_1(w_i,w)\right)\right] f(w) + \delta V_0(t)}{\rho+\delta} - V_0(t)$$

Solving for $V_0(t)$ yields:

$$V_0(t) = \frac{u(w_i) - \lambda_1 \sum_w \left[\ln\left(1-p_1(w_i,w)\right)\right] f(w)}{\rho} - \frac{\rho+\delta}{\rho}\ln\left(\frac{p_0(w_i,t)}{1-p_0(w_i,t)}\right)$$

Note that at this stage everything on the right hand side is known, so that this equality identifies $V_0(t)$ and $\dot{V}_0(t)$. It follows that one can directly solve for and identify $b(t)$ using Equation (2.3):

$$b(t) = \rho V_0(t) + \lambda_0(t)\mathbb{E}_w \log\left(1-p_0(w,t)\right) - \dot{V}_0(t) \tag{3.8}$$

A remarkable implication of these results is that, by exploiting the tight connection between value functions and conditional choice probabilities, we are able to recover the structural parameters of our nonstationary job search model without numerically solving differential equations.

4 Empirical Implementation

4.1 Data

We estimate the model using matched employer-employee data from Hungarian administrative records, provided by the Center for Economic and Regional Studies at the Hungarian Academy of Sciences (CERS-HAS). The dataset used in this analysis combines data from five administrative sources: (i) the National Health Insurance Fund of Hungary; (ii) the Central Administration of National Pension Insurance; (iii) the National Tax and Customs Administration of Hungary; (iv) the Public Employment Service National Labor Office; and (v) the Educational Authority. The sample consists of half of the Hungarian population, i.e., 4.6 million individuals, linked across 900 thousand firms. On the individuals' side, a de facto 50% random sample of the population are observed; every Hungarian citizen born on Jan 1, 1927 and every second day thereafter are included. A key distinctive feature of the Hungarian data is their frequency: individuals are observed on a monthly basis. One individual can be present in at most two work arrangements: labor market measures are observed separately for them. We also have information on demographics, total earnings and days worked (i.e., including tertiary and further work arrangements), as well as benefit payments. On the firm side, all firms are included at which any sampled individuals are observed to have worked. Balance sheet data from the tax authority are available on a yearly basis. Consequently we cannot analyze within-year co-movements of individual and firm measures. However, we can link the yearly information of the old and new firms to a worker who experiences a job-to-job transition.

For estimation we use a sample of employed and unemployed individuals from January 2004 to October 2005. During this time, Hungary had a two-tier unemployment insurance system: only those were eligible for second-tier benefits who had a sufficiently long work history, and benefit payments in the second tier were lower than in the first. Those who exhausted benefits in both tiers were eligible for social assistance. We supplement the main database with detailed information on unemployment status, benefit eligibility, and benefit take-up in both tiers from raw administrative records. From these records, we can directly observe all relevant measures of unemployed individuals. For employed individuals, we observe their monthly earnings and their monthly employment status as well as an anonymous identifier of their primary employers. From these data, we can infer the length of their employment spells, as well as job-to-job transitions from changes in firm identifiers.

4.2 First stage

In the first stage, we estimate hazard rates associated with job-to-job transitions and job separations. Specifically, we estimate the following objects:

1. h_{ij}, the hazard rate of moving from a job with wage w_i to a job with wage w_j;
2. $h_i(t)$, the hazard rate out of unemployment at time t to a job that pays w_i;
3. h_0, the hazard rate of moving from employment to unemployment.

We model these hazards in a competing hazard framework with right-censoring, which may occur when the individual exits the labor force.

To fix ideas, imagine an individual who currently works in a job that pays w_i. At any point in time, four things may happen to this individual. She may switch to a job that pays w_j, which occurs at the rate h_{ij}; she may exit to unemployment which occurs at the rate h_0; she may exit the labor force which we treat as a right-censored employment spell; or none of the above and she stays in her current job. Similarly, at time t an individual who is currently unemployed may transition to a job that pays w_i, which occurs at the rate $h_i(t)$; she may exit the labor force; or she may remain unemployed. Transitions to either of the discrete wage bins or to unemployment are mutually exclusive, and we model them as competing hazards.

4.2.1 Hazard rates of job-to-job transitions

Let $h_{ij}(t)$ denote the hazard of moving from w_i to w_j at time t. We abstract from the time dimension in the case of job-to-job transitions, which is equivalent to assuming that the baseline hazard is constant: $h_{ij}(t) = h_{ij}$ for all t. Consequently, the cumulative hazard of moving from w_i to w_j is

$$H_{ij}(t) = \int_0^t h_{ij}(u)\, du = h_{ij}\, t. \tag{4.1}$$

It then follows that the survival function for spells with the current wage w_i can be written as the product of the destination-specific cumulative hazards:

$$S_i(t) = \prod_j \exp\left(-H_{ij}(t)\right) = \prod_j \exp\left(-h_{ij}\, t\right). \tag{4.2}$$

Assume that we observe data $\{t_s, i_s, j_s, d_s\}_s$: for spell s we know its duration t_s, the wage rates in the origin job i_s and in the destination job j_s, and an indicator d_s of whether the spell is complete. The (log)likelihood of observing these data is

$$L = \prod_s \prod_{i,j} \left[(h_{ij})^{d_s \cdot \mathbb{1}(j_s = j)} \exp\left(-h_{ij}\, t_s\right)\right]^{\mathbb{1}(i_s = i)} \tag{4.3}$$

$$\log L = \sum_s \sum_{i,j} \mathbb{1}(i_s = i) \left[d_s \cdot \mathbb{1}(j_s = j) \log h_{ij} - h_{ij}\, t_s\right], \tag{4.4}$$

which, for a given w_i–w_j transition, is maximized at

$$\sum_s \mathbb{1}(i_s = i) \left[\frac{d_s \cdot \mathbb{1}(j_s = j)}{\hat{h}_{ij}} - t_s\right] = 0. \tag{4.5}$$

Therefore the estimated hazard of switching from w_i to w_j is

$$\hat{h}_{ij} = \frac{\sum_s \mathbb{1}(i_s = i) \cdot d_s \cdot \mathbb{1}(j_s = j)}{\sum_s \mathbb{1}(i_s = i)\, t_s}. \tag{4.6}$$

In words, this hazard is the inverse of the average duration of complete spells among all observations originating from w_i multiplied by the fraction of spells which lead to w_j.

4.2.2 Hazard rates out of unemployment

Let $h_i(t)$ denote the hazard of moving from unemployment to w_i at time t. We assume that the hazard function is Weibull with a wage-dependent shape parameter and a scale parameter normalized to 1 for simplicity. That is, the hazard function is

$$h_i(t) = \alpha_i t^{\alpha_i - 1} \tag{4.7}$$

and the survivor function is

$$S_i(t) = \exp\left(-t^{\alpha_i}\right). \tag{4.8}$$

We observe data $\{t_s, i_s, d_s\}_s$: for unemployment spell s we know its duration t_s, the wage rate in the new job i_s, and an indicator d_s of whether the spell is complete. Given these data, the (log)likelihood is

$$L = \prod_s \prod_i \left(\alpha_i t_s^{\alpha_i - 1}\right)^{d_s \cdot \mathbb{1}(i_s = i)} \exp\left(-t_s^{\alpha_i}\right) \tag{4.9}$$

$$\log L = \sum_s \sum_i d_s \cdot \mathbb{1}(i_s = i)\left(\log \alpha_i + (\alpha_i - 1)\log t_s\right) - t_s^{\alpha_i}, \tag{4.10}$$

which, for a given wage rate i is maximized at

$$\sum_s d_s \cdot \mathbb{1}(i_s = i)\left(\frac{1}{\hat{\alpha}_i} + \log t_s\right) - t_s^{\hat{\alpha}_i} \log \hat{\alpha}_i = 0. \tag{4.11}$$

We can easily solve these first order conditions for $\hat{\alpha}_i$.

4.3 Second stage

The identification strategy presented in Section 3 yields a straightforward sequential estimation procedure. For job-to-job transitions, we first recover the distribution of offered wages from hazard rates of switching jobs within the same wage bin. Then we calculate the offer arrival rate by exploiting the symmetric pattern of the conditional choice probabilities across wage bins. From there, we directly obtain estimates for the conditional choice probabilities and the switching cost. Finally, we recover the flow utility of wages by imposing linearity in the wage rate.

For unemployment-to-employment transitions, we proceed similarly. First, we calculate the arrival rate of offers to the unemployed over their unemployment spell, from which we directly obtain the conditional choice probabilities of accepting offers. Then we estimate the flow utility of unemployment by calculating the value of unemployment and its time derivative. In contrast to the previous literature, none of these steps require numerical procedures: every object has a closed-form solution, hence the estimation routine takes seconds to run.

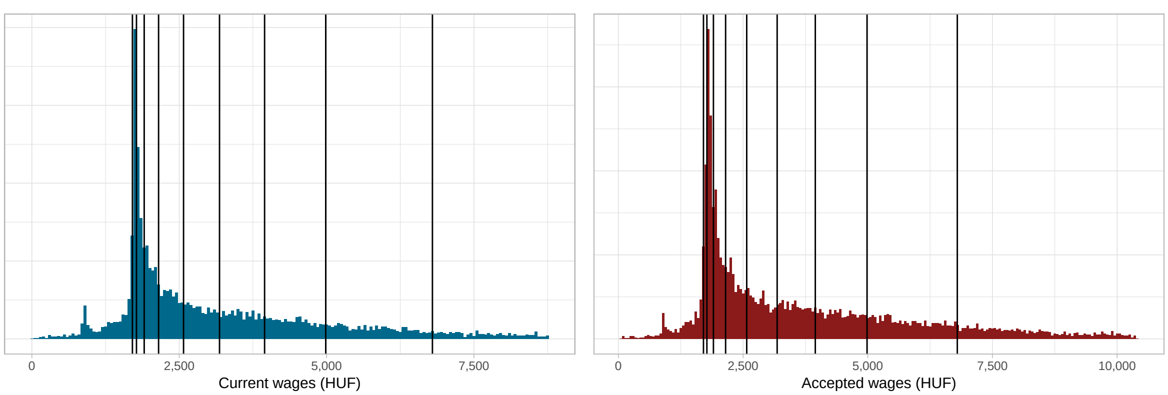
4.4 Results

Our identification strategy relies on discretizing the wage distribution. In this section, we first describe how we discretize current and accepted wages, then we present our hazard estimates.

We create 10 wage bins for current and accepted wages. The cutoffs for these bins are deciles of the distribution of accepted wages for employment spells that lead to a job-to-job transition. Figure 1 plots how these cutoffs partition the empirical distributions of current and accepted wages in our data. Figure 2 plots the resulting discrete distribution of current wages: the left panel contains current wages for employment spells that lead to a job-to-job transition and the right contains current wages for all employment spells. As the right panel suggests, many spells in the highest decile do not lead to job-to-job transitions. Similarly, Figure 3 plots the discrete distribution of accepted wages for job-to-job and unemployment-to-employment transitions. Employed workers switch to higher-paying jobs with an increasing frequency, whereas the distribution of unemployed individuals transitioning to employment is relatively constant across wage bins.

Figure 1. Discretizing observed wages

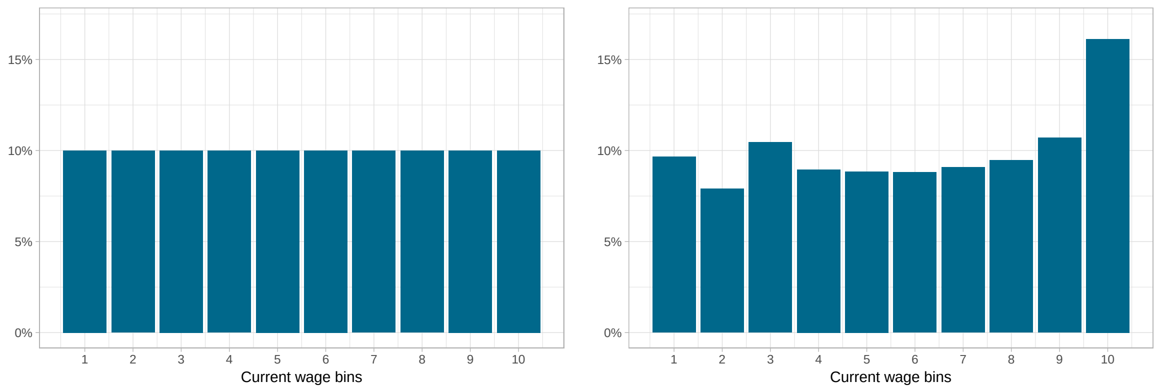
(a) Current wages (b) Accepted wages



Notes: Current and accepted wages for employment spells that lead to a job-to-job transition. Histograms with 50 HUF bin width, truncated at the 95th percentile. Vertical lines denote deciles.

Figure 2. Discrete distribution of current wages

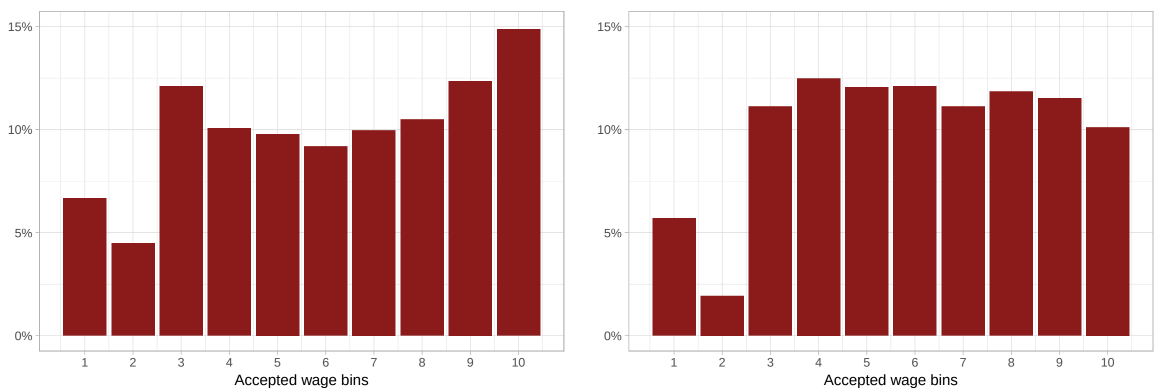
(a) Spells with job-to-job transitions (b) All spells



Notes: Panel 2a: discrete distribution of current wages for employment spells that lead to a job-to-job transition. Panel 2b: discrete distribution of current wages for all employment spells.

Figure 3. Discrete distribution of accepted wages

(a) Job-to-job transitions (b) Unemployment-to-employment transitions



Notes: Panel 3a: discrete distribution of accepted wages for employment spells that lead to a job-to-job transition. Panel 3b: discrete distribution of accepted wages for unemployment spells that lead to an employment spell.

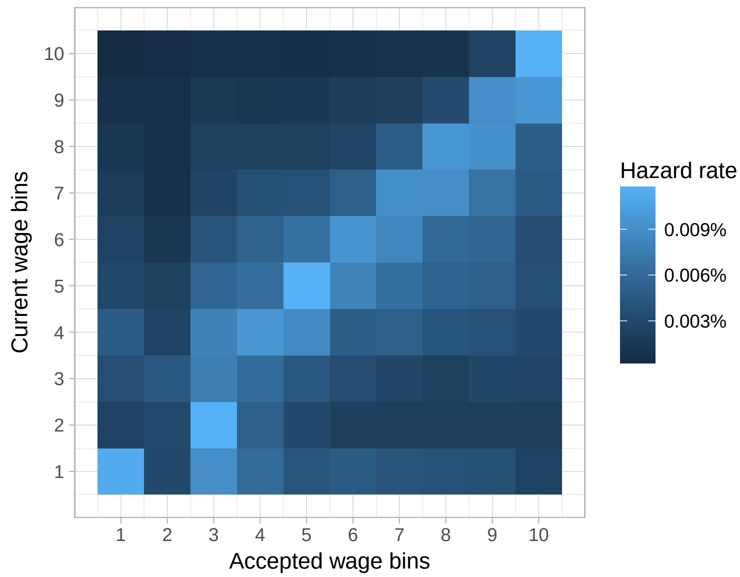
Based on these discrete wage distributions, we estimate hazard rates of job-to-job and unemployment-to-employment transitions, as described in Section 4.2. Figure 4 reports our estimates of constant job-to-job hazard rates. Two main patterns emerge from this figure. First, switching to jobs within the same wage bin is associated with the highest hazard rate among all transitions from the same wage bin. Second, the closer the accepted wage is to the current wage, the higher the hazard of switching. Note that these hazard rates are daily measures: to put them in context, a daily hazard rate of 0.006 percent is equivalent to a 2.2 percent cumulative hazard of job-to-job transitions within 365 days.

Figure 5 plots our estimates of Weibull unemployment-to-employment hazards. Higher accepted wages are mostly associated with a faster transition to employment earlier in the spell. To put the daily measures in context, the cumulative hazard of transitioning from unemployment to a job that pays in the 6th decile is 6.2 percent within 200 days.

4.5 Extension: unobserved heterogeneity

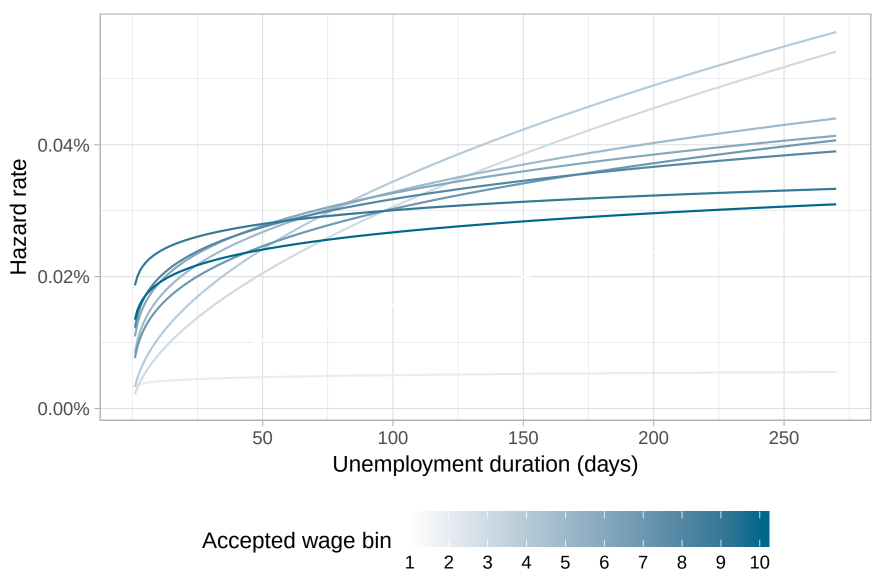
The previous framework can be easily extended to allow for individual-specific unobserved heterogeneity. Following Heckman and Singer (1984), assume that the unobserved heterogeneity follows a discrete distribution with $R \geq 2$ points of support. Individuals know their heterogeneity type $r \in \{1, .., R\}$, which is unobserved to the econometrician. The sequential identification strategy outlined earlier can be easily extended to account for such unobserved heterogeneity, after replacing the hazard rates associated with the various job-to-job (the h_{ij}'s) and unemployment-to-job transitions (the $h_i(t)$'s) by hazard rates that are now conditional on individual unobserved heterogeneity. The previous identification arguments then still apply, resulting in point identification of the structural parameters (arrival rates, switching costs, and wage offer distributions) that are now also a function of unobserved heterogeneity. The initial step of the identification algorithm now consists in identifying the type-specific hazard functions, along with the distribution of heterogeneity types. To do so, we rely on a mixed proportional hazard specification (Elbers and Ridder, 1982, van den Berg, 2001). As is the case for the model without unobserved heterogeneity, the resulting identification strategy is constructive, and can be translated directly into a sequential estimation strategy.

Figure 4. Hazard rates of job-to-job transitions



Notes: Exponential model (constant hazards), maximum likelihood hazard estimates.

Figure 5. Hazard rates of unemployment-to-employment transitions



Notes: Weibull model, maximum likelihood hazard estimates.

5 Conclusion

In this paper, we propose a novel approach to estimating job search models. We extend the traditional continuous-time job search model with on-the-job search to allow for preference shocks which, in turn, allows us to estimate the model using conditional choice probability methods, widely used in the discrete choice literature. We propose a simple two-stage procedure of estimating job search models. In the first stage, we estimate hazard rates of unemployment-to-employment and job-to-job transitions. In the second stage, we uncover the discrete density of accepted wages: these objects inform us on the shape of the probability to accept a job offer as a function of offered wages and duration of unemployment for unemployed workers, and as a function of offered wages and current wages for employed workers. Conditional choice probability methods allow us to obtain closed-form expressions for the remaining structural parameters of the model. As a result, we overcome the computational burden of estimating complicated, such as nonstationary, job search models. We illustrate our method by analyzing the impact of unemployment benefits expiration on the duration of unemployment and wages in Hungary, using administrative data from tax records and the unemployment registry.

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