Information Design with Agency

Jacopo Bizzotto[*] Eduardo Perez-Richet[†] and Adrien Vigier[‡]

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Abstract

We study the problem of a principal who relies on an agent for the production of public information destined to the players of a continuation game. The agent has access to a status quo procedure to generate information. The principal is given the opportunity to design a better procedure, but her design is constrained by the status quo in two ways: First, the new procedure must use the same set of signals as the status quo procedure; Second, the agent incurs a cost of switching to the new procedure. The principal can reward the agent with monetary transfers up to a limited liability constraint, but we assume that procedures are not contractible, only signals are. The principal therefore faces a problem of information design with agency in which she must trade off informativeness about the states of the world to influence the continuation game with informativeness about the choice of the agent to reduce the cost of agency. We provide a general methodology for solving these problems and characterize optimal implementable procedures. We examine comparative statics with respect to switching cost. Finally, we apply our results to information acquisition, leak prevention and persuasion examples.

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[*]Faculty of Social Sciences, OsloMet. Email: jacopobi@oslomet.no.
[†]Department of Economics, Sciences Po and CEPR. Email: eduardo.perez@sciencespo.fr.
[‡]Department of Economics, Norwegian Business School. Email: adrien.h.vigier@bi.no.

1 Introduction

Consider a principal designing a new information production procedure with the aim of influencing subsequent decisions by a group of receivers. For example, the management of a firm may choose an information acquisition procedure to help its own decision making, a university may choose a testing and grading policy to influence the job placement of its students, the head of research and development at a technology firm may choose an internal communication protocol to prevent its innovations from leaking to competitors, or the head of security at an airport may choose the screening protocol of passengers. In many cases, the task of running the procedure is delegated to an agent. For example, consultants or employees acquire information to help managerial decision making, teachers grade students, and employees may or may not follow protocols. Whenever procedures are not contractible, the principal faces a moral hazard problem. We seek to understand how this affects her choice of procedure.

In general, agents may deviate from the prescribed procedure in many ways. We will consider situations in which the agent's only alternative is to revert to a given *status quo* procedure instead of following the principal's recommendation. The procedure chosen by the agent may be observable but is not contractible. But procedures generate contractible outputs: the signals. We assume that the set of signals generated by the designed procedure cannot be freely chosen. Instead, the new procedure and the status quo procedure must share a common language. For example, grading must be on a scale from A to D, or consultants must provide an action recommendation.

Because of the common language, the principal is generally unable to perfectly distinguish across different procedures. She faces an informational trade off between generating information about the states of the world to influence receivers, and making her chosen procedure easy to distinguish from others so as to lower the cost of agency.

One natural interpretation of our setup is that the principal is seeking to improve the information process in her organization. However, agents are used to the current procedure and learning to run the new procedure requires a costly training. Furthermore, adoption of the new procedure is not contractible.[1]

We show that, as in Kamenica and Gentzkow (2011), the optimal information design problem of the principal can be formulated as the problem of choosing a Bayes plausible

[1]This is reminiscent of Atkin, Chaudhry, Chaudry, Khandelwal and Verhoogen (2017), which illustrates how misalignment of interests between management and employees can act as a barrier to technological innovation. We consider technological innovations in information production processes and explore how incentive-payment schemes can help.

distribution of posterior beliefs, where each belief distribution has an agency cost. Contrary to Gentzkow and Kamenica (2014), this agency cost is not linear in the distribution of posterior beliefs, but is rather a decreasing and convex function of the probability of the most likely belief under this distribution. Because of this nonlinearity, the optimization problem of the principal cannot be solved by concavification. However, we show that the optimal belief distribution can be decomposed as follows: First, split the prior belief between a payment belief, which is the only posterior belief realization for which the agent gets paid, and a resplitting belief. Conditional on reaching the payment belief, pay the agent the exact amount needed to make him ex ante indifferent between the two procedures. Conditional on reaching the resplitting belief, redistribute posterior beliefs so as to concavify the objective function of the principal in the continuation game. The problem can therefore be reduced to the problem of choosing a Bayes plausible binary distribution of beliefs.

Example 1. *The ministry of transport is considering whether to build a public transportation infrastructure. To do so the ministry employs a consulting agency to collect data and run a model that produces a recommendation. There are two equally likely states of the world, ω_b (*build*) and ω_n (*do not build*). The ministry seeks to choose an action a_b or a_n matching the realized state. In the past, the consultant has been using a well known model that we will call the status quo procedure, which recommends the right action with probability $\frac{1+q}{2}$, for a given precision parameter q.*

The ministry has recently developed a partnership with a public research center that designs models to make this type of recommendations. They have the opportunity to develop a new recommendation procedure. Switching to this new procedure, however, will cost $c > 0$ to the consultant in terms of effort. The principal has no means of verifying whether the consultant uses the new procedure or sticks to the status quo. The only way to make sure that the consultant runs the new procedure is by committing to transfers (bonus payments) contingent on the recommendation.

As a benchmark, we assume that the principal is free to select any new procedure she wants. If procedures are contractible, the principal selects the new procedure to be perfectly informative, that is to recommend building with probability one when the state is b and not building when the state is n. In sharp contrast, the principal cannot induce the agent to use the perfectly informative procedure if recommendations, but not procedures, are contractible.[2] *The*

[2] Indeed, the expected payoff of the agent with promised payments $t(a_b)$ and $t(a_n)$ is $\frac{1}{2}t(a_b) + \frac{1}{2}t(a_n) - c$ if he chooses the new perfectly informative procedure, and $\frac{1}{2}\left(\frac{1+q}{2}t(a_b) + \frac{1-q}{2}t(a_n)\right) + \frac{1}{2}\left(\frac{1-q}{2}t(a_b) + \frac{1+q}{2}t(a_n)\right) = \frac{1}{2}t(a_b) + \frac{1}{2}t(a_n)$ if he chooses the status quo procedure.

straightforward intuition is that each recommendation is equally likely under either procedure, so the principal cannot provide the agent with incentives by rewarding recommendations that indicate that she has been using the new procedure. In this case, the agency cost associated with the fully informative procedure is infinite.

When designing the new procedure, the principal must strike a balance between being informed about the state of the world, to increase her informational payoff, and being informed about the procedure which the agent uses, to lower the agency cost of inducing the agent to pick the new procedure. In this example, to lower the agency cost, the principal must bias the procedure towards one recommendation which she will use to reward the agent. We find that an optimal procedure[3] *is one that recommends not to build with probability 1 when the state of the world is* n*, and recommends to build with probability* x^* *when the state of the world is* b*, where* x^* *maximizes*[4]

$$\underbrace{\frac{1}{2}(1+x)}_{\textit{informational payoff}} - \underbrace{\left(\frac{2-x}{1-x}\right)c}_{\textit{agency cost}}.$$

This expression clearly reflects the informativeness trade-off: Greater distortions (captured by lower x*) induce an informational loss for the principal, but reduce the agency cost (which tends to infinity as* x *tends to 1).*

2 Model

General Environment. We consider an information design environment in which the final information structure of a continuation game is determined by a principal-agent interaction. The finite set of states of the world is denoted Ω, with typical element ω. The information structure, or *procedure*, provides public information about the realized state to a group of $N \geqslant 1$ receivers that possibly includes the principal, but not the agent. All players share a common prior[5] $\mu_0 \in \Delta\Omega$ with full support about the state. Based on public information generated by the principal-agent interaction, the receivers form a belief $\mu \in \Delta\Omega$ and play an equilibrium action profile of the continuation game that induces a payoff $v(\mu)$ for the principal. This payoff function summarizes all we need to know about the continuation game to analyze

[3]The other optimal procedure is the symmetric one.

[4]We suppose that the principal's payoff is 1 in case she makes the right decision, and 0 otherwise.

[5]This is for simplicity, as the analysis can be extended to the case of heterogeneous priors with full support using the transformation in Alonso and Câmara (2016) or Laclau and Renou (2016).

the design problem with agency. We assume that $v(\cdot)$ is upper semicontinuous.

Two particular cases, each exhibiting a single receiver, will be of specific interest. In the *information acquisition* case, the single receiver is (or has aligned preferences with) the principal, who is trying to obtain information so as to solve a decision problem. In this case, $v(\cdot)$ is known to be a convex function[6], and the optimal procedure in the absence of agency would be fully informative. In the *persuasion* case, the single receiver is a third party whose interests are misaligned with the principal's, as in Kamenica and Gentzkow (2011).

The Agency Problem. The principal (she) can design a procedure to be implemented by the agent (he). In doing so, she is constrained by an existing *status quo* procedure which generates signals in a finite set S (with typical element s) according to conditional distributions given by $\varphi(s|\omega)$. Let $\phi(s) = \sum_{\omega} \mu_0(\omega)\varphi(s|\omega)$ be the probability of signal s under the status quo procedure, and $\underline{\phi} := \min_s \ \phi(s)$. We take S to be the support of $\phi(\cdot)$, thus $\underline{\phi} > 0$. This procedure could be informative or not. We assume that the principal is constrained to choose a procedure that uses the same language as the status quo, that is the same set of signals S.[7] Hence, the principal chooses a procedure $\{\psi(s|\omega)\}_{(s,\omega)\in S\times\Omega}$ that consists of signal distributions conditional on the state of the world. We assume a rich language in the sense that $|S| \geqslant |\Omega|+1$; however, with a single receiver, $|A|$ signals will be sufficient, where A denotes the set of actions which the receiver chooses from.[8]

The agent then chooses whether to use the status quo or the new procedure. To keep things simple, we assume that the relative cost of using the new procedure, $c > 0$, is independent of the nature of the two procedures[9]. The agent's choice of procedure is not observable or not contractible by the principal, giving rise to moral hazard. To solve this problem, the principal can provide the agent with incentives through a signal contingent payment scheme $t : S \to \mathbb{R}_+$, which incorporates limited liability of the agent.[10] Hence, we assume that signals themselves are verifiable and contractible, but procedures are not.

[6]In this case, we can write $v(\mu) = \max_a \ \sum_{\omega} \mu(\omega)u(a,\omega)$, where a denotes the choice of the principal, and $u(a,\omega)$ her conditional payoff.

[7]Additional language constraints are considered in Section 5.2.

[8]See Proposition 9.

[9]There are many ways in which the switching cost could depend on the status quo and proposed procedures. In section 5, we consider the case where the cost of a procedure is given by an expected uncertainty reduction measure as in Gentzkow and Kamenica (2014).

[10]Limited liability is key to our main trade-off. It is possible to show that without it, any procedure ψ inducing a signal distribution different than ϕ is such that a payment scheme t exists ensuring that (IC) holds, the agent's expected payoff is 0, and the principal's expected cost is c.

The timing of the game is as follows. First, the principal designs a procedure $\{\psi(s|\omega)\}_{(s,\omega)\in S\times\Omega}$ and a payment scheme $\{t(s)\}_{s\in S}$. Second, the state of the world is realized, but unobserved by any of the players. Third, the agent selects the status quo or the new procedure, a signal is generated according to the selected procedure and publicly revealed. Fourth, receivers play the continuation game after having observed the contract offered by the principal (so they can infer the procedure the agent must have chosen), and the informative signal generated by the procedure. The principal and the agent are risk-neutral, and the equilibrium concept is subgame perfect equilibrium.

The Program of the Principal. The principal can choose to either implement the optimal procedure that is incentive compatible for the agent, or stick to the status quo. As usual in moral hazard problems, the core of the analysis consists in finding an optimal incentive compatible procedure, that is, in solving the following problem,

$$\max_{\psi,t} \sum_{\omega,s} \mu_0(\omega)\psi(s|\omega)\{v(\mu(s;\psi)) - t(s)\} \tag{P0}$$

$$\text{s.t.} \sum_{\omega,s} \mu_0(\omega)\psi(s|\omega)t(s) - c \geqslant \sum_{s} \phi(s)t(s), \tag{IC}$$

where $\mu(s;\psi) \in \Delta\Omega$ is the Bayes-updated belief[11] of receivers about ω after observing signal s, and knowing that it was generated according to the procedure ψ. One can then compare the value function of this problem to the value of the status quo for the principal (with zero payments). In most of the paper, our focus is on solving (P0). We will only be concerned about the choice between the optimal incentive compatible procedure and the status quo for applications.

Benchmark: No Agency. In the absence of agency (say if $c = 0$), our problem is exactly that of Kamenica and Gentzkow (2011). As a benchmark, and in order to introduce some useful notations, we recall their main results. In this case, they show that: (i) one can focus on the distribution of beliefs $\tau \in \Delta\Delta\Omega$ generated by a procedure; (ii) these belief distributions are *splittings* of μ_0: they satisfy the Bayes plausibility condition $\sum_{\mu\in\text{supp}(\tau)} \tau(\mu)\mu = \mu_0$ (Aumann, Maschler and Stearns, 1995; Kamenica and Gentzkow, 2011); (iii) the optimal splittings

[11] $\mu(s;\psi) = \frac{\mu_0 \otimes \psi(s|\cdot)}{\langle \mu_0 \otimes \psi(s|\cdot), \mathbb{1}\rangle}$, where $\otimes$ is the Hadamard vector product, $\langle , \rangle$ is the scalar product, and $\mathbb{1}$ is an $|\Omega|$-dimensional vector with one on every dimension.

concavify $v(\cdot)$ at μ_0, that is, defining $\hat{v}(\cdot)$ as the concavification[12] of $v(\cdot)$, the value function of the principal is $\hat{v}(\mu_0)$; (iv) there exists an optimal (v-concavifying) splitting τ such that $|\operatorname{supp}(\tau)| \leqslant |\Omega|$.

We will use the notation $T(\mu)$ for the set of splittings of μ, and $T_v(\mu)$ for the set of v-concavifying splittings of μ supported on less than $|\Omega|$ beliefs, that is,

$$T_v(\mu) := \Big\{\tau \in T(\mu) : |\operatorname{supp}(\tau)| \leqslant |\Omega|, \sum_{\mu' \in \operatorname{supp}(\tau)} \tau(\mu')v(\mu') = \hat{v}(\mu)\Big\}.$$

Point (iv) right above ensures that $T_v(\mu)$ is always non-empty, irrespective of μ.

3 Analysis

3.1 Main Characterization in the General Case

Our main general theorem reduces the problem of the principal to the choice of a splitting of μ_0 and relates this choice to the solution of Kamenica and Gentzkow (2011). We show that the problem of the principal is entirely captured by the following simple program:

$$\max_{\substack{p \in (\underline{\phi}, 1] \\ \mu^\dagger, \hat{\mu} \in \Delta\Omega}} \quad pv(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu}) - \tilde{\gamma}(p) \tag{P}$$

$$\text{s.t.} \quad p\mu^\dagger + (1-p)\hat{\mu} = \mu_0, \tag{BP}$$

where

$$\tilde{\gamma}(p) := c + \frac{c\underline{\phi}}{p - \underline{\phi}}.$$

The optimal splitting of the principal consists of a binary splitting between a payment belief $\mu^\dagger$ at which the agent is paid, and a resplitting belief $\hat{\mu}$ at which beliefs are optimally resplit according to some splitting in $T_v(\hat{\mu})$.

The next lemma records some basic properties satisfied by any solution to (P).

Lemma 1. *A solution to* (P) *exists. Moreover, if* $(p, \mu^\dagger, \hat{\mu})$ *solves* (P) *and* $p < 1$*, then,* $\forall\, \alpha \in T_v(\hat{\mu})$*:*

1. $\mu^\dagger \notin \operatorname{supp}(\alpha)$*;*

[12]That is the smallest concave function $\hat{v}(\cdot)$ such that $\hat{v}(\mu) \geqslant v(\mu)$ for all $\mu \in \Delta\Omega$.

2. $p \geqslant (1-p)\alpha(\mu)$, $\forall\ \mu \in \text{supp}(\alpha)$.

Henceforth, say that a splitting τ of μ_0 solves program (P) if $\tau(\mu^\dagger) = p$ while $\tau(\mu) = (1-p)\alpha(\mu)$ for all $\mu \in \text{supp}(\alpha)$, where $(p, \mu^\dagger, \hat{\mu})$ solves (P) and $\alpha \in T_v(\hat{\mu})$. By Lemma 1,

$$\tau \text{ solves (P)} \quad \Longrightarrow \quad \tau(\mu^\dagger) = p = \arg \max_{\mu \in \text{supp}(\tau)} \tau(\mu). \tag{1}$$

We can now state the section's main result.

Theorem 1. *Let τ denote a splitting of μ_0 solving* (P)*, and $\{s_\mu\}_{\mu \in \text{supp}(\tau)}$ a collection of $|\text{supp}(\tau)|$ signals from S satisfying $s_{\mu^\dagger} = \underline{s}$, where $\phi(\underline{s}) = \underline{\phi}$. Then the following procedure and payment scheme solve* (P0)*:*

(i) $t(\underline{s}) = \frac{c}{p - \underline{\phi}}$ *while* $t(s) = 0$ *for all* $s \in S \setminus \{\underline{s}\}$*;*

(ii) $\psi(s_\mu|\omega) = \tau(\mu)\frac{\mu(\omega)}{\mu_0(\omega)}$, $\quad \forall \mu \in \text{supp}(\tau), \forall \omega \in \Omega$.

In view of property (1), the theorem thus yields the following 3-steps approach for solving (P0):

1. find a splitting τ of μ_0 solving (P);
2. derive a procedure ψ generating beliefs distributed according to τ such that the most likely signal under ψ is also least likely under ϕ;
3. pay the agent exclusively upon the realization of the signal in step 2.

We provide the essential steps of the proof of this theorem, as well as additional characterization results in Subsection 3.2. The simple intuition behind steps 2 and 3 lies in the idea that, since the agent is risk-neutral, maximizing the likelihood of the new procedure at one signal also minimizes the agency cost. The principal therefore assigns the least likely signal under ϕ to the most likely signal under ψ, which, combining (1) and Theorem 1, occurs with probability $\sum_\omega \mu_0(\omega)\psi(s_{\mu^\dagger}|\omega) = \sum_\omega \tau(\mu^\dagger)\mu^\dagger(\omega) = \tau(\mu^\dagger) = p$.

Next, to understand step 1, consider the problem of finding the best possible splitting of μ_0 subject to the constraint that one belief is generated with probability at least equal to p. This problem's value function, $I(p)$, henceforth referred to as the principal's *informational* *payoff* function, satisfies

$$I(p) = \max_{\mu^\dagger, \hat{\mu} \in \Delta\Omega} \quad pv(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu})$$
$$\text{s.t.} \quad p\mu^\dagger + (1-p)\hat{\mu} = \mu_0.$$

The optimal p then maximizes $I(p) - \tilde{\gamma}(p)$. The function $\tilde{\gamma}(\cdot)$ is an agency cost function proportional to c. One shows that the informational payoff function $I(\cdot)$ is non-increasing and satisfies

$$I(p) = \hat{v}(\mu_0) \quad \iff \quad p - p_{KG} \leqslant 0,$$

where p_{KG} denotes the maximum probability among all splittings in $T_v(\mu_0)$.[13] Intuitively, $p - p_{KG}$ measures the new procedure's distortion relative to Kamenica and Gentzkow (2011). Increasing p reduces the agency cost function $\tilde{\gamma}(p)$, but lowers the principal's informational payoff. The optimal choice of p balances the conflicting objectives of maximizing the informational payoff while minimizing the agency cost. The next proposition shows that the optimal p is monotone in c.

Proposition 1. *Let $p(c)$ such that, for every $c > 0$, a tuple $\big(p(c), \mu^\dagger, \hat{\mu}\big)$ solves* (P) *at c. Then $p(c)$ is non-decreasing in c, and there exists $\bar{c}$ such that for $c \geqslant \bar{c}$, $p(c) = 1$, and the uninformative procedure is optimal.*

As the switching cost c increases, the principal sacrifices a larger chunk of informational payoff to incentivize the agent to use the new procedure. In Example 1 larger values of c induce the principal to design less informative procedures. As the next example demonstrates, larger values of c may in fact induce the principal to design *more* informative procedures.

Example 2. *Consider the following variant on Example 1. As in that example, the receiver is the ministry of transport, deciding whether to build a public transportation infrastructure. But the principal hiring the consultant is now a local government, with a vested interest in building the infrastructure, irrespective of the state of the world. Abusing notation slightly, in this binary example beliefs will be represented by the probability attached to ω_b. We assume that $\mu_0 \in (0, \frac{1}{4})$. As long as the consultant's report results in $\mu \geqslant 1/2$, the ministry chooses action a_b (build). We also assume that the infrastructure is more valuable to the local government in*

[13] In particular, $I(1) < \hat{v}(\mu_0)$ as long as $v(\mu_0) < \hat{v}(\mu_0)$.

state ω_b than in state ω_n, resulting in the payoff function

$$v(\mu) = \begin{cases} 0 & \text{if } \mu \in [0, \frac{1}{2}) \\ \frac{1}{2} + \eta(\mu - \frac{1}{2}) & \text{if } \mu \in [\frac{1}{2}, 1], \end{cases}$$

where $\eta \in (0,1)$. In the absence of an agency problem, the local government would commission a study splitting μ_0 on 0 and 1/2. We next solve for the optimal strategy when $c > 0$.

We show first that any solution to (P) *satisfies either $p = 1$ or $\hat{\mu} \geqslant \frac{1}{2}$ (call this Claim 1). The details of the proof are in the appendix. The idea is as follows. One shows that any solution to* (P) *with $\mu^\dagger > \mu_0$ must satisfy $\mu^\dagger = \frac{1}{2}$ and $\hat{\mu} = 0$. Yet, for $\mu_0 \in (0, \frac{1}{4})$, choosing $\mu^\dagger = 0$ and $\hat{\mu} = \frac{1}{2}$ induces the same informational payoff as $\mu^\dagger = \frac{1}{2}$ and $\hat{\mu} = 0$ while reducing the agency cost. Therefore any solution to* (P) *must be such that $\mu^\dagger \leqslant \mu_0$. Next, by* (BP)*, any solution such that $p < 1$ must have $\hat{\mu} > \mu_0$. But $\mu^\dagger \leqslant \mu_0$ implies $v(\mu^\dagger) = 0$, while $\hat{v}(\hat{\mu}) = \hat{\mu}$ on the interval $[0, \frac{1}{2}]$. So any tuple $(p, \mu^\dagger, \hat{\mu})$ with $\hat{\mu} \in (\mu_0, \frac{1}{2})$ is dominated by $(p, 0, \frac{1}{2})$.*

Claim 1 enables us to rewrite (P) *as*

$$\max_{\substack{p \in (\underline{\phi}, 1] \\ \mu^\dagger, \hat{\mu} \in \Delta\Omega}} \quad (1-p)\Big(\frac{1}{2} + \eta(\hat{\mu} - \frac{1}{2})\Big) - \tilde{\gamma}(p)$$
$$s.t. \quad p\mu^\dagger + (1-p)\hat{\mu} = \mu_0.$$

Next, let $\Lambda(p) := \{(\mu^\dagger, \hat{\mu}) : p\mu^\dagger + (1-p)\hat{\mu} = \mu_0, \hat{\mu} \geqslant \frac{1}{2}\}$ and $\hat{\mu}(p) := \max\{\hat{\mu} : (\mu^\dagger, \hat{\mu}) \in \Lambda(p)\}$, that is,

$$\hat{\mu}(p) = \begin{cases} 1 & \text{if } p \in [1-\mu_0, 1]; \\ \frac{\mu_0}{1-p} & \text{if } p \in [1-2\mu_0, 1-\mu_0]. \end{cases}$$

The maximand above being increasing in $\hat{\mu}$, we may rewrite the program as

$$\max_{p \in (\underline{\phi}, 1]} \quad (1-p)\Big(\frac{1}{2} + \eta(\hat{\mu}(p) - \frac{1}{2})\Big) - \tilde{\gamma}(p),$$

or, substituting for $\hat{\mu}(p)$,

$$\max_{p \in (\underline{\phi}, 1]} \quad I(p) - \tilde{\gamma}(p),$$

where

$$I(p) = \begin{cases} \frac{1}{2}(1-p)(1+\eta) & \text{if } p \in [1-\mu_0, 1]; \\ \eta\mu_0 + \frac{1}{2}(1-p)(1-\eta) & \text{if } p \in [1-2\mu_0, 1-\mu_0]. \end{cases}$$

This function being concave and $\tilde{\gamma}(p)$ strictly convex, the program therefore admits a unique solution, denoted $p(c)$.

We illustrate in Figure 1 the optimal beliefs $\mu^\dagger$ and $\hat{\mu}$, as well as probability p, as a function of the cost c. Notice that the principal's optimal procedure becomes more informative (in the sense of Blackwell) as c increases from c_1 to c_2 but becomes less informative as c goes from c_3 to c_4.

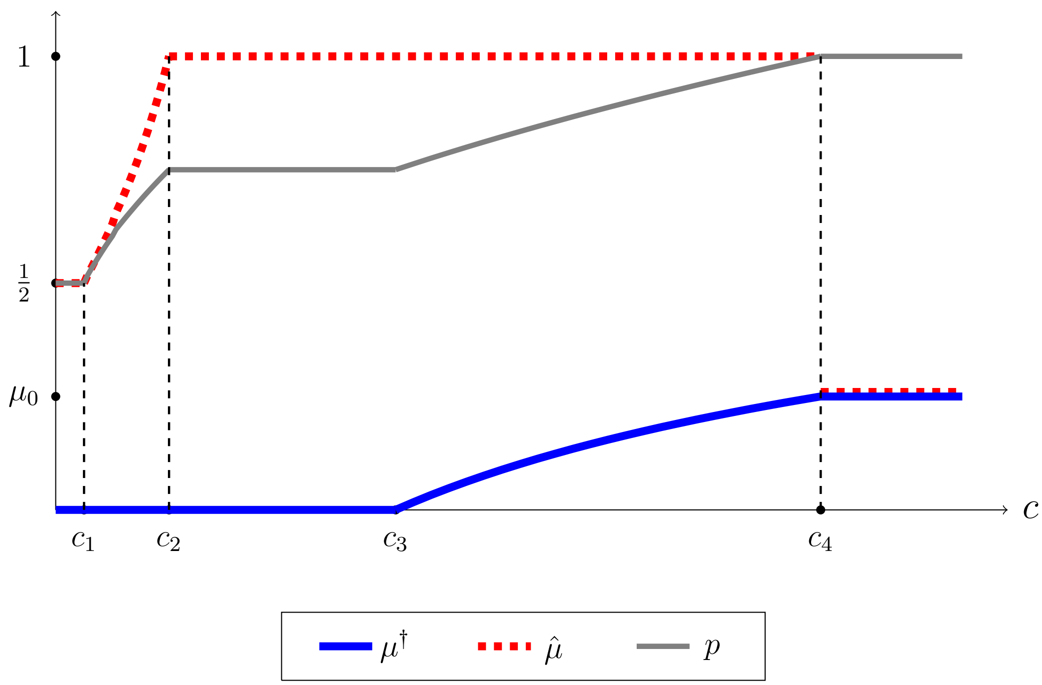


Figure 1: Example 2

Since the principal may have conflicting interests with those of the receivers (as in the previous example), she may find it optimal, even without agency, to be completely uninformative about the state of the world. A direct corollary of Theorem 1 is that, in this case, an uninformative procedure is optimal.

Proposition 2. *If* $v(\mu_0) = \hat{v}(\mu_0)$*, then the uninformative procedure is optimal. Its agency cost is given by* $c + \frac{c\underline{\phi}}{1-\underline{\phi}}$.

Lastly, we noted earlier that the agency cost function is increasing in the switching cost. When c is large, the principal may prefer the status quo to any new procedure. An example of this kind is discussed in Section XX.

3.2 Reducing the Problem

In this subsection, we delineate the steps that lead to the proof of Theorem 1; readers less interested in the technicalities can skip to the next section. We start the analysis by showing that the problem of the principal can be reduced to the problem of choosing a splitting $\tau \in T(\mu_0)$, as in Aumann *et al.* (1995) and Kamenica and Gentzkow (2011). Naturally, this belief distribution must have a finite support of size at most $|S|$. We denote by $\Delta_{|S|}\Delta\Omega$ the set of belief distributions that satisfy this requirement. Furthermore, each of these distributions is associated with an agency cost $\gamma(\tau)$ that the principal must pay to the agent as information rents so as to implement the corresponding procedure.

We first show that the principal cannot gain by duplicating beliefs. While it is clear that no informational loss results from merging signals which give rise to the same belief, the proof proceeds by showing that transfers can be rearranged so as to leave the expected payment to the agent in case of compliance unchanged, while lowering the agent's payoff if he chooses to stick to the status quo.

Lemma 2. *If* (P0) *admits a solution, then it also admits a solution* (ψ, t) *such that for every* $s \neq s'$, $\mu(s;\psi) \neq \mu(s';\psi)$.

As a consequence, we focus on procedures such that signals generated with positive probability induce distinct beliefs. Such a procedure can be fully described by the combination of a Bayes plausible belief distribution τ in $\Delta_{|S|}\Delta\Omega$, and an injective assignment function $\sigma : \text{supp}(\tau) \to S$ that assigns a unique signal in S to each belief induced by τ. This reduces the program of the principal as follows:

$$\max_{\substack{\tau\in\Delta_{|S|}\Delta\Omega\\ \sigma:\mathrm{supp}(\tau)\to S\\ t:S\to\mathbb{R}_+}} \sum_{\mu\in\mathrm{supp}(\tau)} \tau(\mu)\big\{v(\mu) - t\big(\sigma(\mu)\big)\big\} \tag{P1}$$

$$\text{s.t.} \sum_{\mu\in\mathrm{supp}(\tau)} \tau(\mu)\mu = \mu_0 \tag{BP1}$$

$$\sum_{\mu\in\mathrm{supp}(\tau)} t\big(\sigma(\mu)\big)\big\{\tau(\mu) - \phi(\sigma(\mu))\big\} \geqslant c. \tag{IC1}$$

From a triple (τ, σ, t), we can recover an equivalent procedure and transfer pair $(\psi_{\tau,\sigma}, t)$ by letting $\psi_{\tau,\sigma}\big(\sigma(\mu)|\omega\big) = \frac{\mu(\omega)}{\mu_0(\omega)}\tau(\mu)$, for all $\mu \in \mathrm{supp}(\tau)$, and $\psi_{\tau,\sigma}(s|\omega) = 0$, for all $s \in S \setminus \sigma\big(\mathrm{supp}(\tau)\big)$.

Proposition 3. *If* (τ, σ, t) *solves* (P1)*, then the pair* $(\psi_{\tau,\sigma}, t)$ *solves* (P0)*.*

Next we consider the problem of minimizing the agency cost the principal needs to pay to ensure incentive compatibility of a new procedure with associated splitting $\tau \in \Delta_{|S|}\Delta\Omega$. This corresponds to the following cost minimization problem:

$$\gamma(\tau) := \min_{\substack{\sigma:\mathrm{supp}(\tau)\to S\\ t:S\to\mathbb{R}_+}} \sum_{\mu\in\mathrm{supp}(\tau)} \tau(\mu)t\big(\sigma(\mu)\big) \tag{CM$_\tau$}$$

$$\text{s.t.} \sum_{\mu\in\mathrm{supp}(\tau)} t\big(\sigma(\mu)\big)\big\{\tau(\mu) - \phi(\sigma(\mu))\big\} \geqslant c.$$

The value function $\gamma(\tau)$ of this problem is the agency cost corresponding to the splitting τ. This program is easy to solve. Fixing $\sigma(\cdot)$, we have a linear program in $t\big(\sigma(\cdot)\big)$, and we can show that the incentive constraint must bind. Hence, together with the positivity constraints, the binding incentive constraint defines a convex and compact polytope. By the extreme point theorem, this implies that the minimal expected payment can be obtained by paying the agent a positive amount for a single belief realization $\mu^\dagger$, the *payment belief*, and nothing otherwise. The binding constraint implies

$$t\big(\sigma(\mu^\dagger)\big) = \frac{c}{\tau(\mu^\dagger) - \phi\big(\sigma(\mu^\dagger)\big)},$$

and the expected payment by the principal is

$$\frac{\tau(\mu^\dagger)c}{\tau(\mu^\dagger) - \phi\big(\sigma(\mu^\dagger)\big)}.$$

To minimize this expected payment, it is optimal to choose $\mu^\dagger$ to be the most likely belief under τ, and to assign it to the least likely signal under ϕ. To summarize, we have obtained the following solution of the cost minimization problem.

Proposition 4. *The value function of* (CM_τ) *is given by*

$$\gamma(\tau) = c + \frac{c\underline{\phi}}{\overline{\tau} - \underline{\phi}},$$

where

$$\overline{\tau} = \max_{\mu \in \mathrm{supp}(\tau)} \tau(\mu).$$

It is obtained by choosing to pay the agent only if the most likely belief under τ is realized, and by assigning this belief to the least likely signal under ϕ.

The principal wants to pay the agent only when the relative likelihood that the agent has used her proposed procedure rather than the status quo is at the highest. The expected payment by the principal is decreasing in this relative likelihood which is maximized by pairing the most likely outcome under the new procedure with the least likely outcome under the status quo. Note that other belief-signal pairings are inconsequential for the principal. Note also that when ϕ is uniform, the cost of implementing a procedure inducing uniformly distributed beliefs is infinite. Naturally, we can reformulate the original program as follows

$$V(\mu_0) := \max_{\tau \in \Delta_{|S|}\Delta\Omega} \sum_{\mu \in \mathrm{supp}(\tau)} \tau(\mu)v(\mu) - \gamma(\tau) \tag{P2}$$

$$\text{s.t.} \sum_{\mu \in \mathrm{supp}(\tau)} \tau(\mu)\mu = \mu_0, \tag{BP2}$$

yielding the following theorem as an immediate corollary of Proposition 4.

Theorem 2. *The plan* (τ, σ, t) *solves* (P1) *if and only if* τ *solves* (P2)*, and* (σ, t) *solves the cost minimization problem* (CM_τ)*. Furthermore, the value function of the principal in* (P0) *is given by* $V(\mu_0)$*.*

The problem of the principal can therefore be reformulated as a concavification problem in which different splittings of the prior carry an agency cost. The agency cost function is unlike any cost function encountered in the literature on information design with costs (e.g. Gentzkow and Kamenica, 2014). In particular, this program cannot be reformulated as a pure concavification problem, since $\gamma(\tau)$ is not linear in τ. Furthermore, choosing a more informative procedure may either increase or decrease its cost.[14]

The problem can be further reduced to program (P) which highlights the relationship between our problem and the usual information design problem without agency. We know from Proposition 4 that, given any splitting τ, we can restrict attention to payment schemes that reward the agent at a single belief realization $\mu^\dagger \in \text{supp}(\tau)$ which must be the most likely realization of τ. Let p be the probability of this payment belief, and define

$$\hat{\mu} := \sum_{\mu \in \text{supp}(\tau) \setminus \{\mu^\dagger\}} \frac{\tau(\mu)}{1-p} \mu,$$

be the average belief generated by τ conditional on not hitting $\mu^\dagger$. Next, pick some $\alpha \in T_v(\hat{\mu})$, and consider the splitting τ' of μ_0 that puts probability p on $\mu^\dagger$, and probability $(1-p)\alpha(\mu)$ on each $\mu \in \text{supp}(\alpha)$. The language richness assumption implies that $|\text{supp}(\tau')| \leqslant |S|$. By definition of α,

$$\begin{aligned} pv(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu}) &= pv(\mu^\dagger) + (1-p) \sum_{\mu \in \text{supp}(\alpha)} \alpha(\mu) v(\mu) \\ &\geqslant pv(\mu^\dagger) + (1-p) \sum_{\mu \in \text{supp}(\tau) \setminus \{\mu^\dagger\}} \tau(\mu) v(\mu), \end{aligned}$$

so the principal gets a higher informational payoff from τ' than from τ. Furthermore, $\overline{\tau}' \geqslant \overline{\tau}$, so the agency cost of τ' is lower than the agency cost of τ. Hence to find optimal procedures we can focus on splittings of μ_0 across two beliefs, a paying belief $\mu^\dagger$ and a resplitting belief $\hat{\mu}$, and conditional on reaching $\hat{\mu}$, resplit $\hat{\mu}$ as optimal in the pure information design problem so as to concavify v at $\hat{\mu}$. This is exactly the content of Theorem 1.

[14] Suppose $\Omega = \{0, 1\}$, and $\mu_0 = 1/2$. Consider the family of information structures τ_ε that split μ_0 between $1/4$ and $3/4 + \varepsilon$, for $\varepsilon \in [-1/4, 1/4]$. Then the informativeness of τ_ε is increasing in ε, yet $\gamma(\tau_\varepsilon)$ is increasing in ε on $[-1/4, 0)$ and decreasing on $(0, 1/4]$.

4 The Information Acquisition Case

In the information acquisition case, the principal is also the single receiver. She seeks to acquire information about the state of the world ω to optimally choose an action a from a set A, given a utility function $u(a, \omega)$. In this case the belief-contingent payoff function v can be written as[15]

$$v(\mu) = \max_a \sum_\omega \mu(\omega) u(a, \omega), \tag{2}$$

which is a convex and therefore continuous function on $\Delta\Omega$. As we now show, the convexity of the payoff function v simplifies the planner's problem (P). Since v is convex, $\hat{v}(\hat{\mu}) = \sum_\omega \hat{\mu}(\omega) v(\delta_\omega)$, where δ_ω denotes the probability distribution attaching probability 1 to state ω. Let a_ω denote a payoff maximizing action in state ω. Then,

$$\hat{v}(\hat{\mu}) = \sum_\omega \hat{\mu}(\omega) u(a_\omega, \omega). \tag{3}$$

We first establish that, with v convex, we can restrict attention to splittings of the prior such that either $\mu^\dagger$, $\hat{\mu}$ or both lie on the boundary of $\Delta\Omega$.

Proposition 5. *Either an uninformative procedure is optimal, or a solution to* (P) *exists with* $\{\mu^\dagger, \hat{\mu}\} \cap \partial\Delta\Omega \neq \varnothing$.

We show next that the principal's problem in fact simplifies beyond Proposition 5. Combining (2) and (3), we can rewrite (P) as

$$\max_{\substack{p \in (\underline{\phi}, 1] \\ \mu^\dagger \in \Delta\Omega, a^\dagger \in A}} p \sum_\omega \mu^\dagger(\omega) u(a^\dagger, \omega) + (1-p) \sum_\omega \frac{\mu_0(\omega) - p\mu^\dagger(\omega)}{1-p} u(a_\omega, \omega) - \tilde{\gamma}(p).$$

Intuitively, the maximization over $a^\dagger$ represents the principal's choice of action when receiving the signal that corresponds to the payment belief $\mu^\dagger$. Introducing $x = p\mu^\dagger$, the latter program

[15] We assume existence of this value function, for example by compactness of A and continuity of $u(\cdot, \omega)$.

can now be rewritten as

$$\max_{\substack{p\in(\underline{\phi},1]\\ x\geqslant 0, a^\dagger\in A}} \sum_\omega x(\omega)u(a^\dagger,\omega)+\sum_\omega \big(\mu_0(\omega)-x(\omega)\big)u(a_\omega,\omega)-\tilde{\gamma}(p),$$

$$\text{s.t.}\quad \begin{cases}\sum_\omega x(\omega)=p;\\ x\leqslant \mu_0.\end{cases}$$

Rearranging the maximand above then yields

$$\max_{\substack{p\in(\underline{\phi},1]\\ x\geqslant 0, a^\dagger\in A}} \sum_\omega x(\omega)\big[u(a^\dagger,\omega)-u(a_\omega,\omega)\big]-\tilde{\gamma}(p)+\hat{v}(\mu_0),$$

$$\text{s.t.}\quad \begin{cases}\sum_\omega x(\omega)=p;\\ x\leqslant \mu_0.\end{cases}$$

This last program is linear in the variable x. Defining $\ell(a^\dagger,\omega):=u(a_\omega,\omega)-u(a^\dagger,\omega)$, its dual formulation is[16]

$$\max_{\substack{p\in(\underline{\phi},1]\\ a^\dagger\in A}} \min_\lambda \sum_\omega \mu_0(\omega)\big[\lambda-\ell(a^\dagger,\omega)\big]^+ - p\lambda-\tilde{\gamma}(p)+\hat{v}(\mu_0). \tag{4}$$

Now, since $\tilde{\gamma}(p)$ is convex, the maximand in (4) is convex in λ and concave in p. By virtue of the minimax Theorem, we may switch the order of the minimization over λ and the maximization over p without affecting the result.[17] By doing this, we obtain a straightforward optimization problem in p, whose first order condition is $\lambda+\tilde{\gamma}'(p)=0$.[18] Increasing p lowers the agency cost ($\tilde{\gamma}'<0$) but implies an informational loss equal to the shadow price λ, giving $p=\underline{\phi}+\sqrt{\frac{c\underline{\phi}}{\lambda}}$. Substituting for p and ignoring the constant term $\hat{v}(\mu_0)$ then yields the following simple optimization problem:

$$\max_{a^\dagger} \min_\lambda \sum_\omega \mu_0(\omega)\big[\lambda-\ell(a^\dagger,\omega)\big]^+ - \lambda\underline{\phi}-2\sqrt{c\underline{\phi}\lambda}. \tag{5}$$

We sum up our results in the next theorem.

[16] We use the notation $z^+=\max\{z,0\}$. See Lemma 4 in the appendix.
[17] Since $\tilde{\gamma}(\underline{\phi})$ is infinite, $(\underline{\phi},1]$ can be replaced by a compact set without affecting the problem.
[18] The second order condition is trivially satisfied, since $\tilde{\gamma}(\cdot)$ is convex.

Theorem 3. *Let $(a^\dagger, \lambda)$ solve (5). Then $\lambda \geqslant 0$. Moreover, if $\underline{\phi} + \sqrt{c\underline{\phi}/\lambda} \geqslant 1$, an uninformative procedure is optimal. Otherwise, let*

$$p = \underline{\phi} + \sqrt{\frac{c\underline{\phi}}{\lambda}},$$

$$\mu^\dagger = \frac{x}{p},$$

and

$$\hat{\mu} = \frac{1}{1-p}\big(\mu_0 - x\big),$$

where $x(\omega) = 0$ if $\lambda < \ell(a^\dagger, \omega)$ and $x(\omega) = \mu_0(\omega)$ if $\lambda > \ell(a^\dagger, \omega)$. Then $(p, \mu^\dagger, \hat{\mu})$ is a solution to (P)*.*

Example 3. *Consider the following extension of example 1. The ministry of transport must decide whether or not to build a new train line linking the capital city to some regional town. There is uncertainty regarding the growth potential of the region in question. The ministry attaches probability $\mu_0(\omega_1) = \frac{5}{8}$ to the region growing normally, $\mu_0(\omega_2) = \frac{2}{8}$ to the region experiencing an economic and demographic boom, and $\mu_0(\omega_3) = \frac{1}{8}$ to the region failing to develop.*[19] *In state ω_1 the ministry's optimal decision is to build a low speed train line, and in state ω_2 the optimal decision is to build a high speed line. Not building anything is optimal in state ω_3. The ministry's payoffs are summarized by*

$$u(a, \omega) = \begin{bmatrix} 1 & 0 & -8 \\ 0 & 3 & -10.5 \\ 0 & 0 & 0 \end{bmatrix}$$

Thus building a high speed line delivers a large positive payoff in case the region experiences a boom but induces an even larger loss if the region fails to develop.

The optimal procedure is illustrated in Figure 2, in terms of its canonical representation $(p, \mu^\dagger, \hat{\mu})$, as a function of the parameter c. The blue (respectively red) arrows depict $\mu^\dagger$ (resp. $\hat{\mu}$): $\mu^\dagger$ first goes from A to B and then jumps to C before reaching D; $\mu^\dagger$ then jumps to E before ending up at the prior μ_0. The intuition is as follows. For sufficiently small c the optimal procedure is fully informative and induces the ministry to make the correct decision in every single state. Larger c values induce the high speed line to sometimes be dropped in favor

[19] We assume in the narrative that the region's growth potential is independent of the ministry's decision.

of a low speed line (A to B in Figure 2), which initially is more likely to be ex post optimal. At even larger c values the situation is reversed: it is now the low speed line which is sometimes dropped in favor of the high speed line (C to D in Figure 2). Doing so enables the ministry to realize the benefits accruing from building high speed in the event of an economic boom. As c further increases the low speed line is altogether abandoned; the high speed line is sometimes dropped as well in favor of the safer option not to build (E to μ_0 in Figure 2).

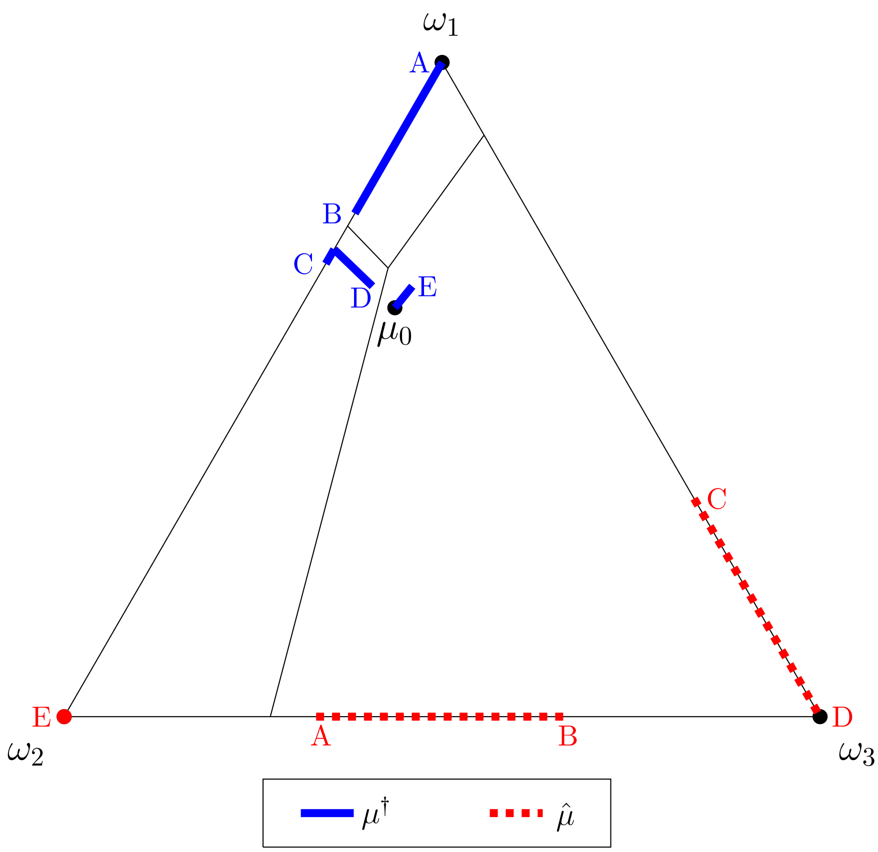


FIGURE 2: EXAMPLE 3

The next lemma provides a simple criterion for comparing the informativeness of optimal procedures. Its proof relies on the convexity of $v(\cdot)$ and is therefore not generalizable beyond the information acquisition case.

Lemma 3. *Consider two procedures, ψ_1 and ψ_2, respectively optimal given switching costs c_1 and c_2. Let $(p_1, \mu_1^\dagger, \hat{\mu}_1)$ and $(p_2, \mu_2^\dagger, \hat{\mu}_2)$ denote their canonical representations. Then $p_2\mu_2^\dagger > p_1\mu_1^\dagger$ if and only if ψ_1 is Blackwell-more-informative than ψ_2.*[20]

When is the principal's optimal procedure Blackwell monotone in c? In what follows say that $\psi(c)$ is ordered according to Blackwell's criterion if $c_2 > c_1$ implies that ψ_1 is (weakly) Blackwell-more-informative than ψ_2. Define for each action $a \in A$ the minimum loss function

$$\mathrm{ML}_a(p) := \min_{x \geqslant 0} \quad \sum_{\omega} x(\omega)\ell(a,\omega) \qquad \text{s.t.} \quad \begin{cases} \sum_{\omega} x(\omega) = p; \\ x \leqslant \mu_0. \end{cases}$$

Let $\mathrm{MLE}(p)$ denote the lower envelope of the minimum loss functions above, that is,

$$\mathrm{MLE}(p) := \min_{a} \mathrm{ML}_a(p).$$

We illustrate in Figure 3 the minimum loss functions corresponding to the last example, as well as their minimum loss envelope.

[20] We use here the standard partial order on $\mathbb{R}^n$, whereby $x > y$ if and only $x_i \geqslant y_i$ for all i with at least one inequality strict.

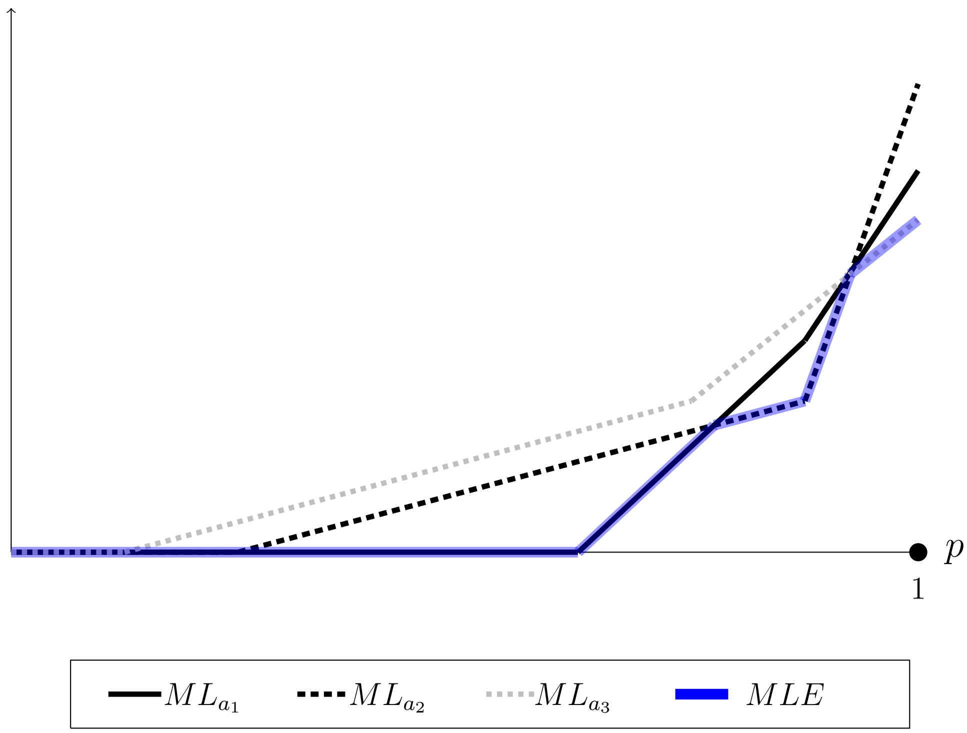


Figure 3: Example 3

Proposition 6. *The following conditions are sufficient for the existence of* $\big(p(c), \mu^\dagger(c), \hat{\mu}(c)\big)$ *solving* (P) *with corresponding procedure* $\psi(c)$ *ordered according to Blackwell's criterion:*

(i) there exists $a_i \in A$ *such that* $a^\dagger(c) = a_i$ *for all* c*;*

(ii) there exists $a_i \in A$ *such that* $MLE(p) = ML_{a_i}(p)$*.*

These conditions are moreover equivalent.

Example 4. *Consider the following example. A company board is looking to recruit a new CEO. The three candidates each have a specialized skill, which may or may not help boost the company's value. We assume that candidate* a_i *creates value* u_i *in state* ω_i *and* 0 *otherwise, where* $\mu_0(\omega_1) > \mu_0(\omega_2) > \mu_0(\omega_3)$ *and* $u_1 > u_2 > u_3$*. Thus, the board's payoffs are given by*

$$u(a, \omega) = \begin{bmatrix} u_1 & 0 & 0 \\ 0 & u_2 & 0 \\ 0 & 0 & u_3 \end{bmatrix}$$

The task of evaluating candidates is left to the company's HR department, which can either stick to the status quo procedure or switch to the board's suggested procedure.

The principal's (i.e. the board, in the context of this example) problem is solved starting with (5)*. Let*

$$g_i(\lambda) := \sum_j \mu_0(\omega_j)\big\{\lambda - \ell(a_i, \omega_j)\big\}^+.$$

A glance at this expression shows that g_i is convex and $g_1 > g_2 > g_3$. Hence $a^\dagger = a_1$, for all values of c. Minimizing $g_1(\lambda) - \lambda\underline{\phi} - \sqrt{c\underline{\phi}\lambda}$ with respect to λ completes the solution to (5)*. Then Theorem 3 yields $(p, \mu^\dagger, \hat{\mu})$ solving* (P)*. We illustrate $\mu^\dagger$ and $\hat{\mu}$ in Figure 4 as a function of the parameter c. The principal's optimal procedure is intuitive. If c is sufficiently small the optimal procedure is such that the company always recruits the most suitable candidate. For larger c values, candidate a_3 is sometimes dropped in favor of a_1. At even larger c values, candidate a_3 is never recruited and a_2 is sometimes dropped in favor of a_1.*

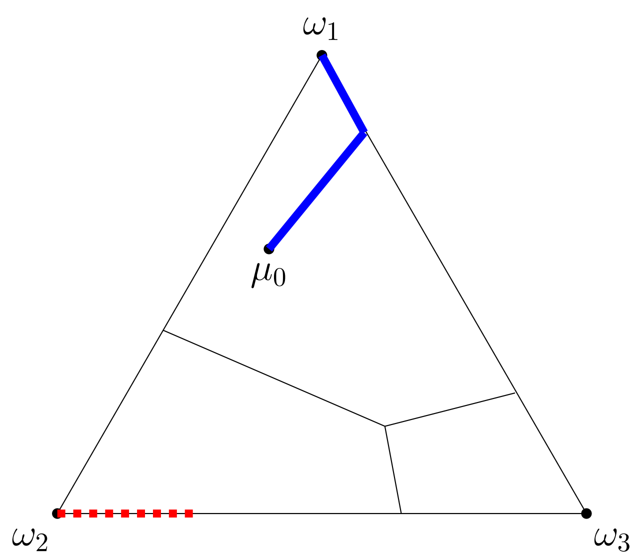


FIGURE 4: EXAMPLE 4

We next show by way of an example that the sufficient conditions uncovered in Proposition 6 are not necessary conditions.

Example 5. *Consider the following example, with three states and three actions:*

$$\mu_0 = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right),$$

and

$$u(a, \omega) = \begin{bmatrix} 12 & 10 & 0 \\ 11 & 12 & 2 \\ 0 & 0 & 10 \end{bmatrix}$$

The minimum loss envelope is illustrated in Figure 5. Clearly, condition (ii) of Proposition 6 is violated. Yet the principal's optimal procedure is Blackwell monotone in c.[21]

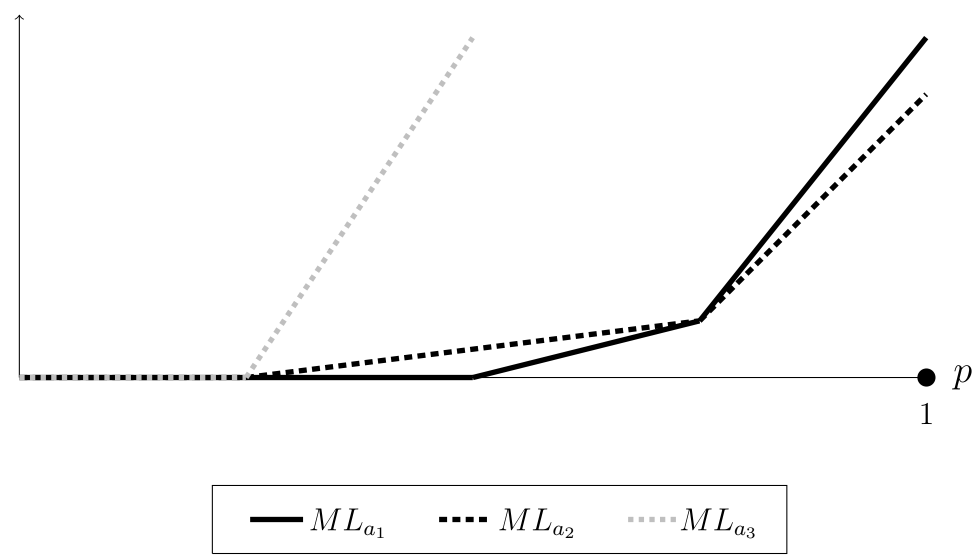


FIGURE 5: EXAMPLE 5

5 Extensions

5.1 Multiple status quo procedures

In this subsection we extend the baseline model by allowing the agent to deviate to one of many status quo procedures. We show that as long as a saddle-point property holds then our

[21] To see this use Lemmata 3 and 5.

main general Theorem easily generalizes. This property captures the condition under which any procedure can be optimally implemented using a single payment signal.

Let K be the set of status quo procedures, with typical element k. The procedure k generates signals according to $\varphi_k(s|\omega)$. We let $\phi_k(s) = \sum_\omega \mu_0(\omega)\varphi_k(s|\omega)$ be the probability of signal s under procedure k, and $\underline{\phi}_k := \min_s \ \phi_k(s)$. The problem of the principal is now

$$\max_{\psi,t} \sum_{\omega,s} \mu_0(\omega)\psi(s|\omega)\{v(\mu(s;\psi)) - t(s)\} \tag{P0}$$

$$\text{s.t.} \sum_{\omega,s} \mu_0(\omega)\psi(s|\omega)t(s) - c \geqslant \sum_s \phi_k(s)t(s), \quad \forall k \in K. \tag{IC$_k$}$$

The next theorem shows that if $\phi_k(s)$ possesses a saddle point then the problem of the principal reduces to finding a solution to (P) with agency cost function $\tilde{\gamma}_k$ replacing $\tilde{\gamma}$, where

$$\tilde{\gamma}_k(p) := c + \frac{c\underline{\phi}_k}{p - \underline{\phi}_k},$$

for some appropriately chosen k.

Theorem 4. *Suppose $\phi_k(s)$ possesses a saddle point (s^*, k^*), that is,*

$$\phi_k(s^*) \leqslant \phi_{k^*}(s^*) \leqslant \phi_{k^*}(s), \quad \forall k \in K, \forall s \in S.$$

Let τ denote a splitting of μ_0 solving (P) *with agency cost function $\tilde{\gamma}_{k^*}$ instead of $\tilde{\gamma}$, and $\{s_\mu\}_{\mu \in \text{supp}(\tau)}$ a collection of $|\text{supp}(\tau)|$ signals from S satisfying $s_{\mu^\dagger} = s^*$. Then the following procedure and payment scheme solve* (P0)*:*

(i) $t(s^*) = \frac{c}{p - \underline{\phi}_{k^*}}$ *while* $t(s) = 0$ *for all* $s \in S \smallsetminus \{s^*\}$*;*

(ii) $\psi(s_\mu|\omega) = \tau(\mu)\frac{\mu(\omega)}{\mu_0(\omega)}, \quad \forall \mu \in \text{supp}(\tau), \forall \omega \in \Omega.$

The logic of the theorem is the following. We found in Theorem 1 that with a single status quo procedure (i) the principal rewards the agent at a single signal $s_{\mu^\dagger}$, (ii) $s_{\mu^\dagger}$ has minimum probability $\underline{\phi}$ under the status quo, (iii) the payment to the agent is an increasing function of $\underline{\phi}$. With multiple status quo procedures, and payment at a single signal, the principal chooses $s_{\mu^\dagger}$ so as to minimize the agent's gain from deviating to one of the status quo procedures, that is, the principal chooses $s_{\mu^\dagger} \in \arg\min_s \max_k \phi_k(s)$. As long as $\phi_k(s)$ possesses a saddle point then $\min_s \max_k \phi_k(s) = \max_k \min_s \phi_k(s)$ and the equilibrium is as if the agent moved first and picked his preferred status quo procedure, that is, as if the single procedure were $k^* \in \arg\max_k \min_s \phi_k(s)$. The principal thus optimally rewards the agent at the single signal $s^* \in \arg\min_s \phi_{k^*}(s)$.

Example 6. *Let φ_a and φ_b be two procedures, and $|S| = n$. We suppose that φ_a generates information about the state, whereas φ_b does not. Furthermore, $\phi_a(s_1) < \cdots < \phi_a(s_n)$ and $\phi_b(s_1) = \cdots = \phi_b(s_n) = \frac{1}{n}$. Each status quo procedure φ_k, $k \in K$, is a convex combination of φ_a and φ_b: $\varphi_k(s|\omega) = q_k\varphi_a(s|\omega) + (1 - q_k)\varphi_b(s|\omega)$. Thus $\phi_k(s) = q_k\phi_a(s) + (1 - q_k)\phi_b(s)$. Let $q_{k^*} = \min_k q_k$. One checks that $\phi_k(s_1) \leqslant \phi_{k^*}(s_1) \leqslant \phi_{k^*}(s)$, $\forall k \in K, \forall s \in S$. Thus $\phi_k(s)$ possesses a saddle point.*

The next proposition shows that the saddle-point property of the previous theorem is in fact (generically) necessary to guarantee optimality of a single payment signal.

Proposition 7. *If $\arg\max_k \phi_k(s)$ contains a single element for all $s \in S$ then the saddle-point property is a necessary condition for all procedures to be optimally implementable using a single payment signal.*

We solve below for the optimal payment scheme in an example where the saddle-point property is violated.

Example 7. *Consider the following example with three signals, $S = \{s_1, s_2, s_3\}$, and two status quo procedures, φ_{k_1} and φ_{k_2}, generating signals in S with probabilities $(\frac{2}{20}, \frac{4}{20}, \frac{14}{20})$ and $(\frac{4}{20}, \frac{1}{20}, \frac{15}{20})$, respectively. Note that $\phi_k(s)$ has no saddle point and that $\arg\max_{k_i} \phi_{k_i}(s)$ contains a single element for all $s \in S$. Assume moreover $c = 1$.*

We look for the optimal payment scheme implementing the procedure ψ generating signals in S with uniform probabilities. As $\max_{k_i} \phi_{k_i}(s_3) > \frac{1}{3}$, the optimum must satisfy $t(s_3) = 0$. Therefore, the cost minimization problem reduces to

$$\min_{t(s_1),t(s_2)\geqslant 0} \quad \frac{1}{3}(t(s_1) + t(s_2))$$

$$\text{s.t.} \quad \sum_{i=1,2} t(s_i)\left(\frac{1}{3} - \phi_{k_1}(s_i)\right) \geqslant 1, \qquad (IC_{k_1})$$

$$\sum_{i=1,2} t(s_i)\left(\frac{1}{3} - \phi_{k_2}(s_i)\right) \geqslant 1. \qquad (IC_{k_2})$$

The set of feasible payments are represented by the gray area in Figure 6. The dashed lines show the principal's indifference curves. Comparing the value function at the three extreme *points of the feasible set (x, y and z) shows that point z with* $t(s_1) = 15/29$ *and* $t(s_2) = 50/29$ *corresponds to the optimal payment scheme. Intuitively, rewarding the agent at a single signal in* $\{s_1, s_2\}$ *enables the agent to save c and still receive full payment with probability* $1/5$*. Spreading rewards reduces the agent's expected payment in case of deviation.*

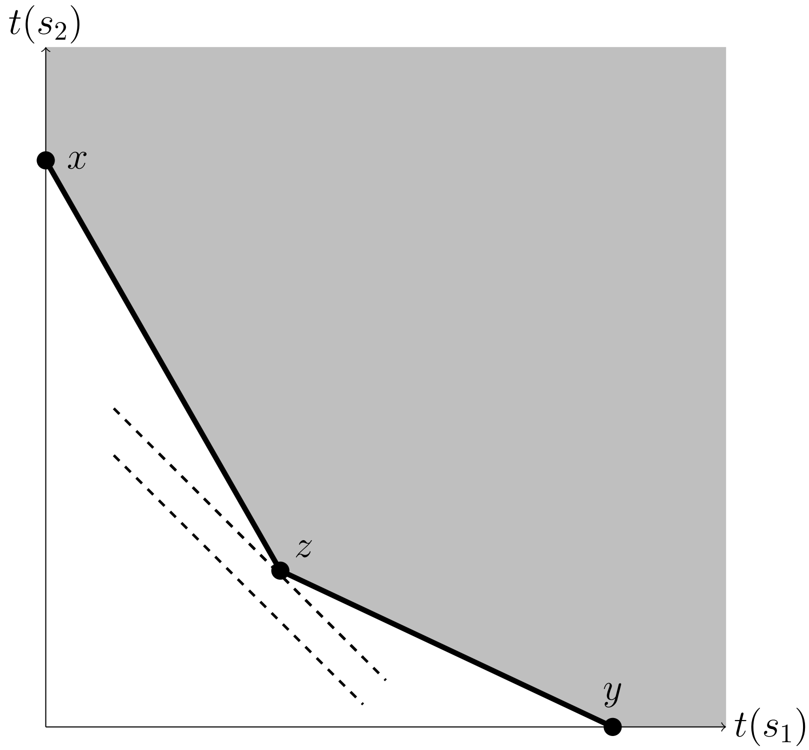


FIGURE 6: EXAMPLE 7

5.2 Language Constraints

The baseline model assumes that the principal is constrained to use the same language as the status quo procedure, that is, the principal must choose a procedure generating signals in $S = \text{supp}(\phi)$. In this section we show that additional language constraints are easily dealt with when, for example, signals correspond to action recommendations to a receiver. In this case, the principal simply solves a family of problems of the kind analyzed in Section 3.

We modify here the baseline setting by assuming that to each signal s corresponds a set $\Delta(s) \subseteq \Delta\Omega$ representing beliefs to which the principal can assign s. We will say that language constraints are regular if the following properties hold:

(i) $\Delta(s)$ is convex for all $s \in S$,

(ii) $\bigcup_s \Delta(s) = \Delta\Omega$,

(iii) the principal's payoff function v is weakly concave on each $\Delta(s)$.

We present in the following example a natural class of problems with regular language constraints.

Example 8. *Consider a setting with a single receiver. For instance, as in Section 4, the single receiver could be the principal. Let A denote the set of actions which the receiver chooses from. The signals corresponds to action recommendations, thus $S = A$. The new procedure is constrained by the requirement that the recommended action be optimal given the beliefs induced by the recommendation. Hence $\Delta(s)$ is the set of beliefs for which action s is optimal for the receiver. One checks that, in this example, language constraints are regular.*

Define the program (P_s) as follows,

$$\max_{\substack{p \in (\phi(s),1] \\ \mu^\dagger \in \Delta(s), \hat{\mu} \in \Delta\Omega}} \quad pv(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu}) - \tilde{\gamma}_s(p) \qquad (\mathrm{P}_s)$$

$$\text{s.t.} \quad p\mu^\dagger + (1-p)\hat{\mu} = \mu_0,$$

where

$$\tilde{\gamma}_s(p) := c + \frac{c\phi(s)}{p - \phi(s)}.$$

Let V_s denote the value function of (P_s), and $s^* \in \arg\max_s V_s$. The next theorem shows that the problem of the principal reduces to solving (P_{s^*}).

Theorem 5. *With regular language constraints, a splitting τ solving (P_{s^*}) exists such that $|\operatorname{supp}(\tau) \cap \Delta(s)| \leqslant 1$ for all $s \in S$. Let $\{s_\mu\}_{\mu \in \operatorname{supp}(\tau)}$ denote a collection of $|\operatorname{supp}(\tau)|$ signals from S containing s^*, and satisfying $\mu \in \Delta(s_\mu)$ for all $\mu \in \operatorname{supp}(\tau)$. Then the following procedure and payment scheme solve* (P0) *with language constraints:*

(i) $t(s^*) = \frac{c}{p - \phi(s^*)}$ *while* $t(s) = 0$ *for all* $s \in S \setminus \{s^*\}$*;*

(ii) $\psi(s_\mu|\omega) = \tau(\mu)\frac{\mu(\omega)}{\mu_0(\omega)}, \quad \forall \mu \in \operatorname{supp}(\tau), \forall \omega \in \Omega.$

The logic is straightforward. One first finds the best possible procedure rewarding the agent exclusively upon the realization of s, for each $s \in S$. In this subproblem the agency cost function is given by $\tilde{\gamma}_s$, and the principal is constrained to pick $\mu^\dagger$ in $\Delta(s)$. Regular language constraints assure that, when resplitting $\hat{\mu}$ according to some splitting $\alpha \in T_v(\hat{\mu})$, the principal is able to assign beliefs in $\mathrm{supp}(\alpha)$ to signals different from s. One then compares across all signals in S and picks the signal giving largest payoff to the principal.

While Theorem 5 shows that our solution method readily extends to incorporate language constraints, the latter may of course drastically affect the chosen procedure of the principal. Intuitively, this will be the case when the agency cost of the unconstrained optimal procedure is substantially increased as a consequence of the language constraints. The next example provides an illustration.

Example 9. *Consider the following modified version of Example 1. The states of the world are n and b, with prior beliefs satisfying $\mu_0(b) < 1/2$. The payoff function of the principal is*

$$v(\mu) = \begin{cases} 1 - 2\mu(b) & \text{if } \mu(b) < 1/2 \\ 2\mu(b) - 1 & \text{if } \mu(b) \geqslant 1/2. \end{cases}$$

The status quo procedure is uninformative, generating signals in $S = \{s_n, s_b\}$ according to $\phi(s_n) = \mu_0(n)$ and $\phi(s_b) = \mu_0(b)$. The language constraints are such that $\Delta(s_n) = \{\mu : \mu(b) \leqslant \frac{1}{2}\}$ and $\Delta(s_b) = \{\mu : \mu(b) \geqslant \frac{1}{2}\}$.

We proceed by solving program (P_{s_n}), followed by (P_{s_b}). To begin with, notice that if $(p, \mu^\dagger, \hat{\mu})$ solves (P_{s_n}) then $\hat{\mu}(b) = 1$. If $\hat{\mu}(b)$ were less than 1 the principal could raise $\hat{\mu}(b)$ and increase p so as to still satisfy Bayes plausibility. The optimal probability p then solves

$$\max_{p \in (\mu_0(n), 1]} \left\{ p(1 - 2\mu^\dagger(b)) + (1 - p) - c - \frac{c\mu_0(n)}{p - \mu_0(n)} \right\},$$

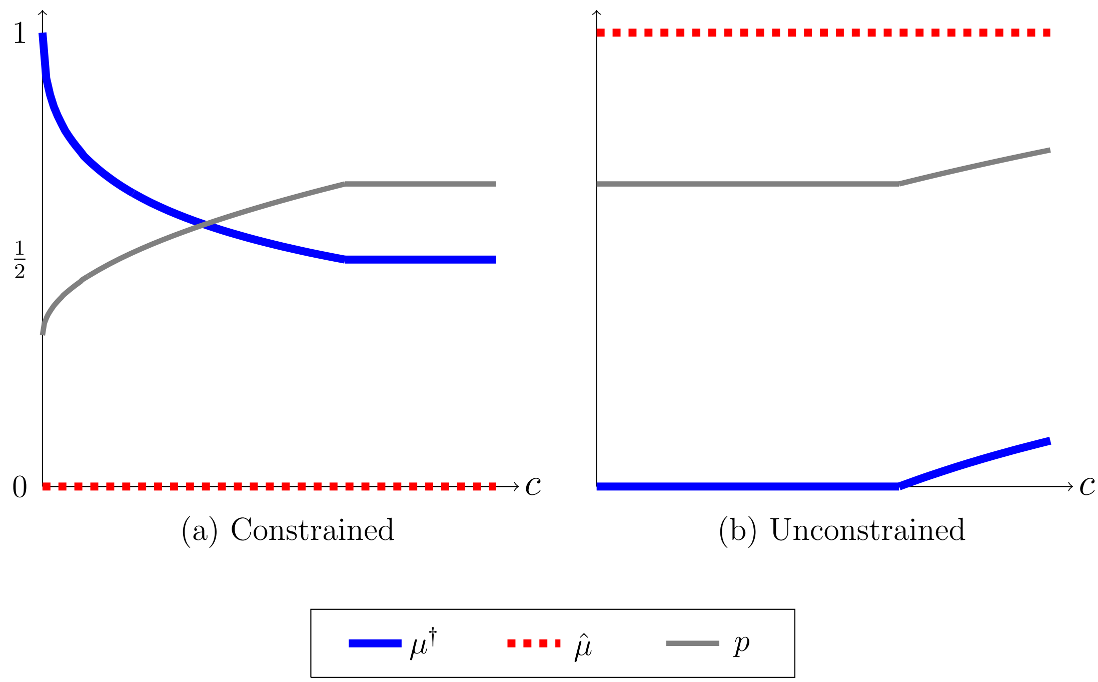
The last two terms represent the agency cost given that the principal rewards the agent upon the realization of s_n; the first three represent the informational payoff $pv(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu})$.

Reasoning as we did above, if $(p, \mu^\dagger, \hat{\mu})$ solves (P_{s_b}) then $\hat{\mu}(b) = 0$, and p solves

$$\max_{p \in (\mu_0(b), 2\mu_0(b)]} \left\{ p(2\mu^\dagger(b) - 1) + (1 - p) - \tilde{\gamma}_{s_b}(p) \right\}.$$

One checks that $V_{s_b} > V_{s_n}$, giving $s^* = s_b$.[22] *Figure 7 compares the optimal procedures with and without language constraints (Panel (a) and Panel (b) respectively) as a function of the switching cost c.*

[22] The intermediate steps can be found in Appendix B.



(a) Constrained (b) Unconstrained

Figure 7: Example 9

We show in the next example that language constraints may overturn Proposition 2, that is, the principal could prefer the new procedure to be informative even though more informative procedures reduce the principal's informational payoff.

Example 10. *The principal is a plant manager. As per an agreement with the unions, the manager hires regularly a certified inspector (the agent) to report on workplace safety. The working conditions could be safe (state ω_1) or unsafe (state ω_0). Prior beliefs satisfy $\mu_0(\omega_1) < 1/2$. For all posterior beliefs such that $\mu(\omega_1) < 1/2$ the company must incur safety-related expenses proportional to the likelihood that the plant is unsafe. The manager aims to minimize expenses, thus*

$$v(\mu) = \begin{cases} 0 & \text{if } \mu(\omega_1) > 1/2 \\ \mu(\omega_1) - 1/2 & \text{if } \mu(\omega_1) \leqslant 1/2. \end{cases}.$$

The status quo procedure is fully informative and generates signals in $S = \{s_0, s_1\}$, with $\phi(s_0) = \mu_0(\omega_0)$ and $\phi(s_1) = \mu_0(\omega_1)$. The language constraints are such that $\Delta(s_0) = \Delta\Omega$, while $\Delta(s_1) = \{\mu : \mu(\omega_1) \geqslant x\}$, for some $x \in [1/2, 1)$ capturing the agent's minimum safety standards in order to report s_1.

We proceed to characterize the principal's optimal procedure, first solving (P_{s_0}) before tackling (P_{s_1}). The solution to (P_{s_0}) is straightforward: $p = 1$ and $\mu^\dagger = \mu_0$, giving

$$V_{s_0} = \mu_0(1) - \frac{1}{2} - \frac{c}{\mu_0(1)}.$$

We next solve (P_{s_1}). To begin with, notice that if $(p, \mu^\dagger, \hat{\mu})$ solves (P_{s_1}) then $\mu^\dagger(\omega_1) = x$. If $\mu^\dagger(\omega_1)$ were greater than x the principal could reduce $\mu^\dagger(\omega_1)$ and increase p so as to still satisfy Bayes plausibility. The optimal probability p then solves

$$\max_{p \in \left(\mu_0(\omega_1), \frac{\mu_0(\omega_1)}{x}\right]} \left\{ \mu_0(\omega_1) - xp - \frac{1}{2}(1-p) - c - \frac{c\mu_0(\omega_1)}{p - \mu_0(\omega_1)} \right\}.$$

The last two terms represent the agency cost given that the principal rewards the agent upon the realization of s_1; the first three represent the informational payoff $pv(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu}) = (1-p)v(\hat{\mu}) = (1-p)v\left(\frac{\mu_0 - p\mu^\dagger}{1-p}\right)$.

Lastly, we compare V_{s_0} and V_{s_1}. By inspection of the maximand above, V_{s_1} is decreasing in x. As V_{s_0} is independent of x, there exists a unique $\tilde{x}$ such that $\arg\max_{s_i} V_{s_i} = s_1$ if $x < \tilde{x}$ and $\arg\max_{s_i} V_{s_i} = s_0$ if $x > \tilde{x}$. It is easy to check that $V_{s_1} > V_{s_0}$ for $x = \frac{1}{2}$. Thus $\tilde{x} > \frac{1}{2}$. In this example the optimal procedure can therefore be informative even though v is concave.

Procedure Dependent Cost Instead of a fixed switching cost, it may be natural to consider a switching cost that depends on the complexity of the procedure designed by the principal. This can be captured by assuming that the cost for the agent of switching to a new procedure is determined by an entropy measure. Here we follow Gentzkow and Kamenica (2014) and assume that the cost of switching to a procedure ψ inducing belief distribution τ_ψ is given by $c + C(\tau_\psi)$, for

$$C(\tau_\psi) = H(\tilde{\mu}) - \sum_{\mu \in \mathrm{supp}(\tau_\psi)} \tau_\psi(\mu) H(m_\mu),$$

where $H : \Delta\Omega \to \mathbb{R}_+$ is strictly concave, $\tilde{\mu}$ is an interior belief, and $m_\mu \in \Delta\Omega$ satisfies:[23]

$$m_\mu(\omega) := \frac{\mu(\omega)\tilde{\mu}(\omega)}{\mu_0(\omega)} \left(\sum_{x \in \Omega} \frac{\mu(x)\tilde{\mu}(x)}{\mu_0(x)} \right)^{-1}.$$

Then we can define the following posterior-dependent agency cost function

$$\Gamma(\mu, p) = H(m_\mu) \left(1 + \frac{\underline{\phi}}{p - \underline{\phi}} \right),$$

and the modified payoff function

$$\Pi(\mu, p) = v(\mu) - \Gamma(\mu, p).$$

Then we can show that the optimal procedure can be obtained as the solution to the program[24]

$$\max_{\substack{p \in (\underline{\phi}, 1] \\ \mu^\dagger, \hat{\mu} \in \Delta\Omega}} \; p\Pi(\mu^\dagger, p) + (1-p)\hat{\Pi}(\hat{\mu}, p) + \gamma_H(p) \qquad (P_\Pi)$$
$$\text{s.t.} \;\; p\mu^\dagger + (1-p)\hat{\mu} = \mu_0,$$

where $\hat{\Pi}(\cdot, p)$ denotes the concavification of $\Pi(\cdot, p)$ and

$$\gamma_H(p) := \left(1 + \frac{\underline{\phi}}{p - \underline{\phi}} \right) (c + H(\tilde{\mu})).$$

5.3 Partially Contractible Signals.

Here we relax our contractability assumption by considering that the principal can only make payments conditional on elements of a partition of the signal space.

Example 11. *The principal is the manager of a company that provides holiday reservation services for movie stars. The set of available destinations is Ω, and the state of the world is the destination chosen by the client. The prior is uniform. The success of the company relies on its discretion. Receivers are paparazzi who seek to find the destination of customers and sell*

[23] As discussed in Gentzkow and Kamenica (2014), if s is a signal that induces posterior μ for an observer with prior μ_0, then the same signal induces posterior m_μ for an observer with prior $\tilde{\mu}$ (see the transformation in Alonso and Câmara (2016) or Laclau and Renou (2016)).

[24] In Appendix C we show that program (P_H) has a solution.

their pictures. They move all together and go to the destination they believe to be most likely. Currently, the company's employees communicate through an insecure channel which means that the paparazzi learn the actual location with probability $1/2$, *and otherwise receive a null signal* $\varnothing$ *that is uninformative. We assume that the manager can only reward her employees conditional on whether the paparazzi go to the right location or not, but cannot condition on the actual location.*[25] *To fix ideas, we assume that the principal gets a payoff of 1 if paparazzi go to a wrong location, and -1 if they go to the right location. With the status quo procedure, the principal gets a payoff of*

$$\frac{1}{2}\big(1 - 2/|\Omega|\big) - \frac{1}{2} = -1/|\Omega|.$$

More secure, and even completely secure, communication channels are available, but they are costly to use for the employees, and the manager cannot control that the employees are indeed using a secure channel. The optimal procedure for the principal is uninformative by Proposition 2, i.e. the completely secure communication channel. The principal can optimally incentivize her employees to use the secure channel by paying them a bonus when the paparazzi go the wrong location. The agency cost of this procedure is

$$c + \frac{c/2}{1 - 1/2} = 2c,$$

and its informational payoff is $1 - 2/|\Omega|$. *Therefore, the principal adopts the secure communication channel if*

$$c \leqslant \frac{1 - 1/|\Omega|}{2}.$$

6 Conclusion

[25] Hence, not all signal realizations are contractible. See Section 5 for an explanation of how our results extend to this case.

Appendix

A Proofs

Proof of Lemma 1: We first show that a solution to (P) exists. By choosing $p = 1$ in (P), it is possible to achieve the value $v(\mu_0) - \tilde{\gamma}(1)$ for the principal. Furthermore $\hat{v}(\mu_0)$ is an upper bound for the informational payoff of the principal. For p sufficiently close to $\underline{\phi}$, the agency cost is so high that a principal would not want to choose p even if she could attain her best informational payoff by doing so. This is the case if

$$\hat{v}(\mu_0) - \tilde{\gamma}(p) < v(\mu_0) - \tilde{\gamma}(1),$$

or equivalently if

$$p < \underline{p} := \underline{\phi} + \frac{c\underline{\phi}}{\hat{v}(\mu_0) - v(\mu_0) + \frac{c\underline{\phi}}{1-\underline{\phi}}}.$$

Hence, if $\underline{p} < 1$, we can rewrite (P) as a maximization problem over the set of triples $(p, \mu^\dagger, \hat{\mu}) \in [\underline{p}, 1] \times \Delta\Omega^2$ that satisfy the constraint (BP), which is a compact set. The objective function in (P) is upper semicontinuous in $(p, \mu^\dagger, \hat{\mu})$, hence it attains its maximum value (see, for example, Aliprantis and Border, 2006, theorem 2.43).

The only remaining case is if $\underline{p} > 1$. In this case, the principal can not do better than choosing the uninformative procedure, which is therefore a solution to (P).

We have shown that a solution to (P) exists; we now prove the rest of the lemma. Suppose a solution $(p, \mu^\dagger, \hat{\mu})$ of (P) exists with $p < 1$. Let $\alpha \in T_v(\hat{\mu})$ and suppose by way of contradiction that $\alpha(\mu^\dagger) > 0$. Note first that $\alpha(\mu^\dagger) = 1$ is impossible, as otherwise $(1, \mu_0, 0)$ would do strictly better than $(p, \mu^\dagger, \hat{\mu})$ in the program (P). We can then write

$$\begin{aligned}\mu_0 &= \big(p + (1-p)\alpha(\mu^\dagger)\big)\mu^\dagger + (1-p)\sum_{\mu \in \text{supp}(\alpha)\setminus\{\mu^\dagger\}} \alpha(\mu)\mu \\ &= p'\mu^\dagger + (1-p')\hat{\hat{\mu}}, \end{aligned} \tag{6}$$

where $p' := p + (1-p)\alpha(\mu^\dagger)$ and $\hat{\hat{\mu}} := \frac{1-p}{1-p'}\sum_{\mu \in \text{supp}(\alpha)\setminus\{\mu^\dagger\}} \alpha(\mu)\mu$. Define $B := 1 - \alpha(\mu^\dagger)$. Then straightforward algebra establishes

$$\hat{\hat{\mu}} = \sum_{\mu \in \text{supp}(\alpha)\setminus\{\mu^\dagger\}} \frac{\alpha(\mu)}{B}\mu.$$

Next, as $\frac{1}{B}\sum_{\mu\in\text{supp}(\alpha)\smallsetminus\{\mu^\dagger\}}\alpha(\mu)=1$,

$$p'v(\mu^\dagger)+(1-p')\hat{v}(\hat{\hat{\mu}})\geqslant p'v(\mu^\dagger)+(1-p')\sum_{\mu\in\text{supp}(\alpha)\smallsetminus\{\mu^\dagger\}}\frac{\alpha(\mu)}{B}v(\mu).$$

Noting that $\frac{1-p'}{B}=1-p$ then gives

$$\begin{aligned}p'v(\mu^\dagger)+(1-p')\hat{v}(\hat{\hat{\mu}})&\geqslant pv(\mu^\dagger)+\Big[(1-p)\alpha(\mu^\dagger)v(\mu^\dagger)+(1-p)\sum_{\mu\in\text{supp}(\alpha)\smallsetminus\{\mu^\dagger\}}\alpha(\mu)v(\mu)\Big]\\&=pv(\mu^\dagger)+(1-p)\sum_{\mu\in\text{supp}(\alpha)}\alpha(\mu)v(\mu),\end{aligned}$$

that is, since $\alpha\in T_v(\hat{\mu})$,

$$p'v(\mu^\dagger)+(1-p')\hat{v}(\hat{\hat{\mu}})\geqslant pv(\mu^\dagger)+(1-p)\hat{v}(\hat{\mu}).\tag{7}$$

As $p'>p$, we have $\tilde{\gamma}(p')<\tilde{\gamma}(p)$, and thus combining (6) and (7) contradicts $(p,\mu^\dagger,\hat{\mu})$ being a solution of (P). Hence $\mu^\dagger\notin\text{supp}(\alpha)$, as claimed in the lemma.

We now show the second part of the lemma. Suppose by way of contradiction that $(p,\mu^\dagger,\hat{\mu})$ is a solution of (P) such that $p<(1-p)\alpha(\mu')$ for some $\mu'\in\text{supp}(\alpha)$ and some $\alpha\in T_v(\hat{\mu})$. Consider the triple $(\tilde{p},\tilde{\mu}^\dagger,\tilde{\hat{\mu}})$ such that: $\tilde{p}=(1-p)\alpha(\mu')$, $\tilde{\mu}^\dagger=\mu'$ and

$$\tilde{\hat{\mu}}=\frac{p\mu^\dagger+\sum_{\mu\in\text{supp}(\alpha)\setminus\{\mu'\}}(1-p)\alpha(\mu)\mu}{p+\sum_{\mu\in\text{supp}(\alpha)\setminus\{\mu'\}}(1-p)\alpha(\mu)}.$$

One can check that $(\tilde{p},\tilde{\mu}^\dagger,\tilde{\hat{\mu}})$ satisfies Bayes Plausibility; moreover $\tilde{p}>p$, hence $\tilde{\gamma}(\tilde{p})<\tilde{\gamma}(p)$, and:

$$\begin{aligned}\tilde{p}v(\tilde{\mu}^\dagger)+(1-\tilde{p})\hat{v}(\tilde{\hat{\mu}})&\geqslant\tilde{p}v(\tilde{\mu}^\dagger)+(1-\tilde{p})\frac{pv(\mu^\dagger)+\sum_{\mu\in\text{supp}(\alpha)\setminus\{\mu'\}}(1-p)\alpha(\mu)v(\mu)}{p+\sum_{\mu\in\text{supp}(\alpha)\setminus\{\mu'\}}(1-p)\alpha(\mu)}\\&=(1-p)\alpha(\mu')v(\mu')+pv(\mu^\dagger)+\sum_{\mu\in\text{supp}(\alpha)\setminus\{\mu'\}}(1-p)\alpha(\mu)v(\mu)\\&=pv(\mu^\dagger)+(1-p)v(\hat{\mu}).\end{aligned}$$

These observations show that $(p,\mu^\dagger,\hat{\mu})$ is not a solution of (P). This contradiction concludes the proof.

∎

Proof of Theorem 1: We claim that if (ψ, t) solves (P0) then the payoff of the principal can be written like the maximand in (P). By virtue of Proposition 4, any value of the maximand in (P0) can be achieved by the principal for some (ψ, t) satisfying (IC). So the theorem will be proven if we can show that the claim holds.

Suppose (ψ, t) solves (P0). Assume moreover that t rewards the agent at a single signal realization (we know that a solution exists which satisfies this property), which we denote by $s^\dagger$. Let $\tau \in \Delta\Delta\Omega$ be the beliefs distribution induced by ψ, $\mu^\dagger$ the payment belief, $p := \tau(\mu^\dagger)$, and $\hat{\mu} := \frac{\mu_0 - p\mu^\dagger}{1-p}$. By Proposition 4, the agency cost to implement ψ is equal to $\tilde{\gamma}(p)$. Therefore, the payoff of the principal under (ψ, t) can be written as

$$pv(\mu^\dagger) + (1-p) \sum_{\mu \in \mathrm{supp}(\tau) \backslash \{\mu^\dagger\}} \frac{\tau(\mu)}{1-p} v(\mu) - \tilde{\gamma}(p),$$

and to show the claim we only need to show that the term multiplying $(1-p)$ equals $\hat{v}(\hat{\mu})$. Suppose, for a contradiction, that it is not equal to $\hat{v}(\hat{\mu})$. Let $\alpha \in T_v(\hat{\mu})$. If $\mu^\dagger \notin \mathrm{supp}(\alpha)$ then define $\tau' \in \Delta\Delta\Omega$ by $\tau'(\mu^\dagger) = p$ and $\tau'(\mu) = (1-p)\alpha(\mu)$ for all $\mu \in \mathrm{supp}(\alpha)$. Let ψ' be any procedure inducing τ' and satisfying $\mu(s^\dagger; \psi') = \mu^\dagger$. Then (ψ', t) satisfies (IC) and yields to the principal a payoff equal to $pv(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu}) - \tilde{\gamma}(p)$, that is, a payoff larger than that obtained from (ψ, t), contradicting the optimality of (ψ, t). Finally, consider the case where $\mu^\dagger \in \mathrm{supp}(\alpha)$. Define then $\tau' \in \Delta\Delta\Omega$ by $\tau'(\mu^\dagger) = p + (1-p)\alpha(\mu^\dagger)$ and $\tau'(\mu) = (1-p)\alpha(\mu)$ for all $\mu \in \mathrm{supp}(\alpha)\backslash\{\mu^\dagger\}$. Let ψ' be any procedure inducing τ' and satisfying $\mu(s^\dagger; \psi') = \mu^\dagger$. Then (ψ', t) satisfies (IC) (strictly, this time) and yields to the principal a payoff equal to $pv(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu}) - \tilde{\gamma}(p')$, where $p' := p + (1-p)\alpha(\mu^\dagger)$. As $p' > p$ this payoff is larger than that obtained from (ψ, t), contradicting again the optimality of (ψ, t).

∎

Proof of Lemma 2: Let (ψ, t) satisfy (IC). Suppose that there exist signals $s_1 \neq s_2$ such that $\mu(s_1; \psi) = \mu(s_2; \psi)$. Pick labels such that $\phi(s_1) \leqslant \phi(s_2)$. Then let $\tilde{\psi}$ be the procedure defined by $\tilde{\psi}(s|\omega) = \psi(s|\omega)$ whenever $s \notin \{s_1, s_2\}$, $\tilde{\psi}(s_1|\omega) = \psi(s_1|\omega) + \psi(s_2|\omega)$ and $\tilde{\psi}(s_2|\omega) = 0$, so that s_2 is never generated under $\tilde{\psi}$. Then we have $\mu(s_1; \tilde{\psi}) = \mu(s_1; \psi) = \mu(s_2; \psi)$. We also choose the new transfer scheme $\tilde{t}$ such that $\tilde{t}(s) = t(s)$ for every $s \notin \{s_1, s_2\}$, $\tilde{t}(s_2) = 0$, while

$$\tilde{t}(s_1)\tilde{\psi}(s_1) = t(s_1)\psi(s_1) + t(s_2)\psi(s_2). \tag{8}$$

By construction, when the agent uses the procedure proposed by the principal, (ψ, t) and $(\tilde{\psi}, \tilde{t})$ deliver the same expected transfer to the agent and the same expected payoff to the principal. Hence, to show the lemma it is sufficient to show that $(\tilde{\psi}, \tilde{t})$ satisfies (IC). To see that, note that we have:

$$\begin{aligned}
\tilde{\psi}(s_1)\tilde{t}(s_1) + \sum_{s \notin \{s_1, s_2\}} \psi(s)t(s) - c &= \sum_s \psi(s)t(s) - c \\
&\geqslant \sum_s \phi(s)t(s) \\
&\geqslant \sum_{s \notin \{s_1, s_2\}} \phi(s)t(s) + \phi(s_1)\big(t(s_1) + t(s_2)\big) \\
&\geqslant \sum_{s \notin \{s_1, s_2\}} \phi(s)t(s) + \phi(s_1)\tilde{t}(s_1)
\end{aligned}$$

where the first equality is by application of (8), the first inequality follows from the assumption that (ψ, t) satisfies (IC), the second inequality uses $\phi(s_1) \geqslant \phi(s_2)$, and the last inequality follows from (8). This concludes the proof.

■

Proof of Proposition 3: Suppose, by contradiction, that there exists another pair (ψ', t') that satisfies (IC) and such that

$$\sum_{\omega, s} \psi'(s|\omega)\big\{v\big((\mu(s;\psi')\big) - t'(s)\big\} > \sum_{\omega, s} \psi_{\tau,\sigma}(s|\omega)\big\{v\big((\mu(s;\psi_{\tau,\sigma})\big) - t(s)\big\}. \tag{9}$$

Let $M = \big\{\mu(s;\psi')\big\}_{s \in S}$. By Lemma 2, we can without loss of generality assume that ψ' generates a distinct belief for each signal. The function $\sigma' : M \to S$ that associates to $\mu \in M$ the unique signal $s \in S$ such that $\mu(s;\psi') = \mu$ is therefore well defined and injective. Next, let $\tau' \in \Delta_{|S|}\Delta\Omega$ be the belief distribution with support M defined by $\tau'(\mu) = \sum_\omega \mu_0(\omega)\psi'\big(\sigma(\mu)|\omega\big)$, for each $\mu \in M$.

Then τ' satisfies (BP1). Furthermore it is immediate that (IC) for (ψ', t') and (IC1) for (τ', σ', t') are the same equations, and that the value of the objective function of (P1) at (τ', σ', t') is equal to the value of the objective function of (P0) at (ψ', t'). But then (9) implies a contradiction to the optimality of (τ, σ, t).

$\blacksquare$

Proof of Proposition 4: Fixing the assignment function σ, it is immediate to see that for any $s \in S \setminus \text{supp}(\sigma)$ lowering $t(s)$ strictly relaxes the constraint and leaves the objective function unchanged, so in the optimum we must have $t(s) = 0$. We are thus left with a linear program in $t \circ \sigma : \text{supp}(\tau) \to \mathbb{R}_+$. Each $t\big(\sigma(\mu)\big)$ is constrained to be positive. If μ is such that $\tau(\mu) \leqslant \phi\big(\sigma(\mu)\big)$, then lowering $t\big(\sigma(\mu)\big)$ strictly lowers the objective function and relaxes the constraint, so at the optimum we must have $t\big(\sigma(\mu)\big) = 0$. If this is the case for all μ, then it is impossible to satisfy the constraint with the assignment σ. Otherwise, the constraint must bind: if the constraint did not bind the principal could lower his payments while still satisfying the constraint. In this case, we can rewrite the constraint with an equality. Then the set over which we can choose the vector $\big(t\big(\sigma(\mu)\big)\big)_{\mu \in \text{supp}\,\tau : \tau(\mu) > \phi(\sigma(\mu))}$ is a compact and convex polytope. Hence the extreme point theorem implies that we can always choose a solution at an extreme point of this polytope, which in this case means setting all payments but one to 0. Let $\mu^\dagger$ be this payment belief. The binding incentive constraint implies that the cost of implementation is then given by

$$\tau(\mu^\dagger) t(\mu^\dagger) = \frac{c}{1 - \frac{\phi(\sigma(\mu^\dagger))}{\tau(\mu^\dagger)}}.$$

This cost is increasing in $\frac{\phi(\sigma(\mu^\dagger))}{\tau(\mu^\dagger)}$, which is bounded below by $\underline{\phi}/\overline{\tau}$. This bound can be attained by choosing $\mu^\dagger \in \arg\max_\mu \tau(\mu)$, and $\sigma(\mu^\dagger) \in \arg\min_s \phi(s)$. Therefore this combination solves (CM_τ).

We still need to deal with the case where for every assignment σ, there is no belief μ such that $\tau(\mu) > \phi\big(\sigma(\mu)\big)$, which can happen only if τ and ϕ are both uniform on their support, and τ has a support of size $|S|$. In this case, the incentive constraint of the agent cannot be satisfied, so the value function of (CM_τ) is infinite, as is $\gamma(\tau)$.

$\blacksquare$

Claim 1. *Consider the setting of Example 2. Any solution to* (P) *satisfies either* $p = 1$ *or* $\hat{\mu} \geqslant \frac{1}{2}$.

Proof: We show that any solution to (P) with $\mu^\dagger > \mu_0$ must satisfy $\mu^\dagger = \frac{1}{2}$ and $\hat{\mu} = 0$. The rest of the proof follows the arguments in the main text. First, suppose a solution to (P) exists such that $\mu^\dagger < \frac{1}{2}$. Then $v(\mu^\dagger) = 0$. Yet $\hat{v}(\cdot)$ is increasing. So increasing $\hat{\mu}$ slightly while decreasing $\mu^\dagger$ so as to maintain equality in (BP) strictly increases the maximand of (P).

Hence, no solution of (BP) has $\mu^\dagger < \frac{1}{2}$. Suppose next that a solution $(p, \mu^\dagger, \hat{\mu})$ exists such that $\mu^\dagger > \frac{1}{2}$. Define $p' \in (0, 1)$ by

$$\mu_0 = p'\frac{1}{2} + (1 - p')\hat{\mu}.$$

Clearly, $p' > p$. For $\mu \geqslant \frac{1}{2}$, $v(\mu) = \hat{\mu}$. We therefore have

$$pv(\mu^\dagger) + (1 - p)\hat{v}(\hat{\mu}) = p\hat{v}(\mu^\dagger) + (1 - p)\hat{v}(\hat{\mu}) \leqslant p'\hat{v}\,(1/2) + (1 - p')\hat{v}(\hat{\mu}),$$

where the inequality follows from concavity of $\hat{v}$. It ensues that $(p', \frac{1}{2}, \hat{\mu})$ strictly dominates $(p, \mu^\dagger, \hat{\mu})$. Hence, no solution of (BP) has $\mu^\dagger > \frac{1}{2}$. Combining the previous steps shows that any solution to (BP) with $\mu^\dagger > \mu_0$ has $\mu^\dagger = \frac{1}{2}$.

We now show that any solution must satisfy $\mu^\dagger = \frac{1}{2}$ and $\hat{\mu} = 0$. Suppose by way of contradiction that $(p, \frac{1}{2}, \hat{\mu})$ is a solution, and $\hat{\mu} > 0$. Let $p' = 2\mu_0$. Clearly, $p' > p$. Furthermore,

$$pv(1/2) + (1 - p)\hat{v}(\hat{\mu}) = p'v(1/2) + (1 - p')\hat{v}(0).$$

Thus $(p', \frac{1}{2}, 0)$ strictly dominates $(p, \frac{1}{2}, \hat{\mu})$, contradicting our initial assumption.

■

Proof of Proposition 2: Note that, when $v(\mu_0) = \hat{v}(\mu_0)$, the lower bound $\underline{p}$ in the proof of Lemma 1 is equal to 1, so the principal must choose $p = 1$, which implies that the solution of (P) is the uninformative procedure. ■

Proof of Proposition 1: Let $W(p)$ be the value function of the program

$$\max_{\mu^\dagger, \hat{\mu} \in \Delta\Omega} \; pv(\mu^\dagger) + (1 - p)\hat{v}(\hat{\mu})$$
$$\text{s.t. } p\mu^\dagger + (1 - p)\hat{\mu} = \mu_0$$

Then we must have $p(c) \in \arg\max_{p \in [\underline{\phi}, 1]} W(p) - c - \frac{c\phi}{p - \underline{\phi}}$. Applying the monotone selection theorem of Milgrom and Shannon (1994) to this program yields the result.

If $\hat{v}(\mu_0) = v(\mu_0)$, we know from Proposition 2 that the $p(c) = 1$ for all c, and the solution is uninformative. Suppose that $\hat{v}(\mu_0) > v(\mu_0)$. Then the definition of $\underline{p}$ implies that $\underline{p} \geqslant 1$ whenever

$$c \geqslant \bar{c} = \frac{(1 - \underline{\phi})\big(\hat{v}(\mu_0) - v(\mu_0)\big)}{\underline{\phi}^2},$$

which concludes the proof of the last point.

■

Lemma 4. *Let* $d(a^\dagger, \omega) := u(a^\dagger, \omega) - u(a_\omega, \omega)$. *Consider*

$$\max_{x \geqslant 0} \sum_\omega x(\omega) d(a^\dagger, \omega)$$

$$s.t. \quad \begin{cases} \sum_\omega x(\omega) = p; \\ x \leqslant \mu_0. \end{cases}$$

Then the value of the program above is equal to the value of

$$\min_\xi \; p\xi + \sum_\omega \mu_0(\omega) \big[d(a^\dagger, \omega) - \xi \big]^+.$$

Moreover, if x *solves the first program and* ξ *solves the second then* $\xi \leqslant 0$ *and*

$$\begin{cases} d(a^\dagger, \omega) - \xi > 0 \;\Rightarrow x(\omega) = \mu_0(\omega); \\ d(a^\dagger, \omega) - \xi < 0 \;\Rightarrow x(\omega) = 0. \end{cases}$$

Proof: It is easily checked that the dual of the first program in the statement of the lemma is

$$\min_{y \geqslant 0, \xi} \; p\xi + \mu_0.y$$
$$\text{s.t.} \;\; \xi + y(\omega) \geqslant d(a^\dagger, \omega), \;\; \text{for all } \omega.$$

Next, as $\mu_0 \gg 0$, if (ξ, y) is a solution of the dual then

$$y(\omega) = \big[d(a^\dagger, \omega) - \xi \big]^+. \tag{10}$$

So the dual can be rewritten as

$$\min_\xi \; p\xi + \sum_\omega \mu_0(\omega) \big[d(a^\dagger, \omega) - \xi \big]^+. \tag{11}$$

Next, let x denote a solution of the primal problem and (ξ, y) a solution of the dual. As $p \geqslant 0$

and $d(a^\dagger,\omega) \leqslant 0$ for all ω, (11) implies $\xi \leqslant 0$. Moreover, by complementary slackness,

$$\begin{cases} y(\omega) > 0 \;\Rightarrow\; x(\omega) = \mu_0(\omega); \\ \xi + y(\omega) > d(a^\dagger,\omega) \;\Rightarrow\; x(\omega) = 0. \end{cases}$$

Combined with (10), the previous conditions give

$$\begin{cases} d(a^\dagger,\omega) - \xi > 0 \;\Rightarrow x(\omega) = \mu_0(\omega); \\ d(a^\dagger,\omega) - \xi < 0 \;\Rightarrow x(\omega) = 0. \end{cases}$$

■

Proof of Proposition 5: Suppose a solution $(p^*, \mu^{\dagger *}, \hat{\mu}^*)$ to (P) exists with $p^* < 1$ but $\{\mu^{\dagger *}, \hat{\mu}^*\} \cap \partial\Delta\Omega = \varnothing$. Let L denote the line through $\mu^{\dagger *}$ and $\hat{\mu}^*$ and $\tilde{T} \subset T(\mu_0)$ the binary splittings τ of μ_0 with $\text{supp}(\tau) = \{\mu^\dagger, \hat{\mu}\} \in L$ and $\tau(\mu^\dagger) = p^*$. Define

$$L^\dagger := \{\mu^\dagger : \exists \tau \in \tilde{T} \;\text{ such that }\; \mu^\dagger \in \text{supp}(\tau)\},$$

and $g : L^\dagger \to L$ by

$$g(\mu^\dagger) = \frac{\mu_0 - p^*\mu^\dagger}{1-p^*}.$$

Now, by (2), v is convex while, by (3), $\hat{v}$ is linear. Hence, g being linear, $p^* v(\mu^\dagger) + (1 - p^*)\hat{v}\big(g(\mu^\dagger)\big)$ is a convex function of $\mu^\dagger$ defined on $L^\dagger$. The proof is complete, since a convex functional is maximized at one of its extreme points.

■

Lemma 5. *Suppose $v(\cdot)$ is convex. Then $(p, \mu^\dagger, \hat{\mu})$ solves* (P) *if and only if $p \in \arg\min_p\{MLE(p) + \tilde{\gamma}(p)\}$ and $\mu^\dagger = \frac{x}{p}$, with x solving $ML_{a^\dagger,p}$ and $MLE(p) = ML_{a^\dagger}(p)$.*

Proof: The belief-contingent payoff function v being convex, $\hat{v}$ is a linear function. The lemma now follows from the following identities:

$$\begin{aligned} pv(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu}) &= p\left[\hat{v}(\mu^\dagger) - \big(\hat{v}(\mu^\dagger) - v(\mu^\dagger)\big)\right] + (1-p)\hat{v}(\hat{\mu}) \\ &= \hat{v}(p\mu^\dagger + (1-p)\hat{\mu}) - p\left(\hat{v}(\mu^\dagger) - v(\mu^\dagger)\right) \\ &= \hat{v}(\mu_0) - p\sum_\omega \mu^\dagger(\omega)\left(u(a_\omega,\omega) - u(a^\dagger,\omega)\right). \end{aligned}$$

■

Proof of Proposition 6: Suppose condition (ii) holds, and let $a^* \in A$ such that $\text{MLE}(p) = \text{ML}_{a^*}(p)$. Let $n := |\Omega|$. Label the elements of Ω in increasing order of the gradient $u(a_\omega, \omega) - u(a^*, \omega)$, that is,

$$u(a_{\omega_1}, \omega_1) - u(a^*, \omega_1) \leqslant \cdots \leqslant u(a_{\omega_n}, \omega_n) - u(a^*, \omega_n).$$

Next, for all $p \in [0,1]$, define $x_p^* \in \mathbb{R}^n$ such that $\sum_\omega x_p^*(\omega) = p$, $0 \leqslant x_p^* \leqslant \mu_0$, and $x_p^*(\omega_i) < \mu_0(\omega_i) \Rightarrow x_p^*(\omega_{i+1}) = 0$ for all $i < n$. Notice that x_p^* is uniquely determined and solves $\text{ML}_{a^*,p}$. Furthermore, it is easy to see that

$$p' > p \Rightarrow x_{p'}^* > x_p^*. \tag{12}$$

Applying Lemma 5, $(p(c), \mu^\dagger(c), \hat{\mu}(c))$ solves (P) if and only if $p(c) \in \arg\min\{\text{MLE}(p) + \tilde{\gamma}(p)\}$ and $\mu^\dagger(c) = \frac{x}{p(c)}$, with x solving $\text{ML}_{a^\dagger, p(c)}$ and $\text{MLE}(p(c)) = \text{ML}_{a^\dagger}(p(c))$. Hence $\left(p(c), \frac{1}{p(c)} x_{p(c)}^*, \frac{1}{1-p(c)}(\mu_0 - x_{p(c)}^*)\right)$ solves (P). Let $\psi(c)$ denote the corresponding procedure; then by (12) and Lemma 3, $\psi(c)$ is ordered according to Blackwell's criterion.

Equivalence between conditions (i) and (ii) in the statement of the proposition follows from Lemma 5.

■

Proof of Theorem 4: Let (k^*, s^*) denote a saddle-point of ϕ and suppose (ψ, t) solves (P0) when k^* is the unique status quo procedure. Then $t(\cdot)$ rewards the agent only when the signal realization is s^*. Since $\max_k \phi_k(s^*) = \phi_{k^*}(s^*)$, then (ψ, t) is also feasible for (P0) when K is the set of status quo procedures. As the value function of (P0) with multiple status quo procedures cannot exceed the value function with a single status quo, (ψ, t) thus solves (P0) when K is the set of status quo procedures.

■

Proof of Proposition 7: Suppose $\phi_k(s)$ has no saddle point. Let $|S| = n$ and consider a procedure ψ generating signals in S with uniform probabilities. Pick an arbitrary signal $\tilde{s} \in S$. We claim that no optimal payment scheme rewards the agents only when the signal realization is $\tilde{s}$. If $\max_k \phi_k(\tilde{s}) \geqslant \frac{1}{n}$ the result is trivial, since no incentive compatible payment scheme rewards the agent only when the signal realization is $\tilde{s}$. Therefore, assume henceforth

$\max_k \phi_k(\tilde{s}) < \frac{1}{n}$. Let t be an optimal payment scheme within the class of payment schemes that reward the agent only when the signal realization is $\tilde{s}$. Then

$$t(\tilde{s}) = \frac{c}{\frac{1}{n} - \phi_{\tilde{k}}(\tilde{s})},$$

where $\phi_{\tilde{k}}(\tilde{s}) = \max_k \phi_k(\tilde{s})$. As ϕ as no saddle point, there exists s' such that $\phi_{\tilde{k}}(s') < \phi_{\tilde{k}}(\tilde{s})$. Next, define the alternative payment scheme t' as follows:

$$\begin{cases} t'(\tilde{s}) = t(\tilde{s}) - \left(\frac{\frac{1}{n} - \phi_{\tilde{k}}(s')}{\frac{1}{n} - \phi_{\tilde{k}}(\tilde{s})}\right)\epsilon; \\ t'(s') = \epsilon; \\ t'(s) = 0 \quad \forall s \notin \{\tilde{s}, s'\}. \end{cases}$$

where $t(\tilde{s})\left(\frac{\frac{1}{n} - \phi_{\tilde{k}}(s')}{\frac{1}{n} - \phi_{\tilde{k}}(\tilde{s})}\right)^{-1} > \epsilon > 0$.

We make three observations. First,

$$\begin{aligned} \frac{1}{n}\big(t'(\tilde{s}) + t'(s')\big) &= \frac{1}{n}\left(t(\tilde{s}) + \frac{\phi_{\tilde{k}}(s') - \phi_{\tilde{k}}(\tilde{s})}{\frac{1}{n} - \phi_{\tilde{k}}(\tilde{s})}\epsilon\right) \\ &< \frac{1}{n}t(\tilde{s}). \end{aligned}$$

Thus the expected payment made by the principal is strictly lower under $t'(\cdot)$ than under $t(\cdot)$. Second,

$$\begin{aligned} t'(\tilde{s})\left(\frac{1}{n} - \phi_{\tilde{k}}(\tilde{s})\right) + t'(s')\left(\frac{1}{n} - \phi_{\tilde{k}}(s')\right) &= \left(t(\tilde{s}) - \frac{\frac{1}{n} - \phi_{\tilde{k}}(s')}{\frac{1}{n} - \phi_{\tilde{k}}(\tilde{s})}\epsilon\right)\left(\frac{1}{n} - \phi_{\tilde{k}}(\tilde{s})\right) + \epsilon\left(\frac{1}{n} - \phi_{\tilde{k}}(s')\right) \\ &= t(\tilde{s})\left(\frac{1}{n} - \phi_{\tilde{k}}(\tilde{s})\right) = c. \end{aligned}$$

Thus $t'(\cdot)$ satisfies ($\text{IC}_{\tilde{k}}$). Third, by assumption, $\arg\max_k \phi_k(\tilde{s}) = \{\tilde{k}\}$. Therefore, for all $k \neq \tilde{k}$, the payment scheme $t(\cdot)$ rewarding the agent only when the signal realization is $\tilde{s}$ satisfies (IC_k) with strict inequality. As $\lim_{\epsilon \to 0} t' = t$, while the left-hand side of each (IC_k) is continuous in the payment scheme, combining the previous observations shows that, by choosing ϵ sufficiently small, $t'(\cdot)$ implements ψ with strictly lower expected payment from the principal. Hence no optimal payment scheme rewards the agents only when the signal realization is $\tilde{s}$. This completes the proof of the proposition, since $\tilde{s}$ was arbitrary.

■

Proof of Theorem 5: The proof is in two steps. First we show that if (ψ, t) solves (P0) with language constraints then the payoff of the principal can be written like the maximand in (P_s) for some $s \in S$. Second we show that any value of the maximand in (P0) can be achieved by the principal for some $(\tilde{\psi}, \tilde{t})$ satisfying (IC) as well as the language constraints.

Suppose (ψ, t) solves (P0) with language constraints. Assume moreover that t rewards the agent at a single signal realization (we know that a solution exists which satisfies this property), which we denote by $s^\dagger$. Let $\tau \in \Delta\Delta\Omega$ be the beliefs distribution induced by ψ, $\mu^\dagger$ the payment belief, $p := \tau(\mu^\dagger)$, and $\hat{\mu} := \frac{\mu_0 - p\mu^\dagger}{1-p}$. The agency cost to implement ψ is equal to $\frac{pc}{p-\phi(s^\dagger)} = c + \frac{\phi(s^\dagger)}{p-\phi(s^\dagger)} = \tilde{\gamma}_{s^\dagger}(p)$. Therefore, the payoff of the principal under (ψ, t) can be written as

$$pv(\mu^\dagger) + (1-p) \sum_{\mu \in \mathrm{supp}(\tau)\backslash\{\mu^\dagger\}} \frac{\tau(\mu)}{1-p} v(\mu) - \tilde{\gamma}_{s^\dagger}(p).$$

To show that this is like the maximand in (P_s), we only need to show that the term multiplying $(1-p)$ equals $\hat{v}(\hat{\mu})$. Suppose that it is not equal to $\hat{v}(\hat{\mu})$. We will consider two cases, and reach a contradiction in each.

Case 1: Suppose that there exists $\alpha \in T_v(\hat{\mu})$ and a collection $\{s_\mu\}_{\mu \in \mathrm{supp}(\alpha)}$ of $|\,\mathrm{supp}(\alpha)|$ signals in $S\backslash\{s^\dagger\}$ satisfying $\mu \in \Delta(s_\mu)$ for all $\mu \in \mathrm{supp}(\alpha)$. If $\mu^\dagger \notin \mathrm{supp}(\alpha)$ then define $\tau' \in \Delta\Delta\Omega$ by $\tau'(\mu^\dagger) = p$ and $\tau'(\mu) = (1-p)\alpha(\mu)$ for all $\mu \in \mathrm{supp}(\alpha)$. Let ψ' be any procedure inducing τ' and satisfying $\mu(s^\dagger; \psi') = \mu^\dagger$, $\mu(s_\mu; \psi') = \mu$ for all $\mu \in \mathrm{supp}(\alpha)$. Then (ψ', t) satisfies (IC) as well as all language constraints, and yields to the principal a payoff equal to $pv(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu}) - \tilde{\gamma}_{s^\dagger}(p)$, that is, a payoff larger than that obtained from (ψ, t), contradicting the optimality of (ψ, t). In a similar way, if $\mu^\dagger \in \mathrm{supp}(\alpha)$ define then $\tau' \in \Delta\Delta\Omega$ by $\tau'(\mu^\dagger) = p + (1-p)\alpha(\mu^\dagger)$ and $\tau'(\mu) = (1-p)\alpha(\mu)$ for all $\mu \in \mathrm{supp}(\alpha)\backslash\{\mu^\dagger\}$. Let ψ' be any procedure inducing τ' and satisfying $\mu(s^\dagger; \psi') = \mu^\dagger$, $\mu(s_\mu; \psi') = \mu$ for all $\mu \in \mathrm{supp}(\alpha)\backslash\{\mu^\dagger\}$. Then (ψ', t) satisfies (IC) (strictly, this time) as well as all language constraints, and yields to the principal a payoff equal to $pv(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu}) - \tilde{\gamma}_{s^\dagger}(p')$, where $p' := p + (1-p)\alpha(\mu^\dagger)$. As $p' > p$ this payoff is larger than that obtained from (ψ, t), contradicting again the optimality of (ψ, t).

Case 2: Suppose now that for any $\alpha \in T_v(\hat{\mu})$, $s^\dagger \in \{s_\mu\}_{\mu \in \mathrm{supp}(\alpha)}$ for all collections of $|\,\mathrm{supp}(\alpha)|$ signals in S satisfying $\mu \in \Delta(s_\mu)$, $\forall \mu \in \mathrm{supp}(\alpha)$. Pick an arbitrary $\alpha \in T_v(\hat{\mu})$, and a collection $\{s_\mu\}_{\mu \in \mathrm{supp}(\alpha)}$ of $|\,\mathrm{supp}(\alpha)|$ signals in S satisfying $\mu \in \Delta(s_\mu)$ for all $\mu \in \mathrm{supp}(\alpha)$. Let $\ddot{\mu} \in \mathrm{supp}(\alpha)$

such that $s_{\ddot{\mu}} = s^\dagger$ and define $\tilde{\mu}^\dagger := \frac{p\mu^\dagger + (1-p)\alpha(\ddot{\mu})\ddot{\mu}}{p+(1-p)\alpha(\ddot{\mu})}$. Notice that, as $\Delta(s^\dagger)$ is convex, $\tilde{\mu}^\dagger \in \Delta(s^\dagger)$. Next, let $\tau' \in \Delta\Delta\Omega$ given by $\tau'(\tilde{\mu}^\dagger) = p + (1-p)\alpha(\ddot{\mu})$ and $\tau'(\mu) = (1-p)\alpha(\mu)$ for all $\mu \in \text{supp}(\alpha)\backslash\{\ddot{\mu}\}$. Let ψ' be any procedure inducing τ' and satisfying $\mu(s^\dagger; \psi') = \tilde{\mu}^\dagger$, $\mu(s_\mu; \psi') = \mu$ for all $\mu \in \text{supp}(\alpha)\backslash\{\ddot{\mu}\}$. Then (ψ', t) satisfies (IC) (strictly) as well as all language constraints, and yields to the principal a payoff equal to

$$p' v(\tilde{\mu}^\dagger) + (1-p') \sum_{\mu \in \text{supp}(\alpha)\backslash\{\ddot{\mu}\}} \frac{\tau'(\mu)v(\mu)}{1-p'} - \tilde{\gamma}_{s^\dagger}(p'), \tag{13}$$

where $p' := p + (1-p)\alpha(\ddot{\mu})$. The principal's payoff function v is weakly concave on $\Delta(s^\dagger)$, so

$$v(\tilde{\mu}^\dagger) = v\left(\frac{p\mu^\dagger + (1-p)\alpha(\ddot{\mu})\ddot{\mu}}{p + (1-p)\alpha(\ddot{\mu})}\right) \geqslant \frac{p}{p'} v(\mu^\dagger) + \frac{(1-p)\alpha(\ddot{\mu})}{p'} v(\ddot{\mu}).$$

Therefore, (13) is at least as large as

$$\begin{aligned} pv(\mu^\dagger) + (1-p)\alpha(\ddot{\mu})v(\ddot{\mu}) + &\sum_{\mu \in \text{supp}(\alpha)\backslash\{\ddot{\mu}\}} \tau'(\mu)v(\mu) - \tilde{\gamma}_{s^\dagger}(p') \\ &= pv(\mu^\dagger) + (1-p) \sum_{\mu \in \text{supp}(\alpha)} \alpha(\mu)v(\mu) - \tilde{\gamma}_{s^\dagger}(p') \\ &= pv(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu}) - \tilde{\gamma}_{s^\dagger}(p'). \end{aligned}$$

Lastly, as $p' > p$ notice that the right-hand side of the last equation is strictly greater than $pv(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu}) - \tilde{\gamma}_{s^\dagger}(p)$. But given our starting assumption $v(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu}) - \tilde{\gamma}_{s^\dagger}(p)$ is larger than the payoff of the principal under (ψ, t). We therefore reached a contradiction with the optimality of (ψ, t).

The previous steps show that the payoff of the principal under (ψ, t) can be written like the maximand in ($\text{P}_{s^\dagger}$). Analogous arguments show that any value of the maximand in (P0) can be (at least) achieved by the principal for some $(\tilde{\psi}, \tilde{t})$ satisfying (IC) as well as the language constraints. Hence, any solution of (P_{s*}) induces a solution of (P0). ■

B Example 9

Consider program (P_{s_n}). As we discussed in the main text, if $(p, \mu^\dagger, \hat{\mu})$ solves (P_{s_n}), then $\hat{\mu}(b) = 1$. The Bayes plausibility condition thus implies $\mu^\dagger(b) = \frac{\mu_0(b)-(1-p)}{p}$, and the optimal probability p solves

$$\max_{p\in(\mu_0(n),1]} \left\{ 2(1-p-\mu_0(b)) + 1 - c - \frac{c\mu_0(n)}{p-\mu_0(n)} \right\}.$$

The maximand is concave in p, and the first order condition corresponds to $p = \mu_0(n) + \sqrt{\frac{c\mu_0(n)}{2}}$, hence the optimal probability p satisfies:

$$p = \begin{cases} \mu_0(n) + \sqrt{\frac{c\mu_0(n)}{2}} & \text{if } c < c_1 \\ 1 & \text{if } c \geqslant c_1, \end{cases}$$

where $c_1 = \frac{2(1-\mu_0(n))^2}{\mu_0(n)}$, yielding:

$$V_{s_n} = \begin{cases} 1 - c - 2\sqrt{2c\mu_0(n)} & \text{if } c < c_1; \\ 1 - 2\mu_0(b) - \frac{c}{1-\mu_0(n)} & \text{if } c \geqslant c_1. \end{cases}$$

Consider now program (P_{s_b}). If $(p, \mu^\dagger, \hat{\mu})$ solves (P_{s_b}), then $\hat{\mu}(b) = 0$. Noting that in this case Bayes plausibility condition implies $\mu^\dagger(b) = \frac{\mu_0(b)}{p}$, the optimal probability p solves

$$\max_{p\in(\mu_0(b),2\mu_0(b)]} \left\{ 2\mu_0(b) + 1 - 2p - \tilde{\gamma}_{s_b}(p) \right\}.$$

The maximand is concave, and the first order condition corresponds to: $p = \mu_0(b) + \sqrt{\frac{c\mu_0(b)}{2}}$. Therefore the optimal probability p satisfies:

$$p = \begin{cases} \mu_0(b) + \sqrt{\frac{c\mu_0(b)}{2}} & \text{for } c < c_2 \\ 2\mu_0(b) & \text{for } c \geqslant c_2 \end{cases}$$

where $c_2 = 2\mu_0(b)$, yielding:

$$V_{s_b} = \begin{cases} 1 - c - 2\sqrt{2c\mu_0(b)} & \text{for } c < c_2 \\ 1 - 2\mu_0(b) - 2c & \text{for } c \geqslant c_2 \end{cases}$$

Note that $c_1 < c_2$. As $\mu_0(b) < \mu_0(n)$ it is immediate that $V_{s_b} > V_{s_n}$ for $c < c_1$. Also, as $\mu_0(n) > \frac{1}{2}$, then $V_{s_b} > V_{s_n}$ for $c \geqslant c_2$. If $c \in [c_1, c_2)$, then $V_{s_b} > V_{s_n}$ is equivalent to:

$$\begin{aligned} 1 - c - 2\sqrt{2c\mu_0(b)} > 1 - 2\mu_0(b) - \frac{c}{1 - \mu_0(n)} \Leftrightarrow \\ \mu_0(b) - \sqrt{2c\mu_0(b)} > -\frac{c\mu_0(n)}{2(1 - \mu_0(n))}, \end{aligned}$$

where the last inequality holds as $c < c_2$ implies $\mu_0(b) - \sqrt{2c\mu_0(b)} > 0$.

C Procedure dependent cost

Lemma 6. *A solution to* (P_Π) *exists.*

Proof: By choosing $p = 1$ in (P_Π), it is possible to achieve the value $\Pi(\mu_0) + \gamma_H(1)$ for the principal. As $C(\tau) = 0$ if τ is uninformative, then $\Pi(\mu_0) + \gamma_H(1) = v(\mu_0) - \tilde{\gamma}(1)$.

Note that

$$\hat{v}(\mu_0) \geqslant \sup_{\substack{p \in (\underline{\phi}, 1] \\ \mu^\dagger, \hat{\mu} \in \Delta\Omega}} \{p\Pi(\mu^\dagger, p) + (1-p)\hat{\Pi}(\hat{\mu}, p)\},$$

and

$$c \leqslant c + C(\tau), \quad \forall \tau.$$

For p sufficiently close to $\underline{\phi}$, the agency cost is so high that a principal would not want to choose p even if she could attain an informational payoff equal to $\hat{v}(\mu_0)$ at a switching cost of c by doing so. This is the case if

$$\hat{v}(\mu_0) - \tilde{\gamma}(p) < v(\mu_0) - \tilde{\gamma}(1),$$

or equivalently if

$$p < \underline{p} := \underline{\phi} + \frac{c\underline{\phi}}{\hat{v}(\mu_0) - v(\mu_0) + \frac{c\underline{\phi}}{1-\underline{\phi}}}.$$

We have shown in the proof of Lemma 1 that this is sufficient to prove existence of a solution.
■

D Additional Results

Proposition 8. *Suppose* $|\Omega| = 2$. *If* $(p, \mu^\dagger, \hat{\mu})$ *solves* (P) *then* $\mu^\dagger \in \text{Conv}(\text{supp}(\alpha_0))$ *for any* $\alpha_0 \in T_v(\mu_0)$.

Proof of Proposition 8: Suppose $(p, \mu^\dagger, \hat{\mu})$ solves (P), and let $\alpha_0 \in T_v(\mu_0)$. Define $\underline{\mu} := \min\{\mu : \mu \in \text{supp}(\alpha_0)\}$ and $\overline{\mu} := \max\{\mu : \mu \in \text{supp}(\alpha_0)\}$. Suppose by way of contradiction that $\mu^\dagger < \underline{\mu}$. Then $\mu_0 = p'\underline{\mu} + (1-p')\hat{\mu}$ with $p' > p$. Moreover,

$$\begin{aligned} pv(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu}) &\leqslant p\hat{v}(\mu^\dagger) + (1-p)\hat{v}(\hat{\mu}) \\ &\leqslant p'\hat{v}(\underline{\mu}) + (1-p')\hat{v}(\hat{\mu}) \\ &= p'v(\underline{\mu}) + (1-p')\hat{v}(\hat{\mu}). \end{aligned}$$

The first inequality is obtained by noting that $v \leqslant \hat{v}$, the second by noting that $\hat{v}$ is concave, while the final equality follows from $\hat{v} = v$ at $\underline{\mu}$. Hence $(p', \underline{\mu}, \hat{\mu})$ does strictly better than $(p, \mu^\dagger, \hat{\mu})$ in the program (P), contradicting the optimality of $(p, \mu^\dagger, \hat{\mu})$. This shows that $\mu^\dagger \geqslant \underline{\mu}$. The proof that $\mu^\dagger \leqslant \overline{\mu}$ is similar and omitted.

■

Our next result shows that in the information acquisition case the principal never needs more signals than the number of actions he chooses from. Intuitively, if the optimal procedure were such that two signals induce beliefs for which the same action is optimal then the principal could design an alternative procedure merging these two signals without losing anything informationally. Moreover, since merging signals weakly increases the most likely signal, the agency cost in the alternative procedure must be weakly less than in the original procedure.

Proposition 9. *In the information acquisition case an optimal procedure exists which uses no more than* $|A|$ *signals.*

Proof: By Theorem 1, we just need to show that a solution to (P) exists with corresponding distribution over beliefs τ satisfying $|\text{supp}(\tau)| \leqslant |A|$. Suppose a solution to (P) has $|\text{supp}(\tau)| > |A|$. Then we can find $\tilde{a} \in A$ as well as μ_1 and μ_2 in $\text{supp}(\tau)$, $\mu_1 \neq \mu_2$, such that $v(\mu_i) = \max_a \sum_\omega \mu_i(\omega)u(a,\omega) = \sum_\omega \mu_i(\omega)u(\tilde{a},\omega)$. Let $\tilde{\mu} = \frac{\mu_1\tau(\mu_1)+\mu_2\tau(\mu_2))}{\tau(\mu_1)+\tau(\mu_2)}$; $v(\cdot)$ being the maximum of linear functions, note that we can write

$$v(\tilde{\mu}) = \sum_\omega \tilde{\mu}(\omega)u(\tilde{a},\omega). \tag{14}$$

Now construct $\tau' \in \Delta\Delta\Omega$ as follows: $\tau'(\mu) = \tau(\mu)$ for all $\mu \notin \{\mu_1, \mu_2, \tilde{\mu}\}$, $\tau'(\mu_1) = \tau'(\mu_2) = 0$ and $\tau'(\tilde{\mu}) = \tau(\mu_1) + \tau(\mu_2)$. Clearly, since τ is a splitting of μ_0, so is τ'. In view of (14), either $\mu^\dagger \notin \{\mu_1, \mu_2\}$ and then the maximand in (P) is the same under τ and τ', or $\mu^\dagger \in \{\mu_1, \mu_2\}$ and then the maximand in (P) is strictly greater under τ' than under τ.

■

As a corollary of Proposition 9, our results can be extended to any setting in which $|S| \geqslant \min\{|A|, |\Omega| + 1\}$. Example 1 is a case in point: in the example $|\Omega| + 1 = 3$ while $|S| = |A| = 2$.

Example. Consider the following example, taken from Kamenica and Gentzkow (2011):

$$v(\mu) = \begin{cases} 0 & \text{if } \mu \in [0, \frac{1}{2}) \\ 1 & \text{if } \mu \in [\frac{1}{2}, 1], \end{cases}$$

and $\mu_0 < \frac{1}{4}$.

As $\mu_0 < \frac{1}{4}$ any solution to (P) satisfies either $p = 1$ or $\hat{\mu} \geqslant \frac{1}{2}$. We can thus rewrite (P) as

$$\max_{\substack{p \in [\underline{\phi}, 1] \\ \mu^\dagger, \hat{\mu} \in \Delta\Omega}} \quad (1 - p) - \tilde{\gamma}(p)$$
$$\text{s.t.} \quad p\mu^\dagger + (1 - p)\hat{\mu} = \mu_0.$$

As $\tilde{\gamma}(p)$ is convex there exists a unique p solving this program, denoted $p(c)$. We obtain, by inspection of the first order condition,

$$p(c) = \max\left\{1 - 2\mu_0, \min\{1, \underline{\phi} + \sqrt{c\underline{\phi}}\}\right\}.$$

Selecting the Blackwell-most-informative solution gives

$$\mu^\dagger(c) := \begin{cases} \mu_0 & \text{if } c \geqslant c_3 \\ 1 - \frac{1 - \mu_0}{\underline{\phi} + \sqrt{c\underline{\phi}}} & \text{if } c \in [c_2, c_3] \\ 0 & \text{if } c \leqslant c_2 \end{cases}$$

and

$$\hat{\mu}(c) := \begin{cases} \mu_0 & \text{if } c \geqslant c_3 \\ 1 & \text{if } c \in [c_2, c_3] \\ \frac{\mu_0}{1-\underline{\phi}-\sqrt{c\underline{\phi}}} & \text{if } c \in [c_1, c_2] \\ \frac{1}{2} & \text{if } c \leqslant c_1, \end{cases}$$

for some constants $c_1 < c_2 < c_3$.[26] In particular, given $c = c_2$ the fully informative procedure solves the designer's problem.

Other Proof of Proposition 1: Let $c < c'$ and suppose that a solution to (P) at c (respectively c') exists which comprises the payment mass p (resp. p'). As p is optimal at c,

$$W(p) - \tilde{\gamma}_c(p) \geqslant W(p') - \tilde{\gamma}_c(p').$$

Hence,

$$W(p) - W(p') \geqslant \Big(\tilde{\gamma}_c(p) - \tilde{\gamma}_c(p')\Big).$$

Suppose by way of contradiction that $p > p'$. The cross derivative of $\tilde{\gamma}_c(p)$ being negative, we obtain

$$W(p) - W(p') > \Big(\tilde{\gamma}_{c'}(p) - \tilde{\gamma}_{c'}(p')\Big),$$

that is,

$$W(p) - \tilde{\gamma}_{c'}(p) > W(p') - \tilde{\gamma}_{c'}(p'),$$

contradicting the optimality of p' at c'.

■

Other Proof of Proposition 4: We proceed in two steps. The first step fixes the assignment function σ, and minimizes the cost of implementing τ given σ. The second step selects σ to minimize the cost of implementing τ.

[26] One checks that $c_1 = \frac{(1-2\mu_0-\underline{\phi})^2}{\underline{\phi}}$, $c_2 = \frac{(1-\mu_0-\underline{\phi})^2}{\underline{\phi}}$, and $c_3 = \frac{(1-\underline{\phi})^2}{\underline{\phi}}$.

Let $\sigma : \text{supp}(\tau) \to S$. Consider the problem

$$\begin{aligned}\gamma_\sigma(\tau) \equiv \quad & \min_{t:S\to\mathbb{R}_+} \sum_{\mu\in\text{supp}(\tau)} \tau(\mu)t\big(\sigma(\mu)\big) \\ & \text{s.t.} \sum_{\mu\in\text{supp}(\tau)} t\big(\sigma(\mu)\big)\big\{\tau(\mu) - \phi(\sigma(\mu))\big\} \geqslant c.\end{aligned}$$

We can rewrite this as

$$\begin{aligned}\gamma_\sigma(\tau) \equiv \quad & \min_{z:\text{supp}(\tau)\to\mathbb{R}_+} \sum_{\mu\in\text{supp}(\tau)} z(\mu) \\ & \text{s.t.} \sum_{\mu\in\text{supp}(\tau)} \left(\frac{\tau(\mu) - \phi(\sigma(\mu))}{\tau(\mu)}\right) z(\mu) \geqslant c.\end{aligned}$$

Any solution to the problem above satisfies $z(\mu) = 0$ for all $\mu \notin \arg\max \left\{\frac{\tau(\mu)-\phi(\sigma(\mu))}{\tau(\mu)}\right\}$, i.e. for all $\mu \notin \arg\min \frac{\phi(\sigma(\mu))}{\tau(\mu)}$. Moreover, defining $\ell_{\tau,\sigma} := \min_{\mu\in\text{supp}(\tau)} \frac{\phi(\sigma(\mu))}{\tau(\mu)}$, either $\ell_{\tau,\sigma} = 1$ in which case $\gamma_\sigma(\tau)$ is infinite, or $\ell_{\tau,\sigma} < 1$ in which case

$$\gamma_\sigma(\tau) = \frac{c}{1 - \ell_{\tau,\sigma}}.$$

Minimizing $\gamma_\sigma(\tau)$ over σ therefore amounts to minimizing $\ell_{\tau,\sigma}$ over σ. It is easy to see that $\ell_{\tau,\sigma}$ is minimized by assigning the most likely belief under τ to the least likely signal under ϕ. ■

Let $\tau_\psi \in \Delta\Delta\Omega$ denote the belief distribution induced by procedure ψ and $\mathcal{V} := \{\delta_\omega : \omega \in \Omega\}$.

Proposition 10. *Assume v convex and $|S| \geqslant |\Omega| + 1$. Then a solution to* (P0) *exists with the following properties:*

1. $|\text{supp}(\tau_\psi)| \leqslant |\Omega|$,
2. $|\text{supp}(\tau_\psi) \cap \mathcal{V}^\complement| \leqslant 1$.

Proof of Proposition 10: Part 2 follows from Theorem **??**. We therefore focus on part 1. Let $n := |\Omega|$. Consider a solution (ψ, t) to the program (P0). Let $\tilde{S} := \{s \in S : \max_{\omega\in\Omega} \psi(s|\omega) > 0\}$. By previous results, we may choose ψ such that $|\tilde{S}| \leqslant n + 1$, with payment at a single signal. If $|\tilde{S}| \leqslant n$ we are done, so assume $|\tilde{S}| = n + 1$. For convenience, label states and signals by $\Omega = \{\omega_1, \ldots, \omega_n\}$ and $\tilde{S} = \{s_1, \ldots, s_{n+1}\}$. As v is convex we can furthermore choose ψ such that the payment signal is s_{n+1} and, for all $i = 1, \ldots, n$: $\psi(s_i|\omega_i) > 0$ while $\psi(s_i|\omega_j) = 0$ for all $j \neq i$.

Note to start with that, by optimality of (ψ, t), there must exist $i_1 \neq i_2$ such that $\psi(s_{n+1}|\omega_{i_1}) > 0$ and $\psi(s_{n+1}|\omega_{i_2}) > 0$ (otherwise, if $\psi(s_{n+1}|\omega_j) = 0$ for all $j \neq i$ the principal could do strictly better by merging s_i and s_{n+1}). Next, define for all feasible $\varepsilon \in \mathbb{R}$ the modified procedure ψ_ε as follows:

$$\begin{cases} \psi_\varepsilon(s_{i_1}|\omega_{i_1}) = \psi(s_{i_1}|\omega_{i_1}) - \frac{\varepsilon}{\mu_0(\omega_{i_1})} \\ \psi_\varepsilon(s_{n+1}|\omega_{i_1}) = \psi(s_{n+1}|\omega_{i_1}) + \frac{\varepsilon}{\mu_0(\omega_{i_1})} \\ \psi_\varepsilon(s_{i_2}|\omega_{i_2}) = \psi(s_{i_2}|\omega_{i_2}) + \frac{\varepsilon}{\mu_0(\omega_{i_2})} \\ \psi_\varepsilon(s_{n+1}|\omega_{i_2}) = \psi(s_{n+1}|\omega_{i_2}) - \frac{\varepsilon}{\mu_0(\omega_{i_2})}. \end{cases}$$

Let E denote the set of feasible ε, that is, $E := \{\varepsilon : \psi_\varepsilon(s|\omega) \geqslant 0 \text{ for all } s \text{ and } \omega\}$. Note that E is an interval $[a, b]$ with $a < 0 < b$. The probability of s_{n+1} being the same under ψ and ψ_ε it ensues that (ψ_ε, t) satisfies (IC) for all $\varepsilon \in E$. Moreover, v being a convex function,

$$\sum_{\omega,s} \mu_0(\omega)\psi_\varepsilon(s|\omega)\big\{v\big(\mu(s;\psi_\varepsilon)\big) - t(s)\big\}$$

is convex in ϵ. The previous remarks imply that either (ψ_a, t) is another solution to the program (P0), or (ψ_b, t) is. Suppose (ψ_a, t) solves (P0) (the other case is analogous). Then either (i) $\psi_a(s_{i_2}|\omega_{i_2}) = 0$ or (ii) $\psi_a(s_{n+1}|\omega_{i_1}) = 0$. If (i) holds we have found a solution to (P0) which uses n signals. If (ii) holds we can repeat the steps above, starting from (ψ_a, t) instead of (ψ, t). Since n is finite, the recursion must end after a finite number of iterations.

■

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