Certification Delegation

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Abstract

This paper provides a theory of product quality certification and characterizes the optimal certification authority. Realistically, the authority puts a larger weight on quality relative to profit if the certifier represents activists or non-governmental organizations rather than an industry association. The larger is this weight, the higher is the equilibrium certification requirement, the larger are corporate incentives to make long-term investments in quality, but the smaller are the incentives to enter the market in the first place, according to our analysis. The relative importance of investments vs. entry determines the socially optimal certification authority and whether the government prefers to delegate to the industry or an NGO. In either case, a larger externality of consuming quality decreases the optimal authority's weight on profit. The predictions are consistent with anecdotal evidence and shed light on the long-term evolution of certifications.

Introduction

For markets to work efficiently, producers must communicate the quality of their products to the consumers. Only if such communication succeeds and is sufficiently credible we can expect consumers to pay more for valuable goods, and producers to invest in raising the quality. If the communication is not sufficiently credible, markets break down and we are left with lemons, as explained by Akerlof (1970).

Consumers might be able to learn product quality for goods that are simple or frequently consumed, but this is harder when consumers care about products' effects on health, risks, the environment, or on other public goods or externalities such as labor conditions. For such goods, there is a demand for third parties that can verify or certify the quality of products. This simple logic explains the raise of certifiers who guarantee that certified products satisfy some minimal requirements, often pre-specified by the certifier.

There is plenty of evidence of certified products being associated with higher product quality and/or lower production externalitites than non-certified ones. On the demand side, the credibility of certificates is reflected by the fact that certified products often command sizeable price premia.[1] On the supply side, large corporate investments undertaken to fullfill certification requirements are evidence of this credibility.[2]

At the same time, anectodal evidence suggests that different stakeholders, such as non-governmental organizations (NGOs) and industry associations, hold conflicting views on the optimal standards of certification. These conflicts reflect the stakeholders' objective functions and result in different standards set by certifiers that represent NGOs and those that represent industry associations, the latter being associated with laxer standards. The forest product industry and the construction industry are two examples of these differences. In both industries, the most commonly used NGO-backed standard is recognized to be more stringent than the industry-sponsored one.[3] These differences raise a host of questions. Which certifiers are socially optimal? Why do we see government/public certifiers only in certain markets? Why is it that some public certifiers let representatives of industry and NGOs decide the standards of their publicly-run certification programs?[4]

[1]For evidence of green labels' positive effects on consumer willingness to pay see Eichholtz *et al.* (2010) (green buildings); Hainmueller *et al.* (2015) and de Janvry *et al.* (2015) (fair-trade coffee); and Teisl *et al.* (2002) (dolphin-safe tuna).

[2]de Janvry *et al.* (2015) provides evidence of costly investments by coffee farmers to obtain fair-trade labels; Eichholtz *et al.* (2013) finds large differences in energy efficiency between labeled and non-labeled buildings.

[3]For forest product certification see Washington Post and GreenBiz.com, for the construction industry see The Portland Tribune.

[4]To the best of our knowledge all public certifier claim to work together with third parties to set up their standards. In the U.S, the EPA runs, at least for now, the Energy Star program; the EPA consults industry representatives but ultimately decides autonomouly

In our view, to answer these questions it is especially important to develop a deeper understanding of the *long-term* consequences of certification. In particular, what is the effect of certifier identity on producers' incentives to make long-term *investments* in product or production quality? How should we expect *entry* and *market size* to be influenced by the certifier identity?

The purpose of this paper is to address these questions head on. We develop a purposefully stylized and simple workhorse model that easily can be built on in future research. Naturally, even the simplest successful such model must include consumers, producers, and a certifier. The certifier specifies a quality requirement and producers decide whether to raise the quality of their products in order to satisfy the requirement. Firms have heterogeneous costs and the more demanding is the requirement, the smaller is the mass of firms finding it worthwhile to satisfy. According to our analysis, and in line with the empirical observations discussed above, a non-governmental organization prefers a more demanding requirement that does the government, who in turn prefers a thougher requirement than does an industry association. More generally, the larger is the certifier's weight on quality relative to profits, the more demanding is the certification requirement, in equilibrium. In the basic model, the government would always prefer to certify personally, rather than to delegate this choice to somebody else.

The analysis becomes more interesting when firms can take early actions. That is, each firm might be able to invest in quality, or in its ability to raise its quality in the future, before the exact certification requirement is known. (??) Since profit functions are naturally convex, a higher expected quality pays off more when the quality requirement is higher. (??) Barring case of unrealistically large non-divisibilities in firms' invetsments, we show that early investments are larger when the certifier is expected to put more weight on quality relative to profit. Consequently, the government prefers to delegate the certification authority to organizations emphasizing quality when these early investments are important, especially when quality also creates large positive externalities on the rest of the society.

We also endogenize the structure of the market. Profits are lower when the certification requirement is very demanding in our model. When entry into the market is costly, this implies that fewer firms find it optimal to

on standards (see xyz). In Europe, all the major public certifiers such as the Nordic Swan and the Blue Angel apply standards set by decision boards, which must at each point in time include a fixed number of representatives of the industry, of consumer associations and of public authorities.

enter when the certifier is an organization expected to emphasize quality over profit, while more firms enter when the certifier is, for example, the profit-maximizing industry association. Encouraging entry is beneficial to everyone in our model, since a larger total mass of firms implies a larger mass of firms able to satisfy any quality requirement at low costs. When entry is important and very sensitive to expected profit, everyone benefits if certification authority is delegated to the industry itself. Altogether, we show how the optimal certification authority depends on the importance of entry, investments, and consumption externalities.

Interestingly, the above predictions lead to a time inconsistency problem when it comes to delegating certification authority. Early on, before several firms have committed to serve the market, it may be beneficial (for everyone) to delegate authority to the industry itself. After the market structure is settled, at the stage when it is instead important to encourage firms to make long-term quality investments, the government benefits to rather delegate to activists or NGOs. When also these investments are sunk, however, the government would prefer to take control over the certification process itself. Our simple model thus sheds light on the evolution of the certifications at the same time as it detects a fundamental time inconsistency problem underlying this evolution. Our observations explain why some public certifier commit to set standards decided by representives of private interests, and can also explain why a public authority might prefer, at least for some time, to leave certification entirely in private hands.

The framework presented in this paper generates a number of testable predictions:

- non-governmental organizations prefer a more demanding requirement that does the government, who in turn prefers a thougher requirement than industry associations do; more demanding requirements are associated with a smaller share of certified firms;

- in the absence of any externality associated with product quality, government and industry prefer the same standards

- non-governmental organization are associated with large investments in the technology necessary to ensure products of high quality;

- industry certification is associated with large profits and increases in market supply.

Although a serious empirical test must await future research, we believe such research is promising because anecdotal evidence seems to be consistent with our predictions.

Our article contributes to the literatures on certification, and on delegation as a commitment device. Within the literature on certification, reviewed in Dranove and Jin (2010), we are closer to the articles that look at the role of certification when firms can determine the quality of their products (Shapiro (1986), Albano and Lizzeri (2001), Miklós-Thal and Schumacher (2013) and Amacher *et al.* (2004)) and to models in which certification affects the market structure (Guo and Zhao (2009) and Board (2009)). In Shapiro (1986) the certifier is the regulator, in Albano and Lizzeri (2001), Miklós-Thal and Schumacher (2013) certfiers are profit-maximizers, and in Amacher *et al.* (2004) the certifier is not a strategic player. The focus on NGO and for-profit certifiers relate our work to previous models of eco-labels, such as Bottega and De Freitas (2009) and Fisher and Lyon (2014). In Bottega and De Freitas (2009) and Fisher and Lyon (2014) there is no early investment and no firm entry. Bottega and De Freitas (2009) focus on coexistence of voluntary labels and minimum quality standards; Fisher and Lyon (2014) focus on competition among certifiers.

The strand of the literature that focuses on the role of certification in solving the "lemons" problem (Akerlof (1970)) is pioneered by Viscusi (1978). In this strand, our work is somewhat parallel to the recent Stahl and Strautz (2017). Stahl and Strautz (2017) look at demand for certificates from seller and from buyers. They look at markets with different business models: they hold fixed the objective of the certifier and change the identity of the subject demanding a certificate (sellers vs. buyers), we hold fixed the identity of the subject demanding a certificate and vary the objective of the certifier. Within this literature the influential work by Lizzeri (1999) suggests that our simplifying assumption that no fees are collected by the certifier is somewhat limiting. Lizzeri (1999) focuses on a certifier that wants to maximize (its own) profits, and show that it is optimal to provide no information. In our case certifiers do provide information, but none of them has the objective to maximize their own profits (however, we discuss the possibility of expanding our model in this direction in Section 5).

The literature on delegation as a commitment device is vast and originated from studies in monetary policy (Kydland and Prescott (1977), Barro and Gordon (1983), Rogoff (1985)). To the best of our knowledge, the only previous work to look at delegation to a private agency of the authority to certify products is Bizzotto *et al.* (2017). In Bizzotto *et al.* (2017), delegating to a certifier with a less precise signal can be optimal as it induces firms to disclose more information. There the issue is inducing the firms to disclose information about the fixed quality of their product, and public and private certifiers have the same objective function.

The paper is organized as follows. Next section presents our workhorse model, where we show and explain why NGOs prefer more demanding certification requirements than do governments, who in turn prefer tougher requirements than does an industry association. Section 3 allows firms to take early actions, before the exact requirements are defined, and we characterize the optimal certification authority and conditions under which the government prefers to delegate certification authority to NGOs. Section 4 endogenizes the market size by allowing for firm entry. We characterize the equilibrium market size as a function of the certifier identity, describe the optimal certification authority, and we divide the parameter space in regions defining whether the optimal certifier is the industry, the government, or an NGO. Section 5 informally discusses a number of extensions before Section 6 concludes. All proofs can be found in the Appendix.

1 The Basic Model

This section presents a simple workhorse model which can be built on in the following sections. The model intends to capture the key trade-offs when it comes to product quality and certification requirements. The model consists of consumers, producers, and a certifier.

Consumers: There is a mass 1 of homogeneous consumers. Each consumer can buy a unit of a good, in which case he gets utility $u = v(x) - p$, where $v(x)$ is the value of a good with quality x and p is the price of such a good; the consumer ensures utility of 0 if he does not buy. The value function v is increasing and concave, satisfies $v(0) \geq 0$, and for the most part we will consider the functional form $v(x) = v(0) + x^{\alpha}$, where $\alpha \in (1/2, 1)$ measures the concavity of $v(\cdot)$. As will be explained below, the outcomes are trivial for $\alpha \notin (1/2, 1)$.

Producers: There is a mass $m > 1$ of heterogenous producers, each producing 0 or 1 unit of a good each. To produce a unit of quality x_i, producer i incurs cost $c_i(x_i) = c(0) + x_i c_i$ where $c(0) \in [0, v(0)]$. We assume that the marginal cost of quality, c_i, is distributed i.i.d. uniformly on the interval $[c, c + \Delta]$. Consequently, if we order firms according to the c_i's, we have $c_i = c + i\Delta/m$ for $i \in [0, m]$. We let firm i's profit be denoted $\pi_i(x_i) = \mathbb{1}_i(p_i - c_i(x_i))$, where p_i denote firm i's revenue and $\mathbb{1}_i$ measures the number of units produced by firm i. Section 3 endogenizes the cost of providing quality, as captured by parameter c, and Section 4 endogenizes the market structure, as captured by m.

Certifier: In many markets consumers cannot observe product qualities. They may thus benefit from a third party guaranteeing that the product is of a given quality. We will refer to this third party as the certifier. The certifier determines and announces a requirement, and certifies all goods that are of this or higher quality.[5] The firms decide on their quality after observing the certification requirement (the next section allows firms to also take early actions). Hence, each firm finds it optimal to either satisfy the requirement exactly, or to set zero quality, so, in equilibrium, $x_i \in \{0, q\}\, \forall i$, if we henceforth let q represent the requirement. We let $n(q) \in [0, m]$ measure the equilibrium mass of producers that meets the requirement q. Section 5 discusses the implications of setting more than one threshold, as well as cases in which there is more than one certifier in a market. For the time being, our main results will not hinge on these complications. We will also assume that the certifier does not negotiate or trade with consumers or producers (by charging certification fees, for example).

Payoffs: The payoffs for consumers and producers are described already. The certifier's objective function may take several aspects into account. One extreme variant is that the certifier chooses q to simply maximize aggregate

[5]Consumers in our model observe the certifier's requirement. While in practice consumers might sometime be confused about the details of a certifier's requirement, regulation is moving in the direction of making these requirements transparent. For example in the US the Federal Trade Commission "allows certifications from trade groups in which a company is a member as long as the process in developing the seal is open and transparent." (New York Times, 2012).**[Could we say that a low standard in our model can be seen as a standard that is hard for consumers to interpret - i.e. a standard "with many loopholes"?]**

profit, which naturally is a function of q,

$$\Pi(q) \equiv \int_0^m \pi_i(q)di.$$

This objective may be reasonable if we have in mind a trade association, consisting of the firms in the market, seeking to define "best practice," as discussed in the Introduction. Such a certifier will be named C (for "Commerce" or "Corporations") and its choice q_C.

Another special case is a certifier that maximizes the aggregate quality

$$U(q) \equiv \int_0^{n(q)} q^\alpha di = n(q) q^\alpha.$$

This objective function may be reasonable if the certifier is a non-governmental organization (NGO) or an activist group: if so, we name the certifier A (for "Activist") and its choice, q_A. The aggregate quality $U(q)$ may represent the industry's total reduction in emissions, pesticides, child labor, or cruel animal treatment. Thus, $U(q)$ may well be a public good enjoyed by all citizens, and the social value of quality can be $U(q)$ multiplied with a large number, s, but s would not influence the choice of q_A. Furthermore, as long as each consumer is infinitely small, the public-good aspect of quality will not influence the consumer's choice beyond how q directly affects the consumer valuation $v(q)$. It is therefore inconsequential to have excluded externalities associated with quality from the consumers' objective function above, since these would not affect the consumers' choices. The analysis is also unchanged if there are negative externalities associated with consuming low-quality products, as discussed in Section 5.[6]

Externalities matter for a government or bureaucracy, B, which may take both profits and consumer welfare into account if B sets the requirement, q_B. That is, if B is the certifier, B may seek to maximize a weighted sum of aggregate profits and aggregate quality:

$$W(q) \equiv bU(q) + (1-b)\Pi(q),$$

where $b \in [0,1]$. Although a large public-good aspect of the qualities will not influence the purchasing decision of an individual consumer, it will naturally

[6]If the negative externality is $s(1 - v(q))$, decreasing in the quality q, the constant s can be ignored (since every consumer buys one product) and we can phrase this as a positive externality of consuming products with large q's.

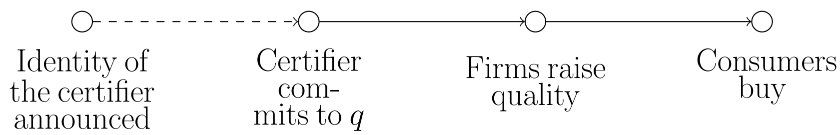

Figure 1: The timing of the game

be reflected in a larger weight b in B's objective function. Thus, the larger is the public-good aspect of q, the larger is b, when $v(q)$ continues to measure the extent to which an individual consumer benefit from its choice of quality.

In addition to discussing the effect of whether the certifier is A, B, or C, we will also allow for a more general certifier identity, D, who maximizes $dU(q)+(1-d)\Pi(q)$, where $d \in [0,1]$ may be different from 0, 1, and b. After all, the bureaucracy B may *delegate* the certification process to a committee consisting of all kinds of representatives, and who places various weights on profits vs. quality. With this, the choice of certifier, as measured by d, may be fine-tuned by B, as a function of b.

The reader may question whether consumer surplus should also be reflected in the certifier's objective function. It may well be, but the next subsection proves that in our model both individual and aggregate consumer surplus is constant (and independent of q) whenever $n(q) \in [0,1]$. Clearly, this constant will not affect anyone's choice of q, and it is thus inconsequential to let the certifier ignore consumer surplus. Section 5 informally discusses how to make consumer surplus relevant; we there also argue that our results continue to hold if we introduced certification fees and private certifiers maximizing the associated revenues: Revenue-maximizers will be equivalent to our industry association, we show.

Timing: Figure 1 illustrates the timing of the game. After the identity of the certifier is known to everyone, the certifier commits to q before the firms simultaneously decide whether to raise their firm-specific qualities enough to be certified. Thereafter, the consumers observe the products that are certified and those that are not, before each consumer buys exactly one product. The prices clears the market, as will be explained now.

1.1 The Market Equilibrium

The game is solved by backward induction, so we start by determining the market prices as a function of q and n. Suppose, for a start, that $n \in (0, 1)$, so that the mass of products that has been certified is smaller than the mass of consumers. Thus, some consumers will purchase certified products while others will not. Those consumers not purchasing certified products benefit from the fact that $m > 1$, implying that competition among the firms drives profits to zero for the firms producing non-certified products. Therefore, their price will be $p(0) = c(0)$, and the consumer surplus is $u(0) = v(0) - c(0)$. Clearly, every consumer is guaranteed at least this consumer surplus. At the same time, there is scarcity for the certified product, and competition among the consumers imply that each of them is willing to pay up to $p(q)$, given by $v(q) - p(q) = u(0)$, so $p(q) = c(0) + q^{\alpha}$. Consequently, the consumer surplus is simply equal to the constant $v(0) - c(0)$, regardless of q. Given the equilibrium price $p(q)$, the profit of certified firm i is $\pi_i(q) = q^{\alpha} - (c + i\Delta/m) q$.

Consider next the stage at which firms decide on whether to seek certification. Firm i benefits from meeting the quality threshold if:

$$\pi_i(q) \geq 0 \Leftrightarrow i \leq n(q) \equiv \frac{m}{\Delta}\left(\frac{q^{\alpha}}{q} - c\right).$$

We assume throughout that firm heterogeneity Δ is sufficiently large to ensure that $n \in (0, 1)$ holds in equilibrium, namely:

$$\Delta > mc\left(\frac{1}{2\alpha - 1} - 1\right),$$

but we relax this assumption at the end of the next subsection.

Naturally, the mass of certified firms, $n(q)$, decreases in the requirement q and the cost c, but increases in the mass of firms, m. With this, we get explicit solutions for the aggregate profit as well as the aggregate quality:

$$\Pi(q) \equiv \int_0^m \pi_i di = \int_0^{n(q)} p(q) - c_i(q)\, di = \frac{qm}{2\Delta}\left(\frac{q^{\alpha}}{q} - c\right)^2,$$

$$U(q) \equiv n(q)\, q^{\alpha} = \frac{mq^{\alpha}}{\Delta q}\left(q^{\alpha} - cq\right).$$

1.2 The Equilibrium Requirement

At the certification stage, certifier D prefers $q_D = \arg\max_q dU(q) + (1-d)\Pi(q)$. Given the explicit formulae for $U(q)$ and $\Pi(q)$, it is straightforward to derive our first benchmark result.

Proposition 1 *The equilibrium requirement is larger if d is large:*

$$q_D(d) = \left(\frac{(2\alpha-1)(1+d)}{c\left(\alpha + \sqrt{\alpha^2 - (2\alpha-1)(1-d^2)}\right)} \right)^{\frac{1}{1-\alpha}}, \text{ implying}$$

$$q_D'(d) > 0, \text{ and } q_A \geq q_B \geq q_C > 0, \text{ with strict inequalities if } b \in (0,1).$$

Consequently,

$$n(q_D(d)) = \frac{mc}{\Delta}\left(\frac{\alpha + \sqrt{\alpha^2 - (2\alpha-1)(1-d^2)}}{(2\alpha-1)(1+d)} - 1 \right) \in (0,1),$$

$$q_A = \left(\frac{2\alpha-1}{c\alpha}\right)^{\frac{1}{1-\alpha}}, \quad n_A = \frac{mc}{\Delta}\frac{1-\alpha}{2\alpha-1},$$

$$q_C = \left(\frac{2\alpha-1}{c}\right)^{\frac{1}{1-\alpha}}, \quad n_C = 2\frac{mc}{\Delta}\frac{1-\alpha}{2\alpha-1} = 2n_A.$$

In other words, A prefers a more demanding certification requirement that does B, who in turn prefers a more demanding requirement than does the commercial industry, C. More generally, the more weight d a certifier D puts on quality relative to profit, the higher is the requirement. This may be intuitive, but it is worth to explain the result in depth.

The scarcity of certified products implies that the firms capture the consumers' value of quality. Thus, the industry's aggregate revenue equals the aggregate quality $U(q)$, plus the constant part of the price, $c(0)$. If certifier C aimed at maximizing revenues alone, it would indeed prefer $q = \arg\max_q U(q)$. However, certifier C maximizes aggregate profit, and profit equals incomes minus costs. Since the costs increase in q, it follows that $q_C < q_A$. Intuitively, then, a convex combination of $U(q)$ and $\Pi(q)$ implies that B's preferred q is between q_A and q_C.

If we relax the assumption that $\Delta > mc\left(\frac{1}{2\alpha-1} - 1\right)$, it is still the case that $q_A \geq q_B \geq q_C$, but these inequalities might hold as equalities even if $b \in (0,1)$.

In particular, if $\Delta < mc\left(\frac{1}{2\alpha-1}-1\right)$, the heterogeneity between the firms is so small that a mass $n(q_C)=1$ of firms prefer to satisfy the requirement when C sets q_C to maximize $\Pi(q)$. A further reduction in q would reduce aggregate profit, however, since then a larger number of certified products would eliminate the scarcity of certified products and thus profits would decline. If Δ is still smaller, then even $n(q_B)=1$, and therefore $q_B=q_C$. Finally, if $\Delta < amc\left(\frac{1}{2\alpha-1}-1\right)$, even q_A implies $n(q_A)=1$, so, in this case, $q_A=q_B=q_C$.

1.3 The (Socially) Optimal Certifier

At the beginning of the game, the identity of the certifier is determined. Assume that the government, or its bureaucracy, B, can influence the choice of certifier. We motivate this assumption at length in the Introduction of the paper. [**Not sure how we motivate this. Why not saying that we just look at the socially optimal to see if there are inefficiences in the markets as they are and to explain why govermnet certifier might let various shareholder decide the standard.**] We here discuss B's optimal choice and let $\succ$ denote B's preference.

If B can freely choose among the certifiers $\{A,B,C\}$, B clearly prefers to certify personally. This leads to the first-best outcome, from B's point of view, while any other choice would imply that the equilibrium q would be either too small or too high. Even if B could delegate to *any* certifier with weight d on quality, B would prefer $d=b$.

Proposition 2
(i) The optimal certifier $d(b)$ is given by $d(b)=b$.
(ii) Consequently, $B \succeq A$ and $B \succeq C$. Furthermore, there exists a threshold $\bar{b}$ such that we have

$$A \succ C \text{ if } b > \bar{b},$$
$$A \sim C \text{ if } b = \bar{b}, \text{ and}$$
$$A \prec C \text{ if } b < \bar{b}.$$

While part (i) of the proposition is explained above, part (ii) is of interest only if B is restricted to choose between A and C as certifiers. This situation might arise if A and C have other characteristics (not modelled here) that makes them better certifiers than B would have been. If so, B would prefer the non-governmental organization to be the certifier rather than the industry in the case in which the social benefit of quality is large. This would be the case if quality is associated with important public goods such as, for example, reduced emissions.

[**I do not understand this paragraph**] The comparative statics above could be discussed at length. In particular, we find the effects of m and c to be quite interesting. The following two sections endogenize and discuss these variables by allowing for corporate early actions and entry of firms. These possibilities dramatically changes the choice of certifier, we will show.

2 Early Action

In reality, corporations make long-term investment decisions that influence their ability to deliver in the future. [**Should we add examples here or in the intro of firms investing in green quality?**] These investments relate to capacity and production technology, and naturally they will also influence their future costs of providing quality. To capture this incentive in the simplest possible way, assume that each firm makes the binary decision of whether or not to invest ex ante, before the certifier has determined the quality threshold. The ex ante investment costs k and reduces the future marginal cost of providing quality. In particular, the above parameter c, measuring the (lowest possible) cost of providing quality, is now assumed to be $\underline{c}$ for firms that invest, and $\overline{c} > \underline{c}$ for firms that do not.[7] Examples of such investments can be the installation of a new technology that is more environmentally friendly or the establishment of a supply chain in countries that are expected to treat employees better.

We allow the investment cost k to be stochastic and uniformly distributed with density σ_k on $[\underline{k} - 1/2\sigma_k, \underline{k} + 1/2\sigma_k]$. The distribution of k is common knowledge and its realization publicly observed. By assuming that the realization of k is perfectly correlated across firms, we can show that all firms invest when the realization of k is sufficiently small but no firm invests when k is sufficiently large. In both these cases, the distribution of the c_i's will be uniform, so the subsequent subgame is exactly as analyzed in Sections 2.1-

[7]As in the previous section, we assume large Δ. In this case the assumption takes the form $\Delta > m\overline{c}\left(\frac{1}{2\alpha-1} - 1\right)$.

2.2. To simplify, we will assume that the investment is relatively marginal (rather than "game-changing"), in that:[8]

$$\frac{\overline{c}-\underline{c}}{\underline{c}}<\frac{(1-\alpha)^2}{\alpha-1/2}.$$

The timing is as follows. After the identity of the certifier is known, k is realized and firms decide whether to invest. Thereafter, the game continues as in the previous section: The certifier sets the quality threshold, each firm decides whether to satisfy the threshold in order to be certified, and finally the consumers purchase products at market-clearing prices.

2.1 Equilibrium Investments

Since there is a continuum of firms, each of them takes the future quality threshold q as given when deciding whether to invest. For any given q, a firm's expected profit is $\mathbb{E}\pi_i=\frac{q}{2\Delta}\left(\frac{q^\alpha}{q}-c\right)^2$, and it is therefore profitable to invest if and only if:

$$k\leq\frac{q}{2\Delta}\left(\frac{q^\alpha}{q}-\underline{c}\right)^2-\frac{q}{2\Delta}\left(\frac{q^\alpha}{q}-\overline{c}\right)^2,$$

where the right-hand side is, one can show, a strictly concave function of q that reaches its maximum at $q^\dagger=\left(\frac{2\alpha}{\underline{c}+\overline{c}}\right)^{\frac{1}{1-\alpha}}$. It follows that the closer is the expected threshold q to $q^\dagger$, the larger is the set of k's such that firms invest, and thus the larger is the expected level of investments. By comparison, we have $q^\dagger>q_A$, as q_A is defined above,[9] suggesting that firms invest for a larger set of k's when the certifier is A than when it is B, and when it is B instead of C.

At the same time, equilibrium investments will influence the certifier's optimal choice of threshold. It is intuitive, and it follows formally from Proposition 1, that any certifier sets a lower quality threshold if firms do not invest: $\partial q_D(d)/\partial c<0$. Therefore, for any certifier characterized by the weight set on quality, d, there exist two relevant thresholds, $\widehat{k}_0(d)$ and

[8] The effect of this assumption is discussed in the next footnote.

[9] This is easy to see when $c=\overline{c}$. Also when $c=\underline{c}$, we have $q^\dagger>q_A$ under the assumption $\frac{\overline{c}-\underline{c}}{\underline{c}}<\frac{(1-\alpha)^2}{\alpha-1/2}$.

$\widehat{k}_1(d) > \widehat{k}_0(d)$: No firm invest when $k > \widehat{k}_0(d)$ and q is set as in Proposition 1 with $c = \overline{c}$, but all firms invest when $k < \widehat{k}_1(d)$ and q is set by the same formulae with $c = \underline{c}$. When $k \in \left(\widehat{k}_0(d), \widehat{k}_1(d)\right)$, we thus have multiple equilibria: If all firms invest, $q_D(d)$ will be high, and investments will indeed be optimal; if no firm invests, $q_D(d)$ would be smaller, and then firms will indeed prefer to not invest.[10]

Proposition 3 *(i) For investment-cost thresholds $k_0(d)$ and $k_1(d) > k_0(d)$, both increasing in d, there is an equilibrium where no firm invests if and only if $k \geq k_0(d)$ and there is an equilibrium where all firms invest if and only if $k \leq k_1(d)$.*
(ii) Whether for $k \in [k_0(d), k_1(d)]$ we assume no firm invests ($\varphi = 0$) or all firms invest ($\varphi = 1$), we have:

$$\mathbb{E}c_A^{\varphi} \leq \mathbb{E}c_B^{\varphi} \leq \mathbb{E}c_C^{\varphi},\ \varphi \in \{0,1\},$$

where, for any certifier D, $\mathbb{E}c_D^{\varphi}$ is the expected cost c, and $\mathbb{E}c_D^0 \leq \mathbb{E}c_D^1$.

Part (ii) states that the firms invest for the largest set of k's when the certifier is A, and for the smallest set when the certifier is C .When k is stochastic, the probability for early action is larger and the expected cost of providing quality is smaller when the certifier is A rather than B, or B rather than C.[11] This fact should be taken into account when it is socially efficient that firms invest.

As explained, part (ii) holds regardless of whether we think that all firms invest ($\varphi = 1$) or that no firm invests ($\varphi = 0$) when both equilibria are possible, as long as this answer is independent of the type of certifier. To fix ideas, we will from now assume that all firms invest when both equilibria are possible, although the analysis is basically analogous otherwise (i.e., when $\varphi = 0$).[12]

[10]When $k \in \left(\widehat{k}_0(d), \widehat{k}_1(d)\right)$, there can also be an equilibrium where some firms invest while others do not. However, one can show that this equilibrium is unstable: If a positive mass of firms changed their decision, it would become strictly optimal for every firm to take the same decision as this mass. For this reason, we have decided to not spend space on this possibility here.

[11]All the inequalities in part (ii) are strict if $k_{\varphi}(d) \in [\underline{k} - 1/2\sigma_k, \underline{k} + 1/2\sigma_k]$ for all φ and all d.

[12]For example, the below comparative static will be similar if instead no firm invests when $k \in [k_0(d), k_1(d)]$, although the exact thresholds in Proposition 4, as they are characterized in the proofs, will be different.

In order to focus on the most interesting parameter region, we assume in what follows that, regardless of the identity of the certifier, for the smallest realization of the investment cost firms do invest, while for the largest realization of the entry cost firms do invest, that is:

$$\underline{k} - 1/2\sigma_k < k_1(0) < k_1(1) < \underline{k} + 1/2\sigma_k. \quad (1)$$

2.2 The Optimal Certifier with Early Action

As in Section 2, suppose that the identity of the certifier is determined at the beginning of the game, i.e., before the realization of k is known. In Section 2, the equilibrium certification requirement were too demanding under A and too lax under C, so B preferred to certify personally. With the possibility to take early action, however, we know that firms are more likely to invest when q is high, and higher than the threshold that will be ex post preferred by B.

To fix ideas, we assume that the investment is socially desirable, that is, for any realization of k and any d welfare increases if firms invest early on:[13]

$$(1-\alpha)\left(2\alpha-1\right)^{\frac{2\alpha-1}{1-\alpha}}\left((1-b)\left(\frac{1-\alpha}{\alpha^{\frac{1}{1-\alpha}}}\right)+4b\right)\left(\underline{c}^{\frac{1-2\alpha}{1-\alpha}}-\overline{c}^{\frac{1-2\alpha}{1-\alpha}}\right)>2\Delta(1-b)(\underline{k}+\frac{1}{2\sigma_k}) \quad (2)$$

To motivate firms to invest, B may thus prefer to delegate certification authority to a certifier that puts a larger weight on quality, since the equilibrium q will then be larger. This type of strategic delegation is particularly valuable when a small increase in q (or in d) will raise the likelihood of investments by a substantial amount. This amount is larger when the density σ_k of the k distribution is large, i.e., when the variance in k is smaller. If the variance in k were very large, it will most of all be the random realization of k that would dictate whether or not firms invested, and a (small) change in q would have a smaller influence.

Proposition 4 *(i) The optimal certifier is given by a function $d_k(\sigma_k, b) > b$, increasing in both σ_k and b.*
(ii) If $A \succ B$ for some σ_k, then $A \succ B$ for all $\sigma'_k > \sigma_k$.

[13]The assumption is equivalent to $(1-b)(\Pi(q_A|\underline{c}) - \Pi(q_A|\overline{c})) + b(U(q_C|\underline{c}) - U(q_C|\overline{c})) > \underline{k} + \frac{1}{2\sigma_k}$.

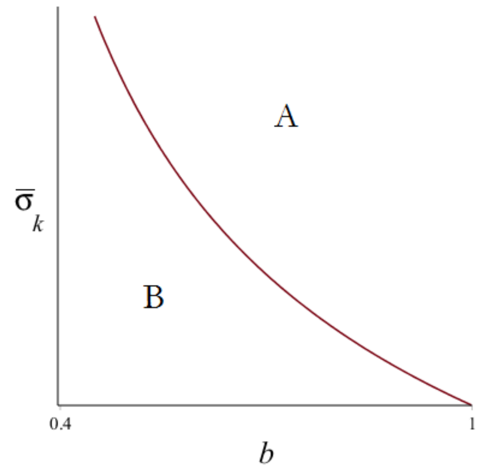

Figure 2: Threshold $\overline{\sigma}_k(b)$.

When quality is beneficial, i.e., when b is large, it is important to motivate firms to invest in their capacities to provide quality. Thus, a larger b is also making it more attractive to delegate to a certifier with a large d. Therefore, the threshold for σ_k is smaller when b is large. Figure 2 captures this intuition and illustrates that $A \succ B$ is optimal when $\sigma_k > \overline{\sigma}_k\,(b)$.

3 Entry

While the previous section endogenized the firms' types, this section endogenizes the market structure, as represented by the mass of firms. After all, although some firms may have entered the market years ago, the willingness of new firms to enter will hinge on the profit they expect. To capture this, suppose that while the mass $\underline{m}$ of firms can be expected to serve the market for any weakly positive expected profit, an additional mass $(\overline{m} - \underline{m})$ can also possibly enter.[14] For each new entrant, the entry cost g must be compared to the expected profit. Thus, in equilibrium, a new firm enters the market if and only if $\mathbb{E}\pi \geq g$.

Analogously to the way we modeled early action, we can allow the entry cost g to be stochastic and uniformly distributed with density σ_g on some interval $\left[\underline{g} - 1/2\sigma_g, \underline{g} + 1/2\sigma_g\right]$. The distribution of g is common knowledge

[14] As in the last two sections, we assume large Δ: $\Delta > \overline{m}\,\overline{c}\left(\frac{1}{2\alpha-1} - 1\right)$.

and its realization is publicly observed. As for the investment cost k, we assume that the realization of g is perfectly correlated across firms.

The timing is as follows. After the identity of the certifier is known, g is observed and firms decide whether to enter the market. Thereafter, the game continues as in the previous section: k is realized, firms decide whether to invest, the certifier sets the quality threshold, each firm decides whether to satisfy the threshold in order to be certified, and finally the consumers purchase products at market-clearing prices.

3.1 Equilibrium Entry

In equilibrium, the mass of firms will be $m \in [\underline{m}, \overline{m}]$. As the analysis above has shown, the equilibrium level of m does not influence any firm's investment decision or the certifier's choice of quality threshold. The expected profit for each firm, $\mathbb{E}\pi_i$, takes into account the option of investing later, and this profit will be independent of m. Thus, depending on the expected quality threshold, which, in turn, depends on the certifier's identity D, there will be a threshold g_D such that the additional $(\overline{m} - \underline{m})$ firms enter if and only if $g \leq g_D$. Consequently, the expected mass of firms is given by $\mathbb{E}m_D = \underline{m} + (\overline{m} - \underline{m}) \Pr(g \leq g_D)$. Since certifier C sets q such as to maximize (ex post) profit, it is intuitive that $\mathbb{E}m_D$ is larger if the certifier is C, or, more generally, if d is small.[15]

Proposition 5 *For any certifier D, the expected equilibrium mass of firms, Em_D, is a decreasing function of the expected q, and thus of d. Consequently,*

$$\mathbb{E}m_A < \mathbb{E}m_B < \mathbb{E}m_C.$$

In order to focus on the most interesting parameter region, we assume in what follows that, regardless of the identity of the certifier, for the smallest realization of the entry cost firms do enter the market, while for the largest realization of the entry cost firms do enter the market, that is:

$$\underline{g} - 1/2\sigma_g < \mathbb{E}m_A < \mathbb{E}m_C < \underline{g} + 1/2\sigma_g. \tag{3}$$

[15]Without early action, profits are maximized at q_C and thus expected entry is maximized when $d = 0$ (i.e., $D = C$). With the possibility to invest in quality, individual firms may in equilibrium invest more than the level that would had maximized aggregate profits since investments will raise q and thus (other) firms' costs. In that case, the profit-maximizing d is negative.

3.2 The Optimal Certifier with Entry and Early Action

Entry is valuable to everyone in our model. A larger mass of firms increases aggregate profit and also the total quality, since there is then a larger mass of firms that finds it affordable to satisfy any quality threshold. Thus, every player in this model has an incentive to encourage entry.[16] Entry is attractive when the expected profit is large, and profit is large when the certifier places a small weight d on quality relative to profits. Thus, it is beneficial to delegate certification authority to someone who places a small weight on quality if entry is very important. Encouraging entry is important when σ_g is large, since then a small change in q (or in d) dramatically influences the probability that additional firms enter. Thus, the larger is σ_g, the smaller is the optimal d. If σ_g is instead small, so that the variance in entry cost is large, then it will mainly be the realization of the random entry cost that determines whether firms enter, and the effect of d will be minor. In this case, it will be relatively more important to motivate firms to invest, and then a larger d will be optimal (and $D = A$ will be better than B and C).

Proposition 6 *(i) The optimal certifier is given by a function* $d_m(\sigma_m, b)$*, decreasing in* σ_g *and increasing in* b*.*
(ii) For any σ_g*:*

$$\text{if } A \succ D \text{ then } A \succ D \text{ for all } \sigma_g' < \sigma_g,$$
$$\text{if } C \succ D \text{ then } C \succ D \text{ for all } \sigma_g' < \sigma_g.$$

As before, a larger b makes it more attractive to delegate to A rather than to C. Thus, the threshold for σ_g, specifying exactly when it is optimal to delegate to an authority with a smaller d, is larger when b is larger. This explains the two lines in Figure 3.2, dividing the parameter space in regions under which it is better to delegate certification authority to A, B, or to C.

With this, our theory can explain when governments prefer to delegate certification authority to private sectors emphasizing profits, as a way to encourage entry, or to non-governmental organizations that instead focus on quality, in order to motivate firms to invest in quality. Although the

[16]This could change if entry itself is associated with negative externalities, for example because firms that fail to sell to the consumers in this market can continue to produce by exporting their goods to other markets. Section 5 discusses this possibility.

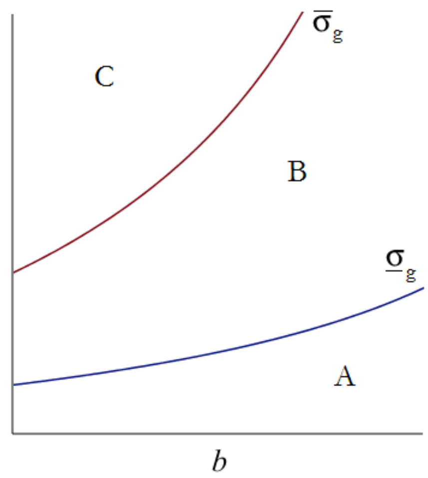

Figure 3: Thresholds $\overline{\sigma}_g(b)$ and $\underline{\sigma}_g(b)$.

results can be interpreted normatively, they also provide testable empirical predictions when governments can influence the structure of the certification market. We discuss the consequences of these predictions in the next section.

4 Extensions and Generalizations

Our workhorse model is stylized and simple exactly because it can then be built on in several directions. Above, we showed how the model could easily permit both entry and long-term investments in the capacities to provide quality. In this section, we informally discuss a number of other possible extensions. In addition to the ones we discuss, there are naturally several alternative ways of extending the model that are both realistic and important. This leaves us with an interesting research agenda in the next few years.

4.1 Developments and Time Inconsistency

Taken together, the analysis above suggests the following development of certification over time. Early on, while it is still possible and important that firms enter the market, it is socially desirable and the government prefers that certification is delegated to someone that places a large weights on profits. At this stage, an industry association may itself be the best provider of certifications. At later stages, however, once the market is further developed, and it remains to motivate existing firms to take the right long-term investments in their capacities to provide quality, it would be socially optimal that the future certification authority instead places more weight on quality. At this stage, therefore, it may be optimal and preferred by the government that an NGO is responsible for certifications. Finally, when many of these investments are sunk and it remains to fully employ the technologies firms have invested in, the government would prefer to take control and itself decide on the quality certification threshold.

The discussion in Section 4 is mostly relevant when the market fully never matures, and when we can always expect the possibility of additional entry and long-term investments. In this case, Figure 3 illustrates how the optimal certification authority depends on parameters in the model.

Insofar as the market does develop by more or less completing the entry stage before firms make long-term commitments to technology, our model provides a theory for how certification should and will evolve over time. The predictions of the theory are testable and, to us, it seems realistic that the industry itself is the certification provider at the beginning, while the government may take a more active role in more developed market. While it would be tempting to speculate further by discussing easily accessible anecdotal evidence, we believe the analysis' predictions deserve a more careful empirical test in future research.

The predicted evolution also reveals an underlying time inconsistency problem when it comes to certification; a problem that has so far been neglected in the literature, as far as we know. While Figure 3 and Proposition 6 describe how it is optimal to delegate authority for the long-run, in order to motivate both entry and investments, the firms may anticipate that the government will later prefer to let the certification authority be represented by agencies that places a smaller weights on profits. When this development is anticipated, entry will be immediately affected. It is simply not sufficient to let an industry association certify today, when the firms can rationally expect that certificates will be more demanding (and profits will be lower) later. Similarly, the right long-term investments are not sufficiently profitable by letting NGOs certify today, if firms can expect that governments will later prefer to take control and relax the quality requirements. Combined, both entry and investments can be negatively affected if the government is unable to commit to future delegation authorities. While our formal analysis presumed that the government could commit in that the certifier's identity was determined at the very beginning of the game, it is simple to see the consequences of relaxing this ability to commit, by moving the delegation stage to a later date in the game.

4.2 Private Certifiers and Certification Fees

If firms must pay some (endogenous) fee f to be certified, firm i is willing to pay at most $q^\alpha - (c + i\Delta/m)\, q$, implying that the total number of firms willing to pay f is

$$m \frac{q^\alpha - cq - f}{q\Delta},$$

and the total revenue from collecting the fees is:

$$fm \frac{q^\alpha - cq - f}{q\Delta}.$$

With this, it is easy to take first-order conditions and derive the revenue-maximizing f and q, and then to compare this q to the q's derived in Section 2. For a given f, the revenue-maximizing q is:

$$-(1-\alpha)\, q^{\alpha-2} + fq^{-2} = 0 \Rightarrow q^\alpha = \frac{f}{1-\alpha},$$

if we assume that the following second-order condition holds:

$$(2-\alpha)(1-\alpha)\, q^{\alpha-3} - 2fq^{-3} < 0 \Rightarrow q^\alpha < \frac{2f}{(2-\alpha)(1-\alpha)},$$

which clearly holds locally when $q^\alpha = \frac{f}{1-\alpha}$. The first-order condition is then sufficient.[17]

For a given q, the revenue-maximizing f is:

$$f = \frac{q^\alpha - cq}{2}.$$

Combined, q is exactly equal to q_C, derived above:

$$q^\alpha = \frac{q^\alpha - cq}{2(1-\alpha)} \Rightarrow q = \left(\frac{2\alpha - 1}{c}\right)^{\frac{1}{1-\alpha}} = q_C.$$

[17]The first-order condition is sufficient because, when the second-order condition does not hold, it would be optimal to set q as large as possible, but then the the number of firms seeking certification (and the associated revenues) become zero.

Consequently, allowing the industry to set q is equivalent to letting a private revenue-maximizing certifier to set q, in our model. It is quite intuitive that a revenue-maximizing certifier, which captures the firms profits, also prefers to set q such as to maximize these profits.

Since the optimal q trade-offs larger revenue per licensed firm with a smaller mass of firms seeking certifications, q_C is smaller than the quality threshold that would have maximized f alone: $q = (\alpha/c)^{\frac{1}{1-\alpha}}$, and this q is also larger than q_A, it is easy to see. This suggest that $f_A > f_B > f_C$, when every certifier $D \in \{A, B, C\}$ sets the fee f_D optimally. At the same time, A and B care about the total quality that is provided, and the total number of firms seeking certification is smaller when f is large. Thus, for any given q, both A and B would have preferred a smaller fee than the revenue-maximizing f, described above. When this effect is taken into account, it is thus not clear whether A and B will charge a higher or a lower fee than does C. What does seem to be clear, however, is that when the the certifier's weight d on quality is larger, q is larger and $n(q)$ is smaller, just as in our analysis above.

If also other certifiers (such as A and B) collect fees and place some weight on these revenues, they will be more similar to C the larger is this weight, and the optimal quality threshold will be smaller. If B, but not A, places a larger weight on collecting the fees, B should be more likely to prefer C as certifier rather than A. If A, but not B, places a larger weight on collecting fees, A becomes more similar to C (and thus to B) when setting the quality threshold, and B should be more likely to prefer A as certifier rather than C, everything else equal.

4.3 Negative Externalities and Consumer Surplus

In our model, a larger weight on quality was justified if high quality had a social value beyond what individual purchasers' could be willing to pay for. This is reasonable for many types of goods, such as when the good's characteristics or production process determine environmental impacts or labor conditions. As explained already, assuming positive externalities associated with consuming high-quality is equivalent to assuming negative externalities associated with consuming low-quality products, since we are assuming that every consumer purchases exactly one good. Under this assumption, it also seems reasonable to ignore any externalities associated with entry itself, since many firms do not produce at all in the equilibrium we have described.

More realistically, however, demand is not as inelastic as we have assumed, and firms that fail to sell in our market may have the opportunity to export their products to other markets and still generate externalities. In this situation, entry itself would generate externalities, and now it would matter a great deal if these externalities are negative or positive. If they are negative, because production leads to emissions, for example, then it is no longer clear that it is socially desirable to encourage market entry.

It would be simple to extend our model to this situation, by allowing for negative externalities associated with entry itself. If these externalities were negative and large, it would be optimal to discourage entry and this is achieved by letting the certifier place higher weights on quality. These externalities are thus having the same impact as the externalities that are already in our model: everything else equal, such externalities would raise the optimal weight the certifier should place on quality and, when the externalities are large, the government is more likely to prefer to delegate certification authority to an NGO than to the industry itself.

It is also straightforward to expand our model such as to make consumer surplus relevant. Above, consumer surplus was constant (and thus irrelevant) whenever the number of certified firms was smaller than the number of consumers, and this, we showed, hold in the model when the certifier did not care about consumer surplus itself. At the same time, it is easy to see that consumer surplus would be larger if one relaxes the quality threshold so much that the number of certified firms is larger than the number of consumers. A certifier that places a large weight on consumer surplus may thus be inclined to lower the quality threshold, as a way to intensity competition among certified firms. This effect implies that certifier's objective function may no longer be single-peaked in the quality threshold, and we thus have to compare the values at different local peaks to determine the equilibrium threshold. While this exercise will generate a few additional interesting results, the intuition for the above qualitative findings, as described by Propositions 1-6, seems quite robust, in our view.

5 Concluding Remarks

Product labels and quality certificates are used in many markets, and many of these are associated with consumption externalities that depend on the characteristics of the good. This paper analyses how quality certificates and the certifier identity affects firms' long-term decisions such as to enter markets and to invest in their abilities to raise the quality. We show that when the certifier puts more weight on quality relative to profit, as if the certifier represents a non-governmental organization rather than an industry association, then the quality requirement is higher, long-term investments higher, but entry and the market size is smaller. The optimal type of certification authority then depends on how important it is to influence investments vs. entry, and how sensitive these decisions are (and this, in turn, is related to the variance in firm costs). For all situations, we also show that the larger is the externality from consuming quality, the smaller is the optimal weight one should place on profit. For large such externalities, an NGO may be the best certifier; for small externalities, an industry association would lead to higher welfare, while the government or its bureaucracy should take control for intermediate values. These predictions also shed light on the development of certification over time, and the underlying time-inconsistency problem associated with this development.

Future research on these issues are [therefore] very promising, in our view. On the theoretical side, the workhorse model in this paper is stylized and deserves to be extended in several directions. To illustrate the results in a minimalistic model, we have assumed specific functional forms, homogeneous consumers, a single quality requirement (rather than a ladder), and a single certification authority (rather than allowing several certifiers to compete). Some of these aspects are discussed in papers that are complementary to ours, as discussed in the Introduction, but future research should combine the approaches in order to get a deeper understanding of product labels and quality improvements. Credible communication between producers and consumers is crucial for markets to work efficiently, and determines whether regulation is warranted.

Appendix

Proof of Proposition 1. We derive here the expression for $q_D(d)$. The rest of the proposition is an immediate consequence. In order to derive $q_D(d)$, we first solve an unconstrained maximization problem, then we show that the candidate $q_D(d)$ satisfies $n(q_D(d)) \leq 1$ for any $d \in [0,1]$.
From our discussion in the main text:

$$\begin{aligned} q_D(d) &= \arg\max_q dU(q) + (1-d)\Pi(q) \\ &= \arg\max_q \frac{m}{2\Delta}(q^{\alpha-1} - c)\left((1+d)q^\alpha - (1-d)qc\right). \end{aligned}$$

Note that $dU(q) + (1-d)\Pi(q) = 0$ in correspondence of the two "corner solutions" ($q = 0$ and $q = \min\{q|n(q) = 0\}$), and $dU(q) + (1-d)\Pi(q)$ is increasing in q for $q = 0$. Thus, the optimal q is interior. The first-order condition corresponds to:

$$(2\alpha-1)(1+d)q^{2(\alpha-1)} - 2\alpha c q^{\alpha-1} + (1-d)c^2 = 0. \tag{4}$$

Equation (4) holds if and only if $q \in \{f_1(d), f_2(d)\}$, where

$$f_1(d) := \left(\frac{c\left(\alpha + \sqrt{\alpha^2 - (2\alpha-1)(1-d^2)}\right)}{(2\alpha-1)(1+d)}\right)^{\frac{1}{\alpha-1}} \text{,and}$$

$$f_2(d) := \left(\frac{c\left(\alpha - \sqrt{\alpha^2 - (2\alpha-1)(1-d^2)}\right)}{(2\alpha-1)(1+d)}\right)^{\frac{1}{\alpha-1}}.$$

For $q = f_1(d)$ the second-order condition holds, while for $q = f_2(d)$ the second-order condition is violated. Thus the only candidate is $q_D(d) = f_1(d)$. Note moreover that $n(f_1(0)) < 1 \Leftrightarrow \Delta > mc(\frac{1}{2\alpha-1} - 1)$. As $n(q)$ is decreasing in q and $f_1(d)$ is increasing in d, then $\Delta > mc(\frac{1}{2\alpha-1} - 1)$ ensures that $n(f_1(d)) \leq 1$ for all $d \in [0,1]$. ■

Lemma 7 *For $q \in [q_C, q_A]$, expected welfare $bU(q) + (1-d)\Pi(q)$ is concave.*

Proof of Lemma 7. The second-order condition $\frac{d^2(bU(q)+(1-d)\Pi(q))}{d^2q}$ corresponds to:

$$\begin{aligned}(2\alpha-1)(1+b)2(a-1)q^{2\alpha-3}-2\alpha c(\alpha-1)q^{\alpha-2}<0 \Leftrightarrow\\ (2\alpha-1)(1+b)q^{\alpha-1}-\alpha c>0 \Leftarrow\\ (2\alpha-1)q^{\alpha-1}-\alpha c>0 \Leftrightarrow\\ q<q_A.\end{aligned}$$

■

Proof of Proposition 2. Part (i) is explained above the proposition. Part (i) implies: $B \succeq A$, $B \succeq C$, and for $b=0$, $A \prec C$, while for $b=1$, $A \succ C$. Note moreover that $U(q_A) > U(q_C)$ while $\Pi(q_A) < \Pi(q_C)$, thus, $(bU(q_A)+(1-b)\Pi(q_A)) - (bU(q_C)+(1-b)\Pi(q_C))$ is increasing in b. These observations imply that there exist a threshold $\bar{b} \in (0,1)$ such that $A \prec C$ if $b \in [0,\bar{b})$, $A \sim C$ if $b=\bar{b}$, and $A \succ C$ if $b \in (\bar{b},1]$. ■

Proof of Proposition 3. For any given d, an equilibrium where no firm invests exists if and only if, conditional on all other firms not investing, it is optimal not to invest, that is, letting $q_D(c)$ refer the requirement chosen by certifier D for expected cost c:

$$\begin{aligned}k \geq k_0(d) :=& f(q_D(\bar{c})), \text{ where}\\ f(q) :=& \frac{q}{2\Delta}\left(q^{\alpha-1}-\underline{c}\right)^2-\frac{q}{2\Delta}\left(q^{\alpha-1}-\bar{c}\right)^2.\end{aligned}$$

Similarly, an equilibrium where all firms invest exists if and only if, conditional on all other firms investing, it is optimal to invest, that is:

$$k \leq k_1(d) := f(q_D(\underline{c})).$$

Note that $f(q)$ is increasing if q is small:

$$\frac{df(q)}{dq} > 0 \Leftrightarrow q < \left(\frac{\underline{c}+\bar{c}}{2\alpha}\right)^{\frac{1}{\alpha-1}};$$

note also that $q_D(c)$ is decreasing in c and increasing in d, thus $q_D(c) < q_A(\underline{c})$. It can be checked that

$$q_A(\underline{c}) \leq \left(\frac{\underline{c}+\bar{c}}{2\alpha}\right)^{\frac{1}{\alpha-1}} \Leftrightarrow \frac{\bar{c}-\underline{c}}{\underline{c}} < \frac{(1-\alpha)^2}{\alpha-1/2},$$

thus both $k_0(\cdot)$ and $k_1(\cdot)$ are increasing in q. As $q_D(c)$ is decreasing in the average cost c, then $f(q_D(\underline{c})) > f(q_D(\overline{c}))$, or, equivalently, $k_1(d) > k_0(d)$ for all d, completing the proof of part (i).
To prove part (ii), note that:

$$\mathbb{E}c_D^0 = \underline{c} + (\overline{c} - \underline{c})\mathbb{P}(k > k_0(d)),$$

thus $\mathbb{E}c_D^0$ is strictly decreasing in d if $k_0(d) \in \left[\underline{k} - \frac{1}{2\sigma_k}, \underline{k} + \frac{1}{2\sigma_k}\right]$, and constant in d otherwise. This, and the observation that $f(q)$ is increasing in q, together imply $\mathbb{E}c_A^0 \leq \mathbb{E}c_B^0 \leq \mathbb{E}c_C^0$. The proof of $\mathbb{E}c_A^1 \leq \mathbb{E}c_B^1 \leq \mathbb{E}c_C^1$ follows the same steps. Finally, $k_1(d) > k_0(d)$ implies $\mathbb{E}c_D^1 \geq \mathbb{E}c_D^0$. ■

Lemma 8 *With ex ante investment, expected welfare is single-peaked in d, and $d_k(\sigma_k, b) \leq b$.*

Proof of Lemma 8. For any d, condition (1) ensures that $k_1(d) \in \left[\underline{k} - \frac{1}{2\sigma_k}, \underline{k} + \frac{1}{2\sigma_k}\right]$, thus expected welfare, $W(d)$, is

$$W(d) := m\mathbb{P}(k > k_1(q))\left(b\mathbb{E}u_i(d, \overline{c}) + (1-b)\mathbb{E}\pi_i(d, \overline{c})\right) +$$
$$\mathbb{P}(k < k_1(q))\left(b\mathbb{E}u_i(d, \underline{c}) + (1-b)\left(\mathbb{E}\pi_i(d, \underline{c}) - \frac{k_1(d) + \left(\underline{k} - \frac{1}{2\sigma_k}\right)}{2}\right)\right),$$

where $\mathbb{E}\pi_i(d, c)$ corresponds to $\mathbb{E}\pi_i(q_D)$ for expected cost c, and $\mathbb{E}u_i(d, c)$ is the expected quality, that is: $\mathbb{E}u_i(d, c) := \frac{q_D^\alpha(q_D^{\alpha-1} - c)}{\Delta}$. W is a continuous function, and it is convenient to write its first derivative as the sum of two expression, $W'(d) = m \cdot w_1(d) + m \cdot w_2(d)$, where

$$w_1(d) := \mathbb{P}(k > k_1(q))\left(b\frac{\partial \mathbb{E}u_i(d, \underline{c})}{\partial d} + (1-b)\frac{\partial \mathbb{E}\pi_i(d, \underline{c})}{\partial d}\right) + \mathbb{P}(k < k_1(q)) \times$$
$$\left(b\frac{\partial \mathbb{E}u_i(d, \overline{c})}{\partial d} + (1-b)\frac{\partial \mathbb{E}\pi_i(d, \overline{c})}{\partial d}\right),$$
$$w_2(d) := \sigma_k k_1'(d)\left(b\left(\mathbb{E}u_i(d, \underline{c}) - \mathbb{E}u_i(d, \overline{c})\right) + (1-b)\left(\mathbb{E}\pi_i(d, \underline{c}) - \mathbb{E}\pi_i(d, \overline{c}) - k_1(d)\right)\right).$$

We proceed by proving two ancillary remarks.

Remark 8.1. *$w_1(d) > 0$ if $d \in [0, b)$, $w_1(d) = 0$ if $d = b$, and $w_1(d) < 0$ if $d \in (b, 1]$.*

Note that:
(i) $b\frac{\partial \mathbb{E}u_i(d,c)}{\partial d} + (1-b)\frac{\partial \mathbb{E}\pi_i(d,c)}{\partial d} = \frac{\partial(b\mathbb{E}u_i(q)+(1-b)\mathbb{E}\pi_i(q))}{\partial q}|_{q=q_D} \cdot q_D'(d)$,
(ii) $q_D'(\cdot) > 0$,
(iii) Lemma 7 ensures that for any expected cost c, $b\mathbb{E}u_i(q) + (1-b)\mathbb{E}\pi_i(q)$ is concave for $q \in [q_C, q_A]$, and maximized at $q = q_B$.
Observations (i), (ii), and (iii) prove Remark 1.

Remark 8.2. *$w_2(d) > 0$ for any $d \in [0, 1]$.*

Condition (2) is equivalent to:

$$b\left(\mathbb{E}u_i(0, \underline{c}) - \mathbb{E}u_i(0, \overline{c})\right) + (1-b)\left(\mathbb{E}\pi_i(1, \underline{c}) - \mathbb{E}\pi_i(1, \overline{c}) - \left(\underline{k} + \frac{1}{2\sigma_k}\right)\right) > 0. \tag{5}$$

Note that $\frac{\partial^2 \mathbb{E}u_i(d,c)}{\partial d \partial c} < 0$, and $\frac{\partial^2 \mathbb{E}\pi_i(d,c)}{\partial d \partial c} < 0$. Therefore $\mathbb{E}u_i(0, \underline{c}) - \mathbb{E}u_i(0, \overline{c}) < \mathbb{E}u_i(d, \underline{c}) - \mathbb{E}u_i(d, \overline{c})$ and $\mathbb{E}\pi_i(1, \underline{c}) - \mathbb{E}\pi_i(1, \overline{c}) < \mathbb{E}\pi_i(d, \underline{c}) - \mathbb{E}\pi_i(d, \overline{c})$ for any $d \in [0, 1]$. Moreover $\underline{k} + \frac{1}{2\sigma_k} \geq k_1(d)$, thus (5) implies:

$$\left(b\left(\mathbb{E}u_i(d, \underline{c}) - \mathbb{E}u_i(d, \overline{c})\right) + (1-b)\left(\mathbb{E}\pi_i(d, \underline{c}) - \mathbb{E}\pi_i(d, \overline{c}) - k_1(d)\right)\right) > 0$$

As $k_1'(d) > 0$ (see Proposition 3), Remark 2 follows.

Remarks 8.1 and 8.2 imply that $W'(d) > 0$ for $d \in [0, b]$, thus $d_k(\sigma_k, b) > b$. All is left to show is that expected welfare is single-peaked for $d \in [b, 1]$. Proposition 1 shows that, for any expected cost c, certifier $D = d$ chooses certification requirement $q = \left(\frac{1}{c}\right)^{\frac{1}{1-a}} f(d)$ where

$$f(d) := \left(\frac{(2a-1)(1+d)}{\left(a + \sqrt{a^2 - (2a-1)(1-d^2)}\right)}\right)^{\frac{1}{1-a}}.$$

Define the following functions; $\tilde{W}(f(d)) := W(d)$, $\tilde{k}_1(f(d)) := k_1(d)$, $\mathbb{E}\tilde{\pi}_i(f(d), c) := \mathbb{E}\pi_i(d, c)$, and $\mathbb{E}\tilde{u}_i(f(d), c) := \mathbb{E}u_i(d, c)$ for any d and any c. The program $\max_{d\in[0,1]} W(d)$ is equivalent to $\max_{x\in[f(0),f(1)]} \tilde{W}(x)$, where

$$\tilde{W}(x) := \left(\underline{k} + \frac{1}{2\sigma_k} - \tilde{k}_1(x)\right)((1-b)\mathbb{E}\tilde{\pi}_i(x,\overline{c}) + b\mathbb{E}\tilde{u}_i(x,\overline{c})) + \tag{6}$$
$$+ \left(\tilde{k}_1(x) - \left(\underline{k} - \frac{1}{2\sigma_k}\right)\right)\left(b\mathbb{E}\tilde{u}_i(x,\underline{c}) + (1-b)\left(\mathbb{E}\tilde{\pi}_i(x,\underline{c}) - \frac{\tilde{k}_1(x) + \left(\underline{k} - \frac{1}{2\sigma_k}\right)}{2}\right)\right)$$

Note that:

$$\tilde{W}''(x) < 0 \Leftrightarrow$$
$$\tilde{k_1}''(x)\left(b\left(\mathbb{E}\tilde{u}_i(x,\underline{c}) - \mathbb{E}\tilde{u}_i(x,\overline{c})\right) + (1-b)\left(\mathbb{E}\tilde{\pi}_i(x,\underline{c}) - \mathbb{E}\tilde{\pi}_i(x,\overline{c}) - \frac{\tilde{k}_1(x) + \left(\underline{k} - \frac{1}{2\sigma_k}\right)}{2}\right)\right) +$$
$$+2\tilde{k_1}'(x)\left((1-b)\frac{\partial\mathbb{E}\tilde{\pi}_i(x,\underline{c})}{\partial x} + b\frac{\partial\mathbb{E}\tilde{u}_i(x,\underline{c})}{\partial x} - \left((1-b)\frac{\partial\mathbb{E}\tilde{\pi}_i(x,\overline{c})}{\partial x} + b\frac{\partial\mathbb{E}\tilde{u}_i(x,\overline{c})}{\partial x}\right)\right) +$$
$$-(1-b)\left(\tilde{k}_1'(x)\right)^2 + \left(\tilde{k}_1(x) - \left(\underline{k} - \frac{1}{2\sigma_k}\right)\right)\left((1-b)\frac{\partial^2\mathbb{E}\tilde{\pi}_i(x,\underline{c})}{\partial^2 x} + b\frac{\partial^2\mathbb{E}\tilde{u}_i(x,\underline{c})}{\partial^2 x}\right) +$$
$$+\left(\underline{k} + \frac{1}{2\sigma_k} - \tilde{k}_1(x)\right)\left((1-b)\frac{\partial^2\mathbb{E}\tilde{\pi}_i(x,\overline{c})}{\partial^2 x} + b\frac{\partial^2\mathbb{E}\tilde{u}_i(x,\overline{c})}{\partial^2 x}\right) < 0 \tag{7}$$

To prove that $\tilde{W}''(x) < 0$ for any $x \in [f(b), f(1)]$ we need some ancillary observations:

(i) $\tilde{k}_1(x) = x\frac{\left(\left(2x^{a-1}-1\right)\underline{c}+\overline{c}\right)(\overline{c}-\underline{c})}{2\Delta\underline{c}^{\frac{1}{1-a}}}$, thus $\tilde{k}_1'(x) > 0 > \tilde{k}_1''(x)$;
(ii) $\mathbb{E}\tilde{\pi}_i(x,c) = \frac{x}{2\Delta}\left(x^{a-1}-1\right)^2 c^{\frac{1-2a}{1-a}}$;
(iii) $E\tilde{u}_i(d,c) = \frac{1}{\Delta}x^a\left(x^{a-1}-1\right)c^{\frac{1-2a}{1-a}}$;

Condition (2) and observation (i) imply that

$$\tilde{k_1}''\times$$
$$\left(b\left(\mathbb{E}\tilde{u}_i(x,\underline{c}) - \mathbb{E}\tilde{u}_i(x,\overline{c})\right) + (1-b)\left(\mathbb{E}\tilde{\pi}_i(x,\underline{c}) - \mathbb{E}\tilde{\pi}_i(x,\overline{c}) - \frac{\tilde{k}_1(x) + \left(\underline{k} - \frac{1}{2\sigma_k}\right)}{2}\right)\right) < 0.$$

Observations (ii) and (iii) imply that

$$\frac{\partial}{\partial c}\left((1-b)\frac{\partial \mathbb{E}\tilde{\pi}_i(x,c)}{\partial x}+b\frac{\partial \mathbb{E}\tilde{u}_i(x,c)}{\partial x}\right)>0 \Leftrightarrow \left((1-b)\frac{\partial \mathbb{E}\tilde{\pi}_i(x,c)}{\partial x}+b\frac{\partial \mathbb{E}\tilde{u}_i(x,c)}{\partial x}\right)<0. \tag{8}$$

Lemma 7 ensures that $bU(q)+(1-b)\Pi(q)$ has a single peak at $q=b$; as q_D is increasing in $f(d)$, then $b\left(\mathbb{E}\tilde{u}_i(x,c)\right)+(1-b)\left(\mathbb{E}\tilde{\pi}_i(x,c)\right)$ is single peaked at $x=f(b)$, thus (8) holds for $x\in[f(b),1]$. This observation, together with (i) imply that

$$2\tilde{k_1}'(x)\times \left((1-b)\frac{\partial \mathbb{E}\tilde{\pi}_i(x,\underline{c})}{\partial x}+b\frac{\partial \mathbb{E}\tilde{u}_i(x,\underline{c})}{\partial x}-\left((1-b)\frac{\partial \mathbb{E}\tilde{\pi}_i(x,\overline{c})}{\partial x}+b\frac{\partial \mathbb{E}\tilde{u}_i(x,\overline{c})}{\partial x}\right)\right)<0.$$

Finally, (ii) and (iii) also imply, respectively, that $\frac{\partial^2 \mathbb{E}\tilde{\pi}_i(x,c)}{\partial^2 x}<0$ and $\frac{\partial^2 \mathbb{E}\tilde{u}_i(x,c)}{\partial^2 x}<0$, which, together with (i) ensure that

$$\left(\tilde{k}_1(x)-\left(\underline{k}-\frac{1}{2\sigma_k}\right)\right)\left((1-b)\frac{\partial^2 \mathbb{E}\tilde{\pi}_i(x,\underline{c})}{\partial^2 x}+b\frac{\partial^2 \mathbb{E}\tilde{u}_i(x,\underline{c})}{\partial^2 x}\right)+ + \left(\underline{k}+\frac{1}{2\sigma_k}-\tilde{k}_1(x)\right)\left((1-b)\frac{\partial^2 \mathbb{E}\tilde{\pi}_i(x,\overline{c})}{\partial^2 x}+b\frac{\partial^2 \mathbb{E}\tilde{u}_i(x,\overline{c})}{\partial^2 x}\right)<0,$$

proving that (7) holds for $x\in[f(b),f(1)]$. As f is increasing in d this proves that welfare is single-peaked for $d\in[b,1]$ and concludes the lemma.
■

Proof of Proposition 4. If $d_k(\sigma_k,b)$ is a corner solution (i.e. $d_k(\sigma_k,b)=1$) then $d_k(\sigma_k,b)$ is constant in σ_k and b. If instead $d_k(\sigma_k,b)$ satisfies the first-order condition $W'(d_k(\sigma_k,b))=0$, then for $d=d_k(\sigma_k,b)$ we have

$$\frac{\partial W'(d)}{\partial \sigma_k}= -\frac{m}{2\sigma_k}\left(b\frac{\partial \mathbb{E}u_i(d,\underline{c})}{\partial d}+(1-b)\frac{\partial \mathbb{E}\pi_i(d,\underline{c})}{\partial d}+b\frac{\partial \mathbb{E}u_i(d,\overline{c})}{\partial d}+(1-b)\frac{\partial \mathbb{E}\pi_i(d,\overline{c})}{\partial d}\right),$$

where $\mathbb{E}u_i(d,\overline{c})$ and $\mathbb{E}u_i(d,\overline{c})$ are defined in Lemma 8. As Lemma 7 shows that $bU(q)+(1-b)\Pi(q)$ is decreasing in q for $q>q_B$, and as q_D is increasing

in d, then $b\mathbb{E}u_i(d,c) + (1-b)\mathbb{E}\pi_i(d,c)$ is decreasing in d for $d \in [b,1]$. This observation, together with the observation that $d_k(\sigma_k, b) > b$ (Lemma 8), implies $\frac{\partial W'(d)}{\partial \sigma_k} > 0$, which in turn implies that $d_k(\sigma_k, b)$ is increasing in σ_k
Next, note that:

$$\frac{\partial W'(d_k(\sigma_k, b))}{\partial b} > 0 \Leftrightarrow$$

$$\left(k_1(d) - \left(\underline{k} - \frac{1}{2\sigma_k}\right)\right)\left(\frac{\partial \mathbb{E}u_i(d,\underline{c})}{\partial d} - \frac{\partial \mathbb{E}\pi_i(d,\underline{c})}{\partial d}\right) +$$
$$+\left(\underline{k} + \frac{1}{2\sigma_k} - k_1(d)\right)\left(\frac{\partial \mathbb{E}u_i(d,\overline{c})}{\partial d} - \frac{\partial \mathbb{E}\pi_i(d,\overline{c})}{\partial d}\right) > 0. \tag{9}$$

As $\frac{\partial \mathbb{E}u_i(d,c)}{\partial d} > 0 > \frac{\partial \mathbb{E}\pi_i(d,c)}{\partial d}$ for any d and any c, then (9) holds, implying that $d_k(\sigma_k, b)$ is increasing in b, and concluding the proof of part (i).
To prove part (ii) recall that $k - \frac{1}{2\sigma_k}, < k_1(b) < k_1(1) < k + \frac{1}{2\sigma_k}$, thus

$$\frac{d\left(W(1) - W(b)\right)}{d\sigma_k} = \frac{W(1) - W(b)}{\sigma_k} + g(1) - g(b), \text{ where}$$

$$g(d) := \frac{-m}{2\sigma_k}\left(b\left(\frac{\partial \mathbb{E}u_i(d,\underline{c})}{\partial d} + \frac{\partial \mathbb{E}u_i(d,\overline{c})}{\partial d}\right) + (1-b)\left(\frac{\partial \mathbb{E}\pi_i(d,\underline{c})}{\partial d} + \frac{\partial \mathbb{E}\pi_i(d,\overline{c})}{\partial d}\right)\right).$$

As, for any c, $\mathbb{E}u_i(d,c) + (1-b)\mathbb{E}\pi_i(d,c)$ is concave for $q \in [q_C, q_A]$ and peaks at $q = q_B$, then $g(b) = 0 < g(1)$. Thus if $\overline{\sigma}_k$ such that $W(1) = W(b)$ exists, then $W(1) - W(b)$ is increasing for any $\sigma_k > \overline{\sigma}_k$. This observation implies that $\overline{\sigma}_k$ is unique and $W(1) < W(b)$ for any $\sigma_k < \overline{\sigma}_k$. ■

Proof of Proposition 5.
For any d, expected profits of a firm in the market are

$$\mathbb{P}(k < k_1(d))\left(\mathbb{E}\pi_i(d,\underline{c}) - \frac{k_1(d) + \left(\underline{k} - \frac{1}{2\sigma_k}\right)}{2}\right) + \mathbb{P}(k > k_1(d))\mathbb{E}\pi_i(d,\overline{c}),$$

where $\mathbb{E}\pi_i(d,\overline{c})$ is defined in Lemma 8. These profits are decreasing in d if and only if:

$$k_1'(d)\left(\mathbb{E}\pi_i(d,\underline{c}) - k_1(d) + \mathbb{E}\pi_i(d,\overline{c})\right) +$$
$$+\left(k_1(d) - \left(\underline{k} - \frac{1}{2\sigma_k}\right)\right)\frac{\partial E'\pi_i(d,\underline{c})}{\partial d} + \left(\underline{k} + \frac{1}{2\sigma_k} - k_1(d)\right)\frac{\partial \mathbb{E}\pi_i(d,\overline{c})}{\partial d} < 0.$$

This inequality holds as: $k_1'(d) > 0$, $\mathbb{E}\pi_i(d,\underline{c}) - k_1(d) + \mathbb{E}\pi_i(d,\overline{c}) < 0$ (by condition (2)), $\underline{k} - \frac{1}{2\sigma_k} < k_1(d) < \underline{k} + \frac{1}{2\sigma_k}$, and, for any c, $\frac{\partial \mathbb{E}\pi_i(d,c)}{\partial d} < 0$. ■

Proof of Proposition 6. The socially optimal d maximizes social welfare W^E, defined as follows:

$$W^E(d) := \sigma_g \int_{\underline{g}-\frac{1}{2\sigma_g}}^{\overline{\pi}(d)} \left(\overline{m}\left((1-b)\overline{\pi}(d) + b\overline{u}(d)\right) - (1-b)(\overline{m}-\underline{m})g\right) dg +$$
$$+\underline{m}\sigma_g \int_{\overline{\pi}(d)}^{\underline{g}+\frac{1}{2\sigma_g}} \left((1-b)\overline{\pi}(d) + b\overline{u}(d)\right) dg =$$
$$\left(m + \sigma_g\left(\overline{m}-\underline{m}\right)\left(\overline{\pi}(d) - \left(\underline{g} - \frac{1}{2\sigma_g}\right)\right)\right)\left((1-b)\overline{\pi}(d) + b\overline{u}(d)\right) +$$
$$-(\overline{m}-\underline{m})\frac{\sigma_g(1-b)}{2}\left(\left(\overline{\pi}(d)\right)^2 - \left(\underline{g} - \frac{1}{2\sigma_g}\right)^2\right),$$

where:

$$\overline{u}(d) := \mathbb{P}(k < k_1(d))\mathbb{E}u_i(d,\underline{c}) + \mathbb{P}(k > k_1(d))\mathbb{E}u_i(d,\overline{c}), \text{ and}$$
$$\overline{\pi}(d) := \mathbb{P}(k < k_1(d))\left(\mathbb{E}\pi_i(d,\underline{c}) - \frac{k_1(d) + (\underline{k} - \frac{1}{2\sigma_k})}{2}\right) + \mathbb{P}(k > k_1(d))\mathbb{E}\pi_i(d,\overline{c}),$$

and $\mathbb{E}\pi_i(d,c)$ as well as $\mathbb{E}\pi_i(d,c)$ are defined in Lemma 8. For $d \in [0,1]$, function W^E is continuous and bounded. This implies that a global maximum $d_m(\sigma_g, b) \in [0,1]$ exists and is generically unique. Moreover:

$$W^{E'}(d) = \sigma_g\left(\overline{m}-\underline{m}\right)\overline{\pi}'(d)b\overline{u}(d) + \tag{10}$$
$$+\left(m + \sigma_g\left(\overline{m}-\underline{m}\right)\left(\overline{\pi}(d) - \left(\underline{g} - \frac{1}{2\sigma_g}\right)\right)\right)\left((1-b)\overline{\pi}'(d) + b\overline{u}'(d)\right).$$

If $d_m(\sigma_g, b)$ satisfies the first-order condition, then, for $d = d_m(\sigma_g, b)$, (10) implies:

$$\frac{\partial W^{E'}(d)}{\partial b} = \frac{W^{E'}(d)}{b} - \frac{1}{b}\left(m + \sigma_g\left(\overline{m}-\underline{m}\right)\left(\overline{\pi}(d) - \left(\underline{g} - \frac{1}{2\sigma_g}\right)\right)\right)\overline{\pi}'(d) =$$
$$-\frac{1}{b}\left(m + \sigma_g\left(\overline{m}-\underline{m}\right)\left(\overline{\pi}(d) - \left(\underline{g} - \frac{1}{2\sigma_g}\right)\right)\right)\overline{\pi}'(d) > 0.$$

If the maximization of social welfare yields a corner solution, then $\frac{\partial d_m(\sigma_g,b)}{\partial b} = 0$. If the maximization of social welfare yields an interior solutions $d_m(\sigma_g, b) \in (0,1)$, then $\frac{\partial W^{E'}(d_m(\sigma_g,b))}{\partial b} > 0$ implies that $\frac{\partial d_m(\sigma_g,b)}{\partial b} > 0$.
The next remark will be used to prove that $d_m(\sigma_g, b)$ is decreasing in σ_g.

Remark 6.1. $d_m(\sigma_g, b) \leq d_k(\sigma_k, b)$.

Pick a $\tilde{d}$ that satisfies $\tilde{d} > d_k$ where $d_k = d_k(\sigma_k, b)$. Then:

$$\begin{aligned}
W^E(d_k) =& \left(\underline{m} + (\overline{m} - \underline{m})\, \mathbb{P}(g < \overline{\pi}(d_k))\right) \left((1-b)\overline{\pi}(d_k) + b\overline{u}(d_k)\right) + \\
& - (\overline{m} - \underline{m})(1-b)\mathbb{P}(g < \overline{\pi}(d_k)) \frac{\overline{\pi}(d_k) + (\underline{g} - \frac{1}{2\sigma_g})}{2} \\
>& \left(\underline{m} + (\overline{m} - \underline{m})\, \mathbb{P}(g < \overline{\pi}(\tilde{d}))\right) \left((1-b)\overline{\pi}(d_k) + b\overline{u}(d_k)\right) + \\
& - (\overline{m} - \underline{m})(1-b)\mathbb{P}(g < \overline{\pi}(\tilde{d})) \frac{\overline{\pi}(\tilde{d}) + (\underline{g} - \frac{1}{2\sigma_g})}{2} \\
\geq& \left(\underline{m} + (\overline{m} - \underline{m})\, \mathbb{P}(g > \overline{\pi}(\tilde{d}))\right) \left((1-b)\overline{\pi}(\tilde{d}) + b\overline{u}(\tilde{d})\right) + \\
& - (\overline{m} - \underline{m})(1-b)\mathbb{P}(g < \overline{\pi}(\tilde{d})) \frac{\overline{\pi}(\tilde{d}) + (\underline{g} - \frac{1}{2\sigma_g})}{2} = W^E(\tilde{d}),
\end{aligned} \tag{11}$$

The first inequality in (11) holds as $\tilde{d} > d_k \Rightarrow \overline{\pi}(d_k) > \overline{\pi}(\tilde{d}) \Rightarrow$
$\mathbb{P}(g < \overline{\pi}(d_k)) > \mathbb{P}(g < \overline{\pi}(\tilde{d}))$ and

$$\begin{aligned}
(1-b)\overline{\pi}(d_k) + b\overline{u}(d_k) - (1-b)\frac{\overline{\pi}(d_k) + \overline{\pi}(\tilde{d})}{2} &= \\
= \left(\frac{1-b}{2}\right)\left(\overline{\pi}(d_k) - \overline{\pi}(\tilde{d})\right) + b\overline{u}(d_k) &> 0,
\end{aligned}$$

The second inequality in (11) holds as, by definition of $d_k(\sigma_k, b)$: $(1-b)\overline{\pi}(d_k) + b\overline{u}(d_k) \geq (1-b)\overline{\pi}(\tilde{d}) + b\overline{u}(\tilde{d})$. The remark follows.
If $d_m(\sigma_g, b)$ satisfies the first-order condition, then, for $d = d_m(\sigma_g, b)$, (10) implies:

$$\frac{\partial W^{E'}(d)}{\partial \sigma_g} =$$

$$\frac{W^{E'}(d)}{\sigma_g} - \frac{1}{\sigma_g}\left(m + (\overline{m} - \underline{m})\left(\frac{1}{2\sigma_g}\right)\right)((1-b)\overline{\pi}'(d) + b\overline{u}'(d)) =$$

$$-\frac{1}{\sigma_g}\left(m + (\overline{m} - \underline{m})\left(\frac{1}{2\sigma_g}\right)\right)((1-b)\overline{\pi}'(d) + b\overline{u}'(d)) < 0.$$

The last inequality is a consequence of Remark 6.1 and Lemma 8.
If the maximization of social welfare yields a corner solution, then $\frac{\partial d_m(\sigma_g,b)}{\partial \sigma_g} = 0$. If the maximization of social welfare yields an interior solution, $\frac{\partial W^{E'}(d)}{\partial \sigma_g} < 0$ implies $\frac{\partial d_m(\sigma_g,b)}{\partial \sigma_g} < 0$.
We now prove part (ii) of the proposition. Suppose that for some d and $d' > d$, it is the case that $W^E(d'^E(d)$ as long as $\sigma_g = \sigma$. We want to show that $W^E(d'^E(d)$ for any $\sigma_g < \sigma$. First of all notice that $W^E(d'^E(d)$ for $\sigma_g = \sigma$ is equivalent to

$$\begin{aligned} &\frac{\overline{m} + \underline{m}}{2\sigma}((1-b)\overline{\pi}(d') + b\overline{u}(d')) + \\ &+ (\overline{m} - \underline{m})\left(\left(\overline{\pi}(d') - \underline{g}\right)((1-b)\overline{\pi}(d') + b\overline{u}(d')) - \frac{(1-b)}{2}(\overline{\pi}(d'))^2\right) \geq \\ &\frac{\overline{m} + \underline{m}}{2\sigma}((1-b)\overline{\pi}(d) + b\overline{u}(d)) + \\ &+ (\overline{m} - \underline{m})\left(\left(\overline{\pi}(d') - \underline{g}\right)((1-b)\overline{\pi}(d') + b\overline{u}(d')) - \frac{(1-b)}{2}(\overline{\pi}(d'))^2\right). \end{aligned} \tag{12}$$

The last inequality holds only if

$$(1-b)\overline{\pi}(d') + b\overline{u}(d') > (1-b)\overline{\pi}(d) + b\overline{u}(d). \tag{13}$$

To see that this is the case, suppose instead that (13) does not hold, then:

$$
\begin{aligned}
W^E(d') \leq f(\overline{\pi}(d')) :=& \\
\left(m + \sigma_g \left(\overline{m} - \underline{m}\right)\left(\overline{\pi}(d') - \left(\underline{g} - \frac{1}{2\sigma_g}\right)\right)\right)&\left((1-b)\overline{\pi}(d) + b\overline{u}(d)\right) + \\
-\sigma_g(\overline{m} - \underline{m})(1-b)&\frac{\left(\overline{\pi}(d')\right)^2 - \left(\underline{g} - \frac{1}{2\sigma_g}\right)^2}{2} < \\
f(\overline{\pi}(d)) = W^E(d)&
\end{aligned}
$$

where the last inequality holds as

$$
\begin{aligned}
f'(\overline{\pi}(d')) = \sigma_g \left(\overline{m} - \underline{m}\right)\left((1-b)\overline{\pi}(d) + b\overline{u}(d)\right) - \sigma_g(\overline{m} - \underline{m})(1-b)\overline{\pi}(d') >& \\
\sigma_g \left(\overline{m} - \underline{m}\right)\left((1-b)\overline{\pi}(d) + b\overline{u}(d)\right) - \sigma_g(\overline{m} - \underline{m})(1-b)\overline{\pi}(d) =& \\
= \sigma_g \left(\overline{m} - \underline{m}\right) b\overline{u}(d) > 0.&
\end{aligned}
\tag{14}
$$

As (14) contradicts (12), we conclude that (12) implies (13).
Inequalities (12) and (13) imply that $W^E(d'^E(d)$ for any $\sigma_g < \sigma$.
Thus we have established that if $d' > d$, and $W^E(d'^E(d)$ for $\sigma_g = \sigma$, then $W^E(d'^E(d)$ for any $\sigma_g < \sigma$. This also implies that if $d' > d$, and $W^E(d'^E(d)$ for $\sigma_g = \sigma$ then $W^E(d'^E(d)$ for any $\sigma_g > \sigma$.
Thus if $A \succ B$ and $A \succ C$ for $\sigma_g = \sigma$ then $A \succ B$ and $A \succ C$ for any $\sigma_g < \sigma$. And if $A \prec C$ and $B \prec C$ for $\sigma_g = \sigma$ then $A \prec C$ and $B \prec C$ for any $\sigma_g > \sigma$. ■

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