When Being Thrifty is Risky: A Paradox of Precaution in International Saving

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Abstract

In partial equilibrium, a small open economy that accumulates savings during good times can mitigate consumption falls during bad times. This paper shows that, in general equilibrium, the opposite may be true if the amount of savings is large enough. More savings require more borrowing and higher leverage in the rest of the world, making it more prone to a financial crisis. Crises in the rest of the world then feed back to the saving economies and destabilize them. If the saving economies are collectively large but individually small, their national policy makers will not fully internalize the negative general equilibrium effect. Thus, in equilibrium, there will be excessive global imbalances and excessive volatility for the savers themselves.

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1 Introduction

The emerging Asian economies recently became major contributors to the global imbalances. After the 1997 Asian financial crisis, they switched from running small current account deficits to large surpluses (Figure 1). While there are multiple explanations behind this "global saving glut" (Bernanke, 2005), the self-insurance motive is a key driving factor (Aizenman and Lee (2007); Calvo et al. (2012); Ghosh et al. (2014)). By contrast, the Anglo-American economies at the center of the international financial system have run large current account deficits. Their current account positions started to deteriorate significantly after 1997 and bottomed at -5% of their GDP in 2006 just before the global financial crisis. Thereafter, they borrowed an extra 2.5% of their GDP (approximately) each year, which is twice the pre-1997 level. In part, this could reflect their superior financial development, including a comparative advantage in producing riskless assets (Gourinchas et al., 2017). Their assets provide saving vehicles for consumption smoothing (Caballero et al., 2008) and insurance instruments for risk-sharing (Mendoza et al., 2009) to the less financially developed rest of the world.

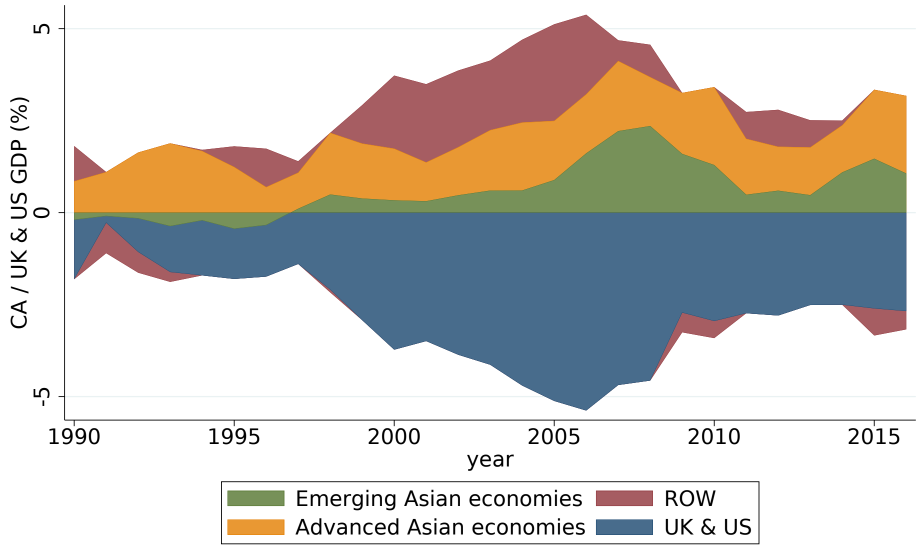

Figure 1: Global saving glut: surplus periphery and deficit center

This chapter asks the following question: to what extent is the strategy of saving for self-insurance effective in a world of large global imbalances? There are two competing forces when the savers are not small. First, more savings provide larger buffers against bad shocks and enhance economic stability directly. This is the positive direct effect as conventionally understood. Second, more savings require more borrowing and higher leverage in the rest of the world, rendering it more prone to a financial crisis. Crises in the rest of the world, when materialize, feed back to the saving economies and destabilize them indirectly. This negative general equilibrium (GE) effect, highlighted in this chapter, can overturn the positive direct effect of extra buffers. That is, higher savings can increase rather than decrease the volatility of the saving economies, making the self-insurance strategy self-defeating. I call this case a "paradox of precaution" in international saving.

This chapter leads to both positive and normative results. From a positive perspective, it shows that there is a paradox of precaution if the saving economies are not small. Intuitively, the negative GE effect is large if the saving economies are large, when their savings drive up leverage ratios in the rest of the world to a crisis-prone level. My quantitative model suggests that this might happen in the late 2010s.

From a normative perspective, it calls for international cooperation among the national policymakers in the surplus economies to rebalance their external positions for their own economic stability. An individual surplus economy such as China, Germany, or Japan only contributes to a fraction of the global imbalances. Therefore, each national policymaker does not fully internalize the negative GE effect. As a result, the saving glut economies acting individually saves more than acting collectively. The symptom is excessive global imbalances, and the consequence is excessive volatility for the savers themselves.

The 2008 global financial crisis could be a suggestive example of a large GE effect. As shown in Figure 2, Germany and Japan, which were among the main contributors to the global imbalances, fell into recessions in 2009. The recessions were even deeper than in the US, which was the epicenter of the global financial crisis. The growth rate of mainland China, which was another main contributor to the global imbalances, dropped by more than 5% from its pre-crisis peak. The European debt crisis in the early 2010s is yet another suggestive example of a large GE effect at work, even though the savers are more financially developed than the borrowers. Following the introduction of the Euro, Germany saved significantly in the southern European economies, expecting high returns. However, the consequent debt crisis in Greece and the uncertainty about the sustainability of the Eurozone drove Germany itself to the brink of recession during 2012 and 2013.

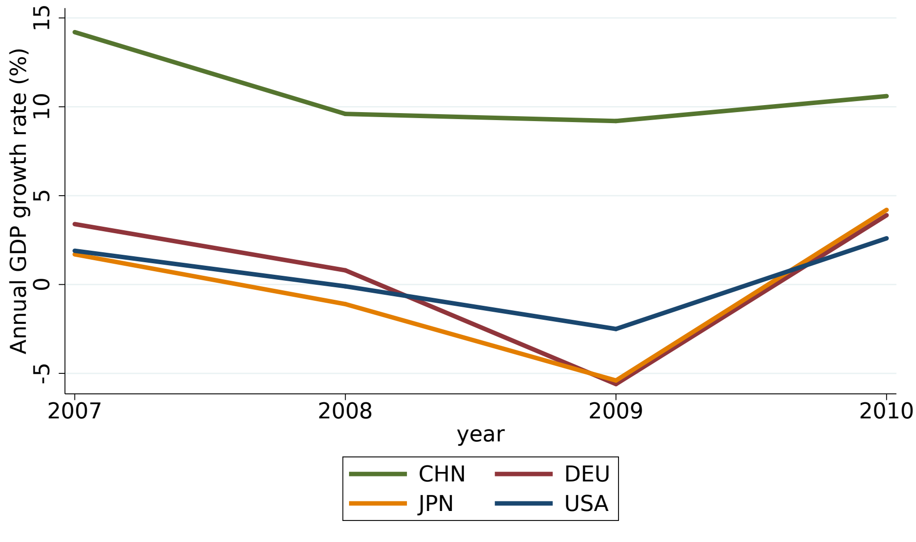

Figure 2: Growth rate for CHN, DEU, JPN and the USA in the global financial crisis

To begin with, this chapter presents an analytical two-country Lucas-tree model with collateral constraints to disentangle the direct and GE effects. The analytical model is then enriched with endogenous labor supply and production for quantitative analysis. In the model, financial crises in the borrowing economies, which come from the financial accelerator of the binding financial constraints reinforced by the fire sale of collateral *a la* Kiyotaki and Moore (1997), destabilize the saving economies through a contraction of the bond supply. More specifically, a decrease in the supply of bond prevents the saving economies from rolling over their savings. It disrupts consumption smoothing over time and across different states. The borrowing economies are more likely to deleverage in the first place, and in such a situation they deleverage more heavily, if additional savings from the saving economies drive up their average leverage ratios. As a result, higher savings increase the volatility of the saving economies through the general equilibrium effect. Consistently, Table 1 shows that there were large declines in the US riskless bond supplies around the global financial crisis when presumably riskless assets lost their "safe haven" status. The same applied to the early 2010s European debt crisis when even some government bonds were reconsidered as risky.

Table 1: A list of riskless assets: pre- and post-crisis in billions of US dollars

	Year	
	2007	2011
US Federal government debt held by public	5,136	10,692
Held by the Federal Reserve	736	1,700
Held by private investors	4,401	8,992
Government-sponsored enterprise (GSE) obligations	2,910	~~2,023~~
Agency- and GSE-backed mortgage pools	4,464	~~6,283~~
Private-issue asset-backed security (ABS)	3,901	~~1,277~~
Total safe asset held by private investors	15,676	8,992

Source: Barclays Capital (2012). The data come from Federal Reserve Flow of Funds, Haver Analytics and Barclays Capital. Numbers are struck through if the corresponding securities are believed to have lost their "safe haven" status after 2007.

It is worth noting that there are various spillback channels that count toward the negative GE effect other than the contraction of bond supply. For example, the collapse of trade, the flight of capital, the interruption of financial services provided by the core countries, or the loss from an outright default all transmit crises from the borrowing to the saving economies. Adding these additional spillback mechanisms will only enhance the GE effect and therefore make the paradox of precaution more likely, which strengthens my result. These channels are not included in this chapter for simplicity. Rather than emphasizing a specific new spillback channel *per se*, this chapter aims to highlight the possibility that the negative GE effect can dominate the positive direct effect.

This chapter contributes to four strands of literature. First, it extends the sudden-stop and capital control literature by going beyond the small open economy framework. The sudden-stop literature models infrequent crises using occasionally binding collateral constraint, which is also developed in this chapter. The literature generally draws the conclusion that the economies borrow too much or save too little ex-ante due to a fire-sale externality (for example, Bianchi (2011), Jeanne and Korinek (2010), Mendoza (2010). Benigno et al. (2013) and Jeanne and Korinek (2013) draw opposite conclusions by emphasizing ex-post policies.). For tractability, the literature uses the small open economy framework in which the international conditions are taken as given. This chapter shows that, the opposite may be true if the interactions between home and foreign economies are modeled explicitly.

Second, the chapter enriches the literature on global imbalances by explicitly studying government policies. The global imbalances literature proposes some explanations in which the imbalances are desirable outcomes from the asymmetry in the development of the financial system and other dimensions (for example, Caballero et al. (2008), Jin (2012), Mendoza et al. (2009)). In particular, the economies at the center of the international financial system provide insurance to the economies at the periphery, fulfilling an "exorbitant duty", in exchange for the "exorbitant privilege" of low financing cost (Gourinchas et al., 2017). In this chapter, a lack of international cooperation among the periphery economies leads to excessive savings and risks for themselves. The global imbalances are undesirably large for the savers.

Third, it supplements the paradox of global thrift literature by providing an alternative channel of self-defeating savings. In the literature following Keynes, large savings in a few economies lead to an undesirable global liquidity trap that reduces global output (Caballero et al. (2016), Caballero and Farhi (2017), Fornaro and Romei (2018)). This chapter does not involve the zero lower bound of nominal interest rates. It deviates from the literature further by focusing on the consumption volatility rather than the output levels, for the savers rather than the borrowers, as a consequence of additional savings.

Fourth, it complements the Triffin dilemma literature (Farhi and Maggiori (2017), Bordo and McCauley (2018)), which focuses on the deficit economies that provide reserve/safe assets to the rest of the world. A Triffin dilemma suggests that the global demand for reserve/safe assets will either remain dangerously unsatisfied or force excessive US money/debt, which is self-defeating for the reserve/safe asset status. My model argues that large savings are also self-defeating for the surplus economies. The more indebted the US is, the more likely it is to deleverage and reduce safe asset supply.

This chapter labels with a new term "paradox of precaution" the situation that the self-insurance by accumulating assets becomes self-defeating through the GE effect. It is a different concept to the seemingly related "paradox of prudence" coined in Brunnermeier and Sannikov (2016). The "paradox of prudence" refers to the situation that a fire sale of risky assets by the prudent investors endogenously increases the riskiness of the asset.

The rest of the chapter will be organized as follows: in Section Two, I introduce an analytical model and disentangle the direct and GE effects; in Section Three I construct a quantitative model and discuss the numerical method and calibration; in Section Four I conduct a positive analysis of the effect of a saving tax on economic volatility; in Section Five I conduct a normative analysis of the optimal saving tax, depending on whether the GE effect is ignored or internalized; and Section Six concludes.

2 Analytical model

This section builds a tractable model to disentangle the direct and general equilibrium (GE) effects of additional savings on the consumption smoothing of the saver. The environment is a two-country-Lucas-tree economy without any random shock. By assuming away shocks, the model becomes analytically tractable. This assumption will be relaxed in the next section to allow a generalization of the intuition from consumption smoothing to economic stabilizing.

The world economy consists of two countries: home and foreign. In each of the two countries, a representative consumer lives for three periods: today, tomorrow, and the future. The representative consumer makes consumption, domestic investment, and international saving decisions. She can invest in dividend-paying trees (with fixed supply) domestically. She can also trade a one-period riskless bond internationally. An international bond issuance must be backed by domestic tree collateral. Restricting the tree market to be domestic not only simplifies the analysis but also reflects the equity home bias in reality.

The two economies differ by the dividend processes of their trees. In particular, the foreign trees yield little dividend in the future, causing a potentially low future consumption that motivates savings today and tomorrow. To clear the market, the home economy borrows. This leads to a global imbalance between home and foreign. In reality, the fast aging economies such as China, Japan and Germany, and the oil exporters receiving windfall can be captured by such dividend processes. They are indeed large savers. The ultimate source of the global imbalances is, however, inessential for the mechanism highlighted in this section. I call the borrowing economy "home" and the saving economy "foreign" to follow the convention of calling the US "home".

2.1 Home consumer's problem

Formally, the representative home consumer solves the following problem.

$$\max \log C_1 + \beta \log C_2 + \beta^2 \log C_3$$

s.t.

$$C_1 + (K_1 - K_0) Q_1 + B_1 P_1 = K_0 d_1 + B_0 \tag{1}$$

$$C_2 + (K_2 - K_1) Q_2 + B_2 P_2 = K_1 d_2 + B_1 \tag{2}$$

$$-B_2 P_2 \leq \phi K_2 Q_2 \tag{3}$$

$$C_3 = K_2 d_3 + B_2 \tag{4}$$

Each tree yields d_t units of output at time t. The consumer uses the dividends from the tree $K_{t-1} d_t$ and the bond repayment B_{t-1} to consume C_t, to purchase new trees $K_t - K_{t-1}$ at price Q_t and to purchase new bond B_t at price P_t. The initial bond position is zero

$B_0 = 0$ and the initial quantity of trees is K_0. If $B_t > 0$, the consumer purchases bond, and if $B_t < 0$, the consumer issues bond. The future period is the final period. Therefore, trees have zero residual value and bond cannot be issued.

The home economy cannot issue riskless bond $-B_2 P_2$ tomorrow[1] more than a small[2] fraction ϕ of its tree collateral $K_2 Q_2$. The collateral constraint comes from a limited enforcement problem *a la* Kiyotaki and Moore (1997). The home borrower cannot commit to repay so they pledge their trees as collateral. While foreign lenders cannot operate home trees, they can temporarily seize a fraction ϕ of the defaulting home borrower's trees and sell them at the market price. To ensure repayment, the foreign lenders will not lend more than the liquidation value of the collateral $\phi K_2 Q_2$. In reality, safe bonds can be issued by the private sector and the government subject to explicit and implicit collateral constraints. I do not distinguish them in the model.

The trees are in fixed supply. Normalize the number of home trees to be 1. The tree market clears as follows.

$$K_t = 1, \forall t = 0, 1, 2.$$

The Euler equations for tree and bond tomorrow are the following, respectively.

$$Q_2 \left(1 - \phi \mu_2\right) = \beta \frac{C_2}{C_3} d_3 \tag{5}$$

$$P_2 \left(1 - \mu_2\right) = \beta \frac{C_2}{C_3} \tag{6}$$

$\mu_2 \geq 0$ is the shadow price of relaxing the collateral constraint (normalized by the shadow price of relaxing the budget constraint). If the collateral constraint is not binding, the shadow price is zero $\mu_2 = 0$. The asset prices $Q_{2,}, P_2$ are the discounted values of future payoffs. The future payoff for the trees only involves the future dividend d_3 because the trees have zero residual value after the final period. If the collateral constraint is binding, the shadow price is positive $\mu_2 > 0$. The asset prices are higher than the discounted future payoffs because an additional unit of tree or bond holding relaxes the collateral constraint.

When the collateral constraint is not binding, eliminating C_2 and C_3 from equations (2), (4) and (6) using $\mu_2 = 0$, the bond position tomorrow relates to bond position today as follows.

$$-B_2 P_2 = \frac{d_3 P_2 + \beta \left(-B_1 - d_2\right)}{\left(1 + \beta\right)} \tag{7}$$

[1] For simplicity, assume there is no constraint for today's issuance. Alternatively, assume the constraint for today is not binding.

[2] The pledgeability ϕ cannot be too large. In this model, $\phi < 1/\left(1 + \beta\right)$. Otherwise, an equilibrium might not exist when the legacy debt is large. To show this, assume the home collateral constraint is not binding and solve the bond position and the collateral price. It is a contradiction as the collateral constraint is violated. Intuitively, to roll over a large legacy debt, current borrowing has to be large, which is not feasible. Then assume the home collateral constraint is binding. However, the equilibrium Lagrangian multiplier μ_2 is negative. It is again a contradiction. Intuitively, the consumer values the collateral so much given its large pledgeability that the collateral constraint becomes slack.

From the expression, the economy issues more bond ($-B_2P_2$ larger) this period if it is more indebted ($-B_1$ larger) in the previous period. It rolls over the debt.

When the collateral constraint is binding and $\mu_2 > 0$, eliminating μ_2, Q_2, C_2, C_3 from equations (2), (3 with equality), (4), (5) and (6), the bond position tomorrow relates to bond position today as follows.

$$-B_2P_2 = \frac{\beta\left(d_2 - (-B_1)\right)}{1/\phi - (1+\beta)} \tag{8}$$

The denominator is positive from the assumption that the pledgeability is not too large $\phi < 1/(1+\beta)$. So, the economy borrows less ($-B_2P_2$ smaller) this period if it is more indebted ($-B_1$ larger) in the previous period. It has a debt consolidation.

Intuitively, to pay back a larger maturing debt ($-B_1$ larger), the economy has to sell trees. But the quantity of the trees is fixed in equilibrium. Consequently, the tree price falls (Q_2 lower) and the collateral constraint tightens. Even less debt can be issued ($-B_2P_2$ smaller). It triggers another round of fire sale, so on and so forth. The economy is in a financial crisis *a la* Kiyotaki and Moore (1997). More specifically, from equations (2) and (8), when the collateral constraint is binding, tomorrow's consumption C_2 is a multiple $\frac{1/\phi-1}{1/\phi-(1+\beta)} > 1$ of resources available $B_1 + d_2$.

$$C_2 = \frac{1/\phi - 1}{1/\phi - (1+\beta)}\left(B_1 + d_2\right)$$

Consumption falls more than one-to-one with a fall of dividend d_2 or resources available $B_1 + d_2$.

From equations (2), (3), (4), (5) and (6), the collateral constraint is binding when

$$d_2 < -B_1 + \left(1/\phi - (1+\beta)\right)\phi/\beta P_2 d_3. \tag{9}$$

If the economy is more indebted ($-B_1$ larger), a deleverage is more likely (the constraint is binding when $-B_1$ is large enough from equation 9) and more heavily (the deleverage $1 - B_2P_2/B_1$ is larger from equation 8). This is a key building block of the GE effect.

2.2 Foreign consumer's problem

The representative foreign consumer solves a similar problem.

$$\max \log C_1^* + \beta \log C_2^* + \beta^2 \log C_3^*$$

s.t.

$$C_1^* + \left(K_1^* - K_0^*\right)Q_1^* + B_1^*P_1\left(1+\tau^*\right) = K_0^*d_1^* + B_0^* + T^* \tag{10}$$

$$C_2^* + \left(K_2^* - K_1^*\right)Q_2^* + B_2^*P_2 = K_1^*d_2^* + B_1^* \tag{11}$$

$$C_3^* = K_2^*d_3 + B_2^* \tag{12}$$

The variables of the foreign consumer are denoted with asterisk superscripts. The foreign problem resembles the home problem except for two differences. First, the consumer faces a saving tax that is fully rebated today.

$$B_1^* P_1 \tau^* = T^*.$$

The saving tax affects the equilibrium saving choice today B_1^* directly and the consumption in the future C_3^* indirectly. The instrument is introduced to allow the analysis of additional savings, which is ultimately induced by the tax. Second, as a saver, the consumer's collateral constraints are never binding and are therefore dropped.

The consumer saves tomorrow if the output d_3 is small in the future. Formally, a not very restrictive sufficient condition for $B_2^* > 0$ is $d_3^*/d_3 < \min\left(d_1^*/d_2, d_2^*/d_2\right)$. I assume so. If the foreign-to-home relative output in the future is lower than today and tomorrow, the foreign consumer should save at least tomorrow for the future. For ease of interpretation, I also assume that d_1^* is not too small so that the foreign consumer saves both today and tomorrow. Both assumptions will be relaxed in the quantitative model.

The trees are in fixed supply K^*. It captures the relative size of the foreign economy to the home economy, in which the quantity of trees is normalized to 1. The tree market clears as follows.

$$K_t^* = K^*, \forall t = 0, 1, 2$$

The Euler equation for bond tomorrow is the following.

$$P_2 = \beta \frac{C_2^*}{C_3^*} \tag{13}$$

Likewise, eliminating C_2^* and C_3^* from equations (11), (12) and (13), the bond position tomorrow relates to the bond position today as follows.

$$B_2^* P_2 = \frac{\beta \left(B_1^* + K^* d_2^*\right) - K^* d_3^* P_2}{(1+\beta)} \tag{14}$$

The world bond market clearing condition closes the model.

$$B_t + B_t^* = 0, \forall t = 1, 2. \tag{15}$$

For a given foreign saving tax τ^*, a general equilibrium is defined as a set of domestic tree prices Q_t, Q_t^* and international bond prices P_t such that (1) the consumption C_t, C_t^*, the bond positions B_t, B_t^* and the tree positions K_t, K_t^* solve the home and foreign consumers' problem; (2) the two domestic tree markets and the world bond market clears.

2.3 Results

The foreign savings today B_1^* are endogenous for an exogenously given τ^*. But it is equivalent and more convenient to treat τ^* as endogenous for an exogenously given B_1^*.

The consumption smoothing involves saving today B_1^* and rolls over the savings tomorrow for future consumption C_3^*. This section shows the consequence of an additional unit of foreign bond B_1^* today (induced by τ^* change) on the future foreign consumption C_3^*.

Lemma 1. *The foreign economy cannot roll over its savings from today to tomorrow if the home economy deleverages and fails to roll over its debt* ($\mathrm{d}B_2^*/\mathrm{d}B_1^* < 0$ *if* $\mu_2 > 0$)*. The foreign economy can roll over its savings from today to tomorrow otherwise* ($\mathrm{d}B_2^*/\mathrm{d}B_1^* > 0$ *if* $\mu_2 = 0$)*.*

Proof. See Appendix A.

□

The lemma reveals that more savings today lead to fewer savings tomorrow for the foreign economy if the home economy deleverages. The home deleverage prevents it from rolling over its debt. This, in turn, creates an asset shortage and prevents the foreign economy from rolling over the savings.

Proposition 2. *Additional savings today exacerbate the foreign economy consumption fall in the future if the home economy deleverages tomorrow* ($\mathrm{d}C_3^*/\mathrm{d}B_1^* < 0$ *if* $\mu_2 > 0$)*. Additional savings today mitigate the foreign economy consumption fall in the future otherwise* ($\mathrm{d}C_3^*/\mathrm{d}B_1^* > 0$ *if* $\mu_2 = 0$)*.*

Proof. From equation (12), $\mathrm{d}C_3^*/\mathrm{d}B_2^* > 0$. The results follow Lemma 1.

□

The proposition reveals the possibility of a foreign paradox of savings. Additional foreign savings today can lead to lower consumption in the future. It is self-defeating for the purpose of consumption smoothing.

Lemma 3. *In the laissez-faire equilibrium, the home economy deleverages tomorrow* ($\mu_2 > 0$) *if the foreign economy is relatively large* $K^* > \hat{K}^*$*, where* $\hat{K}^*$ *solves*

$$\frac{1}{1+\beta+\beta^2}\left((1+\beta) - \frac{(1+K^*d_3^*/d_3)}{(1+K^*d_1^*/d_1)} - \frac{\beta\,(1+K^*d_3^*/d_3)}{(1+K^*d_2^*/d_2)}\right) = \phi$$

Proof. See Appendix A.

□

If the foreign economy is large it demands lots of home assets. The home economy is, in turn, more indebted today and more likely to deleverage tomorrow.

Proposition 4. *Additional savings than the laissez-faire equilibrium today exacerbate the foreign economy consumption fall in the future if its size is relatively large* ($\mathrm{d}C_3^*/\mathrm{d}B_1^* < 0$ *if* $K^* > \hat{K}^*$)*. Additional savings than the laissez-faire equilibrium today mitigate the foreign economy consumption fall in the future otherwise* ($\mathrm{d}C_3^*/\mathrm{d}B_1^* > 0$ *if* $K^* < \hat{K}^*$)*.*

Proof. Combine Proposition 2 and Lemma 3.

□

A paradox of saving appears when the foreign economy is relatively large. In reality, the savers are growing faster than the borrowers, as shown in Appendix B. Following the proposition, a paradox of saving will be increasingly relevant in the future, if not already now.

Due to the rare nature of a financial crisis and identification problems, it is difficult to test the paradox empirically. For anecdotal evidence, the 2008 global financial crisis is a recent scenario in which the financial crises from borrowing economies (the US and the UK), to which the global saving glut (from China, Japan, Germany, and the oil exporters) contributed[3], destabilized the saving economies themselves.

Theorem 5. *The net effect of additional savings today on the foreign economy future consumption can be decomposed to a direct effect and a general equilibrium (GE) effect.*

$$\frac{\mathrm{d}C_3^*}{\mathrm{d}B_1^*} = \underbrace{\left.\frac{\partial C_3^*}{\partial B_1^*}\right|_{P_2}}_{\text{direct effect}} + \underbrace{\frac{\partial C_3^*}{\partial B_2^*}\frac{\partial B_2^*}{\partial P_2}\frac{\mathrm{d}P_2}{\mathrm{d}B_1^*}}_{\text{GE effect}}$$

The GE effect is negative if the home economy deleverages tomorrow and zero otherwise ($\frac{\partial C_3^*}{\partial B_2^*}\frac{\partial B_2^*}{\partial P_2}\frac{\mathrm{d}P_2}{\mathrm{d}B_1^*} < 0$ if $\mu_2 > 0$ and $\frac{\partial C_3^*}{\partial B_2^*}\frac{\partial B_2^*}{\partial P_2}\frac{\mathrm{d}P_2}{\mathrm{d}B_1^*} = 0$ if $\mu_2 = 0$). The direct effect is always positive ($\left.\frac{\partial C_3^*}{\partial B_1^*}\right|_{P_2} > 0$).

Proof. The decomposition results from applying the chain rule. $\frac{\partial C_3^*}{\partial B_2^*}\frac{\partial B_2^*}{\partial P_2} < 0$ comes from equations (12) and (14). The $\mathrm{d}P_2/\mathrm{d}B_1^* > 0$ for $\mu_2 > 0$ case results from eliminating B_2 and B_2^* from equations (8), (14) and (15). The $\mathrm{d}P_2/\mathrm{d}B_1^* = 0$ for $\mu_2 = 0$ case results from eliminating B_2 and B_2^* from equations (7), (14) and (15). $\partial C_3^*/\partial B_1^*|_{P_2} > 0$ results from equations (12) and (14).

□

The direct effect is the effect of additional savings with an infinitely elastic bond supply at a fixed bond price P_2. The direct effect always improves consumption smoothing. Intuitively, if the bond price is fixed, nothing discourages the foreign economy from rolling over its savings tomorrow.

The GE effect is the effect of additional savings from a potentially higher tomorrow bond price P_2 induced by the deleverage and contraction of asset supply in a financial crisis of the rest of the world. The GE effect is negative or zero. Intuitively, the lower tomorrow's interest rate is (P_2 larger), the smaller the savings (B_2^* smaller) are and hence the lower future consumption (C_3^* smaller) is. If the home economy deleverages tomorrow, the safe asset shortage drives up the bond price. Additional foreign savings today unintendedly trigger a more severe home deleverage tomorrow and hence a higher bond price. If the

[3]The financial innovation and regulation failures also contributed to the crisis

home economy does not deleverage tomorrow, the perfect consumption co-movements of the home and foreign economies between tomorrow and the future pins down the world bond price, regardless of the saving decision today.

The GE effect can overturn the conventional wisdom of saving for a rainy day. The net effect depends crucially on the relative magnitude of the direct and GE effects. In this model, the GE effect is so large that it always dominates the direct effect if the home economy deleverages tomorrow.

What's more, the decomposition reveals the possibility of excessive global imbalances. If the negative GE effect is ignored, a foreign policymaker thinks that additional savings always increase its future consumption. It saves more than that if it internalizes the negative GE effect[4]. In reality, the saving economies are collectively large but individually small. The major contributors of the global imbalances such as China, Germany, and Japan all have similar surpluses. The policymakers in these economies are likely to not fully internalize the negative GE effect, creating excessive global imbalances.

3 Quantitative model

I set up an international business cycle model with financial frictions. It extends the discussion from consumption smoothing to economic stabilizing, and quantifies the conditions for the paradox of precaution to appear.

The environment is a stochastic two-country-production economy with an infinite horizon. There is, again, one home economy and one foreign economy of which variables are denoted with asterisk superscripts. But they have infinite horizons. The output is not the exogenous dividend from the trees anymore, but is produced with land and labor. Land, like the tree in the analytical model, is traded domestically. A one-period riskless bond is traded internationally. The total debt cannot exceed a fraction of the land collateral for the same enforcement reasons. The two economies have similar consumption smoothing problems subject to similar budget constraints and credit constraints. They are calibrated asymmetrically to reflect the global imbalances from (1) tighter financial constraint in the foreign economy and (2) higher variance of productivity shock in the foreign economy with details in the calibration section.

3.1 Consumer-entrepreneur's problem

3.1.1 Home economy

The representative home consumer-entrepreneur solves the following problem.

$$V_t = \max \mathbb{E}_j \sum_{t=j}^{\infty} \beta^{t-j} \frac{\left(C_t - \chi \frac{(L_t^s)^{1+\omega}}{1+\omega}\right)^{1-\sigma} - 1}{1-\sigma}$$

[4] The home consumption smoothing is also interrupted from too much debt.

s.t.

$$C_t + Q_t\left(K_t - K_{t-1}\right) + P_t B_t = W_t L_t^s + B_{t-1} + \left(Z_t K_{t-1}^{\alpha}\left(L_t^d\right)^{\gamma} - W_t L_t^d\right)$$

$$-P_t B_t + \theta W_t L_t^d \leq \phi Q_t K_t$$

The consumer-entrepreneur values consumption C_t and dislikes working L_t^s following the Greenwood–Hercowitz–Huffman specification (Greenwood et al. (1988)). This formulation of preferences removes the income effect on the labor supply by making the marginal rate of substitution between consumption and labor depends on the labor only. The consumer-entrepreneur receives income from working L_t^s at wage W_t, bond repayment B_{t-1}, and revenue from operating the firm $Z_t K_{t-1}^{\alpha}\left(L_t^d\right)^{\gamma} - W_t L_t^d$. The firm produces output using land K_{t-1} and labor L_t^d with a Cobb-Douglas production function of productivity Z_t. The proceeds are used to consume, purchase new land $K_t - K_{t-1}$ at price Q_t and purchase new bond B_t at price P_t. The intertemporal debt $-P_t B_t$, plus the intratemporal debt to finance the working capital requirement $\theta W_t L_t^d$, cannot exceed a fraction ϕ of the land value $Q_t K_t$. If the bond position is positive, the financial constraint should be interpreted as that the working capital debt is collateralized against the domestic land and the international reserves in bond $\theta W_t L_t^d \leq \phi Q_t K_t + P_t B_t$.

3.1.2 Foreign economy

Likewise, the representative foreign consumer-entrepreneur has a problem with the same structure as in the home economy.

$$V_t^* = \max \mathbb{E}_j \sum_{t=j}^{\infty} \left(\beta^*\right)^{t-j} \frac{\left(C_t^* - \chi^* \frac{\left(L_t^{s*}\right)^{1+\omega^*}}{1+\omega^*}\right)^{1-\sigma^*} - 1}{1-\sigma^*}$$

s.t.

$$C_t^* + Q_t^*\left(K_t^* - K_{t-1}^*\right) + P_t\left(1+\tau^*\right)B_t^* = W_t^* L_t^{s*} + B_{t-1}^* + \left(Z_t^*\left(K_{t-1}^*\right)^{\alpha^*}\left(L_t^{d*}\right)^{\gamma^*} - W_t^* L_t^{d*}\right) + T_t^*$$

$$-P_t B_t^* + \theta^* W_t^* L_t^{d*} \leq \phi^* Q_t^* K_t^*$$

The only difference is that their savings are taxed at a constant rate τ^*, chosen by the government. A positive τ^* raises the effective price of bond and discourages saving. A negative τ^*, instead, reduces the effective price of bond and encourages saving. The tax revenue is fully rebated through a lump-sum transfer.

$$T_t^* = \tau^* P_t B_t^*.$$

In practice, a positive τ^* captures the effect of capital outflow controls. A negative τ^* captures the reserve accumulations by the central bank plus capital inflow controls. More specifically, the central bank issues papers or sells government bond to domestic agents and uses the proceeds to invest in international reserve assets. The interest rate of international reserve assets is usually lower than the domestic government bond or central bank liabilities. The difference is the saving subsidy τ^*. The capital inflow control rules out arbitrage.

3.1.3 Market clearing conditions

Each economy's land is in fixed supply. The land market clears in the home and foreign economies as follows.

$$K_t = K$$

$$K_t^* = K^*$$

The labor market clears in the home and foreign economies as follows.

$$L_t^s = L_t^d$$

$$L_t^{s*} = L_t^{d*}$$

The world bond market clears as follows.

$$B_t + nB_t^* = 0,$$

where n is the population of the foreign economy as a multiple of the home economy.

3.1.4 Equilibrium conditions

The optimal conditions for the foreign economy are the following.

$$W_t^* = \chi^* \left(L_t^{s*}\right)^{\omega^*}$$

The foreign consumer supply labor until its disutility from working equals the utility from extra consumption. The labor supply increases with the wage.

$$\gamma^* Z_t^* \left(K_{t-1}^*\right)^{\alpha^*} \left(L_t^{d*}\right)^{\gamma^*-1} = \left(1 + \mu_t^* \theta^*\right) W_t^*$$

The foreign entrepreneur's labor demand equals the marginal output and marginal cost of labor. $\mu_t^* \geq 0$ is the shadow price of relaxing the financial constraint (normalized by the shadow price of relaxing the budget constraint). The marginal cost of labor could be higher than wage because it tightens the financial constraint.

$$Q_t^* \left(1 - \phi^* \mu_t^*\right) = \mathbb{E}_t \beta^* \left(\frac{C_{t+1}^* - \chi^* \frac{\left(L_{t+1}^{s*}\right)^{1+\omega^*}}{1+\omega^*}}{C_t^* - \chi^* \frac{\left(L_t^{s*}\right)^{1+\omega^*}}{1+\omega^*}} \right)^{-\sigma^*} \left(Q_{t+1}^* + \alpha^* Z_{t+1}^* \left(K_t^*\right)^{\alpha^*-1} \left(L_{t+1}^{d*}\right)^{\gamma^*} \right)$$

The land price equals the discounted future land price and rents if the financial constraint is not binding. It exceeds the discounted future payoffs if the financial constraint is binding because land provides additional benefit of relaxing the financial constraint.

$$P_t \left(1 + \tau_t^* - \mu_t^*\right) = \mathbb{E}_t \beta^* \left(\frac{C_{t+1}^* - \chi^* \frac{\left(L_{t+1}^{s*}\right)^{1+\omega^*}}{1+\omega^*}}{C_t^* - \chi^* \frac{\left(L_t^{s*}\right)^{1+\omega^*}}{1+\omega^*}} \right)^{-\sigma^*}$$

The bond price equals discounted future unit repayment if the financial constraint is not binding. It exceeds the discounted future payoff if the financial constraint is binding because bond provides additional benefit of relaxing the financial constraint. A tax makes the bond less attractive and its price is lower.

The complementary slackness condition for the financial constraint is the following.

$$\mu_t^* \left(\phi^* Q_t^* K_t^* + P_t B_t^* - \theta^* W_t^* L_t^{d*}\right) = 0 \text{ and } \mu_t^* \geq 0$$

The shadow price is positive if the constraint is binding and is otherwise zero.

The home consumer-entrepreneur's maximizing problem yields the same sets of optimal conditions without a saving tax in the bond Euler equation.

$$W_t = \chi \left(L_t^s\right)^{\omega}$$

$$\gamma Z_t \left(K_{t-1}\right)^{\alpha} \left(L_t^d\right)^{\gamma-1} = \left(1 + \mu_t \theta\right) W_t$$

$$Q_t \left(1 - \phi \mu_t\right) = \mathbb{E}_t \beta \left(\frac{C_{t+1} - \chi \frac{\left(L_{t+1}^s\right)^{1+\omega}}{1+\omega}}{C_t - \chi \frac{\left(L_t^s\right)^{1+\omega}}{1+\omega}} \right)^{-\sigma} \left(Q_{t+1} + \alpha Z_{t+1} \left(K_t\right)^{\alpha-1} \left(L_{t+1}^d\right)^{\gamma}\right)$$

$$P_t \left(1 - \mu_t\right) = \mathbb{E}_t \beta \left(\frac{C_{t+1} - \chi \frac{\left(L_{t+1}^s\right)^{1+\omega}}{1+\omega}}{C_t - \chi \frac{\left(L_t^s\right)^{1+\omega}}{1+\omega}} \right)^{-\sigma}$$

$$\mu_t \left(\phi Q_t K_t + P_t B_t - \theta W_t L_t^d\right) = 0 \text{ and } \mu_t \geq 0$$

An equilibrium for a given saving tax τ^* is defined by a set of international bond price P_t, domestic land price Q_t, Q_t^* and wage W_t, W_t^* such that (1) the consumption C_t, C_t^*, the bond positions B_t, B_t^* the land positions K_t, K_t^*, the labor supply L_t^s, L_t^{s*}, and labor demand L_t^d, L_t^{d*}solve the problems of the consumer-entrepreneurs in the home and foreign economies; (2) the domestic land markets and labor markets clear; and (3) the international bond market clears.

3.2 Numerical Algorithm

The model is solved by a global, nonlinear solution method for two reasons. First, financial crises are infrequent events during which the binding financial constraint reinforces the initial fall in consumption and collateral price from a bad shock. Second, the consumer-entrepreneurs make portfolio choices between the safe bond and the risky land so that a nonstochastic steady state cannot be well defined.

The global solution is solved by iterating on the optimal conditions following Coleman (1990)'s time iteration algorithm. The algorithm starts by guessing the policy functions for the next period variables and solves the policy functions for current period variables. It then updates the policy functions for the next period variables using the solution and repeats the process until it converges. The details of the algorithm are described in Appendix D.

For the tricky issue of two occasionally binding constraints, which leads to four binding-nonbinding combinations for each state in each iteration of the time iteration algorithm, I apply a transformation of the complementary slackness conditions to get rid of it. The complementary slackness condition in the form of $\mu = 0, \mu \geq 0, X \geq 0$ is transformed to $\mu = \max(0, \hat{\mu}^3), X = \max(0, -\hat{\mu}^3)$ by introducing an auxiliary variable $\hat{\mu} \in (-\infty, \infty)$. The transformation satisfies the complementary slackness condition by construction and the new auxiliary variable does not have any restrictions of its value. The forward-looking equation system can then be treated as those without inequality conditions. It is worth noting that the auxiliary variable is raised to the cubic power to ensure that the second-order derivatives almost always exist. This facilitates root-finding using the fast Newton methods in each iteration.

I further implement an adaptive grid method to improve precision and use parallelization to improve speed. The model is hence solved fairly accurately according to the Euler equation error within a reasonable time. This allows me to proceed to the comparative static analysis, in which the model needs to be solved many times for different parameterizations. The details are in Appendix D.

3.3 Calibration

I calibrate the model to the US and China annually. The home economy refers to the deficit economies represented by the US and the foreign economy refers to the surplus economies represented by China. Table 2 summarizes the calibration.

A few parameters are standard in quantitative DSGE models. The discount rate β is set to 0.96, the relative risk aversion σ is set to 2, and the labor share γ is set to 2/3. The corresponding foreign variables β^*, σ^* and γ^* are set to the same value as home. The Frisch elasticity of labor supply $(1/\omega, 1/\omega^*)$ is set to equal to 1, in line with Kimball and Shapiro (2008).

A few parameters can be normalized. The labor disutility coefficients (χ, χ^*) are both normalized to the same as labor shares (γ, γ^*). The quantities of fixed assets (K, K^*) are both normalized to 1. Even if China have different disutility from working $(\chi^* \neq \chi)$ or different fixed assets for collateral $(K^* \neq K)$ in reality, the effects will be completely picked up through the calibration of the productivity processes (Z_t, Z_t^*). The fixed asset

Table 2: Calibration

Parameter	Description	Value	Source/Target
β,β^*	discount rate	0.96	standard
σ, σ^*	relative risk aversion	2	standard
γ ,γ^*	labor share	2/3	standard
χ, χ^*	labor disutility coefficient	2/3	normalization
K, K^*	quantity of asset	1	normalization
α,α^*	asset share	0.05	normalization, US $QK/GDP = 1.25$
ω, ω^*	inverse Frisch elasticity of labor	1	Kimball and Shapiro (2008)
ϕ	pledgeability	0.29	tar. US freq. crisis 0.03
ϕ^*	pledgeability	0.1	tar. $NIIP^*/GDP^* = 0.4$
$\theta\gamma$	working capital coefficient	0.15	cal. $M_1/GDP = 0.15$
$\theta^*\gamma^*$	working capital coefficient	0.5	cal. $M_1^*/GDP^* = 0.5$
$\overline{Z}$	mean productivity	1	normalization
$\overline{Z}^*$	mean productivity	0.25	tar. $GDP^*/GDP = 0.5$
n	foreign population	4	cal. CN/US population
ρ	persistence of log productivity	0.55	tar. US log GDP autocorr 0.54
σ_Z	stderr of shock to log productivity	0.012	tar. US log GDP stderr 0.021
ρ^*	persistence of log productivity	0.79	tar. CN log GDP autocorr 0.73
σ_Z^*	stderr of shock to log productivity	0.0155	tar. CN log GDP stderr 0.030

share coefficients (α, α^*) are set to be 0.05, so the mean fixed asset value to GDP (approximately $\alpha/(1-\beta)$ in the model) is about 1.25, which is roughly the value of residential housing collateral to GDP in the US. This calibration is neither essential because for any normalization, the effects will be fully absorbed through calibration of the pledgeability coefficient (ϕ, ϕ^*).

Four coefficients for the working capital and pledgeability constraints $(\theta, \theta^*, \phi, \phi^*)$ are specific to my model. The working capital coefficients (θ, θ^*) are calibrated so that the working capital to GDP ratios ($\theta\gamma, \theta^*\gamma^*$ in the model) match the cash equivalents M1 to GDP ratios, following Schmitt-Grohe and Uribe (2007). It captures the fact that firms need cash or credit lines to pay wage bills and suppliers in advance and the households need cash or credit cards to purchase goods and services. M1 includes currency or assets that can be quickly converted to cash and are therefore an appropriate proxy. In the US, the M1 to GDP ratio is around 0.15, and in China, it is around 0.5. The high demand for cash equivalents in China provides a reason why it is a net creditor.

The pledgeability coefficient ϕ^* for China is difficult to calibrate to micro-level data due to the high level of aggregation of the model and the wide dispersion in the loan-to-value restrictions in reality. Instead, it is chosen so that the mean bond to GDP ratio B^*/Y^* matches the net international investment position (NIIP) to GDP for China. It is around 0.4 in the early 2010s[5]. China's GDP is about half of the US in the early 2010s,

[5] The NIIP of China under consideration sums the separately reported data from mainland China, the Hong Kong special administrative region of China, the Macau special administrative region of China, and the Taiwan province of China.

this target implies that the US NIIP/GDP will be -0.2 in the absence of other economies. It is indeed the case for the US. The two-country model works well for the US and China. This approach leads to $\phi^* = 0.1$.

The pledgeability coefficient ϕ for the US suffers from the same difficulty to use micro-level data. Instead, it is calibrated to match the frequency of financial crises in the US as in Mendoza (2010). To be consistent with the empirical literature, a financial crisis is an event during which the financial constraint is binding, and the deleverage exceeds its one standard deviation. Over the last century, the US has encountered three major financial crises: the great depression, the savings and loan crisis, and the great recession. By targeting a frequency of crisis of 0.03, the value of ϕ is calibrated to be 0.29. It is consistent with the measures of the household and corporate leverage in the US, varying from 0.2 to 0.45.

The relative population of China to the US is 4 so $n = 4$. The mean productivity of the US $\overline{Z}$ is normalized to be 1 and the mean productivity of China $\overline{Z}^*$ is calibrated to be 0.25 to target a Chinese GDP as 1/2 of that of the US, which was the case in the early 2010s.

The log productivity processes are set to follow first-order autoregressive processes.

$$\log\left(Z_t - \overline{Z}\right) = \rho_Z \log\left(Z_t - \overline{Z}\right) + \epsilon_Z$$

$$\log\left(Z_t^* - \overline{Z}^*\right) = \rho_Z^* \log\left(Z_t^* - \overline{Z}^*\right) + \epsilon_Z^*$$

Each process is approximated using the quadrature procedure of Tauchen and Hussey (1991) with 3 nodes. The two shocks are assumed to be completely independent. The persistence and standard deviation are set to target the business cycle statistics of log GDP in both economies. The log GDP autocorrelation and standard deviation are calculated from HP filtered log real GDP data in a constant local currency unit. For the post-war period 1947-2017, the US log GDP has the autocorrelation 0.54 and standard deviation 0.021. For 1980-2017 the Chinese log GDP has the autocorrelation 0.73 and standard deviation 0.030. The pre-1980s data are not used as the Chinese economy was mostly central planning during that time. The targets lead to $\rho_z = 0.55, \sigma_z = 0.012, \rho_z^* = 0.79, \sigma_z^* = 0.0155$. The log output volatilities are larger than that of the shocks with the amplification from the financial frictions. The log output processes are, however, less persistent than that of the shocks because I abstract away from capital accumulation or capital adjustment cost.

The equilibrium equation system is summarized in Appendix C. In Table 3, I summarize the model moments and the data moments from simulations. The simulated moments match the corresponding targets fairly well. I also report the relative volatility of consumption to output for a double-check. While the relative volatility does not closely follow the longest data series available, which was used as target of the productivity shock processes, they match the data fairly well for the recent three decades 1990-2017.

The calibration leads to a mean international bond price of 0.965 and mean world interest rate of 3.6%. This is roughly in line with the long-run real interest rate.

Table 3: Model and data moments

	Model	Data (detrended with HP filter) 1947-2017	1980-2017	1990-2017
$std\,(\log Y)$	0.0215	0.0212		0.0160
$autocorr\,(\log Y)$	0.5387	0.5375		0.5942
$std\,(\log Y^*)$	0.0302		0.0301	0.0266
$autocorr\,(\log Y^*)$	0.7341		0.7393	0.8588
$std\,(\log C)\,/std\,(\log Y)$	1.0283	0.8286		1.0393
$std\,(\log C^*)\,/std\,(\log Y^*)$	0.8664		1.1437	0.8233

4 Positive analyses

This section reports the effect of a saving tax on its consumption volatility for the foreign economy and how that changes with its relative size. The consumption volatility is calculated as the standard deviation of the log consumption adjusted with the disutility from working ($\hat{C}_t^* \equiv C_t^* - \chi\,(L_t^*)^{1+\omega^*}/\,(1+\omega^*)$) by simulating a long time series after solving the model globally[6]. I do not track the probability of a foreign financial crisis because the foreign economy is almost always a large creditor from the baseline calibration (NIIP/GDP=40%) and a crisis is hard to define.

4.1 Paradox of precaution

To understand the effect of a foreign saving tax, Figure 3 plots adjusted consumption volatility against different saving taxes, for two scenarios. The left panel is for a "foreign-small" scenario in which the average foreign/home GDP ratio is 0.5 as in the baseline calibration and the right panel is for a "foreign-large" scenario in which the average foreign/home GDP ratio is 0.8. There are two main results.

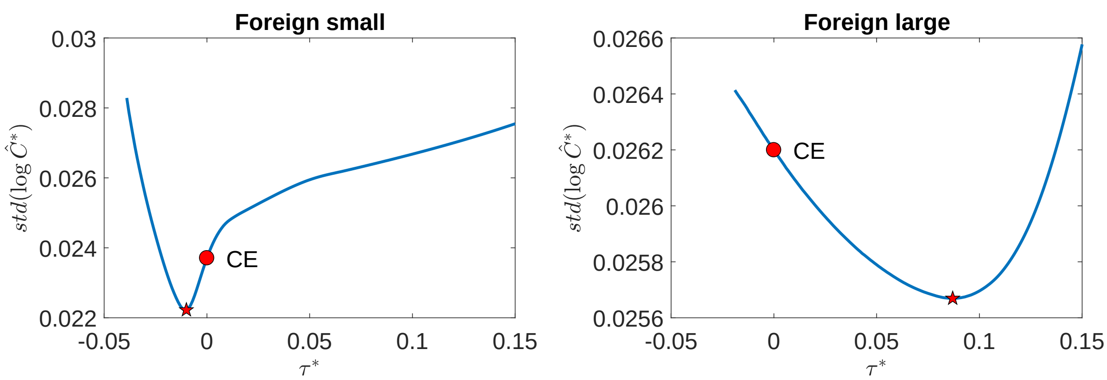

Figure 3: Effect of a foreign saving tax on foreign consumption volatility.
Left panel: foreign-small scenario, $\overline{Z}^*/\overline{Z} = 0.125$ so $nY^*/Y = 0.5$. Right panel: foreign-large scenario, $\overline{Z}^*/\overline{Z} = 0.342$ so $nY^*/Y = 0.8$. Circle marker: laissez-faire equilibrium. Star marker: the tax for volatility minimizing.

[6]The effect on the unadjusted standard deviation of log consumption is similar. However, the adjusted ones are more appropriate targets for consumption smoothing in a model with endogenous labor supply.

First, around the laissez-faire equilibrium (no tax, marked with a circle) the consumption volatility increases with the saving tax in the foreign-small scenario but decreases with the saving tax in the foreign-large scenario. The former is consistent with the conventional wisdom that a saving subsidy enhances economic stability. The latter shows a paradox of precaution: a saving subsidy increases economic volatility. This is consistent with the prediction of the analytical model that additional savings are bad for consumption smoothing when the saving economy is large.

Second, both curves are V-shaped. The consumption volatility is decreasing with the saving tax in the left-part of both panels, while it is increasing in the right-part. The lowest point of the curve is marked with a star in each panel. When the foreign economy is small, a saving subsidy of around 0.8 percentage points leads to the minimum volatility. When the foreign economy is large, a saving tax around 8.7 percentage points leads to the minimum volatility.

To understand the two results, Figures 4 and 5 augment Figure 3 by providing three additional statistics for the two scenarios: the average external saving position B^*/Y^*, the average bond price P, and the probability of a home crisis. To be consistent with the empirical literature, a home financial crisis is defined as an event when the financial constraint is binding and the deleverage ($\frac{B_{t-1}-B_t}{B_{t-1}}$) exceeds more than one standard deviation.

An increase of the saving tax reduces the external savings, as shown in the upper-right panel. The international bond price decreases consequently as in the lower-left panel and this induces lower leverage ratios in the home economy. Therefore, the probability of a home crisis decreases as shown in the lower-right panel.

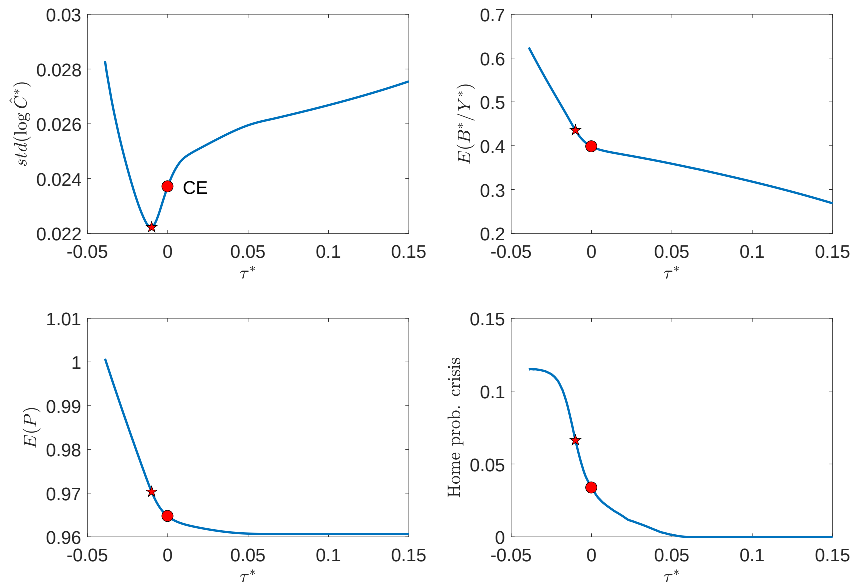

Figure 4: Effect of a foreign saving tax, foreign-small scenario
Circle marker: laissez-faire equilibrium. Star marker: the tax for volatility minimizing. Foreign small: $\overline{Z}^*/\overline{Z} = 0.125$ so $nY^*/Y = 0.5$.

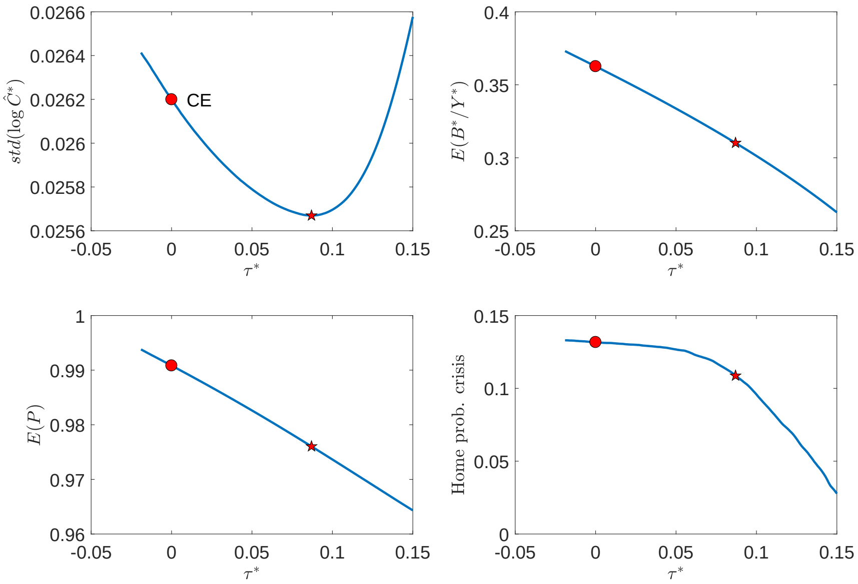

Figure 5: Effect of a foreign saving tax, foreign-large scenario

Circle marker: laissez-faire equilibrium. Star marker: the tax for volatility minimizing. Foreign large: $\overline{Z}^*/\overline{Z} = 0.342$ so $nY^*/Y = 0.8$

In both figures, when the probability of a home crisis (as shown in the lower-right panel) is relatively small, which is true when the tax rate is relatively high and the foreign saving/GDP ratio (home debt/GDP ratio) is relatively low, the volatility-tax curve is upward-sloping (as shown in the upper-left panel). An increase in the savings (as shown in the upper-right panel), which is possible when the tax decreases, reduces the economic volatility (as shown in the upper-left panel). When the probability of a home crisis is large, which is true when the tax rate is relatively low and the foreign saving/GDP ratio (home debt/GDP ratio) is relatively high, the volatility-tax curve is downward-sloping. An increase in the savings, which is possible when the tax rate further decreases, increases economic volatility.

The probability of a home financial crisis (3% as calibrated) is small around the laissez-faire equilibrium when the foreign economy is small (GDP 50% of the home economy). The home debt/GDP ratio is 20% as calibrated. By contrast, the probability of a home financial crisis (13%) is large around the laissez-faire equilibrium when the foreign economy is large (GDP 80% of the home economy). The home debt/GDP ratio, which equals the foreign saving/GDP ratio divided by the foreign/home GDP ratio to clear the international bond market, is as high as 45%. Therefore, the volatility-tax curve is upward-sloping around the laissez-faire equilibrium with small home leverage and crisis probability. It is instead downward-sloping around the laissez-faire equilibrium with large home leverage and crisis probability.

Following the insights from the analytical model, the probability of a home financial crisis is large if the foreign economy is large, or a low tax (a large subsidy) induces the foreign economy to save a lot. There is a large spillover effect from additional foreign savings to home financial stability when the home leverage is already high. The destabilizing spillback effect is large, in turn, when the probability of a home financial crisis is large. As a result, the GE effect is large if the global imbalances are large and the home economy is crisis-prone.

Why is the magnitude of the spillback effect closely related to the probability of a home financial crisis? First, a home deleverage event prevents the foreign saver from rolling over its savings for consumption smoothing. The foreign economy is forced to increase consumption immediately but reduce consumption thereafter. Second, the reduced buffers leave the foreign economy vulnerable to the realizations of its shocks. Third, low saving and low output reinforce each other in the foreign economy when its financial constraint is binding. The foreign economy, which maintains about 40% external savings to GDP in the baseline calibration and 36% when its GDP is 80% of the home economy, is effectively using both the land and the international savings as collateral for its large working capital requirement by calibration ($\theta^* W_t^* L_t^{d*} \leq \phi^* Q_t^* K_t^* + P_t B_t^*$). A low supply of asset from the home economy tightens the working capital constraint and depresses the output in the foreign economy. Being more indebted, the home economy is more likely to deleverage and in such a situation deleverages more heavily. As a result, the spillback effect is larger.

Figure 6 plots the ergodic distribution of the home debt position, normalized by its mean for the foreign-large scenario (the foreign-small scenario is similar). The normalization facilitates the comparison of the two distributions. Light blue bars refer to the probability density distribution with the foreign saving tax $\tau^* = 0.087$ and light red bars refer to the probability distribution for the laissez-faire equilibrium $\tau^* = 0$. The foreign saving tax is the tax that minimizes foreign consumption volatility.

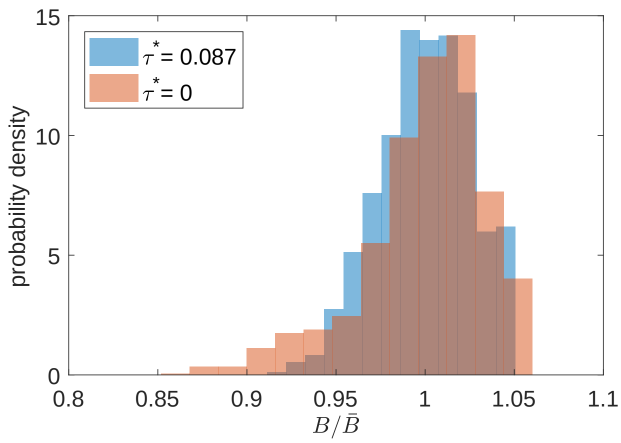

Figure 6: Distribution of normalized bond position, foreign-large scenario
Light blue: saving tax $\tau^* = 0.087$. Light red: laissez-faire equilibrium.

The distribution of the normalized home debt position is skewed. Its fat tail comes from the home deleverages reinforced by the fall of consumption and land price. The deleverages lead to an acute shortage of safe assets and it interrupts foreign consumption smoothing. The volatility of the home asset supply introduces volatility into the foreign economy.

Importantly, the variance of the normalized home bond position is smaller with a foreign saving tax. From Figure 5, the foreign saving tax reduces the foreign saving to GDP ratios from 36% to 31%. This means that the home leverage ratio reduces from 45% to 39%. As a result, a home deleverage is less likely and less heavy, which improves foreign economic stability. In the foreign-large scenario, an 8.7 percentage saving tax reduces the volatility of home asset supply to such a degree that it dominates the adverse direct effect of lower buffers, and the foreign volatility reduces from 0.0262 to 0.0257.

To further understand how a negative home shock destabilizes the foreign economy around the laissez-faire equilibrium and the equilibrium with a foreign saving tax, I conduct an event study for the foreign-large scenario. The event window covers two years before and after the event. In the calibration, the realizations of the productivity Z and Z^* are each discretized to three nodes. So, naturally, there are good, normal, and bad states for the home economy. I define negative home shock events as those in the simulations when (1) the state of the current period is in the bad and (2) the state of the previous period is either normal or good. The events account for 9.8% of all simulations. The medians of the selected variables are reported in Figure 7.

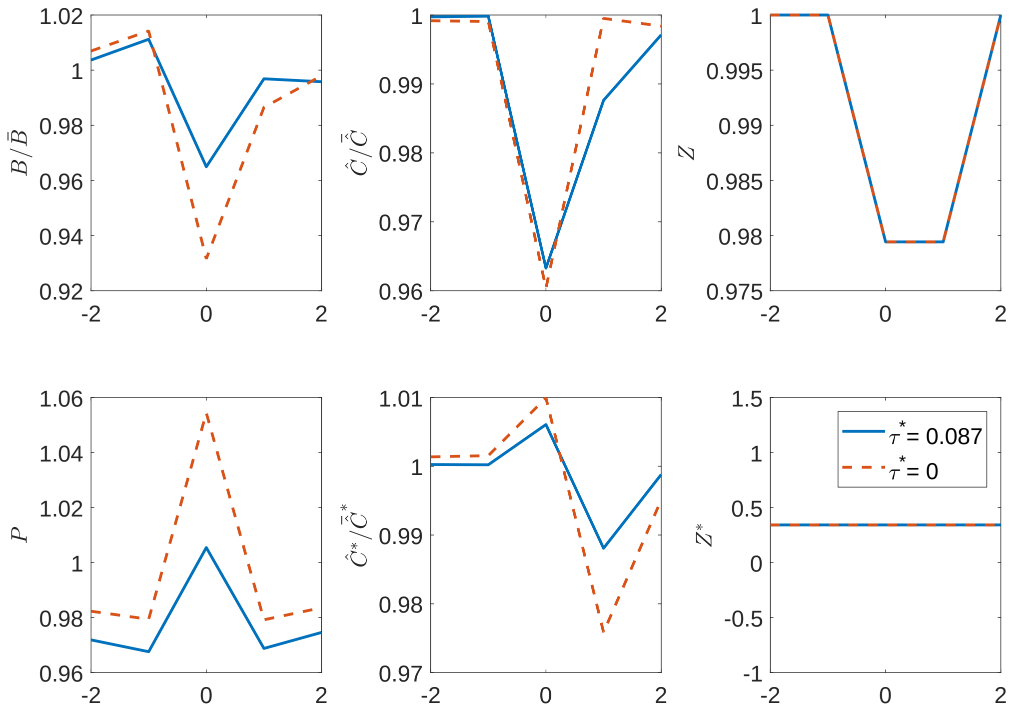

Figure 7: Event study: home bad shock at year 0, foreign-large scenario
Solid blue: saving tax $\tau^* = 0.087$. Dash red: laissez-faire equilibrium.

The right panel tracks the median productivity of the economies. From the upper-right panel, the event is indeed a bad shock to the home economy, starting from a normal state and lasting for one period (in median). The lower-right panel shows that the median productivity state for the foreign economy around the event is normal. This comes from the assumption that both shocks are perfectly uncorrelated.

The dashed red line represents the laissez-faire equilibrium. From the upper-left panel, the home economy accumulates debt gradually prior to the bad shock and deleverage heavily for about 8% when the bad shock hits. The home economy starts to increase borrowing again after the shock. In line with the bond deleverage at the shock, the home consumption drops for about 4% from the upper-middle panel. From the lower-left panel, the bond price shoots up at the shock due to the limited asset supply from the home deleverage.

The foreign consumption rises slightly at the shock and drops close to 3% one year after the shock, from the lower-middle panel. The slight rise of consumption results from low asset supply after the home deleverage. Without enough assets to postpone consumption, the foreign economy has to consume more. But why does foreign consumption drop significantly one year later in the absence of a domestic shock? First, it reflects the mechanical effect to restore bond positions to the previous level. Second, the low level of external assets tightens the foreign working capital constraint and generates an output loss. In other words, the shortage of safe assets exacerbates foreign financial frictions and amplifies the shock transmission.

The solid blue line is for the same low-home productivity event, but the foreign saving tax is fixed at $\tau^* = 0.087$. Compared with the responses in the laissez-faire economy, the home economy is less indebted during good times and deleverages less at the shock. As a result, the consumption fall in the home economy is smaller, the bond price is lower, and the consumption adjustment for the foreign economy is also shallower. In one word, the GE effect is smaller.

To further understand the effect of the relative country size, Figure 8 plots the foreign consumption volatility deviation from the laissez-faire equilibrium by a 1% foreign saving tax, for different foreign/home GDP ratios.

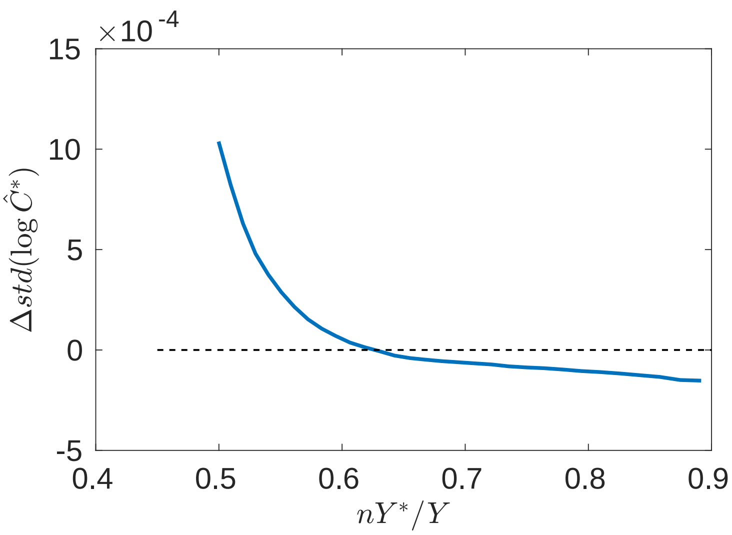

Figure 8: Effect of a foreign saving tax $\tau^* = 0.01$ on foreign consumption volatility for different relative GDP

The figure confirms that a paradox of precaution appears when the foreign economy is relatively large. What's more, the turning point is when the foreign economy is about 63% of home GDP. This is approximately China's relative GDP to the US in 2017. Therefore, a paradox of precaution might appear by the late 2010s.

The late 2010s estimation can be radical. The Chinese financial system is improving, and the US also enhances financial regulations. Mapping to the model, the working capital coefficient for China θ^* is decreasing, the pledgeability for China ϕ^* is increasing, and the US might have imposed a prudential tax themselves on borrowing.

The late 2010s estimation can, however, also be conservative. First, the model abstracts away other forceful channels of crisis contagion from the home economy to the foreign economy, for example, through the collapse of trade, the flight of capital, the interruption of financial services provided by the core countries, or the loss from an outright default. The GE effect should be stronger in reality than in the model. Second, the GE effect can also be stronger with reproducible capital that this model abstracts away. Reproducible capital propagates financial crises in the home economy. This exacerbates the overall asset shortage problem from a foreign perspective. The shortage of safe assets further forces the foreign economy to increase in its portfolio the risky domestic capital.

The net bias is unclear. A safe conclusion is that we are increasingly likely to see a paradox of precaution over time as the economies that save, which comprises many financially underdeveloped emerging markets, generally grow faster than the economies that borrow, which comprises mainly financially developed advanced economies.

4.2 Paradox of precaution - borrower version

I briefly discuss in this section the effect of a home borrowing tax. When the foreign economy is large, the home borrowing tax reduces home volatility around the laissez-faire equilibrium, consistent with the conventional wisdom. However, when the foreign economy is small, the home borrowing tax increases home volatility around the laissez-faire equilibrium. It also comes from the GE effect dominating the direct effect. More borrowing increases the probability of a home crisis from home shocks directly. But it also increases the asset supply to the foreign economy. The foreign economy becomes safer from its shocks and is less likely to withdraw its savings in large amounts. As a result, the home economy has a more stable source of finance and becomes less vulnerable. This is a borrower's version of the paradox of precaution.

5 Normative analyses

The laissez-faire equilibrium is hardly optimal. There are inefficiencies both at the domestic level and the international level. However, an individual surplus economy such as China, Germany, or Japan only contributes to a fraction of the global imbalances, so a national policymaker does not fully internalize the negative GE effect.

I restrict the policy instrument to be a constant saving tax. It is feasible in practice through capital control and/or reserve accumulation. To study the optimal policies for the savers, I consider two extremes of the foreign policymaker's problems.

First, the policymaker completely ignores the GE effect. For this purpose, I extend the model to assume a continuum of identical atomistic foreign economies. There is a policymaker in each of them. The policymaker in economy i chooses τ_i^* to maximize the unconditional expected welfare $\mathbb{E}V_i^*$, taking the world bond price P_t processes as given. The detailed policymaker's problem ignoring the GE effect is described in Appendix E, as well as the numerical algorithm for this complex problem. In a symmetric equilibrium, $\tau_i^* = \tau^*$.

The setting resembles the policymaker's problem in a small open economy with one key difference: the GE effect exists but is ignored. The policymakers take the P_t processes as given and choose τ_i^* optimally. But the P_t processes are an equilibrium outcome affected by τ^*.

From the policymaker i's perspective, there is a domestic pecuniary externality that matters because the consumer-entrepreneur takes the land price $Q_{i,t}^*$ and wage $W_{i,t}^*$ as given but these prices affect the tightness of the collateral constraint when it is binding. In other words, the consumer-entrepreneur fails to internalize the fire-sale externality when they borrow. The policymaker knows that a saving subsidy induces more precautionary savings at good times. It mitigates the consumption and land price fall in bad times. The higher land price, in turn, relaxes the financial constraint and make the economy better off. The policymaker, therefore, wants to subsidize savings, as in Bianchi and Mendoza (2011); Bianchi (2011); Bianchi and Mendoza (2018).

Second, the policymaker completely internalizes the GE effect. For this purpose, the original model in section 3.1 is used. This is the case when a single saver is dominatingly large or the foreign policymakers cooperate. A foreign policymaker who internalizes the GE effect chooses τ^* to maximize the unconditional expected welfare $\mathbb{E}V^*$ of the competitive equilibrium. From the perspective of a foreign policymaker that fully internalizes the GE effect, there are two additional inefficiencies internationally. Firstly, the foreign policymaker understands that a saving tax reduces foreign savings. It reduces the home leverage and the home economy deleverages less frequently and heavily. So, the home supply of safe assets and the equilibrium bond price P_t are less volatile, which is good for the foreign economy. The foreign policymaker, therefore, would like to tax savings to internalize this GE effect I emphasized. Secondly, the foreign policymaker also knows that it can use its monopoly power on the supply of savings. A saving tax reduces foreign savings and raises the interest rate. The foreign policymaker, therefore, would like to tax savings to extract the monopoly rents from the home economy. This extra GE effect that I do not emphasize also drives the optimal choice toward a saving tax for nonstability reasons. The net effect is ambiguous in theory.

Figure 9 adds two markers to the previous Figure 3. The square marker shows the foreign policymaker's choice if the GE effect is fully ignored. The triangle marker shows the foreign policymaker's choice if the GE effect is fully internalized. The results are quite different for the two extremes of the policymakers problem.

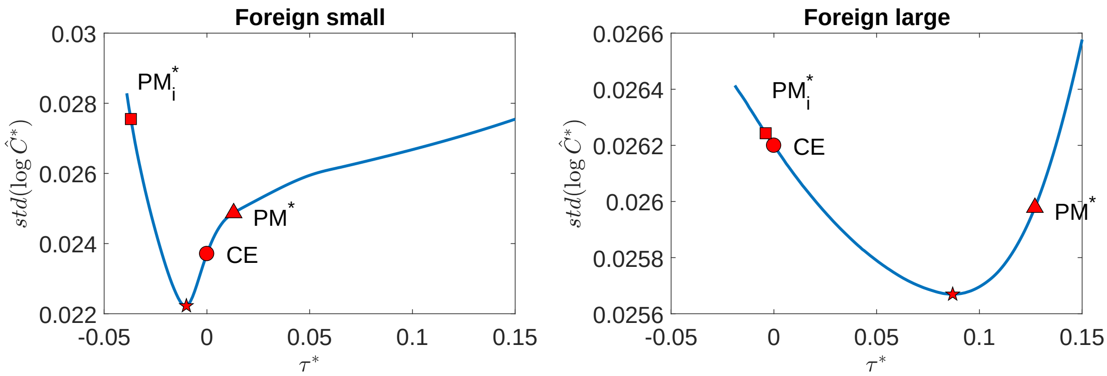

Figure 9: Effect of a foreign saving tax on foreign consumption volatility

Left: foreign-small scenario, $\overline{Z}^*/\overline{Z} = 0.125$ so $nY^*/Y = 0.5$. Right: foreign-large scenario, $\overline{Z}^*/\overline{Z} = 0.342$ so $nY^*/Y = 0.8$. Circle marker: laissez-faire equilibrium. Star marker: the tax for volatility minimizing. Square marker: policymaker's choice ignoring GE effect ignored. Triangle marker: policymaker's choice considering GE effect.

First, a foreign policymaker subsidizes saving if the GE effect is ignored because the choice is driven solely by domestic concerns. However, it leads to an even worse than the laissez-faire equilibrium for the foreign economies from the ignored negative GE effect. When foreign economies are collectively half the size of the home as shown in the left panel, the standard deviation of log adjusted consumption $std\left(\log \hat{C}^*\right)$ increases from 0.024 in laissez-faire to 0.028 with intervention by policymakers. The expected external saving position B^*/Y^* increases from 0.4 in laissez-faire to 0.6. This means the debt-to-GDP ratio for the home economy increases from 0.2 in laissez-faire to 0.3, a 50% increase. Foreign welfare, not shown in the figure, is actually lower. When the foreign economy is large, the pattern is similar but the magnitude is more limited. This is because, facing a low interest rate the policymaker does not want to subsidize saving too much.

The result has strong policy implications. Global imbalances are not necessarily desirable outcomes. It can be a symptom of a lack of cooperation between policymakers in the surplus economies. The consequence is excessive volatility for the savers themselves. While it is a Pareto improvement for the surplus economies to reduce savings collectively, no policymaker wants to do this individually. Cooperation between the surplus economies is thus necessary.

Second, the foreign policymaker taxes saving if the GE effect is fully internalized. The saving tax is larger than that minimizes the consumption volatility, reflecting the extra GE effect from a higher interest rate. For the foreign-small scenario, the policymaker would rather choose a saving tax to reap the monopoly rents at a higher interest rate despite the economy being more volatile than the laissez-faire. The result echoes the Lucas criticism on the small welfare costs of economic volatility in this kind of model.

The reality likely lies closer to the first result that the GE effect is completely ignored.

In addition to the fact that the surplus economies are individually small but collectively large, policymakers in reality may also ignore the GE effect emphasized in this chapter, simply because they are unaware of it when they choose policies.

6 Conclusion

This chapter studies two competing forces of additional savings on economic stability in a two-country framework. The direct effect of a larger buffer tends to enhance the saving economy's consumption smoothing and reduce its consumption volatility. The general equilibrium (GE) effect does the opposite to the saving economy by increasing the probability and depth of financial crises in the borrowing economy.

The policy implications of the chapter are twofold[7]. First, it calls for global rebalances when saving economies are so large that a paradox of precaution appears. The quantitative model suggests that, for the stability of their economies, policymakers in saving economies should switch from encouraging saving to discouraging saving from the late 2010s, given their size. While the estimation is approximate given the model and parameter uncertainty, it is safe to say that the paradox of precaution is increasingly likely over time because the developing savers tend to grow faster than the developed borrowers. Policymakers in the saving economies should switch from the saving subsidy to a saving tax in the foreseeable future, if not now. Second, the chapter calls for international cooperation between the policymakers in the surplus economies. Without global cooperation, there are excessive global imbalances and excessive volatility for the savers themselves.

[7]In practice, two extra policies are worth consideration for global rebalances beyond the model. First, better global safety nets from a more powerful global lender of last resort and bilateral currency swap arrangements reduce the need for precautionary savings. Second, liberalizing the trade of service to also helps economies at the center of the international financial system to narrow down their current account deficits.

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Appendices

A Lemma 1 and 3

To prove Lemma 1, eliminate B_2 and P_2 from equations (8), (14) and (15) for the $\mu_2 > 0$ case and from equations (7), (14) and (15) for the $\mu_2 = 0$ case. In the $\mu_2 > 0$ case,

$$\frac{\beta\left(d_2 - B_1^*\right)}{1/\phi - (1+\beta)} = \frac{\beta\left(B_1^* + K^* d_2^*\right) - K^* d_3 \beta\left(d_2 - B_1^*\right)/\left(\left(1/\phi - (1+\beta)\right) B_2^*\right)}{1+\beta}.$$

In equilibrium, $B_1^* < d_2$ as $B_2^* > 0$ from the global imbalance assumption. If $B_1^* \uparrow \Rightarrow B_2^* \uparrow$, the left-hand-side is smaller but the right-hand-side is larger, which is a contradiction. In the $\mu_2 = 0$ case, P_2 is a constant $\beta\left(d_2 + K^* d_2^*\right)/\left(d_3 + K^* d_3^*\right)$. From equation (14), $B_1^* \uparrow \Rightarrow B_2^* \uparrow$.

To prove Lemma 3, solve the laissez-faire equilibrium with $\tau^* = 0$ assuming the collateral constraint is not binding. From the goods market clearing condition $C_t + C_t^* = d_t + K^* d_t^*, \forall t = 1, 2, 3$ and the bond Euler equations of both economies for today

$$P_1 = \beta\frac{C_1}{C_2} = \beta\frac{C_1^*}{C_2^*}$$

and tomorrow

$$P_2 = \beta\frac{C_2}{C_3} = \beta\frac{C_2^*}{C_3^*},$$

the bond pricing equations are straightforward as follows.

$$P_1 P_2 = \beta^2\frac{d_1 + K^* d_1^*}{d_3 + K^* d_3^*} \tag{16}$$

$$P_2 = \beta\frac{d_2 + K^* d_2^*}{d_3 + K^* d_3^*} \tag{17}$$

From the home bond Euler equations and the budget constraints (1), (2), and (4)

$$B_2 = \frac{\beta^2}{1+\beta+\beta^2}\left(\frac{d_1}{P_1 P_2} + \frac{d_2}{P_2} + d_3\right) - d_3 \tag{18}$$

From the above three equations

$$B_2 = \frac{d_3}{1+\beta+\beta^2}\left(\frac{\left(1 + K^* d_3^*/d_3\right)}{\left(1 + K^* d_1^*/d_1\right)} + \frac{\beta\left(1 + K^* d_3^*/d_3\right)}{\left(1 + K^* d_2^*/d_2\right)} - (1+\beta)\right)$$

From the home bond and tree Euler equations, the collateral constraint is not binding if

$$-B_2 < \phi d_3. \tag{19}$$

Subsitute with the expression of B_2,

$$\frac{1}{1+\beta+\beta^2}\left((1+\beta)-\frac{(1+K^*d_3^*/d_3)}{(1+K^*d_1^*/d_1)}-\frac{\beta\,(1+K^*d_3^*/d_3)}{(1+K^*d_2^*/d_2)}\right)<\phi.$$

The left-hand side is increasing in K^* when $d_3^*/d_3 < \min\left(d_1^*/d_2, d_2^*/d_2\right)$, which I assume to generate the global imbalances in the first place.

B Size of the savers

The economic growth rate is generally faster in the financially underdeveloped emerging markets, which run overall current account surpluses, than the financially developed advanced economies, which run overall current account deficits. As a result, savers are likely to gain relative importance compared with borrowers over time, as shown in Figure 10.

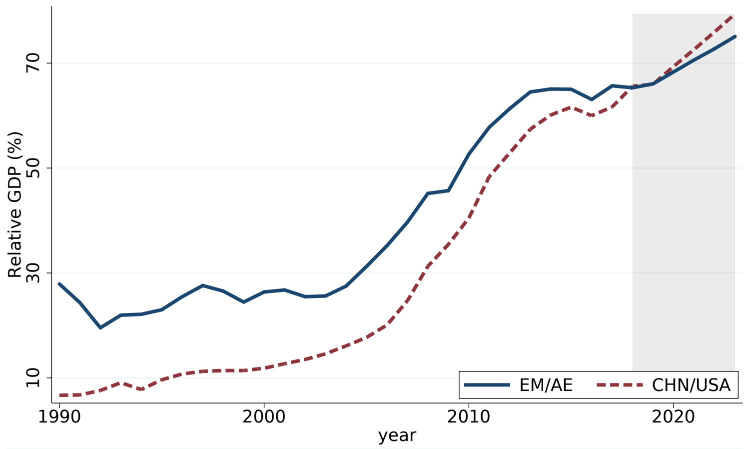

Figure 10: Relative size of emerging markets to advanced economies
Data and definition of emerging markets and advanced economies from IMF World Economic Outlook 2018. Shaded area: projections.

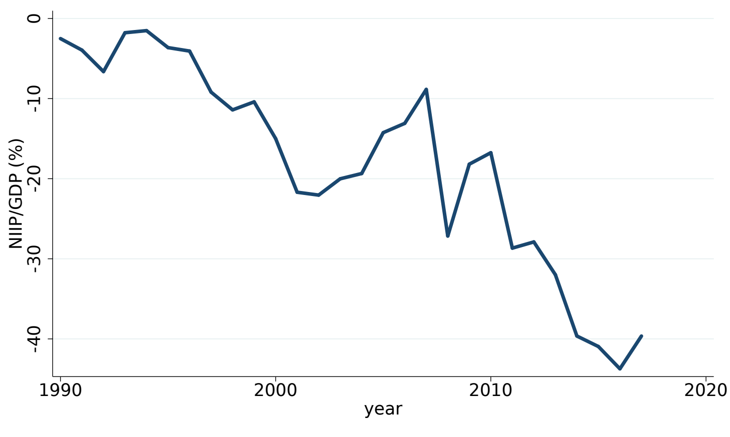

Figure 11: Net international investment position for the US
Data from FRED

While a small open economy has previously been a useful framework to study the emerging markets, it is less apparent recently. The home economy mainly refers to the US, and, to some extent, the UK, which are at the center of the international financial system with the comparative advantage to provide safe assets for the rest of the world. The US international investment position is evidently deteriorating over time as shown in Figure 11. The same is true for the UK. While a financial crisis is not necessarily imminent, the risk of adjustment is increasing.

C Equation system

The normalization of land to unit discussed in the calibration section is already implemented. The complementary slackness conditions are transformed with new variables $\hat{\mu}_t, \hat{\mu}_t^* \in (-\infty, \infty)$ as discussed in the numerical algorithm section.

$$V_t = \frac{\left(C_t - \chi \frac{L_t^{1+\omega}}{1+\omega}\right)^{1-\sigma} - 1}{1-\sigma} + \mathbb{E}_t \beta V_{t+1} \tag{20}$$

$$C_t + P_t B_t = Z_t L_t^{\gamma} + B_{t-1} \tag{21}$$

$$\gamma Z_t = (1 + \mu_t \theta) \chi L_t^{\omega+1-\gamma} \tag{22}$$

$$Q_t (1 - \phi \mu_t) = \mathbb{E}_t \beta \left(\frac{C_{t+1} - \chi \frac{L_{t+1}^{1+\omega}}{1+\omega}}{C_t - \chi \frac{L_t^{1+\omega}}{1+\omega}} \right)^{-\sigma} \left(Q_{t+1} + \alpha Z_{t+1} L_{t+1}^{\gamma}\right) \tag{23}$$

$$P_t (1 - \mu_t) = \mathbb{E}_t \beta \left(\frac{C_{t+1} - \chi \frac{L_{t+1}^{1+\omega}}{1+\omega}}{C_t - \chi \frac{L_t^{1+\omega}}{1+\omega}} \right)^{-\sigma} \tag{24}$$

$$P_t B_t + \phi Q_t - \theta \chi L_t^{\omega+1} = \max\left(0, -\hat{\mu}^3\right) \tag{25}$$

$$\mu_t = \max\left(0, \hat{\mu}_t{}^3\right) \tag{26}$$

$$V_t^* = \frac{\left(C_t^* - \chi^* \frac{(L_t^*)^{1+\omega^*}}{1+\omega^*}\right)^{1-\sigma^*} - 1}{1-\sigma^*} + \mathbb{E}_t \beta^* V_{t+1}^* \tag{27}$$

$$C_t^* + P_t B_t^* = Z_t^* (L_t^*)^{\gamma^*} + B_{t-1}^* \tag{28}$$

$$\gamma^* Z_t^* = (1 + \mu_t^* \theta^*) \chi^* (L_t^*)^{\omega^*+1-\gamma^*} \tag{29}$$

$$Q_t^* (1 - \phi^* \mu_t^*) = \mathbb{E}_t \beta^* \left(\frac{C_{t+1}^* - \chi^* \frac{\left(L_{t+1}^*\right)^{1+\omega^*}}{1+\omega^*}}{C_t^* - \chi^* \frac{(L_t^*)^{1+\omega^*}}{1+\omega^*}} \right)^{-\sigma^*} \left(Q_{t+1}^* + \alpha^* Z_{t+1}^* \left(L_{t+1}^*\right)^{\gamma^*}\right) \tag{30}$$

$$P_t (1 + \tau^* - \mu_t^*) = \mathbb{E}_t \beta^* \left(\frac{C_{t+1}^* - \chi^* \frac{\left(L_{t+1}^*\right)^{1+\omega^*}}{1+\omega^*}}{C_t^* - \chi^* \frac{(L_t^*)^{1+\omega^*}}{1+\omega^*}} \right)^{-\sigma^*} \tag{31}$$

$$P_t B_t^* + \phi^* Q_t^* - \theta^* \chi^* \left(L_t^*\right)^{\omega^*+1} = \max\left(0, -\left(\hat{\mu}^*\right)^3\right) \tag{32}$$

$$\mu_t^* = \max\left(0, \left(\hat{\mu_t}^*\right)^3\right) \tag{33}$$

$$B_t + nB_t^* = 0 \tag{34}$$

D Numerical algorithm

Time iteration algorithm

I use a time iteration algorithm as in Coleman (1990), which directly operates on first-order conditions, modified to address occasionally binding constraints. To solve the dynamic equation system $\mathbb{E}_t f\left(X_{t+1}, s_{t+1}, X_t, s_t, X_{t-1}, \tau^*\right) = 0$ defined as equations 20 to 34 where X_t are the endogenous variables, $s_t \in Z_t \times Z_t^*$ are the shock variables, do the following:

1, Generate a discrete grid for the economy's state variable X_{t-1} and current shock variable s_t. I will use adaptive grids to be explained in more detail.

2, Make an initial guess of the time-invariant policy function $X_t = \mathcal{X}^0\left(X_{t-1}, s_t\right)$. The policy functions are approximated with the finite element method to take care of the kinks arising from the occasionally binding constraints. Linear interpolation is used.

3, Iteration m

(a) Solve X_t^m such that $\mathbb{E}_t f\left(\mathcal{X}^{m-1}\left(X_t^m, s_{t+1}\right), s_{t+1}, X_t^m, s_t, X_{t-1}, \tau^*\right) = 0$ for each grid point.

(b) Update the guess of policy function $\mathcal{X}^m$ so that $X_t^m = \mathcal{X}^m\left(X_{t-1}, s_t\right)$

(c) Go to (a) if the update is larger than a threshold $\left|\left|\mathcal{X}^m - \mathcal{X}^{m-1}\right|\right| > \epsilon$ and set $m = m + 1$.

Adaptive grids

The adaptive grids are implemented in the following way.

1, The model is solved using a coarse equidistant grid with few points.

2, The accuracy of the solution is then evaluated according to the Euler equation errors at the middle of any two grid points.

3, New grid points are placed at those inaccurate parts according to the following the two rules.

(a) The number of new grid points is proportional to the Euler equation errors between existing grid points.

(b) No new grid points will be placed on inaccurate parts that will not be visited in the ergodic distribution of variables. The ergodic states are calculated directly from the policy functions.

4, Steps 1-3 are repeated in several rounds.

More specifically, the initial grid points are $80 \times 3 \times 3 = 720$, in which 80 are for the continuous state B_{t-1} and 3 for home productivity and 3 for foreign productivity. I add two rounds of extra points $80 \times 3 \times 3$ and $160 \times 3 \times 3$ to those states that are not accurate.

The maximum Euler equation error is usually below 10^{-4} from the solution. The adaptive grid method significantly accelerates the solution and improves the precision for a given number of final grid points.

Simulation

The solutions are policy functions. To simulate, generate a series of productivity shocks and choose an initial state. Evaluate the policy function to get new states and other variables of interest. Repeat this to simulate forward. Discard the initial 20% of simulations with the potential transition from the choice of the initial state. The simulations are then used to generate statistics for the ergodic distribution and for event analysis.

Parallelization

The time iteration step 3(a) is the most costly because roots need to be found for a non-linear equation system for a large number of states. The task is parallelized to accelerate. The policy functions can be solved fairly quickly and accurately. To give some sense, for one set of parameterizations the model can be solved in 10 minutes with 8 CPU cores on the LSE Fabian high-performance computing platform.

E Foreign policymaker's problem ignoring GE effect

For simplicity, assume foreign economies receive the same productivity shock Z_t^*. The welfare V_i^* for a given state solves the foreign representative consumer-entrepreneur's utility maximization problem, taking the foreign policymaker's choice τ_i^* as given.

$$V_i^*\left(B_{i,t-1}^*, \overline{B}_{t-1}^*, Z_t^*, Z_t\right) = \max \frac{\left(C_{i,t}^* - \chi \frac{\left(L_{i,t}^{s*}\right)^{1+\omega^*}}{1+\omega^*}\right)^{1-\sigma^*} - 1}{1-\sigma^*} + \mathbb{E}_t V_i^*\left(B_{i,t}^*, \overline{B}_t^*, Z_{t+1}^*, Z_{t+1}\right)$$

s.t.

$$\begin{aligned} &C_{i,t}^* + Q_{i,t}^*\left(K_{i,t}^* - K_{i,t-1}^*\right) + P_t\left(1+\tau_i^*\right) B_{i,t}^* \\ &= W_{i,t}^* L_{i,t}^{s*} + B_{i,t-1}^* + \left(Z_t^*\left(K_{i,t-1}^*\right)^{\alpha^*}\left(L_{i,t}^{d*}\right)^{\gamma^*} - W_{i,t}^* L_{i,t}^{d*}\right) + T_{i,t}^* \end{aligned}$$

$$-P_t B_{i,t}^* + \theta^* W_{i,t}^* L_{i,t}^{d*} \leq \phi^* Q_{i,t}^* K_{i,t}^*$$

The state variables for the foreign economy i's problem are $B_{i,t-1}^*, \overline{B}_{t-1}^*, Z_t^*, Z_t$. The representative consumer-entrepreneur takes as given the rest of the foreign economies' bond policy function $\overline{B}_t^* = \mathcal{B}^*\left(\overline{B}_{t-1}^*, Z_t^*, Z_t\right)$ and the bond price from the GE $P_t = \mathcal{P}\left(\overline{B}_{t-1}^*, Z_t^*, Z_t\right)$. Her individual choices such as $B_{i,t}^*$ have no effect on the international bond prices.

The saving tax is fully rebated $\tau_i^* P_t B_{i,t}^* = T_{i,t}^*$ through lump-sum transfers. The domestic land market clears with normalization $K_{i,t}^* = 1$ and domestic labor market clears $L_{i,t}^{s*} = L_{i,t}^{d*}$.

The policymaker maximizes the unconditional expected welfare $\mathbb{E}V_i^*$ of the representative consumer-entrepreneur by choosing the saving tax τ_i.

General equilibrium

In a symmetric equilibrium $B_{i,t}^* = \overline{B}_t^* = B_t^*$ and $\tau_i^* = \tau^*$. The policy functions for economy i degenerates. For example, the bond policy function $B_{i,t}^* = \mathcal{B}_i^* \left(B_{i,t-1}^*, \overline{B}_{t-1}^*, Z_t^*, Z_t\right)$ degenerate to $B^* = \mathcal{B}^* \left(B_{t-1}^*, Z_t^*, Z_t\right)$. Although the individual consumer-entrepreneur's problem in economy i takes both the individual and aggregate states, the two states become identical in a symmetric equilibrium.

The rest of the foreign economies' bond policy function $\overline{B}_t^* = \overline{\mathcal{B}}^* \left(\overline{B}_{t-1}^*, Z_t^*, Z_t\right)$ and the bond price from the general equilibrium $P_t = \mathcal{P} \left(\overline{B}_{t-1}^*, Z_t^*, Z_t\right)$ solve the general equilibrium of competitive equilibrium defined in section 3.1, given the optimal choice of the saving tax τ^* by symmetric foreign policymakers.

Numerical algorithm

The problem can be solved by repeating two steps. First, for given τ^* solve the problem in section 3.1 for the processes of P_t and B_t. Second, given P_t and B_t processes, solve the policymaker's problem in section E for τ_i and set $\tau = \tau_i$. Repeat the two steps until it converges. The algorithm within each step follows section D.

The second step involves an extra continuous state. To mitigate the curse of dimension, I use a set of efficient grids for the aggregate state $\overline{B}_{t-1}^*$. More specifically, one grid point is placed exactly at the state where the home constraint is marginally binding, and the rest of the grid points are placed sparingly. By doing this, a few grid points are sufficient to capture the aggregate policy function well.