

DEMOGRAPHIC CHANGE, COLLECTIVE WAGE BARGAINING AND PAY-AS-YOU-GO SOCIAL SECURITY*

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Abstract

This paper investigates how demographic change affects the financial sustainability of a defined benefit pay-as-you-go social security system in an environment with collective bargaining on the labor market. Temporary equilibrium analysis shows that the contribution rate decreases, if the old-age dependency ratio rises. The government balances the social security budget by aiming indirectly at a higher level of employment. In the intertemporal equilibrium the opposite applies. The government increases the contribution rate due to additional effects of demographic change on capital accumulation and labor demand. In contrast to a perfect labor market scenario, the imposed financing burden from an aging society is overcompensated by favorable labor market effects on the social security budget.

JEL-Codes: E24, H55, J11, J51

Keywords: demographic change, PAYG pension, social security, trade union, collective bargaining, unemployment

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1 Introduction

Increasing longevity and low fertility are phenomena which are highly relevant for the member states of the European Union (EU). Together both measures determine the old-age dependency ratio (AQ), which is defined as the proportion of the number of individuals above the age of 65 to the number of individuals between 15 - 65 years old. Figure 1 depicts projections from the European Commission for several countries of the EU. It shows that on average (EU 28) there will be a sharp rise of the AQ from around 32% in the year 2020 to 50% in 2040. Only in 2070 will the upward movement stop and peak at a value of around 55%. The severity of the development will differ between countries. While for Greece a peak of the AQ is projected only for the year 2050 at a ratio of 70%, the AQ in France is projected to be around 45% in the same year.

This demographic development has implications not only on how societies organize themselves but in particular also on capital accumulation and labor markets as well as the local social security systems. As [Schmähl \(1990\)](#) already notes, demographic developments do not only affect the social security system directly, e.g. by higher expenditures due to a higher number of benefit recipients, but also to a great extent indirectly via the effects of population aging on earnings and employment. Most importantly, these relations are interdependent.

The theoretical standard model to analyze pension systems is the overlapping generations model (OLG) developed by [Samuelson \(1958\)](#) and [Diamond \(1965\)](#). Within this framework relevant studies which consider the impact of demographic change on the sustainability of a pay-as-you-go pension system are [Fanti and Gori \(2008, 2010, 2012\)](#) and [Cipriani \(2014\)](#). Furthermore, [Artige \(2014\)](#), [Tabata \(2015\)](#) and [Dedry et al. \(2018\)](#) put a special comparative focus on the performance of defined-benefits and defined-contribution systems as well as on the effect of demographic aging on economic growth.

In particular, [Tabata \(2015\)](#) identifies a so-called antidilution and a life-span effect of aging on capital accumulation. Population aging increases the capital intensity either due to less dispersion of capital among households or increased savings. Furthermore, [Tabata \(2015\)](#) finds that an increase of the old-age dependency ratio positively affects the social security tax rate such that individuals save less and capital accumulation decreases (the so-called tax burden effect). The total effect of population aging on economic growth then depends on the relative strength of these competing effects.

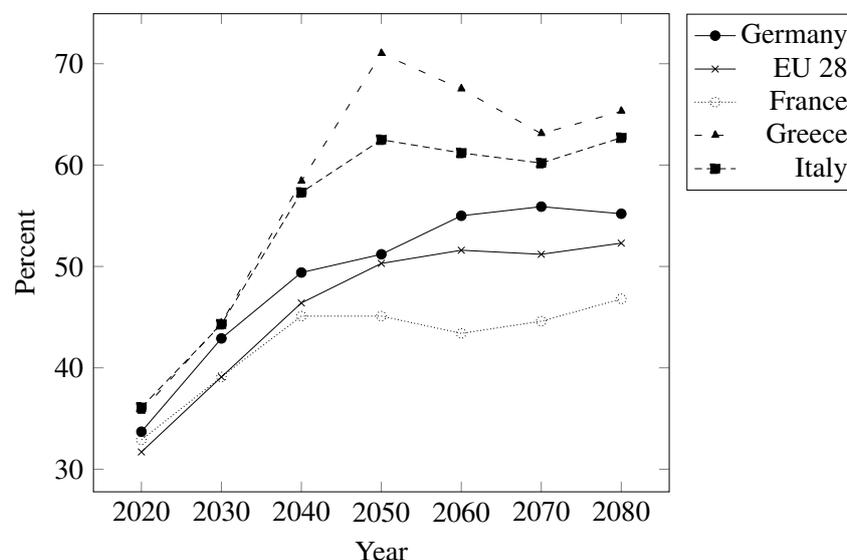


Figure 1: Projected old-age dependency ratio. Source: [Eurostat \(2019\)](#)

However, the aforementioned studies do only consider perfect labor markets which excludes indirect labor market effects on the extensive margin. To take account of these effects, this paper in turn considers imperfect labor markets with collective wage bargaining among trade unions and firms. In contrast to the general expectation that demographic change induces higher contribution rates due to an increased financing burden, it can be shown that this does not necessarily hold true in a framework with labor market imperfections, causing an interplay of wage setting, employment and social security contributions.

Only a few papers have considered the effect of demographic change and the social security system in a framework with imperfect labor markets already: e.g. [Keuschnigg and Keuschnigg \(2004\)](#), [de la Croix et al. \(2013\)](#), [Morimoto et al. \(2018\)](#). In particular, [de la Croix et al. \(2013\)](#) emphasize that labor market imperfections matter when evaluating pension systems and reforms under demographic pressure. If demographic change causes an expansion of capital supply, it may induce labor demand such that relatively more contributors to the social security system are available. However, the aforementioned papers consider equilibrium unemployment caused by search and matching. In contrast, this paper relies on collective wage bargaining, which is of particular relevance for European countries (see e.g. [Boeri et al., 2001](#)). As in e.g. [Corneo and Marquardt \(2000\)](#) or [Ono \(2007\)](#), a link between the social security system and collective wage negotiations is implemented. This allows for the analysis of the direct and indirect effects of demographic change on employment, savings and the per capita burden of the social security system. In order to focus on the role of the trade unions, a single closed economy is considered. It will be shown that the wage setting behavior of the bargaining parties strongly matters for the sustainability of the social security system.

The paper is organized as follows. Chapter 2 develops the model. Chapter 3 defines the temporary and intertemporal equilibria of the economy. Chapter 4 presents a numerical example to consider general equilibrium effects. Conclusions are drawn in chapter 5.

2 The model

The basic structure of the model refers to the standard OLG framework developed by [Samuelson \(1958\)](#) and [Diamond \(1965\)](#), and is extended by collective wage bargaining on the labor market. Time is discrete and the economy is closed. A large number of identical households and identical firms exist in the economy. Households have perfect foresight, do not leave bequests and supply labor inelastically. Lifetime uncertainty is offset by a perfect insurance market. The social security system is characterized by proportional labor income taxes and a unitary social benefit which is transferred to the unemployed and the retirees.

2.1 Demography

In any discrete time period t , the population of the economy divides into two generations: the young and the old. All young individuals survive the first period of their lifetime, but on average only a fraction $0 < \pi_t \leq 1$ of the second period. The variable $Z_{y,t}$ denotes the number of young individuals and $Z_{o,t}$ the number of old individuals. Total population then amounts to

$$Z_t = Z_{y,t} + Z_{o,t} = Z_{y,t-1}(1 + x_t) + Z_{y,t-1}\pi_t \quad (1)$$

with $x_t > -1$ reflecting the young population's growth rate from period $t - 1$ to period t . The old-age dependency ratio is defined as the ratio of the number of the old to the young:

$$\frac{Z_{o,t}}{Z_{y,t}} = \frac{\pi_t}{1 + x_t} = AQ \quad (2)$$

Furthermore, the group of the young divides into the number of employed N_t and the number of unemployed U_t : $Z_{y,t} = N_t + U_t$.

2.2 Individuals

Young individuals are identical ex ante and supply one unit of labor each. An individual earns the gross wage w_t the share τ_t of which is taxed away, if employed with probability n_t . If unemployed with probability $u_t = 1 - n_t$, an individual receives the social benefit b_t . The disposable income is used to finance consumption $c_{y,t}$ and to build up savings s_t . In the second period of their lifetime, individuals retire and use their savings and their pension b_t to finance consumption $c_{o,t+1}$ during retirement. For simplicity social benefits are assumed to be identical for the unemployed and the retirees. The flow budget constraints of both lifetime periods are then given by

$$c_t = w_t(1 - \tau_t)n_t + b_t u_t - s_t \quad (3)$$

$$c_{t+1} = \frac{R_{t+1}}{\pi_{t+1}} s_t + b_{t+1} \quad (4)$$

The variable $R_{t+1} = 1 + r_{t+1}$ describes the return on income invested in firms' securities, with r_{t+1} representing the capital market price in period $t + 1$.

Lifetime uncertainty is offset by insurance companies which offer securities promising fixed payments in the retirement period. Following the interpretation of the survival probability π_t as reflecting the average individual length of the second lifetime period, the payments could be visualized as annuities that are paid at regular fractions of the respective time period. For the formal implementation of such an insurance it is assumed that the insurance companies are risk neutral and operate on competitive private annuity markets (see also [Yakita, 2001](#), [Cipriani, 2014](#)). The yield of the security then amounts to R_{t+1}/π_{t+1} .

A young individual optimizes life-time utility over both periods by choosing the optimal levels of consumption while young, $c_{y,t}$ and while old, $c_{o,t+1}$ as well as savings s_t . The time preference is denoted by β and utility $U(\cdot)$ is assumed to be additive, separable and logarithmic. The maximization problem is given by

$$\max_{c_t, c_{t+1}, s_t} u_{y,t}(c_t, c_{t+1}) = \ln(c_t) + \pi_{t+1} \beta \ln(c_{t+1}) \quad (5)$$

subject to the budget constraints (3) and (4). The following first-order condition results:

$$\frac{\partial u_{y,t}}{\partial s_t} = -\frac{1}{w_t(1 - \tau_t)n_t + b_t u_t - s_t} + \pi_{t+1} \beta \frac{\frac{1+r_{t+1}}{\pi_{t+1}}}{\frac{1+r_{t+1}}{\pi_{t+1}} s_t + b_{t+1}} \stackrel{!}{=} 0 \quad (6)$$

Solving for the optimal amount of savings s_t yields

$$s_t = \frac{\beta \pi_{t+1}}{1 + \beta \pi_{t+1}} \left[w_t(1 - \tau_t)n_t + (1 - n_t)b_t - \frac{b_{t+1}}{\beta(1 + r_{t+1})} \right] \quad (7)$$

Total savings in period t and thus total capital supply in period $t + 1$ are then given by

$$Z_t^y s_t = K_{t+1} \quad (8)$$

2.3 Labor market

The labor market in period t is characterized by a large number $i = 1, 2, \dots, I$ of respectively identical firms and trade unions. Each firm-union pair has symmetric bargaining power and negotiates the local gross wage w_t^i for a given number of union members $Z_{y,t}^i = Z_{y,t}/I$. It is assumed that each young individual is a member of a union in period t and that membership is equally distributed among all unions.

2.3.1 Firms

The production technology is of Cobb-Douglas type and yields the output $Y_t = F(K_t^i, N_t^i) = A(K_t^i)^\alpha (N_t^i)^{1-\alpha}$ by employing the amounts K_t^i of capital and N_t^i of labor. The parameter $A > 0$ denotes the total factor productivity and the parameter $0 < \alpha < 1$ the elasticity of output with respect to capital. Capital and labor are so-called q-complements, e.g. higher utilization of capital in production increases the demand for labor due to higher marginal productivity: $F_{K_t^i N_t^i} > 0$ (see also [Seidman, 1989](#)). Firms issue securities to gather the required capital for production. For one unit of capital investment, firms pay an interest of r_t on the capital market. Furthermore, capital depreciates completely at the end of each period. The representative firm's objective function then is to maximize profit P_t^i by choosing the amounts K_t^i and N_t^i :

$$\max_{K_t^i, N_t^i} P_t^i = A(K_t^i)^\alpha (N_t^i)^{1-\alpha} - (1 + r_t)K_t^i - w_t^i N_t^i \quad (9)$$

The first-order conditions for a profit maximum in firm i are

$$\frac{\partial P_t^i}{\partial K_t^i} = \alpha A \left(\frac{N_t^i}{K_t^i} \right)^{1-\alpha} \stackrel{!}{=} 1 + r_t \quad (10)$$

$$\frac{\partial P_t^i}{\partial N_t^i} = (1 - \alpha) A \left(\frac{K_t^i}{N_t^i} \right)^\alpha \stackrel{!}{=} w_t^i \quad (11)$$

Condition (11) implies profit maximizing labor demand $N_t^i(w_t)$ and allows for the derivation of the local employment rate at the firm-union pair i :

$$n_t^i = \left(\frac{A(1 - \alpha)}{w_t^i} \right)^{\frac{1}{\alpha}} k_t^i \quad (12)$$

with $n_t^i = N_t^i/Z_{y,t}^i$ and $k_t^i = K_t^i/Z_{y,t}^i$.

2.3.2 Wage setting

The local trade union's objective in the Nash-bargain is given by the sum of utilities $SU_{y,t}^i$ of its members. Employed members at firm i earn the gross wage w_t^i the share τ_t of which is taxed away, and unemployed members of trade union i receive the social benefit b_t . With logarithmic utility, the following objective function results

$$SU_{y,t}^i = N_t^i \ln(w_t^i(1 - \tau_t)) + (Z_{y,t}^i - N_t^i) \ln(b_t) \quad (13)$$

Several assumptions are implied in equation (13). First, old individuals are excluded from membership and unions care only about those young individuals who are members of the respective trade union (compare e.g. [Lingens, 2012](#), ch. 2).¹ Second, no intertemporal trade-off occurs for

¹Alternatively and without loss of generality, it can be assumed that due to capital investment the old own the firms, and wage negotiations in any period t take place between the old and the young.

any trade union, i.e. the effect of wage setting on savings in period t is not taken into account. Third, the macroeconomic variables τ_t and b_t as well as membership $Z_{y,t}^i$ are taken as given from a single union's perspective. This implies that the outside option of a local trade union is given by $\overline{SU}_t^i = Z_{y,t}^i \ln(b_t)$.

The objective function of a firm is its profit P_t^i . It is assumed that the stock of capital is installed in advance of wage setting, which means that the outside option of the firm is $\overline{P}_t^i = -(1 + r_t)K_t^i$ (see also [Devereux and Lockwood, 1991](#)).

The symmetric Nash bargaining problem at the firm-union pair i is subject to implicit optimal local labor demand (11) and reads as:

$$\max_{w_t^i} V_t^i = (A(K_t^i)^\alpha (N_t^i)^{1-\alpha} - w_t^i N_t^i) (N_t^i [\ln(w_t^i(1 - \tau_t)) - \ln(b_t)]) \quad (14)$$

with the first-order condition

$$\begin{aligned} \frac{dV_t^i}{dw_t^i} = & \left(A(K_t^i)^\alpha (1 - \alpha) (N_t^i)^{-\alpha} \frac{\partial N_t^i}{\partial w_t^i} - N_t^i - w_t^i \frac{\partial N_t^i}{\partial w_t^i} \right) (N_t^i [\ln(w_t^i(1 - \tau_t)) - \ln(b_t)]) \\ & + (A(K_t^i)^\alpha (N_t^i)^{1-\alpha} - w_t^i N_t^i) \left(\frac{\partial N_t^i}{\partial w_t^i} [\ln(w_t^i(1 - \tau_t)) - \ln(b_t)] + N_t^i \frac{1}{w_t^i} \right) \stackrel{!}{=} 0 \end{aligned} \quad (15)$$

The following local wage setting curve results:

$$w_t^i = \frac{b_t}{1 - \tau_t} \mu \quad (16)$$

where $\mu = e^{\frac{\alpha}{2-\alpha}}$ is a mark-up on the alternative income b_t , with e as Euler's number. The wage level at the firm-union pair i depends on aggregate variables and the output elasticity of capital only.² Therefore, the local wage level (16) is equal among all firm-union pairs such that $w_t^1 = w_t^2 = \dots = w_t^I = w_t$ holds. Note that the local as well as the aggregate wage level depend positively on the contribution rate and the benefit level.

2.4 Social security budget

Expenditures for social security are financed via a pay-as-you-go transfer system, implying inter- and intra-generational redistribution. The level of the social benefit is dependent on the aggregate wage level w_t and determined by

$$b_t = C + \rho w_t \quad (17)$$

where $C > 0$ represents a fixed component and $0 < \rho \leq 1$ describes the replacement rate. The budget of the government balances tax revenues $\tau_t w_t \sum_{i=1}^I N_t^i$ with total expenditures $b_t(U_t + Z_{o,t})$. Because $w_t^i = w_t$, and $n_t^i = n_t$ as well as $k_t^i = k_t$ hold due to the symmetry assumption, the following per young capita budget equation results:

$$w_t \tau_t n_t = (C + \rho w_t) (1 - n_t + A Q_t) \quad (18)$$

²Note that the level of membership $Z_{y,t}^i$ does not affect the local wage level.

3 Equilibria

3.1 Temporary equilibrium

Labor market The following three equations define the temporary equilibrium on the labor market in period t :

$$\text{Aggregate wage level:} \quad w_t = \frac{\mu C}{1 - \tau_t - \mu \rho} \quad (19)$$

$$\text{Aggregate employment rate:} \quad n_t = \left(\frac{A(1 - \alpha)}{w_t} \right)^{\frac{1}{\alpha}} k_t \quad (20)$$

$$\text{Social security budget:} \quad \tau_t = \frac{C + \rho w_t}{w_t n_t} (1 - n_t + A Q_t) \quad (21)$$

where (19) is obtained by substituting the social benefit (17) into the local wage setting curve (16). The system of non-linear equations (19) - (21) can be solved implicitly for the contribution rate τ_t :

$$F \equiv w_t(\tau_t) n_t(w_t(\tau_t), k_t) \tau_t - b_t(w_t(\tau_t)) (1 - n_t(w_t(\tau_t), k_t) + A Q_t) = 0 \quad (22)$$

with $A Q_t = \pi_t / (1 + x_t)$. For plausible parameter values, a unique value $\tau_t(k_t)$ solves equation (22).³ Thus, it defines the temporary labor market equilibrium in dependency of the capital per capita ratio k_t and yields the equilibrium values $w_t(\tau_t(k_t))$ and $n_t(k_t, w_t(\tau_t(k_t)))$.

In each period t the government balances the social security budget (21) by adjusting the contribution rate τ_t . Thereby it takes into account how aggregate wage setting and aggregate labor demand respond to changes of the contribution rate. The government's response to an exogenous change of the old-age dependency ratio $A Q_t$ is obtained by applying the implicit function theorem on (22) with respect to τ_t and $A Q_t$: $d\tau_t/dA Q_t = -F_{A Q_t}/F_{\tau_t}$. In the following, this implicit derivative is called the *demographic effect* on the contribution rate. The corresponding partial derivatives are given by

$$F_{A Q_t} = -b_t < 0 \quad (23)$$

$$F_{\tau_t} = \underbrace{w_t n_t}_{\text{direct revenue effect}} + \frac{dw_t}{d\tau_t} \frac{\tau_t}{w_t} \left[\underbrace{1 + \frac{dn_t}{dw_t} \frac{w_t}{n_t}}_{\text{indirect revenue effects}} - \underbrace{\frac{db_t}{dw_t} \frac{w_t}{b_t} + \frac{b_t}{w_t \tau_t} \frac{dn_t}{dw_t} \frac{w_t}{n_t}}_{\text{expenditure effects}} \right] w_t n_t \quad (24)$$

Equation (23) shows that a rising old-age dependency ratio $A Q_t$ increases the financial burden in the social security budget by the amount of additional benefits to be financed. Equation (24) in turn shows how an adjustment of the contribution rate affects revenues and expenditures of a balanced budget. The direct effect on revenues is of positive sign, because for a given tax base $w_t n_t$, e.g. a higher proportional contribution rate increases the revenue. However, the indirect revenue effects and the indirect expenditure effects are of negative sign in total. First, a higher value of the contribution rate decreases the total wage bill (tax base), because employment decreases relatively stronger than wages increase. Second, expenditures increase due to a higher level of the social benefit, and because the number of unemployed (beneficiaries) increases, too. Summarizing, a rising old-age dependency ratio increases the contribution rate, $d\tau_t/dA Q_t > 0$, if the direct revenue effect outweighs the indirect effects, and vice versa. In the special case that the indirect effects countervail the direct effect, the implicit derivative $d\tau_t/dA Q_t$ is not defined, because the balance of the budget is not affected by an adjustment of the contribution rate.

³Assuming $0 < \alpha \leq 0.63$ guarantees a unique solution. See appendix Lemma 1.

Now, consider the effect of the capital stock on the contribution rate: $d\tau_t/dk_t = -F_{k_t}/F_{\tau_t}$. In the following, this implicit derivative is called the *capital supply effect* on the contribution rate. The denominator is given by equation (24) while the numerator is given by

$$F_{k_t} = (w_t \tau_t + b_t) \frac{n_t}{k_t} > 0 \quad (25)$$

According to the partial derivative (25), an increase of the capital stock per capita increases the tax revenues, because the level of employment increases, and vice versa. For the very same reason expenditures simultaneously decrease as less unemployed have to be granted a social benefit. In total, a rising capital intensity then decreases the contribution rate, $d\tau_t/dk_t < 0$, if the direct revenue effect outweighs the indirect effects in equation (24), and vice versa.

Capital market Given the equilibrium on the labor market, the temporary equilibrium on the capital market in period t is defined by the market price for capital, which equalizes capital supply $k_t = s_{t-1}/(1+x_t)$ from the old and capital demand of the firms:

$$1 + r_t = \alpha A \left(\frac{n_t(k_t)}{k_t} \right)^{1-\alpha} \quad (26)$$

Goods market Given the temporary equilibria on the labor and the capital market, the equilibrium on the goods market in period t requires that production Y_t equals consumption of the young and the old in period t plus the young's savings:

$$Y_t = Z_{t-1} \pi_t c_t^o + Z_t^y c_t^y + Z_t^y s_t \quad (27)$$

Rewriting (27) in terms of per young capita yields

$$F_t(k_t, n_t) = (1 + r_t)k_t + w_t n_t \quad (28)$$

which holds true by Euler's theorem.

Temporary equilibrium Given the variable s_{t-1} from period $t-1$ and the expected variables b_{t+1} and r_{t+1} , the temporary equilibrium of period t is defined by

1. the wage level w_t and the return on capital R_t ,
2. the aggregate variables N_t, n_t, K_t, k_t, Y_t ,
3. the social security variables τ_t, b_t
4. and the individual variables c_t, c_{t+1}, s_t

that satisfy the four optimality conditions (6), (10), (11), (15), and the three equilibrium conditions (22), (26) and (28).⁴

⁴Definition and analysis of temporary and intertemporal equilibria for a basic two-period OLG model can be found in [de la Croix and Michel \(2002\)](#).

3.2 Intertemporal equilibrium

Any two successive periods t and $t + 1$ are linked via the law of motion of the capital stock, equation (8). Taking account of the optimality and equilibrium conditions yields:

$$k_{t+1} = \frac{1}{1 + x_{t+1}} \frac{\pi_{t+1}\beta}{1 + \pi_{t+1}\beta} \underbrace{\left(w_t(k_t)n_t(k_t) - \frac{\pi_t}{1 + x_t}b_t(k_t) - \frac{b_{t+1}(k_{t+1})}{1 + r_{t+1}(k_{t+1})} \frac{1}{\beta} \right)}_{g(k_t, k_{t+1})} \quad (29)$$

An intertemporal equilibrium with perfect foresight then is a sequence of temporary equilibria that satisfies the accumulation rule (29) of capital and the formation of expectations with respect to r_{t+1} and b_{t+1} .

4 Numerical example

The simulation shows how demographic change affects the intertemporal equilibrium of the model presented in section 2. In particular it considers how simultaneous adjustments of the contribution rate and the capital stock affect the labor market and the financing of social security.

4.1 Calibration

Table 1 lists the assumptions for the exogenous variables. They do not replicate the conditions of a certain country. The values of the total factor productivity (TFP), A and the fixed part of the social benefit, C guarantee that the endogenous level of employment is below unity. Table 2 presents the different demographic scenarios. Until period 1 the economy is in a steady state. In period 2 the economy experiences a demographic shift. Scenario a) describes the case of rising longevity while fertility stays at a constant level. In scenario b) the same applies vice versa. Both scenarios imply a rise of the old-age dependency ratio from $AQ_1 = 0.4$ to $AQ_2 = 0.5$.

| Variable | Value | Variable | Value |
|-------------------------------|----------------|-----------------------|----------|
| Output elasticity of capital: | $\alpha = 1/3$ | TFP: | $A = 20$ |
| Subjective discount rate: | $\beta = 1$ | | |
| Replacement rate: | $\rho = 1/4$ | Basic social benefit: | $C = 5$ |

Table 1: Exogenous variables

| Scenarios | Longevity | Population growth rate | Implied AQ |
|----------------------|-----------------|------------------------|--------------|
| Initial steady state | $\pi_1 = 0.3$ | $x_1 = -0.25$ | $AQ_1 = 0.4$ |
| a) Longevity | $\pi_2 = 0.375$ | $x_2 = -0.25$ | $AQ_2 = 0.5$ |
| b) Fertility | $\pi_2 = 0.3$ | $x_2 = -0.4$ | $AQ_2 = 0.5$ |

Table 2: Demographic scenarios

4.2 Results

Figure 2 illustrates the simulation results for both considered scenarios. On one hand, it shows that both types of aging have the same qualitative impact on the variables of the social security system and the labor market. On the other hand, it reveals that differences emerge with respect

to the savings behavior of the young individuals and their lifetime utility. In the following, these similarities and differences will be explained in detail. Firstly, consider scenario a) with a rise of longevity in period 2.

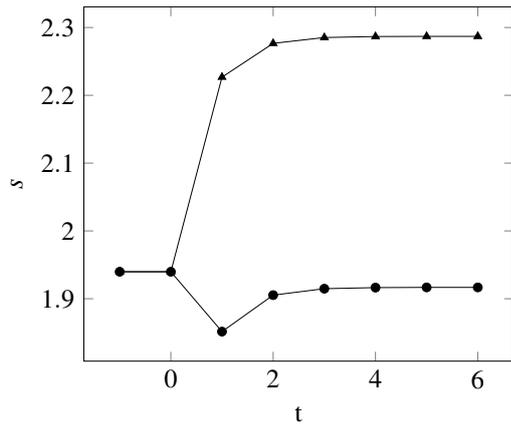
Figure 2(a) shows that per capita savings increase in period 1. The young anticipate the higher average length of the second lifetime period, which increases the weight of old-age consumption in the lifetime utility function. The simulation results imply that the positive effect of higher longevity on savings in period 1 outweighs the negative effect of a higher pension level in period 2. Accordingly, figure 2(b) shows that due to the increased amount of savings in period 1, the amount of productive capital in period 2 increases, too.

For setting of the contribution rate in period 2, two developments are decisive. First, the old-age dependency ratio increases. Second, more capital is used in production, which induces additional labor demand. For the chosen calibration of the model ($0 < \alpha \leq 0.63$), the indirect revenue and expenditure effects in equation (24) outweigh the direct revenue effect. Thus, the demographic effect induces the government to set a lower contribution rate while the capital supply effect does the opposite (see equations (23) and (25)).⁵ If both effects were equally strong, employment would increase while the level of the contribution rate would not change. However, figures 2(c) and 2(d) show that both, the employment rate as well as the contribution rate, take higher levels in period 2. This means that the capital supply effect does not only cover the inflicted financial burden from demographic change, but itself leads to an imbalance of the social security budget: *ceteris paribus*, revenues are higher than expenditures. To balance the budget again, the government is induced to partially reverse the favorable effect of higher capital supply on revenues and expenditures. This behavior is driven by the fact that the government is only concerned with a balanced budget, and does not redistribute the windfall profits. Therefore, the contribution rate increases such that the trade unions negotiate a higher wage level (as illustrated by figure 2(e)), and part of the induced additional labor demand is crowded out.

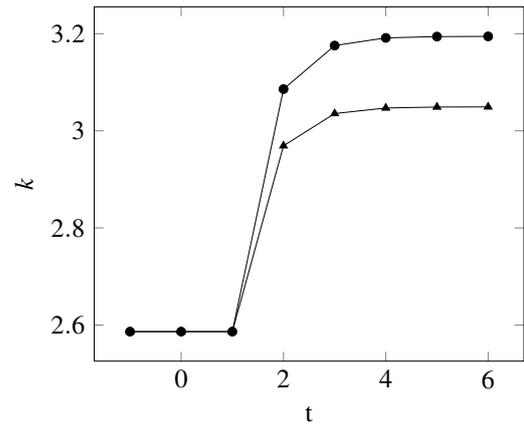
Due to the higher probability of being employed as well as the higher wage level, savings in period 2 increase again. Thereby, the negative effects on capital supply of higher pensions and unemployment benefits in period 3 are overcompensated. With a higher amount of savings in period 2, the amount of productive capital in period 3 increases, too. As before, this implies a capital supply effect on the contribution rate via additional labor demand. *Ceteris paribus*, revenues increase while expenditures decrease. To balance the budget, the government then aims at a higher contribution rate such that wages rise and employment decreases. However, there is no demographic effect which increases expenditures. Therefore, the imbalance of the budget in period 3 is relatively more severe than in period 2. The negative effect of the contribution rate on the level of employment overcompensates the positive effect of a higher amount of productive capital per capita. Thus, equilibrium employment in period 3 does not increase but decreases.

With respect to lifetime utility, longevity driven aging of the society does not show negative effects, as illustrated by figure 2(f). In contrast, fertility driven aging implies a loss in lifetime utility for the young generation in period 1. The forward looking individuals of this young generation decrease savings, because they recognize that pensions are going to increase in the period of their retirement. In contrast to longevity driven aging, old-age consumption does not get a higher weight in the lifetime utility function. However, the antidilution effect increases capital utilization per capita such that the interest rate in period 3 decreases. The higher pension level in period 2 then does not suffice to cover the lower return on savings, and lifetime utility decreases. In the following, all generations are better off, because fertility decreases only once.

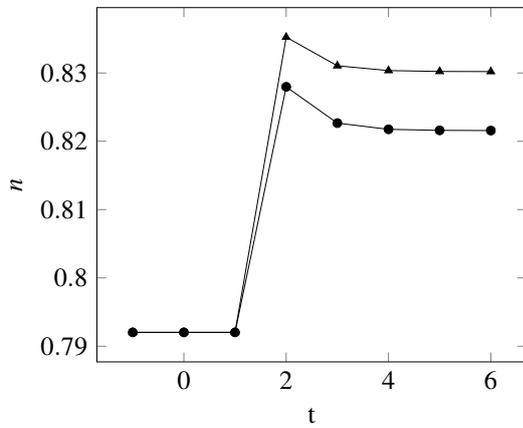
⁵Note that, *ceteris paribus*, the demographic effect does only increase expenditures while the capital supply effect simultaneously increases revenues and decreases expenditures via induced labor demand.



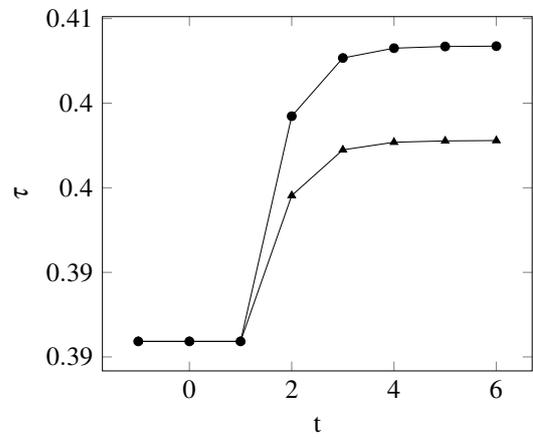
(a) Savings



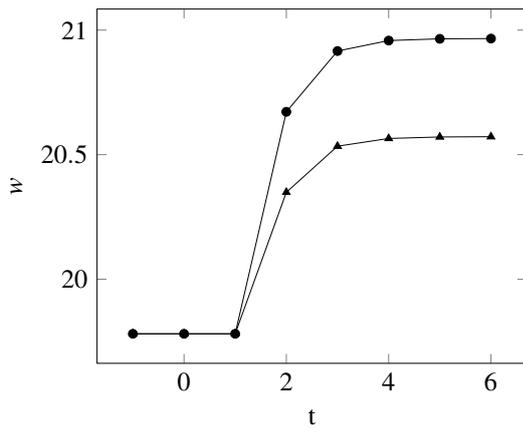
(b) Capital intensity



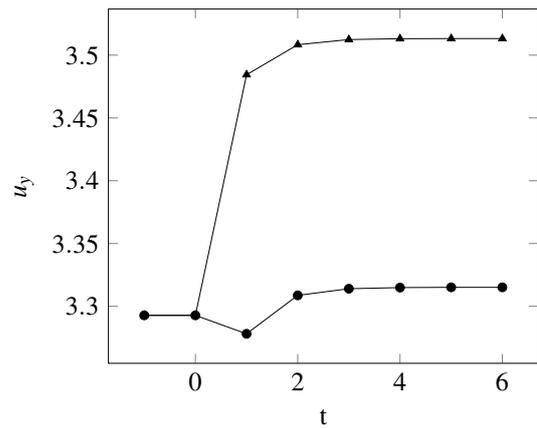
(c) Employment



(d) Contribution rate



(e) Wage level



(f) Utility young generation

—▲— Longevity —●— Fertility

Figure 2: Simulation results. Demographic shock in period $t = 2$.

5 Conclusion

This paper investigated how demographic change affects the financial sustainability of a pay-as-you-go social security system in an environment with collective bargaining on the labor market. Analytically, the following could be shown.

First, in a temporary equilibrium a higher old-age dependency ratio decreases the proportional contribution rate to the social security system. *Ceteris paribus*, the budget is unbalanced, because expenditures increase. The government can then increase tax revenues and simultaneously decrease social expenditure by adjusting the contribution rate downward. That result is obtained, because the negative indirect wage and employment effects on revenues and expenditures are stronger than the positive direct revenue effect of a contribution rate adjustment.

Second, in a temporary equilibrium a higher level of capital intensity leads to a higher contribution rate. Due to increased capital supply labor demand increases such that more contributors to the social security system are available. Simultaneously expenditures decrease due to a smaller number of beneficiaries. *Ceteris paribus*, the budget is unbalanced, because revenues increase and expenditures decrease. In this situation, a lower contribution rate would increase revenues and decrease expenditure even further. Therefore, the government adjusts the contribution rate upward, which attenuates labor demand and therefore balances the budget. As before, this result is obtained, because the negative indirect effects of a contribution rate adjustment are stronger than the positive direct effect.

General equilibrium effects of demographic change were investigated by calculating steady state values and transition paths for a calibrated economy. In contrast to the temporary equilibrium effect, it can be shown that the contribution rate ultimately increases, if the old-age dependency ratio rises. The reason for that rise is much different in comparison to a perfect labor market scenario where the wage and employment levels may be unaffected by adjustments of the contribution rate. With a perfect labor market individual contributions had to increase in order to cover the additional financing burden. In the setup of this paper with an imperfect labor market due to collective bargaining, an adjustment of the contribution rate causes indirect effects on the labor market which in turn affect revenues and expenditures of the government. The two aforementioned temporary equilibrium effects apply simultaneously in the period of the demographic shock. The net effect of both ultimately increases the contribution rate. Furthermore, due to the continued upward adjustment of the capital stock, the capital supply effect is at work in each successive period until the new steady state is reached. In the long-run, the economy then is characterized by a higher contribution rate, but also by a higher employment rate and a higher gross wage level.

A general insight of this paper is that demographic change may well cover a substantial part of its own financing burden, if labor market imperfections are taken into account. Furthermore, if demographic change induced labor demand is less severe than in the calibrated economy, the contribution rate may even decrease in order to cover the financing burden. This had the side effect that the potentially distorting effects of income related contribution rates have a lower impact on the economy as a whole.

Appendix

A unique solution of equation (22) is obtained, if the absolute value of the wage elasticity of labor demand is higher than or equal to the wage mark up: $-\frac{dn}{dw} \frac{w}{n} \geq \mu$. This necessary condition is fulfilled for $0 < \alpha \leq 0.63$. In the case of $1 > \alpha > 0.63$, more than one solution may be feasible, which is excluded in the following. Rewriting the government's budget (22) yields

$$(1 - \tau - \rho\mu) \left(\frac{1 + \tau(\mu - 1)}{1 - \tau} \right)^\alpha = \left(\frac{1 + \frac{\pi}{1+x}}{k} \right)^\alpha \frac{C\mu}{A(1 - \alpha)} \quad (30)$$

Figure 3 illustrates the existence of a unique solution of the government's budget. In the upper quadrant it depicts the left-hand-side (LHS) as well as the right-hand-side (RHS) of equation (30). While the RHS takes a constant value, the LHS is non-linear in τ . For the assumed range of α , the value of the LHS is falling with an increasing contribution rate. The lower quadrant shows the aggregate labor demand (20), which depends on τ . For the unique contribution rate that balances the budget, it gives the corresponding equilibrium employment rate.

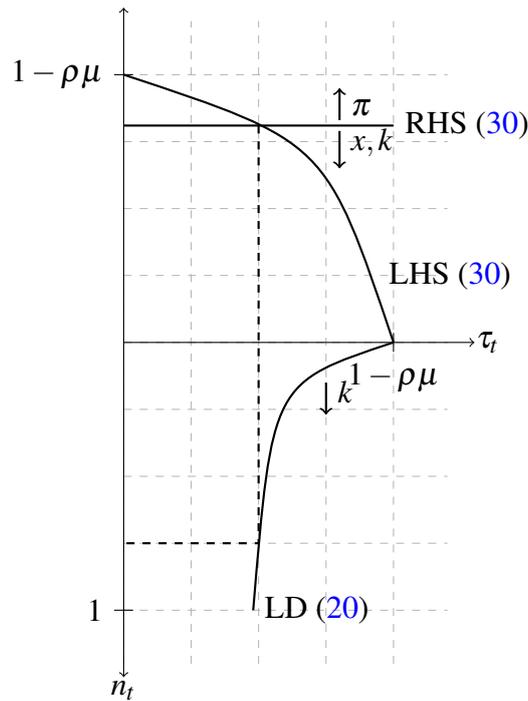


Figure 3: Aggregate labor market equilibrium

Lemma 1. *The government's budget (30) solves for a unique value of the contribution rate τ , if the output elasticity of capital is in the range of $0 < \alpha \leq 0.63$.*

Proof. The LHS of equation (30) is given by

$$LHS = (1 - \tau - \rho\mu) \left(\frac{1 + \tau(\mu - 1)}{1 - \tau} \right)^\alpha$$

Taking the derivative with respect to τ yields

$$\frac{dLHS}{d\tau} = (1 - \tau - \rho\mu) \alpha \left(\frac{1 + \tau(\mu - 1)}{1 - \tau} \right)^{\alpha-1} \frac{\mu}{(1 - \tau)^2} - \left(\frac{1 + \tau(\mu - 1)}{1 - \tau} \right)^\alpha < 0$$

$$(1 - \tau)(\alpha\mu - 1 - \tau(\mu - 1)) < \rho\alpha\mu^2$$

If $\alpha\mu - 1 - \tau(\mu - 1) \leq 0$, the LHS of equation (30) is a decreasing function of the contribution rate. This necessary condition is fulfilled for

$$\frac{\alpha\mu - 1}{\mu - 1} \leq \tau$$

which in turn holds true for any value of $0 < \alpha \leq 0.63$ or $\mu \leq 1/\alpha$. For $RHS < 1 - \rho\mu$ then, the decreasing LHS and the constant RHS take the same values for a unique $0 < \tau < 1 - \rho\mu$, which balances the government's budget (30). ■

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