Rational Bubbles and Middlemen*

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Very Preliminary

Abstract

This paper develops a finite-period model of rational bubbles where trade of an asset takes place through a chain of middlemen. We show that there exists a unique equilibrium, and a bubble and a burst can occur due to higher-order uncertainty. The equilibrium price is increasing and accelerating during bubbles although the fundamental value is constant over time. Bubbles may be detrimental to the economy; however, bubble-bursting policies affect agents' beliefs and it turns out that they have no effect on welfare. We also demonstrate that the possibility that middlemen obtain more information leads to larger bubbles.

Keywords: Rational bubbles; Middlemen; Higher-order uncertainty; Asymmetric information; Flippers

JEL Classification Numbers: D82, D83, D84, G12, G14

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1 Introduction

Bubbles refer to asset prices that exceed the fundamental value of an asset. Famous historical examples are the Dutch tulip mania (1634-7), the Mississippi Bubble (1719-20), the South Sea Bubble (1720), and the Roaring Twenties followed by the 1929 crash. A recent example is the housing bubble that preceded the 2008 financial crisis. During the time of asset market fluctuations, continuous price increases, interrupted by a sudden market crash, often occur through chains of intermediaries. These intermediaries, or middlemen, are engaged in flipping, i.e., purchasing an asset at a low price and quickly reselling it at a higher price.

This paper develops a finite-period model of rational bubbles where trade of an asset takes place through a chain of middlemen. We assume a simple network of agents, or a simple form of search frictions, where agents are located on a straight line and each agent can meet and trade only with his nearest neighbors. An agent located on the one edge of the network is the initial owner of an indivisible asset in fixed supply, whereas an agent located on the other edge is the final user of this asset. Between the owner and the user, there exist middlemen who do not consume the asset but have storage technologies that allow for the circulation of this asset through the network. Everyone can produce and consume a divisible good, which can be used to buy the asset.

In this setup, we consider the following information structure: all parameters describing utilities, costs, etc., are common knowledge, except for the consumption value of the asset for the final user. We focus on states where the consumption value is zero, in a model where the consumption value is positive in the other state. Prior to trade, all agents except for the final user observe the consumption value. When the consumption value is zero, with some probability, the final user receives a signal that tells him that the asset is worthless, and then the information that the final user knows that the asset is worthless spreads from the final user to the initial owner, but it is subject to loss between any two agents. When the final user receives the signal, there is a situation where everyone knows that the consumption value is zero, but this is not common knowledge—one may not know if others know that the asset is worthless. This opens a room for a bubble—one acquires an asset, knowing it is overpriced, in hope of finding a greater fool who believes that he can find an even greater fool.

We show that there exists a unique equilibrium. In our model, agents are rational and share a common prior distribution, but a bubble and a burst can occur in equilibrium due to higher-order uncertainty. A middleman buys the asset only if he believes that he will be able to find a greater fool who also believes to find an even greater fool. In this process, despite the fundamental value of zero, the asset is exchanged for a positive amount of the divisible good, and hence a bubble is occurring. However, if one encounters another who is pessimistic to find a greater fool, he refuses to buy the asset, and then the bubble bursts. Note that, if the fact that the fundamental value of the asset is zero were common knowledge, then bubbles would not occur. The key here is that each middleman cares less about the fundamental value of the asset, but more about how much the other agents value it. Therefore, there is a room for higher-order uncertainty to play a role for the occurrence of bubbles. Middlemen are essential for bubbles in the sense that bubbles do not occur without middlemen. Hence, this suggests that they can be a source of fragility of the economy.

The equilibrium price has the following properties. First, it is increasing over time during bubbles, and hence middlemen actually act as flippers. This is because each middleman always faces risk that he cannot find a greater fool. The price not only increases but also accelerates. This is because the probability that one can find a greater fool decreases over time. In other words, middlemen who trade in later periods are exposed to bigger risk.

Models with rational agents permit the use of standard tools to analyze welfare of the underlying economy. We show that bubbles are beneficial to the economy when agents enjoy sufficiently high utility from consuming the divisible good but detrimental otherwise. Based on this result, we discuss policy implication. “Irrational exuberance” is the phrase used by Alan Greenspan in a 1996 speech, “The Challenge of Central Banking in a Democratic Society.” The speech was given during the dot-com bubble, and the Tokyo market moved down sharply by his speech and other markets followed. Hence, his speech was interpreted as a warning that assets were overpriced. In our model, the central bank considers policy that deflates overpriced assets by revealing information about this overpricing. We assume that the central bank knows that the asset is worthless only when every agent knows that the asset is worthless, and then it announces the information before trade takes place. Then, its inaction can affect agents’ beliefs and hence prices of the asset because it reveals the information that the final user does not know that the asset is worthless. This induces a “side effect” when bubbles are detrimental, and we show that the side effect offsets the welfare gain of the bubble-bursting policy, and as a result the policy has no effect on welfare. Therefore, if the central bank has to pay some cost to announce the information, it should not employ the policy, or should keep the information secret.

We also investigate the relationship between the size of bubbles and the amount of information. In our baseline environment, with some probability, middlemen obtain information that the final user knows that the asset is worthless. Comparing the environments with and without such possibility, we demonstrate that it increases the size of bubbles. In other words, if there is the possibility that middlemen can obtain more information about the underlying economy, then prices deviate more from the fundamental value of the asset. This is because, when each middleman does not receive information, he calculates his expected utility based on the probability that he can find a greater fool conditional on the event that he does not receive information. Hence, this result suggests that the development of information technology for financial intermediaries may make the economy more fragile as long as it is incomplete in the sense that they may not receive information.

Related Literature

There are several strands of models in the literature of bubble. First, monetary models by Samuelson (1958), Tirole (1982), and others study rational bubbles in an economy with symmetric information, and thereby they need an infinite horizon to show the occurrence of bubbles.[1] Second, the models building on Allen, Morris, and Postlewaite (1993) consider rational bubbles in an economy with asymmetric information and show the occurrence of bubbles even in a finite horizon. Third, bubbles due to limited arbitrage are examined for example by DeLong, Shleifer, Summers, and Waldmann (1990) and Abreu and Brunnermeier (2003). Fourth, Harrison and Kreps (1978), Scheinkman and Xiong (2003), and others investigate heterogeneous-beliefs bubbles. In our model, agents are rational and asymmetrically informed, and share a common prior distribution. Hence, our model is included in the second strand. Asymmetric information creates a lemons problem, and thus we need some motivation for agents to trade assets. The models starting from Allen, Morris, and Postlewaite (1993) through Liu and Conlon (2018) assume risk-sharing as the motive for trade, and Liu and White (2018) employ intertemporal consumption-smoothing as the motive for trade. In our model, since the consumption value of the asset is zero for all agents except for the final user, when they obtain the asset, they are subject to risk that each of them cannot sell the asset to the next agent. In other words, when agents trade, sellers shift the risk to buyers. This is the motivation for trade in our model, which is studied by Allen and Gorton (1993), Allen and Gale (2000) and Barlevy (2014) among others. We study the relationship between bubbles and higher-order uncertainty. In this respect, our paper is also related to Morris, Postlewaite, and Shin (1995), Abreu and Brunnermeier (2003), Conlon (2004, 2015), Doblas-Madrid (2012, 2016), and Matsushima (2013).[2] To the best of our knowledge, our contribution is as follows. First, the existing models use irrationality of

[1]Search-theoretic models of money in the line of Kiyotaki and Wright (1989) are also included in this strand. See Lagos, Rocheteau, and Wright (2017) for a recent survey on these models.

[2]See Brunnermeier and Oehmke (2013) for a recent survey on bubbles.

agents or the growth of the fundamental value to explain price increases. In contrast, we show that the equilibrium price is increasing and, moreover, accelerating during bubbles although there is no irrational agent and the fundamental value is constant at zero over time. Second, in the models building on Allen, Morris, and Postlewaite (1993), buyers do not know whether they are buying from "good" or "bad" sellers, and hence it is important for the two types of sellers behave the same way. This depends on a coincidence, which means that the models are not robust. In contrast, our model is robust to small changes in most parameters. Third, again in the models building on Allen, Morris, and Postlewaite (1993), bubbles are a symptom of asymmetric information but do not hurt welfare. In contrast, in our model, they may be detrimental to the economy.[3] Fourth, it turns out that the possibility that agents obtain more information leads to larger bubbles. This is a novel insight on the size of bubbles that has never been pointed out.

Since the seminal work by Rubinstein and Wolinsky (1987), models of middlemen have been developed to study the role of middlemen not only in goods markets but also in financial markets (e.g., Duffie, Gârleanu, and Pedersen (2005)). Our environment is akin to Wright and Wong (2014), who develop a model of intermediation chains. They show that there is a bubble only when there are an infinite number of middlemen, or time is infinite. Our innovation is to provide a different information structure from theirs by relaxing the assumption of common knowledge and show that there is a bubble and a burst in finite periods. Moreover, we find that, for economies with middlemen to be active, the asset in question does not need to be valuable in the fundamental value. Gofman (2014), Glode and Opp (2016), Choi, Galeotti, and Goyal (2017), Condorelli, Galeotti, and Renou (2017), Farboodi (2017), and Manea (2018) also propose models of intermediation chains and study how intermediaries affect prices and efficiency, while we investigate how intermediaries cause bubbles due to higher-order uncertainty and

[3] Grossman and Yanagawa (1993) and Miao and Wang (2014) show that bubbles may reduce welfare in models of endogenous growth, but their bubbles are included in a different strand from ours.

discuss their implications on price changes and welfare.[4]

Hirshleifer (1971) studies the idea that it may be optimal to keep information secret. The idea is also examined by Diamond and Verrecchia (1991), Kaplan (2006), Andolfatto and Martin (2013), Andolfatto, Berentsen, and Waller (2014), Dang, Gorton, Holmström, and Ordoñez (2017), and Monnet and Quintin (2017). Our model has a case where the central bank should keep the information that the asset is worthless secret. Hence, we add a new model to this literature as well.

The Structure of the Paper

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 shows the existence and uniqueness of equilibrium. Section 4 derives implications on price changes and welfare, and discusses policy implication. Section 5 investigates the relationship between the size of bubbles and the amount of information. Section 6 provides two examples of the common prior distribution. Section 7 concludes.

2 The Model

In this section, we describe the environment of the economy in the first subsection and the knowledge structure of agents in the second subsection.

2.1 The Environment

Time is discrete and continues for $N-1$ periods from 1 to $N-1$, where $2 < N < \infty$. There are N agents $A_1, A_2, \cdots, A_N$. They are spatially separated in the following fashion: A_n can meet, and hence trade with A_{n-1} and A_{n+1} but no one else. Therefore,

[4]See the references of Wright and Wong (2014) to find more papers on middlemen. We also refer to Allen and Babus (2009) for a survey on networks in financial markets and Condorelli and Galeotti (2016) for a survey on strategic intermediation in networks.

trade between A_{n-1} and A_{n+1} must go through A_n. We assume that trade is sequential, A_n and A_{n+1} trade in period n, and A_n exits the economy after trading with A_{n+1}.

There are two objects in this economy. One is an indivisible asset x in fixed supply, and the other is a divisible good y that every agent can produce at unit cost. Only A_1 is endowed with x, that is, A_1 is the initial owner of x. He can try to trade it to A_2 in exchange for some amount of y, say y_1. We assume the consumption value of x for A_1 is zero. More generally, if A_n acquires x from A_{n-1}, he can try to trade it to A_{n+1} for y_n, which generates a payoff

$$U(y_n) = \kappa y_n$$

where κ is some positive constant.[5] The consumption value of x for A_n is 0 for each $n < N$. That for the final user A_N is $v > 0$ with some probability and 0 with the remaining probability. For simplicity, agents do not discount utilities between any two periods. Middlemen $A_2, A_3, \cdots, A_{N-1}$ are a necessary part of the process of getting x from the initial owner A_1 to the final user A_N.

We employ the generalized Nash bargaining to determine the terms of trade $(y_n)_{n=1}^{N-1}$, where agents' utilities are zero if they disagree to trade. In trade between A_n and A_{n+1}, let θ be the bargaining power of A_n with $0 < \theta < 1$.

[5]If $\kappa > 1$, it would be natural to consider a situation where A_n and A_{n+1} cannot obtain any utility from consuming the divisible good produced by A_n. Otherwise, A_n would produce an infinite amount of the divisible good. In other words, there are $N-1$ divisible goods, and A_n specialized in producing one of them that A_{n-1} can enjoy positive utility from consuming but A_n and A_{n+1} cannot. In this case, agents have specialized tastes and technologies as in search-theoretic models of money in the line of Kiyotaki and Wright (1989). Then, it would be more legitimate to regard the indivisible asset x as money rather than the divisible good y. We will see that our implications on price changes and welfare depend on whether $\kappa > 1$.

2.2 Knowledge

All parameters describing utilities, costs, etc., are common knowledge, *except* for the consumption value of x for A_N. We will consider states of the world where the consumption value is zero, and all agents including A_N know this, but this is *not* common knowledge—one may not know if others know that x is worthless. This opens a room for a bubble—one acquires an asset, knowing it is overpriced, in hope of finding a greater fool who believes that he can find an even greater fool. We employ a model of knowledge reminiscent of Rubinstein's (1989) Email game.

Prior to trade, all agents except for A_N observe the consumption value of x for A_N. When it is zero, then A_N receives a signal with some probability. Otherwise, A_N does not receive any signals. Thus, if he receives a signal, then A_N is sure that the consumption value is zero, and in this event, every agent knows that x is worthless. Moreover, if A_N receives a signal, he (non-strategically) sends a signal (email in the terminology of Rubinstein (1989)) to A_{N-1}. The signal reaches A_{N-1} with some probability but is lost with the remaining probability. Thus, if A_{N-1} receives a signal, he is sure that A_N knows that the consumption value is zero. Similarly, if A_{N-1} receives a signal from A_N, he (non-strategically) sends a signal to A_{N-2}. The signal reaches A_{N-2} with some probability but is lost with the remaining probability. This process continues until a signal is lost between some two agents or the initial owner A_1 receives a signal. In words, the signal (rumor) that A_N knows that x is worthless spreads from downstream agents to upstream agents, but it is subject to loss between any two agents.[6] We assume all these occur *prior to trade.*

To describe the above situation formally, we introduce $N + 2$ states of the world.

[6]For simplicity, we say that A_n "non-strategically" sends a signal to A_{n-1} when A_n receives a signal. However, actually, A_n does not care whether A_{n-1} receives the signal. It will turn out that, if A_{n-1} receives the signal, the only effect on A_n is that A_n does not receive an offer from A_{n-1}, but it is always optimal for A_n to reject the offer. Hence, A_n does not care whether to send a signal to A_{n-1}, and might therefore use a mixed strategy, or even just let the information randomly leak out.

The consumption value of x for A_N is $v > 0$ at state ω_v and 0 at the other states. When the state is ω_ϕ, no agent receives a signal. On the other hand, for each $n = 1, \cdots, N$, the state ω_n corresponds to the case where $A_N, A_{N-1}, \cdots, A_n$ receive signals, while the others do not. Hence, the set of the states is

$$\Omega = \{\omega_v, \omega_\phi, \omega_N, \omega_{N-1}, \cdots, \omega_1\}$$

Let μ be the common prior distribution over Ω, and assume that $\mu(\omega) > 0$ for each $\omega \in \Omega$.

We represent agents' knowledge by partitions of Ω. Agent A_N's partition is

$$\mathcal{P}_N = \{\{\omega_v, \omega_\phi\}, \{\omega_N, \omega_{N-1}, \cdots, \omega_1\}\}$$

The first element, $\{\omega_v, \omega_\phi\}$, corresponds to the case where A_N does not receive a signal and cannot find out whether x is worthless. The second element, $\{\omega_N, \omega_{N-1}, \cdots, \omega_1\}$, corresponds to the case where A_N receives a signal and knows that x is worthless. For each $n < N$, agent A_n's partition is

$$\mathcal{P}_n = \{\{\omega_v\}, \{\omega_\phi, \omega_N, \omega_{N-1}, \cdots, \omega_{n+1}\}, \{\omega_n, \omega_{n-1}, \cdots, \omega_1\}\}$$

The first element, $\{\omega_v\}$, corresponds to the case where the consumption value of x for A_N is v. The second element, $\{\omega_\phi, \omega_N, \omega_{N-1}, \cdots, \omega_{n+1}\}$, corresponds to the case where the consumption value is zero and A_n does not receive a signal. The third element, $\{\omega_n, \omega_{n-1}, \cdots, \omega_1\}$, corresponds to the case where the consumption value is zero and A_n receives a signal. Each agent can distinguish any two states if each of them belongs to a different element of his partition, but cannot otherwise.

Since we are interested in bubble, we assume the economy is at state $\omega \neq \omega_v$. Then, if A_N does not receive a signal, the posterior probability that the consumption value of x for A_N is v is $\mu(\omega_v)/[\mu(\omega_v) + \mu(\omega_\phi)]$, and hence the expected value is

$$v_e = \frac{\mu(\omega_v)}{\mu(\omega_v) + \mu(\omega_\phi)} v > 0$$

It will be useful to calculate the probability ψ_n that A_{n+1} does not receive a signal conditional on the event that A_n does not receive a signal. The probability is as follows: for $N-1$,

$$\psi_{N-1} = \frac{\mu(\omega_\phi)}{\mu(\omega_\phi) + \mu(\omega_N)}$$

and for each $n = 2, \cdots, N-2$,

$$\psi_n = \frac{\mu(\omega_\phi) + \mu(\omega_N) + \cdots + \mu(\omega_{n+2})}{\mu(\omega_\phi) + \mu(\omega_N) + \cdots + \mu(\omega_{n+2}) + \mu(\omega_{n+1})}$$

Note that $0 < \psi_n < 1$ for each $n = 2, \cdots, N-1$ because $\mu(\omega) > 0$ for each $\omega \in \Omega$. It will turn out that ψ_n is the probability that A_n can find a greater fool.

3 Equilibrium

In this section, we derive equilibrium $(y_n)_{n=1}^{N-1}$ in the economy. In the first subsection, we display the main result of the paper; in the second subsection, we prove the result.

3.1 Life of a Bubble

We will show the existence and uniqueness of equilibrium. To this end, define a sequence $(\hat{y}_n)_{n=1}^{N-1}$ as follows: for $N-1$,

$$\hat{y}_{N-1} = \theta v_e$$

and for each $n = 1, \cdots, N-2$,

$$\hat{y}_n = \theta\kappa\psi_{n+1}\hat{y}_{n+1}$$

Note that $\hat{y}_n > 0$ for each $n = 1, \cdots, N-1$ because $\theta > 0$, $v_e > 0$, $\kappa > 0$, and $\psi_{n+1} > 0$ for each $n = 1, \cdots, N-2$. We obtain the following characterization of equilibrium.

Lemma 1. *Assume $\omega \neq \omega_v$. Given the common prior distribution μ, there exists a unique equilibrium $(y_n)_{n=1}^{N-1}$. In equilibrium, if agent A_{n+1} receives a signal, agent A_{n+1} does not trade with agent A_n, or $y_n = 0$; if agent A_{n+1} does not receive a signal, agent A_{n+1} trades with agent A_n and obtains x in exchange for $y_n = \hat{y}_n$.*

If $\omega \in \{\omega_1, \omega_2\}$, a bubble does not occur because A_2 receives a signal and does not trade with A_1. If $\omega = \omega_\phi$, trade takes place, but this is simply because A_N does not know that x is worthless. If $\omega \in \{\omega_N, \omega_{N-1}, \cdots, \omega_3\}$, this lemma describes a life of a bubble.

To see this, suppose the economy is at ω_{n^*} where $n^* > 2$. Then, agents $A_N, \cdots, A_{n^*}$ receive signals, while the others do not. Given this realization, every agent knows that x is worthless, and hence the fundamental value of x is zero. Yet, the asset x is exchanged for a positive amount of the good y for $n^* - 2$ periods. In this sense, a bubble is occurring. Obviously, if the fact that the fundamental value of x is zero were common knowledge, then x would not be traded. At period $n^* - 1$, agent A_{n^*} refuses to trade with agent A_{n^*-1}, and then the bubble bursts. We summarize this result as follows.

Theorem 1. *A bubble occurs when state ω belongs to $\{\omega_N, \omega_{N-1}, \cdots, \omega_3\}$.*

Consider a fictitious situation where there is no middleman. Then, the initial owner may not be able to trade with the final user without middlemen. Even if they can trade directly, the final user refuses to trade with the initial owner because the final user receives a signal and knows that the asset is worthless. Hence, middlemen are essential for the occurrence of bubble.

During a bubble, agent A_{n+1} "buys" the asset x at the "price" $\hat{y}_n$ and tries to "resell" it at the "price" $\hat{y}_{n+1}$. Each agent is exposed to risk that he may be the greatest fool when he buys the asset. However, the final user A_N is the only "real" greater-fool in the sense that A_N can be the only agent who buys the asset from an agent who has more pessimistic information than A_N himself has. In trade among the other agents, if the seller knows that the asset is worthless, then the buyer also knows that it is worthless. Hence, buyers' risk is not that they are buying from "bad" sellers, who knows that buyers will be hurt, but that the next potential buyer may know that the final user A_N knows that the asset is worthless.

This feature makes our model robust to small changes in most parameters. In the models building on Allen, Morris, and Postlewaite (1993), buyers do not know whether they are buying from "good" or "bad" sellers, and hence it is important for the two types of sellers behave the same way. This depends on a coincidence, which means that the models are not robust.[7] In contrast, in our model, there are no good or bad sellers except when A_{N-1} is selling to A_N. Even in that case, agent A_{N-1} behaves the same way whether or not the asset is worthless because the consumption value of x for A_{N-1} is always zero.

Wright and Wong (2014) show the following two results on intermediation bubbles. First, bubbles occur if there are an infinite number of middlemen, or time is infinite, and the utility function U is nonlinear. Second, bubbles never occur if $U(y_n) = y_n$. They need positive potential gains from trade in terms of y to make bubbles occur, and hence bubbles do not occur with the linear utility function. Hence, our innovation is not only relaxing the assumption of common knowledge and showing the occurrence of bubbles in finite periods but also showing that there is a bubble even when the potential gains from trade is zero or negative. This will lead us to a different welfare implication from theirs.

3.2 Proof of Lemma 1

Proof is by backward induction.

Trade between A_{N-1} and A_N:

If A_N receives a signal, A_N knows that x is worthless, and hence A_N does not trade with A_{N-1}, or $y_{N-1} = 0$.

If A_N does not receive a signal, his expected value of x is v_e. Then, A_N and A_{N-1} negotiate the terms of trade:

$$\max_{y_{N-1}} (\kappa y_{N-1})^{\theta} (v_e - y_{N-1})^{1-\theta}$$

[7] See Liu and Conlon (2018) for a discussion of the robustness.

subject to incentive constraints: $\kappa y_{N-1} \geq 0$ and $v_e - y_{N-1} \geq 0$. Note that, if they disagree to trade, they do not obtain any utilities. The solution is

$$\hat{y}_{N-1} = \theta v_e$$

Hence, A_N obtains x in exchange for $\hat{y}_{N-1}$.

Trade between A_{N-2} and A_{N-1}:

If A_{N-1} receives a signal, A_{N-1} knows that A_N receives a signal. Then, as we have shown above, A_N does not trade with A_{N-1}, and hence A_{N-1} does not trade with A_{N-2}, or $y_{N-2} = 0$.

If A_{N-1} does not receive a signal, there are exactly two possibilities:

1. both A_{N-1} and A_N do not receive signals; and
2. A_N receives a signal, but A_{N-1} does not.

In the first case, A_N trades with A_{N-1}. In the second case, however, A_N does not trade with A_{N-1}. The first case occurs with probability ψ_{N-1}, and the second case occurs with the remaining probability. Hence, A_{N-1}'s expected utility of obtaining x is $\psi_{N-1}\kappa\hat{y}_{N-1}$. Then, A_{N-1} and A_{N-2} negotiate the terms of trade:

$$\max_{y_{N-2}} (\kappa y_{N-2})^{\theta} (\psi_{N-1}\kappa\hat{y}_{N-1} - y_{N-2})^{1-\theta}$$

subject to incentive constraints: $\kappa y_{N-2} \geq 0$ and $\psi_{N-1}\kappa\hat{y}_{N-1} - y_{N-2} \geq 0$. The solution is

$$\hat{y}_{N-2} = \theta\kappa\psi_{N-1}\hat{y}_{N-1}$$

Therefore, A_{N-1} obtains x in exchange for $\hat{y}_{N-2}$.

Induction Hypothesis:

Suppose that

1. if A_{n+1} receives a signal, A_{n+1} does not trade with A_n, or $y_n = 0$;
2. if A_{n+1} does not receive a signal, A_{n+1} trades with A_n and obtains x in exchange for $y_n = \hat{y}_n$.

Then, we will show that

1. if A_n receives a signal, A_n does not trade with A_{n-1}, or $y_{n-1} = 0$;
2. if A_n does not receive a signal, A_n trades with A_{n-1} and obtains x in exchange for $y_{n-1} = \hat{y}_{n-1}$.

Trade between A_{n-1} and A_n:

If A_n receives a signal, A_n knows that A_{n+1} receives a signal. Then, by induction hypothesis, A_{n+1} does not trade with A_n, and hence A_n does not trade with A_{n-1}, or $y_{n-1} = 0$.

If A_n does not receive a signal, there are exactly two possibilities:

1. both A_n and A_{n+1} do not receive signals; and
2. A_{n+1} receives a signal, but A_n does not.

In the first case, by induction hypothesis, A_{n+1} trades with A_n and obtains x in exchange for $\hat{y}_n$. In the second case, however, A_{n+1} does not trade with A_n again by induction hypothesis. The first case occurs with probability ψ_n, and the second case occurs with the remaining probability. Hence, A_n's expected utility of obtaining x is $\psi_n \kappa \hat{y}_n$. Then, A_{n-1} and A_n negotiate the terms of trade:

$$\max_{y_{n-1}} (\kappa y_{n-1})^{\theta} (\psi_n \kappa \hat{y}_n - y_{n-1})^{1-\theta}$$

subject to incentive constraints: $\kappa y_{n-1} \geq 0$ and $\psi_n \kappa \hat{y}_n - y_{n-1} \geq 0$. The solution is

$$\hat{y}_{n-1} = \theta \kappa \psi_n \hat{y}_n$$

Therefore, A_n obtains x in exchange for $\hat{y}_{n-1}$.

Uniqueness:

In each trade, both agents obtain positive expected utilities because $0 < \theta < 1$. Moreover, when A_{n+1} does not trade with A_n, A_{n+1} has strict incentives to refuse the trade, or choose $y_n = 0$. Hence, there is no indifference among choices of each agent, which implies the uniqueness of equilibrium.

4 Implications

In this section, we investigate the properties of equilibrium. In the first subsection, we study how the equilibrium price changes over time; in the second subsection, we derive welfare implication; in the third subsection, we discuss policy implication.

4.1 Price Changes

We will show that $\hat{y}_n$, the price that A_{n+1} has to pay to obtain the asset x, is not only *increasing* but also *accelerating* in n during a bubble. To this end, we show the following technical lemma.

Lemma 2. *If $\mu(\omega_2) \leq \cdots \leq \mu(\omega_N)$, the probability ψ_n is decreasing in n, that is,*

$$\psi_{n+1} - \psi_n < 0$$

Proof. For $n = 1, \cdots, N-3$, letting $\mathcal{M} = \mu(\omega_\phi) + \mu(\omega_N) + \cdots + \mu(\omega_{n+3})$,

$$\begin{aligned}
\psi_{n+1} - \psi_n &= \frac{\mathcal{M}}{\mathcal{M} + \mu(\omega_{n+2})} - \frac{\mathcal{M} + \mu(\omega_{n+2})}{\mathcal{M} + \mu(\omega_{n+2}) + \mu(\omega_{n+1})} \\
&= \frac{\mathcal{M}[\mu(\omega_{n+1}) - \mu(\omega_{n+2})] - [\mu(\omega_{n+2})]^2}{[\mathcal{M} + \mu(\omega_{n+2})][\mathcal{M} + \mu(\omega_{n+2}) + \mu(\omega_{n+1})]} \\
&< 0
\end{aligned}$$

The last inequality holds since we have $\mu(\omega_{n+1}) \leq \mu(\omega_{n+2})$ for each $n = 1, \cdots, N-3$ and $\mu(\omega) > 0$ for each $\omega \in \Omega$ by assumption. A similar argument holds to the case between $N-1$ and $N-2$. □

Now, we obtain the following result on price changes.

Proposition 1. *Assume $\theta\kappa \leq 1$. Then, $\hat{y}_n$ is increasing in n, that is,*

$$\hat{y}_{n+1} - \hat{y}_n > 0$$

Moreover, if $\mu(\omega_2) \leq \cdots \leq \mu(\omega_N)$, it is accelerating in n, that is,

$$\hat{y}_{n+2} - \hat{y}_{n+1} > \hat{y}_{n+1} - \hat{y}_n$$

Proof. To see that $\hat{y}_n$ is increasing, we obtain

$$\begin{aligned} \hat{y}_{n+1} - \hat{y}_n &= \hat{y}_{n+1} - \theta\kappa\psi_{n+1}\hat{y}_{n+1} \\ &= (1 - \theta\kappa\psi_{n+1})\hat{y}_{n+1} \\ &> 0 \end{aligned}$$

Note that the first equality follows from the definition of $\hat{y}_n$.

To see that $\hat{y}_n$ is accelerating, we obtain

$$\begin{aligned} & (\hat{y}_{n+2} - \hat{y}_{n+1}) - (\hat{y}_{n+1} - \hat{y}_n) \\ = \;& (1 - \theta\kappa\psi_{n+2})\hat{y}_{n+2} - (1 - \theta\kappa\psi_{n+1})\hat{y}_{n+1} \\ = \;& \hat{y}_{n+2} - \hat{y}_{n+1} + \theta\kappa(\psi_{n+1}\hat{y}_{n+1} - \psi_{n+2}\hat{y}_{n+2}) \end{aligned}$$

Here, we have

$$\begin{aligned} \psi_{n+1}\hat{y}_{n+1} - \psi_{n+2}\hat{y}_{n+2} &= \psi_{n+1}\hat{y}_{n+1} - \psi_{n+2}\hat{y}_{n+1} + \psi_{n+2}\hat{y}_{n+1} - \psi_{n+2}\hat{y}_{n+2} \\ &= (\psi_{n+1} - \psi_{n+2})\hat{y}_{n+1} - \psi_{n+2}(\hat{y}_{n+2} - \hat{y}_{n+1}) \end{aligned}$$

Combining these,

$$\begin{aligned} & (\hat{y}_{n+2} - \hat{y}_{n+1}) - (\hat{y}_{n+1} - \hat{y}_n) \\ = \;& (1 - \theta\kappa\psi_{n+2})(\hat{y}_{n+2} - \hat{y}_{n+1}) + \theta\kappa(\psi_{n+1} - \psi_{n+2})\hat{y}_{n+1} \end{aligned}$$

The first term is positive because $\hat{y}_n$ is increasing in n as we have shown above. The second term is also positive because the probability ψ_n is decreasing in n by Lemma 2. Therefore, $\hat{y}_n$ is accelerating. □

The fact that $\hat{y}_n$ is increasing is a consequence that each middleman may be the greatest fool with positive probability, $1 - \psi_n$. During a bubble, middlemen "flip"—agent A_{n+1} buys the asset x at the price $\hat{y}_n$ and tries to resell it at the price $\hat{y}_{n+1} > \hat{y}_n$. The fact that $\hat{y}_n$ is accelerating is a consequence that the probability that one can find a greater fool, ψ_n, is decreasing over time. In other words, flippers who trade in later periods are exposed to bigger risk, and the prices are determined in such a way that they compensate for the risk.

We need a sufficiently small κ for this result. If $\kappa \leq 1$, the assumption is necessarily satisfied. Otherwise, if κ is big, there are a lot of gains from trade in terms of y, and hence each agent is willing to produce a large amount of y to obtain x.[8] For price acceleration, we impose the additional assumption, which means that a state with more agents who receive signals realizes with smaller probability. In Section 6, we will provide simple examples that satisfy this assumption.

4.2 Welfare

Before discussing welfare implications, to make our analysis meaningful, we consider agents' incentives to participate in the economy. To this end, we will demonstrate that expected utility of each agent is nonnegative in both ex ante and interim stages. The ex ante stage is before the state is determined, and the interim stage is after the state was determined and before trade takes place. From the equilibrium prices, we can see that the interim expected utilities are positive for agents who do not receive signals and zero for the other agents. Hence, every agent has incentives to participate in the economy at the interim stage.[9] This further implies that the ex ante utility of each agent is positive,

[8] As discussed in footnote 5, in this case, it would be natural to consider a situation where there are $N - 1$ divisible goods, and then y cannot be regarded as the price of x because it is no longer the numeraire to define a price relative to. Therefore, we could say that, as long as y can be regarded as the price of x, the price is increasing and accelerating during bubbles.

[9] When the state is ω_v, agents $A_1, \cdots, A_{N-1}$ know that the consumption value of the asset x for the final user A_N is $v > 0$, and the final user A_N does not know the fact since he never receives a signal.

and thus every agent has incentives to participate in the economy at the ex ante stage as well. Therefore, before trade takes place, it is optimal for each agent to participate in the economy.

Now, we derive welfare implication. Our welfare criterion is utilitarian, that is, welfare is the sum of all agents' utilities. Note that, in trade between A_n and A_{n+1}, the gains from trade are

$$\kappa y_n - y_n = (\kappa - 1) y_n$$

because we normalized the production cost of y to be 1. Thus, when the economy is at ω_{n^*} with $n^* > 2$, welfare is

$$(\kappa - 1) \sum_{n=1}^{n^*-2} \hat{y}_n$$

This is because a bubble continues for $n^* - 2$ periods, and the terms of trade are $y_n = \hat{y}_n$ during a bubble and $y_n = 0$ when and after a bubble bursts. We obtain the following result on welfare.

Proposition 2. *Bubbles are beneficial to the economy if $\kappa > 1$ but detrimental if $\kappa < 1$.*

Consider a fictitious situation where the asset x is traded at its fundamental value, that is, zero. In this case, welfare is zero. Hence, whether bubbles are beneficial or detrimental depends on whether $\kappa > 1$ or $\kappa < 1$. Consider another fictitious situation where there is no middleman. Then, bubbles do not occur, and as a result welfare is zero. Hence, we can reinterpret this result as follows: When the asset is worthless, middlemen are beneficial to the economy if $\kappa > 1$ but detrimental if $\kappa < 1$.

When $\kappa > 1$, there are positive gains from trade in terms of the divisible good y, but those gains from trade can only be realized if the asset x has a positive price. Hence, bubbles are beneficial to the economy in this case. When $\kappa < 1$, there are efficiency losses from trading the divisible good y. This case could be regarded as the case of

Then, each trade takes place between any two adjacent agents, and each agent enjoys positive expected utility at the interim stage.

inflation. If we have inflation, the value of "money" y depreciates, and then bubbles are detrimental to the economy.

4.3 Policy

The central bank considers policy that deflates overpriced assets by revealing information about this overpricing. We assume that the central bank knows that the asset is worthless only when each agent knows that the asset is worthless, and then it announces the information before trade takes place. More precisely, the knowledge of the central bank is the same as the final user's:

$$\mathcal{P}_c \equiv \{\{\omega_v, \omega_\phi\}, \{\omega_N, \omega_{N-1}, \cdots, \omega_1\}\} = \mathcal{P}_N$$

Under this policy, when the state is ω_v, the central bank does not announce the information, and the final user A_N still does not know that the consumption value of the asset x is $v > 0$. When the state is ω_ϕ, the central bank again does not announce the information, and in this case, all agents except for A_N knows that A_N does not receive a signal because the central bank would announce the information if A_N received a signal. In other words, the inaction of the central bank affects agents' beliefs. More precisely, for each $n < N$, agent A_n's partition is changed by the inaction to

$$\mathcal{P}'_n = \{\{\omega_v\}, \{\omega_\phi\}, \{\omega_N, \omega_{N-1}, \cdots, \omega_{n+1}\}, \{\omega_n, \omega_{n-1}, \cdots, \omega_1\}\}$$

Then, there is no risk that each middleman cannot sell the asset to the next agent, and thus the terms of trade increase. More specifically, the terms of trade are changed to $(\hat{y}'_n)_{n=1}^{N-1}$ defined as follows: for $N-1$,

$$\hat{y}'_{N-1} \equiv \theta v_e = \hat{y}_{N-1}$$

and for each $n = 1, \cdots, N-2$,

$$\hat{y}'_n \equiv \theta\kappa\hat{y}'_{n+1} = \frac{1}{\psi_{N-1}\psi_{N-2}\cdots\psi_{n+1}}\hat{y}_n$$

When the state belongs to $\{\omega_N, \omega_{N-1}, \cdots, \omega_1\}$, the central bank announces the information, and trade does not take place.

Suppose $\kappa < 1$. Then, bubbles are detrimental to the economy, and hence the central bank wants to burst bubbles. Since it can burst bubbles when the state belongs to $\{\omega_N, \omega_{N-1}, \cdots, \omega_3\}$, the welfare gain of this policy is

$$-\sum_{n^*=3}^{N} \mu(\omega_{n^*}) \sum_{n=1}^{n^*-2} (\kappa - 1)\hat{y}_n = (1-\kappa) \sum_{n^*=3}^{N} \mu(\omega_{n^*}) \sum_{n=1}^{n^*-2} \hat{y}_n$$

However, there is a "side effect." This policy increases the terms of trade at state ω_ϕ, and thus the welfare loss is

$$-\mu(\omega_\phi) \sum_{n=1}^{N-1} (\kappa - 1)(\hat{y}'_n - \hat{y}_n) = (1-\kappa)\mu(\omega_\phi) \sum_{n=1}^{N-2} \left(\frac{1}{\psi_{N-1}\psi_{N-2}\cdots\psi_{n+1}} - 1 \right) \hat{y}_n$$

Hence, if we have

$$\sum_{n^*=3}^{N} \mu(\omega_{n^*}) \sum_{n=1}^{n^*-2} \hat{y}_n > \mu(\omega_\phi) \sum_{n=1}^{N-2} \left(\frac{1}{\psi_{N-1}\psi_{N-2}\cdots\psi_{n+1}} - 1 \right) \hat{y}_n$$

the central bank should employ the bubble-bursting policy. When $\kappa > 1$, the opposite happens, that is, it should employ the policy if the above inequality is reversed. However, regardless of the value of κ, it turns out that the side effect offsets the welfare gain.

Proposition 3. *The bubble-bursting policy has no effect on welfare.*

Proof. Since, for each $n = 1, \cdots, N-2$,

$$\psi_{N-1}\psi_{N-2}\cdots\psi_{n+1} = \frac{\mu(\omega_\phi)}{\mu(\omega_\phi) + \mu(\omega_N) + \cdots + \mu(\omega_{n+2})}$$

we have

$$\begin{aligned}
& \mu(\omega_\phi) \sum_{n=1}^{N-2} \left(\frac{1}{\psi_{N-1}\psi_{N-2}\cdots\psi_{n+1}} - 1 \right) \hat{y}_n \\
= \; & \mu(\omega_\phi) \sum_{n=1}^{N-2} \left[\frac{\mu(\omega_\phi) + \mu(\omega_N) + \cdots + \mu(\omega_{n+2})}{\mu(\omega_\phi)} - 1 \right] \hat{y}_n \\
= \; & \sum_{n=1}^{N-2} [\mu(\omega_N) + \cdots + \mu(\omega_{n+2})] \hat{y}_n \\
= \; & \sum_{n^*=3}^{N} \mu(\omega_{n^*}) \sum_{n=1}^{n^*-2} \hat{y}_n
\end{aligned}$$

The last equality holds by the following argument. In the sum $\sum_{n=1}^{n^*-2} \hat{y}_n$, we have $\hat{y}_n$ if and only if $n \leq n^* - 2$, or $n^* \geq n + 2$. Thus, in the sum $\sum_{n^*=3}^{N} \mu(\omega_{n^*}) \sum_{n=1}^{n^*-2} \hat{y}_n$, the coefficient of $\hat{y}_n$ is $[\mu(\omega_N) + \cdots + \mu(\omega_{n+2})]$, which is the same as that in the sum $\sum_{n=1}^{N-2} [\mu(\omega_N) + \cdots + \mu(\omega_{n+2})]\hat{y}_n$. □

Even though the side effect occurs only at state ω_ϕ, it is enough large to offset the welfare gain. Therefore, if the central bank has to pay some cost to announce the information that the asset is worthless, it should not employ the bubble-bursting policy, or should keep the information secret. Conlon (2015) studies the same sort of bubble-bursting policy and shows that its effect on welfare is ambiguous, that is, there are cases where it improves welfare and other cases where it worsens welfare.[10]

5 Bubbles and Information

In this section, we investigate the relationship between the size of bubbles and the amount of information. Suppose that the final user knows that the asset is worthless. Then, in the model that we have studied so far, the information spreads from downstream agents to upstream agents although it is subject to loss between any two agents. What if there is no possibility that agents $A_1, \cdots, A_{N-1}$ know that the final user A_N knows that the asset is worthless?

As before, prior to trade, all agents except for A_N observe the consumption value of x for A_N. When it is zero, then A_N receives a signal with some probability. Otherwise,

[10] Conlon (2015) also considers the case where the central bank knows that the asset is worthless whenever it is worthless. We can study this case as well. When the state is ω_v, the central bank announces the information that the consumption value of the asset is $v > 0$, and hence all agents including A_N know that the consumption value is $v > 0$. Then, the terms of trade are increased by this policy because A_N's expected utility increases. When the state is $\omega \neq \omega_v$, the central bank announces the information that the asset is worthless, and thus trade does not take place. Even in this case, we can show that the policy has no effect on welfare.

A_N does not receive any signals. Thus, if he receives a signal, then A_N is sure that the consumption value is zero, and in this event, every agent knows that x is worthless. However, now, A_N is the only agent who receives a signal. In words, the signal (rumor) that A_N knows that x is worthless *never* spreads.

To describe the difference between the information structures formally, we consider the three states that we have used before. The set of the states is

$$\Omega^0 = \{\omega_v, \omega_\phi, \omega_N\}$$

In other words, states $\omega_{N-1}, \cdots, \omega_1$ do not realize. Let μ^0 be the common prior distribution over Ω^0, and assume that $\mu^0(\omega) > 0$ for each $\omega \in \Omega^0$. Agent A_N's partition is

$$\mathcal{P}_N^0 = \{\{\omega_v, \omega_\phi\}, \{\omega_N\}\}$$

For each $n < N$, agent A_n's partition is

$$\mathcal{P}_n^0 = \{\{\omega_v\}, \{\omega_\phi, \omega_N\}\}$$

Note that middlemen $A_2, \cdots, A_{N-1}$ never know that the final user A_N knows that the asset is worthless.

Assume the economy is at state $\omega \neq \omega_v$. If A_N does not receive a signal, the expected value of x is

$$v_e^0 = \frac{\mu^0(\omega_v)}{\mu^0(\omega_v) + \mu^0(\omega_\phi)} v > 0$$

For A_{N-1}, the probability that A_N does not receive a signal is

$$\psi_{N-1}^0 = \frac{\mu^0(\omega_\phi)}{\mu^0(\omega_\phi) + \mu^0(\omega_N)}$$

The other middlemen $A_2, \cdots, A_{N-2}$ are not exposed to risk that each of them cannot find a greater fool. Define a sequence $(\hat{y}_n^0)_{n=1}^{N-1}$ as follows: for $N-1$,

$$\hat{y}_{N-1}^0 = \theta v_e^0$$

for $N-2$,

$$\hat{y}^0_{N-2} = \theta\kappa\psi^0_{N-1}\hat{y}^0_{N-1}$$

and for each $n = 1, \cdots, N-3$,

$$\hat{y}^0_n = \theta\kappa\hat{y}^0_{n+1}$$

Then, by a similar argument to Lemma 1, we can show the following.

Lemma 3. *Assume $\omega \neq \omega_v$. Given the common prior distribution μ^0, there exists a unique equilibrium $(y^0_n)^{N-1}_{n=1}$. In equilibrium, if agent A_N receives a signal, he does not trade with agent A_{N-1}, or $y^0_{N-1} = 0$; if agent A_N does not receive a signal, he trades with agent A_{N-1} and obtains x in exchange for $y^0_{N-1} = \hat{y}^0_{N-1}$. For $n = 1, \cdots, N-2$, agent A_{n+1} always trades with agent A_n and obtains x in exchange for $y^0_n = \hat{y}^0_n$.*

Now, we compare two economies with the different information structures. To make a fair comparison between the two economies with Ω and Ω^0, we assume $\mu(\omega_v) = \mu^0(\omega_v)$ and $\mu(\omega_\phi) = \mu^0(\omega_\phi)$ in the following result. This assumption means that the probabilities that the consumption value of the asset x is $v > 0$ are the same across the two economies and the probabilities that the final user A_N knows that the asset x is worthless are also the same across the two economies.

Proposition 4. *Assume $\mu(\omega_v) = \mu^0(\omega_v)$ and $\mu(\omega_\phi) = \mu^0(\omega_\phi)$. Then, $\hat{y}_n$ and $\hat{y}^0_n$ satisfy the following: for $N-1$,*

$$\hat{y}_{N-1} = \hat{y}^0_{N-1}$$

and for each $n = 1, \cdots, N-2$,

$$\hat{y}_n > \hat{y}^0_n$$

Proof. For $N-1$, we have

$$\hat{y}_{N-1} = \theta v_e = \theta\frac{\mu(\omega_v)}{\mu(\omega_v) + \mu(\omega_\phi)}v = \theta\frac{\mu^0(\omega_v)}{\mu^0(\omega_v) + \mu^0(\omega_\phi)}v = \theta v^0_e = \hat{y}^0_{N-1}$$

For $N-2$, we have

$$\begin{aligned}
\hat{y}_{N-2} - \hat{y}^0_{N-2} &= \theta\kappa\psi_{N-1}\hat{y}_{N-1} - \theta\kappa\psi^0_{N-1}\hat{y}^0_{N-1} \\
&= v_e\theta^2\kappa(\psi_{N-1} - \psi^0_{N-1})
\end{aligned}$$

Note that

$$\psi_{N-1} = \frac{\mu(\omega_\phi)}{\mu(\omega_\phi) + \mu(\omega_N)} > \frac{\mu(\omega_\phi)}{\mu(\omega_\phi) + \mu^0(\omega_N)} = \frac{\mu^0(\omega_\phi)}{\mu^0(\omega_\phi) + \mu^0(\omega_N)} = \psi^0_{N-1}$$

Hence, we obtain $\hat{y}_{N-2} > \hat{y}^0_{N-2}$.

For $n = 3, \cdots, N-1$, we have

$$\begin{aligned}
\hat{y}_{N-n} - \hat{y}^0_{N-n} &= \theta\kappa\psi_{N-n+1}\hat{y}_{N-n+1} - \theta\kappa\hat{y}^0_{N-n+1} \\
&= v_e\theta^n\kappa^{n-1}(\psi_{N-n+1}\psi_{N-n+2}\cdots\psi_{N-1} - \psi^0_{N-1})
\end{aligned}$$

Note that

$$\begin{aligned}
\psi_{N-n+1}\psi_{N-n+2}\cdots\psi_{N-1} - \psi^0_{N-1} &= \frac{\mu(\omega_\phi)}{\mu(\omega_\phi) + \mu(\omega_N) + \cdots + \mu(\omega_{N-n+2})} - \frac{\mu(\omega_\phi)}{1 - \mu(\omega_v)} \\
&> 0
\end{aligned}$$

Hence, we obtain $\hat{y}_{N-n} > \hat{y}^0_{N-n}$. □

This result means that, during bubbles, the deviation of prices from the fundamental value of the asset is greater in the economy with Ω than in the economy with Ω^0. Hence, the possibility that middlemen know that the final user knows that the asset is worthless increases the size of bubbles. In the economy with Ω, agent A_{N-1} considers the probability that agent A_N does not receive a signal conditional *not only* on the event that the consumption value of x for A_N is zero *but also* on the event that A_{N-1} does not receive a signal. On the other hand, in the economy with Ω^0, agent A_{N-1} considers the probability that agent A_N does not receive a signal conditional *only* on the event that the consumption value of x for A_N is zero. Hence, when A_{N-1} does not know that A_N knows that the asset is worthless, A_{N-1}'s expected utility of obtaining x is greater in the economy with Ω. Note that, in the economy with Ω^0, all agents except for A_N are not exposed to risk that each of them may be the greatest fool. However, the effect by A_{N-1}'s expectation outweighs the effect by the reduction of the risk for all agents except for A_{N-1}, and hence we have $\hat{y}_n > \hat{y}_n^0$ for each $n = 1, \cdots, N-2$. This result suggests that the development of information technology for financial intermediaries may make the economy more fragile as long as it is incomplete in the sense that they may not receive information.

The possibility that middlemen obtain more information leads to larger bubbles. This further has implications on the occurrence of bubbles. In our model, a bubble occurs with probability $\mu(\omega_N) + \mu(\omega_{N-1}) + \cdots + \mu(\omega_3)$ in the economy with Ω and with probability $\mu^0(\omega_N)$ in the economy with Ω^0. Hence, under the assumption that $\mu(\omega_v) = \mu^0(\omega_v)$ and $\mu(\omega_\phi) = \mu^0(\omega_\phi)$, a bubble occurs with higher probability in the economy with Ω^0. However, if each middleman has to pay the fixed storage cost when he buys the asset, there is a case where bubbles are more likely to occur in the economy with Ω due to larger bubbles.

6 Examples

In this section, we provide two simple examples of the common prior distribution μ. Both examples satisfy the assumptions that we have imposed in the previous sections.

Example 1. The distribution μ is uniform, that is, for each $\omega \in \Omega$,

$$\mu(\omega) = \frac{1}{N+2}$$

It is obvious that $\mu(\omega) > 0$ for each $\omega \in \Omega$ and $\mu(\omega_2) \leq \cdots \leq \mu(\omega_N)$. The probability ψ_n is

$$\psi_n = \frac{N-n}{N-n+1}$$

and the price $\hat{y}_n$ is

$$\hat{y}_n = \frac{1}{N-n}\theta^{N-n}\kappa^{N-n-1}v_e$$

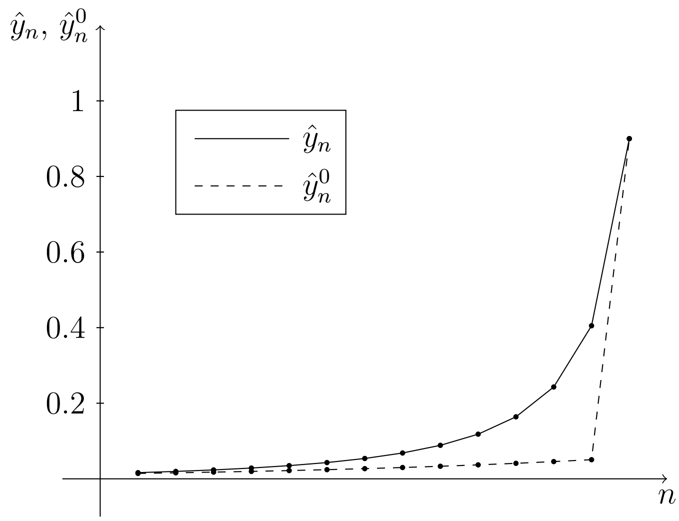

Figure 1: Graph of $\hat{y}_n$ and $\hat{y}_n^0$ for Example 1 ($N = 15$, $\theta = 0.9$, $\kappa = 1$, and $v = 2$)

where

$$v_e = \frac{1}{2}v$$

Finally, when $\mu(\omega_v) = \mu^0(\omega_v)$ and $\mu(\omega_\phi) = \mu^0(\omega_\phi)$, the price $\hat{y}_n^0$ is as follows: for $N - 1$

$$\hat{y}_{N-1}^0 = \theta v_e$$

and for each $n = 1, \cdots, N-2$,

$$\hat{y}_n^0 = \frac{1}{N+1}\theta^{N-n}\kappa^{N-n-1}v_e$$

See Figure 1 for the graph of $\hat{y}_n$ and $\hat{y}_n^0$. □

Example 2. Consider a situation where the signal to A_N is lost with probability ε, and moreover, between any two adjacent agents, each signal is lost with the same probability, ε. In this case, we have

$$\mu(\omega_\phi) = [1 - \mu(\omega_v)]\varepsilon$$

for each $n = 2, \cdots, N$,

$$\mu(\omega_n) = [1 - \mu(\omega_v)](1 - \varepsilon)^{N-n+1}\varepsilon$$

and for $n = 1$,

$$\mu(\omega_1) = [1 - \mu(\omega_v)](1 - \varepsilon)^N$$

Assume $0 < \mu(\omega_v) < 1$ and $0 < \varepsilon < 1$. Then, it is again obvious that $\mu(\omega) > 0$ for each $\omega \in \Omega$ and $\mu(\omega_2) \leq \cdots \leq \mu(\omega_N)$. The probability ψ_n is

$$\psi_n = \frac{1 - (1 - \varepsilon)^{N-n}}{1 - (1 - \varepsilon)^{N-n+1}}$$

and the price $\hat{y}_n$ is

$$\hat{y}_n = \frac{\varepsilon}{1 - (1 - \varepsilon)^{N-n}} \theta^{N-n} \kappa^{N-n-1} v_e$$

where

$$v_e = \frac{\mu(\omega_v)}{\mu(\omega_v) + [1 - \mu(\omega_v)]\varepsilon} v$$

Finally, when $\mu(\omega_v) = \mu^0(\omega_v)$ and $\mu(\omega_\phi) = \mu^0(\omega_\phi)$, the price $\hat{y}_n^0$ is as follows: for $N - 1$

$$\hat{y}_{N-1}^0 = \theta v_e$$

and for each $n = 1, \cdots, N - 2$,

$$\hat{y}_n^0 = \varepsilon \theta^{N-n} \kappa^{N-n-1} v_e$$

See Figure 2 for the graph of $\hat{y}_n$ and $\hat{y}_n^0$. □

7 Conclusion

We developed a finite-period model of intermediaries and, assuming neither irrational agents nor heterogeneous priors, showed that a bubble and a burst can occur in a unique equilibrium. The equilibrium price is increasing and accelerating during bubbles although the fundamental value of the asset is constant at zero over time. Bubbles may be detrimental to the economy; however, it turned out that the bubble-bursting policy has no effect on welfare. Moreover, we investigated the relationship between the size of bubbles and the amount of information and showed that the possibility that agents

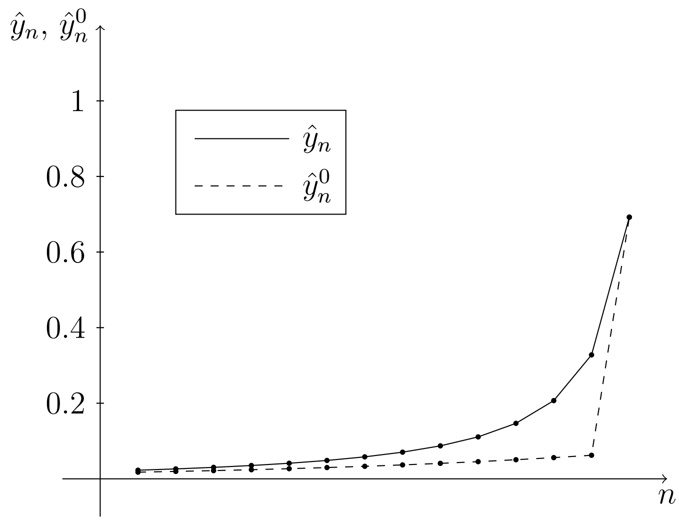

Figure 2: Graph of $\hat{y}_n$ and $\hat{y}_n^0$ for Example 2 ($N = 15$, $\theta = 0.9$, $\kappa = 1$, $\upsilon = 2$, $\mu(\omega_\upsilon) = 1/17$, and $\varepsilon = 0.1$)

obtain more information about the underlying economy increases the size of bubbles. We focused on the simple network, bilateral trade, and bargaining. It would be interesting to extend our model to more complicated networks, different matching technologies, and different pricing mechanisms. We leave these as future works.

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