Home ownership as self-insurance for long-term care - A dynamic simulation analysis

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February 2019

Abstract

Housing wealth and long-term care (LTC) risk are important drivers of the savings behavior in retirement. In a world with incomplete insurance provision, home ownership may act as a self-insurance device since in need of care one may either prefer to stay at home or sell the property in order to afford a better nursing home. The present paper captures these links by introducing health-dependent utility from housing consumption in a general equilibrium overlapping generation model. Our model also accounts for the negative intra-cohort correlation between home ownership and LTC risk which strengthens inequities in LTC insurance demand. Simulation results with the model calibrated for Germany highlight the significant intra-cohort distributional effects as well as the growth consequences of public LTC insurance.

JEL Classifications: C61, D15, G11, H55

Keywords: OLG model, tenure choice, old-age saving, LTC risk and insurance

Financial support from the German Research Foundation (DFG) grant no. FE-377/8-1 is gratefully acknowledged.

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1 Introduction

Long-term care (LTC) cost are rising steadily in almost all societies of the Western world. The main driver of this development is the fact that people tend to live longer from year to year and LTC risk is dramatically increasing in old age. While health expenses in most developed countries are covered by public and/or private insurance systems, most countries only provide a partial LTC coverage as part of the public insurance system. Since private insurances are either expensive or hardly existent, households are left with a substantial out-of-pocket cost. This risk is usually not distributed uniformly within a cohort. In almost all European countries, households with a lower socio-economic status (SES) face a higher probability to receive LTC services, see Ilinca et al. (2017) for a recent study using data from the Survey of Health, Ageing and Retirement in Europe (SHARE). The first channel is of course the fact that older people of lower SES have on average poorer health in general. However, there are also other factors such as the household structure (marital status, number of children and contacts with them, etc.) which give an advantage to receive informal care for higher income groups. Inequities in the use of LTC may also depend on the fact whether the household lives in a rented home or owns the place. In the latter case there are more options to provide informal care at home so that the home equity might be interpreted as some form of self-insurance device, see Laferrere (2012). Rouwendal and Thomese (2013) consequently find a negative effect of housing tenure on the probability of moving to a nursing home in the Netherlands. Since home ownership is also concentrated among richer households, LTC risk for lower SES households is amplified: they have poorer health and they have less options to self-insure. The provision of public LTC insurance therefore not only improves the risk allocation in society, it also has a strong redistributive component, which has only recently been emphasized in the literature, see for example Hussem et al. (2016) or Wouterse et al. (2018).

Our study applies a dynamic general equilibrium model calibrated for Germany to quantify the economic effects of LTC insurance coverage. Germany may serve as an interesting case study since it combines many of the features discussed above. First, the German statutory social security system provides an LTC insurance which was established in 1995 where the coverage of cost is only limited. Depending on the severeness of the need for care, annual private co-payments may sum up to an average pension income. Since children are forced to pay for their parents needs in case they have not enough resources, LTC risk is among the most important explanations why German elderly do not draw down their savings even in very old age. LTC risk is dramatically rising with age. While currently only 3.2 percent of cohorts aged 65-70 receive some form of care, this number reached 21 percent and 40 percent for the cohorts aged 80-85 and 85-90, respectively. Second, the German home ownership rate is fairly low at 43 percent and strongly concentrated among the more affluent households. Very few households in the lower tail of the income distribution can afford to own their house. Third, as documented by Rodrigues et al. (2018) there is a strong SES gradient in the use of home care where the rates double between the richest and the poorest wealth and income quintiles. Inequalities in the health conditions also affect the life expectancy within a cohort. For currently retired cohorts the longevity gap between the top and the bottom lifetime income decile reaches seven years according to a recent study by Haan et al. (2017).

Our applied numerical overlapping generation model tries to capture the link between the housing tenure decision at younger ages and the self-insurance provision at older age. When entering the labour market households are assigned to a specific skill (or education) class which determines their income shocks during employment and their LTC shocks during retirement. Labour supply is fixed but households have to decide in each period whether to buy a house or live in a rented home. As long as they are healthy, home owners receive a higher utility from housing consumption than renters, which may reflect the so-called "pride of ownership", see Hu (2005) or Nakajima and Telyukova (2013). After retirement, households face modest and severe LTC shocks which both reduce life expectancy and induce costs which are not fully covered by insurance. In the case of a modest LTC shock they may be able to receive home care in an owned house so that the consumption utility from housing is further increased. In the case of a severe LTC shock (think of dementia etc.) the advantages of the owned house are completely lost and may even turn into a burden since the owner is no longer able to maintain the house. Consequently, the consumption utility from owned housing decreases dramatically even below those of renters so that the house is sold immediately. In contrast to owners, the consumption utility from rented homes does not change in case of LTC shocks. Our model reflects the German LTC insurance provision, but for simplicity we do not allow children to give care or provide financial support to their parents. In addition we account for the German progressive income tax system and the pension system which both provide an implicit insurance.

We combine three different strands of the literature on savings behavior in old age: First, following Yang (2009), Chen (2010), Kindermann and Kohls (2018), Kaas et al. (2017) or Sommer and Sullivan (2018) we introduce a tenure choice in a stochastic OLG model. However, our main focus is not the tenure decision but the insurance value of owner occupied housing. Therefore a second strand of related literature are papers by Kopecky and Koreshkova (2014), Bueren (2018), Hussem et al. (2016) or Wouterse et al. (2018) which quantify the LTC risks and/or the implied distributional effects of LTC insurance reforms in life-cycle models. A third strand of literature are recent contributions which suggest that marginal utility of consumption may depend on one's health status. Especially when health shocks – as in the case of LTC shocks – lead to permanent disabilities, changes in preferences are more likely. Finkelstein et al. (2009) argue that such health shocks could induce a rise or a fall in marginal consumption utility, while Finkelstein et al. (2013) estimate a decline of marginal utility of consumption from a chronic disease. Using strategic survey questions to understand the state dependence of utility related to LTC disabilities, Ameriks et al. (2018) find a roughly 30 percent drop in marginal consumption utility after a LTC shock. Brown et al. (2016) provide similar survey-based measures of the variation of marginal utility of consumption by health status. Consistent with previous studies they find that individuals tend to value consumption in unhealthy states less than in healthy states. However, there exists a strong variation so that in some states marginal utility might even increase.

In the following we will present our numerical simulation model that captures the interaction between tenure choice and LTC risks in retirement. This model is calibrated to reflect the current situation in Germany with low and highly concentrated home ownership rates and only partial coverage of LTC risks so that households seek for self-insurance devices. Starting from that initial steady-state we perform three central simulations where we either eliminate the insurance system completely or introduce a full coverage or a mixed reform which only covers severe LTC shocks.

2 The model economy

In the following we describe the theoretical set-up of a general equilibrium OLG model with housing tenure choice which captures the distributional effects of social security reforms along the transition path between two steady-states. Within this framework households are heterogenous with respect to their productivity during the working life and their long-term care status in retirement. They have to decide in each period over consumption of goods and housing services, their savings in financial assets and their future housing tenure. The production sector of the economy is modeled by a representative firm with a standard neo-classical production function. Following Gervais (2002), the concept of a financial intermediary is used to derive the equilibrium on the rental market. Due to incomplete insurance of income and LTC risks, individuals have to form expectations about future events. We therefore formulate a dynamic programming problem and solve the model recursively.

2.1 Demographic structure and intracohort heterogeneity

We consider an economy populated by overlapping generations of individuals which may live up to a maximum possible lifespan of J periods. At each date, a new generation is born at a population growth rate n that replaces the oldest cohort from the previous period whose members die with certainty after having lived for J periods. When entering the labour market agents are assigned to a specific skill class θ which might be interpreted as a permanent shock that affects the income and LTC risks later on. Individuals face lifespan uncertainty with $\psi_{j,s} < 1$ the time-invariant conditional survival probability from age $j-1$ to age j, and $\psi_{J+1,s} = 0$. The survival probability varies with age j and depends on the current health status s.

Since our model is solved recursively, agent of age j in a specific period are characterized by the state vector $z_j = (j, \theta, x_j, h_j, ep_j, s_j, \eta_j)$ where $j \in \mathcal{J} = \{1, \dots, J\}$, $\theta \in \mathcal{E} = \{1, \dots, E\}$ denotes the permanent skill (or education) class, $x_j \in \mathcal{X} = [0; \infty]$ denotes cash-on-hand at the beginning of age j, $h_j \in \mathcal{H} = [0, \infty]$ denotes the size of the owned house held at the beginning of age j, $ep_j \in \mathcal{P} = [0, \infty]$ defines the agent's accumulated earning points for public pension claims, $s_j \in \mathcal{S} = \{0, 1, 2\}$ denotes the current health status and $\eta_j \in \mathcal{N} = [\underline{\eta}, \overline{\eta}]$ is the individual productivity at age j.

Since income and health are uncertain the productivity and health state are assumed to follow a first-order Markov process described in more detail below. Consequently, each age-j cohort is fragmented into subgroups $\xi(z_j)$, according to the initial distribution (i.e. at $j = 1$), the Markov process and optimal decisions. Let $\Xi(z_j)$ be the corresponding cumulated measure to $\xi(z_j)$. Hence,

$$\int_{\mathcal{E} \times \mathcal{X} \times \mathcal{H} \times \mathcal{P} \times \mathcal{S} \times \mathcal{N}} d\Xi(z_j) = 1 \quad \text{for all} \quad j = 1, \dots, J \tag{1}$$

must hold, as $\xi(z_j)$ is not affected by cohort sizes but only gives densities within cohorts. For later reference we define $\mathcal{Z} = \mathcal{J} \times \mathcal{E} \times \mathcal{X} \times \mathcal{H} \times \mathcal{P} \times \mathcal{S} \times \mathcal{N}$. In the following, we concentrate on the long-run equilibrium and omit the time index t, the state index z and the age j for every variable whenever possible. Future variables are then denoted by the index "$^+$".

2.2 The household side

Risk exposure over the life-cycle

Labour supply is inelastic and equal to unity, so that changes in labour productivity translate one-to-one into changes in labour income y_j. Labour productivity is a function of a deterministic, skill-specific age-profile of earnings $e_j(\theta)$ and the transitory component $\eta_j(\theta)$. The latter has an AR(1) autoregressive structure so that

$$\eta_j = \rho \eta_{j-1} + \epsilon_j \quad \text{with} \quad \epsilon_j \sim N(0, \sigma_\epsilon^2) \quad \text{and} \quad \eta_1 = 0.$$

The idiosyncratic innovation term ϵ_j is normally distributed with mean zero and variance σ_ϵ^2. Following Fehr et al. (2013), the autoregressive correlation term ρ as well as the variance of the innovation term σ_ϵ^2 are assumed to be skill-specific. Household's labour income is therefore

$$y_j = \begin{cases} we_j(\theta) \cdot \exp(\eta_j(\theta)) & \text{for } j < J_r \\ 0 & \text{otherwise.} \end{cases}$$

Households are forced to retire at age J_r and receive periodic pension payment pen_j which is computed from the sum of accumulated earning points ep_{J_r} until retirement multiplied with the accrual κ of the average income $\bar{y}$ in the economy, i.e.

$$pen_j = ep_{J_r} \times \kappa \times \bar{y}$$

The concept of earning points ensures that the pensions households receive are proportional to the contributions they make during their working life: In each period until retirement individuals pay contributions which are proportional to their income up to the contribution ceiling into the pension system. Given the contribution rate τ^p and a contribution ceiling which amounts to twice the average income, periodical contributions are given by $\tau^p \max[y_j, 2\bar{y}]$. Since average income earner receive exactly one earning point, accumulation of earning points is computed from

$$ep_{j+1} = ep_j + \min\left[\frac{y_j}{\bar{y}}, 2\right]. \tag{2}$$

During retirement, an individual's uncertainty on labour productivity is replaced by uncertainty over the need for long-term care. We distinguish three different states: Individuals can continue to be in good health ($s = 0$), they might be subject to a moderate long-term care shock ($s = 1$) or, in the worst case, are exposed to a severe long-term care shock ($s = 2$). The transition between the different states occurs with probability $v_{j,s,\theta}$ which depends on age j, the current long-term care status s and skill level θ. As already pointed out above, LTC affects mortality $\psi_{j,s}$ and an individual's utility from owner-occupied real-estate. Individuals in good health do not face any cost, while those subject to a moderate or severe long-term care shock face expenses

$$m_s = \varpi_s \bar{y} \quad \text{with} \quad \varpi_0 = 0.$$

We assume that $\varpi_2 > \varpi_1$ holds, so that individuals with a moderate LTC shock are better off than those with a severe LTC shock. The government provides a partial insurance for these expenses with a coverage ς_s that depends on the LTC state. Similar to the pension system, it is mandatory for individuals to pay contributions $\tau^s y_j$ into the LTC insurance until retirement.

Housing tenure choice

In each period, households have to decide how much to consume, how much to save and whether to become a renter or owner in the next period. Households who want to buy a house have to choose a specific house size h^+ and the amount of financial assets a_f^+ they want to hold. The difference between financial assets a_f^+ and maximum mortgage ζh^+ the household could take out against the house determines either the current mortgage debt or the amount of liquid assets available. The maximum mortgage is fixed by the maximum loan-to-value ratio ζ which is specified exogenously. As a consequence, the households split up their aggregate savings a^+ into the selected house size h^+ (net of mortgage debt), the resulting transaction cost $tr(h, h^+)$ of changing the house size and liquid assets, i.e. $a^+ = (1-\zeta)h^+ + tr(h, h^+) + a_f^+$. For a specific house investment, the household chooses ω^+ as a fraction of his total savings a^+ so that we obtain the next period's house value as $h^+ = \frac{\omega^+ a^+}{1-\zeta}$. Financial assets are then determined as $a_f^+ = (1-\omega^+)a^+ - tr(h, h^+)$. Households who become renter (i.e. $\omega^+ = h^+ = 0$) can invest all their savings net of transaction cost on the financial market, i.e. $a_f^+ = a^+ - tr(h, 0)$. Following Yang (2009) homeowners may change their housing consumption by undertaking housing renovation or by allowing depreciation up to a fraction of φ their house size. If the size is reduced or appreciated more than this fraction, it is assumed that the household has moved. In this case the household has to pay transaction costs which are a fraction ϕ_1 of its selling value and a fraction ϕ_2 of its buying value. As we normalize the prices for real estate to unity this implies

$$tr(h, h^+) = \begin{cases} 0 & \text{if} \quad h^+ \in [(1-\varphi)h; (1+\varphi)h], \\ \phi_1 h + \phi_2 h^+ & \text{otherwise.} \end{cases}$$

Beside transaction costs, the purchase of housing properties is subject to a minimum size h_{min}, which should capture the indivisibility of real estate. If an individual can not afford the minimum housing size, he/she is forced to rent housing services. Real estate comes with periodical maintenance costs δ_h and implies opportunity cost in the form of foregone interest.

Household's optimization problem

Our model assumes a preference structure that is represented by a time-separable, nested CES utility function. The maximization problem of a representative consumer at age j and state z is formulated recursively given the set of control variables $\Omega = (c, a^+, \omega^+)$ as

$$V(z) = \max_{\Omega} \frac{\left[c^{\nu} (\chi_{h,s} c_h)^{1-\nu}\right]^{1-\frac{1}{\gamma}}}{1-\frac{1}{\gamma}} + \beta\psi_{j+1} \left(\mathbf{1}_{\omega^+>0} E\left[V(z^+)\right] + \mathbf{1}_{\omega^+=0} E\left[V(z^+)\right]\right), \qquad (3)$$

where the variables c and c_h denote ordinary and housing consumption, respectively. The parameters ν, γ and β denote the Cobb-Douglas expenditure share of ordinary consumption, the intertemporal elasticity of substitution and the discount rate, respectively. Consumption of housing services may be derived from owner-occupied property or from rental housing. Homeowners consume their property h as housing services (i.e. $c_h = h$), while renters (where $h = 0$) have to purchase housing services on the rental market. The scaling parameter $\chi_{h,s}$ controls for the utility drawn from owner-occupied real estate and is dependent on the current home ownership h and long-term care status s. $\mathbf{1}_{k=x}$ is an indicator function that returns 1 if $k = x$ and 0 if $k \neq x$. The expectation operators E in equation (3) are with respect to the stochastic processes of productivity η during working age and the uncertainty of LTC need during retirement. Note that we abstract from a bequest motive in our set-up, so that all bequests are accidental.

Current homeowners (i.e. where $h > h_{min}$) are subject to the constraints

$$\begin{aligned} a^+ &= x - (1+\tau^c)c, \quad 0 \le \omega^+ \le 1, \quad a^+ \ge tr(h,h^+), \quad h^+ = \frac{\omega^+ a^+}{1-\xi} \ge h_{min}, \\ a_f^+ &= (1-\omega^+)a^+ - tr(h,h^+) \\ x^+ &= (1+r)(a_f^+ - \zeta h^+) + y^+(1-\tau^s) + pen^+ + b^+ + (1-\delta_h)h^+ - \\ & \quad \tau^p \max[y^+, 2\bar{y}^+] - T_y(y^+, pen^+, a_f^+) - (1-\varsigma_s)m_s^+ \end{aligned} \tag{4}$$

with x being the available wealth of an individual and τ^c the consumption tax-rate. Current resources are split between consumption and aggregate assets. Households who buy a house are restricted by transaction cost, the maximum loan-to-value ratio and the minimum house size. Future resources are derived from financial assets (including interest r), labour or pension income (net of social security contributions), housing assets and bequest net of LTC cost and income taxes. As annuity markets are excluded from this model, households obtain bequests b from previous generations. These bequests are left in the form of foregone future financial assets a_f^+ and the foregone stock of future real estate h^+ from the previous period's population which is distributed equally across all individuals in the current workforce in the current period.

The objective function of a current renter is similar, except that we have $h = 0$ so that the current constraint (4) changes to

$$a^+ = x - (1+\tau^c)c - r_h c_h.$$

Renters have to pay rent r_h per housing unit c_h. Of course, they face the same restrictions on future resources, future house size and borrowing as current owners. The rental price r_h is linked to the return of financial assets through

$$r_h = r + \delta_h, \tag{5}$$

which makes sure that renters implicitly bear the maintenance cost of the house and compensate the landlord for the opportunity cost of foregone interest.

When looking at the optimization problem (3), we recognize that it has a particular structure that allows us to solve it in the following three steps:

1. *Wealth exposure in real estate:* Given a current state $\tilde{z} = (j, \theta, h, ep, s, \eta)$ and total savings a^+ we need to split them between financial and housing assets which yields an allocation $\omega^+ = \omega(\tilde{z}, a^+)$. In case of a future renter household we simply set $\omega^+ = \omega(\tilde{z}, a^+) = 0$.
2. *The consumption-savings decision:* Given a current state z and the optimal split between financial and housing assets ω^+, we can solve the consumption savings decision depending on future home ownership status o^+ in order to get $c(z, o^+)$ and $a^+(z, o^+)$ for a (current) homeowner and in addition $c_h(z, o^+)$ for a (current) renter.

3. *The tenant decision:* Finally, given optimal consumption and savings for both ownership options $o^+ = O$ and $o^+ = R$, we can determine the respective value functions and select the optimal future home ownership $o^+(z)$.

In order to save space the optimization procedure is explained in Appendix A.1. in more detail.

3 The production side

The production sector in the economy only produces one output good according to its production function and receives capital K as well as labour L as input from perfectly competitive markets. The production function is in the following of Cobb-Douglas type so that

$$Y = \Phi K^\alpha L^{1-\alpha}$$

with Φ as the level of technology and α as the share of capital used in the production process. In a closed economy the interest rate r is given by the marginal product of the production function with respect to capital K, i.e.

$$r = \alpha\Phi \left(\frac{L}{K}\right)^{1-\alpha} - \delta_k \tag{6}$$

with δ_k as the depreciation rate of capital. The wage rate w is independently from the type of the economy given as the marginal product of the production function with respect to labour, i.e.

$$w = (1-\alpha)\Phi \left(\frac{K}{L}\right)^{\alpha}. \tag{7}$$

4 Financial intermediaries

The supply side on the rental market of the economy is fully provided by financial intermediaries (Gervais 2002). Financial intermediaries are modeled as two-period lived institutions and receive deposits D from households in the first period of their life, which are fully used for the purchase of rental properties RP and to issue mortgages M to households. The rental properties RP are immediately rented out in return for a periodical rent $r_h RP$ that is obtained at the beginning of the second period. Similarly the mortgages issued in the first period yield an interest payment of rM at the beginning of the next period, whereas the financial institution has to pay interest rD itself on the deposits received in the first period and faces maintenance cost δ_h for the housing stock RP. At the same time the stock of real estate RP is liquidated and sold to a new institution in order to use the proceeds together with the loans issued M in the first period to pay back deposits D received from households. The respective optimization problem of a financial intermediary is then given by

$$\max_{RP,M,D} r_h RP + rM - \delta_h RP - rD \quad \text{s.t.} \quad RP \leq D - M$$

The constraint ensures that financial intermediaries do not purchase more rental properties RP than their available wealth, i.e. their net position on the capital market. Under perfect competition a zero profit condition has to hold for the rental agency, i.e. the periodical price r_h for rental properties for households has to be equal to marginal costs, so that (5) holds.

5 Government

The role of the government in this model is to provide public goods and a social security system to cover longevity and long-term care risk. Social security is provided in two separate pay-as-you-go financed systems. Consequently, the sum of pension contributions made by working age cohorts have to finance the aggregated pension benefits of retired households in a certain period, so that

$$\tau^p \int_{\mathcal{Z}} \min[y, 2\bar{y}] \mathrm{d}\Xi(z) = \int_{\mathcal{Z}} pen \mathrm{d}\Xi(z) \tag{8}$$

holds and the contribution rate τ^p is adjusted to balance the budget.

Similarly, individuals contribute during working life into the LTC insurance and are eligible for benefits in the case that they receive an LTC shock. Again, the contribution rate τ^s balances the budget

$$\tau^s \int_{\mathcal{Z}} y \mathrm{d}\Xi(z) = \int_{\mathcal{Z}} \varsigma_s m_s \mathrm{d}\Xi(z). \tag{9}$$

In each period t the government issues new debt $(1+n)B_{G,t+1} - B_{G,t}$ and collects taxes from workers in order to finance the provision of a public good G which is fixed per capita as well as interest payments on its debt. The steady state budget therefore reads

$$nB_G + T_y + \tau^c C = G + rB_G, \tag{10}$$

where revenues of income taxation are computed from

$$T_y = \int_{\mathcal{Z}} T^{GER}(y, pen, a_f - \zeta h) \mathrm{d}\Xi(z)$$

In the initial long-run equilibrium we specify the debt-to-output ratio B_G/Y as well as the public consumption-to-output ratio G/Y and adjust the consumption tax rate τ^c endogenously to balance the budget. We assume that contributions to public pensions are exempted from tax, while benefits are fully taxed. Given taxable labour income, we apply a progressive tax code that mimics the one in Germany and add the proportional tax τ^r on capital income $\max[a_f - \zeta h; 0]$. Note that we do not allow individuals in this formulation to deduct mortgage interest-payments from their tax-payments.

6 Equilibrium definition

A stationary equilibrium is given by a policy scheme $\{\tau_t^c, \tau_t^p, \tau_t^s, g_y, b_y, \zeta\}\forall t$, a set of prices $\{r, w\}$, value function $V(z)$, a household's decision rules $c(z)$, $a_f^+(z)$, $h^+(z)$, or $c(z)$, $a^+(z)$, $\omega^+(\tilde{z}, a^+)$ equivalently, a distribution of bequests $b(z)$ and the measure of households $\xi(z)$ at all states $z = (j, \theta, x, h, ep, s, \eta)$ of the state space $\mathcal{Z}$ such that the following conditions are satisfied:

1. households' decision rules solve the households decision problem (3);
2. factor prices are competitive, i.e. (6) and (7) hold;
3. individual and aggregate behavior are consistent:

$$\begin{aligned}
A_F &= \int_{\mathcal{Z}} a_f(z) \mathrm{d}\Xi(z) \\
C &= \int_{\mathcal{Z}} c(z) \mathrm{d}\Xi(z) \\
H_O &= \int_{\mathcal{Z}} \mathbf{1}_{h>0} h(z) \mathrm{d}\Xi(z) \\
H_R &= \int_{\mathcal{Z}} \mathbf{1}_{h=0} c_h(z) \mathrm{d}\Xi(z) \\
L &= \int_{\mathcal{Z}} e_j(\theta) \exp(\eta_j(\theta)) \mathrm{d}\Xi(z) \\
M_S &= \int_{\mathcal{Z}} m(z) \mathrm{d}\Xi(z) \\
T_R &= \int_{\mathcal{Z}} tr(z) \mathrm{d}\Xi(z);
\end{aligned}$$

4. the initial distribution (1) and the law of motion for the measure of households

$$\xi_{t+1}(z^+) = \int_{\mathcal{Z}} \mathbf{1}_{x^+=x^+(z)} \times \mathbf{1}_{h^+=h^+(z)} \times \mathbf{1}_{ep^+=ep^+(z)} \pi^s(s^+|s) \pi^\eta(\eta^+|\eta) \mathrm{d}\Xi(z).$$

   holds;

5. unintended bequests satisfy

$$(1+n) \int_{\mathcal{Z}} b(z) \mathrm{d}\Xi(z) = \int_{\mathcal{Z}} (1 - \psi_{j+1,s}) \big((1+r) a^+(z) + (1 - \delta_h) \mathbf{1}_{h^+>0} h^+(z) \big) \mathrm{d}\Xi(z);$$

6. the budgets (8), (9) and (10) of the pension, the LTC and the tax systems are balanced;
7. financial intermediaries optimize under perfect competition, i.e. (5) holds and the rental market clears;
8. the capital market clears, i.e. $K = A_F - \zeta H_O - H_R$
9. the goods market clears, i.e. $Y = C + G + (n + \delta_k)K + (n + \delta_h)(H_O + RP) + T_R + M_S$

7 Calibration

In this section we explain the baseline calibration of our model and discuss the parameterization shown in Table 1. In order to reduce computational time, each model period covers five years. Agents start their life at age 20 ($j = 1$), are forced to retire at age 65 ($j_r = 10$) and face a maximum possible life span of 100 years ($J = 16$). Following Fehr et al. (2013), we assign them to three skill levels ($E = 3$), where we use the International Standard Classifaction of Education (ISCED) of the UNESCO. We merge levels 0 to 2 (primary and lower secondary education), levels 3 and 4 (higher secondary education) and levels 5 and 6 (tertiary education). The initial probability distribution is calculated using data from the German Socio-Economic Panel (SOEP), a description of which can be found in Wagner, Frick and Schupp (2007). The consumption share $\nu = 0.6$ in the Cobb-Douglas utility function is slightly lower than in Kaas et al. (2017) and is used together with the time-discount factor ($\beta = 0.97$) to obtain realistic life-cycle profiles for the home ownership and asset accumulation. With respect to the scaling factor $\chi_{h,s}$ for utility from housing-services we assume that individuals in good health ($s = 0$) prefer living in their own property over renting ($\chi_{0,s} = 0.5$ and $\chi_{h>0,0} = 1.0$). This is justified by the usual assumption that a homeowner is able to modify a property according to subjective tastes without being restricted by a landlord. A value of $\chi_{0,s} = 0.5$ is slightly lower than as it is suggested for instance by Hu (2005) and might be justified by the highly customized properties in Germany. If an individual is subject to a moderate long-term care shock, we assume that living in the own property gets even more valuable and hence set $\chi_{h>0,1} = 1.50$. This can be justified by the fact that it is easier to implement necessary modifications that ensure accessibility in owned properties than in rented accommodations. In addition there are more options to provide formal care in a owner-occupied house. In the case of a severe long-term care shock, physical limitations or dementia make it hard for individuals to continue to maintain their property so that we set $\chi_{h>0,2} = 0.2$. For the intertemporal elasticity of substitution we use a standard value of $\gamma = 0.5$ from the literature. For the the production function of the representative firm we assume a capital share in the production function of $\alpha = 0.36$ and set the annual deprecation rate $\delta_k = 1.50\%$ to generate a realistic capital-output ratio of 3.9. The parameters for the auto-regressive productivity shock are taken from Fehr et al. (2013) and converted to 5 year periods. With respect to the LTC transition probabilities, we initialize them to match the population shares Germany that are subject to moderate and severe long-term care shocks above age 65. The respective probabilities depend on age and the shocks are not reversible. Once an individual is hit by an LTC shock, the probability for being in good health again is zero so that the LTC state can only stay constant or deteriorate in the future. We set the cost for a moderate LTC care to be equal to the average annual income and for a severe care to be equal to twice the average annual income, i.e. $\varpi_1 = \bar{y}$ and $\varpi_2 = 2\bar{y}$ respectively.

The survival probabilities in our model are based on the 2012/14 Life Tables for Germany

Table 1: Baseline parameter values

Symbol	Definition	Value
	Preferences	
ν	Ordinary consumption share	0.60
$\chi_{h,s}$	Scaling of utility from housing services	
	... of renters	0.5
	... of homeowners (good health)	1
	... of homeowners (moderate LTC)	1.5
	... of homeowners (severe LTC)	0.2
β	Time discount factor	0.98
γ	Intertemporal elasticity of substitution (IES)	0.50
	Production Parameters	
α	Capital share in Production	0.36
δ_k	Capital depreciation rate	0.08
	Productivity	
$e_j(\theta)$	Productivity at age j	[1]
ρ	AR(1) correlation	
	low-skilled	0.95666
	middle-skilled	0.95687
	high-skilled	0.95828
σ^2_ϵ	Transitory variance	
	low-skilled	0.02321
	middle-skilled	0.02812
	high-skilled	0.03538
	Mortality and Long-term care	
$v_{j,s,\theta}$	LTC Probabilities	[2]
$\psi_{j,s}$	Survival Probabilities	[2]
	LTC cost	
ϖ_1	... for a moderate shock	$\bar{y}$
ϖ_2	... for a severe shock	$2{\cdot}\bar{y}$
	Housing market	
h_{min}	Minimum house size	$2.25{\cdot}\bar{y}$
δ_h	Depreciation rate	0.015
ζ	Loan-to-value ratio	0.70
	Transaction cost	
ϕ_1	... of selling price	0.06
ϕ_2	... of buying price	0.12
φ	... free fraction	0.2
	Government	
τ_r	Tax-rate on capital income	0.25
b_y	Government debt (in % of GDP)	0.80
g_y	Government spending (in % of GDP)	0.20
κ	Pension accrual rate	0.05
	Social security coverage of cost	
ς_1	... from a moderate LTC shock	0.50
ς_2	... from a severe LTC shock	0.50

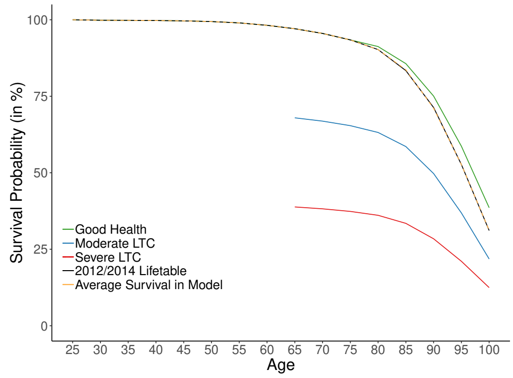
[1] Taken from Fehr et al. (2013).

[2] More Details in Appendix A.2.

Table 2: Population-shares in LTC (%)

	Modest LTC		Severe LTC	
Age	Model	Data	Model	Data
65	0.00	1.35	0.00	0.62
70	1.84	2.16	0.90	1.00
75	4.02	3.51	2.25	1.88
80	7.56	6.11	3.80	3.77
85	14.36	12.55	7.29	8.59
90	21.16	22.21	14.14	17.50
95	35.91	33.79	29.11	32.35

Figure 1: Survival Probabilities over the Life-cycle



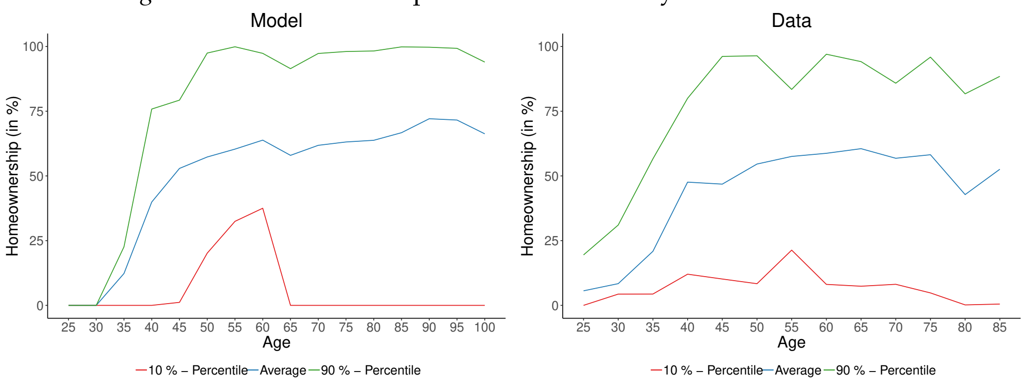
reported by Statistisches Bundesamt (2016) and adjusted to 5 year periods, see Table 2. As the mean time spent in long-term care are merely 51.4 months for men and 36.6 months for women (Ehing et al. 2015) we lower the survival probabilities for both moderate and severe long-term care shocks by 30% and 60%, respectively. The survival probabilities for healthy individuals are then adjusted in order to match the 2012/14 Life Tables for Germany again on average.[1] Figure 1 compares the resulting survival probabilities and compares the average with the actual values from the German Life Table.

The minimum housing size is set to twice the average annual income and used to obtain a very low home ownership rate for the lower end of the wealth distribution as illustrated in Figure 2. The minimum down payment requirement of $\zeta = 0.3$ is above other values in the literature and reflects the very prudent financing regulations for real-estate in Germany. The annual maintenance cost for real-estate is set to $\delta_h = 1.5\%$ per year and generates a realistic value of 12.2% of the GDP per year which are invested into maintaining the stock of real-estate.

[1] A more detailed discussion of this procedure can be found in appendix A.3.

The population growth is set to $n = 0.01$ per year and should be interpreted rather as growth rate in labour productivity than actual population growth. The accrual rate for the pension system is initialized as $\kappa = 0.05$ so that after forty years of contribution one would receive a benefit which is about forty percent of average income. Social security is covering half of the expenses of LTC cost independent of the shock, i.e. $\varsigma_1 = \varsigma_2 = 0.5$. Wage income is taxed with a progressive tax-system that mimics the implementation in Germany. The fraction of government spending is initialized to be $g_y = 0.19$ of the total output and government debt to $b_y = 0.8$ which are realistic values for Germany.

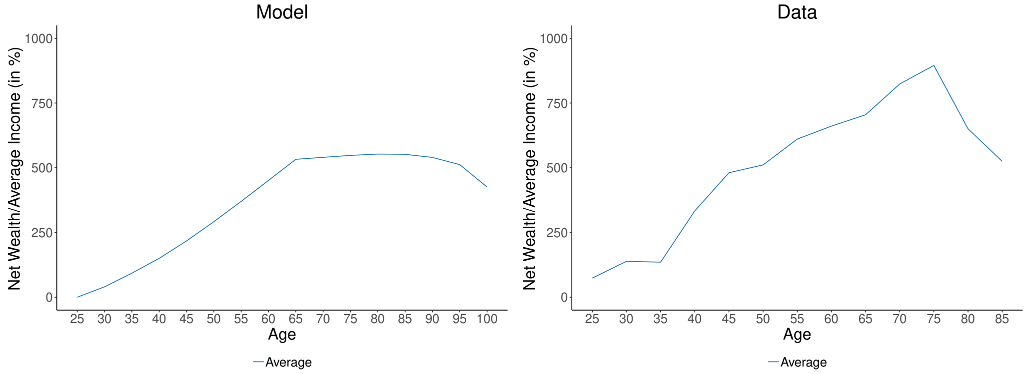
Figure 2: Home ownership rates over the life-cycle: Model and data



With this parameterization of our model we obtain a home ownership rate of 41.5% that is similarly low as the value of 43.0% we can observe in the data. In order to calibrate the life-cycle profiles to the Germany, we use the HFCS dataset from the ECB. In this dataset we group our observations into 13 data cells of 5-year age groups ranging from 25-29 until 85-89. Then we split each data cell into net wealth quintiles and compute the home ownership rates for the whole data cell as well as for the wealth-richest and the wealth-poorest individuals. Individuals start purchasing real-estate at age 30, the average home ownership rate then increases until age 50 and then remains constant until very high ages. The bottom wealth decile of each cohort hold no real-estate, while the top wealth decile have a home ownership rate close to 100% as it is illustrated in Figure 2. The left part shows the simulated profiles, while the right part shows the profiles from the data.

The lifecycle-pattern of net-wealth that is illustrated in Figure 3. Individuals accumulate wealth until retirement and hold it constant throughout their remaining lifespan. As we abstract from bequest motives in our set-up, this behavior is explained purely from a precautionary savings motive. The estimated income processes we use from from Fehr et al. (2013) comes

Figure 3: Net Wealth over the Life-cycle



close to explaining Gini-coefficient for gross-income, however has difficulties in matching Gini-coefficients for the wealth distribution. This is a well known issue in this type models and for instance addressed by Kaas (2017) as well as De Nardi and Fella (2017). With respect to the social security system we obtain a pension contribution rate of 14.0% and a contribution rate to the LTC insurance of 2.0% which are realistic values for Germany. Pension benefits paid out by the pension system are slightly lower with 8.6% in the model than with 11.0% in the data, while total amount of LTC benefits paid amount to 1.0%, see Table 3.

Table 3: The initial equilibrium

Calibration target	Model	Germany 2015
Life-expectancy	80.6	80.6
Pension benefits (% of GDP)	8.6	11.0
Pension contribution rate (in %)	14.3	18.9
Tax revenues (% of GDP)	20.5	22.0
LTC benefits (% of GDP)	1.0	0.9
LTC contribution rate (in %)	2.0	3.0
Capital-output ratio	3.9	3.8
Investment in real estate (% of GDP)	12.2	10.9[1]
Home ownership rate	41.5	45.0[2]
Other benchmark coefficients		
Interest rate p.a. (in %)	1.61	–
Gini index gross income	0.395	0.50[3]
Gini index wealth	0.494	0.790[3]

*Source: IdW(2016), [1] ZIA, [2] HFCS, [3] IW Koeln.

8 Simulation results

This section discusses the macroeconomic dynamics and distributional effects along the transition-path for three different reforms scenarios:

(1) A complete elimination of the LTC insurance (i.e. $\varsigma_1 = \varsigma_2 = 1.0$);

(2) A complete coverage of LTC cost (i.e. $\varsigma_1 = \varsigma_2 = 0$);

(3) A mixed coverage regime with full coverage for severe LTC shocks (i.e. $\varsigma_1 = 0, \varsigma_2 = 1$).

8.1 Elimination of LTC insurance

When the LTC insurance is eliminated in the first period of the transition, all households will increase their savings in order to better self-insure against LTC shocks. Consequently, wealth increases significantly throughout the transition. Not surprisingly, particularly low-skilled individuals build up more savings since they have higher risk and less options for self-insurance. Due to higher wealth accumulation also home ownership rates increase for high- and middle-skilled households who seek more self-insurance. Low-skilled, on the other hand, reduce their housing wealth, since it has less value for them as a precautionary device. Especially on impact, those who are hit by a LTC shock have to sell their housing equity in order to finance LTC needs. The aggregate home ownership rate therefore hardly changes on impact, but in the medium run there is a significant increase in home ownership due to the rising wealth. Higher savings also tend to reduce the interest rate slightly during the transition. Of course, the contribution rate for LTC insurance immediately falls to zero, so that the income tax base increases. Higher savings also increase income taxes, but the drop in aggregate consumption dominates so that consumption taxes have to rise during the transition.

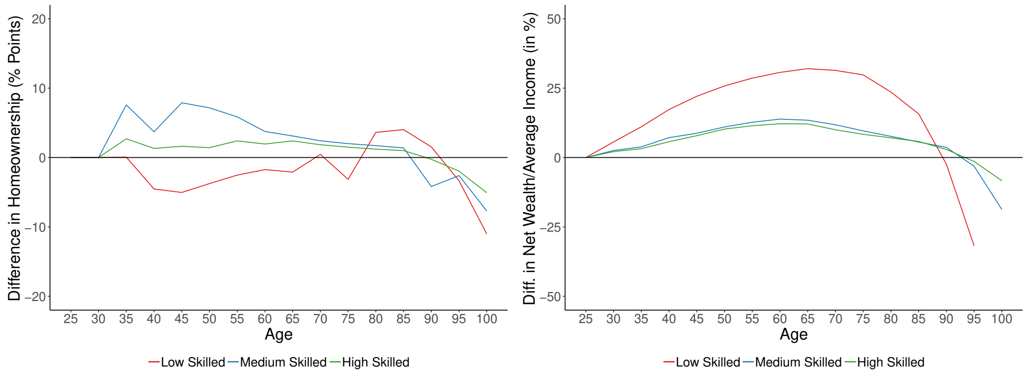
To highlight the differences in saving patterns between the skill classes, Figure 4 compares the long-run differences in home ownership rates and relative net wealth over the life cycle. Although the increase their wealth much stronger than middle- and high-skilled, low-skilled reduce home ownership already al younger ages significantly while the two other skill classes increase home ownership. Of course, in later years of retirement all households reduce home ownership significantly, when hit by severe LTC shocks.

Table 4: Macroeconomic effects of LTC insurance elimination (in %)

Period	t = 1	t = 3	t = 5	t = 7	∞
Macroeconomic aggregates					
Home ownership rate[a]	0.12	1.12	1.62	1.77	1.75
low-skilled[a]	-2.93	-1.71	-1.56	-1.54	-1.54
medium-skilled[a]	1.30	2.43	3.22	3.46	3.42
high-skilled[a]	0.77	1.14	1.27	1.32	1.32
Wealth ($A_F + (1-\zeta)H_O$)	2.16	3.71	3.91	3.95	3.96
low-skilled	3.48	6.31	6.58	6.65	6.65
medium-skilled	1.74	2.89	3.08	3.08	3.08
high-skilled	1.44	2.21	2.44	2.50	2.53
Prices					
Interest rate[a]	-0.23	-0.25	-0.26	-0.26	-0.26
Consumption tax rate[a]	0.78	0.95	0.89	0.87	0.87

[a]In percentage points.

Figure 4: Steady state comparison of net wealth and home ownership rates



8.2 Full coverage of LTC shocks

Next we consider a reform where the government introduces a full coverage of all LTC cost. As a consequence, LTC expenditures in % of GDP almost double from 1 to 2% and the contribution rate rises from 2 to 3.8 percentage points. The full coverage of LTC risks will dampen precautionary savings of all skill types, so that aggregate savings are decreased considerably during the transition. The reduction is especially strong for low-skilled individuals, who experience a much lower risk exposure than other skill types. However, the home ownership rate of low-skilled now increases while it falls for the two other skill types. The reason is that the former group no longer needs high liquidity to be hedged against adverse LTC shocks at very high ages. For medium- and high-skilled households home ownership is less interesting, since they need no device for self insurance any more. Due to lower savings the capital stock decreases which in turn increases interest rates throughout the transition. Finally, the shift from savings towards consumption expenditures allows the government to reduce consumption tax rates despite the lower income tax base.

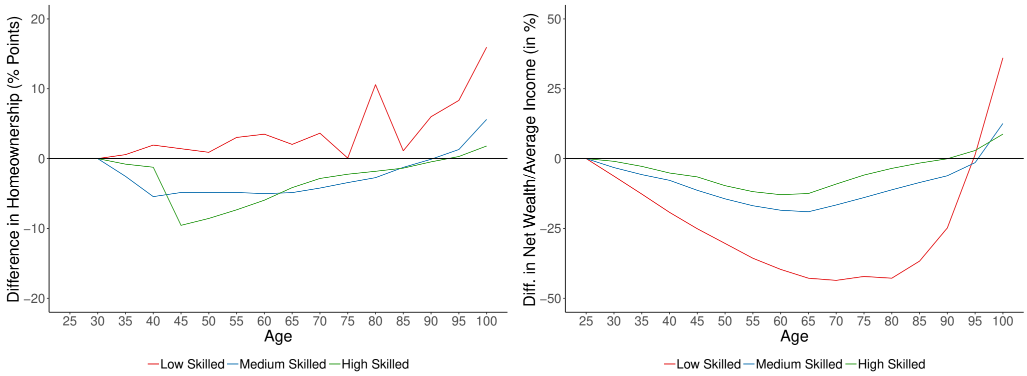
Table 5: Macroeconomic effects of full LTC coverage (in %)

Period	t = 1	t = 3	t = 5	t = 7	∞
Macroeconomic aggregates					
Home ownership rate[a]	0.03	-1.64	-1.90	-1.92	-1.93
low-skilled[a]	5.24	2.09	1.90	1.88	1.88
medium-skilled[a]	-1.19	-2.82	-3.14	-3.18	-3.18
high-skilled[a]	-2.20	-3.16	-3.33	-3.35	-3.35
Wealth ($A_F + (1-\zeta)H_O$)	-2.66	-4.78	-5.07	-5.10	-5.10
low-skilled	-4.67	-8.60	-8.80	-8.84	-8.84
medium-skilled	-2.09	-3.84	-4.17	-4.20	-4.20
high-skilled	-1.32	-1.93	-2.16	-2.19	-2.19
Prices					
Interest rate[a]	0.28	0.34	0.35	0.35	0.35
Consumption tax rate[a]	-0.18	-1.70	-1.90	-1.92	-1.92

[a]In percentage points.

Figure 5 shows exactly the opposite changes in home ownership and relative net wealth than Figure 4. Now the low-skilled reduce their savings the most in the long run, while they at the same time they increase their home ownership rates during the whole life cycle. In contrast, middle- and high-skilled individuals reduce their home ownership rates.

Figure 5: Steady state comparison of net wealth and home ownership rate



8.3 Mixed coverage of LTC shocks

Finally, we consider a reform where the government focusses on those who are hit by a severe LTC shock. These cost are now completely covered while those who experience a modest LTC shock now have to finance the full cost. As a consequence, the contribution rate increases from 2% to 2.7% and public LTC expenditures rise from 1% to 1.6% of GDP. Similar as the previous reform with full coverage, this reform is in particular beneficial for low-skilled individuals. Consequently, they will reduce their savings, but less than before since they still need to build up precautionary savings to hedge against moderate LTC shocks. As before, home ownership rates change quite differently across skill classes: While medium and high-skilled reduce home ownership, more low-skilled individuals than before now buy houses. The latter group needs less liquidity as a hedge against adverse LTC shocks at very high ages. Reduced precautionary savings will induce a falling capital stock and a slightly rising interest rate. The rising consumption spending of especially high-skilled will reduce consumption taxes during the transition, see Table 6.

Table 6: Macroeconomic effects mixed coverage of LTC shocks (in %)

Period	t = 1	t = 3	t = 5	t = 7	∞
Macroeconomic aggregates					
Home ownership rate[a]	0.09	-0.44	-0.88	-0.86	-0.76
low-skilled[a]	2.22	1.36	0.39	0.39	0.39
medium-skilled[a]	-0.48	-0.92	-1.06	-1.09	-1.04
high-skilled[a]	-1.05	-1.40	-2.04	-1.83	-1.47
Assets ($A_F + (1-\zeta)H_O$)	-0.98	-1.71	-1.77	-1.80	-1.80
low-skilled	-1.86	-3.44	-3.34	-3.35	-3.39
medium-skilled	-0.70	-1.21	-1.36	-1.40	-1.37
high-skilled	-0.50	-0.58	-0.65	-0.67	-0.72
Prices					
Interest rate[a]	0.08	0.09	0.10	0.10	0.10
Consumption tax rate[a]	-0.06	-0.46	-0.89	-0.83	-0.83

[a]In percentage points.

Consequently, less risk leads to lower wealth accumulation over the life-cycle for all skill levels. Especially the low-skilled increase home ownership at early stages of the life cycle and draw it down again in retirement. Low-skilled are now more willing to hold a non-interest bearing housing assets and therefore allocate more resources to the early stage of life. During retirement low-skilled individuals are then still subject to moderate LTC shocks that come with cost. However they hold less savings now which forces them to sell their real-estate despite the utility premium they have in the case of a positive LTC shock, see Figure 6.

Figure 6: Steady state comparision of net wealth and home ownership rate

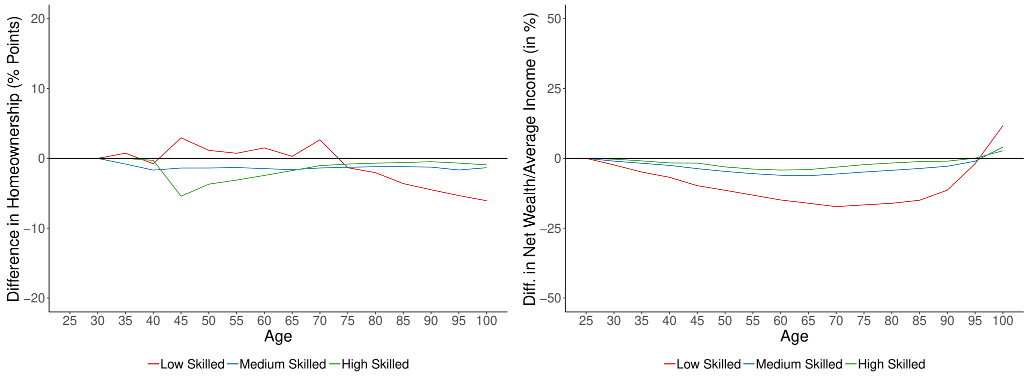
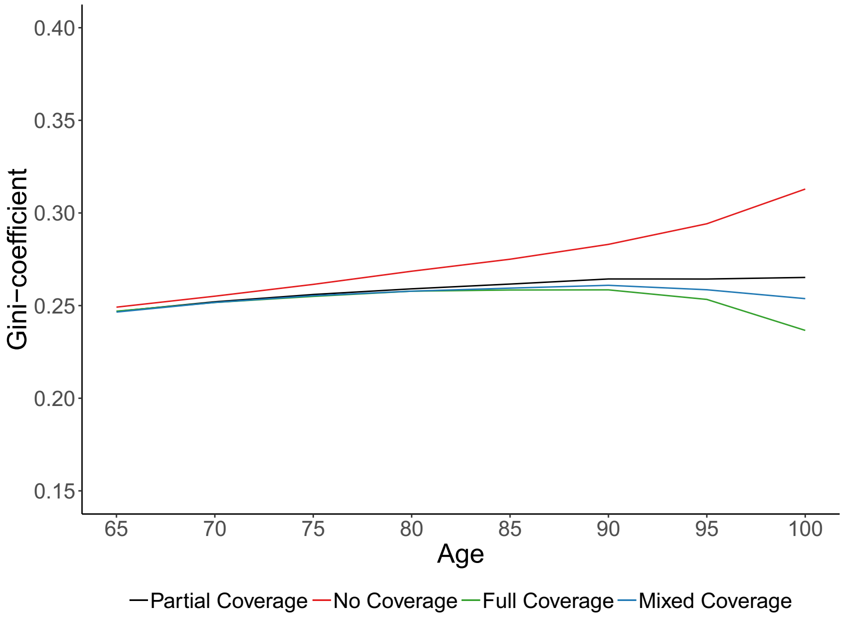


Figure 7 compares the intra-cohort Gini-coefficients of net resources for retired cohorts of all LTC systems considered. In the baseline with a partial coverage of LTC cost the Gini coefficient almost remains constant with rising age. If the LTC insurance is eliminated the intra-cohort Ginis rise significantly with age indicating a rising inequality for older households. The opposite happens when the government introduces a full coverage of LTC cost and the mixed system remains close to the baseline situation.

Figure 7: Intra-cohort Gini coefficients over the life-cycle



9 Conclusion and outlook

This paper applies a general equilibrium model with overlapping generations in order to quantify the link between home ownership and LTC risk. More specifically, our model tries to capture the self-insurance properties of home ownership by state-dependent utility of housing consumption. In addition we also capture the negative correlation between home ownership and LTC risk. Our simulation results indicate that the LTC insurance has significant effects on aggregate savings and growth. However, the composition of assets changes quite differently for different skill-classes. While low-skilled individuals hardly can use housing assets as self-insurance device, they need liquid assets to hedge against LTC risk. Medium- and high-skilled individuals, on the other hand, could fully apply thew self-insurance properties of real estate so that they increase (or downsize) housing when the LTC insurance coverage is reduced (increased).

Of course, the simulations of this paper are only a first step. In the next step we intend to introduce variable labour supply as an additional self-insurance device. Another extension concerns the modeling of Epstein-Zin (1991) preferences which allow to distinguish between intertemporal substitution and relative risk aversion. In the present setting this is especially relevant since there exists some evidence that risk aversion is higher for low-skill households. Finally, following Fehr et al. (2013), we also plan to apply the concept of a lump-sum redistribution authority (LSRA) in order to separate redistribution from efficiency effects of policy reforms.

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Numerical Appendix

A.1 - Three-step solution procedure

At any state $z = (j, \theta, x_j, h_j, ep_j, s_j, \eta_j)$, the household has to decide how to split up resources their into ordinary consumption $c(z)$ and total wealth $a^+(z)$ and total wealth into financial assets $a_f^+ = (1 - \omega^+)a^+ - tr(h, h^+)$ and future real estate $h^+ = \frac{\omega^+ a^+}{1-\zeta}$. Note that the current renter is identified whenever $h = 0$ while $h^+ = 0$ for the future renter. The corresponding optimization problem can be solved in three steps:

1. *Real-estate exposure in wealth:* Given a specific state $\tilde{z} = (j, \theta, h_j, ep_j, s_j, \eta_j)$, and savings to the next period a^+, a future homeowner has to solve

$$S(\tilde{z}, a^+, o^+ = O) = \max_{\underline{\omega}(a^+) \leq \omega^+ \leq 1} \psi_{j+1,s} E\left[\ V(z^+)\ \right]$$

subject to

$$a_f^+ = (1 - \omega_h^+)a^+ - tr(h, h^+) \geq 0.$$

We need to make sure that the minimum house requirement is fulfilled and again that the transaction cost are taken into account. Note that we can define an implicit minimum housing share $\underline{\omega}(a^+) = \frac{(1-\zeta)h_{min}}{a^+}$. The solution to this problem gives us $\omega^+ = \omega(\tilde{z}, a^+, o^+ = O)$. The household who wants to become a renter simply has to sell his house (in case he/she is a owner) and pay the resulting transaction cost, the second optimization step is no longer necessary and we obtain

$$S(\tilde{z}, a^+, o^+ = R) = \psi_{j+1}(s) E\left[\ V(z^+)\ \right]$$

subject to We need to make sure that

$$a_f^+ = a^+ - tr(h, 0) \geq 0.$$

with $tr(h, 0)$ defining the transaction cost of moving to a rented home. Note that all combinations where $a^+ < tr(h, 0)$ are not feasible!

2. *The consumption-savings decision:* Finally knowing how much wealth to allocate to real estate and how much financial assets to hold, it is possible to set up the consumption savings problem for *current* homeowners and renters separately. The former own a positive housing stock $c_h = h \geq h_{min}$ that is consumed. The maximization problem then reads

$$W(z, o^+) = \max_{c, a^+} \frac{\left[c^\nu (\chi_{h,s} c_h)^{1-\nu}\right]^{1-\frac{1}{\gamma}}}{1 - \frac{1}{\gamma}} + \beta S(\tilde{z}, a^+, o^+) \quad \text{s.t.} \quad a^+ = x - (1 + \tau^c)c.$$

Current renters (i.e. where $h = 0$) have to decide how to split their resources between ordinary consumption, housing consumption and savings. They therefore maximize

$$W(z, o^+) = \max_{c, c_h, a^+} \frac{[c^\nu (\chi_{h,s} c_h)^{1-\nu}]^{1-\frac{1}{\gamma}}}{1 - \frac{1}{\gamma}} + \beta S(\tilde{z}, a^+, o^+) \quad \text{s.t.} \quad a^* = x - (1 + \tau^c)c - r_h c_h$$

Substituting the first order conditions

$$c = \frac{\nu}{1-\nu} \frac{r_h}{1+\tau^c} c_h$$

into the above budget constraint we derive the optimal housing choice

$$c_h = \frac{(1-\nu)(x - a^+)}{r_h}.$$

The solution yields the consumption and rental housing demand as well as the savings functions $c(z, o^+), c_h(z, o^+)$ and $a^+(z, o^+)$, respectively.

3. *The tenure decision:* Finally substituting the consumption and housing demand $c(z, o^+), c_h(z, o^+)$ and savings functions $a^+(z, o^+)$ from the last step into the respective objective functions we can derive the values $W(z, o^+ = R)$ and $W(z, o^+ = O)$ for future renters and owners, respectively. The final value function is then simply derived from

$$V(z) = \max\, [W(z, o^+ = R), W(z, o^+ = O)].$$