Why Echo Chambers are Useful*

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Abstract

Why do people appear to forgo information by sorting into “echo chambers”? We construct a highly tractable multi-sender, multi-receiver cheap talk game in which players choose with whom to communicate. We show that segregation into small, homogeneous groups can improve everybody’s information and generate Pareto-improvements. Polarized preferences create a need for segregation; uncertainty magnifies this need. Using data from Twitter, we examine the relationship between the informativeness of debate and the political distance between a Twitter user and his likely audience.

JEL: D72, D82 (Asymmetric Information), D83 (Learning, Communication), D85 (Network Formation and Analysis)

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Large parts of society are organized around the (non-market) exchange of information and opinions: People gather around breakfast and dinner tables, in meeting rooms and committees, cafés and bars, while keeping in permanent touch with friends, co-workers and strangers through electronic messaging and social media. But while people constantly seek out others’ views and knowledge, they do not seek out a wide range of different viewpoints. Instead, they segregate into homogeneous communities and limit the number of views they are exposed to.\footnote{Jann: Nuffield College and Department of Economics, University of Oxford; ole.jann@economics.ox.ac.uk. Schottmüller: Department of Economics, University of Cologne; c.schottmueller@uni-koeln.de. We are grateful for helpful comments by James Best, Vince Crawford, Ben Golub, Sanjeev Goyal, Paul Klemperer, Meg Meyer, Kyle Woodward and Peyton Young, as well as audiences at the universities of Konstanz and Oxford.

\footnote{See, for example, studies on segregation in blogs (Lawrence et al., 2010), on Facebook (Del Vicario et al., 2016; Quattrociocchi et al., 2016), on Twitter (Barberá et al., 2015) and in online and offline contexts in general (Gentzkow and Shapiro, 2011).}
This poses a theoretical puzzle: If people put so much energy into seeking and exchanging information, why do they artificially limit both the diversity and the amount of information available to themselves? It also poses a practical problem for society: The segregation into “echo chambers” has widely been decried as being responsible for recent populist insurgencies in the Western world.²

In this paper, we develop a general model of how people with different preferences and different information rationally communicate in groups, and how they sort into groups while anticipating what communication within the group will be like. This allows us to argue that segregation into small, homogeneous groups can be a rational choice that maximizes the amount of information available to an individual. In fact, homophilic segregation can be efficient and even Pareto-optimal for society.

The activity of sorting into groups and communicating within them is highly complex and does not easily lend itself to strategic analysis. Every speaker chooses his message based on how he thinks different messages will be perceived – which, in turn, depends on how the listeners expect a speaker to choose his message and on what other knowledge the listeners have, which in turn may depend on who else is speaking to them and what their messages are. And all these implications have to be considered when deciding which group to interact with (which table to join, which room to enter).

In sections 1 to 5, we develop a highly tractable model of strategic information transmission (i.e. cheap talk) between many individuals, who all have different information, who all have the ability to send and receive messages, and who all freely choose within which group of people they wish to communicate.

In section 6, we use empirical evidence from the micro-blogging service Twitter to examine our predictions that people interact more with others who hold similar political views, and that the character of interaction depends on the distance between the political views of the sender and his expected audience.

Theoretical Results We conduct our theoretical analysis in a general model in which individuals sort into groups, communicate within groups, and finally make choices under aggregate uncertainty and with different preferences. Consider the following example: A group of voters has to decide on a level of taxation and redistribution. There are two sources of disagreement: Knowledge and preferences. People disagree over how bad taxation is for economic growth, i.e. they have different information about the state of the world. But they also have different preferences: Even if they all agreed on the state of the world, rich people would still prefer lower taxes than poor people.

We assume (for now) that people’s preferences are common knowledge, while their information is private and cannot verifiably be communicated. Before they each vote for

²See, for example, articles on the role of echo chambers in the “Brexit” referendum (Chater, 2016) or the rise of Donald Trump (Hooton, 2016).
a level of taxation, people can communicate the information they have about the harmful-
ness of taxes. But differences in preferences interfere with the exchange of information: If
a rich man says that taxes aren’t harmful, is that because he really thinks so, or because
he is trying to fool people into voting for lower taxes, which benefits him personally? It
depends on his audience: If speaking to a group of other rich people, he wants to give
them accurate information, given that they will then vote for a tax policy that is close
to what he prefers. If he speaks to a group of paupers instead (who are inclined to vote
for what he views as “too high” taxation), he will try to convince them that taxes aren’t
hurtful, and the paupers hence have no reason to pay any attention.

What if he speaks to a mixed audience, or one that includes members from even more
groups? What if those other people also speak simultaneously, possibly submitting informa-
tion to the speaker and the rest of the audience? Our multiple-sender, multiple-receiver
cheap talk game has an intuitive, geometric solution (theorem 1): Whether someone tells
the truth in equilibrium depends only on the distance between the speaker’s preference
parameter and the average preferences of his audience.

With this understanding of communication within arbitrary groups, we can turn to
the question of how people rationally sort into groups, and when such equilibrium sorting
is optimal. We assume that before any communication takes place, people can enter
one of many “rooms”. Each message is heard by everyone within the same room but
can not be heard outside the room. Entering or leaving a room can have many effects:
Disciplining those whose preferences are close to one’s own (making them more willing to
tell the truth), destroying truth-telling between people which otherwise existed, providing
information to others in the room (if the entrant tells the truth), giving more information
to the entrant (if he comes from a room in which he learned less) – or any combination
of these. Since every player cares about his own information (to make a precise choice)
and that of others (because he cares about their choices, which they make based on their
information), the analysis may at first seem to be quite complex.

We show, however, that all of these considerations simplify to one: In choosing a room,
a player wants to maximize the weighted sum of pieces of information that is generated
by subsequent communication (proposition 1). A “piece of information”, in this context,
is simply the fact that the information of one player is available to another player. In our
set-up, we can measure this information generation in bits, the basic unit of information.
The only differences in motivation between players arise because they each value their
own information more than that of others.

Our analysis focuses on two closely interrelated questions: What is the welfare-maximizing
allocation of people into rooms, and which room allocations can emerge as equilibria from
individual behavior? The simplest way to think about these two problems is to consider
a polarized society that consists of two groups of players who differ in their preferences.
Our results immediately allow us to generally solve this case (proposition 4). We charac-
terize the optimal room allocation and, when it is not an equilibrium, the welfare-optimal equilibrium of the room choice game.

More generally, we think of polarization as “clustering” of preferences around certain values. We parametrize this notion of polarization and show that for high polarization, full segregation by preferences is always welfare-optimal and an equilibrium, whereas integration is optimal and an equilibrium for low polarization (theorem 2). We provide a lower bound for how polarized a society would need to be before segregation becomes both optimal and an equilibrium phenomenon, by showing that if preferences are evenly distributed, segregation is neither optimal nor an equilibrium (proposition 3).

In our example of choosing a tax policy, this may mean that society optimally splits into two political parties: One bringing together the rich, the other the poor. Within each party, members can truthfully discuss their thoughts and knowledge on how the world works – while a meaningful discussion involving members of both parties would be impossible. Depending on how preferences are distributed, other results are possible. Fully integrated debate may be feasible and optimal if there is no polarization in preferences. If there is stronger polarization, society could fragment into even more parties. (Of course, the assumption that preferences are simply a reflection of wealth is highly stylized. The argument would equally apply to any other polarization in preferences, as long as people were to disagree about what was the right thing to do even if they could agree on the particular fact they are currently discussing.)

Overall, our results suggest that segregation into homogeneous “echo chambers” is a rational and often Pareto-optimal response to polarized preferences. Segregation is caused by polarization, not the other way around. However, these results do not mean that polarization is in any way good for society – in fact, we can show that polarization persistently lowers welfare (proposition 5). Segregation, as a rational response to polarization, mitigates the corrosive effects of polarization, and can hence be seen as an indicator as well as a countermeasure of society against polarization.

In section 5, we consider what happens if preferences as well as information are private. This hinders communication even more, since players are more skeptical and scrutinize each message both for information about the state and the sender’s motivation. This leads to even more segregation in equilibrium and welfare optimum. Indeed, we find that if we start with any situation in which full integration is optimal and an equilibrium, and increase uncertainty in preferences (while keeping expected preferences constant), we can reach a situation in which full segregation by preference types is the only equilibrium and also welfare-optimal. We suggest that this is relevant for thinking about interactions and debates on the Internet, where the precise type of one’s conversational partner as well as audience is often unclear. Under such uncertainty, there is a stronger need for segregation than there may be with off-line interactions, and hence further reduced welfare.
Empirical Evidence  In the last part of our paper, we provide evidence for some of the results of our theoretical considerations. We do not try to argue that we directly “take our model to the data” or even try to estimate model parameters. We understand our results to be at a high level of abstraction and not every aspect should be taken at face value.

One of the main predictions of our model – that people segregate into homogeneous echo chambers – does not seem to require any additional efforts: This has been shown by other studies (see citations above) and was indeed part of the motivation for this paper. However, we think that our prediction that the informativeness of interactions varies with the ideological difference between the participants can be verified by observing actual behavior. For this, we turn to the online messaging and networking platform Twitter. On Twitter, users can send different kinds of messages (“tweets”), which are seen (in expectation) by different types of audiences.

We interpret a Twitter user’s ideological stance similar to the “bias” of our model, and develop a novel method to score random Twitter users on their ideological stance based on word-usage frequencies. Having done that, and knowing that Twitter users tend to have networks that are like themselves, we can explore the influence of audience on the informativeness of messages, especially when political topics are discussed. Preliminary results, shown in section 6, suggest that exchanges between more ideologically distant people, and in front of more ideologically mixed audiences, differ from exchanges between ideologically similar people and in front of more homogeneous audiences.

Relation to other research  Our work closely relates to four different methodological approaches, and ties into a wider-ranging literature on segregation, isolation and echo chambers.

In methodological terms, we develop a highly tractable model of many-to-many cheap-talk. Our simple geometrical solution avoids much of the exponential complexity that usually appears in models with multiple senders or receivers. As such, our model can reproduce and simplify some insights from other multi-sender or multi-receiver models. For example, similarly to the classical analysis by Farrell and Gibbons (1989), the presence of other receivers can discipline the sender or subvert truth-telling. In contrast to most other papers, we allow for an arbitrary number of agents who are both receivers and senders and add a first stage in which agents decide whom to communicate with. In our main analysis, we restrict ourselves to binary signals and messages, but show in the supplementary material that our main results are robust to the introduction of an arbitrary finite number of states and signals.

3While our novel setup allows us to vastly simplify the analysis of many-to-many cheap talk, our main arguments are not dependent on this particular setup and can be derived in a more classical cheap-talk setting, as we show in the supplementary material.
While the rooms of our analysis are a novel modeling device, they can in principle be thought of as fully connected, disjoint networks. A related paper by Galeotti et al. (2013) analyses communication in networks by agents who face a decision problem similar to ours, but in their setup the most informative (or welfare optimal) equilibrium can be in mixed strategies. Such mixed equilibria are a common occurrence in similar models but are generally intractable. In our model, however, the most informative equilibrium is always in pure strategies. There is, of course, a much larger literature on endogenous network formation. The principal differences to our paper are that we consider cheap talk, do not focus on directed networks, and construct a tractable model of room choice, which allows us to study (efficient) segregation.

The room choice in our model can also be seen as an information design problem: How can an information designer induce information exchange between several agents, if these agents have an incentive to manipulate others through lies, and commitment (as in the literature on Bayesian Persuasion) is not available? The right construction of mixed groups can induce truth-telling. Rooms endogenously create costs to lying (the main instrument of discipline in Kartik 2009), and they induce truth-telling despite the fact that different senders’ information is orthogonal to each other and there hence exists no mechanism (as in e.g. Krishna and Morgan 2001) to elicit information by playing them off against each other.

Third, we show that bias uncertainty has a corrosive effect on truth-telling. This is similar to Morgan and Stocken (2003), who consider financial analysts who are biased in a known direction, but whose precise bias is unknown. Such one-sided uncertainty leads to one-sided losses in informativeness (i.e. one of two messages becomes more common but less informative). Our analysis extends to general distributions of players’ biases and hence considers uncertainty about the size and the sign of the sender’s bias, which may be continuously or discretely distributed. What turns out to matter is the concentration of probability mass around certain values, and hence we can show that two-sided uncertainty does not necessarily help with information transmission (as it does in Li and Madarász, 2008). Our results and methods generalize without loss to large groups of players and general distributions of biases. Of course, we are mostly interested in these results as a preliminary for room choice, as rooms are optimally and in equilibrium more segregated for higher uncertainty. To our knowledge, we are the first to generally analyze how uncertainty about bias influences whom people want to associate and communicate with.

Finally, in our empirical work, we develop a novel way to score Twitter users on a partisan left-to-right scale, based only on their tweets. The method is similar to how Gentzkow and Shapiro (2010) score newspaper editorials; we demonstrate that such a method is valid for scoring arbitrary Twitter users. The main differences to this earlier work are in the size of our partisan dictionary (which is about 18 times the size of Gentzkow and Shapiro’s dictionary) and the causal agnosticism with which it is compiled: While
earlier works have focused on phrases with clear ideological content, our dictionary also contains non-obvious (but informative) entries such as hashtags, names and locations.

The debate about echo chambers has recently been given urgency by several studies and popular treatises on how the internet changes the way societies debate. Sunstein (2001, 2017) prominently makes the case that the internet has been increasing ideological segregation and that this endangers democracy. Gentzkow and Shapiro (2011) point out, however, that the segregation of “offline” interactions is larger than that of “online” interactions. But while such offline segregation can happen simply because we live close to people who are like us in many socio-economic aspects, segregation on the internet is driven more by choice. Lawrence et al. (2010), for example, show that blog readers tend to read blogs that agree with their own ideological bias. Our model allows us to analyze the informational effects of any kind of segregation or integration, as well as predicting which communication structures arise from optimizing behavior, and whether they are optimal. Most importantly, we argue that those who see in segregation the ruin of societies are focusing on a symptom, not the cause. Polarization of preferences and mutual mistrust are the real culprits; informational segregation is a rational behavior that mitigates the harm they do.

1. Model

There is an unknown state of the world \( \theta = \sum_{k=1}^{n} \theta_k \). Each \( \theta_i \) is independently drawn to be 0 or 1 with equal probabilities, so that \( \theta \) is binomially distributed on \( \{0, 1, \ldots, n\} \). \( n \) individuals each make an observation about the state. In particular, individual \( i \) receives a private signal \( \sigma_i \in \{\sigma^l, \sigma^h\} \) of accuracy \( p \) about \( \theta_i \), i.e. \( Pr(\sigma_i = \sigma^h|\theta_i = 1) = Pr(\sigma_i = \sigma^l|\theta_i = 0) = p > 1/2 \). Before observing his signal, a player can access one of \( n \) “rooms”. There are no costs to entering a room, and rooms have no capacity constraints – but each player can only be in exactly one room. After observing his signal, a player sends a cheap-talk message \( m_i \in \{m^l, m^h\} \) that is received by all players in the same room. Finally, each player takes an action \( a_i \).

The payoff of player \( i \) is

\[
    u_i(a, b_i, \theta) = -(a_i - b_i - \theta)^2 - \alpha \sum_{j \neq i} (a_j - b_i - \theta)^2
    = -\left(a_i - b_i - \sum_{k=1}^{n} \theta_k\right)^2 - \alpha \sum_{j \neq i} \left(a_j - b_i - \sum_{k=1}^{n} \theta_k\right)^2
\]

(1)

where \( a \) denotes the vector of actions of all players and \( b_i \in \mathbb{R} \) is a commonly known “bias” of player \( i \). That is, actions of all players affect \( i \)’s payoff, and \( i \) would like that all players choose the action \( b_i + \theta \). The parameter \( \alpha \) measures the relative weight players assign to other players’ behavior. Players maximize their expected payoff.
The timing of the game is:

1. Players simultaneously decide which room to enter.

2. Players privately observe their signals \( \sigma_i \), and room choices become common knowledge. Players simultaneously send messages \( m_i \) that are observable by everyone in the same room \( R_i \).

3. Players simultaneously take actions \( a_i \); payoffs are realized.

We analyze the model by backwards induction: First we characterize optimal choice of action given messages, then the optimal choice of message given a room allocation, and then we analyze the game in which players choose which room to enter. The solution concept used throughout is Perfect Bayesian Equilibrium.\(^4\)

2. Equilibrium Behavior Within a Room

2.1. Choice of Action

We can immediately see that only the first part of expression 1 matters for determining \( i \)'s optimal action \( a_i^* \). Taking the first-order condition, we get that

\[
    a_i^* = b_i + \mathbb{E}[\theta] = b_i + \sum_{j=1}^{n} \mathbb{E}[\theta_j],
\]

(2)

i.e. the optimal action is simply \( i \)'s bias plus his conditional expectation of the state.

In the following, we will denote by \( \mu_{ij} = \mathbb{E}_i[\theta_j] \) \( i \)'s belief about \( \theta_j \), so that expression (2) becomes

\[
    a_i^* = b_i + \sum_{j=1}^{n} \mu_{ij}.
\]

2.2. Choice of Message

Now that we have established the agents’ optimal action choice given a set of beliefs, we can consider the optimal choice of message. For this, we focus on a single room, and consider the equilibria of the cheap talk game in this room. This means that when we speak of “equilibrium” in this section, we mean the equilibrium in a specific room (with a given set of members with given biases), and not the overall equilibrium of the game. We can do this because once players have sorted into rooms, the messages in other rooms are unobservable and the actions of players in other rooms are orthogonal to a player’s optimization problem. Hence, an equilibrium of the subgame after room choice can be disassembled into one equilibrium of the cheap talk game for each room.

**Definition 1.** We call a messaging strategy \( m_i \) …

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\(^4\)All messages occur in equilibrium and there is no hidden information at the time that people choose rooms, so that our results are insensitive to assumptions about off-path beliefs.
• babbling if \( m_i \) is independent of \( i \)'s observed signal \( \sigma_i \) and therefore nobody learns anything from \( m_i \).

• truthful if \( m_i(\sigma^l) = m^l \) and \( m_i(\sigma^h) = m^h \).

• lying if \( m_i(\sigma^h) = m^l \) or \( m_i(\sigma^l) = m^h \).

• pure if \( m_i \) is either babbling or truthful.

• mixed if for some signal \( \sigma^k \), \( k \in \{l, h\} \), both messages are sent in equilibrium and the strategy is not babbling.

The cheap talk game within a room can – as usual – have several equilibria. For each player \( i \), there always exists an equilibrium in which \( i \) babbles. (Consequently, there also always exists an equilibrium in which all players babble.) In line with the cheap talk literature, we will focus on the most informative equilibrium.\(^5\) The following lemma allows us to show that the most informative equilibrium is in pure strategies.

**Lemma 1.** Let \((m_1, \ldots, m_n)\) be equilibrium strategies. If \( m_i \) is a mixed strategy, then there also exists an equilibrium with strategies \((m^t_i, m^*_{-i})\), where \( m^t_i \) is the truthful strategy. (Proof on page 29.)

What is the intuition for this result? Imagine an equilibrium in which player \( i \) mixes between messages after observing signal \( \sigma^h \). That is, \( i \) is indifferent between sending a high message that induces high actions by the other players in his room and a low message that induces lower actions by the players in his room. This means that the low actions induced by \( m^l \) are somewhat too low from \( i \)'s point of view and the high actions induced by \( m^h \) are somewhat too high from \( i \)'s point of view. Note that \( i \) will always send the low message in case he observes a low signal in such an equilibrium because the actions \( i \) would like the other players to take are increasing in his signal. Consequently, a high message perfectly reveals \( i \)'s high signal. Now consider switching to an equilibrium in which \( i \) uses the truthful strategy. When \( i \) now observes a high signal, sending the high message will lead to exactly the same actions by the other players as in the original equilibrium. However, sending a low message will lead to a lower belief than in the original equilibrium and therefore to lower actions by the other players. Player \( i \) will then strictly prefer the high message as these lower actions are too low (given that \( i \) was indifferent in the original equilibrium).

The main implication of lemma 1 is that the most informative equilibrium is always in pure strategies: Starting from any mixed equilibrium we can switch the mixing players one by one to truthful – and therefore more informative – strategies and the resulting strategy profile remains an equilibrium.

\(^5\)The concept of “most informative” equilibrium is not necessarily well defined in multi-sender cheap talk games. However, the following paragraphs will make clear that this concept is straightforward in our model.
Corollary 1. *The most informative equilibrium in a room is always in pure strategies.*

We can now characterize the most informative equilibrium. Intuitively, we might expect that the distance of $b_i$ to the biases of the other players is crucial for $i$’s incentive to tell the truth, since $i$ becomes more interested in misleading the other players if their biases differ by a lot. We formalize this intuition and specify the most informative equilibrium in the following result, which is illustrated by figure 1:

**Theorem 1.** Let $\bar{b} = \frac{\sum_{k \in R} b_k}{n_R}$ be the mean bias of players in room $R$. In the most informative equilibrium in this room, a player $i$ tells the truth if and only if

$$b_i \in \left[ \bar{b} - \frac{n_R - 1}{n_R} (p - \frac{1}{2}), \bar{b} + \frac{n_R - 1}{n_R} (p - \frac{1}{2}) \right]$$

and babbles otherwise. (Proof on page 30.)

The size of the truth-telling interval increases in both $n_R$, the number of people in the room, and $p$, the precision of individual signals. The increase in $n_R$ can be seen as a correction term: What really matters for the motivation of a player is his distance from the average bias of the other players in the room. Hence, if we write a symmetric interval around $\bar{b}$ (which includes $b_i$), we have to add this correction. When $p$, the precision of signals, is higher, each truthful signal causes a greater change in the actions of others. People communicate truthfully if they are disciplined by the danger of influencing others’ actions too much by lying. Hence, if $p$ is higher, this disciplining force is stronger and a player can be further away from the average bias of others and still tell the truth.

Figure 1: Finding the most informative equilibrium in a room consisting of players 1 to 5. We find the average bias and construct a symmetric interval around it. Players 1 and 5 babble in the most informative equilibrium, since their biases are too far from $\bar{b}$. Players 2, 3 and 4 tell the truth.

### 3. Room Choice

We can now analyze room choice, under the assumption that the most informative equilibrium will be played in any room (i.e. in each subgame). Recall that rooms are secluded

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6Intuitively, one could also think that the average room bias “stabilizes” for larger $n_R$, so that a player can be further away from the average room bias and have the same distance from the average bias of other players in the room.
from each other, but do not influence a player’s payoff function – in other words, each player cares the same about all other players’ actions regardless which room they are in. We will first derive some results about the welfare-optimal room allocation, and then analyze under which conditions this optimal room allocation is in fact an equilibrium.

### 3.1. Welfare-Optimal Room Allocation

Given the expression for individual payoff (1), overall welfare in the model is given by

$$W(a, b, \theta) = \sum_{i=1}^{n} u_i(a_i, b_i, \theta) = -\sum_{i=1}^{n} \left( a_i - b_i - \sum_{k=1}^{n} \theta_k \right)^2 - \alpha \sum_{i=1}^{n} \sum_{j \neq i} \left( a_j - b_i - \sum_{k=1}^{n} \theta_k \right)^2.$$ 

This expression, of course, is not yet very helpful in trying to compare different room allocations. However, we can show that in our model, welfare can be reduced to a purely informational problem.

Consider the information that is available to a single player. A player always receives his own signal $\sigma_i$. We can call this one piece of information. Assume that $i$ also receives truthful signals from two other players; then we can say that $i$ has three pieces of information about $\theta$. Let $\zeta_i \in \{1, 2, \ldots, n\}$ be the number of pieces of information available to player $i$ which are either his own signal or truthful messages from other players. Given that each $\sigma_j$ has two possible values (high or low), $\zeta_i$ in fact measures player $i$’s information in bits, the unit of information. We can show the following result, which reduces all welfare comparisons to informational accounting in bits:

**Proposition 1.** Welfare is linearly increasing in $\sum_i \zeta_i$, i.e. the sum of pieces of information, and only depends on $\sum_i \zeta_i$ and model parameters. (Proof on page 31.)

For this result, we make use of the fact that we can additively separate a player’s payoff into (i) losses through preference differences and (ii) losses from variance due to lack of information. In an equilibrium of the messaging game, the former losses are unavoidable, but the latter can be mitigated by increasing the flow of information between players. We can measure this flow of information simply by counting the pieces of information that each player has when making their decision. Since every player $i$ has exclusive knowledge about $\theta_i$, there are no decreasing marginal returns to information, and the sum of all $\zeta_i$ is indeed a sufficient statistic for welfare.

This result means that we can quickly compare any two room allocations. Consider, for example, the room allocation in figure 1. Having everybody in the same room generates 17 pieces of information: 3 players have 3 pieces of information each, while two players (those who babble) have 4 pieces each. Would it be possible to improve on this allocation? We can immediately see that this cannot be achieved by splitting players up into two rooms with 3 and 2 players, respectively: Even if everybody in these rooms was telling the truth, only $3^2 + 2^2 = 13$ pieces of information would be produced. The same is true for splitting
them into a higher number of even smaller rooms. But even if we somehow could get 4 people in one room to tell the truth by putting one of the players into a separate room, the total number of pieces of information would be $4^2 + 1 = 17$ – the same as with full integration. Hence the room allocation shown in the figure is welfare-optimal.

Of course, we may often not be able to make such quick deductions in general cases and might have to consider many possible room allocations before concluding what the optimal one is. This problem gets more complex as $n$ grows, since the number of possible partitions of a set (given by the Bell sequence) grows quite rapidly in the size of the set. We will derive general results on optimal allocations in section 4 below.

3.2. When is the Welfare-Optimal Allocation an Equilibrium?

Finding out which room allocation is welfare-optimal is only half the story. We are interested in situations where people can freely decide which room they want to be in. Understanding how a social planner would assign people to rooms is a useful fiction for one side of the problem; now we need to analyze which room allocations can result from actual behavior.

We begin by rewriting player $i$’s payoff analogously to our result on welfare (proposition 1; the derivation is in the proof of that proposition):

$$U_i = \left( \frac{1}{4} - p(1-p) \right) \left[ \zeta_i + \alpha \sum_{j \neq i} \zeta_j - \alpha \sum_{j \neq i} \{(b_j - b_i)\} - \frac{1}{4} [n + \alpha(n-1)n] \right].$$

This redefines $i$’s choice of room in purely informational terms: When choosing a room, $i$ wishes to minimize a weighted sum of his own uncertainty and that of other players. When he considers switching from, say, room $R_A$ to $R_B$, $i$ will consider how much more he can learn in room $R_B$, as well as how much more or less the other people in both rooms will learn after his switch. How exactly $i$ is willing to trade off these informational effects against each other depends on $\alpha$. For $\alpha = 1$, each agent simply maximizes welfare, and we can hence derive the following result:

**Proposition 2.** For any configuration of biases, there exist $\underline{\alpha}$ and $\overline{\alpha}$ such that $\underline{\alpha} \leq 1 \leq \overline{\alpha}$ and a welfare-optimal room allocation is also an equilibrium of the room choice game if $\alpha \in [\underline{\alpha}, \overline{\alpha}]$. (Proof on page 33.)

Depending on the deviations that are possible in the welfare-optimal allocation, $\underline{\alpha}$ and $\overline{\alpha}$ will often be strictly below and above 1. We can intuitively analyze the situation by considering figure 2, which shows all possible deviations for all players in the welfare-optimal room allocation of an exemplary game. Deviations in the shaded area cannot exist if players are allocated to rooms in a welfare-optimal way. The condition $\Delta \zeta_i + \alpha \Delta \sum_{j \neq i} \zeta_j \leq 0$ translates to $\Delta \sum_{j \neq i} \zeta_j \leq -\frac{1}{\alpha} \Delta \zeta_i$: Any line through the origin that has
no points (i.e. deviations) above it corresponds to an $\alpha$ for which the welfare-optimum is also an equilibrium. In the given example, this is true for all $\alpha \in [0.75, 1]$.

\[ \Delta \sum_j \zeta_j \leq -\frac{1}{\alpha} \Delta \zeta_i \]  

\[ \alpha = 0.75 \]

\[ \Delta \sum_j \zeta_j \]

\[ \Delta \zeta_i \]

Figure 2: Possible deviations for all players in the welfare optimal room allocation. Deviations in the shaded area cannot exist in the welfare optimum. Any $\alpha$ for which $\Delta \sum_{j \neq i} \zeta_j \leq -\frac{1}{\alpha} \Delta \zeta_i$ means that the welfare-optimal allocation is also an equilibrium of the room choice game.

4. Polarization and Segregation

We have now shown that the messaging problem inside each room has a simple geometrical interpretation, and that the room choice game reduces to a problem in which all players wish to reduce their own uncertainty and that of the other players. Using these results, we can analyze the problem of how people segregate and when such segregation is optimal for society: How does the composition of the set of biases influence which room allocations are optimal, and which allocations can be achieved in equilibrium?

Despite the complexity of the model and the discrete nature of optimal communication and room choice, the tools and simplifications we have derived in the preceding sections allow us to gain general insights. The main insight is that segregation is welfare-optimal and an equilibrium if players are sufficiently polarized. By polarization, we mean that biases are clustered in two or more groups, instead of being clustered in one group or evenly spread out.
We approach polarization in three ways. First, we will show that if we start out with any bias configuration and increase polarization by “stretching” the set of biases, total segregation will become the welfare-optimal room allocation and an equilibrium. If we instead carry out the opposite transformation, i.e. contract any set of biases, full integration will eventually become welfare-optimal and the unique equilibrium of the room choice game.

Second, we establish a lower bound for how polarized biases need to be for segregation to be optimal. We show that if biases are evenly spread on some interval of the real line, then full integration (or a very similar room allocation) is the welfare-optimal room allocation and an equilibrium.

Third, we narrow our analysis to the case in which there are only two bias types, and fully characterize the set of all optimal allocations for different distances between the two biases as well as different proportions between the two groups, and whether these allocations are equilibria.

We will begin with a simple graphical example of how segregation can be a welfare-optimal equilibrium if players are polarized, and then introduce our general results.

4.1. A Simple Example

Consider a set of biases as in panel (i) of figure 3: A group of 6 players, 3 of which have relatively small biases, while the other 3 have relatively large biases. If all players are within the same room (panel i), the truth-telling interval within this fully integrated room does not cover any of the players’ biases, which means that in the most informative equilibrium none of them reveal any information. The number of pieces of information generated is hence 6.

Now, however, the players could segregate by bias type into two separate rooms – see panel (ii). The truth-telling interval in both rooms covers all the players in the respective rooms, which means that all players reveal their information truthfully. In each room, 9 pieces of information are generated, which means that overall this allocation generates 18 pieces of information.

Is this segregation an equilibrium? We can consider the most profitable deviation of player 3 (which is symmetric to the most profitable deviation of player 4 and better than the best deviations of any other players) – see panel (iii). If player 3 moves into the other room, he will move the average in this room so that players 5 and 6 no longer tell the truth in any equilibrium. (The lengthening of the truth-telling interval that results from 3’s move is not enough to compensate for the change in average bias.) The resulting room allocation generates $2^2 + 4 + 3 = 11$ pieces of information, which clearly leads to lower welfare. But it is also inferior for player 3, since he now has 2 pieces of information (his own and the message from player 4) instead of 3. Hence this deviation can never be optimal for player 3, and no player has a profitable deviation from two segregated rooms.
– which means that this allocation is not only welfare-optimal, but also an equilibrium.

\[
\begin{align*}
\text{(i)} & \quad b_1 b_2 b_3 b \quad b_i \\
\text{(ii)} & \quad b_A b \quad b_i \\
\text{(iii)} & \quad b_A b \quad b_i \\
\end{align*}
\]

Figure 3: Truth-telling intervals for (i) the fully integrated room, (ii) two segregated rooms, (iii) player 3’s best deviation from the segregated room.

4.2. When is Segregation optimal?

Consider any bias configuration \( B \), and let \( B_\eta \) be the bias configuration that contains \( \eta b_i \) for each \( b_i \) in \( B \). Then the following is true:

\textbf{Theorem 2.} (i) If \( \eta \) is sufficiently close to 0, full integration is welfare-optimal and a room-choice equilibrium.

(ii) If \( \eta \) is sufficiently large, full segregation by bias types is generically welfare-optimal and a room-choice equilibrium. (Proof on page 33.)

Figure 4 summarizes the result. We can intuitively explain it in the following way: If biases are clustered very closely, having all players in one room would result in universal truth-telling. This cannot be improved upon in welfare terms, and it is also an equilibrium since any player would lose by leaving the fully integrated room.

On the opposite end of the spectrum, we consider the case where biases are clustered very widely and we do not assume special, non-generic properties such as that one bias

\[
\begin{align*}
\text{Full integration optimal and equilibrium} & \quad \eta \\
\text{Full segregation optimal and equilibrium} & \quad \eta \\
\end{align*}
\]

Figure 4: Welfare-optimal allocations that are also equilibria for large and small \( \eta \).
is the exact average of two other biases. Then truth-telling will be impossible in any room that contains two or more players with different biases. Hence there exists no room allocation that can improve welfare compared to full segregation by bias types. Similarly, no player has an incentive to deviate from full segregation, since such a deviation cannot provide more information to the player himself or any other player.

4.3. A Lower Bound for Polarization

To find out how polarized biases need to be so that segregation is optimal and an equilibrium, we can consider the stylized case of biases that are evenly distributed on an interval of the real line. We can think of this case as having “zero polarization”, whereas clustering of biases around certain values exhibits positive polarization.\(^7\) What is the welfare-optimal allocation in this case? Given that biases could be evenly distributed on a very large interval, we might expect that the optimal allocation involves separating the players into several rooms. But in fact, the opposite is true: If biases are evenly distributed, the welfare-optimal allocation often involves a single integrated room that is also an equilibrium. In some special cases, an allocation that places one player outside the room is welfare-optimal, and this allocation is an equilibrium if \(\alpha\) is large enough.

**Proposition 3.** Let \(b_i = (i - 1) \cdot k/(n - 1)\) for \(i = 1, \ldots, n\). Then one single room with all players is both welfare optimal and an equilibrium if either

\[
\frac{k}{2(n - 1)} \leq b - \left[\frac{k/2 - (p - 1/2)n - 1}{n} \right] \quad (3)
\]

or

\[
k(1 - 1/n)/2 - (p - 1/2)n - 2 > b - \frac{k}{n - 1}. \quad (4)
\]

If neither of these two conditions holds, isolating player \(n\) in one room and all other players in one room is welfare optimal. This is only an equilibrium if

\[
\alpha \geq \frac{\left\lfloor \frac{n - 1}{n} (2p - 1)/(k/(n - 1)) \right\rfloor - 1}{n - 2}. \quad (5)
\]

(Proof on page 34.)

Why is this? We can start by considering the fully integrated room, in which some people whose biases are close to the overall average tell the truth, and the rest babbles and learns from the truth-tellers. Since biases are evenly distributed by assumption, there is little welfare to be gained by moving the bias average around by allocating people to another room. (This can only work because of integer effects – i.e. because changes in the average bias have discrete effects on who tells the truth – which is precisely what gives

\(^7\)If biases are tightly clustered around a central value, we could think of this as “negative polarization” – we consider this case in the supplementary material.
us the exceptions in the second half of the proposition.) Any room that includes only part of the players will have a shorter truth-telling interval, which (again, absent integer effects) means fewer truth-tellers. But if we cannot increase the number of truth-tellers by segregating into smaller rooms, then the fully integrated room must be welfare-optimal and hence also an equilibrium.

4.4. Bipolar Polarization

We now focus on the case where there are two bias groups, i.e. \( b_i \in \{0, b\} \) for some \( b > 0 \). This “bipolar polarization” is often used synonymously with the word polarization. Our results allow us to generally solve this setting for all possible parameter values. In the main text, we will derive solutions for the case where both groups have equal size, while we solve the more general case of arbitrary group sizes in the supplementary material.

If all players are in one room, the average bias will be \( b/2 \) and all players send truthful messages if and only if \( b/(p - 1/2) \leq 2(n - 1)/n \), see theorem 1. Clearly, such a room with all players will then be both welfare optimal and an equilibrium. At the other extreme, consider the case where the presence of one player of bias \( b \) in a room containing all players with bias 0 will lead to babbling by all players. The average bias in such a room is \( b/(n/2 + 1) \) and by theorem 1 babbling even by the players with bias 0 is inevitable if and only if \( b/(p - 1/2) > n/2 \). In this case, any room containing players of both bias types will lead to babbling. Segregating the two groups is consequently both welfare optimal and an equilibrium.

This illustrates our point that segregation is optimal and an equilibrium if polarization is high (i.e. if \( b \) is large), and full integration is optimal and an equilibrium if polarization is low (if \( b \) is sufficiently low). For intermediate levels of polarization, the welfare optimal room allocation need not be an equilibrium. More precisely, the two groups may not be segregated enough in any equilibrium. Intuitively, if segregation is welfare optimal, players might have an incentive to switch to the room where the players with the opposite bias are because this allows them to receive more messages. They neglect the negative externality of this deviation, namely the loss of their own truthful message for players of their own bias. Altogether, this gives us the following result:

**Proposition 4.** Let \( b_i \in \{0, b\} \) and \( n_0 = n_b = n/2 \). Then, for

- \( b/(p - 1/2) \leq 2(n - 1)/n \) one room in which all players send truthful messages is welfare optimal and an equilibrium,
- \( b/(p - 1/2) \in (2(n - 1)/n, n/2] \) segregation is welfare optimal but only an equilibrium if \( \alpha \geq 2/(n - 2) \),
- \( b/(p - 1/2) > n/2 \) segregation is welfare optimal and an equilibrium.
When the welfare-optimal room allocation is not an equilibrium, the welfare-maximizing equilibrium is straightforward: All players of one type, say bias 0, are in one room and are joined by \( m \) players of bias \( b \). The players with bias 0 tell the truth, while the \( m \) players with bias \( b \) babble. All other bias \( b \) players are in a separate room, where they tell the truth. The number of babbling players, \( m \), is such that one additional bias \( b \) player in the mixed room would lead to babbling of the players with bias 0.\(^8\) From a welfare perspective, there is too little segregation in this equilibrium, and the resulting babbling constitutes a socially undesirable information loss.

We can also describe how the most informative equilibrium evolves when polarization, i.e. \( b \), increases and \( \alpha < 2/(n - 2) \): For low \( b \) there is one integrated room in which everyone is truthful. At some point there would be babbling in such a room and therefore one player, say of bias \( b \), voluntarily isolates himself so that all bias 0 players will tell the truth again. As \( b \) increases further, more and more players of bias 0 have to withdraw from the mixed bias room in order to maintain truthful behavior by the players with bias 0. Eventually, we obtain full segregation. In a nutshell, higher polarization gradually increases segregation, though it may be inefficiently slow in doing so.

4.5. Polarization Destroys Welfare

We have argued that segregation is a rational and Pareto-optimal response to polarization. This does not mean that polarization in itself increases welfare – quite the opposite. If we return to the \( \eta \)-parametrization under which we derived our general results on integration and segregation in section 4.2, we can show that welfare is weakly decreasing in \( \eta \), i.e. the measure of polarization.

**Proposition 5.** Denote expected welfare in the welfare optimal room assignment with bias configuration \( B_\eta \) by \( W(\eta) \). \( W(\eta) \) is decreasing in \( \eta \). (Proof on page 36.)

We should be very precise about the mechanism by which higher polarization decreases welfare. It is not through segregation, even though higher polarization causes more segregation, which ultimately causes less information to be exchanged. But saying “segregation lowers welfare” would ignore the crucial intermediate step, which is that polarization in itself causes an informational breakdown. In fact, segregation mitigates this breakdown, without of course being able to restore communication between people that are now in separate rooms.

One could think of echo chambers as society’s (decentralized) defense mechanism against polarization. Like fever in a human body, segregation occurs as the effect of an underlying problem, and its presence hence indicates that polarization is at problematic levels. Echo chambers, and segregation more generally, are a hence symptom of polarization. And just like artificially lowering fever, treating the symptom without addressing

\(^8\)That is, \( m \) is the integer such that
\[
\frac{bm}{(n/2 + m)} - \frac{(p - 1/2)(n/2 + m - 1)/(n/2 + m)}{< 0 < b(m + 1)/(n/2 + m + 1) - (p - 1/2)(n/2 + m)/(n/2 + m + 1)}
\]

by theorem 1.
the cause can in fact exacerbate the situation. Reducing polarization will weakly improve welfare; reducing segregation may not.

5. Uncertainty

So far, we have assumed that all biases $b_i$ are common knowledge. In real-life situations, the type of a sender is often not known, so that the receiver is drawing inferences about the state of the world and the type of the sender at the same time. This can make informative communication much harder. In this section, we consider the effect of uncertainty about biases on the existence of within-room equilibria, and on the room choice game.

Let all biases $b_i$ be randomly distributed on $\mathbb{R}$ according to distribution $F_i$. Each player observes his own bias $b_i$, but only knows the distributions of the biases of other players. Let $b_i^e = \int_{-\infty}^{\infty} b_i dF_i$ be the expected value of $b_i$. This can be thought of as a generalization of the previous sections, in which all biases were always identical to their expected value. When we talk about “introducing” or “adding” uncertainty in this section, we think of starting with the model in which all biases are known with certainty, and replacing each bias with a bias distribution that has the same expected value. Throughout this section, we will be comparing across distributions that have the same expected value. The following paragraphs intuitively analyze the model with uncertainty; the corresponding formal statements and analysis are in part B of the appendix.

To find the messaging equilibria within a room, we need to consider $i$’s problem of choosing a message $m_i$ after observing $b_i$ and $\sigma_i$, but only knowing $F_j$ for all $j \in R_i$. We can show that this problem is very similar to knowing all biases with certainty. In particular, recall that $i$’s willingness to tell the truth only depended on the distance between $b_i$ and the average of all other $b_j$’s in the model with certainty. This insight applies analogously to a model in which all biases are unknown: Now $i$ only cares about the difference between $b_i$ and the average of all $b_j^e$, i.e. the expected values of other people’s bias.

A difference in describing equilibria with uncertainty arises since $i$ may want to tell the truth for some values of $b_i$ and not for others, and the other players are unsure about $b_i$ when interpreting $m_i$. Their belief about how likely $i$ is to tell the truth hence depends on how $b_i$ is distributed. For each possible probability with which $i$ tells the truth, there exists an interval around $\sum_{j \in R_i, j \neq i} b_j^e$ such that $i$ wants to tell the truth if the realized $b_i$ lies within this interval. Since the distribution of $b_i$ is common knowledge, that gives us the following equilibrium condition: The beliefs of all other players about $i$’s probability of truth-telling need to give rise to a truth-telling interval for $i$ around the average of all $b_j$ such that $i$ wants to tell the truth with exactly the probability with which the other players believe that he tells the truth.

This translates into a slightly generalized version of theorem 1 which, for any distribution of $b_i$, gives us the highest probability with which $i$ can tell the truth in any
equilibrium. Intuitively, the more concentrated $F_i$ is around $\frac{\sum_{j \in R_i, j \neq i} b_j}{n_{R_i} - 1}$, the higher the probability with which $i$ can tell the truth in equilibrium. Interestingly, only the probability mass of $F_i$ that is sufficiently close to $\frac{\sum_{j \in R_i, j \neq i} b_j}{n_{R_i} - 1}$ matters; whether or not $b_i$ itself is close to the average or not is not directly relevant for whether $i$ is able to tell the truth in equilibrium.

In particular, this means that we can choose any set of expected biases, regardless of how close they are to each other, and construct bias distributions such that none of the players ever wants to tell the truth to anyone in any room allocation. This shows that for any bias configuration, uncertainty has the potential to completely destroy truth-telling and hence any chance of creating any room in which information is exchanged.

**Proposition 6.** Take a set of $n$ players with biases $\{b_1, b_2, \ldots, b_n\}$ such that there exists a room allocation in which some (or all) players tell the truth. Then there exists a set of probability distributions $\{F_1, F_2, \ldots, F_n\}$ of biases with expected values $\{b_1, b_2, \ldots, b_n\}$ such that in any room allocation of the $n$ players, no player will tell the truth in any equilibrium. (Proof on page 39.)

This is, of course, a very stark result. Uncertainty need not always destroy communication. It can, in fact, make communication possible where it was previously impossible, by moving probability mass of $b_i$’s distribution closer to the average of other biases. This effect, however, is more limited and can never lead to full truth-telling if there is no full truth-telling in a model with certain biases and identical expected values.

**Proposition 7.** If $b_i$ is such that there exists no equilibrium in room $R_i$ where $i$ tells the truth, there exists a distribution $F_i$ with expected value $b_i = b_i$ such that there exists an equilibrium in $R_i$ where $i$ tells the truth with positive probability. However, there exists no $F_i$ such that $i$ tells the truth with probability 1 in any equilibrium. (Proof on page 40.)

While uncertainty can make some truth-telling possible where it was not possible with certainty, large amounts of uncertainty will always destroy any truth-telling and make all messages arbitrarily uninformative unless they preserve sufficient probability mass in the neighborhood of $\frac{\sum_{j \in R_i, j \neq i} b_j}{n_{R_i} - 1}$. Because of the large space of possible distributions and possible orderings on uncertainty, we show this result in two ways. First, we consider any continuous bias distribution and show that by “stretching” it, any equilibrium will become arbitrarily uninformative. Then we consider discrete bias distributions with bounded support, and show that any way of increasing the variance of such a distribution will likewise eventually erode all informative equilibria. In the following propositions, $\mu_{ji}$ is $j$’s belief about $\theta_i$, given that $i$ has sent the signal $m_i$; the other expressions involving $\mu$ are defined analogously.

**Proposition 8.** Let $F$ be a continuous distribution function that is continuous at its expected value $b_i$ and symmetric around $b_i$. Let $F^\kappa(x) = F(b_i + \kappa(x - b_i))$, i.e. $b_i = b_i$. 

20
almost surely for \( \lim_{\kappa \to \infty} F^\kappa \). For any \( F \) and \( \varepsilon > 0 \), there exists a \( \bar{\kappa} > 0 \) such that \( \mu_{ji}^F - \mu_{ji}^1 < \varepsilon \) if \( F_i = F^\kappa \) and \( \kappa \leq \bar{\kappa} \). (Proof on page 40.)

**Proposition 9.** Fix the expected bias \( b_e_i \) of all players in a given room and a bounded support for all bias distributions \( F_i \). Assume that there is at least one element in the support that is smaller than \( \sum_{j \in R_i, j \neq i} b_j^{n_i} \frac{n-1}{n} - (2p - 1) \) and at least one element that is larger than \( \sum_{j \in R_i, j \neq i} b_j^{n_i} \frac{n-1}{n} + (2p - 1) \). Then for each \( \varepsilon > 0 \) there exists some \( \sigma F_i \) such that for all such \( F_i \) with \( \text{Var} (b_i) \geq \sigma F_i^2 \), \( \mu_{ji}^F - \mu_{ji}^1 \leq \varepsilon \). (Proof on page 40.)

These results already contain statements about room choice with uncertainty: If truth-telling is greatly reduced or becomes impossible, there is not much to be gained from being in one room. Of course, truth-telling between people with identical bias distributions is not necessarily easier – note that proposition 6 contained no assumption that people differ in how their biases are distributed. So are the effects of uncertainty simply to make communication hard in general? Not necessarily. Consider a model where full integration is welfare-optimal and an equilibrium if biases are known. We can show that for any such model, uncertainty can cause segregation between groups to become Pareto-superior to integration, and such segregation is also an equilibrium of the room choice game.

**Proposition 10.** Let the number of players be weakly larger than 4 and let \( b_i^e \in \{0, b\} \), with \( b \in (0, \frac{n-1}{n} (2p - 1)] \). Let the two bias groups be of equal size, i.e. \( n_0 = n_b = n/2 \). Then the following is true:

- If \( b_i = b_i^e \) with certainty, the fully integrated room is welfare-optimal and an equilibrium.
- If biases are uncertain, we can find distributions \( F_i \) that keep all \( b_i^e \) constant such that full segregation between the two bias groups is welfare-optimal. For \( \alpha \geq \frac{2}{n-2} \), this is also an equilibrium.

(Proof on page 40.)

To illustrate this result, let us return to the example on taxation from the introduction, and assume that the world consists of liberals and conservatives. Liberals generally prefer higher taxes than conservatives, but everybody is aware that the optimal tax level depends on how bad taxes are for economic growth. If the exact political preference of each person is known, an informative exchange is possible even across party lines as long as preferences are not too different. But now assume that instead, each member of each political group is either a moderate or an extremist. It is only observable whether anyone is liberal or conservative, not whether they are extremists or moderates. Both have equal probability, so that in expectation each person is still an “average” liberal or conservative.

Consider the problem of a liberal who is unsure whether he is listening to a moderate conservative or a conservative extremist. He knows that a conservative extremist would
always tell a liberal that taxes are bad for the economy, regardless of what his information is. Any statement about the damages of taxes has hence become less informative, while being more likely to be made, than if the liberal was talking to an average conservative. The same is true for a conservative listening to a liberal. Yet while discussion across party lines has become less informative, this is not true for discussion within parties: The possible biases within groups are still close enough so that both moderates and extremists want to truthfully reveal their knowledge to other members of their party. It is hence better for liberals to only talk to other liberals and for conservatives to only talk to conservatives, than for any cross-party discussion to take place – not because of inherent differences in preferences, but because of uncertainty about who one’s interlocutor is.

6. Empirical Evidence from Twitter

The most important mechanism behind our results is that the content of communication depends on the audience in a systematic way: The same person communicates in different ways with audiences of different ideology. All other results are logical consequences of this premise. This section gives an illustration that this basic premise holds true in a real life communication context. Using data from the micro-blogging service Twitter, we will investigate whether users send different messages depending on the likely audience of their messages.

Twitter allows its users to send short messages of 140 characters either to people who have followed them (“tweets”), or to specific receivers (“replies”). Replies are also public, and are especially visible to followers of the sender, the receiver, or to people who are reading a specific “thread” that was started by a message.

This coexistence of different audiences creates a natural environment to study the messages that individuals send when they believe they are talking to an audience of mostly like-minded people, or to a mixed audience, or to an audience of people they disagree with. In particular, we attempt to show that messages change with the audience that the sender (reasonably) expects to be addressing. We will be focusing on political communication and in particular on partisan slant in messages. In the following we describe our data collection and analysis step by step.

**First step: Building a dictionary** We analyzed the tweets of all 535 current members of the U.S. Congress (100 senators and 435 members of the house of representatives) to build a dictionary of partisan words and bigrams (groups of two words). For that, we counted how often each word or bigram was used by Democratic and Republican members of Congress, and isolated the words whose usage was (i) high enough and (ii) different enough between parties.

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9Since November 2017: 280 characters. However, our data was collected before this change in Twitter’s
Table 1: Left: Most partisan words among the words that were used very often (more than 1000 times) in our sample. “#senatemajldr” is a hashtag for the (Republican) Senate Majority Leader McConnell; Susan Collins is a Republican senator.
Right: Most partisan words among words that were used at least 10 times in our sample. (Note that these expressions are stemmed.)

Table 1 has some examples for partisan words. Note that the differences in usage might derive from using different words for the same thing (talking about “Obamacare” vs “affordable care act”) or from different focuses (talking about “Iran” vs talking about “women”). We are agnostic about where the differences come from.

The table also shows that the most intensely partisan words are often those used less frequently – in fact, all the words in the table on the right are used only by one side. We weight words according to their frequency to avoid over-extrapolating from small samples.

**Second step: Scoring accounts** Armed with this partisan dictionary, we can identify a person’s political leanings purely based on their twitter feed. For each word or bigram that this person uses and which is found in our dictionary, we assign a score based on how differently the term is used between parties. In the end, we arrive at an overall score for that person, based on all partisan terms they have used.

To demonstrate the effectiveness of our partisan dictionary, we have created scores for a number of political journalists and pundits, whose political leaning is known but who are not part of our sample.\(^\text{10}\) If our dictionary works well at scoring, we should be able to separate the journalists and pundits into partisan camps, only based on their twitter feed. Table 3 on page 43 of the appendix shows that we are able to do so with about 80% accuracy.

**Third step: Sampling random twitter users** We randomly sampled a number of twitter users who (i) mostly or exclusively tweeted in English, (ii) had tweeted at least

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\(^{10}\)We used the list of the 20 most influential journalists and blogger on the right and left, respectively, from StatSocial (2015).
3000 times, (iii) had at least 1000 followers, (iv) wrote some tweets of their own (and did not only re-tweet other people’s tweets), and (v) tweeted sufficiently often about political topics (i.e. used enough terms from our political dictionary).

We scored these random twitter users based on their original tweets, i.e. all tweets that were not replies to or retweets of other tweets, so that each user is assigned a location on a left-right scale.

Fourth step: Making use of different visibilities When “tweeting”, users’ texts are read by different audiences, based on what type of tweets they are. Simple tweets by user X are shown in the timelines of all users who follow X. A reply by user X to user Y is shown in the timelines of users who follow either X or Y.

Given that we have scored random twitter users based on their original tweets (which are only shown to their own followers), we can now examine how these twitter users interact with other twitter users, given that such interactions (if they are replies) are visible to a different audience than the tweets based on which we have scored the user.

Fifth step: Actual difference-in-difference analysis Using our work from the previous steps, we have generated a data set containing 12,043 reply tweets sent from 87 senders to 3,730 receivers. For each of these interactions, we can determine the political score of the sender and the receiver, as well as the properties of the interaction itself. This allows us to ask questions like: How is the interaction between a Democratic-leaning Twitter user and another Democratic-leaning Twitter user different from an interaction between a Democratic-leaning twitter user and a Republican-leaning twitter user?

First, we should establish how many interactions across the political spectrum actually take place, compared to interactions between people with similar political leaning. Figure 6 shows this relationship including a linear best fit; table 4 in the appendix shows the exact regression results. The relationship is strongly positive, meaning that the more right-wing a Twitter user is, the more he will on average interact with other right-wing Twitter users. If we split the writers of tweets in our sample into quartiles depending on their political score, the lowest quartile will on average reply to tweets by people with a score of 0.4234, while the highest quartile will respond to people with an average score of 0.4752. This supports the existence of “echo chambers” in how people interact in our dataset.\footnote{Of course, we are not the first to show such segregation on twitter – consider, for example, the studies by Barberá et al. (2015) or Krasodomski-Jones (2017).}

Next, we can examine what actually happens when people with different political opinions interact. Our model predicts that the content of an interaction depends on the expected audience. Regarding the political score of tweets in our data, that would mean

\footnote{See here for how the company itself describes visibility: https://help.twitter.com/en/using-twitter/types-of-tweets}
two things: (a) If a user replies to someone with similar political leaning, the political content of the reply (as measured by the score of the tweet) should be close to the user’s usual tweets. (b) If a user replies to someone who has a higher (lower) political score, the tweet should have a higher (lower) score than usual. Formally, if we estimate the equation

$$(\text{tweet score} - \text{sender score}) = \text{intercept} + \beta (\text{receiver score} - \text{sender score}),$$

then (a) implies that the intercept should be close to 0, while (b) implies that $\beta$ is positive and not small. Table 2 shows that this is the case. The coefficient estimate of 0.3582 means that the average sender adjusts the political content of an average reply tweet to bridge more than a third of the gap towards the receiver.

|                          | Estimate | Std. Error | t value | Pr(>|t|) |
|--------------------------|----------|------------|---------|----------|
| (Intercept)              | -0.0190  | 0.0022     | -8.78   | 0.0000   |
| (receiver score - sender score) | 0.3582   | 0.0202     | 17.70   | 0.0000   |

Table 2: The political score of a reply tweet changes with the distance between the political score of the sender and the receiver.
7. Discussion

7.1. Who provides the Rooms?

In our model, we have assumed that the rooms are available in sufficient quantity so that players who want to segregate themselves can do so. In reality, that is of course not guaranteed. Information exchange could literally be impossible for lack of an empty room, such as when co-workers find themselves unable to discuss sensitive questions in an open-plan workspace. Bernstein and Turban (2018) have shown that the creation of open-plan offices tends to decrease the number of (public) face-to-face interactions and increase the number of (segregated) electronic interactions among colleagues. Or the shortage of rooms could be more figurative, such as when a politician may want to discuss his doubts of a policy with colleagues but cannot find a forum in which to do so without potentially giving ammunition to his political opponents.

In both cases, we have seen that segregation may be in the interest of everybody involved. It benefits not just the sender and the receiver in the segregated room, but even those who end up being excluded – since their inclusion would render communication impossible and thus not benefit anyone. Since rooms provide such clear benefits and are not automatically available, those in need of them should be willing to pay for whoever can provide them. We could imagine a group of agents who are sufficiently polarized and caught together in one place, which makes them unable to exchange any information. If now a plucky entrepreneur opened a separate room and took a small entrance fee, it would be an equilibrium for one group of agents to each pay the fee, enter the room – and improve their own and everybody else’s situation.

We think that this fable provides a way to understand the success of social messaging platforms such as Facebook, Twitter, WhatsApp and Snapchat. Each of these allows its users to send messages (and other content) to certain groups of others, with varying possibilities of exclusion. It can seem from the outside as if the service that is provided is to connect people with each other, but our model suggests it is just as much to exclude some people and not others, while providing sophisticated ways to determine who should and should not be excluded.\textsuperscript{13} This has a strict economic logic to it: Once the Internet is available and ubiquitous, simply connecting people is not a scarce resource or service. But connecting them in such a way that they want to communicate truthfully, and can exchange the information they want to exchange, is much harder, and those who do it well can make a profit.

\textsuperscript{13}Facebook, for example, allows its users among other things to (i) choose which of their data is visible to search engines, (ii) choose for each post and image whether it is visible to everybody or just friends or friends of friends or even select group of friends (iii) block individual other users from seeing certain content (iv) create public or private events or groups to which members can be invited, (v) message directly with selected users or groups of users. All of these are tools of intelligent segregation, not connection.
7.2. Political Parties and “Safe Spaces”

Of course, the room structure need not be provided by the market, it could be created by the agents themselves so that they can communicate with others who share their interests and world view. Besides the obvious examples of clubs and societies, we think that this is one rationale for the existence of political parties. In a society that is polarized enough, political parties can help solve the problem of aggregating political views and opinions.

We should also note that while messages are meaningless if a player is not truth-telling in equilibrium, the messages that he is most reluctant to send are those that could be seen as being counter to his own interest. For example, if an agent’s $b_i$ is much lower than the average of all $b_j$, he has no problem truthfully reporting $\sigma^l$, but is more reluctant after $\sigma^h$. This is how political parties can be useful: by providing a secluded forum in which, for example, members of a party can discuss the flaws and merits of their own candidates or programs. They would not be able to have this kind of discussion in the presence of members from other parties, where they would become overly defensive of “their” candidates and programs.

But the problem of defensiveness also provides an argument for so-called “safe spaces”, i.e. spaces in which minorities or marginalized groups can communicate without outside interference. Informationally, such safe spaces may provide opportunities to communicate that would otherwise not exist. Consider the problem of two vegetarians who privately doubt whether vegetarianism is indeed a sensible choice – yet they find themselves defending it whenever they talk to (or in the presence of) non-vegetarians. Providing a “safe space” for vegetarians would allow them to discuss freely, and would hence provide a Pareto-improvement.

7.3. Rooms as Commitment and Information Design

The main unit of groups in our model is the room, in which everybody communicates with everybody and which is disjoint from other rooms. This allows us not just to examine the influence of bias differences on communication, but also the disciplining as well as the destructive effects of room composition.

Consider, for example, a setting in which person 1 is willing to tell the truth to 2, and 2 is willing to tell the truth to 3, but 1 is not willing to tell the truth to 3 (and all relationships hold vice versa). If we were to simply allow 1, 2 and 3 to form bi-directional communication links, that would be the end of it. But if they need to communicate within rooms, the effects are more interesting: 2’s presence could discipline 1 or 3’s message, and hence make communication possible between people who would not otherwise be able to exchange information. Or 3’s presence could prove to be a centrifugal force to the whole room and make communication between 1 and 2 impossible without inducing 2 to tell the truth to 3, thus shutting down any communication.

We can hence think of rooms as an information design tool, and room allocation as an
information design problem – where other room members provide a source of commitment. The results we have derived in section 2.2 show exactly how and when that is possible. For example, a social planner that wants to achieve a maximum of communication between ideologically distant groups could rely on the addition of moderates. In situations where individual information can never be observed – neither ex ante nor ex post – this may well be the only information design tool available.

8. Conclusion

Modern democratic societies have three main mechanisms to aggregate information: Debates, markets, and votes. Of the three, debate is arguably the oldest – and while the other two require an organized framework and somebody who can enforce the rules, debate just needs an ability to speak and to listen.

But when will people speak truthfully (and hence have reason to listen)? In this paper, we have argued that if people have different preferences as well as different information, segregation into like-minded, homogeneous groups can be individually rational and Pareto-efficient. Echo chambers are not necessarily as destructive as popular discourse can make them seem. But even more importantly, we have shown that if segregation happens, it is not in itself the cause of an inability to debate. Instead, the existence of echo chambers is the consequence of differences in preferences, and of uncertainty and mistrust about other people’s motives.

This has implications for how to improve debate. Society has a lot to gain from getting people with diverse backgrounds, experiences and opinions to exchange their views. But this can not simply be achieved by forcing or cajoling people to interact who would not do so out of their own choosing. In fact, that could be counter-productive, as it could destroy disjoint echo chambers in which communication works, in favor of large integrated groups in which it does not. Our research, which we have set out in this paper, suggests that meaningful debate can only happen if the participants feel that they have sufficiently much in common and they trust each others’ motives. That may be a taller order than simply putting people into a room and expecting them to come out smarter and in agreement. But functioning debate requires consideration for the motivations of the debaters.
Appendix

A. Proofs for the Model with Certainty

Proof of lemma 1 on page 9.

Let \((m_1, \ldots, m_n)\) be an equilibrium. Player \(i\)'s expected payoff when sending message \(m_i\) to players in room \(R_i\) can be written as

\[
U_i(m_i|\sigma_i) = \mathbb{E} \left[ -\left( a_i(m_{-i,R_i}, \sigma_i) - b_i - \sum_{k=1}^{n} \theta_k \right)^2 - \alpha \sum_{j \notin R_i} \left\{ \left( a_j(m_{-i,R_j}, \sigma_j) - b_i - \sum_{k=1}^{n} \theta_k \right)^2 \right\} \right]
\]

\[
-\alpha \sum_{j \in R_i, j \neq i} \left\{ \left( a_j(m_i, m_{-i,R_i}, \sigma_j) - b_i - \sum_{k=1}^{n} \theta_k \right)^2 \right\} \left| \sigma_i \right|
\]

which can be split in a part that is independent of \(i\)'s message \(m_i\) and a part that depends on \(m_i\):

\[
U_i(m_i) = \mathbb{E} \left[ \text{const} - \alpha \sum_{j \in R_i, j \neq i} \left( a_j(m_i, m_{-i,R_i}, \sigma_j) - b_i - \sum_{k=1}^{n} \theta_k \right)^2 \left| \sigma_i \right] \right.
\]

Specifically, sending message \(m^h\) gives expected payoff

\[
U_i(m^h) = \mathbb{E} \left[ \text{const} - \alpha \sum_{j \in R_i, j \neq i} \left( b_j - b_i + \mu_{ji}^h + \sum_{k \neq i} \mu_{jk} - \theta_i - \sum_{k \neq i} \theta_k \right)^2 \left| \sigma_i \right] \right.
\]

where \(\mu_{ji}^h = \mathbb{E}[\theta_i|m_i = m^h]\), i.e. \(\mu_{ji}^h\) is the belief of a player \(j\) in the same room as \(i\) concerning \(\theta_i\) if player \(i\) sends message \(m^h\). Note that this belief is the same for all players \(j \neq i\) in the same room as \(i\). Sending message \(m^l\) gives

\[
U_i(m^l) = \mathbb{E} \left[ \text{const} - \alpha \sum_{j \in R_i, j \neq i} \left( b_j - b_i + \mu_{ji}^l + \sum_{k \neq i} \mu_{jk} - \theta_i - \sum_{k \neq i} \theta_k \right)^2 \left| \sigma_i \right] \right.
\]
where $\mu^b_{ji} = \mathbb{E}[\theta_i | m_i = m^b]$. The difference in expected payoff is then

$$
\Delta U_i(\sigma_i) = \left( U_i(m^h) - U_i(m^l) \right) / \alpha
$$

$$
= - \sum_{j \in R_i, j \neq i} \mathbb{E} \left[ \mu^b_{ji} - \mu^l_{ji} \right]^2 + 2(\mu^b_{ji} - \mu^l_{ji}) \left( b_j - b_i + \sum_{k \neq i} \mu_{jk} - \theta_i - \sum \theta_k \right) \sigma_i
$$

$$
= -2(\mu^b_{ji} - \mu^l_{ji}) \sum_{j \in R_i, j \neq i} \left[ \frac{\mu^b_{ji} + \mu^l_{ji}}{2} + b_j - b_i - \mathbb{E} [\theta_i | \sigma_i] \right]
$$

$$
= 2(\mu^b_{ji} - \mu^l_{ji})(n_{R_i} - 1) \left[ - \frac{\mu^b_{ji} + \mu^l_{ji}}{2} - \sum_{j \in R_i, j \neq i} b_j \right. \left. / (n_{R_i} - 1) + b_i + \mathbb{E} [\theta_i | \sigma_i] \right]
$$

(6)

where $n_{R_i}$ denotes the number of players in room $R_i$.

Player $i$ is only willing to choose a mixed strategy after receiving signal $\sigma_i$ if $\Delta U_i(\sigma_i) = 0$. From expression (6) it is clear that this can only be true for at most one signal as $\mathbb{E} [\theta_i | \sigma_i]$ varies in the $\sigma_i$. Furthermore, $U_i(\sigma^h) = 0$ implies $U_i(\sigma^l) < 0$ and similarly $U_i(\sigma^l) = 0$ implies $U_i(\sigma^h) > 0$.

Now suppose $i$’s equilibrium strategy $m_i$ is mixed after signal $\sigma^h$. Then, $U_i(\sigma^h) = 0$ implies $U_i(\sigma^l) = 2(\mu^b_{ji} - \mu^l_{ji})(n_{R_i} - 1)(1 - 2p) < 0$ and therefore $m_i(\sigma^l) = m^l$ which implies $\mu^b_{ji} = p$ as a $m^h$ is only sent by $i$ after receiving signal $\sigma^h$. This implies $(\mu^b_{ji} + \mu^l_{ji})/2 \geq 1/2$ as $\mu^l_{ji} \geq 1 - p$. Now consider the equilibrium candidate $(m^l_i, m_{-i})$. With the truthful strategy $m^l_i, \mu^b_{ji} = p$ and $\mu^l_{ji} = 1 - p$ and therefore $(\mu^b_{ji} + \mu^l_{ji})/2 = 1/2$. This implies that $U_i(\sigma^h) > 0$ in the equilibrium candidate $(m^l_i, m_{-i})$, i.e. truthful reporting is optimal for $i$ after receiving signal $\sigma^h$. In the equilibrium candidate $(m^l_i, m_{-i})$, truthful messaging is still optimal after signal $\sigma^l$ as well: From $p > 1/2$, $\mu^b_{ji} \leq p$ and $\mu^l_{ji} \leq 1/2$ it follows that $-1/2 + (1 - p) < - (\mu^b_{ji} + \mu^l_{ji})/2 + p$. As in the original equilibrium $(m_i, m_{-i})$ we had $\Delta U_i(\sigma^h) = 0$ and therefore $- (\mu^b_{ji} + \mu^l_{ji})/2 + p = \sum_{j \in R_i, j \neq i} b_j / (n_{R_i} - 1) + b_i$, we get that $-1/2 + 1 - p < \sum_{j \in R_i, j \neq i} b_j / (n_{R_i} - 1) + b_i$ and therefore $U_i(\sigma^l) < 0$ in the truthful equilibrium candidate $(m^l_i, m_{-i})$. Hence, truthful messaging is $i$’s best response in the equilibrium candidate $(m^l_i, m_{-i})$. Finally, note that the $\Delta U_i(\sigma_j)$ for $j \neq i$ is not affected by changing $i$’s strategy from $m_i$ to $m^l_i$. Hence, $(m^l_i, m_{-i})$ is an equilibrium.

The argument in case $i$’s strategy is mixed after signal $\sigma^l$ is analogous. 

Proof of theorem 1 on page 10.

Consider again the difference between lying and truth-telling for player $i$ that we considered in equation (6) in the proof of lemma 1. Following corollary 1, we only consider pure strategies and therefore for every non-babbling player $\mu^b_{ji} = p$ and $\mu^l_{ji} = 1 - p$ which
implies that $\Delta U_i(\sigma^*) \geq 0$ simplifies to
\[
\frac{1}{n_R - 1} \sum_{j \in R_i, j \neq i} (b_i - b_j) \geq \frac{1}{2} - p
\]
\[
b_i - \frac{1}{n_R - 1} \sum_{j \in R_i, j \neq i} b_j \geq \frac{1}{2} - p
\]
\[
\frac{n_R}{n_R - 1} b_i - \frac{1}{n_R - 1} \sum_{k \in R_i} b_k \geq \frac{1}{2} - p
\]
\[
b_i \geq \bar{b} - \frac{n_R - 1}{n_R} \left( p - \frac{1}{2} \right).
\]
If this inequality does not hold, player $i$ will not use the truthful strategy in the most informative equilibrium and by corollary 1 this implies that he will babble in the most informative equilibrium.

We can analogously solve for $\Delta U_i(\sigma^*)$ and get the interval used in the proposition. \qed

**Proof of proposition 1 on page 11.**

Denote the sets of babbling and truthful players in room $R_j$ as $R_j^{bab}$ and $R_j^{truth}$, respectively. For a given room allocation, the expected payoff of player $i$ in room $R_i$ is
\[
U_i = -\mathbb{E} \left[ \left( \sum_{j \in R_i^{truth} \cup \{i\}} (\mu_{ij} - \theta_j) + \sum_{j \notin R_i^{truth} \cup \{i\}} (\frac{1}{2} - \theta_j) \right)^2 \right] + \alpha \sum_{j \in R_i, j \neq i} \left( b_j - b_i + \sum_{k \in R_i^{truth} \cup \{j\}} (\mu_{jk} - \theta_k) + \sum_{k \notin R_i^{truth} \cup \{j\}} (\frac{1}{2} - \theta_k) \right)^2 \right] + \alpha \sum_{j \notin R_i} \left( b_j - b_i + \sum_{k \in R_i^{truth} \cup \{j\}} (\mu_{jk} - \theta_k) + \sum_{k \notin R_i^{truth} \cup \{j\}} (\frac{1}{2} - \theta_k) \right)^2 \right].
\]

For any $i \neq j$, the two values of $\theta_i$ and $\theta_j$ are independent; the same is true for $\mu_{ij}$ and $\mu_{ik}$. Hence $\mathbb{E} [\mu_{ij} - \theta_j] = 0$ and $\mathbb{E} [(\mu_{ij} - \theta_j) (\mu_{ik} - \theta_k)] = 0$, which means that the above expression can be rewritten as
\[
U_i = -\sum_{j \in R_i^{truth} \cup \{i\}} \mathbb{E} [(\mu_{ij} - \theta_j)^2] - \sum_{j \notin R_i^{truth} \cup \{i\}} \mathbb{E} \left[ (\frac{1}{2} - \theta_j)^2 \right] - \frac{\alpha}{n_R - 1} \sum_{j \in R_i, j \neq i} (b_j - b_i) - \frac{\alpha}{n_R - 1} \sum_{j \in R_i, j \neq i} \sum_{k \in R_i^{truth} \cup \{j\}} \mathbb{E} [(\mu_{jk} - \theta_k)^2] - \frac{\alpha}{n_R} \sum_{j \notin R_i} \sum_{k \in R_i^{truth} \cup \{j\}} \mathbb{E} \left[ (\frac{1}{2} - \theta_k)^2 \right].
\]
Now note that $\mathbb{E}[(\mu_{jk} - \theta_k)^2]$ can have two possible values: If $k \in R_j^{\text{truth}} \cup \{j\}$, i.e. if $j$ has received information about $\theta_k$, then $\mathbb{E}[(\mu_{jk} - \theta_k)^2] = p(1-p)$. If $j$ has not received information about $\theta_k$, then $\mathbb{E}[(\mu_{jk} - \theta_k)^2] = \frac{1}{4}$. (We can check that information always reduces variance and increases welfare since $p > \frac{1}{2}$ and hence $p(1-p) < \frac{1}{4}$.)

This means that if $i$ is telling the truth, we can write

$$U_i^{\text{truth}} = -\alpha \sum_{j \neq i} \{(b_j - b_i)^2\} - \frac{1}{4} [n + \alpha(n-1)n]$$

$$+ \left(\frac{1}{4} - p(1-p)\right) \left[ n_{R_i}^{\text{truth}} + \alpha \sum_R \{ n_{R}^{\text{truth}} n_{R}^{\text{truth}} + (n_R - n_R^{\text{truth}})(1 + n_R^{\text{truth}}) \} - \alpha n_{R_i}^{\text{truth}} \right]$$

The first term represents the loss that $i$ suffers because other players choose a decision that is by $b_j - b_i$ too high from $i$’s point of view. The second term represents the (theoretical) loss that would result if no player had any information and all $\mu$’s were simply $\frac{1}{2}$. The factors $n$ and $(n-1)n$, which sum up to $n^2$, represent the total number of possible pieces of information in the model: If everybody’s signal was available to everyone, $n$ people would receive $n$ pieces of information. The term hence represents, for each potential piece of information, the loss to $i$ of that information not being available.

This loss is mitigated by information, which we see in the second line: $i$ receives his signal and $n_{R_i}^{\text{truth}} - 1$ truthful messages, which means that instead of $\frac{1}{4}$, on each of these pieces of information $i$ loses only $p(1-p) < \frac{1}{4}$. Other players, about whose decisions $i$ cares with weight $\alpha$, also receive some signals/messages: in any given room $R$, $n_R^{\text{truth}}$ players receive their own signal and $n_R^{\text{truth}} - 1$ truthful messages while $n_R - n_R^{\text{truth}}$ players (those that babble in $R$) receive $n_R^{\text{truth}}$ truthful messages and their own signal. (We have to subtract the correction term $-\alpha n_{R_i}^{\text{truth}}$ for room $R_i$ in which there are only $n_R^{\text{truth}} - 1$ other players who tell the truth – in other words, $i$ cannot count himself again as one of the players who receive information.) Analogously, we can write

$$U_i^{\text{bab}} = -\alpha \sum_{j \neq i} \{(b_j - b_i)^2\} - \frac{1}{4} [n + \alpha(n-1)n]$$

$$+ \left(\frac{1}{4} - p(1-p)\right) \left[ 1 + n_{R_i}^{\text{truth}} + \alpha \sum_R \{ n_{R}^{\text{truth}} n_{R}^{\text{truth}} + (n_R - n_R^{\text{truth}})(1 + n_R^{\text{truth}}) \} - \alpha (1 + n_{R_i}^{\text{truth}}) \right]$$

In both the expressions for $U_i^{\text{truth}}$ and $U_i^{\text{bab}}$, the second lines are adjusting the (pessimistic) expression in the first line for the reduction in variance by information. We can simplify both expressions by simply writing

$$U_i = -\alpha \sum_{j \neq i} \{(b_j - b_i)^2\} - \frac{1}{4} [n + \alpha(n-1)n] + (\frac{1}{4} - p(1-p)) \left[ \zeta_i + \alpha \sum_{j \neq i} \zeta_j \right] \quad (9)$$

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and express welfare as

$$W = \sum_i U_i = \sum_i \left[ -\alpha \sum_{j \neq i} \{(b_j - b_i)^2\} - \frac{1}{4} n + \alpha (n - 1) n + (1/4 - p(1 - p)) \left[ \zeta_i + \alpha \sum_{j \neq i} \zeta_j \right] \right]$$

$$= -\alpha \sum_{i=1}^n \sum_{j \neq i} \{(b_j - b_i)^2\} - \frac{1}{4} n^2 \left[ 1 + \alpha (n - 1) \right] + (p - \frac{1}{2})^2 \left( 1 + \alpha (n - 1) \right) \sum_i \zeta_i.$$ 

In this expression, all terms are model parameters except for the sum over all $\zeta_i$, which shows that welfare is linearly increasing in $\sum_i \zeta_i$.

Proof of proposition 2 on page 12.

Let $\Delta \zeta_i$ be the change in $\zeta_i$ that results from a deviation, and $\Delta \sum_{j \neq i} \zeta_j$ the change in $\sum_{j \neq i} \zeta_j$ resulting from the same deviation. We know that in the welfare-optimal room allocation, there can be no deviation by a single player that would increase welfare. That means that in the welfare-optimum, it must be

$$\Delta \zeta_i + \Delta \sum_{j \neq i} \zeta_j \leq 0. \quad (10)$$

The condition that there is no profitable deviation for player $i$ is

$$\Delta \zeta_i + \alpha \Delta \sum_{j \neq i} \zeta_j \leq 0, \quad (11)$$

which is identical except for the factor $\alpha$. From this we can immediately see that 10 implies 11 if $\alpha = 1.$

Proof of theorem 2 on page 15.

Recall that a truth-telling equilibrium exists if and only if for every player $i$ it is

$$\left| \sum_{k \neq i} \{b_k / (n - 1)\} - b_i \right| \leq \frac{1}{2}.$$ 

This can be rewritten as $|\sum_k \{b_k\} - nb_i| / (n - 1) \leq \frac{1}{2}$. If $\eta$ is sufficiently small, this inequality holds for all players and all signals. Clearly, having all players in one room and telling the truth is welfare optimal whenever it is feasible, and no player can gain from leaving the room.

If $\left| \sum_{k \in R_i, k \neq i} \{b_k / (n - 1)\} - b_i \right| > \frac{1}{2}$, then $i$ will not be truthful when receiving either signal $\sigma^l$ or $\sigma^h$. Generically, $\left| \sum_{k \in R_i, k \neq i} \{b_k / (n - 1)\} - b_i \right| \neq 0$ for any room configuration containing players from more than one bias group. (This follows from the finiteness of players which implies that the number of such room configurations is finite.) Now observe
that the left hand side of the non-truth telling inequality is scaled by $\eta$ while the right hand side is not. That is, for $\eta$ sufficiently high, player $i$ will report the highest (lowest) signal in all rooms in which $\sum_{k \in R_i, k \neq j} b_k < n_{R_i} b_i (\sum_{k \in R_i, k \neq j} b_k > n_{R_i} b_i)$. Put differently, any room that contains one or more players of a bias not equal to $b_i$ will lead to totally uninformative messages by $i$ if $\eta$ is sufficiently high. For high enough $\eta$, this holds true for all players and it is then obvious that full separation is both welfare maximizing and an equilibrium.

\[ \blacksquare \]

**Proof of proposition 3 on page 16**

Theorem 1 states that in the most informative equilibrium of the messaging subgame players in room $R$ will tell the truth if and only if $b_i \in \left[ \bar{b} - \frac{n_{R_i} - 1}{n_R}(p - \frac{1}{2}), \bar{b} + \frac{n_{R_i} - 1}{n_R}(p - \frac{1}{2}) \right].$ If this interval covers $[0, k]$, then one room leads to truth telling by all players and one single room is clearly optimal. In the remainder of this proof, we therefore assume that this is not the case. The length of the interval $[\bar{b} - \frac{n_{R_i} - 1}{n_R}(p - \frac{1}{2}), \bar{b} + \frac{n_{R_i} - 1}{n_R}(p - \frac{1}{2})]$ is $\frac{2(n_{R_i} - 1)}{n_R}$. The number of players telling the truth in any room is consequently bounded from above by $\lceil \frac{2(n_{R_i} - 1)}{n_R} (2p - 1) / (k/(n - 1)) \rceil + 1$ as the players’ biases are equally spaced with distance $k/(n - 1)$ between two consecutive players’ biases. This bound may not be attained by any feasible room due to the discrete nature of the problem. More specifically, if we take the fully integrated room, then the number of truth telling players will be either $\lceil \frac{2(n_{R_i} - 1)}{n_R} (2p - 1) / (k/(n - 1)) \rceil + 1$ or $\lfloor \frac{2(n_{R_i} - 1)}{n_R} (2p - 1) / (k/(n - 1)) \rfloor$.

Let $t^*$ be the maximal number of truth telling players in any possible room. From the above, it is clear that $t^* \in \{ \lceil \frac{2(n_{R_i} - 1)}{n_R} (2p - 1) / (k/(n - 1)) \rceil + 1, \lfloor \frac{2(n_{R_i} - 1)}{n_R} (2p - 1) / (k/(n - 1)) \rfloor \}$. Suppose $t^*$ is the number of truth telling players if all players are in the same room. Then the number of pieces of information generated in this room is $t^* n + n - t^*$. We will show that in this case no other room configuration generates more pieces of information: The total number of pieces of information in $r$ rooms is: $\sum_R t_R n_R + n_R - t_R = \sum_R t_R (n_R - 1) + n_R \leq \sum_R t^*(n_R - 1) + n_R = t^*(n - r) + n \leq t^* n + n - t^*$. By proposition 1, one big room with all players is then welfare optimal if this leads to $t^*$ truth telling players.

Next consider the situation where one integrated room with all players leads not to $t^*$ but only to $t^* - 1$ truth telling players. Suppose that there is some room $R^*$ with $n - 1$ players in which $t^*$ players are truth telling. We show that in this case the room configuration $(R^*, \{1, \ldots, n \} \setminus R^*)$ is welfare optimal. This will lead to $t^*(n - 1) + n - 1 - t^* + 1 = t^*(n - 1) + n$ pieces of information. The big integrated room leads to only $(t^* - 1)n + n - t^* + 1 = t^*(n - 1) < t^*(n - 1) - t^* + n$ pieces of information and is therefore welfare inferior. Any other room configuration with $r$ rooms leads to $\sum_R t_R n_R + n_R - t_R = \sum_R t_R (n_R - 1) + n_R \leq \sum_R t^*(n_R - 1) + n_R = t^*(n - r) + n \leq t^*(n - 2) + n$ pieces of information which is also (weakly) less than $(R^*, \{1, \ldots, n \} \setminus R^*)$. Hence, in this case $(R^*, \{1, \ldots, n \} \setminus R^*)$ is welfare optimal.

Finally, we show that the conditions in the proposition lead to either of the two just
described cases. Note that in the fully integrated room $\bar{b} = k/2$. Hence, condition (3) states that the distance from the lowest player’s bias who tells the truth to the lower boundary of the truthtelling interval is less than $1/2$ the distance between two consecutive players’ biases. By symmetry of the truthtelling interval around $\bar{b}$ and the equal spacing of biases, this is also true for the distance of the highest bias player telling the truth and the upper boundary of the truthtelling interval. First, let (3) hold strictly. Then it is clear that shifting the truthtelling interval (by changing $\bar{b}$) cannot lead to more players being truthtelling. Furthermore, the length of the truthtelling interval is strictly decreasing in the number of players in the room. Hence, in no other room can there be more truthtelling players than in the fully integrated room. This holds also if (3) holds with equality as the length of the truthtelling inequality is strictly decreasing in the number of players in the room. Consequently, $t^*$ is achieved by the fully integrated room and the argument two paragraphs above shows that then the fully integrated room is welfare optimal.

Now consider the case where (3) does not hold. Start from the fully integrated room. If (3) does not hold, shifting the truthtelling interval by $k/(2(n-1))$ down (by – for now magically – reducing $\bar{b}$ by this amount), will imply that this interval contains 1 more player than in the fully integrated room. Furthermore, the distance of this lowest truthtelling player after the shift to the lower boundary of the truthtelling interval will be less than $k/(2(n-1))$ by the assumption that (3) did not hold. Now note that removing player $n$ from the fully integrated room will reduce $\bar{b}$ by exactly $k/(2(n-1))$ (from $k/2$ to $(k-k/(n-1))/2$). But note that removing this player also implies that $n_R = n-1$ and therefore the length of the truthtelling interval is reduced. Condition (4) states that due to the shrinking of the interval when moving from $n$ to $n-1$ players the one player whose truthtelling was gained by shifting the interval down is lost again. Furthermore, the “shrinking” occurs at the upper as well as the lower boundary to the same extent. This implies that also at the upper boundary one truthtelling player is lost due to the shrinking (while the shifting did not lose anyone as (3) was violated by assumption). Consequently, the room without player $n$ will have one less truthtelling player than the fully integrated room if (3) is violated and (4) holds. In this case, no room with $n-1$ (or less) players can have more truthtelling players than the fully integrated room and therefore $t^*$ is attained in the fully integrated room. Consequently, the fully integrated room is by the results above welfare optimal.

If neither (3) nor (4) holds, then the “shrinking” argument above implies that the room allocation $\{\{1, \ldots , n-1\}, \{n\}\}$ leads to one more truthtelling player in $R^* = \{1, \ldots , n-1\}$ than in the fully integrated room. Consequently, $t^*$ is attained in $R^*$ and $\{\{1, \ldots , n-1\}, \{n\}\}$ is welfare optimal by the results above.\(^\text{14}\)

In terms of equilibrium, it is immediate that no player wants to deviate from the fully

\(^{14}\)It should be noted that similar arguments as above, with an upward instead of a downward shift, lead to the optimality of $\{\{1\}, \{2, \ldots , n\}\}$ which will also attain $t^*$ if (3) and (4) are violated.
integrated room by isolating himself as self-isolation leads to less information for himself and no more information for other players. The same argument applies for players in room $R^*$ in case (3) and (4) are violated. However, the isolated player might have an incentive to join $R^*$: This would reduce the amount of information as only $t^* - 1$ instead of $t^*$ players would be truthtelling in the resulting fully integrated room reducing the number of pieces of information of all other players in this room from $t^*(n-1) + n - 1 - t^*$ to $(t^* - 1)(n - 1) + n - t^*$. However, the deviating player would gain more information for himself, i.e. the number of pieces of information he observes is $t^*$ instead of 1. From 9, it follows that the deviation is profitable if and only if $\alpha < \frac{t^* - 1}{n - 2}$. Note that $t^* = \left\lfloor \frac{n - 1}{n} \left(2p - 1\right) / (k/(n - 1)) \right\rfloor$ in the here analyzed case where one integrated room is not optimal. This gives the condition in the proposition. 

**Proof of proposition 5 on page 18**

Take two values of $\eta$, namely $\eta'$ and $\eta'' > \eta'$. Denote a welfare optimal room assignment under $\eta''$ by $R''$. Consider the same room assignment $R''$ with biases $\eta'$. In each room the number of pieces of information is weakly higher with set of biases $B_{\eta'}$ than with set of biases $B_{\eta''}$: By theorem 1 a player $i$ is truthtelling if and only if $\eta_{b} - \eta_{R''} - 1(n - 1) / (p - 1) \leq \eta_{b} \leq \eta_{b} + \eta_{R''} - 1(n - 1) / (p - 1)$. Hence, player $i$ will be truthtelling in room $R''$ with biases in $B_{\eta'}$ if he is truthtelling in $R''$ with biases $B_{\eta''}$ by $\eta' < \eta''$. Consequently, there is weakly more information transmitted in every room given assignment $R''$ under $\eta'$ than under $\eta'' > \eta'$. This implies $W(\eta') \geq W(\eta'')$ by proposition 1.

**B. Detailed Analysis and Proofs for the Model with Uncertainty**

**B.1. Preliminary Analysis**

Similarly to the derivation of expression (6), we can write

$$U_{i}(m^{h}) = \mathbb{E} \left[ \text{const} - \alpha \sum_{j \in R, j \neq i} \left( b_{j} - b_{i} + \mu^{h}_{ji} + \sum_{k \neq i} \mu_{jk} - \theta_{i} - \sum_{k \neq i} \theta_{k} \right)^{2} \right]$$

$$U_{i}(m^{l}) = \mathbb{E} \left[ \text{const} - \alpha \sum_{j \in R, j \neq i} \left( b_{j} - b_{i} + \mu^{l}_{ji} + \sum_{k \neq i} \mu_{jk} - \theta_{i} - \sum_{k \neq i} \theta_{k} \right)^{2} \right] .$$

Note that we are interested in the difference of the two expressions. Hence, while all $b_{j}$s are now unknown, this uncertainty only matters where $b_{j}$ is multiplied by $\mu^{h}_{ji}$ and $\mu^{l}_{ji}$.
probability and

We can also write the following two terms, which both appear in equation (12):

\[
\Delta U_i(\sigma_i) = \frac{(U_i(m^h) - U_i(m^l))}{\alpha} = 2(\mu_{ji}^h - \mu_{ji}^l)(n_{R_i} - 1) \left[ -\frac{\mu_{ji}^h + \mu_{ji}^l}{2} - \frac{\sum_{j \in R_i, j \neq i} b_j^i}{n_{R_i} - 1} + b_i + \mathbb{E}[\theta_i | \sigma_i] \right],
\]

which is identical to (6) except that we have substituted \( b_j^i \) for \( b_j \). i’s problem remains virtually unchanged, except that he now considers the expected value of biases of other people within the room.

Now consider i’s messaging strategy. In the following, let

\[
\lambda^h = \Pr(m_i = m^h | \sigma_i = \sigma^h) \quad \text{and} \quad \lambda^l = \Pr(m_i = m^l | \sigma_i = \sigma^l)
\]

i.e. \( \lambda^h \) and \( \lambda^l \) are the marginal probabilities with which i truthfully reveals his signal, averaging over all possible bias types. For example, if \( b_i \) has two possible values with equal probability and i only reveals \( \sigma^h \) truthfully for one of them, then \( \lambda^h = \frac{1}{2} \). The resulting beliefs of player j are

\[
\mu_{ji}^h = \frac{p\lambda^h + (1 - p)(1 - \lambda^l)}{1 + \lambda^h - \lambda^l}, \quad \mu_{ji}^l = \frac{p(1 - \lambda^h) + (1 - p)\lambda^l}{1 - \lambda^h + \lambda^l}.
\]

We can also write the following two terms, which both appear in equation (12):

\[
\mu_{ji}^h - \mu_{ji}^l = \frac{2p\lambda^h + 2p\lambda^l - 2p - \lambda^h - \lambda^l + 1}{(\lambda^h - \lambda^l + 1)(\lambda^l - \lambda^h + 1)}
\]

\[
= (2p - 1) \frac{\lambda^h + \lambda^l - 1}{(\lambda^h - \lambda^l + 1)(\lambda^l - \lambda^h + 1)} \tag{13}
\]

\[
\mu_{ji}^h + \mu_{ji}^l = \frac{2p\lambda^h - 2p(\lambda^h)^2 - 2p\lambda^l + 2p(\lambda^l)^2 - 2(\lambda^h)^2 - \lambda^h + \lambda^l + 2\lambda^h \lambda^l + 1}{(\lambda^h - \lambda^l + 1)(\lambda^l - \lambda^h + 1)}
\]

\[
= 4p(\lambda^l)^2 - 2(\lambda^h)^2 - 4p\lambda^h \lambda^l + 2\lambda^h \lambda^l + 2p\lambda^h - \lambda^h - 2p\lambda^l + \lambda^l - 2p + 1
\]

\[
= (2p - 1) \frac{2(\lambda^l)^2 - 2\lambda^h \lambda^l + \lambda^h - \lambda^l - 1}{(\lambda^h - \lambda^l + 1)(\lambda^l - \lambda^h + 1)} + 2p
\]

\[
= (2p - 1) \left( \frac{(\lambda^l)^2 - \lambda^h \lambda^l - \lambda^l}{(\lambda^h - \lambda^l + 1)(\lambda^l - \lambda^h + 1)} + \frac{(\lambda^l)^2 - \lambda^h \lambda^l + \lambda^h - 1}{(\lambda^h - \lambda^l + 1)(\lambda^l - \lambda^h + 1)} \right) + 2p
\]

\[
= (2p - 1) \left( \frac{\lambda^l}{\lambda^h - \lambda^l + 1} + \frac{\lambda^l - 1}{\lambda^h - \lambda^l + 1} \right) + 2p. \tag{14}
\]
From (13), we can see that the condition $\mu_{ji}^h \geq \mu_{ji}^l$ translates to $\lambda^h + \lambda^l \geq 1$. We can distinguish two cases:

- $\lambda^h + \lambda^l = 1$. Then $\mu_{ji}^h - \mu_{ji}^l = 0$ and i’s messages are completely uninformative.
- $\lambda^h + \lambda^l > 1$. We will focus on this case, in which messages by $i$ have some informative content.

We can intuitively see that if i’s messages are believed to contain some information about $\sigma_i$, $i$ should never want to misrepresent $\sigma^h$ if $b_i$ is high compared to the average bias of other players (and vice versa if $b_i$ is low). In fact, we can show the following result:

**Lemma 2.** Assume that $\lambda^h + \lambda^l > 1$. Then i always strictly prefers to truthfully reveal (i) $\sigma^h$ if $b_i \geq \mathbb{E} \left[ \frac{\sum_{j \in R_i, j \neq i} b_{ji}^h}{n_{R_i} - 1} \right]$ and (ii) $\sigma^l$ if $b_i \leq \mathbb{E} \left[ \frac{\sum_{j \in R_i, j \neq i} b_{ji}^l}{n_{R_i} - 1} \right]$.

**Proof.** Consider case (i) and assume that the opposite was true, i.e. $\Delta U_i(\sigma^h) \leq 0$ for some $b_i \geq \mathbb{E} \left[ \frac{\sum_{j \in R_i, j \neq i} b_{ji}^h}{n_{R_i} - 1} \right]$. Then, since $(\mu_{ji}^h - \mu_{ji}^l) > 0$ by assumption and $b_i \geq \mathbb{E} \left[ \frac{\sum_{j \in R_i, j \neq i} b_{ji}^h}{n_{R_i} - 1} \right]$, it must be that $\mu_{ji}^h + \mu_{ji}^l - \mathbb{E} [\theta_i | \sigma_i] > 0$ or $\mu_{ji}^h + \mu_{ji}^l - p > 0$, which means $\left( \frac{\lambda^l}{\lambda^h - \lambda^l - 1} + \frac{\lambda^l - 1}{\lambda^h - \lambda^l + 1} \right) > 0$. But we know that $\lambda^h - \lambda^l - 1 < 0$ and $\lambda^h - \lambda^l + 1 > 0$ from $\lambda^h + \lambda^l > 1$, which implies that $\left( \frac{\lambda^l}{\lambda^h - \lambda^l - 1} + \frac{\lambda^l - 1}{\lambda^h - \lambda^l + 1} \right) < 0$. We can analogously prove (ii).

Now we can consider which conditions need to be in place for an equilibrium to exist in which $i$ tells the truth with probabilities $\lambda^h$ and $\lambda^l$. To be clear: We are still considering pure equilibria, since $i$ has a strict preference for lying or telling the truth for any $b_i$ except for non-generic boundary cases. However, given $F_i$ (the distribution of $b_i$), we can determine how often $i$’s messages will be truthful once we have established for which $b_i$ $i$ wants to tell the truth and for which he wants to lie. We can think of $\lambda^h$ and $\lambda^l$ as the marginal probabilities of truth-telling by $i$.

**Lemma 3.** There exists an equilibrium in which $i$ truthfully reveals $\sigma^h$ with marginal probability $\lambda^h$ and truthfully reveals $\sigma^l$ with marginal probability $\lambda^l$ if and only if

$$1 - F_i \left( \frac{\sum_{j \in R_i, j \neq i} b_{ji}^h}{n_{R_i} - 1} + \left( p - \frac{1}{2} \right) \cdot \left( \frac{\lambda^l}{\lambda^h - \lambda^l - 1} + \frac{\lambda^l - 1}{\lambda^h - \lambda^l + 1} \right) \right) \leq \lambda^h$$

and

$$F_i \left( \frac{\sum_{j \in R_i, j \neq i} b_{ji}^l}{n_{R_i} - 1} + \left( p - \frac{1}{2} \right) \left( \frac{\lambda^h - 1}{\lambda^h - \lambda^l - 1} + \frac{\lambda^h}{\lambda^h - \lambda^l + 1} \right) \right) \geq \lambda^l.$$

Both inequalities hold with equality if $F_i$ is continuous at the argument.

**Proof.** From equation 12 we get that $\Delta U_i(\sigma_i) \geq 0 \iff$

$$b_i - \frac{\sum_{j \in R_i, j \neq i} b_{ji}^h}{n_{R_i} - 1} \geq \frac{\mu_{ji}^h + \mu_{ji}^l}{2} - \mathbb{E} [\theta_i | \sigma_i].$$
Recall that \( \mathbb{E} [\theta_i | \sigma_i = \sigma^h] = p \) and \( \mathbb{E} [\theta_i | \sigma_i = \sigma^l] = 1 - p \). We can make use of the expression for \( \mu_{ji}^h + \mu_{ji}^l \) that we have derived in (14) to get \( \Delta U_i(\sigma^h) \geq 0 \equiv
\begin{align*}
  b_i - \sum_{j \in R_i, j \neq i} b_j^* \geq \left( p - \frac{1}{2} \right) \left( \frac{\lambda^l}{\lambda^l - 1} + \frac{\lambda^l - 1}{\lambda^h - \lambda^l + 1} \right)
\end{align*}
\]
and \( \Delta U_i(\sigma^l) \leq 0 \equiv
\begin{align*}
  b_i - \sum_{j \in R_i, j \neq i} b_j^* \leq \left( p - \frac{1}{2} \right) \left( \frac{\lambda^h - 1}{\lambda^h - \lambda^l + 1} + \frac{\lambda^h}{\lambda^h - \lambda^l + 1} \right) .
\end{align*}
\]

In an equilibrium, the beliefs of the receivers of \( m_i \) must be correct on average. In this case, this means that it must be sufficiently likely for \( b_i \) to fulfill either of the two inequalities, which gives us the conditions from the proposition. If \( F_i \) is continuous at the argument, correct beliefs require that the inequalities hold with equality. If it is not, there could potentially be mixed equilibria in which for the borderline type, \( i \) mixes between different messages and beliefs are correct on average. \( \square \)

Note that that \( \left( \frac{\lambda^h - 1}{\lambda^h - \lambda^l + 1} + \frac{\lambda^h}{\lambda^h - \lambda^l + 1} \right) - \left( \frac{\lambda^l}{\lambda^l - \lambda^l - 1} + \frac{\lambda^l - 1}{\lambda^h - \lambda^l + 1} \right) = 2 \). Lemma 3 consequently describes conditions on the distribution function \( F \) at two points that are \( 2p - 1 \) apart.

In particular if \( F_i \) is continuous at these two points the conditions state that probability mass in the interval between these two points has to equal \( \lambda^l + \lambda^h - 1 \). More importantly, the conditions can be used to show that player \( i \) babbles in a given room if \( F_i \) does not have enough probability mass around the average bias of the other players in the room. To be precise, if \( F_i \) has no probability mass in \( \frac{\sum_{j \in R_i, j \neq i} b_j^*}{n_{R_i} - 1} \pm (2p - 1) \), then the conditions of lemma 2 imply \( \lambda^l + \lambda^h = 1 \) and therefore uninformative messages. \(^{15}\)

B.2. Proofs

Proof of proposition 6 on page 20.

Without loss of generality, let \( b_1 \) and \( b_n \) be the smallest and largest biases respectively. We can represent each bias as the expected value of a distribution that only places density on the values \( b_1 - (2p - 1) \) and \( b_n + (2p - 1) \). For this set of distributions \( \{F_1, F_2, \ldots, F_n\} \), the conditions of lemma 3 imply \( \lambda^h + \lambda^l = 1 \), and hence there exists no equilibrium in which any of the players tells the truth. \( \square \)

\(^{15}\)To be precise, both points at which \( F_i \) is evaluated in lemma 2 lie in the interior of the interval \( \left[ \frac{\sum_{j \in R_i, j \neq i} b_j^*}{n_{R_i} - 1} - (2p - 1), \frac{\sum_{j \in R_i, j \neq i} b_j^*}{n_{R_i} - 1} + (2p - 1) \right] \) and therefore \( F_i \) will be continuous at both points and equal to the same value if there is no probability mass in this interval. As the conditions in lemma 2 then hold with equality, they imply \( \lambda^h + \lambda^l = 1 \) which in turn implies \( \mu_{ji}^h - \mu_{ji}^l = 0 \).
Proof of proposition 7 on page 20.

We can construct a distribution $F_i$ that has positive density on $\frac{\sum_{j \in R_{i,j} \setminus i} b_{ji}^*}{n_{R_i} - 1}$, which means that the conditions of lemma 3 imply that there exists an equilibrium in which a message by $i$ is informative.

To achieve full truth-telling (i.e. $\lambda^h = \lambda^l = 1$), lemma 3 implies we would have to be able to construct an $F_i$ that only has density inside the interval $\frac{\sum_{j \in R_{i,j} \setminus i} b_{ji}^*}{n_{R_i} - 1} \pm (p - \frac{1}{2})$. However, this would contradict our starting assumption that if $b_i$ is $b_i^*$ for sure, there exists no equilibrium in which $i$ tells the truth. \qed

Proof of proposition 8 on page 20.

By the symmetry of $F$, all $F^\kappa$ have the same expected value. We can find a $\bar{\kappa}$ small enough so that $F^\kappa$ has less than $\varepsilon' > 0$ probability mass within $\frac{\sum_{j \in R_{i,j} \setminus i} b_{ji}^*}{n_{R_i} - 1} \pm (2p - 1)$ for any $\kappa \leq \bar{\kappa}$. Then it follows from lemma 3 that there exists no equilibrium for which $\lambda^l + \lambda^h > 1 + \varepsilon'$. The result follows now from the continuity of (14) and the fact that $\mu^h_{ji} - \mu^l_{ji} = 0$ if $\lambda^h + \lambda^l = 1$. \qed

Proof of proposition 9 on page 21.

Let the lower (upper) bound of the support be $b_{\bar{i}}$ ($\bar{b}_{i}$). Note that by assumption $b_i \leq \frac{\sum_{j \in R_{i,j} \setminus i} b_{ji}^*}{n_{R_i} - 1} - (2p - 1)$ and $\bar{b}_i \geq \frac{\sum_{j \in R_{i,j} \setminus i} b_{ji}^*}{n_{R_i} - 1} + (2p - 1)$ which implies by lemma 2 that player $i$ sends uninformative messages in equilibrium. Now fix $b_i^\varepsilon = \frac{\sum_{j \in R_{i,j} \setminus i} b_{ji}^*}{n_{R_i} - 1} - (2p - 1)$ and $\bar{b}_i^\varepsilon = \frac{\sum_{j \in R_{i,j} \setminus i} b_{ji}^*}{n_{R_i} - 1} + (2p - 1)$. This implies that $\mu^h_{ji} - \mu^l_{ji} \leq \varepsilon$ whenever the probability that $b_i \geq b_i^\varepsilon$ plus the probability that $b_i < \bar{b}_i^\varepsilon$ is more than $1 - \varepsilon'$ for some $\varepsilon' > 0$ (by lemma 2 and the continuity of $\mu^h_{ji}$ in $\lambda^h$ and $\lambda^l$). Let $\sigma^2_{F_i}$ be defined by

$$\sigma^2_{F_i} = (1 - \varepsilon') \left( \frac{\bar{b}_i - b_i}{\bar{b}_i - \underline{b}_i} (\bar{b}_i - \underline{b}_i)^2 + \frac{b_i - \underline{b}_i}{\bar{b}_i - \underline{b}_i} (\bar{b}_i - b_i)^2 \right) + \varepsilon' \left( \frac{\bar{b}_i^* - b_i^*}{\bar{b}_i^* - \underline{b}_i^*} (\bar{b}_i^* - \underline{b}_i^*)^2 + \frac{b_i^* - \underline{b}_i^*}{\bar{b}_i^* - \underline{b}_i^*} (\bar{b}_i^* - b_i^*)^2 \right).$$

Any distribution with variance above $\sigma^2_{F_i}$ has to have more than $\varepsilon'$ probability mass above $\bar{b}_i^\varepsilon$ or below $\underline{b}_i^\varepsilon$ as $\sigma^2_{F_i}$ is the variance of the distribution maximizing variance under the constraint that only $1 - \varepsilon'$ probability mass is outside the interval $[\underline{b}_i^\varepsilon, \bar{b}_i^\varepsilon]$. Consequently, any distribution with variance above $\sigma^2_{F_i}$ will lead to $\mu^h_{ji} - \mu^l_{ji} \leq \varepsilon$. \qed

Proof of proposition 10 on page 21.

Fix 0 and a $b > 0$. Consider the distributions putting probability 1/2 on $-(p - 1/2)$ and 1/2 on $p - 1/2$ instead of 0 for sure and 1/2 on $b - (p - 1/2)$ and 1/2 on $b + (p - 1/2)$. Under segregation everyone is (just!) truth-telling. In any room including at least 1 player with another bias than the own one, a bias 0 ($b$) player will however lie if his bias is the lower (higher) element of the support:
Take for example a player with bias $b + p - 1/2$ that got a low signal. Then $\Delta U(\sigma^i) > 0$ can be written as $b + p - 1/2 - \frac{\sum_{j \in R_i \cap \sigma^i \neq i} \lambda_{ij}}{n_{R_i - 1}} > (\mu_{ij}^b + \mu_{ij}^l)/2 - (1-p)$. The right hand side of this inequality is bounded from above by $p - 1/2$ because $\mu_{ij}^b \leq p$ and $\mu_{ij}^l = 1 - p$ by lemma 2 according to which $\lambda^b = 1$. As $b - \frac{\sum_{j \in R_i \cap \sigma^i \neq i} \lambda_{ij}}{n_{R_i - 1}} > 0$, the claim follows.

To compute welfare under a non-segregated scenario, we need to compute $\mathbb{E}[(\mu_{ij} - \theta_j)^2]$. Take, for example, a player $j$ with biases in $\{b - p + 1/2, b + p - 1/2\}$. We showed that this player always sends the high signal if $b_i = b + p - 1/2$ if at least one player of the other group is in his room. The most informative messaging strategy of such a player in such a room is therefore truth telling when $b_i = b - p + 1/2$ and sending the high message otherwise. This implies $\lambda^h = 1$ and $\lambda^l = 1/2$ and therefore $\mu_{ij}^h = (1+p)/3$ and $\mu_{ij}^l = 1-p$. In this case,

$$
\mathbb{E}[(\mu_{ij} - \theta_j)^2] = \frac{1}{2} \left[ \frac{1}{2} \left\{ p \left( \frac{1 + p}{3} - 1 \right)^2 + (1-p)(-p)^2 \right\} + \frac{1}{2} \left\{ p(1-p)^2 + (1-p) \left( \frac{1 + p}{3} \right)^2 \right\} \right] + \frac{1}{2} \left\{ \frac{1}{2} \left( \frac{1 + p}{3} - 1 \right)^2 + \frac{1}{2} \left( \frac{1 + p}{3} \right)^2 \right\}
$$

$$
= \frac{1}{4} \left[ (1 + p) \frac{p^2 - 4p + 4}{9} + (1-p)p^2 + p(1-p)^2 + (2-p) \frac{1 + 2p + p^2}{9} \right]
$$

$$
= \frac{1}{4} \left[ \frac{2}{3} + \frac{4}{3}p - \frac{4}{3}p^2 \right].
$$

Following the derivations of player $i$’s utility in a room that contains players of both groups, see the proof of proposition 1, we can write player $i$’s utility if all players are in the same fully integrated room – and choose the best possible messaging strategy corresponding to $\lambda^h = 1$ ($\lambda^l = 1/2$) and $\lambda^l = 1/2$ ($\lambda^l = 1$) for players with expected bias $b^*_i = b$ ($b^*_i = b$) as

$$
U^\text{int}_i = -\alpha \sum_{j \neq i} \{ (b_j - b_i)^2 \} - [n + \alpha(n-1)n] / 4 + (1/4 - p(1-p))(1 + \alpha(n-1))
$$

$$
+ (1/4 - [2/3 + p4/3 - p^24/3]/4) [n - 1 + \alpha \sum_{j \neq i} \{n - 1\}]
$$

while his expected payoff under full segregation is

$$
U^\text{seg}_i = -\alpha \sum_{j \neq i} \{ (b_j - b_i)^2 \} - [n + \alpha(n-1)n] / 4 + (1/4 - p(1-p))(n/2 + \alpha(n-1)n/2).
$$
$U_i^\text{seg}$ exceeds $U_i^\text{int}$ if and only if

\[
(1/4 - p(1 - p))(1 + \alpha(n - 1))(n/2 - 1) \geq (1/4 - [2/3 + p4/3 - p^24/3]/4) [n - 1 + \alpha(n - 1)^2]
\]

\[
\iff (1 - 4p + 4p^2))(1 + \alpha(n - 1))(n/2 - 1) \geq (1/3 - p4/3 + p^24/3) [n - 1 + \alpha(n - 1)^2]
\]

\[
\iff 3(1 + \alpha(n - 1))(n/2 - 1) \geq n - 1 + \alpha(n - 1)^2
\]

\[
\iff \frac{3}{2}(1 + \alpha(n - 1))\frac{n - 2}{n - 1} \geq 1 + \alpha(n - 1)
\]

\[
\iff \frac{n - 2}{n - 1} \geq \frac{2}{3}
\]

which is true for $n \geq 4$. As the payoffs do not differ across players in each of the two scenarios, welfare is higher under segregation than under integration given that $n \geq 4$.

To see that other room configurations cannot improve welfare, start from full segregation. Moving $k$ players from room 1 to room 2 will lead to less information for the remaining players in room 1. Suppose nevertheless that this move was welfare increasing. Then players in the new room 2 must have better information than under segregation. Note that by assumption the most informative strategy players could possibly adopt in the new room is $\lambda^h = 1$ ($\lambda^b = 1/2$) and $\lambda^l = 1/2$ ($\lambda^e = 1$) for players with expected bias $b_i = b$ ($b_i^e = b$). Assume that this strategy is an equilibrium in the new room 2 (if it is not, this step increases the welfare gain over segregation). But then it is clearly optimal to move the remaining players from room 1 to room 2 as well (if this strategy remains an equilibrium): This improves information for all players. But this would imply $U_i^\text{int} > U_i^\text{seg}$ which contradicts what we showed above. \qed
C. Empirical Work: Tables and Figures

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Table 3: The scores of the twitter feeds of 40 prominent American political pundits. The higher the score, the more Republican-leaning a pundit is.
|            | Estimate | Std. Error | t value | Pr(>|t|) |
|------------|----------|------------|---------|----------|
| (Intercept)| 0.3336   | 0.0044     | 75.27   | 0.0000   |
| scoreOriginalRel | 0.2399 | 0.0103     | 23.19   | 0.0000   |

Table 4: The political scores of sender and receiver are correlated. The table shows the results of estimating the equation receiver score = intercept + \( \beta \) sender score.

References


Quattrociocchi, W., A. Scala, and C. R. Sunstein (2016). Echo chambers on Facebook. Available on SSRN.

