Pricing with Cookies: Behavior-Based Price Discrimination and Spatial Competition*

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Abstract

We study a two-period model of spatial competition with two symmetric firms where firms learn customers' preferences from the first-period purchase, which they use for personalized pricing in the second period. With product choice exogenously fixed with maximal differentiation, we show that, unlike Fudenberg and Tirole (2000), there exist two asymmetric equilibria and customer switching is only from one firm to the other. Firms are worse off with such behavior-based price discrimination than when they compete in uniform pricing. When product choice is also made optimally, there continue to exist two asymmetric equilibria, none of which features maximal differentiation. More customer information hurts firms, and more so when they make both product choice and pricing decisions.

Keywords: Spatial competition, behavior-based price discrimination, personalized pricing, endogenous product choice
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1 Introduction

Firms can tightly target pricing, marketing and even product characteristics to individual consumers, using information technology and large datasets of customer-level information (big data). For example, tracking tools such as cookies, web beacons, or Etags, allow individual sellers or data brokers to record consumers' browsing histories on the Internet.[1] This information can be used by websites to target their offerings based on consumers' purchasing history, location, referring sites and even computer operating systems.[2] Traditional retailers, such as supermarkets, use loyalty schemes to gather personal and shopping data and offer consumers personalized discounts.[3]

The ability of firms to use big data to price discriminate and raise profits has spurred active marketing research on one hand.[4] On the other hand, it has also led to concerns about competition, privacy and equity. For example, Shiller (2013, p. 3) concludes that, from his simulation, "[s]ubstantial equity concerns may arise . . . [as] some consumers may be charged twice as much as others are for the same product".[5] Also the US Council of Economic Advisers (2015, p. 17) note that "[s]ome consumer advocates suggest that we should . . . limit the use of personalized pricing to offline settings or require its disclosure to buyers".[6]

Despite these concerns, economic research shows that access to consumer information may intensify competition and benefit consumers *once the information has been gathered*.[7] As Thisse and Vives (1988, p. 124) note, because of their access to consumer information, "firms may get trapped into a Prisoner's Dilemma-type situation and end

[1]A cookie is a file placed on a browser's computer by a website to allow the website owner to track the browser's interactions with the site. See "Little Brother", *The Economist* (September 13, 2014), and "How companies learn your secrets", *The New York Times* (February 16, 2012). Bergemann and Bonatti (2015) provide a formal analysis of data brokers.

[2]See "On Orbitz, Mac users steered to pricier hotels", *The Wall Street Journal*, (August 23, 2012).

[3]See "Supermarkets offer personalized pricing", *Bloomberg Business*, (November 145, 2013).

[4]For example, using 2006 data, Schiller (2013) estimates that Netflix could have raised its profit by 1.4% if it had used customer data to set personalized prices rather than using second-degree price discrimination. See also Shaffer and Zhang (2002), Dewan et al. (2003), Syam et al. (2005), and Weisstein et al. (2013).

[5]Similarly, Hannak et al. (2014, p. 305) state that "personalization on e-commerce sites may also be used to the users disadvantage by manipulating the products shown (price steering) or by customizing the prices of products (price discrimination)".

[6]Consumers' attitudes to personalized pricing appear mixed. While they appear willing to provide personal information, for example, in return for free on-line services (Council of Economic Advisers, 2015, p. 15), they appear to dislike this information being used to provide personalized prices (Richards et al., 2016).

[7]See for example Thisse and Vives (1988), Fudenberg and Tirole (2000), and Esteves (2010, 2014). Villas-Boas (1999) shows that competition can also intensify in an infinite horizon, overlapping generations framework. Chen (2005) and Fudenberg and Villas-Boas (2007) survey the early literature. Chen and Zhang (2009), however, show that the intensification of competition may be avoided if there are some 'loyal' customers.

up with lower profits due to the intense competition unleashed".[8] This need not be the case, however, when firms are actively engaged in gathering the information. Gathering information may lead to either less intense[9] or more intense competition.[10]

Further, the effects of personalized pricing can depend on the type of information gathered by sellers. Previous work has focused on symmetric information. Either all sellers know the specific characteristics of all buyers as in Thisse and Vives (1988). Or, in a dynamic context, sellers know (or can infer) where all consumers previously purchased, but not any finer details about customers' characteristics, as in Fudenberg and Tirole (2000).[11]

Information collection by sellers, however, will often be asymmetric. For example, a cookie can provide highly personalized information about a consumer. But that information is only available to the seller who installs the cookie. Similarly, loyalty programs provide extensive histories about a customer's shopping preferences at a particular retailer. But this information is not available to other retailers. While 'minimal' information about a particular customer may be available to a firm that fails to sell to that customer, the successful seller may gather significantly more information about the same customer.

In this paper, we analyze competition with asymmetric information acquisition and personalized prices using the two-firm/two-period framework of Fudenberg and Tirole (2000) with quadratic transportation costs. A firm that sells to a particular customer in the first period gathers precise information about that customer. However, the other firm only knows that the customer chose the rival seller. Using two versions of our model (exogenous product choice and endogenous product choice), we show that this asymmetric information acquisition leads to multiple asymmetric pure-strategy equilibria.

With exogenously fixed product choice, there are two asymmetric, pure-strategy equilibria, each favoring a firm with more aggressive pricing in the first period. In each equilibrium, the firm with more aggressive first-period pricing gains a larger market share. In the second period, the other firm retains all of its own first-period customers and poaches some customers back from its larger rival. However, it will continue to sell to less than half of the customers in the second period. This is in contrast to the

[8]Subsequent studies show that introducing some heterogeneity among firms (Shaffer and Zhang, 2002; Matsumura and Matsushima, 2015) or quality choice by firms (Choudhary et al. 2005; Ghose and Huang, 2009) can resolve this prisoner's dilemma. However, these papers do not consider the active gathering of information and, as such, provide limited insight into situations where firms endogenously gather information.

[9]For example, Fudenberg and Tirole (2000) and Esteves (2010).

[10]For example, Zhang (2011) and this paper.

[11]Zhang (2011, p. 173) refers to this as the "minimum information assumption about consumer purchase histories; a consumer's product choice reveals her relative preference between the two firms but not the precise strength of her preference".

situation with the minimum *symmetric* information assumption, as in Fudenberg and Tirole (2000) and Zhang (2011) where two-way poaching occurs in the second period with each firm stealing some of its rival's first-period customers.

When firms make product choice endogenously in the first period, then there continue to exist two asymmetric, pure-strategy equilibria where one firm chooses extreme differentiation, as in the standard one-period game. The other firm, however, makes a more aggressive product choice, reducing differentiation. This aggressive firm then sets a relatively high price in the first period and has a smaller share of customers. In the second period it then poaches customers from its larger rival, but not vice-versa. Indeed, poaching is significant enough that the firm with smaller sales in the first period sells to more than half of the market in the second period.

In each case, the more aggressive seller, either low first-period price given exogenous product choice or aggressive product choice when it is endogenous, makes larger profits over the two periods than the other seller. However, the ability to gather information and use personalize prices reduces firms' total profits. In particular, and in contrast to the results of Fudenberg and Tirole (2000) and Esteves (2010), firms set lower prices to all customers in the first period than in the standard one-period game.

The potential for asymmetric equilibria has been noted previously in the literature. For example, Esteves (2010, Section 6) notes that an asymmetric pure strategy equilibrium may arise in her model where one firm gains the entire market in the first period. In this situation, no information is created through first period sales, which leads to higher second period profits. The asymmetric equilibrium is driven by one firm avoiding any sales in the first period in order to eliminate the creation of information. In contrast, in our model, both sellers are always active in the market and information is created. Unlike Esteves (2010), it is the asymmetry of the information that drives asymmetric behavior in the first period.

Zhang (2011, Section 5.1) considers a situation with the same information assumptions as in our model. But she allows costless personalization of products as well as prices. This means that once one firm has customer-specific information, the other firm cannot effectively serve that customer. The result is a symmetric equilibrium with highly aggressive pricing in the first period and perfect price/product discrimination in the second period. Thus her results differ substantially from our own. Further, while her assumption of costless product personalization may be relevant in some settings, in many situations it is reasonable to expect some limits to product variety. In that sense, our analysis is complementary to hers. Our model takes the opposite product assumption to hers (each firm only chooses one product), leading to very different but, in our opinion, widely applicable results.

Our paper significantly extends the existing literature on behavior-based price discrimination in a number of ways. The asymmetric information structure we analyze captures key features of actual information gathering by firms. But it has not been widely considered in the literature. As a result, and in contrast to existing literature, our analysis shows that asymmetric equilibria can arise even when there are two *ex ante* symmetric firms. This asymmetry feeds into all elements of the competitive process: product choice where relevant; pricing in both the first and second periods of the game; and customer poaching in the second period. While our model captures key features of the existing literature, such as the prisoner's dilemma nature of competition with customer information, we show how competition can also be intensified in the first period through asymmetric behavior. Unlike previous work, our analysis shows that there can be an aggressive firm that reaps greater profits over the two periods than its passive rival. However, the relationship between total profits and market share depends on whether product choice is exogenous or endogenous for the firms.

The rest of this paper proceeds as follows. Section 2 describes the model. Section 3 analyzes the case with exogenously fixed product choice while Section 4 studies the case where product choice is also endogenous. Section 5 summarizes our main findings and concludes. Proofs not provided in the main text are presented in the appendix.

2 The Model

Consider a Hotelling linear city with consumers distributed uniformly over $[0, 1]$. Consumer located at x will be simply called consumer x. Each consumer buys one unit of good in each period for two periods, and derives utility v from each unit. We assume v is sufficiently large so that the entire market is covered in equilibrium. There are two firms indexed by $i = A, B$. Both firms have constant marginal cost of production, which is normalized to zero. Consumers have quadratic transportation costs.[12] Thus if firm i is located at z and sets a price $P_i(x)$ for consumer x, then consumer x gets a payoff of $v - P_i(x) - t(x - z)^2$ if she buys from firm i. Our main analysis assumes that consumers are myopic in the sense that they make purchase decisions separately in each period. But we also offer discussions on how our results change when consumers are forward-looking in that they make purchase decisions to maximize the sum of utilities over the two periods. The total payoff to each firm is the sum of their profits over the two periods, to be discussed below. To simplify notation, we assume firms do not discount future.

There are two periods in the game, indexed by $\tau = 1, 2$. The $\tau = 1$ is the standard

[12]We assume quadratic transport costs so that the one period game with endogenous product choice involves a unique pure strategy equilibrium where firms choose maximal differentiation (Anderson et al., 1992). This makes our results easily comparable to other standard results.

Hotelling model: firms simultaneously choose locations which are fixed over two periods, after which they compete in price. The prices set by each firm in $\tau = 1$ are non-discriminatory: each firm sets a single price and sells to all consumers who wish to purchase at that price. Consumers observe these prices and choose to buy from one firm. In $\tau = 1$, each firm also uses 'cookies' to track consumers. Let $\mathcal{A}$ be the set of consumers that choose firm A in $\tau = 1$ and $\mathcal{B}$ be the set of consumers that choose firm B in $\tau = 1$. By assumption, all consumers are members of only one of these sets. At the end of $\tau = 1$, firm A knows, for each consumer x: (i) whether $x \in \mathcal{A}$ or $x \in \mathcal{B}$; and (ii) if $x \in \mathcal{A}$, then the location x. Similarly, at the end of $\tau = 1$, firm B knows, for each consumer y: (i) whether $y \in \mathcal{A}$ or $y \in \mathcal{B}$; and (ii) if $y \in \mathcal{B}$, then the location y.

In $\tau = 2$, firms again simultaneously set prices. However, each firm can now set its prices to discriminate between consumers based on the information acquired in $\tau = 1$. Thus firm A can set individual prices $P_A(x)$ to each consumer $x \in \mathcal{A}$, to be called personalized pricing. Firm A can only set a single uniform price, $P_A(\mathcal{B})$ to the set of consumers in $\mathcal{B}$. Similarly, firm B chooses individual prices $P_B(y)$ for each consumer $y \in \mathcal{B}$ and a single uniform price, $P_B(\mathcal{A})$ for the set of consumers in $\mathcal{A}$.Consumers again observe the prices and make their purchase decisions and firms receive their $\tau = 2$ profits.

For future reference, we discuss below the results from the three benchmark models adapted to our framework. First, suppose firms do not store customer information in $\tau = 1$. In this case, the second period is a mere repetition of the first period. Thus the standard Hotelling outcome with quadratic transportation cost obtains in each period: one firm chooses location 0 and serves $[0, \frac{1}{2}]$ while the other chooses location 1 and serves $[\frac{1}{2}, 1]$; firms charge the same price t, earn profit $\frac{t}{2}$, and the average distance traveled by a consumer is $\frac{1}{4}$.

Second, consider Thisse and Vives (1998) where both firms employ personalized pricing for all potential customers. Since firms' locations are exogenously fixed in their model, we can assume that firm A is at 0 and firm B at 1. Then in equilibrium, firm A serves $[0, \frac{1}{2}]$ with prices $P_A(x) = (1 - 2x)t$ for all $x < \frac{1}{2}$, firm B serves $[\frac{1}{2}, 1]$ with prices $P_B(y) = (2y - 1)t$ for all $y > \frac{1}{2}$, and each firm earns profit $\frac{t}{4}$. The average distance traveled by a consumer is again $\frac{1}{4}$.

Third, in Fudenberg and Tirole (2000), firms exercise third-degree price discrimination in $\tau = 2$ based on customers' purchase behavior in $\tau = 1$. With firm A located at 0 and firm B at 1, the equilibrium is symmetric with price equal to $\frac{4t}{3}$ and firm A's market share is $[0, \frac{1}{2}]$ in $\tau = 1$. In $\tau = 2$, both firms charge $\frac{2t}{3}$ to their $\tau = 1$ customers and $\frac{t}{3}$ to their rival's customers and, as a result, consumers in $[\frac{1}{3}, \frac{1}{2}]$ switch from firm A to firm B, and those in $[\frac{1}{2}, \frac{2}{3}]$ switch from firm B to firm A. The $\tau = 2$ profit for each firm is then $\frac{5t}{18}$. The average distance traveled by a consumer in $\tau = 2$ is $\frac{11}{36} > \frac{1}{4}$, hence social welfare is lower in $\tau = 2$ compared to the two previous cases because of inefficient customer switching.[13]

In sum, all three models lead to a symmetric equilibrium in which the two firms have the same market share each period. In the period when firms can exercise price discrimination, profits are the smallest with personalized pricing and the largest with uniform pricing. Thus the more customer information firms use to devise finer pricing strategies, the more intense competition becomes, which hurts firm profitability.

3 Exogenously Fixed Locations

We start by analyzing the case in which the firms' locations are fixed at 0 and 1, i.e., maximal differentiation. Without loss of generality, suppose firm A is located at 0. Analyzing this case helps us understand the second-period pricing game more clearly than when the location choice is also endogenous. This will in turn facilitate solving the whole game with endogenous location choice. Note that the equilibrium in the standard Hotelling model with quadratic transportation cost also has maximal differentiation.

3.1 Second Period

Let us begin with the pricing game in $\tau = 2$. Since a standard revealed preference argument shows that each of $\mathcal{A}$ and $\mathcal{B}$ is a connected interval, we can define a unique value $z \in [0, 1]$ such that $x \in \mathcal{A}$ iff $x \le z$. Then the equilibrium prices in $\tau = 2$ are described in the following lemmas.

Lemma 1 *(i) If $z \le \frac{1}{2}$, then the unique equilibrium in $\tau = 2$ is given by*

$$P_A(x) = \begin{cases} (1-2x)t & \text{if } x \in [0, z], \\ \dfrac{(1-2z)t}{2} & \text{if } x \in [z, 1], \end{cases}$$

$$P_B(y) = \begin{cases} 0 & \text{if } y \in [0, \dfrac{1+2z}{4}], \\ \dfrac{(4y-2z-1)t}{2} & \text{if } y \in [\dfrac{1+2z}{4}, 1]. \end{cases}$$

[13]The average travel distance is $\frac{1}{6}$ for a consumer staying with the same firm and $\frac{7}{12}$ for a switching consumer. Thus the average travel distance for a consumer is $\frac{1}{6} \times \frac{2}{3} + \frac{7}{12} \times \frac{1}{3} = \frac{11}{36}$.

(ii) If $z \geq \frac{1}{2}$, then the unique equilibrium in $\tau = 2$ is given by

$$P_A(x) = \begin{cases} \dfrac{(2z-4x+1)t}{2} & \text{if } x \in [0, \dfrac{1+2z}{4}], \\ 0 & \text{if } x \in [\dfrac{1+2z}{4}, 1], \end{cases}$$

$$P_B(y) = \begin{cases} \dfrac{(2z-1)t}{2} & \text{if } y \in [0, z], \\ (2y-1)t & \text{if } y \in [z, 1]. \end{cases}$$

Proof: See the appendix.

Based on the above, we can calculate the $\tau = 2$ profit for each firm. Consider first the case $z \leq \frac{1}{2}$. Then consumers in $[0, z]$ continue to purchase from firm A, consumers in $[z, \frac{1+2z}{4}]$ switch from firm B to firm A, and consumers in $[\frac{1+2z}{4}, 1]$ continue to purchase from firm B. Thus firm A's $\tau = 2$ profit from its repeat customers is $\int_0^z (1-2x)tdx = (1-z)zt$ and its profit from switching customers is $\frac{(1-2z)t}{2}\left(\frac{1+2z}{4} - z\right) = \frac{(1-2z)^2 t}{8}$. So if $z \leq \frac{1}{2}$, firm A will make $\tau = 2$ profit equal to:

$$\pi_A^2 = (1-z)zt + \frac{(1-2z)^2 t}{8} = \frac{(1+4z-4z^2)t}{8}.$$

If $z > \frac{1}{2}$, then consumers in $[\frac{1+2z}{4}, z]$ switch from firm A to firm B. Thus firm A serves only those in $[0, \frac{1+2z}{4}]$ by charging personalized prices $P_A(x) = \frac{(2z-4x+1)t}{2}$. So it makes profit:

$$\pi_A^2 = \int_0^{\frac{1+2z}{4}} \frac{(2z-4x+1)t}{2} dx = \frac{(1+2z)^2 t}{16}.$$

Due to symmetry, firm B's profit is the same as firm A's profit in the relevant region when z is replaced by $1-z$. Summarizing the above, we have

Lemma 2 *Equilibrium profits in $\tau = 2$ are given by*

$$\pi_A^2 = \begin{cases} \dfrac{(1+4z-4z^2)t}{8} & \text{if } z \leq \dfrac{1}{2}, \\ \dfrac{(1+2z)^2 t}{16} & \text{if } z \geq \dfrac{1}{2}, \end{cases}$$

$$\pi_B^2 = \begin{cases} \dfrac{(3-2z)^2 t}{16} & \text{if } z \leq \dfrac{1}{2}, \\ \dfrac{(1+4z-4z^2)t}{8} & \text{if } z \geq \dfrac{1}{2}. \end{cases}$$

It is easy to verify that both profit functions are continuous, firm A's profit increases in z, and firm B's profit decreases in z. At $z = \frac{1}{2}$, the two firms' profits are the same and are equal to $\frac{t}{4}$. Thus firm A has incentives to increase z and firm B has incentives to decrease z, which would intensify price competition in the first period. However, a change in $\tau = 1$ market share affects each firm's $\tau = 2$ profit in an asymmetric way. As can be checked easily from Lemma 2, as z increases from $\frac{1}{2}$, firm A's $\tau = 2$ profit increases more than (the absolute value in) the decrease in firm B's $\tau = 2$ profit: for z arbitrarily close to, but larger than, $\frac{1}{2}$, we have $\frac{d\pi_A^2}{dz} \approx 1/2$ and $\frac{d\pi_B^2}{dz} \approx 0$. As we will see below, this asymmetry leads to asymmetric equilibria in the $\tau = 1$ price game.

3.2 First period

In $\tau = 1$, firm A's profit is $\pi_A^1 = P_A^1 z$ while firm B's profit is $\pi_B^1 = P_B^1(1 - z)$ where P_i^1 is firm i's $\tau = 1$ price, $i = A, B$. Total profit to firm A from the two periods is then

$$\Pi_A = \begin{cases} P_A^1 z + \dfrac{t}{8}\left(1 + 4z - 4z^2\right) & \text{if } z \leq \dfrac{1}{2}, \\ P_A^1 z + \dfrac{t}{16}\left(1 + 2z\right)^2 & \text{if } z \geq \dfrac{1}{2}. \end{cases}$$

Since consumers are myopic, their $\tau = 1$ purchase decisions depend only on the comparison of P_A^1 and P_B^1. Thus we have $z = \frac{P_B^1 - P_A^1 + t}{2t}$ and $z \geq \frac{1}{2}$ if and only if $P_A^1 \leq P_B^1$. Substituting z into Π_A and Π_B, we can express these profit functions in terms of P_A^1 and P_B^1. Based on these, the equilibria of the $\tau = 1$ price game can de derived as follows.

Lemma 3 *The price game in $\tau = 1$ has two equilibria:*

(i) $P_A^1 = \dfrac{10t}{13}$ *and* $P_B^1 = \dfrac{8t}{13}$ *with* $z = \dfrac{11}{26}$*;*

(ii) $P_A^1 = \dfrac{8t}{13}$ *and* $P_B^1 = \dfrac{10t}{13}$ *with* $z = \dfrac{15}{26}$*.*

Proof: See the appendix.

The above result is interesting in that asymmetric equilibria obtain even though the two firms are symmetric and their $\tau = 1$ locations are fixed exogenously at a maximal distance. This is in contrast to the symmetric equilibrium in the standard Hotelling model: $P_A^1 = P_B^1 = t$ and $z = \frac{1}{2}$. Even when firms use third-degree price discrimination in $\tau = 2$ as in Fudenberg and Tirole (2000), the $\tau = 1$ equilibrium is unique with equal market share for each firm.

The reason for the asymmetric equilibria in our case is the use of personalized pricing in the second period. Personalized pricing enables firms to effectively protect its market in $\tau = 2$ and, as a result, each firm's $\tau = 2$ profit increases if it has a larger market share in $\tau = 1$.[14] As shown previously, however, the effect of a change in market share on firms'

[14] When firms use third degree price discrimination as in Fudenberg and Tirole (2000), each firm's equilibrium profit in $\tau = 2$ is independent of its market share in $\tau = 1$. This is discussed in detail in Section 3.5.

$\tau = 2$ profits is asymmetric, which breaks down the symmetric Hotelling equilibrium. Starting from the Hotelling price t and $z = \frac{1}{2}$, a small increase in z increases firm A's $\tau = 2$ profit more than it decreases firm B's $\tau = 2$ profit. Likewise, a small decrease in z decreases firm A's $\tau = 2$ profit less than it increases firm B's $\tau = 2$ profit. Thus when firm B chooses the Hotelling price t, firm A's best response is to undercut it to $\frac{5}{7}t$. But firm B does not gain by undercutting firm A: its best response is to lower its $\tau = 1$ price to $\frac{29}{35}t > \frac{5}{7}t$. It is followed by further price cuts by both firms, each time firm B's price remaining higher than firm A's. This leads to the equilibrium with $z = \frac{15}{26}$. The adjustment process can be understood with help of Figure 1 where $\hat{P}_A^1$ and $\tilde{P}_A^1$ ($\hat{P}_B^1$ and $\tilde{P}_B^1$, resp.) represent firm A's (firm B's, resp.) reaction function.

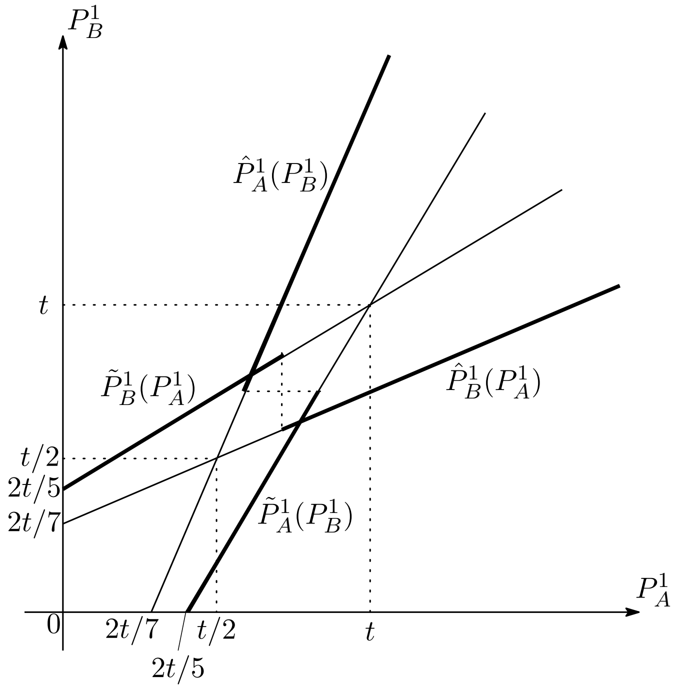

Figure 1: Equilibrium Prices in $\tau = 1$ Given Locations at (0, 1)

3.3 Equilibria and discussions

We are now ready to describe the equilibria for the whole game. By substituting the value of z from the $\tau = 1$ equilibrium back into the $\tau = 2$ prices, we have:

Proposition 1 *The equilibrium prices for the two periods are given by:*

(i) $P_A^1 = \dfrac{10t}{13}$ *and* $P_B^1 = \dfrac{8t}{13}$ *with* $z = \dfrac{11}{26}$,

$$P_A(x) = \begin{cases} (1-2x)t & \text{if } x \in [0, \frac{11}{26}], \\ \frac{1}{13}t & \text{if } x \in [\frac{11}{26}, 1], \end{cases}$$

$$P_B(y) = \begin{cases} 0 & \text{if } \ y \in [0, \dfrac{6}{13}], \\ \left(2y - \dfrac{12}{13}\right)t & \text{if } \ y \in [\dfrac{6}{13}, 1]. \end{cases}$$

(ii) $P_A^1 = \dfrac{8t}{13}$ *and* $P_B^1 = \dfrac{10t}{13}$ *with* $z = \dfrac{15}{26}$,

$$P_A(x) = \begin{cases} \left(\dfrac{14}{13} - 2x\right)t & \text{if } \ x \in [0, \dfrac{7}{13}], \\ 0 & \text{if } \ x \in [\dfrac{7}{13}, 1], \end{cases}$$

$$P_B(y) = \begin{cases} \dfrac{1}{13}t & \text{if } \ y \in [0, \dfrac{15}{26}], \\ (2y-1)t & \text{if } \ y \in [\dfrac{15}{26}, 1]. \end{cases}$$

A number of interesting observations emerge from the above proposition. We discuss them below focusing on the equilibrium where firm A has larger market share in $\tau = 1$, i.e., $z = \frac{15}{26}$.[15] Calculating equilibrium profits in this case, we have $\pi_A^1 = 0.355t, \pi_A^2 = 0.290t, \pi_B^1 = 0.325t$, and $\pi_B^2 = 0.247t$.

First, although firm A secures a larger market share by pricing below firm B in $\tau = 1$, its market share shrinks in $\tau = 2$ since its $\tau = 1$ customers in $[\frac{7}{13}, \frac{15}{26}]$ switch to firm B. However firm A is better off having switching customers since they help firm A to fend off firm B's aggressive pricing in $\tau = 2$ and use personalized pricing for the remaining customers that continue to purchase from firm A. Indeed firm A's most loyal customers, i.e., $x \in [0, \frac{1}{26})$, are charged price higher than t, the Hotelling price. But the maximum price firm B charges is t. Thus firm A has larger profit than firm B in both periods.

Second, the dynamic consideration and the accompanied personalized pricing in $\tau = 2$ intensify price competition in $\tau = 1$. As a result, both firms choose their $\tau = 1$ prices below t. Consequently their $\tau = 1$ profits are smaller than $\frac{t}{2}$, the Hotelling profit.

Third, the dynamic consideration also differentiates our $\tau = 2$ equilibrium from that in Thisse and Vives (1988). In the latter adapted to our setup, the unique equilibrium is symmetric where $z = \frac{1}{2}$ and each firm earns profit equal to $\frac{t}{4}$. In contrast, we have an asymmetric equilibrium in $\tau = 2$ with firm A's market share is larger than firm B's. In addition firm A's profit is larger than in Thisse and Vives (1988) while firm B's profit is smaller: $\pi_A^2 = 0.290t > \frac{1}{4}t > \pi_B^2 = 0.247t$.

Fourth, the $\tau = 2$ profits are smaller than the $\tau = 1$ profits for both firms. This is generally consistent with Thisse and Vives (1988) that the ability to price-discriminate harms profitability by intensifying competition, although in our setup firms use personalized pricing only for their repeat customers. With price discrimination, each firm can

[15] The explanations for the other case are the same with firm A replaced by firm B.

price aggressively in the other's turf while protecting its own turf through personalized pricing. With uniform pricing, firms cannot price too aggressively because the same price applies to all their customers.

Indeed one can verify that access to customer information creates the prisoner's dilemma for the two firms. When both firms can choose between personalized pricing and uniform pricing in $\tau = 2$, both are better off choosing uniform pricing but choosing personalized pricing is a dominant strategy. To see this, suppose both firms commit not to use cookies and play the standard Hotelling equilibrium, hence $z = \frac{1}{2}$. We consider firm A's deviation in $\tau = 2$ when firm B charges $P_B = t$. If firm A employs personalized pricing, then it sets personalized price $P_A(x)$ for its $\tau = 1$ customer $x \in [0, \frac{1}{2}]$ and a uniform price P_A for all customers $y \in [\frac{1}{2}, 1]$ who purchased from firm B in $\tau = 1$. For $x \in [0, \frac{1}{2}]$, firm A's optimal pricing requires $P_A(x) + x^2 t = t + (1-x)^2 t$, hence $P_A(x) = 2t(1-x)$. For $y \in [\frac{1}{2}, 1]$, let $\tilde{y}$ be the consumer who is indifferent between the two firms given P_A. Then $\tilde{y} = 1 - \frac{P_A}{2t}$ and firm A's profit from the segment $[\frac{1}{2}, \tilde{y}]$ is $(\tilde{y} - \frac{1}{2})P_A$. Maximizing this leads to $P_A = \frac{t}{2}$ and $\tilde{y} = \frac{3}{4}$. That is, firm A can poach additional customers $y \in [\frac{1}{2}, \frac{3}{4}]$ from firm B. Firm A's profit when it employs personalized pricing in $\tau = 2$ is $\pi_A^2 = \int_0^{\frac{1}{2}} 2t(1-x)dx + (\frac{3}{4} - \frac{1}{2})\frac{t}{2} = \frac{7t}{8} > \frac{t}{2}$. Thus the commitment not to use personalized pricing is not credible. Similarly, it is easy to show that firm A's best response when firm B employs personalized pricing is also to employ personalized pricing

So far, we have seen that both firms are strictly worse off in each period when they use personalized pricing than when they use only uniform price. But are consumers better off under price discrimination? To answer this, we first note that, in the Hotelling equilibrium, price is t, all consumers $x \leq \frac{1}{2}$ choose firm A in both periods at total cost $2(t + x^2 t)$, and all consumers $y > \frac{1}{2}$ choose firm B in both periods at total cost $2(t + (1-y)^2 t)$.

Let us consider again the equilibrium with $z = \frac{15}{26}$. We have shown previously that both firms charge price strictly below t in $\tau = 1$. In $\tau = 2$, only consumers in $[0, \frac{1}{26})$ pay price higher than t. For all other consumers, price is strictly lower than t in both periods. In addition, these consumers have an option to choose the firms in the Hotelling equilibrium and pay the same transportation costs. Thus all consumers in $[\frac{1}{26}, 1]$ are strictly better off under price discrimination. Consider now $x \in [0, \frac{1}{26})$, who chooses firm A in both periods, pays the $\tau = 1$ price $P_A^1 = \frac{8}{13}t < t$ and the $\tau = 2$ price $P_A(x) = \left(\frac{14}{13} - 2x\right)t > t$. But $P_A^1 + P_A(x) < \frac{22}{13}t < 2t$ for all $x \in [0, \frac{1}{26})$, hence total cost for x is $P_A^1 + P_A(x) + 2x^2 t < 2(t + x^2 t)$. Thus all consumers $[0, \frac{1}{26})$ are strictly better off under price discrimination.

But welfare is strictly lower in both periods than in Hotelling equilibrium since firms have asymmetric market shares in both periods and there is inefficient customer switching in $\tau = 2$. In our equilibrium with $z = \frac{15}{26}$, the average distance traveled is $\frac{173}{676} > \frac{1}{4}$ in $\tau = 1$ and $\frac{85}{338} > \frac{1}{4}$ in $\tau = 2$. On the other hand, welfare in $\tau = 2$ is higher than in Fudenberg and Tirole (2000) since there is only one-way switching in our case.[16] The following proposition summarizes the discussions so far.

Proposition 2 *In equilibrium where firms employ personalized pricing in $\tau = 2$, firms are worse off in each period, all consumers are strictly better off, but social welfare is lower compared to when firms use uniform price.*

3.4 Forward-looking consumers

In this section, we discuss the case where consumers are forward-looking and make purchase decisions in $\tau = 1$ based on the comparison of the total utilities over the two periods. Firms' problems in $\tau = 2$ do not change given the marginal consumer z. Thus the equilibria in $\tau = 2$ remain the same as before.

To solve for each firm's pricing decision in $\tau = 1$, we need to express each firm's $\tau = 1$ market share in terms of the $\tau = 1$ prices. Consider first the equilibrium with $z \leq \frac{1}{2}$. In this equilibrium, the marginal consumer z is indifferent between choosing firm A in both periods, and choosing firm B in $\tau = 1$ while switching to firm A in $\tau = 2$. Thus we have

$$P_A^1 + z^2 t + P_A(z) + z^2 t = P_B^1 + (1 - z)^2 t + P_A(\mathcal{B}) + z^2 t.$$

Substituting $P_A(z) = (1 - 2z)t$ and $P_A(\mathcal{B}) = \frac{(1-2z)t}{2}$ into the above, we obtain $z = \frac{t-2(P_A^1 - P_B^1)}{2t}$ and $z \leq \frac{1}{2}$ if and only if $P_A^1 \geq P_B^1$. Similarly, in the equilibrium with $z \geq \frac{1}{2}$, the marginal consumer z is indifferent between choosing firm B in both periods, and choosing firm A in $\tau = 1$ while switching to firm B in $\tau = 2$. Proceeding as before, we again obtain the same z. Repeating the analysis as in the previous sections, one can show that the $\tau = 1$ price game has two equilibria: (i) $P_A^1 = \frac{3}{14}t$ and $P_B^1 = \frac{1}{14}t$ with $z = \frac{5}{14}$; (ii) $P_A^1 = \frac{1}{14}t$ and $P_B^1 = \frac{3}{14}t$ with $z = \frac{9}{14}$.[17]

Compared to the case with myopic consumers, both firms charge lower prices in $\tau = 1$. For example, in equilibrium where firm B has a larger market share in $\tau = 1$, $P_A^1 = \frac{10}{13}t$, $P_B^1 = \frac{8}{13}t$ with myopic consumers, and $P_A^1 = \frac{3}{14}t$, $P_B^1 = \frac{1}{14}t$ with forward-looking consumers. Moreover the difference in the two firms' prices is also smaller when consumers are forward-looking. In this sense, competition in $\tau = 1$ becomes more intense when consumers are forward-looking. Such intense competition results in lower profit

[16] In $\tau = 1$, the average distance traveled is $\left(\frac{15}{26}\right)^2 \times \frac{1}{2} + \left(\frac{11}{26}\right)^2 \times \frac{1}{2} = \frac{173}{676}$. In $\tau = 2$, it is $\left(\frac{7}{13}\right)^2 \times \frac{1}{2} + \left(\frac{11}{26}\right)^2 \times \frac{1}{2} + \frac{23}{26} \times \frac{1}{26} \times \frac{1}{2} = \frac{85}{338}$. Both are smaller than $\frac{11}{36}$, the average distance traveled in $\tau = 2$ in Fudenberg and Tirole (2000).

[17] The detailed calculation is available from the authors.

than when consumers are myopic. Indeed it can be checked easily that both firms are strictly worse off when consumers are forward-looking. These results are in contrast to Fudenberg and Tirole (2000) where firms are better off when consumers are forward-looking. In the next section, we discuss how and why our results are different from those in Fudenberg and Tirole (2000).

3.5 Comparison with Fudenberg and Tirole (2000)

The current case with exogenously fixed locations is identical to the setup in Fudenberg and Tirole (2000), to be called FT henceforth, except the only difference that firms use personalized pricing in $\tau = 2$ in our model while they use third-degree price discrimination in FT. Thus we offer detailed discussions of why equilibrium changes drastically in our model when pricing strategies become more sophisticated.

First, let us recall that when the FT model is adapted to our case of uniform distribution and quadratic transportation costs, there is a unique symmetric equilibrium where, in $\tau = 1$, both firms charge $\frac{4}{3}t$ and firm A's market share is $[0, \frac{1}{2}]$. In $\tau = 2$, both firms charge $\frac{2}{3}t$ to their $\tau = 1$ customers and $\frac{1}{3}t$ to their rival's customers. Consumers in $[\frac{1}{3}, \frac{1}{2}]$ switch from firm A to firm B while those in $[\frac{1}{2}, \frac{2}{3}]$ switch from firm B to firm A. As a result, the two firms continue to have the same market share in $\tau = 2$. These results are based on FT's assumption of forward-looking consumers. If consumers are myopic, only the $\tau = 1$ price changes from $\frac{4}{3}t$ to t and the rest remains the same. Because prices in $\tau = 1$ are lower with myopic consumers, firms are better off when consumers are forward-looking.

In FT, one can verify that the equilibrium profit in $\tau = 2$ is the same for both firms *regardless* of firm A's market share $[0, z]$ in $\tau = 1$.[18] This is because customer switching is two-way and, as firm A's market share in $\tau = 1$ increases, more customers switch to firm B in $\tau = 2$. Specifically, the fraction of customers switching from firm A to firm B and the fraction of those switching from firm B to firm A are exactly the same if and only if $z = \frac{1}{2}$, and the former (latter) is larger if $z > (<)\frac{1}{2}$. Such two-way customer switching is due to the assumption that firms use third-degree price discrimination. Because each firm has to charge the same price to all its $\tau = 1$ customers, it cannot price too aggressively to protect its turf in $\tau = 2$. If a firm wants to continue to serve its marginal customer, it has to reduce price for its most loyal customers as well. Likewise, firms cannot price too aggressively to poach their rival's customers. The end result is that both firms poach some of their rival's customers.

In contrast, the ability to use personalized pricing in our model allows each firm to protect its turf better. Specifically, if $z \leq \frac{1}{2}$, then firm A can continue to serve

[18] This argument applies as long as $z \in [\frac{1}{4}, \frac{3}{4}]$, which is true in equilibrium.

all its $\tau = 1$ customers while poaching some customers from firm B. As discussed previously, however, such customer switching benefits firm B since it allows firm B to use personalized pricing for its remaining customers and extract larger surplus than when firm B has a smaller market share in $\tau = 1$. Thus firm B has a larger profit than firm A in each period even though it loses some of its customers to firm A in $\tau = 2$. Similarly if $z \geq \frac{1}{2}$, customer switching is only from firm A to firm B but firm A has a larger profit than firm B. The flip side of the ability to use personalized pricing is that firms choose more aggressive poaching offers than in FT. In both equilibria in our model, each firm's successful poaching offer is $\frac{1}{13}t$, lower than $\frac{1}{3}t$ as in FT.

Because of the one-way customer switching in our model, we have two asymmetric equilibria which feature tougher price competition in $\tau = 1$ compared to FT. In the latter, a larger market share in $\tau = 1$ does not lead to a larger profit in $\tau = 2$. In addition, the firm with a larger market share in $\tau = 1$ loses more customers to its rival in $\tau = 2$. Thus price competition in $\tau = 1$ is softer than in the static Hotelling model. As a result, both firms charge their $\tau = 1$ price above the Hotelling price t. In our model, a larger market share implies a larger profit in each period, which makes price competition in $\tau = 1$ tougher than in the static Hotelling model. As a result, both firms charge their $\tau = 1$ price below the Hotelling price.

As mentioned previously, firms in FT are better off when consumers are forward-looking than when they are myopic. This is because forward-looking consumers anticipate favorable poaching offers in $\tau = 2$, which makes them less sensitive to prices charged in $\tau = 1$. These poaching offers, equal to $\frac{1}{3}t$, are the same whether consumers are forward-looking or myopic because of symmetric two-way customer switching. Given that there is a unique symmetric equilibrium in FT, the only effect forward-looking customers have on the firm's behavior is thus to soften competition in $\tau = 1$. In our results, customer switching is one-way and there are two asymmetric equilibria, one favoring one firm over the other. This intensifies first-period competition relative to FT, and more so when consumers are forward-looking.

4 Endogenous Location Choice

We now turn to the full game where firms optimally choose their locations. The first period is the standard Hotelling game in which each firm chooses location and a uniform price. As before, the locations are fixed over two periods. In the second period, firms compete by using personalized prices whenever possible. An equilibrium consists of each firm's location and prices in the two periods. We focus on the case where consumers are myopic.[19] Denote firm A's location by a and firm B's location by b with $0 \leq a \leq b \leq 1$. Once again it is easy to see that each firm's $\tau = 1$ market segment is a connected set. Without loss of generality, we thus denote firm A's $\tau = 1$ market segment by $\mathcal{A} = [0, z]$ with $a \leq z \leq b$.[20]

The game can be solved backwards in three steps. First, given (a, b, z), we find the equilibrium of the $\tau = 2$ pricing game and denote corresponding profits by $\pi_i^2(a, b, z)$, $i = A, B$. Second, given (a, b), we solve for the $\tau = 1$ prices P_i^1, $i = A, B$, such that P_A^1 maximizes firm A's total profit over the two periods

$$\Pi_A(P_A^1, P_B^1) = P_A^1 z(P_A^1, P_B^1) + \pi_A^2 \left[a, b, z(P_A^1, P_B^1)\right]$$

and P_B^1 maximizes

$$\Pi_B(P_A^1, P_B^1) = P_B^1 \left[1 - z(P_A^1, P_B^1)\right] + \pi_B^2 \left[a, b, z(P_A^1, P_B^1)\right].$$

This gives us first-period prices $P_A^1(a, b)$, $P_B^1(a, b)$ and firm A's market share $z(a, b)$. Third, we solve for the equilibrium location choice where firm A chooses a to maximize $\Pi_A(a, b) = P_A^1(a, b) z(a, b) + \pi_A^2 [a, b, z(a, b)]$ and firm B chooses b to maximize $\Pi_B(a, b) = P_B^1(a, b) [1 - z(a, b)] + \pi_B^2 [a, b, z(a, b)]$.

4.1 Second period

Analogous to the previous case with maximal differentiation, we divide analysis into two cases depending on where consumers lie relative to the midpoint between the two firms: $z \leq \frac{a+b}{2}$ and $z \geq \frac{a+b}{2}$. Following the argument used previously, we can show the following.

Lemma 4 *Suppose $z \leq \frac{a+b}{2}$. Then the unique equilibrium in $\tau = 2$ is given by*

$$P_A(x) = \begin{cases} (a + b - 2x)(b - a)t & \text{if } x \in [0, z], \\ \dfrac{(a + b - 2z)(b - a)t}{2} & \text{if } x \in [z, 1], \end{cases}$$

$$P_B(y) = \begin{cases} 0 & \text{if } y \in [0, \frac{a + b + 2z}{4}], \\ \dfrac{(4y - 2z - a - b)(b - a)t}{2} & \text{if } y \in [\frac{a + b + 2z}{4}, 1]. \end{cases}$$

[19] With forward-looking consumers, the qualitative results are similar in that maximal differentiation never appears in equilibrium. The details are available from the authors.

[20] To see that $a \leq z \leq b$ holds in equilibrium, we note that firm A's (firm B's, resp.) $\tau = 2$ profit increases (decreases, resp.) in z. In addition if $z < a$, then firm A can increase its $\tau = 1$ profit by lowering its $\tau = 1$ price, hence increasing z. Similarly if $z > b$, then firm B can increase its $\tau = 1$ profit by lowering its $\tau = 1$ price, thereby decreasing z.

The corresponding profits are

$$\pi_A^2 = \frac{t}{8}(b-a)\left[(a+b)^2 + 4(a+b)z - 4z^2\right],$$
$$\pi_B^2 = \frac{t}{16}(b-a)(4-a-b-2z)^2.$$

Proof: See the appendix.

Lemma 5 *Suppose $z \geq \frac{a+b}{2}$. Then the unique equilibrium in $\tau = 2$ is given by*

$$P_A(x) = \begin{cases} \dfrac{(2z-4x+a+b)(b-a)t}{2} & \text{if } x \in [0, \dfrac{a+b+2z}{4}], \\ 0 & \text{if } x \in [\dfrac{a+b+2z}{4}, 1], \end{cases}$$
$$P_B(y) = \begin{cases} \dfrac{(2z-a-b)(b-a)t}{2} & \text{if } y \in [0, z], \\ (2y-a-b)(b-a)t & \text{if } y \in [z, 1]. \end{cases}$$

The corresponding profits are

$$\pi_A^2 = \frac{t}{16}(b-a)(a+b+2z)^2,$$
$$\pi_B^2 = \frac{t}{8}(b-a)\left[8(1-a-b) + (a+b)^2 + 4(a+b)z - 4z^2\right].$$

Proof: See the appendix.

As before, one can verify that both firms' $\tau = 2$ profit functions are continuous in z, π_A^2 increases in z, and π_B^2 decreases in z. In equilibrium with $z \leq \frac{a+b}{2}$, all consumers in $[0, z]$ choose firm A in both periods, those in $[z, \frac{a+b+2z}{4}]$ choose firm B in $\tau = 1$ but switch to firm A in $\tau = 2$, and the rest choose firm B in both periods. In the other equilibrium, some consumers switch from firm A to firm B.

4.2 First period: Price

Next we solve for the equilibrium prices in $\tau = 1$ given locations fixed at a and b. Given prices P_A^1 and P_B^1, marginal consumer z satisfies $P_A^1 + (z-a)^2 t = P_B^1 + (z-b)^2 t$, hence $z = \frac{a+b}{2} - \frac{P_A^1 - P_B^1}{2(b-a)t}$. Firm A chooses P_A^1 to maximize $\Pi_A(P_A^1, P_B^1) = P_A^1 z(P_A^1, P_B^1) + \pi_A^2\left[a, b, z(P_A^1, P_B^1)\right]$ and firm B chooses P_B^1 to maximize $\Pi_B(P_A^1, P_B^1) = P_B^1\left[1 - z(P_A^1, P_B^1)\right] + \pi_B^2\left[a, b, z(P_A^1, P_B^1)\right]$. Proceeding similarly as before, we can show the following.

Lemma 6 *Given fixed locations a and b with $a \leq b$, the price game in $\tau = 1$ has two equilibria:*

(i) $P_A^1 = \dfrac{2(3+2a+2b)(b-a)t}{13}$ *and* $P_B^1 = \dfrac{2(5-a-b)(b-a)t}{13}$ *with* $z = \dfrac{4+7a+7b}{26}$*;*

(ii) $P_A^1 = \dfrac{2(3+a+b)(b-a)t}{13}$ *and* $P_B^1 = \dfrac{2(7-2a-2b)(b-a)t}{13}$ *with* $z = \dfrac{8+7a+7b}{26}$.

1. If $a+b > \frac{84}{13\sqrt{70}-28} \simeq 1.04$*, then only the first equilibrium exists. 2. If* $\frac{84}{13\sqrt{70}-28} \geq a+b \geq \frac{2(13\sqrt{70}-70)}{13\sqrt{70}-28} \simeq 0.96$*, then both equilibria exist. 3. If* $\frac{2(13\sqrt{70}-70)}{13\sqrt{70}-28} > a+b$*, then only the second equilibrium exists.*

Proof: See the appendix.

4.3 First period: Location

Let us now turn to the equilibrium location choice. Lemma 6 shows that the $\tau = 1$ pricing game has different equilibria depending on the range of $a + b$. In addition there are multiple equilibria in the intermediate range of $a+b$. Thus each firm's location choice depends on which of these equilibria each firm anticipates in the subgame following its location choice. The equilibrium location choice in turn should be consistent with the anticipated equilibrium of the pricing subgame.

Lemma 7 *The location game in* $\tau = 1$ *has two equilibria:*

(i) $a = \dfrac{2\sqrt{56029}-347}{621} \simeq 0.2$ *and* $b = 1$*, which is followed by the equilibrium of the pricing subgame where* $z = \dfrac{4+7a+7b}{26}$*;*

(ii) $a = 0$ *and* $b = \dfrac{968-2\sqrt{56029}}{621} \simeq 0.8$*, which is followed by the equilibrium of the pricing subgame where* $z = \dfrac{8+7a+7b}{26}$.

Proof: See the appendix.

It is worth noting that equilibrium product choice does not result in maximal differentiation, i.e., $a = 0, b = 1$, in either of the two equilibria. This is due to the presence of the second period when firms can employ personalized pricing. We offer detailed discussions in the following section where, for clarity of exposition, we round equilibrium locations to the first decimal point, i.e., $a = 0.2$ in the first equilibrium and $b = 0.8$ in the second. This simplification does not change our qualitative results and discussions in any meaningful way.

4.4 Equilibria and discussions

Collecting the results from Lemmas 4-7, we have

Proposition 3 *With endogenous location choice, there are two equilibria given by:*

(i) $a = 0.2,\ b = 1;\ P_A^1 = \frac{216t}{325},\ P_B^1 = \frac{152t}{325}$ *with* $z = \frac{31}{65}$;

$$P_A(x) = \begin{cases} \frac{8(3-5x)t}{25} & \text{if } x \in [0, \frac{31}{65}], \\ \frac{32}{325}t & \text{if } x \in [\frac{31}{65}, 1], \end{cases}$$

$$P_B(y) = \begin{cases} 0 & \text{if } y \in [0, \frac{7}{13}], \\ \frac{8(13y-7)t}{65} & \text{if } y \in [\frac{7}{13}, 1]. \end{cases}$$

(ii) $a = 0,\ b = 0.8;\ P_A^1 = \frac{152t}{325}t,\ P_B^1 = \frac{216t}{325}$ *with* $z = \frac{34}{65}$;

$$P_A(x) = \begin{cases} \frac{8(7-13x)t}{65} & \text{if } x \in [0, \frac{7}{13}], \\ 0 & \text{if } x \in [\frac{7}{13}, 1], \end{cases}$$

$$P_B(y) = \begin{cases} \frac{32t}{325} & \text{if } y \in [0, \frac{34}{65}], \\ \frac{8(5y-2)t}{25} & \text{if } y \in [\frac{34}{65}, 1]. \end{cases}$$

In Section 3 where locations were exogenously fixed at $a = 0, b = 1$, we found that there are two equilibria in the $\tau = 1$ pricing game. When locations are chosen endogenously, the above proposition shows that there are two equilibria in the $\tau = 1$ location game, each associated with a unique equilibrium in the ensuing pricing subgame. Thus the multiplicity of equilibrium prices changes to the multiplicity of equilibrium locations when location choice is endogenous.

It is worth noting that neither of these equilibria features maximal differentiation. The reasoning is as follows. While the maximal differentiation can soften the first-period price competition, it reduces the firm's ability to extract surplus from its most loyal customers in $\tau = 2$. To see this, suppose firm B chooses $b = 1$. We will show that $a = 0$ cannot be firm A's best response. Suppose to the contrary that firm A chooses $a = 0$. Then there are two possibilities in the subsequent pricing games based on the equilibria found in Lemma 6 (also Proposition 1). First, firm A can price less aggressively and have a smaller market share in $\tau = 1$, i.e., $z = \frac{11}{26}$ (Lemma 6(i)). In this case, firm A's profits are $\pi_A^1 = 0.325t, \pi_A^2 = 0.247t$. In contrast, by choosing $a = 0.2$, firm A's profits change to $\pi_A^1 = 0.317t, \pi_A^2 = 0.282t$. Firm A's $\tau = 1$ profit decreases since it is now closer to firm B. But its $\tau = 2$ profit increases more due to its ability to exercise personalized pricing more effectively thanks to its position closer to the center. As a result, firm A's total profit is larger when $a = 0.2$. Thus $a = 0$ cannot be firm A's best response to $b = 1$. Second, firm A can price more aggressively with a view to securing a larger market share, $z = \frac{15}{26}$ (Lemma 6(ii)). But in this case, firm B can deviate by choosing $b = 0.8$ and increase its profits. This shows that $a = 0,\ b = 1$ cannot be equilibrium location choice.

Comparing the above equilibria with those from the previous section leads to several observations. We discuss them below based on the equilibrium with $a = 0.2, b = 1$ and $z = \frac{31}{65}$.[21] In this equilibrium, firm A's $\tau = 1$ market share is $[0, z]$ but its $\tau = 2$ market share increases to $[0, \tilde{y}]$ where $\tilde{y} = \frac{7}{13}$. In $\tau = 2$, firm A continues to serve all its $\tau = 1$ customers with personalized price $P_A(x)$ that decreases from $\frac{24}{25}t$ to $\frac{64}{325}t$ on $x \in [0, z]$, and poaches firm B's $\tau = 1$ customers in $[z, \tilde{y}]$ with a uniform price $P_A(\mathcal{B}) = \frac{32}{325}t$. In $\tau = 2$, firm B charges a uniform price 0 to all firm A's $\tau = 1$ customers as well as its $\tau = 1$ customers in $[z, \tilde{y}]$. But the latter switch to firm A. For its remaining $\tau = 1$ customers, firm B uses personalized price $P_B(y)$ that increases from 0 to $\frac{48}{65}t$ on $y \in [\tilde{y}, 1]$. Calculating equilibrium profits in this case, we have $\pi_A^1 = 0.317t, \pi_A^2 = 0.282t, \pi_B^1 = 0.245t, \pi_B^2 = 0.170t$. Thus firm A has larger profit than firm B in both periods. Figure 2 shows each firm's pricing strategies in $\tau = 2$ and how market shares change over time in this equilibrium.

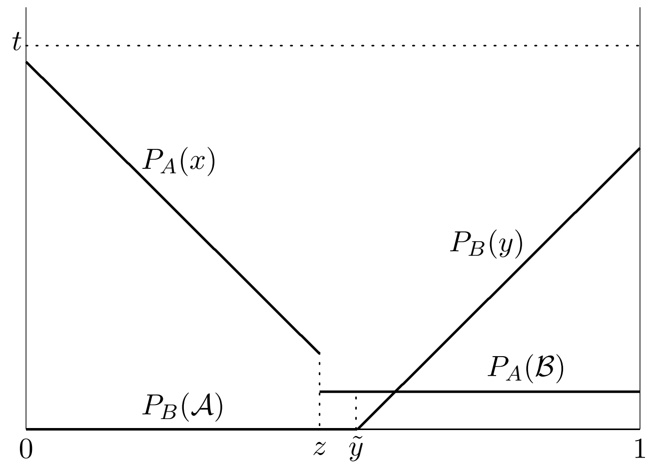

Figure 2: Equilibrium in $\tau = 2$ with Endogenous Location Choice ($a = 0.2$)

First, firm A has a smaller market share in $\tau = 1$ although its location is closer to the center compared to firm B. To see why, note that firm A's $\tau = 1$ price needs to be the same for all its customers but those to the left of firm A are in firm A's backyard and are much more locked in to firm A than to firm B. Firm A can extract larger surplus

[21]For the other case, we can simply swap firm A and firm B and the same explanations apply.

from these customers by charging higher than firm B and, as a result, has a smaller market share. In $\tau = 2$, however, firm A can leverage its location to poach firm B's customers and secure a larger market share than firm B. In this equilibrium, firm A has larger profit than firm B in both periods, which is due to its ability to choose more aggressive positioning than firm B. In the other equilibrium with $a = 0, b = 0.8$, exactly the opposite is the case where firm B benefits by choosing more aggressive positioning. This is in contrast to the case where locations were fixed at 0 and 1: in that case, the firm with a larger market share in $\tau = 1$ continues to have a larger market share in $\tau = 2$ and obtains larger profit in both periods although its market share decreases in $\tau = 2$ due to customer switching.

Second, the $\tau = 1$ prices are lower than the Hotelling price, t, once again confirming the intuition that the competition in personalized pricing in $\tau = 2$ intensifies competition in $\tau = 1$. Compared to the case where locations are fixed at 0 and 1, one firm charges a higher price while the other charges a lower price. Of course we need to be more precise in the comparison since there are two equilibria given fixed locations. Since the above equilibrium is the one that favors firm A, a meaningful comparison would be with the equilibrium given fixed locations that also favors firm A, i.e., the second equilibrium in Proposition 1. In the latter, the first-period prices are $P_A^1 = \frac{8}{13}t < \frac{216}{325}t$ and $P_B^1 = \frac{10}{13}t > \frac{152}{325}t$. That firm A chooses a higher $\tau = 1$ price when its location is at 0.2 is again due to the existence of customers in its backyard.

Third, profits are smaller for both firms in each period when location choice is endogenous. Comparing the same pair of equilibria as before, firm A's profit changes from $0.355t$ to $0.317t$ in $\tau = 1$ and from $0.290t$ to $0.282t$ in $\tau = 2$. Firm B's profit changes from $0.325t$ to $0.245t$ in $\tau = 1$ and from $0.247t$ to $0.170t$ in $\tau = 2$. The decrease in profits is primarily due to the fact that, given endogenous location choice, the two products are less than maximally differentiated, which intensifies competition. It is easy to verify that there is more customer switching with endogenous location choice. In the equilibrium with $a = 0.2, b = 1$, the fraction of customers who switch from firm B to firm A is $\frac{7}{13} - \frac{31}{65} \approx 0.061$. In the equilibrium with maximal differentiation that favors firm A, the fraction of customers who switch from firm A to firm B is $\frac{15}{26} - \frac{7}{13} \approx 0.038$. In sum, firms are worse off when they choose locations than when locations are fixed exogenously at 0 and 1.

Finally, compared to the case with maximal differentiation, some consumers are better off and some worse off when firms choose locations optimally. For example, consider the equilibrium with $a = 0.2, b = 1$. It is easy to verify that consumer $x = 0$ is worse off than when locations are fixed at 0 and 1. It is because this consumer is in the deepest territory of firm A when its location is at 0.2, which allows firm A to extract more surplus by charging higher $\tau = 1$ price than when its location is at 0. In addition consumer $x = 0$ incurs positive transportation cost in both periods when firm A's location is at 0.2. On the other hand, consumer $y = 1$ is better off in equilibrium with $a = 0.2, b = 1$ than when locations are fixed at 0 and 1. But welfare is higher with endogenous location choice because the average distance traveled by a consumer is smaller than when locations are fixed at 0 and 1. Specifically, in equilibrium with $a = 0.2, b = 1$, the average distance traveled by a consumer is around 0.195 in $\tau = 1$ and 0.184 in $\tau = 2$.[22] When locations are fixed at 0 and 1, the minimum average distance traveled by a consumer is 0.25.

4.5 Endogenous location choice in Fudenberg and Tirole (2000)

The analysis in the previous section leads to two main findings. First, firms are worse off when they make product choice optimally than when it is exogenously fixed with maximal differentiation. Second, when product choice is endogenously made, maximal differentiation is never an equilibrium outcome. In this section, we will argue that both of these results are driven by personalized pricing firms use in the second period. To this end, we revisit Fudenberg and Tirole (2000) where firms use third degree price discrimination in the second period, consumers are forward-looking, and firms' locations are fixed at 0 and 1. We relax the third assumption and allow firms to choose locations optimally. The following proposition shows that the equilibrium outcome in Fudenberg and Tirole (2000) remains the same even when location choice is endogenous.

Proposition 4 *Suppose firms choose location optimally in the first period, use third degree price discrimination in the second period, and consumers are forward-looking. Then there is a unique equilibrium where location choice is 0 and 1, and the resulting prices and the distribution of market share in each period are exactly the same as in Fudenberg and Tirole (2000).*

Proof: See the appendix.

In a sense, the above proposition shows that it is without loss of generality to assume maximal differentiation when firms compete in third degree price discrimination, hence providing a theoretical justification to Fudenberg and Tirole (2000) in fixing locations exogenously at 0 and 1. Nevertheless it is somewhat surprising that the results change dramatically when firms compete in personalized pricing. The intuition is that, compared to third degree price discrimination, personalized pricing allows each firm to protect its

[22] In equilibrium with $a = 0.2, b = 1$, consumers in $[0, 0.477]$ purchase from firm A in $\tau = 1$. Thus the average distance traveled is $\frac{0.2^2}{2} + \frac{(0.477-0.2)^2}{2} + \frac{(1-0.477)^2}{2} \approx 0.195$. In $\tau = 2$, consumers in $[0, 0.538]$ purchase from firm A. Thus the average distance traveled is $\frac{0.2^2}{2} + \frac{(0.538-0.2)^2}{2} + \frac{(1-0.538)^2}{2} \approx 0.184$.

turf better, which allows it to price more aggressively in its rival's turf. The end result is more intense competition in both pricing and product choice.

5 Conclusion

This paper has studied a two-period model of differentiated duopoly where firms compete à la Hotelling in the first period, and compete using personalized pricing for their repeat customers in the second period. A key departure in our paper from most of the existing literature is that information is asymmetric and personalized. If a firm sells to a customer in the first period, it can gather highly specific information about that customer, which allows the firm to use personalized pricing for that customer. But if a customer buys from a firm's rival in the first period, the firm has no further information about that customer. Thus the firm charges a uniform price for all its new customers.

This information structure mimics actual information gathering by many businesses. Firms know a lot about their own customers, whether through loyalty programs, IT mechanisms such as 'cookies', or other intelligence gathering such as through shopping apps. However, this is proprietary information. They do not share it with competitive rivals and, as a result, firms know much more about their own customers than about consumers who frequent their competitive rivals.

As this paper shows, asymmetric information significantly alters the competitive outcomes. In contrast to the existing literature, gathering consumer information results in asymmetric equilibria in pricing and (when endogenous) in product choice. We summarize our main findings below.

First, when product choice is exogenously fixed with maximal differentiation, there are two asymmetric equilibria. A firm with more aggressive pricing in the first period secures a larger market share in the first period and continues to have a larger market share in the second period although some of its customers switch to the rival. The dynamic consideration intensifies the first period competition in that both firms charge first period prices lower than the Hotelling equilibrium price. As a result, firms are worse off and consumers are better off compared to the static Hotelling equilibrium. Unlike Fudenberg and Tirole (2000) where firms use third degree price discrimination and customer switching is two-way, personalized pricing in the second period results in one-way customer switching in our model.

Second, when firms also make product choice optimally, there continue to exist two asymmetric equilibria, none of which features maximal differentiation. With endogenous product choice, firms now use product choice as the main tool for aggressive posturing in the first period: a firm with product choice closer to the center secures larger profit in both periods; although it has a smaller market share in the first period, it uses its advantageous location to poach customers from its rival and secures a larger market share in the second period. But endogenous product choice intensifies competition further and leaves firms worse off than when product choice is fixed with maximal differentiation.

By highlighting the importance of the asymmetric structure of customer information for competing firms, our paper significantly advances the previous literature. However, there remain important limitations. One such limitation is that consumers in our model cannot act strategically, for example, by delaying purchasing, in order to interfere with the information acquisition by firms. Villas-Boas (2004) and Acquisti and Varian (2005) have both shown how strategic behavior by consumers can undermine personalized pricing. Introducing strategic customer behavior into our model remains as further research.

Similarly, as with most of the economics literature in this area, we have avoided issues of privacy or 'ownership' of information. These issues, however, tend to dominate public debate. Further, they have real economic consequences, if consumers eschew buying from a business that engages in personalized pricing. For example, in a well publicized incident in 2000, Internet retailer Amazon apologized for experimenting with personalized pricing due to customer backlash.[23] These types of behavioral responses by consumers to personalized pricing, while important, remain areas for future theoretical research.

Appendix

Proof of Lemma 1: Suppose $z \leq 1/2$. First, consider the segment $\mathcal{A} = [0, z]$. Since $z \leq 1/2$, firm A has a location advantage over firm B on this segment. Moreover firm A can use personalized prices $P_A(x)$ while firm B can use only a uniform price $P_B(\mathcal{A})$. Consumer x chooses firm A if $P_A(x) + x^2 t \leq P_B(\mathcal{A}) + (1-x)^2 t$ or $P_A(x) \leq P_B(\mathcal{A}) + (1-2x)t$. Thus the Bertrand competition on this segment leads to $P_B(\mathcal{A}) = 0$ and $P_A(x) = (1-2x)t$. Note that $P_A(x) \geq 0$ since $x \leq z \leq 1/2$ on $\mathcal{A}$.

Next, consider the segment $\mathcal{B} = [z, 1]$ for which firm A chooses a uniform price $P_A(\mathcal{B})$ while firm B uses personalized prices $P_B(y)$. Consumer y will choose firm B so long as $P_A(\mathcal{B}) + y^2 t > P_B(y) + (1-y)^2 t$ or $P_B(y) < P_A(\mathcal{B}) + (2y-1)t$. However, firm B will not want to sell to consumer y if $P_B(y) < 0$. Thus for any $P_A(\mathcal{B})$ we can define a critical value of y, denoted by $\tilde{y}$ such that $P_A(\mathcal{B}) = (1-2\tilde{y})t$ or $\tilde{y} = \frac{t - P_A(\mathcal{B})}{2t}$. Then for any price $P_A(\mathcal{B})$, consumer $y \in [z, \tilde{y}]$ chooses firm A (if $\tilde{y} > z$). On the other hand, firm B can choose nonnegative prices to serve all consumers $y \in [\tilde{y}, 1]$.

For the segment $[z, \tilde{y}]$, profit for firm A is $\int_z^{\tilde{y}} P_A(\mathcal{B}) dy = (\tilde{y} - z) P_A(\mathcal{B})$. Substituting for $\tilde{y}$ and maximizing, the optimal value of $P_A(\mathcal{B})$ for firm A is $P_A(\mathcal{B}) = \frac{(1-2z)t}{2}$, which is

[23] See "Amazon's prime suspect", *The New York Times Magazine* (August 6, 2010).

the price charged to all the consumers in $\mathcal{B}$ since firm A cannot price-discriminate these consumers. Given $P_A(\mathcal{B}) = \frac{(1-2z)t}{2}$, we have $\tilde{y} = \frac{1+2z}{4}$. It is easy to verify $z \leq \tilde{y} \leq 1/2$.

Given $P_A(\mathcal{B})$ derived above, firm B sets personalized prices for the segment $[\tilde{y}, 1]$. They are given by $P_B(y) = (2y-1)t + P_A(\mathcal{B}) = \frac{(4y-2z-1)t}{4}$. Thus the relevant prices constitute an equilibrium.

For the case $z > 1/2$, the same argument can be applied. $\square$

Proof of Lemma 3: Firm A's profit function is

$$\Pi_A = \begin{cases} P_A^1 \dfrac{P_B^1 - P_A^1 + t}{2t} + \dfrac{2t^2 - (P_B^1 - P_A^1)^2}{8t} & \text{if } P_A^1 \leq P_B^1, \\ P_A^1 \dfrac{P_B^1 - P_A^1 + t}{2t} + \dfrac{(2t + P_B^1 - P_A^1)^2}{16t} & \text{if } P_A^1 \geq P_B^1. \end{cases}$$

The first-order conditions for profit maximization are then

$$\frac{\partial \Pi_A}{\partial P_A^1} = \begin{cases} \dfrac{3P_B^1 - 5P_A^1 + 2t}{4t} & \text{if } P_A^1 \leq P_B^1, \\ \dfrac{3P_B^1 - 7P_A^1 + 2t}{8t} & \text{if } P_A^1 \geq P_B^1. \end{cases}$$

From the first-order conditions, we have the following local optimal prices of firm A and the respective profit:

$$P_A^1(P_B^1) = \begin{cases} \tilde{P}_A^1(P_B^1) \equiv \dfrac{3P_B^1 + 2t}{5} & \text{if } P_B^1 \leq t, \\ \hat{P}_A^1(P_B^1) \equiv \dfrac{3P_B^1 + 2t}{7} & \text{if } P_B^1 \geq t/2. \end{cases}$$

$$\Pi_A(P_B^1) = \begin{cases} \dfrac{2(P_B^1)^2 + 6t(P_B^1) + 7t^2}{20t} & \text{when } P_A^1(P_B^1) = \tilde{P}_A^1(P_B^1), \\ \dfrac{2(P_B^1)^2 + 5t(P_B^1) + 4t^2}{14t} & \text{when } P_A^1(P_B^1) = \hat{P}_A^1(P_B^1). \end{cases}$$

For $P_B^1 \in [t/2, t]$, firm A has two local optimal prices, $\tilde{P}_A^1(P_B^1)$ and $\hat{P}_A^1(P_B^1)$. Comparing firm A's profits for each case, we have

$$\frac{2(P_B^1)^2 + 6t(P_B^1) + 7t^2}{20t} \geq \frac{2(P_B^1)^2 + 5t(P_B^1) + 4t^2}{14t} \quad \Leftrightarrow \quad P_B^1 \leq \frac{(\sqrt{70} - 4)t}{6} \simeq 0.7278t.$$

Thus firm A's reaction function is.

$$P_A^1(P_B^1) = \begin{cases} \tilde{P}_A^1(P_B^1) = \dfrac{3P_B^1 + 2t}{5} & \text{if } P_B^1 \leq \dfrac{(\sqrt{70} - 4)t}{6}, \\ \hat{P}_A^1(P_B^1) = \dfrac{3P_B^1 + 2t}{7} & \text{if } P_B^1 \geq \dfrac{(\sqrt{70} - 4)t}{6}. \end{cases}$$

Applying the same argument to firm B's best response problem, we can derive its reaction function as:

$$P_B^1(P_A^1) = \begin{cases} \tilde{P}_B^1(P_A^1) \equiv \dfrac{3P_A^1 + 2t}{5} & \text{if } P_A^1 \leq \dfrac{(\sqrt{70}-4)t}{6}, \\ \hat{P}_B^1(P_A^1) \equiv \dfrac{3P_A^1 + 2t}{7} & \text{if } P_A^1 \geq \dfrac{(\sqrt{70}-4)t}{6}. \end{cases}$$

Solving these reaction functions simultaneously, we can derive the equilibrium prices given in Lemma 3. □

Proof of Lemma 4: First, $x \in [0, z]$ chooses firm A if $P_A(x) + (x-a)^2 t \leq P_B(\mathcal{A}) + (x-b)^2 t$ or $P_A(x) \leq P_B(\mathcal{A}) + (a+b-2x)(b-a)t$. Noting that $a+b \geq 2x$, the Bertrand competition on this segment leads to $P_A(x) = (a+b-2x)(b-a)t$ and $P_B(\mathcal{A}) = 0$. Second, since $z \leq \frac{a+b}{2}$, firm A can serve additional customers on the segment $[z, \tilde{y}]$ where $\tilde{y}$ satisfies $P_A(\mathcal{B}) + (2\tilde{y} - a - b)(b-a)t = 0$ or $\tilde{y} = \frac{a+b}{2} - \frac{P_A(\mathcal{B})}{2(b-a)t}$. Firm A chooses $P_A(\mathcal{B})$ to maximize profit from this segment given by $(\tilde{y} - z)P_A(\mathcal{B})$. This leads to $P_A(\mathcal{B}) = \frac{(a+b-2z)(b-a)t}{2}$. Substituting $P_A(\mathcal{B})$ back into $\tilde{y}$, we obtain $\tilde{y} = \frac{a+b+2z}{4}$. On $[z, \tilde{y}]$, firm B's best response to $P_A(\mathcal{B})$ is $P_B(y) = 0$. Finally $y \in [\tilde{y}, 1]$ chooses firm B if $P_B(y) + (y-b)^2 t \leq P_A(\mathcal{B}) + (y-a)^2 t$. Thus firm B's optimal pricing on this segment is $P_B(y) = P_A(\mathcal{B}) + (2y - a - b)(b-a)t = \frac{(4y-2x-a-b)(b-a)t}{2}$.

Firm A's second-period profit in this equilibrium is $\pi_A^2 = \int_0^z (a+b-2x)(b-a)t dx + (\tilde{y} - z)P_A(\mathcal{B})$. Firm B's profit is $\int_{\tilde{y}}^1 \frac{(4y-2z-a-b)(b-a)t}{2} dy$. Straightforward calculation leads to the desired results. □

Proof of Lemma 5: The same argument used in Lemma 4 applies. □

Proof of Lemma 6: Substituting $z = ((b^2 - a^2)t + P_B^1 - P_A^1)/(2(b-a)t)$ into firm A's profit function, we have

$$\Pi_A = \begin{cases} P_A^1 \dfrac{P_B^1 - P_A^1 + (b^2 - a^2)t}{2(b-a)t} + \dfrac{(b-a)t((b+a)(a+b+4z) - 4z^2)}{8} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } P_A^1 \leq P_B^1, \\ P_A^1 \dfrac{P_B^1 - P_A^1 + (b^2 - a^2)t}{2(b-a)t} + \dfrac{(b-a)t(a+b+2z)^2}{16} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{if } P_A^1 \geq P_B^1. \end{cases}$$

The first-order conditions for profit maximization are

$$\frac{\partial \Pi_A}{\partial P_A^1} = \begin{cases} \dfrac{3P_B^1 - 7P_A^1 + 2(b^2 - a^2)t}{8(b-a)t} & \text{if } P_A^1 \leq P_B^1, \\ \dfrac{3P_B^1 - 5P_A^1 + 2(b^2 - a^2)t}{4(b-a)t} & \text{if } P_A^1 \geq P_B^1. \end{cases}$$

From the above first-order conditions, we have the following local optimal prices and respective profit of firm A:

$$P_A^1(P_B^1) = \begin{cases} \tilde{P}_A^1(P_B^1) \equiv \dfrac{3P_B^1 + 2(b^2-a^2)t}{5} & \text{if } P_B^1 \le (b-a)(b+a)t, \\ \hat{P}_A^1(P_B^1) \equiv \dfrac{3P_B^1 + 2(b^2-a^2)t}{7} & \text{if } P_B^1 \ge (b-a)(b+a)t/2. \end{cases}$$

$$\Pi_A(P_B^1) = \begin{cases} \dfrac{2(P_B^1)^2 + 6(b^2-a^2)tP_B^1 + 7(b^2-a^2)^2t^2}{20(b-a)t} & \text{when } P_A^1(P_B^1) = \tilde{P}_A^1(P_B^1), \\ \dfrac{2(P_B^1)^2 + 5(b^2-a^2)tP_B^1 + 4(b^2-a^2)^2t^2}{14(b-a)t} & \text{when } P_A^1(P_B^1) = \hat{P}_A^1(P_B^1). \end{cases}$$

For $P_B^1 \in [(b-a)(b+a)t/2, (b-a)(b+a)t]$, firm A has two local optimal prices, $\tilde{P}_A^1(P_B^1)$ and $\hat{P}_A^1(P_B^1)$. Comparing the profits from each case, we have

$$\frac{2(P_B^1)^2 + 6(b^2-a^2)tP_B^1 + 7(b^2-a^2)^2t^2}{20(b-a)t} \ge \frac{2(P_B^1)^2 + 5(b^2-a^2)tP_B^1 + 4(b^2-a^2)^2t^2}{14(b-a)t}$$
$$\Leftrightarrow P_B^1 \le \frac{(\sqrt{70}-4)(b^2-a^2)t}{6}.$$

Thus firm A's reaction function is given by

$$P_A^1(P_B^1) = \begin{cases} \tilde{P}_A^1(P_B^1) = \dfrac{3P_B^1 + 2(b^2-a^2)t}{5} & \text{if } P_B^1 \le \dfrac{(\sqrt{70}-4)(b^2-a^2)t}{6}, \\ \hat{P}_A^1(P_B^1) = \dfrac{3P_B^1 + 2(b^2-a^2)t}{7} & \text{if } P_B^1 \ge \dfrac{(\sqrt{70}-4)(b^2-a^2)t}{6}. \end{cases}$$

Applying the same argument to firm B's optimization problem, we can derive firm B's reaction function:

$$P_B^1(P_A^1) = \begin{cases} \tilde{P}_B^1(P_A^1) \equiv \dfrac{3P_A^1 + 2(b-a)(2-(a+b))t}{5} \\ \qquad \text{if } P_A^1 \le \dfrac{(\sqrt{70}-4)(b-a)(2-(a+b))t}{6}, \\ \hat{P}_B^1(P_A^1) \equiv \dfrac{3P_A^1 + 2(b-a)(2-(a+b))t}{7} \\ \qquad \text{if } P_A^1 \ge \dfrac{(\sqrt{70}-4)(b-a)(2-(a+b))t}{6}. \end{cases}$$

Using the above reaction functions, we derive the equilibrium prices. First, we solve for the intersection of the following two reaction functions:

$$\begin{cases} P_A^1(P_B^1) = \dfrac{3P_B^1 + 2(b^2-a^2)t}{5} & \text{if } P_B^1 \le \dfrac{(\sqrt{70}-4)(b^2-a^2)t}{6}, \\ P_B^1(P_A^1) = \dfrac{3P_A^1 + 2(b-a)(2-(a+b))t}{7} & \text{if } P_A^1 \ge \dfrac{(\sqrt{70}-4)(b-a)(2-(a+b))t}{6}. \end{cases}$$

The resulting prices and the value of z are given as

$$({P_A^1}^*, {P_B^1}^*) = \left(\frac{2(b-a)(3+2(a+b))t}{13}, \frac{2(b-a)(5-(a+b))t}{13}\right), \quad z^* = \frac{4+7(a+b)}{26}.$$

The prices satisfy the above two inequalities if and only if

$$a+b \geq \frac{2(13\sqrt{70}-70)}{13\sqrt{70}-28} \simeq 0.96.$$

Second, we solve for the intersection of the following two reaction functions:

$$\begin{cases} P_A^1(P_B^1) = \dfrac{3P_B^1 + (b^2-a^2)2t}{7} & \text{if } P_B^1 \geq \dfrac{(\sqrt{70}-4)(b^2-a^2)t}{6}, \\ P_B^1(P_A^1) = \dfrac{3P_A^1 + 2(b-a)(2-(a+b))t}{5} & \text{if } P_A^1 \leq \dfrac{(\sqrt{70}-4)(b-a)(2-(a+b))t}{6}. \end{cases}$$

The resulting prices and the realized value of z are

$$({P_A^1}^*, {P_B^1}^*) = \left(\frac{2(b-a)(3+(a+b))t}{13}, \frac{2(b-a)(7-2(a+b))t}{13}\right), \; z^* = \frac{8+7(a+b)}{26}.$$

The prices satisfy the above inequalities if and only if

$$a+b \leq \frac{84}{13\sqrt{70}-28} \simeq 1.04.$$

Combining these two cases gives us Lemma 6. □

Proof of Lemma 7: Firm A's profit is $\Pi_A = P_A^1 z + \pi_A^2$ and firm B's profit is $\Pi_B = P_B^1(1-z) + \pi_B^2$. If $a+b > 84/(13\sqrt{70}-28)$, then Lemma 6 shows that the $\tau = 1$ pricing subgame has a unique equilibrium with $z = \frac{4+7a+7b}{26} < \frac{a+b}{2}$, which we call E1. If $a+b < 2(13\sqrt{70}-70)/(13\sqrt{70}-28)$, then the $\tau = 1$ pricing game has a unique equilibrium with $z = \frac{8+7a+7b}{26} > \frac{a+b}{2}$, which we call E2. If $84/(13\sqrt{70}-28) \geq a+b \geq 2(13\sqrt{70}-70)/(13\sqrt{70}-28)$, both E1 and E2 are possible. In this case, each firm's location choice depends on which of the two pricing equilibria they expect in the subgame. Given that E1 follows when $a+b > 84/(13\sqrt{70}-28)$ and E2 follows when $a+b < 2(13\sqrt{70}-70)/(13\sqrt{70}-28)$, by continuity we assume that both firms expect E1 if and only if $a+b > k$ for some $k \in (2(13\sqrt{70}-70)/(13\sqrt{70}-28), 84/(13\sqrt{70}-28))$. In what follows, we assume $k = 1$. But it is easy to verify that our argument applies for any $k \in (2(13\sqrt{70}-70)/(13\sqrt{70}-28), 84/(13\sqrt{70}-28))$. Given $k = 1$ and the stipulated expectation, each firm's profit function can be written as

$$\Pi_A = \begin{cases} \dfrac{(b-a)\left[207(a+b)^2+140(a+b)+40\right]}{676} & \text{if } a+b>1, \\ \dfrac{(b-a)\left[32(a+b)^2+49(a+b)+28\right]}{169} & \text{if } a+b\le 1, \end{cases}$$

$$\Pi_B = \begin{cases} \dfrac{(b-a)\left[32(a+b)^2-177(a+b)+254\right]}{169} & \text{if } a+b>1, \\ \dfrac{(b-a)\left[207(a+b)^2-968(a+b)+1148\right]}{676} & \text{if } a+b\le 1, \end{cases}$$

with the corresponding derivatives

$$\frac{\partial \Pi_A}{\partial a} = \begin{cases} \dfrac{-621a^2-(280+414b)a-(40-207b^2)}{676} & \text{if } a+b>1, \\ \dfrac{-2\left[14-16b^2+(49+32b)a+48a^2\right]}{169} & \text{if } a+b\le 1, \end{cases}$$

$$\frac{\partial \Pi_B}{\partial b} = \begin{cases} \dfrac{2\left[127-16a^2-(177-32a)b+48b^2\right]}{169} & \text{if } a+b>1, \\ \dfrac{1148-207a^2-(1936-414a)b+621b^2}{676} & \text{if } a+b\le 1. \end{cases}$$

Solving the above, we have the following candidate equilibria:

$$\begin{cases} a=\dfrac{2\sqrt{56029}-347}{621}\simeq 0.2,\ b=1 & \text{if } a+b>1, \\ a=0,\ b=\dfrac{968-2\sqrt{56029}}{621}\simeq 0.8 & \text{if } a+b\le 1. \end{cases}$$

The first equilibrium is consistent with E1 since $a+b>84/(13\sqrt{70}-28)$ in E1. Given $b=1$, firm A's best response problem is over the entire range of $[0,1]$. Thus firm A does not have an incentive to deviate from $a\simeq 0.2$. On the other hand, firm B may choose to deviate by locating at b such that E2 is realized in the pricing subgame. However we can easily show that firm B does not have an incentive to change its location: plotting Π_B given $a=0.2$ shows that Π_B is indeed maximized when $b=1$. Thus $a=0.2, b=1$ constitute an equilibrium. Similarly one can verify that $a=0, b=0.8$ also constitute an equilibrium, which is followed by E2 in the pricing subgame. □

Proof of Proposition 4: Fix firm A's location at a and firm B's location at b. Suppose that, in $\tau=1$, the indifferent consumer is located at z_1. Then firm A's market share is $[0,z_1]=\mathcal{A}$ and firm B's market share is $[z_1,1]=\mathcal{B}$.

In $\tau=2$, firm A sets $P_A(\mathcal{A})$ to the set of consumers in $\mathcal{A}$ and $P_A(\mathcal{B})$ to the set of consumers in $\mathcal{B}$. Similarly, firm B sets $P_B(\mathcal{A})$ to the set of consumers in $\mathcal{A}$ and $P_B(\mathcal{B})$ to the set of consumers in $\mathcal{B}$.

In the set of consumers in $\mathcal{A}$, the indifferent consumer $z_{\mathcal{A}}$ is derived from the following equation:

$$P_A(\mathcal{A}) + t(z_{\mathcal{A}} - a)^2 = P_B(\mathcal{A}) + t(z_{\mathcal{A}} - b)^2.$$

Solving it, we have

$$z_{\mathcal{A}} = \frac{(b^2 - a^2)t - (P_A(\mathcal{A}) - P_B(\mathcal{A}))}{2(b-a)t}.$$

Similarly, in the set of consumers in $\mathcal{B}$, the indifferent consumer $z_{\mathcal{B}}$ is derived from the following equation:

$$P_A(\mathcal{B}) + t(z_{\mathcal{A}} - a)^2 = P_B(\mathcal{B}) + t(z_{\mathcal{A}} - b)^2.$$

Solving it, we have

$$z_{\mathcal{B}} = \frac{(b^2 - a^2)t - (P_A(\mathcal{B}) - P_B(\mathcal{B}))}{2(b-a)t}.$$

Then profits in $\tau = 2$ are[24]

$$\begin{aligned} \Pi_{A2} &= P_A(\mathcal{A})z_{\mathcal{A}} + P_A(\mathcal{B})(z_{\mathcal{B}} - z_1), \\ \Pi_{B2} &= P_B(\mathcal{A})(z_1 - z_{\mathcal{A}}) + P_B(\mathcal{B})(1 - z_{\mathcal{B}}). \end{aligned}$$

The first-order conditions for profit maximization are

$$\begin{aligned} \frac{\partial \Pi_A^2}{\partial P_A(\mathcal{A})} &= \frac{(b-a)(a+b)t + p_B(\mathcal{A}) - 2p_A(\mathcal{A})}{2(b-a)t} = 0, \\ \frac{\partial \Pi_A^2}{\partial P_A(\mathcal{B})} &= \frac{(b-a)((a+b) - 2z)t + p_B(\mathcal{B}) - 2p_A(\mathcal{B})}{2(b-a)t} = 0, \\ \frac{\partial \Pi_B^2}{\partial P_B(\mathcal{A})} &= \frac{(b-a)(2z - (a+b))t + p_A(\mathcal{A}) - 2p_B(\mathcal{A})}{2(b-a)t} = 0, \\ \frac{\partial \Pi_B^2}{\partial P_B(\mathcal{B})} &= \frac{(b-a)(2 - (a+b))t + p_A(\mathcal{B}) - 2p_B(\mathcal{B})}{2(b-a)t} = 0. \end{aligned}$$

Solving the above leads to the following prices in $\tau = 2$:

$$\begin{aligned} P_A(\mathcal{A}) &= \frac{(b-a)(2z_1 + a + b)t}{3}, \ P_B(\mathcal{A}) = \frac{(b-a)(4z_1 - (a+b))t}{3}, \\ P_A(\mathcal{B}) &= \frac{(b-a)(2 + a + b - 4z_1)t}{3}, \ P_B(\mathcal{B}) = \frac{(b-a)(4 - 2z_1 - (a+b))t}{3}. \end{aligned}$$

Plugging these prices back into the locations of the indifference consumers, we have

$$z_{\mathcal{A}} = \frac{a + b + 2z_1}{6}, \quad z_{\mathcal{B}} = \frac{2 + a + b + 2z_1}{6}.$$

[24]To be precise, we need to check the possibility of corner solution. Fortunately, the interior solution always exists as shown below.

Substituting the above prices, $z_{\mathcal{A}}$ and $z_{\mathcal{B}}$ into each firm's $\tau = 2$ profit, we have

$$\begin{aligned}\Pi_{A2} &= \frac{(b-a)((a+b+2z_1)^2 + (2+a+b-4z_1)^2)t}{18},\\ \Pi_{B2} &= \frac{(b-a)((4z_1-(a+b))^2 + (4-(a+b)-2z_1)^2)t}{18}.\end{aligned}$$

Note that $0 < z_{\mathcal{A}} < z_1$ and $z_1 < z_{\mathcal{B}} < 1$ if and only if

$$\frac{a+b}{4} < z_1 < \frac{2+a+b}{4}.$$

As we will show below, the equilibrium in $\tau = 1$ satisfies these inequalities.

Since consumers are forward-looking, the indifferent consumer z_1 in $\tau = 1$ can be derived from the following equation:

$$P_A^1 + t(z_1-a)^2 + P_B(\mathcal{A}) + t(z_1-b)^2 = P_B^1 + t(z_1-b)^2 + P_A(\mathcal{B}) + t(z_1-a)^2.$$

Solving it, we have

$$z_1 = \frac{2(b-a)t(1+a+b) - 3(P_A^1 - P_B^1)}{8(b-a)t}.$$

The total profits over the two periods are

$$\begin{aligned}\Pi_A &= P_A^1 z_1 + \frac{(b-a)((a+b+2z_1)^2 + (2+a+b-4z_1)^2)t}{18},\\ \Pi_B &= P_B^1(1-z_1) + \frac{(b-a)((4z_1-(a+b))^2 + (4-(a+b)-2z_1)^2)t}{18}.\end{aligned}$$

Thus the first-order conditions for profit maximization are

$$\begin{aligned}\frac{\partial \Pi_A}{\partial P_A^1} &= \frac{2(b-a)t(3+a+b) + P_B^1 - 7P_A^1}{16(b-a)t} = 0,\\ \frac{\partial \Pi_B}{\partial P_B^1} &= \frac{2(b-a)t(5-(a+b)) + P_A^1 - 7P_B^1}{16(b-a)t} = 0.\end{aligned}$$

Solving the above, we have the following optimal prices of firms A and B and, after substituting them into z_1, we find the equilibrium location of the indifferent consumer:

$$P_A^1 = \frac{(b-a)(13+3(a+b))t}{12}, \quad P_B^1 = \frac{(b-a)(19-3(a+b))t}{12}, \quad z_1 = \frac{7+a+b}{16}.$$

It is easy to check the above z_1 satisfies $0 < z_{\mathcal{A}} < z_1$ and $z_1 < z_{\mathcal{B}} < 1$ for any $a \in [0,1]$ and $b \in [0,1]$.

Using the above prices and locations, we can express each firm's total profit in terms of only a and b as follows:

$$\Pi_A^* = \frac{(b-a)(599+354(a+b)+135(a+b)^2)t}{1152},$$
$$\Pi_B^* = \frac{(b-a)(1847-894(a+b)+135(a+b)^2)t}{1152}.$$

The derivatives of the above profit functions are:

$$\frac{\partial \Pi_A^*}{\partial a} = -\frac{(599-135b^2+(708+270b)a+405a^2)t}{1152} < 0 \ \forall\ a,b \in [0,1],$$
$$\frac{\partial \Pi_B^*}{\partial b} = \frac{(1847-135a^2-(1788-270a)b+405b^2)t}{1152} > 0 \ \forall\ a,b \in [0,1].$$

Therefore, the equilibrium locations are $a = 0,\ b = 1$.

Putting these equilibrium locations back into equilibrium prices and the locations of indifferent consumers in each period, we have $P_A^1 = P_B^1 = (4t)/3$, $P_A(\mathcal{A}) = P_B(\mathcal{B}) = (2t)/3$, $P_A(\mathcal{B}) = P_B(\mathcal{A}) = t/3$, and $z_1 = 1/2, z_{\mathcal{A}} = 1/3, z_{\mathcal{B}} = 2/3$. □

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