On Multitasking and Job Design in Relational Contracts

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Abstract

We investigate the optimal job design in relational contract with multiple tasks where the performance measurement is distorted, aggregated, and nonverifiable. We compare task bundling where all the tasks are assigned to a single agent with task separation where the tasks are split and assigned to two agents. Compared to task bundling, task separation mitigates misallocation of efforts among the tasks but requires more commitment due to dispersion of informal bonuses to multiple agents. As a result, task separation is better than task bundling if and only if the discount factor of the parties is high. We furthermore show that the optimal job design may exhibit task exclusion, in which only a single agent is employed but the assigned tasks are limited.

1 Introduction

In recent complex work places, uncertain contingencies and states are hard to be described ex ante and/or verified ex post. As a result, it is often impossible to specify ex ante what a worker must do through a formal contract. Rather, management practices supporting high productivity are often based on relational contracts (Gibbons and Henderson, 2012). Long-term relationship allows the parties to share and support informal rules that cannot be specified through a formal contract, which often contributes to substantial improvement of organizational performance.

The difficulty of specification of working rule arises especially when there are multiple tasks involved in the working place. One leading example is product innovation in firms, where, in various dimensions, there are multiple conflicting activities that must be cared simultaneously. For instance, at a broad level, the literature of innovation management argues when a growing firm with continuous innovation of new products must pursue both exploration of new idea for the future business and exploitation

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of old idea for the current profit. Such ambidextrous management is not easy and current successful firms tend to fail into the so-called “competency trap”, meaning that the firms are inclined to exploitation and their future performance is declined due to the lack of exploration. Henderson (1994) and Gibbons and Henderson (2012) describe a more specific example in the recent leading pharmaceutical firms like Merck. Since 1990s, several pharmaceutical firms have attempted to discover drugs by “science-based” approach, which requires the workers academic-oriented science research (that includes attending academic conferences and publishing articles in academic journals) as well as discovery of drugs. Since it is difficult for other parties to understand ex ante what creative workers do, managers attempts to utilize long-term interaction and to build informal rules that do not rely on ex ante specification. Nevertheless, not all the pharmaceutical firms successfully have implemented the science-based approach since they failed to establish a relational contract that support an appropriate informal working rule.

Do firms have other ways to deal with the difficulty of multiple conflicting tasks? An organization may specify an activity to be focused on for each innovative worker by separating or decentralizing the organization. In cases of innovation management, a research and development division in a firm may be separated into research sub-division that focuses on basic research and development sub-division that focuses on developing new products. To separate or spin off a division of new generation product from the existing division may reduce the innovation gap caused by tensions between businesses of old and new products. Actually O’Reilly and Tushman (2004) report that in USA today and Ciba such an organizational design had a great influence on exploiting and exploring activities in the firm.¹ Furthermore, Henderson (2006) argues that the competency trap is caused by inappropriate organizational design, which suggests that the working rule and workers incentives as well as intrinsic motivation of creative workers are potential sources of the competency trap. Actually task assignment through organization design has a potential impact on the workers incentive for multiple activities through change of the set of the worker’s choice and that of way to evaluate them.

The purpose of this article is to clarify the effect of the job design in such complex environments relying on relational contracts. We consider a repeated moral hazard

¹Another interesting empirical result is reported by Igami (2014), who quantifies the effect of creative destruction in the HDD industry. His counterfactual simulation finds that separation of existing and new businesses shrinks a significant part of the innovation gap caused by the cannibalization concern between existing and new products.
model in which there are multiple tasks and the performance measure is aggregated, distorted, and nonverifiable. Nonverifiability of the performance measurement implies that a relational contract is indispensable for incentive provision. In addition to designing a relational contract, the organization chooses a mode of job design: either task bundling in which a single agent performs all the tasks; or task separation in which the tasks are split and assigned to two agents. In our setup, the tasks do not exhibit any technological complementarity or substitutes so that the job design does not influence any efficiency due to technological reasons. Although we recognize the importance of technological aspect through job design, our analysis exclusively sheds light on the incentive effect of the job design.

Our analysis point out an intuitive trade-off between task bundling and task separation. When the aggregated performance measurement is distorted, the allocation of efforts made by an agent must also be distorted. Under task separation, each agent performs fewer tasks than task bundling, which mitigates misallocation of efforts. However, under task separation, since there are multiple agents, the organization must pay discretionary bonuses to all the agents. The dispersion of discretionary bonuses under task separation makes it harder to sustain relational contracts. In other words, task bundling has commitment advantage relative to task separation. Consequently, we obtain a clear-cut relationship between the modes of job design: task separation (resp. bundling) is chosen if and only if the parties are patient (resp. impatient).

The optimal job design also depends on the proportion of the magnitude of the effect of marginal bonus on the benefit and on the performance measure. Given task separation, each agent has a marginal effect of a bonus on the organization benefit, which means the degree of improvement of the principal’s benefit from adding one unit of discretionary bonus paid to the agent. Similarly, each agent also have a marginal effect of a bonus on the performance measure. If the agents are balanced in the sense that the ratio of the marginal effect on the principal’s benefit to the marginal effect on the performance measurement is vary similar between the agents, then shifting to task bundling does not make misallocation of efforts more seriously. Hence task bundling tends to be preferred. By contrast, if the agents are unbalanced in that the ratios mentioned above are different between agents, then the agent under task bundling makes excess efforts on tasks with larger marginal effect of bonus on the performance measures and smaller marginal effect on the principal’s benefit. Hence task bundling substantially boosts distortion of effort allocation and tends to be avoided.

Furthermore, if the ratios are extremely different, then task separation may exhibit
task exclusion, meaning that one of the agents does not perform the tasks at all. Note that task exclusion can also be interpreted as task bundling with restriction to a limited number of tasks to be performed. Restriction to important tasks can mitigate a distortion of effort allocation without providing additional incentives.

This article relates to a broad strand of theoretical literature on moral hazard and job design. As in the present article, the analysis of job design is often motivated by multitasking problem that causes misallocation of effort due to distortion of the performance measures as Holmström and Milgrom (1991) and Baker (2002) point out. The existing literature typically discusses a trade-off between task bundling and task separation, which provides a richer insight on organization design. On the one hand, mitigation of misallocation of efforts is usually a bright side of task separation. On the other hand, the dark side of task separation can be: generation of additional risk premium payments (Itoh, 1994, 2001; Corts, 2007) in case of risk-averse agents; or excess rent provision due to limited liability (Kragl and Schöttner, 2014). All of these articles analyse a one-shot moral hazard model with multiple tasks and verifiable measurements. Our analysis is different from them in that we consider a repeated and relational incentive contract and find a new insight on job design in cases of relational contracting. Especially, we first point out dispersion of informal bonuses as a cost of task separation when incentive provision must be relational.

The present article is also positioned in the literature of relational incentive contracts and organizational design since Baker et al. (2002) and Levin (2003). Recently, there are a number of articles which study relational contracting in a multitasking environment (Schmidt and Schnitzer, 1995; Daido, 2006; Kvaløy and Olsen, 2008; Schöttner, 2008; Mukherjee and Vasconcelos, 2011; Ishihara, 2014). Among them, the most related article to our analysis is Schöttner (2008), who considers a setup similar to our model. She highlights a trade-off between mitigating effort misallocation under task separation and strengthening the punishment under task bundling. Consequently, she argues that task bundling is preferred if and only if the parties are patient, which is totally opposite to our result. This conflict of the arguments comes from slightly different assumptions of information and commitment ability. We discuss the detail of Schöttner (2008) and the difference from our analysis in Section 5. Mukherjee and Vasconcelos (2011) also consider a relational contracting model with job design and argue a trade-off similar to

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2Itoh (1994, 2001) also investigate another kind of job design such that the principal can perform a part of tasks, which we abstract.
3Kragl and Schöttner (2014) also demonstrate the optimality of task exclusion as in our analysis.
4See also Malcomson (2012) for a recent survey.
our result in a conceptually different setup. Specifically, in their model, the number of agents and assigned tasks for each agent must be fixed and the modes of job design is either: individual assignment such that the discretionary bonus is contingent only on a single performance measure; or team assignment such that the discretionary bonus is contingent on multiple measures. They demonstrate that individual assignment has a commitment advantage while team assignment can mitigation misallocation of efforts. A theoretical advantage of our model is that our model can treat asymmetric environments between agents whereas Mukherjee and Vasconcelos (2011) focus on symmetric agents.

The rest of the article is organized as follows. The next section describes the preliminary of our model. Section 3 derives an optimal contract under each job design and section 4 compares the optimal contract of each job design to derive the pattern of the optimal job design. Section 5 discusses what the difference of Schöttner (2008)’s model is and how this difference plays a role for demonstrating the opposite result. The final section concludes. The Appendix contains all the proofs of the propositions.

2 The Model

2.1 Environment

Our model is based on a canonical multitasking model, studied by Baker (2002), with long-lived players and modified verifiability assumptions. There are three players, principal, agent 1, and 2, all of whom are risk-neutral and live in $t = 0, 1, \ldots$ until infinity with discount factor $\delta \in (0, 1)$. There are $N$ productive tasks, denoted by $n = 1, \ldots, N$. In each period $t$, for each task $n \in \mathcal{N}$, a level of effort $e_{nt} \in [0, \infty)$ is chosen by an agent.

Given $e_t \equiv (e_{t1}, \ldots, e_{tN})$, the principal stochastically receives private benefit: either 1 with probability $f(e_t) \equiv \min\{\sum_{n \in \mathcal{N}} \alpha_n e_{nt}, 1\}$; or 0 with probability $1 - f(e_t)$. We assume that the benefit is privately observable to the principal. In addition, the parties observe a stochastic signal, which we call the performance measurement, either $x_t = s$ with probability $p(e_t) \equiv \min\{\sum_{n \in \mathcal{N}} \mu_n e_{nt}, 1\}$; or $x_t = F$ with probability $1 - p(e_t)$.

Let $\mathcal{N} \equiv \{1, \ldots, N\}$ be the set of the tasks and $(\mathcal{N}_1, \mathcal{N}_2)$ be a partition of $\mathcal{N}$ satisfying $\mathcal{N}_i \neq \emptyset$ for $i = 1, 2$. In each period $t$, if the parties engage in a formal contract, the

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5This setup is based on the static model of Corts (2007).
6We can relax the specification of the private benefit as long as the principal’s expected benefit is linear in $e_t$ and she cannot perfectly identify $e_t$ from observing the private benefit.
principal chooses either task bundling or task separation as the job design, which is denoted by $z_t \in \{TB, TS\}$. Under task bundling, all the tasks in $\mathcal{N}$ are assigned to agent 1, which is essentially equal to a situation where only a single agent is employed. Under task separation, for each $i = 1, 2$, the tasks in $\mathcal{N}_i$ are assigned to agent $i$. Assume that the partition $(\mathcal{N}_1, \mathcal{N}_2)$ is exogenously given and fixed in all periods, meaning that the parties cannot arrange the task allocation between the agents under task separation. Nevertheless, it still allows another kind of job design, which we hereafter call task exclusion, that under task separation an agent does not make any effort for any assigned tasks. It is interpreted as employing a single agent who is prohibited to perform some tasks.

The agent incurs costs from efforts of the assigned tasks. Specifically, an agent who is responsible to task $n$ incurs cost $c_n(e_{nt}) \equiv \gamma_n e_{nt}^2/2$ from task $n$. The total cost of the agent is expressed as an additive separable way. Then agent 1’s total cost under task bundling is $\sum_{n \in \mathcal{N}} c_n(e_{nt})$ and agent $i$’s cost under task separation is $\sum_{n \in \mathcal{N}_i} c_n(e_{nt})$. We assume that both the effort level $e_{nt}$ and cost $c_n(e_{nt})$ of task $n$ is privately observable to the assigned agent.

In each period $t$, the principal offers a short-term formal contract. We assume that the mode of job design is verifiable while the stochastic signal $x_t$ is not verifiable. Then the formal contract stipulates a fixed amount of monetary transfer $w_{it} \in \mathbb{R}$ to agent $i$ and job design, either task bundling or task separation. In addition, the parties implicitly promise a discretionary bonus $b_{it} \in \mathbb{R}$ that may be contingent on commonly observable performance measure $x_t$. Since the discretionary bonus is implicit, the principal can renge on it even after realization of $x$.

As Figure 1 explains, the parties play the following game in period $t$.

1. The principal offers a formal contract (and implicit promise) to the agents.

2. Each agent chooses to accept or reject the contract. If at least either of the agents chooses rejection, then each party receive value 0 from the outside option and period $t$ ends.

3. If both agents accept the contract, then according to the stipulated job design, each agent chooses $e_{nt}$ for each $n \in \mathcal{N}$ and the stochastic signal $x_t$ is realized.

4. The principal chooses to honour the discretionary bonus or not.

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7Some related articles (e.g., Schöttner (2008) and Mukherjee and Vasconcelos (2011)) assume that the principal chooses and commits to a job design before the repeated game begins while the principal chooses a job design each period. In our model, the result does not change at all even if the principal commits before period 0. The role of commitment to a job design is briefly discussed in Section 5.
Each party’s payoff is quasi-linear in monetary transfer. Specifically, given that the principal pays $W_i t_i$ to agent $i$ for each $i = 1, 2$, her (expected) payoff in period $t$ is $f(e_t) - \sum_{i=1}^{2} W_i t_i$ and each agent’s (ex post) payoff is $W_1 t - \sum_{n \in N_1} c_n(e_{nt})$ and $W_2 t - \sum_{n \in N_2} c_n(e_{nt})$ under task bundling and $W_1 t - \sum_{n \in N_1} c_n(e_{nt})$ and $W_2 t - \sum_{n \in N_2} c_n(e_{nt})$ under task separation. We assume that for each $n \in N$, $\alpha_n > 0$, $\mu_n > 0$, and $\gamma_n > 0$. Furthermore, $\alpha_n$ and $\mu_n$ are sufficiently small relative to $\gamma_n$, which guarantees that we can assume $f(e_t) \equiv \sum_{n \in N} \alpha_n e_{nt}$ and $p(e_t) \equiv \sum_{n \in N} \mu_n e_{nt}$ throughout the analysis. Let $\gamma(e_t) \equiv \sum_{n \in N} \alpha_n e_{nt} - c_n(e_{nt})$ be the total surplus shared by the parties, which is independent of job design.

As in the literature of relational contracting, we characterize a perfect public equilibrium that is optimal for the principal.\(^8\) By applying Levin (2003)’s analogy, we obtain that without loss of generality an optimal equilibrium can be characterized by stationary contracts such that a pair of fixed transfer $(w_1, w_2)$, job design $z$, a pair of discretionary bonus rule $(b_1(\cdot), b_2(\cdot))$ contingent on $x$, and effort profile $e \equiv (e_1, \ldots, e_N)$ are time-invariant on the equilibrium path.\(^9\) Thanks to this simplification, in what follows, let the time script for each variable be dropped.

Since the job design is persistent on the equilibrium path, in what follows, we first characterize an optimal stationary contract under each job design, task bundling and task separation, given that the job design is fixed over time. Then, we compare the performance of these modes of job design.

3 Optimal Contract

3.1 Task Bundling

We first suppose that the principal chooses task bundling on the equilibrium path. Under task bundling, it is obvious that no task is assigned to agent 2 and then providing

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\(^8\)The formal definition of strategy and equilibrium in this repeated game is in Appendix A.1.

\(^9\)The formal proof is omitted.
agent 2 with incentives based on the performance measure does not improve efficiency at all. Hence the principal focuses on the incentive problem of agent 1. Let $\bar{\beta}$ be the discretionary bonus paid to agent 1 when the performance measurement exhibits $x = S$. Then, thanks to Levin (2003, Theorem 3), an optimal stationary contract can be characterized by the following procedure.\(^1\)

Lemma 1 Suppose that the job design on an optimal stationary contract is task bundling. Then a solution to the following optimization problem is the effort $e$ and discretionary bonus $\bar{\beta}$ paid to agent 1 for $x = S$ on the optimal stationary equilibrium:

\[
\begin{align*}
\max_{\bar{\beta}, e} & \quad Y(e) \\
\text{subject to} & \quad e \in \arg \max_{e' \in \mathbb{R}_+^N} \left[ \bar{\beta} p(e') - \sum_{n \in N} c_n(e'_n) \right], \\
& \quad -\bar{\beta} + \frac{\delta}{1 - \delta} Y(e) \geq 0.
\end{align*}
\]

Furthermore, the optimized value is the principal’s payoff on the optimal equilibrium.

Constraints (1) and (2) are corresponding to incentive compatibility constraint and dynamic enforcement constraint, respectively. The incentive compatibility constraint guarantees that agent 1 voluntarily chooses the targeted effort profile $e$. The dynamic enforcement constraint guarantees that the principal credibly honours the discretionary bonus $\bar{\beta}$ rather than reneges after $x = S$. The left hand side of (2) is the principal’s value when keeping the promise: paying $\bar{\beta}$ ensures the future relationship generating the optimal continuation payoff. If the principal reneges on the promise, then she does not have to pay anything as an informal bonus, which causes termination of the relationship and generates the outside value 0 as a continuation payoff.

Note that provided that $\bar{\beta} \geq 0$, the incentive compatibility constraint can be simplified by the first order condition:

\[
\forall n \in N, \quad e_n = \frac{\mu_n}{\gamma_n}.
\]

Then substituting (3) into the objective function and (2) transforms the optimization problem to:

\[
\begin{align*}
\max_{\bar{\beta} \in \mathbb{R}_+} & \quad A\bar{\beta} - \frac{M - 2}{2} \beta \quad \text{subject to} \quad \frac{1}{r} \left[ A\bar{\beta} - \frac{M - 2}{2} \beta \right] \geq \bar{\beta},
\end{align*}
\]

\(^1\)The formal proof is omitted.
where $r \equiv (1 - \delta)/\delta$ is the discount rate, $A \equiv \sum_{n \in N} \alpha_n \mu_n / \gamma_n$, and $M \equiv \sum_{n \in N} \mu_n^2 / \gamma_n$. The solution implies the following proposition on the optimal stationary contract.

**Proposition 1** Suppose that the job design on an optimal stationary contract is task bundling. Then the optimal stationary contract satisfies the following property.

1. For $r < A/2$, the dynamic enforcement constraint is not binding.

2. For $A/2 \leq r < A$, the dynamic enforcement constraint is binding and the discretionary bonus is positive.

3. For $r \geq A$, the discretionary bonus is zero.

4. The principal’s payoff is non-increasing in $r$ and strictly decreasing in $r \in (A, A/2)$.

As a summary of Proposition 1, Figure 2 illustrates the principal’s payoff on the optimal equilibrium, denoted by $Y^B$, where the horizontal axis is the discount rate, meaning the degree of impatience of the parties. Not surprisingly, the principal’s payoff is monotonic in the parties patience. If the parties are sufficiently patient, then the credibility problem is innocuous so that the dynamic enforcement constraint is not binding. If the parties’ patience is intermediate, then the principal’s commitment becomes an issue and in such cases the principal’s payoff is decreasing in the discount rate. Finally, if the parties are sufficiently impatient, then the principal cannot commit to informal bonuses at all and the relationship has no value.

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\[11\text{Strictly speaking, the principal can choose negative } \beta, \text{ in which the agent chooses zero effort for all the tasks. However, it is easy to check that this outcome is never strictly optimal and therefore we focus on non-negative } \beta.\]
3.2 Task Separation

We next suppose that the principal chooses task separation on the equilibrium path. Obviously, under task separation, the principal must deal with the incentive problems of both agents. For each \(i = 1, 2\), let \(\beta_i\) be the discretionary bonus paid to agent \(i\) when the performance measurement exhibits \(x = S\). Then, similar to task bundling, our problem can be summarized by the following optimization problem.

**Lemma 2** Suppose that the job design on an optimal stationary contract is task separation. Then a solution to the following optimization problem is the effort \(e\) and the pair of the discretionary bonuses \((\beta_1, \beta_2)\) for \(x = S\) on the optimal stationary equilibrium:

\[
\begin{align*}
\max_{\beta_1, \beta_2, e \in (e^1, e^2)} & \quad Y(e) \\
\text{subject to} & \quad e^i \in \arg\max_{\tilde{e}^i \in \mathbb{R}^N_i} \left[ \beta_i p(\tilde{e}^i, e^{-i}) - \sum_{n \in N_i} c_n(\tilde{e}^i_n) \right], \ \forall i = 1, 2, \\
& \quad -2 \sum_{i=1}^2 \beta_i + \frac{\delta}{1 - \delta} Y(e) \geq 0.
\end{align*}
\]

(5) (6)

Similar to Lemma 1, the optimized value is the principal’s payoff on the optimal equilibrium.

Constraints (5) and (6) are again incentive compatibility constraint and dynamic enforcement constraint, respectively. Note that the left hand side of the dynamic enforcement constraint captures the sum of the discretionary bonuses over the agents since the principal now must pay bonuses to both agents.

Under task separation, provided that \(\beta_i \geq 0\), the incentive compatibility constraint (5) can be simplified by the first order condition:

\[
\forall i = 1, 2, \forall n \in N_i, \quad e_n = \beta_i \frac{\mu_n}{\gamma_n}.
\]

(7)

Plugging (7) transforms the optimization problem into:

\[
\max_{(\beta_1, \beta_2) \in \mathbb{R}_+^2} \quad \sum_{i=1}^2 \left[ a_i \beta_i - m_i \beta_i^2 \right] \quad \text{subject to} \quad \frac{1}{r} \sum_{i=1}^2 \left[ a_i \beta_i - m_i \beta_i^2 \right] \geq \sum_{i=1}^2 \beta_i,
\]

(8)

where \(a_i \equiv \sum_{n \in N_i} \alpha_n \mu_n / \gamma_n \) and \(m_i \equiv \sum_{n \in N_i} \mu_n^2 / \gamma_n.\) Under task separation, the solution to the problem depends on the new parameters \(a_i\) and \(m_i\). Hereafter, we assume without loss of generality \(a_1\) is at least equal to \(a_2.\)

\[\text{Similar to task bundling, the principal can choose negative } \beta_i, \text{ in which agent } i \text{ chooses zero effort for all of his assigned tasks. We again ignore this possibility since this is never strictly optimal.}\]
Assumption 1 \( a_1 \geq a_2 \).

To characterize the solution, for each \( i = 1, 2 \), let \( \rho_{Ai} \equiv a_i/A \), \( \rho_{Mi} \equiv m_i/M \). Since \( \sum_{i=1}^{2} a_i = A \), \( \sum_{i=1}^{2} m_i = M \), and all of these parameters are positive, Assumption 1 implies \( 1 > \rho_{A1} \geq 1/2 \geq \rho_{A2} > 0 \). Introducing \( \rho_{Ai} \) and \( \rho_{Mi} \) helps us characterize an optimal contract under task separation easily.

Proposition 2 Suppose that Assumption 1 holds and the job design on an optimal stationary contract is task separation. Then the optimal stationary contract satisfies the following property.

1. For \( r < A(\rho_{M1}\rho_{A2}^2 + \rho_{M2}\rho_{A1}^2)/2(\rho_{M1}\rho_{A2} + \rho_{M2}\rho_{A1}) \), the dynamic enforcement constraint is not binding.

2. For \( A(\rho_{M1}\rho_{A2}^2 + \rho_{M2}\rho_{A1}^2)/2(\rho_{M1}\rho_{A2} + \rho_{M2}\rho_{A1}) \leq r < A/2 \), the dynamic enforcement constraint is binding and \( \beta_i > 0 \) for both \( i = 1, 2 \).

3. For \( A/2 \leq r < \rho_{A1}A \), the dynamic enforcement constraint is binding, \( \beta_1 > 0 \), and \( \beta_2 = 0 \).

4. For \( r \geq \rho_{A1}A \), \( \beta_i = 0 \) for both \( i = 1, 2 \).

5. The principal’s payoff is non-increasing in \( r \) and strictly decreasing in \( r \in (A(\rho_{M1}\rho_{A2}^2 + \rho_{M2}\rho_{A1}^2)/2(\rho_{M1}\rho_{A2} + \rho_{M2}\rho_{A1}), \rho_{A1}A) \).

Figure 3 illustrates the principal’s payoff on the optimal equilibrium under task separation, denoted by \( Y^S \). Similar to task bundling, the principal’s payoff is monotonic in the parties patience. One specific feature is that the optimal contract exhibits task exclusion for \( A/2 < r \leq \rho_{A1}A \). Assumption 1 guarantees that if tasks are excluded, then those assigned to agent 2 must be excluded. In this sense, an agent with higher \( a_i \) is more important than the other agent with lower \( a_i \).

4 Optimal Job Design

In this section, we derive the optimal job design by comparing the performance of each job design. Before providing the formal result, we first explain the key trade-off behind the result and the interpretation of the parameters introduced above.

4.1 Trade-Off between the Modes of Job Design

Each job design has cost and benefit. First we investigate the benefit of task separation. As the existing literature has already pointed out, task separation can mitigate
misallocation of efforts caused by multitasking problem with distorted performance measures. From the incentive compatibility constraint under task bundling, (3), it is obvious to see that a pair of effort chosen by agent 1 must be \( e = \bar{\beta}(\mu_1/\gamma_1, \ldots, \mu_N/\gamma_N) \), namely, a point on vector \( (\mu_1/\gamma_1, \ldots, \mu_N/\gamma_N) \). On the other hand, task separation may implement other pairs of efforts. For explanatory simplicity, suppose in this paragraph \( N_1 = \{1, 2, \ldots, \hat{N}\} \) and \( N_2 = \{\hat{N} + 1, \ldots, N\} \) for some \( \hat{N} = 2, \ldots, N-1 \). The incentive compatibility constraint under task separation, (7), implies that agent 1 and 2 choose a pair of effort \( e^1 = \beta_1(\mu_1/\gamma_1, \ldots, \mu_{\hat{N}}/\gamma_{\hat{N}}, 0, \ldots, 0) \) and \( e^2 = \beta_2(0, \ldots, 0, \mu_{\hat{N}+1}/\gamma_{\hat{N}+1}, \ldots, \mu_N/\gamma_N) \). Thus the effort choice of each agent is dependent on his own bonus but independent of the other’s bonus. In other words, by choosing a pair of bonuses \( (\beta_1, \beta_2) \) appropriately, the principal can implement a pair of effort \( e \) that is on the plane with basis \( (\mu_1/\gamma_1, \ldots, \mu_{\hat{N}}/\gamma_{\hat{N}}, 0, \ldots, 0) \) and \( (0, \ldots, 0, \mu_{\hat{N}+1}/\gamma_{\hat{N}+1}, \ldots, \mu_N/\gamma_N) \). Note that vector \( (\mu_1/\gamma_1, \ldots, \mu_N/\gamma_N) \) must be on this plane, meaning that all the pairs of effort that can be implemented under task bundling can also be implemented under task separation as long as a pair of bonuses \( (\beta_1, \beta_2) \) can be freely chosen. Since the principal’s payoff, \( Y(e) \), is maximized at \( e = (\alpha_1/\gamma_1, \ldots, \alpha_N/\gamma_N) \), which is in general not on vector \( (\mu_1/\gamma_1, \ldots, \mu_N/\gamma_N) \), task separation may help the principal implement the principal’s more desirable effort pair.

Nevertheless, task separation is also costly, relative to task bundling, due to the commitment concern. As the dynamic enforcement constraints (2) and (6) exhibit, the amount of discretionary bonus must be limited from above due to the principal’s reneging temptation. Then the principal must consider how the organization can achieve high performance with smaller bonuses. Now suppose that the principal can commit to \( \beta \) units of bonus to pay. If this bonus is paid to agent 1 under task separation, the
objective function of the optimization problem after plugging the incentive compatibility constraints implies \( a_1 \beta - m_1 \beta^2 / 2 \). On the other hand, under task bundling, the same amount of bonus yields the principal’s payoff \( A \beta - M \beta^2 / 2 = \sum_{i=1,2} [a_1 \beta - m_1 \beta^2 / 2] \), meaning that the agent performs the tasks as if a pair of bonus \((\beta, \beta)\) is paid under task separation. In other words, broader task assignment induces efforts without additional bonuses and a performance under task separation may be achieved with smaller bonuses.

4.2 Interpretation of Parameters

We now provide interpretation of parameters \( A, M, \rho_{Ai}, \) and \( \rho_{Mi} \), which affect the optimal contract under each job design. First, consider task bundling. Taking the first derivative of (3) with respect to \( \beta \) implies \( d\alpha_n / d\beta = \mu_n / \gamma_n \) for each \( n \in N \). Since (3) is the incentive compatibility described by the first order condition, the term \( \mu_n / \gamma_n \) expresses the marginal effort of increasing bonus on task \( n \). Furthermore, the principal’s expected benefit is given by \( \sum_{n \in N} \alpha_n e_n \), implying that the marginal benefit of increasing effort of task \( n \) is \( \alpha_n \). Since under task bundling one unit of bonus increases effort by \( \mu_n / \gamma_n \) for each \( n \in N \), parameter \( A \equiv \sum_{n \in N} \alpha_n (\mu_n / \gamma_n) \) is interpreted as the principal’s marginal benefit of bonus. Similarly, since the marginal success probability of effort of task \( n \) is \( \mu_n \), parameter \( M \equiv \sum_{n \in N} \mu_n (\mu_n / \gamma_n) \) is interpreted as the marginal success probability of bonus under task bundling.

Similarly, the first derivative of (7) with respect to \( \beta_i \) implies \( d\alpha_i / d\beta_i = \mu_n / \gamma_n \), meaning again the marginal effort of bonus. Nevertheless, under task separation, one unit of bonus to agent \( i \) increases the effort only for the assigned tasks \( n \in N_i \). Hence, one unit of bonus to agent \( i \) increases the principal benefit only by \( a_i \equiv \sum_{n \in N_i} \alpha_n (\mu_n / \gamma_n) \) and the success probability only by \( m_i \equiv \sum_{n \in N_i} \mu_n (\mu_n / \gamma_n) \). In other words, parameters \( a_i \) and \( m_i \) are interpreted as the marginal benefit and success probability through agent \( i \) under task separation.

The objective function in the optimization problems (4) and (8) indicates that the optimal discretionary bonus should take into account the marginal benefit of bonus, \( A \) and \( a_i \), and the marginal success probability of bonus, \( M \) and \( m_i \). Intuitively, the former indicates the degree of the principal’s benefit reflected by increasing bonuses while the latter indicates the agent’s effort sensitivity to bonuses. For instance, when the marginal success probability is high relative to the marginal benefit, the agent is inclined to make efforts more by increase of bonuses since the measure is sensitive to
the effort, which might induce excess efforts from the perspective of the principal’s benefit. In this case, the optimal level of bonus should be relatively lower.

Under task separation, each agent has his own marginal benefit $a_i$ and marginal success probability $m_i$ according to the assigned tasks. Based on these parameters, the principal may pay discretionary bonuses to each agent differently. However, such bonus discrimination is infeasible under task bundling since all the tasks are assigned to a single agent and the evaluation is based on only a single aggregated performance measurement. Hence, as already pointed out in Section 4.1, the allocation of efforts is distorted more under task bundling.

The degree of misallocation of efforts is determined by $\rho_{Ai}$, the share of agent $i$’s marginal benefit in the total marginal benefit, and $\rho_{Mi}$, the share of agent $i$’s marginal success probability in the total marginal success probability. In what follows, based on $\rho_{A1}$ and $\rho_{M1}$, we classify circumstances into three cases.

**Definition 1** The agents are:

- (perfectly) balanced when $\rho_{A1} = \rho_{M1}$;
- weakly unbalanced when $\rho_{A1} \neq \rho_{M1}$ and $1/2 \leq \rho_{A1} \leq (1 + \rho_{M1})/2$; and
- strongly unbalanced when $\rho_{A1} > (1 + \rho_{M1})/2$.

When the agents are perfectly balanced, we further obtain $\rho_{A2} = 1 - \rho_{A1} = 1 - \rho_{M1} = \rho_{M2}$. In this case, under task separation, the proportion of the effect of marginal bonus on the principal’s benefit to the agent’s effort sensitivity to bonuses is the same between the agents. As $\rho_{A1}$ departs from $\rho_{M1}$, the effect of marginal bonus on the principal’s benefit differs from the agent’s effort sensitivity to bonuses. Then under task separation, the principal pays more bonuses to the agent whose effort sensitivity to bonuses is small relative to the other agent. It in turn implies that misallocation of efforts under task bundling is serious relative to task separation.

We should mention a technical digression that the interpretation here may be useful for other multitasking problems with a general number of tasks. Many of theoretical articles on multitasking problems and job design usually assume a specific number of tasks, actually two tasks in most cases. Many of theoretical articles on multitasking problems and job design usually assume a specific number of tasks, actually two tasks in most cases. These analyses may be successfully extended.

---

13 Itoh (1994, 2001) and Kragl and Schöttner (2014) assume two tasks in total. Schöttner (2008) assumes three tasks, which is somewhat crucial assumption for her result as we point out later. In articles being interested in issues of team, the number of tasks are typically assumed to be four (Corts, 2007; Kvaløy and Olsen, 2008; Mukherjee and Vasconcelos, 2011; Ishihara, 2014).
to a general number of multiple tasks by summarizing a bunch of parameters into the marginal benefit of bonus and success probability of bonus for each agent.

As seen in the next section, the case of balanced agents are and special in that task bundling is weakly dominating task separation. In other cases, there is a trade-off between task bundling and task separation and task bundling is preferred if and only if the parties are impatient.

4.3 Balanced Agents

The following proposition shows that task bundling is weakly dominating task separation when the agents are balanced.

**Proposition 3** Suppose that Assumption 1 holds and the agents are balanced.

1. For $A/4 < r < A$, task bundling is strictly preferred to task separation.

2. Otherwise, task separation and bundling are indifferent.

Figure 4 illustrates the principal’s optimal payoff under task bundling, $Y^B$, and that under task separation, $Y^S$, in a unified diagram. When the agents are balanced, the effort allocation of the agents cannot be improved at all by splitting the tasks: the optimal effort allocation implemented under task separation can also be implemented under task bundling. In addition, the required bonus for implementing the targeted efforts can be smaller under task bundling than task separation. As a result, task separation never outperforms task bundling.
4.4 Unbalanced Agents

When the agents are (weakly or strongly) unbalanced, we obtain a different implication. In particular, task separation may mitigate misallocation of efforts caused under task bundling. The following proposition argues that task separation is actually optimal if the parties are sufficiently patient.

**Proposition 4** Suppose that Assumption 1 holds and the agents are weakly unbalanced.

1. For $r < \max_{i=1,2}[\rho_{Mi}/\rho_{Ai}]A/4$, task separation is strictly preferred to task bundling.
2. For $\max_{i=1,2}[\rho_{Mi}/\rho_{Ai}]A/4 < r < A$, task bundling is strictly preferred to task separation.
3. Otherwise, task separation and bundling are indifferent.

**Proposition 5** Suppose $\rho_{A1} > (1 + \rho_{M1})/2$.

1. For $r < (\rho_{A1} - \rho_{M1})A/(1 - \rho_{M1})$, task separation is strictly preferred to task bundling. Furthermore, for $A/2 \leq r < (\rho_{A1} - \rho_{M1})A/(1 - \rho_{M1})$, tasks $n \in N_2$ are excluded.
2. For $(\rho_{A1} - \rho_{M1})A/(1 - \rho_{M1}) < r < A$, task bundling is strictly preferred to task separation.
3. Otherwise, task separation and bundling are indifferent.

Figures 5 and 6 illustrate the principal’s optimal payoff in a case of weakly and strongly unbalanced agents, respectively. In both cases, task separation may achieve higher surplus than task bundling as long as any bonus scheme is feasible. Actually, when the parties are sufficiently patient, the principal’s commitment is not a concern and then task separation with ideal bonus payments is optimal. However, as the parties become impatient, such a bonus payment becomes infeasible. Then, instead of task separation, task bundling exhibits is the optimal job design due to its commitment advantage.

In cases of strongly unbalanced agents, the optimal job design sometimes exhibits task exclusion, which is never optimal in cases of weakly unbalanced agents. It is easy to check that $(\rho_{A1} - \rho_{M1})A/(1 - \rho_{M1})$, the threshold value of the discount rate in cases of strongly unbalanced agents, is greater than $A/2$. Since Proposition 2 shows that the optimal contract under task separation exhibits task exclusion for $r \in [A/2, \rho_{A1}A)$, the principal chooses task exclusion for $r \in [A/2, (\rho_{A1} - \rho_{M1})A/(1 - \rho_{M1}))$. Interestingly, the principal optimally chooses task exclusion though she has an option of task bundling. As pointed out in Section 4.1, assigning additional tasks by choosing task bundling
induces the agent to make an effort of the additionally assigned tasks \( n \in \mathcal{N}_2 \) without any additional bonuses. However, when the agents are strongly unbalanced and agent 2 receives no bonus under task separation, the effort level of the new tasks is inefficiently high under task bundling.

To see this more concretely, the condition of strongly unbalanced agents guarantees that \( \rho_{A1} \) is substantially high relative to \( \rho_{M1} \). It implies that under task separation the principal implements substantial bonus discrimination in that the bonus agent 1 receives is much large relative to agent 2. Especially, for \( r \in [A/2, (\rho_{A1} - \rho_{M1})A/(1 - \rho_{M1})] \), only agent 1 receives a positive amount of bonus. Given such \( \rho_{A1}, \rho_{M1} \), and \( r \), we obtain the following corollary.

**Corollary 1** Suppose that Assumption 1 holds, the agents are strongly unbalanced, and \( r \in [A/2, (\rho_{A1} - \rho_{M1})A/(1 - \rho_{M1})] \). Let \( \bar{\beta} \) be the optimal solution of (4). Then \( a_2 \bar{\beta} - m_2 \bar{\beta}^2 / 2 < 0 \).

Recall from the discussion in Section 4.1 that given bonus \( \bar{\beta} \) under task bundling, the contribution to the principal’s benefit from tasks in \( \mathcal{N}_2 \) is \( a_2 \bar{\beta} - m_2 \bar{\beta}^2 / 2 \), which is now negative. The intuition is as follows. Since \( \rho_{A1} \) is quite high relative to \( \rho_{M1} \), the principal is almost exclusively interested in tasks in \( \mathcal{N}_1 \). Under task bundling, if the principal chooses an appropriate bonus to induce efforts of tasks in \( \mathcal{N}_1 \), then the agent rather chooses high efforts of tasks in \( \mathcal{N}_2 \) since such tasks are effective to increase the success probability of the performance measure. From the efficiency view based on the principal’s benefit and the agent’s cost, the induced efforts of tasks in \( \mathcal{N}_2 \) are excessive. Therefore it is optimal for the principal to deliberately exclude tasks in \( \mathcal{N}_2 \) by which the excess efforts are prevented.
The optimality of task exclusion is also pointed out by Kragl and Schöttner (2014), who consider a one-shot relationship in which the performance measurement is verifiable and the agents are protected by wage floors. They find a kind of monotonic relationship between minimum wage and task exclusion in that a task is inclined to be excluded as the wage floor rises up given the degree of distortion of the performance measurement fixed. Increase of wage floors is a source of inefficiency of incentive provision due to positive rents received by the agent. Given that the performance measure is sufficiently distorted, hiring only a single agent and excluding a task can reduce the rents and mitigate distortion of the effort allocation. By contrast, in our setup of relational contracting, being parties more impatient does not necessarily imply task exclusion: the parties who are sufficiently impatient choose task bundling even though the agents are strongly unbalanced.

4.5 Effect of the Balancedness

We have seen the optimal job design for each $r$, patience of the parties. The following proposition provides another illustration of the condition for the optimal job design.

**Proposition 6** Suppose that Assumption 1 holds.

1. When $\rho_{M1} \leq 1/2$:

   (a) for $r \leq (1 - \rho_{M1})A/2$, task separation is strictly preferred for any $\rho_{A1} \in [1/2, 1) \setminus \{\rho_{M1}\}$.

   (b) for $r \in ((1 - \rho_{M1})A/2, A)$, there exists a threshold $\hat{\rho}_{A1} \in (1/2, 1)$ such that task separation (resp. bundling) is strictly preferred if and only if $\rho_{A1} > \hat{\rho}_{A1}$ (resp.
\[ \rho_{A1} < \hat{\rho}_{A1}. \] Furthermore, for \( r \in [A/2, A) \), if \( \rho_{A1} > \hat{\rho}_{A1} \), then task \( n \in N_2 \) is excluded.

2. When \( \rho_{M1} > 1/2 \):

(a) for \( r \leq A/4 \), task separation is strictly preferred for any \( \rho_{A1} \in [1/2, 1) \setminus \{\rho_{M1}\} \).

(b) for \( r \in (A/4, \rho_{M1}A/2) \), there exist two thresholds \( \rho_{A1} \in [1/2, \rho_{M1}) \) and \( \bar{\rho}_{A1} \in (\rho_{M1}, 1] \) such that task separation (resp. bundling) is strictly preferred if and only if \( \rho_{A1} < \rho_{A1} \) or \( \rho_{A1} > \bar{\rho}_{A1} \) (resp. \( \rho_{A1} \in (\rho_{A1}, \bar{\rho}_{A1}) \)).

(c) for \( r \in (\rho_{M1}A/2, A) \), there exists a threshold \( \hat{\rho}_{A1} \in (\rho_{M1}, 1) \) such that task separation (resp. bundling) is strictly preferred if and only if \( \rho_{A1} > \hat{\rho}_{A1} \) (resp. \( \rho_{A1} < \hat{\rho}_{A1} \)).

Furthermore, for \( r \in [A/2, A) \), if \( \rho_{A1} > \hat{\rho}_{A1} \), then task \( n \in N_2 \) is excluded.

Proposition 6 is explained in an intuitive way by Figures 7 and 8, which illustrate the optimal job design characterized by \( r \) and \( \rho_{A1} \) given \( \rho_{M1} \) fixed. In both figures, the dash-dot curve, namely,

\[
r = r(\rho_{A1}) \equiv \begin{cases} 
\max \left\{ \frac{\rho_{M1}}{\rho_{A1}} \left( \frac{1 - \rho_{M1}}{1 - \rho_{A1}} \right) \frac{A}{4} \right\} & \text{if } \rho_{A1} \in \left[ \frac{1}{2}, 1 + \rho_{M1} \right], \\
\frac{A(\rho_{A1} - \rho_{M1})}{1 - \rho_{M1}} & \text{if } \rho_{A1} \in \left( \frac{1}{2}, 1 \right], 
\end{cases}
\]

is the boundary of the optimal job design. Consistent to the results shown above, task separation is preferred for lower area in the figures. Here we additionally see that task bundling is inclined to be preferred as the agents are getting less unbalanced. Specifically, if \( \rho_{A1} \) closes to \( \rho_{M1} \), then the range of \( r \) in which task bundling is optimal becomes broader. As pointed out in Section 4.1, when \( \rho_{A1} \) is far away from \( \rho_{M1} \), the importance of efforts on the principal’s benefit is different between the agents under task separation. In such a case, discrimination of bonus payment between the agents under task separation has a large room for improvement of efficiency. Thus, the principal facing with unbalanced agents tends to choose task separation.

5 Role of Distortion and Verifiability of Performance Measurements

In this section, we discuss the relationship between our result and the related article by Schöttner (2008). The implication of our result is opposite to her result. In the following, we point out several subtle differences between the models, which substantially
Figure 7: Optimal Job Design when $\rho_{M1} < 1/2$

Figure 8: Optimal Job Design when $\rho_{M1} \geq 1/2$

differentiates the implication of the result. This discussion also point out effects of distortion and verifiability of performance measurements on the optimal job design.

Schöttner (2008) considers a similar model of relational contracting with multiple tasks and discusses the optimal job design as we also do. The important differences from our model are: (i) observability and verifiability of the performance measures; (ii) the number of tasks; and (iii) the principal’s commitment ability to job design. In her model, as summarized in Table 1, the parties informational asymmetry is less severe than our model. Specifically, the performance measurement is verifiable and the principal’s benefit, either 1 or 0, can be observed by the parties.

These different assumptions imply the following results that do not occur in our model. First of all, it is possible to provide incentives at least partly through a formal contract contingent on the verifiable measurement. Nevertheless, since the verifiable measurement is distorted from the principal’s benefit, informal bonuses sustained by relational contracting is still useful to complement the incentive provision via formal
contracting. More importantly, since the principal’s benefit is observable, an appropriate bonus payment contingent on the principal’s benefit can implement the first best allocation of efforts even when the agent performs multiple tasks. In such cases, when the parties are sufficiently patient, the job design does not matter: under both task separation and task bundling, an appropriate relational incentive can perfectly resolve inefficiency caused by the multitasking problem and implement the first best effort.

Given that the principal’s benefit can be involved in an informal agreement of discretionary bonus, relatively patient parties prefer the job design which guarantees high-powered incentives via relational contracts. In Schöttner (2008), the reason of the commitment advantage of task bundling is different from our model. Specifically, since the number of the tasks are three in her model, one agent (agent 1 in her model) must perform only a single task under task separation. Since there exists a verifiable performance measure, the principal can provide agent 1 with the first best incentive through a formal contract. It then implies that a discretionary bonus is paid only to a single agent, agent 2 here, even under task separation. Thus the principal’s problem under task separation is essentially interpreted as designing explicit and relational incentives for a single agent with two tasks. In other words, dispersion of discretionary bonuses between agents under task separation, pointed out in our model, never happens in her model. Therefore, in general, reducing the payment of discretionary bonuses under task bundling is not the reason of relative commitment advantage.

Nevertheless, task bundling is still advantageous from a commitment perspective due to the third assumption, the principal’s commitment ability. In her model, the principal chooses the job design at the beginning of the repeated game and can commit to the job design throughout the entire game. The commitment to job design affects the loss from reneging on informal bonuses. It can be confirmed that in a spot formal contracting task separation is more profitable than task bundling due to the effect of mitigating misallocation efforts. Under assumption that the parties have a relationship without informal agreements once the principal reneges on the relational contract, the

<table>
<thead>
<tr>
<th>(Information)</th>
<th>x</th>
<th>Principal’s benefit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Schöttner</td>
<td>Verifiable</td>
<td>Observable Nonverifiable</td>
</tr>
<tr>
<td>This Article</td>
<td>Observable Nonverifiable</td>
<td>Unobservable</td>
</tr>
</tbody>
</table>

Table 1: Assumption about Information
retaliation payoff is worse under task bundling than task separation. It, in turn, implies that task bundling can support larger informal bonuses in a relational contract due to fear of severe punishment.

As a result, task bundling, which has a commitment advantage, can achieve the first best outcome for broader range of patience than task separation. However, for impatient parties, since relational incentives are not sufficiently effective for incentive provision, the principal prefers task separation combined with explicit incentives in order to utilize the mitigation of misallocation of effort. Consequently, the relationship between patience and job design in her model is completely opposite to our result.

By contrasting our model with Schöttner (2008), we see that it is important to clarify the nature of the subjective performance measurement. In our model, unless the agents are perfectly balanced, the nonverifiable performance measurement is not perfect in that task separation can mitigate misallocation of efforts even for patient parties. Nevertheless, since task separation also has a disadvantageous aspect in terms of commitment, only patient parties enjoys task separation. As a result, the relationship between patience and job design is opposite in our model.

We should also pay attention to the existence of verifiable performance measurement. In our model, since there is no verifiable performance measure, the principal provides the agents with incentives only through informal bonuses. Then task bundling is indifferent to task separation as long as the principal cannot provide any relational incentives. By contrast, if there is a verifiable performance measure and it is possible to provide explicit incentives, then job design can influence the incentive provision through explicit contracts. In particular, impatient parties who cannot credibly pay informal bonuses strictly prefers task separation since task separation can mitigate the misallocation of efforts caused by explicit incentives.

Based on the above discussion, we obtain a conjecture on relationship between patience and job design when there exists a stochastic and verifiable performance measure in addition to subjective performance measure $x$. Suppose that the verifiable performance measure is distorted both from the principal’s benefit and $x$ and that the principal hopes to provide both parties with relational incentives in order to complement explicit incentives under task separation. Then task separation is optimal if and only if the parties are sufficiently patient or sufficiently impatient. Sufficiently patient parties prefer task separation for the same reason as demonstrated in our main model. As long as relational incentives and splitting tasks are important for mitigating misallocation of efforts, task separation has a nonnegligible advantage for improving
the surplus. Nevertheless, since task separation requires dispersion of relational incentives, less patient parties would prefer task bundling due to reducing the informal bonus. However, if the parties become impatient further so that informal bonuses are quite ineffective, then the parties again switches to task separation in order to mitigate misallocation efforts through explicit incentives and splitting tasks.\(^{14}\) A similar non-monotonic relationship is also found in Mukherjee and Vasconcelos (2011). In their second part of comparison of two modes of task assignment, when both verifiable and unverifiable performance measurement exist, team assignment, which can resolve misallocation of effort, is preferred to individual assignment, which has a commitment advantage, by the parties that are sufficiently patient or sufficiently impatient due to the reason similar to the above. Although we leave the formal analysis due to the analytical complexity, we argue that the conjecture here is not far away from the point.

6 Conclusion

Recent productive organization does not necessarily take a homogeneous form of job design: in some firms, broader task assignment is prevalent while division of labour supports high productivity in other firms. In order to clarify the difference of the optimality of task assignment from an incentive perspective, we investigate the optimal job design in a multitasking environment where relational incentives are necessary. We find an intuitive trade-off between misallocation of effort and dispersion of informal bonuses. Task separation can potentially mitigate misallocation of efforts. However, since the principal must pay informal bonuses to all the parties, it becomes harder to establish the credibility of honouring the informal bonuses as the number of agents increases. Consequently, a clear-cut relationship emerges: task separation is preferred if and only if the parties are patient. The degree of misallocation of efforts under task bundling relative to task separation in general depends on how the performance measure is distorted from the principal’s benefit. When the degree of misallocation of efforts is sufficiently high, as in cases called strongly unbalanced agents, the principal may exclude some tasks to focus on more important tasks.

There may be various way of extension and application of job design problem in re-

\(^{14}\)If the parties can commit to job design at the beginning of the game as assumed in Schöttner (2008), it is easy to predict that task bundling becomes more prevalent as the optimal job design. Nevertheless, we guess that the qualitative property of the relationship between patience and the optimal job design would not be drastically changed.
lational contracts. Theoretically, it is possible to consider generalization of more agents and/or performance measures. We guess that this extension makes the analysis significantly harder but finds few additional insights. One may be interested in comparison of several patterns of task assignment under task separation. Since we successfully characterize the optimal contract under task separation as an explicit solution, two different task allocations can be compared at least computationally. Finally, we totally abstract technological effects of changing job design. To disentangle and quantify the incentive effect we pointed out and other technological effects of job design would be an important empirical question.

A Appendix: Proofs

A.1 Definition of Strategies and Equilibrium

A strategy in the repeated game is defined as follows. Let a profile of observable events (or simply events) in period \( \tau \) by \( h_\tau \), which consists of: (i) transfer \((w_1^\tau, w_2^\tau)\) and job design \( z_\tau \in \{TB, TS\} \) stipulated by an offered formal contract; (ii) the agents’ decisions on whether to accept or reject the contract, \( q_i^\tau \in \{ac, re\} \); and (iii) given \((q_1^\tau, q_2^\tau) = (ac, ac)\), a performance measurement \( x_\tau \in \{S, F\} \) and discretionary bonuses paid to the agents, \((b_1^\tau, b_2^\tau)\). Denote the set of all possible events by \( \mathcal{H} \). A public history up to period \( \tau \) is defined as \( h_\tau \in \mathcal{H}^{\tau-1} \) for \( \tau \geq 1 \) and \( h^0 \equiv \emptyset \). A public strategy of the principal is a function of public history, \( \sigma^p(h^\tau) \), which stipulates after \( h^\tau \): (i) a formal contract \((w_1^\tau, w_2^\tau, z_\tau)\) offered to the agents; and (ii) discretionary bonuses paid to agent \( i \) for each \( i = 1, 2 \) given a formal contract, acceptance by both agents \((q_1^\tau, q_2^\tau) = (ac, ac)\), and a performance measurement \( x_\tau \). A public strategy of agent 1 in period \( \tau \) is also a function of public history, \( \sigma^i(h^\tau) \), which stipulates after \( h^\tau \): (i) decision of acceptance or rejection to a formal contract; and (ii) effort \( e_{nt} \) for each assigned task given a formal contract which was accepted. A strategy profile \((\sigma^p, \sigma^1, \sigma^2)\) is a perfect public equilibrium if all \( \sigma^p, \sigma^1, \) and \( \sigma^2 \) are public strategies and the profile is a sequential equilibrium.

A.2 Proof of Proposition 1

As discussed in the main text, an optimal stationary contract satisfies the solution to problem (4). If the constraint is ignored, then the first order condition implies that the optimal solution is \( \bar{\beta} = A/M \). This solution satisfies the constraint with strict inequality
if and only if

\[ \frac{1}{r} A^2 > \frac{A}{M} \iff r < \frac{A}{2}. \]

In this case, the value is \( A^2/2M \), which is independent of \( r \).

Otherwise the constraint must be binding, which implies that \( \bar{\beta} \) is either \( 2(A - r)/M \) or 0. Since the constraint is binding, the objective function satisfies \( A\bar{\beta} - M\bar{\beta}^2 / 2 = r\bar{\beta} \), implying that \( \bar{\beta} \) should be higher. Then when \( r < A \), the optimal solution satisfies \( \bar{\beta} = 2(A - r)/M \) and the value is \( 2r(A - r)/M \). Since the derivative of the value with respect to \( r \) is \( 2(A - 2r)/M \), the value \( 2r(A - r)/M \) is decreasing in \( r \in (A/2, A) \). On the other hand, when \( r \geq A \), the solution implies that the discretionary bonus \( \bar{\beta} \) is zero.

Therefore, the solution \( \bar{\beta} \) and the value \( Y_B \) are summarized as follows:

\[
(\bar{\beta}, Y_B) = \begin{cases} 
\left( \frac{A}{M}, \frac{A^2}{2M} \right) & \text{if } r < \frac{A}{2}, \\
\left( \frac{2(A - r)}{M}, \frac{2r(A - r)}{M} \right) & \text{if } \frac{A}{2} \leq r < A, \\
(0, 0) & \text{if } r \geq A.
\end{cases}
\]

\( Y_B \) is continuous due to the theorem of the maximum. The enforcement constraint is binding if and only if \( r \geq A/2 \).

A.3 Proof of Proposition 2

Similar to Proposition 1, an optimal stationary contract satisfies the solution to problem (8). If the constraints are ignored, then the first order condition implies that the optimal solution is \( \beta_i = a_i/m_i = (\rho_{Ai}/\rho_{Mi})(A/M) \) for each \( i = 1, 2 \). This solution satisfies the constraint with strict inequality if and only if

\[
\frac{1}{r} \left[ \frac{A^2}{2M} \sum_{i=1}^{2} \frac{\rho_{Ai}^2}{\rho_{Mi}} \right] > \frac{A}{M} \sum_{i=1}^{2} \frac{\rho_{Ai}}{\rho_{Mi}} \iff r < \frac{A}{2} \sum_{i=1}^{2} (\rho_{Ai}^2/\rho_{Mi}) = A(\rho_{A1}^2\rho_{M2} + \rho_{A2}^2\rho_{M1})/2(\rho_{A1}\rho_{M2} + \rho_{A2}\rho_{M1}).
\]

In this case, the value is \( A^2 \sum_{i=1}^{2} (\rho_{Ai}^2/\rho_{Mi})/2M \), which is independent of \( r \).

Otherwise at least either the dynamic enforcement constraint or the non-negative constraint of \( \beta_i \) must be binding. To investigate which constraint is binding, Define the Lagrangian as

\[
\mathcal{L} = \sum_{i=1}^{2} \left[ a_i\beta_i - \frac{m_i}{2} \beta_i^2 \right] + \lambda \left[ \frac{1}{r} \sum_{i=1}^{2} \left[ a_i\beta_i - \frac{m_i}{2} \beta_i^2 \right] - \sum_{i=1}^{2} \beta_i \right] + \sum_{i=1}^{2} \eta_i \beta_i
\]
where $\lambda$ is the multiplier for the dynamic enforcement constraint and $\eta_i$ is the multiplier for the non-negative constraint. The first order condition with respect to $\beta_i$ is

$$\frac{\partial L}{\partial \beta_i} = \left[1 + \frac{\lambda}{r}\right][a_i - m_i \beta_i] - \lambda + \eta_i = 0 \iff \beta_i = \frac{a_i}{m_i} - \frac{\lambda - \eta_i}{m_i(1 - \lambda/r)}.$$  

If $\lambda = 0$ and $\eta_i = 0$ for all $i = 1, 2$, then $\beta_i = a_i/m_i$ for $i = 1, 2$ and as seen above, this satisfies the constraint if and only if (10) is satisfied (with weak inequality). If $\lambda = 0$ and $\eta_i > 0$ for some $i = 1, 2$, then the first order condition implies $\beta_i > a_i/m_i > 0$ whereas the complementary slackness condition implies $\beta_i = 0$, a contradiction. Therefore, in the rest of the cases, suppose $\lambda > 0$, which implies by the complementary slackness condition that the dynamic enforcement constraint is binding.

First suppose that $\eta_1 = \eta_2 = 0$. Then combining the first order condition for both $i = 1, 2$ yields $a_1 - m_1 \beta_1 = a_2 - m_2 \beta_2$. Plugging this equation into the binding dynamic enforcement constraint eliminates $\beta_2$ as:

$$\beta_1 = \frac{\rho_{A1}A - r}{\rho_{M1}M} \pm \sqrt{\left(\frac{\rho_{A1}A - r}{\rho_{M1}M}\right)^2 + \frac{A(\rho_{A2} - \rho_{A1})(A - 2r)}{\rho_{M1}M^2}} = \frac{\rho_{A1}A - r \pm K(r)}{\rho_{M1}M}$$

where $K(r) = \sqrt{(\rho_{M2}\rho_{A1} + \rho_{M1}\rho_{A2})A - r^2 + \rho_{M1}\rho_{M2}(1 - 4\rho_{A1}\rho_{A2})A^2}$.\footnote{The fact that $\sum_{i=1}^2 \rho_{Ai} = \sum_{i=1}^2 \rho_{Mi} = 1$ is used for the derivation.} Note that $K(r)$ is a real and positive number for any $r$ since $1 - 4\rho_{A1}\rho_{A2} \geq 0$. Plugging this $\beta_1$ yields

$$\beta_2 = \frac{a_2 - a_1 + m_1 \beta_1}{m_2} = \frac{\rho_{A2}A - r \pm K(r)}{\rho_{M2}M}.$$  

Recall that the dynamic enforcement constraint is binding. Then the objective function satisfies $\sum_{i=1}^2 (a_i\beta_i - m_i \beta_i^2/2) = r\sum_{i=1}^2 \beta_i$, implying that $\beta_i$ should be higher. Then the optimal solution should be $\beta_i = (\rho_{Ai}A - r + K(r))/\rho_{Mi}M$ for each $i = 1, 2$, which yields a payoff $r\sum_{i=1}^2 (\rho_{Ai}A - r + K(r))/\rho_{Mi}M$. We now check the condition under which $\beta_i \geq 0$ for $i = 1, 2$. For $r < \rho_{Ai}A$, $\beta_i$ is positive and the non-negative constraint is satisfied. For $r \geq \rho_{Ai}A$, there are two sub cases two be considered.

**Case of $\rho_{A1} = \rho_{A2} = 1/2$:** Since we now suppose $r \geq A/2$, $K(r) = \sqrt{(A/2 - r)^2} = r - A/2$. Then $\beta_i = [K(r) - r + A/2]/\rho_{M2}M = 0$ for any $r \geq A/2$.  

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Case of $\rho_{A1} > 1/2 > \rho_{A2}$: $\beta_1 \geq 0$ is satisfied if and only if

$$K(r) \geq r - \rho_{A1} A$$

$$\iff [(\rho_{M2}\rho_{A1} + \rho_{M1}\rho_{A2})A - r]^2 + \rho_{M1}\rho_{M2}(1 - 4\rho_{A1}\rho_{A2})A^2 \geq (r - \rho_{A1} A)^2$$

$$\iff 2(\rho_{M2}\rho_{A1} + \rho_{M1}\rho_{A2} - \rho_{A1})A r \leq [(\rho_{M2}\rho_{A1} + \rho_{M1}\rho_{A2})^2 + \rho_{M1}\rho_{M2}(1 - 4\rho_{A1}\rho_{A2}) - \rho_{A1} A^2]A^2$$

$$\iff 2\rho_{M1}(\rho_{A2} - \rho_{A1})r \leq [\rho_{M1}(\rho_{M2}\rho_{A1} + \rho_{M1}\rho_{A2} + \rho_{A1})(\rho_{A2} - \rho_{A1}) + \rho_{M1}\rho_{M2}(\rho_{A2} - \rho_{A1})^2]A$$

$$\iff r \geq \frac{A}{2}[(\rho_{M2}\rho_{A1} + \rho_{M1}\rho_{A2} + \rho_{A1}) + \rho_{M2}(\rho_{A2} - \rho_{A1})] = \frac{A}{2},$$

which is satisfied since $r \geq \rho_{A1} A > A/2$. The similar procedure implies that $\beta_2 \geq 0$ is satisfied if and only if $r \leq A/2$. Therefore $\beta_i \geq 0$ for both $i = 1, 2$ if and only if $r \leq A/2$.

By replacing weak inequalities with strict inequality, we also see that $\beta_i > 0$ for both $i = 1, 2$ if and only if $r < A/2$.

As the non-negative constraint is innocuous when $\rho_{A1} = \rho_{A2} = 1/2$, the rest of the proof supposes $\rho_{A1} > 1/2 > \rho_{A2}$ and $r \geq A/2$. When $\lambda > 0$ and $\eta_1 > 0$, the complementary slackness condition implies $\beta_1 = 0$. Then the binding dynamic enforcement constraint implies

$$\frac{1}{r}[\alpha_2\beta_2 - \frac{m_2}{2}\beta_2^2] = \beta_2 \iff \beta_2 = 0, \frac{2(\rho_{A2} A - r)}{\rho_{M2} M}.$$

Due to the non-negative constraint on $\beta_2$ and $r \geq A/2 > \rho_{A2} A$, the bonus $\beta_2$ must also be zero and as a result the value is also zero. On the other hand, if $\lambda > 0$ and $\eta_2 > 0$, then the complementary slackness condition implies $\beta_2 = 0$. Then the binding dynamic enforcement constraint implies

$$\frac{1}{r}[\alpha_1\beta_1 - \frac{m_1}{2}\beta_1^2] = \beta_1 \iff \beta_1 = 0, \frac{2(\rho_{A1} A - r)}{\rho_{M1} M}.$$

Similar to the previous argument, $\beta_1$ should be higher for attaining higher value. For $r \in [A/2, \rho_{A1} A)$, a positive bonus $\beta_1 = 2(\rho_{A1} A - r)/\rho_{M1} M$ is feasible and yields a positive payoff $2r(\rho_{A1} A - r)/\rho_{M1} M$, which is strictly preferred to $\eta_1 > 0$. On the other hand, for $r \geq \rho_{A1} A$, the bonus for agent 1 is also zero, yielding zero payoff for the principal.
Therefore, the solution \((\beta_1, \beta_2)\) is
\[
(\beta_1, \beta_2) = \begin{cases} 
\left( \frac{\rho_{A1}A}{\rho_{M1}M}, \frac{\rho_{A2}A}{\rho_{M2}M} \right) & \text{if } r < \frac{A\left(\rho_{A1}^2\rho_{M2} + \rho_{A2}^2\rho_{M1}\right)}{2\left(\rho_{A1}\rho_{M2} + \rho_{A2}\rho_{M1}\right)}, \\
\frac{\rho_{A1}A - r + K(r)}{\rho_{M1}M} & \text{if } \frac{A\left(\rho_{A1}^2\rho_{M2} + \rho_{A2}^2\rho_{M1}\right)}{2\left(\rho_{A1}\rho_{M2} + \rho_{A2}\rho_{M1}\right)} \leq r < \frac{A}{2}, \\
\frac{2(\rho_{A1}A - r)}{\rho_{M1}M} & \text{if } \frac{A}{2} \leq r < \rho_{A1}A, \\
0 & \text{if } r \geq \rho_{A1}A,
\end{cases}
\]
and the value \(Y^s\) is
\[
Y^s = \begin{cases} 
\frac{A^2}{2M} \sum_{i=1}^{2} \frac{\rho_{Ai}^2}{\rho_{M_i}} & \text{if } r < \frac{A\left(\rho_{A1}^2\rho_{M2} + \rho_{A2}^2\rho_{M1}\right)}{2\left(\rho_{A1}\rho_{M2} + \rho_{A2}\rho_{M1}\right)}, \\
\frac{r}{M} \left(2A + \frac{K(r) - r}{\rho_{M1}\rho_{M2}}\right) & \text{if } \frac{A\left(\rho_{A1}^2\rho_{M2} + \rho_{A2}^2\rho_{M1}\right)}{2\left(\rho_{A1}\rho_{M2} + \rho_{A2}\rho_{M1}\right)} \leq r < \frac{A}{2}, \\
\frac{2r}{M} \left[A - \frac{r}{\rho_{M1}}\right] & \text{if } \frac{A}{2} \leq r < \rho_{A1}A, \\
0 & \text{if } r \geq \rho_{A1}A.
\end{cases}
\]

\(Y^s\) is continuous due to the theorem of the maximum. For \(r \in \left(A\left(\rho_{M1}\rho_{A2}^2 + \rho_{M2}\rho_{A1}^2\right)/2\left(\rho_{M1}\rho_{A2} + \rho_{M2}\rho_{A1}\right), \rho_{A1}A\)\), since \(Y^s > 0\) and \(\lambda > 0\) the envelope theorem implies
\[
\frac{dY^s}{dr} = -\frac{\lambda}{r^2} Y^s < 0,
\]
meaning that \(Y^s\) is decreasing in \(r \in \left(A\left(\rho_{M1}\rho_{A2}^2 + \rho_{M2}\rho_{A1}^2\right)/2\left(\rho_{M1}\rho_{A2} + \rho_{M2}\rho_{A1}\right), \rho_{A1}A\)\).

### A.4 Proof of Proposition 3

When \(\rho_{A1} = \rho_{M1}\), from Proposition 2 the principal’s payoff under task separation is
\[
Y^s = \begin{cases} 
\frac{A^2}{2M} & \text{if } r < \frac{A}{4}, \\
\frac{r}{M} \left(2A + \frac{K(r) - r}{\rho_{M1}\rho_{M2}}\right) & \text{if } \frac{A}{4} \leq r < \frac{A}{2}, \\
\frac{2r}{M} \left[A - \frac{r}{\rho_{M1}}\right] & \text{if } \frac{A}{2} \leq r < \rho_{A1}A, \\
0 & \text{if } r \geq \rho_{A1}A.
\end{cases}
\]
For } r \leq A/4, Y^s \text{ is equal to } Y^b. \text{ For } r \in (A/4, A/2), \text{ since } Y^s \text{ is decreasing in } r, Y^s \text{ is strictly less than } A^2/2M = Y^b. \text{ For } r \in [A/2, \rho_{A1}A),
\begin{align*}
Y^b - Y^s &= \frac{2r(A - r)}{M} - \frac{2r}{M} \left[ A - \frac{r}{\rho_{M1}} \right] = \frac{2r}{M} \left[ \frac{1}{\rho_{M1}} - 1 \right] > 0.
\end{align*}
For } r \in [\rho_{A1}A, A), Y^b \text{ is positive while } Y^s \text{ is zero, which implies } Y^b > Y^s. \text{ Finally, for } r \geq A, Y^b = Y^s = 0.

A.5 Proof of Proposition 4

First note that \((\rho_{M1}\rho_{A1}^2 + \rho_{M2}\rho_{A1}^2)/(\rho_{M1}\rho_{A2} + \rho_{M2}\rho_{A1}) < 1\) since \(\rho_{A1} \in (0,1)\) and \(\rho_{A2} \in (0,1)\). Then, for } r < A(\rho_{M1}\rho_{A2}^2 + \rho_{M2}\rho_{A1}^2)/2(\rho_{M1}\rho_{A2} + \rho_{M2}\rho_{A1}),
\begin{align*}
Y^s - Y^b &= \frac{A^2}{2M} \left[ \sum_{i=1}^2 \frac{\rho_{Ai}^2}{\rho_{Mi}} - 1 \right] = \frac{A^2}{2M} \frac{\rho_{M2}\rho_{A1}(\rho_{A1} - \rho_{M1}) + \rho_{M1}\rho_{A2}(\rho_{A2} - \rho_{M2})}{\rho_{M1}\rho_{M2}} \\
&= \frac{A^2}{2M} \frac{\rho_{M2}\rho_{A1}}{\rho_{M1}} \left( \frac{1}{\rho_{M1}} - \frac{\rho_{A1}}{\rho_{M1}} \right) (\rho_{A1} - \rho_{M1}).
\end{align*}
Note that \(\rho_{A1} > \rho_{M1}\) if and only if \(1 - \rho_{A1} < 1 - \rho_{M1}\). Hence both terms in parenthesis are positive when \(\rho_{A1} > \rho_{M1}\) and both are negative when \(\rho_{A1} < \rho_{M1}\). Therefore \(Y^s - Y^b > 0\).

For } r \in A(\rho_{M1}\rho_{A2}^2 + \rho_{M2}\rho_{A1}^2)/2(\rho_{M1}\rho_{A2} + \rho_{M2}\rho_{A1}), A/2], \text{ the following claim is useful for the result.}

Claim 1 Under Assumption 1, \(Y^s \leq Y^b\) at } r = A/2 \text{ if and only if } \rho_{A1} \leq (\rho_{M1} + 1)/2.

Proof From Propositions 1 and 2, \(Y^s \leq Y^b\) at } r = A/2 \text{ if and only if }

\[ A \left[ A(\rho_{A1}\rho_{M2} + \rho_{A2}\rho_{M1}) - A/2 + K(A/2) \right] \leq \frac{A^2}{2M} \]
\[ \iff \sqrt{[\rho_{M2}\rho_{A1} + \rho_{M1}\rho_{A2} - 1/2]^2 + \rho_{M1}\rho_{M2}(1 - 4\rho_{A1}\rho_{A2})} \leq \rho_{M1}\rho_{M2} + \frac{1}{2} - (\rho_{M2}\rho_{A1} + \rho_{M1}\rho_{A2}), \]
which is satisfied only if the right hand side is non-negative. Given that the right hand side is non-negative, this is satisfied if and only if
\[ \left[ \rho_{M2}\rho_{A1} + \rho_{M1}\rho_{A2} - \frac{1}{2} \right]^2 + \rho_{M1}\rho_{M2}(1 - 4\rho_{A1}\rho_{A2}) \leq \left[ \rho_{M1}\rho_{M2} + \frac{1}{2} - (\rho_{M2}\rho_{A1} + \rho_{M1}\rho_{A2}) \right]^2 \]
\[ \iff (\rho_{M1} - 2\rho_{A1})(\rho_{M2} - 2\rho_{A2}) \geq 0 \]
\[ \iff (\rho_{M1} - 2\rho_{A1})(2\rho_{A1} - \rho_{M1} - 1) \geq 0. \]
Since $\rho_{M1} < 1 \leq 2\rho_{A1}$, this is satisfied if and only if $\rho_{A1} \leq (\rho_{M1} + 1)/2$. Finally, given $\rho_{A1} \leq (\rho_{M1} + 1)/2$,
\[
\rho_{M1}\rho_{M2} + \frac{1}{2} - (\rho_{M2}\rho_{A1} + \rho_{M1}\rho_{A2}) = \rho_{M1}\rho_{M2} + \frac{1}{2} - ((1 - \rho_{M1})\rho_{A1} + \rho_{M1}(1 - \rho_{A1})) = -\rho_{M1}^2 + \frac{1}{2} - (1 - 2\rho_{M1})\rho_{A1}.
\]

For $\rho_{M1} \geq 1/2$, this is non-negative since
\[
-\rho_{M1}^2 + \frac{1}{2} - (1 - 2\rho_{M1})\rho_{A1} \geq -\rho_{M1}^2 + \frac{1}{2} - (1 - 2\rho_{M1})\frac{1}{2} = \rho_{M1}(1 - \rho_{M1}) > 0.
\]

Similarly, $\rho_{M1} \leq 1/2$, this is non-negative since
\[
-\rho_{M1}^2 + \frac{1}{2} - (1 - 2\rho_{M1})\rho_{A1} \geq -\rho_{M1}^2 + \frac{1}{2} - (1 - 2\rho_{M1})\frac{1}{2} = \frac{\rho_{M1}}{2} > 0.
\]

From Claim 1, $Y^S \leq Y^B$ at $r = A/2$ when the agents are weakly unbalanced. Recall that from Propositions 1 and 2 $Y^B$ is constant in $r$ while $Y^S$ is continuous and decreasing in $r$. Then there uniquely exists $\hat{r} \in (A(\rho_{M1}\rho_{A2}^2 + \rho_{M2}\rho_{A1}^2)/2(\rho_{M1}\rho_{A2} + \rho_{M2}\rho_{A1}), A/2]$ such that $Y^S > Y^B$ for $r < \hat{r}$, $Y^S = Y^B$ for $r = \hat{r}$ and $Y^S < Y^B$ for $r > \hat{r}$. Such $\hat{r}$ must satisfy
\[
\hat{r} \left[ A(\rho_{A1}\rho_{M2} + \rho_{A2}\rho_{M1}) - \hat{r} + K(\hat{r}) \right] - \frac{A^2}{2M} = 0 \iff K(\hat{r}) = \frac{\rho_{M1}\rho_{M2}A^2}{2\hat{r}} - A(\rho_{A1}\rho_{M2} + \rho_{A2}\rho_{M1}) + \hat{r},
\]
which implies
\[
K(\hat{r})^2 = \left[ \frac{\rho_{M1}\rho_{M2}A^2}{2\hat{r}} - A(\rho_{A1}\rho_{M2} + \rho_{A2}\rho_{M1}) + \hat{r} \right]^2 \iff \hat{r} = \frac{A\rho_{M1}}{4\rho_{A1}}, \frac{A\rho_{M2}}{4\rho_{A2}}.
\]

Let $\ell \in \{1, 2\}$ be such that $\min\{\rho_{M1}/\rho_{A1}, \rho_{M2}/\rho_{A2}\} = \rho_{ML}/\rho_{AL}$ and suppose $\hat{r} = \rho_{ML}/\rho_{AL}$. Since it is easy to check that $\rho_{AL} > \rho_{ML}$\(^{16}\) and
\[
\frac{A(\rho_{M1}\rho_{A2}^2 + \rho_{M2}\rho_{A1}^2)}{2(\rho_{M1}\rho_{A2} + \rho_{M2}\rho_{A1})} - \frac{A\rho_{ML}}{4\rho_{AL}} = \frac{A[2(\rho_{ML}(1 - \rho_{AL})^2\rho_{AC} + 2(1 - \rho_{ML})\rho_{M1}^2) - \rho_{ML}(\rho_{ML}(1 - \rho_{AL}) + (1 - \rho_{ML})\rho_{AC})]}{4\rho_{AC}(\rho_{ML}\rho_{AC} + \rho_{M2}\rho_{A1})} = \frac{A(\rho_{AC} - \rho_{ML})[2\rho_{ML}(\rho_{AC} - \rho_{ML}) + \rho_{MC}]}{4\rho_{AC}(\rho_{ML}\rho_{AC} + \rho_{M2}\rho_{A1})} > 0,
\]
which contradicts $\hat{r} \in (A(\rho_{M1}\rho_{A2}^2 + \rho_{M2}\rho_{A1}^2)/2(\rho_{M1}\rho_{A2} + \rho_{M2}\rho_{A1}), A/2]$. Therefore $\hat{r} = \max\{\rho_{M1}/\rho_{A1}, \rho_{M2}/\rho_{A2}\}A/4$.

\(^{16}\)Recall that the agents are assumed to be weakly unbalanced and then $\rho_{AC} \neq \rho_{ML}$.
For $r \in (A/2, \rho_{A1}A)$,

$$Y^S - Y^B = \frac{2r(\rho_{A1}A - r)}{M} - \frac{2r(A - r)}{M} = \frac{2r}{M} \left[ \left( \frac{\rho_{A1}}{\rho_{M1}} - 1 \right) A - \left( \frac{1}{\rho_{M1}} - 1 \right) r \right]$$

$$< \frac{2r}{M} \left[ \left( \frac{\rho_{A1}}{\rho_{M1}} - 1 \right) A - \left( \frac{1}{\rho_{M1}} - 1 \right) \frac{A}{2} \right] = \frac{rA}{\rho_{M1}M} \left[ (2\rho_{A1} - \rho_{M1} - 1) \right] \leq 0,$$

where the last inequality is due to $\rho_{A1} \leq (1 + \rho_{M1})A/2$.

For $r \in [\rho_{A1}A, A)$, $Y^B$ is positive while $Y^S$ is zero, which implies $Y^B > Y^S$. Finally, for $r \geq A$, $Y^B = Y^S = 0$.

### A.6 Proof of Proposition 5

The procedure of the proof is similar to Proposition 4.

For $r < A(\rho_{M1}\rho_{A2}^2 + \rho_{M2}\rho_{A1}^2)/2(\rho_{M1}\rho_{A2} + \rho_{M2}\rho_{A1})$, the argument same as Proposition 4 implies $Y^S > Y^B$.

For $r \in (A(\rho_{M1}\rho_{A2}^2 + \rho_{M2}\rho_{A1}^2)/2(\rho_{M1}\rho_{A2} + \rho_{M2}\rho_{A1}), A/2]$, Claim 1 implies that $Y^S > Y^B$ at $r = A/2$ when the agents are strongly unbalanced. Since $Y^B$ is constant in $r$ while $Y^S$ is continuous and decreasing in $r$, we obtain $Y^S > Y^B$ for all $r \in (A(\rho_{M1}\rho_{A2}^2 + \rho_{M2}\rho_{A1}^2)/2(\rho_{M1}\rho_{A2} + \rho_{M2}\rho_{A1}), A/2]$.

For $r \in (A/2, \rho_{A1}A]$,

$$Y^S - Y^B = \frac{2r}{M} \left[ \left( \frac{\rho_{A1}}{\rho_{M1}} - 1 \right) A - \left( \frac{1}{\rho_{M1}} - 1 \right) \frac{A}{2} \right],$$

which is positive if $r < (\rho_{A1} - \rho_{M1})A/(1 - \rho_{M1})$, zero if $r = (\rho_{A1} - \rho_{M1})A/(1 - \rho_{M1})$, and negative if $r > (\rho_{A1} - \rho_{M1})A/(1 - \rho_{M1})$. Note that

$$\rho_{A1}A - \frac{(\rho_{A1} - \rho_{M1})A}{1 - \rho_{M1}} = \frac{\rho_{M1}(1 - \rho_{A1})A}{1 - \rho_{M1}} > 0,$$

$$\frac{A}{2} - \frac{(\rho_{A1} - \rho_{M1})A}{1 - \rho_{M1}} = \frac{(1 + \rho_{M1} - 2\rho_{A1})A}{2(1 - \rho_{M1})} < 0,$$

where the latter is satisfied since the agents are strongly unbalanced. Then $(\rho_{A1} - \rho_{M1})A/(1 - \rho_{M1}) \in (A/2, \rho_{A1}A]$. Note furthermore that whenever $Y^S > Y^B$, tasks $n \in N_2$ are excluded due to Proposition 2.

The rest of the proof is the same as Proposition 4: for $Y^B > Y^S = 0$ for $r \in [\rho_{A1}A, A)$ and $Y^B = Y^S = 0$ for $r \geq A$. 

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A.7 Proof of Corollary 1

Since \(r \in [A/2, (\rho_{A1} - \rho_{M1})A/(1 - \rho_{M1})]\) and \(\rho_{A1} < 1\), the proof of Proposition 1 implies \(\bar{\beta} = 2(A - r)/M\). Then

\[
a_2\beta - \frac{m_2\beta^2}{2} = \frac{2a_2(A - r)}{M} - \frac{2m_2(A - r)^2}{M^2} = \frac{2(A - r)}{M} [-(\rho_{A1} - \rho_{M1})A + (1 - \rho_{M1})r],
\]

which is negative since \(r < (\rho_{A1} - \rho_{M1})A/(1 - \rho_{M1}) < A\).

A.8 Proof of Proposition 6

Let \(\hat{\beta}(\rho_{A1})\) be defined as (9). It is easy to check that \(\hat{\beta}(\rho_{A1})\) is continuous in \(\rho_{A1}\).

Suppose first \(\rho_{M1} \leq 1/2\). Then Propositions 3, 4, and 5 imply that task separation is strictly preferred if \(r < \hat{\beta}(\rho_{A1})\) unless the agents are perfectly balanced (i.e., \(\rho_{A1} \neq \rho_{M1}\)) and task bundling is strictly preferred if \(r \in (\hat{\beta}(\rho_{A1}), A)\). Note that since \(\rho_{M1} \leq 1/2 \leq \rho_{A1}\), \(\max\{\rho_{M1}/\rho_{A1}, (1 - \rho_{M1})/(1 - \rho_{A1})\} = (1 - \rho_{M1})/(1 - \rho_{A1})\). Then \(\hat{\beta}(\rho_{A1})\) is increasing in \(\rho_{A1}\). Furthermore, since \(\rho_{A1} \in [1/2, 1)\), \(\hat{\beta}(\rho_{A1}) \in [(1 - \rho_{M1})A/2, A)\). Suppose \(r \leq (1 - \rho_{M1})A/2\). Then \(r \leq \hat{\beta}(\rho_{A1})\) for any \(\rho_{A1} \in [1/2, 1)\) and it holds with equality only if \(\rho_{A1} = 1/2\), which is never satisfied for \(\rho_{A1} \neq \rho_{M1}\) since \(\rho_{M1} \leq 1/2\). Therefore, we have \(r < \hat{\beta}(\rho_{A1})\) for any \(\rho_{A1} \in [1/2, 1) \setminus \{\rho_{M1}\}\), in which task separation is strictly preferred. For \(r \in ((1 - \rho_{M1})A/2, A)\), let \(\hat{\beta}(\rho_{A1}) = \hat{\beta}^{-1}(r) \in (1/2, 1)\). Then for \(\rho_{A1} > \hat{\beta}(\rho_{A1})\), we obtain \(r < \hat{\beta}(\rho_{A1})\), implying that task separation is strictly preferred. Similarly, for \(\rho_{A1} < \hat{\beta}(\rho_{A1})\), since \(r > \hat{\beta}(\rho_{A1})\), task bundling is strictly preferred. For \(r \in [A/2, A)\), since \(\hat{\beta}((1 + \rho_{M1})/2) = A/2 \leq r\) and \(\hat{\beta}^{-1}(\cdot)\) is increasing, \(\hat{\beta}^{-1}(r) \geq \hat{\beta}^{-1}(A/2) = (1 + \rho_{M1})/2\). Hence \(\rho_{A1} > \hat{\beta}(\rho_{A1})\) implies \(\rho_{A1} > (1 + \rho_{M1})/2\). Furthermore, since \(\rho_{A1} > \hat{\beta}(\rho_{A1})\) is equivalent to \(\hat{\beta}(\rho_{A1}) > r\), \(\rho_{A1} > (1 + \rho_{M1})/2\) implies \(r < A(\rho_{A1} - \rho_{M1})/(1 - \rho_{M1})\). Then Proposition 5 implies that task \(n \in N_2\) is excluded for \(r \in [A/2, A)\).

Suppose next \(\rho_{M1} > 1/2\). Then \(\hat{\beta}(\rho_{A1})\) is expressed as

\[
\hat{\beta}(\rho_{A1}) = \begin{cases} \frac{\rho_{M1}A}{4\rho_{A1}} & \text{if } \rho_{A1} \in \left[\frac{1}{2}, \rho_{M1}\right], \\ \frac{(1 - \rho_{M1})A}{4(1 - \rho_{A1})} & \text{if } \rho_{A1} \in \left(\rho_{M1}, \frac{1 + \rho_{M1}}{2}\right], \\ \frac{A(\rho_{A1} - \rho_{M1})}{1 - \rho_{M1}} & \text{if } \rho_{A1} \in \left(\frac{1 + \rho_{M1}}{2}, 1\right). \end{cases}
\]

which is decreasing in \(\rho_{A1} \in [1/2, \rho_{M1})\) and increasing in \(\rho_{A1} \in (\rho_{M1}, 1)\). Note that \(\hat{\beta}(1/2) = \rho_{M1}A/2\), \(\hat{\beta}(\rho_{M1}) = A/4\), and \(\lim_{\rho_{A1} \to 1} \hat{\beta}(\rho_{A1}) = A\). For \(r \leq A/4\), since \(r < \hat{\beta}(\rho_{A1})\) for any \(\rho_{A1} \in [1/2, 1) \setminus \{\rho_{M1}\}\), Propositions 4 and 5 imply that task separation is strictly
preferred. For $r \in (A/4, \rho_{M1}A/2]$, there exactly exist two values, $\rho_{A1} \in [1/2, \rho_{M1})$ and $\bar{\rho}_{A1} \in (\rho_{M1}, 1)$ such that $\hat{r}(\rho_{A1}) = r$ for $\rho_{A1} = \rho_{A1}' = \bar{\rho}_{A1}$. By the shape of $\hat{r}(\rho_{A1})$, $r < \hat{r}(\rho_{A1})$ if and only if $\rho_{A1} < \rho_{A1}'$ or $\rho_{A1} > \bar{\rho}_{A1}'$, in which task separation is strictly preferred. Similarly, we see that $r > \hat{r}(\rho_{A1})$ if and only if $\rho_{A1} < \rho_{A1}' < \rho_{A1}'$, in which task bundling is strictly preferred.

For $r \in (\rho_{M1}A/2, A)$, the shape of $\hat{r}(\rho_{A1})$ implies that there uniquely exists $\hat{\rho}_{A1} \equiv \hat{r}^{-1}(r) \in (\rho_{M1}, 1)$ and for $\rho_{A1} > \hat{\rho}_{A1}$, we obtain $r < \hat{r}(\rho_{A1})$, implying that task separation is strictly preferred. Similarly, for $\rho_{A1} < \hat{\rho}_{A1}$, since $r > \hat{r}(\rho_{A1})$, task bundling is strictly preferred. Finally, for $r \in [A/2, A)$, the same proof as the case of $\rho_{M1} \leq 1/2$ shows that task $n \in \mathcal{N}_2$ is excluded if $\rho_{A1} > \hat{\rho}_{A1}$.

References


