Does a narrower product range mean lower welfare? Horizontal merger analysis with endogenous product range choice*

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Abstract

We examine how a bilateral merger between multi-product large firms with the endogenous choice on product range affects the market structure and consumer welfare. In the presence of single product small firms, we show that on certain conditions, low merger efficiencies in the form of a moderate marginal cost synergy for the merging firms (insider), which lead to the shrinkage of the insider’s product range, are beneficial to consumer welfare. By contrast, high merger efficiencies, which induce the expansion of the insider’s product range, are detrimental to consumer welfare. Our findings suggest that high merger efficiencies may be anti-competitive in the market with large and small firms.

Keywords: merger; product choice; big firms; small firms
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1 Introduction

Antitrust agencies usually regard merger efficiencies and the likelihood of post-merger entry as two important elements to counteract the harmful effects on competition in the competitiveness assessment. As the European Commission’s horizontal merger guidelines (2004, paragraph 79) require,

the relevant benchmark in assessing efficiency claims is that consumers will not be worse off as a result of the merger. For that purpose, efficiencies should be substantial...

Moreover, the US Horizontal Merger Guidelines consider entry into the relevant market as part of the full assessment of competitive effect:

The prospect of entry into the relevant market will alleviate concerns about adverse competitive effects only if such entry will deter or counteract any competitive effects of concern so the merger will not substantially harm customers.

These merger guidelines suggest that a merger is likely to be procompetitive if merger efficiencies are substantial or the prospect of entry is high in the relevant market.

In this paper, we challenge the conventional wisdom that associates competitiveness with the degree of merger efficiency in a mixed market structure. We establish a theoretical framework to characterize the market with the coexistence of multi-product (MP) large firms and single-product (SP) small firms. Each large firm is a multi-product one that strategically supplies a continuum of varieties, and the variety range it offers is a choice variable. A small firm is modeled as an infinitesimal SP firm with free entry and exit. We examine how a bilateral merger between MP firms with post-merger product choice changes the mixed market structure and impacts consumer welfare. We find that merger efficiencies generate non-monotonic effects on consumer welfare. Specifically, high merger efficiencies harm consumer welfare, while moderate merger efficiencies enhance consumer welfare. This surprising result arises from the endogenous post-merger product choice by the MP large firm, the endogenous behavior of SP small firms and the different
competition patterns between large and small firms. We explain the details as follows.

First, if the marginal cost is the same across MP large firms and SP small firms, a SP small firm behaves more aggressively than a large firm, that is, a small firm’s quantity is larger than a large firm’s per-variety quantity. A small firm’s decision has a negligible impact on the aggregate output, so it does not internalize the negative effect of its own production. On the contrary, a large firm has a non-negligible impact on the aggregate quantity of the market, hence behaving like an oligopolist and internalizing the negative effect of its own production on the aggregate output. Furthermore, as an MP firm, a large firm also internalizes the externalities among the varieties. These two internalization effects make each MP large firm less aggressive than each SP small firm.

Second, the comparison between the marginal cost synergy and fixed cost savings from product range shrinkage determines whether the merged large firm (insider) expands or shrinks its variety range. On the one hand, the marginal cost synergy reduces the cost per unit of good and hence encourages an expansion of the insider’s variety range. On the other hand, a shrinkage of variety range would save per-variety fixed cost and weaken the cannibalization effect. Therefore, if the marginal cost synergy is moderate compared to the per-variety fixed cost, it is more profitable for the insider to shrink its product range; and if the marginal cost synergy is sufficiently large, the insider would make more profit by expanding its product range.

Third, the change in consumer welfare varies with the endogenous product choice of the insider. The change in consumer welfare depends on the replacement effect and the variety effect. With a moderate marginal cost synergy, the large insider shrinks its variety range, then the competitive fringe expands, and thus, a portion of large firms’ product range is replaced by that of small firms. As shown earlier, a small firm behaves more aggressively than a large firm. Consequently, the expansion of the competitive fringe generates a positive replacement effect on consumer welfare. However, the shrinkage of the insider’s variety range reduces the total product range, hence resulting in a negative variety effect. We show that the replacement effect dominates the variety effect, so a small marginal cost synergy, which leads to a shrinkage of the large insider’s variety range, actually
improves consumer welfare in spite of the reduction in the number of varieties. This is because small firms, which are more aggressive than large ones, occupy a larger share of the market after merger. In contrast, a large marginal cost synergy, which expands the insider’s variety range but induces the exit of small firms, is harmful to consumer welfare. Taking the endogenous post-merger product choice of the large insider into account, our welfare results imply that a moderate post-merger cost synergy may improve consumer welfare owing to the presence of more aggressive small firms.

Our results also explain the varying post-merger product choice in the convenience store market in Japan, which is featured with the mixed market structure. Among the famous merger cases in the Japanese convenience store market, firms made different choices on the product range after merger. Circle K and Sunkus, which used to be the fourth and fifth largest convenience store in Japan, merged in 2004. The merged entity, which was named the Uny Group Holdings Co., maintained both brands. Nevertheless, the Uny Group did not operate well after the merger, and Circle K and Sunkus did not see much growth in their profit.\(^1\) In 2016, FamilyMart merged with the Uny Group and shrank its product range, consolidating stores under the Circle K and Sunkus chains into FamilyMart. The brand consolidation is still in progress, but the merged group has already benefited from the resulting reduction in cost and rise in profits per store.\(^2\) As indicated from our model, it is profitable for the merged entity to shrink its product range when merger efficiencies are moderate. This result fits with the sluggish growth in profit after the merger between Circle K and Sunkus and the success of the merger between FamilyMart and the Uny Group.

In the literature, theoretical analyses of merger efficiencies are typically conducted in the framework of Cournot oligopoly.\(^3\) Davidson and Mukherjee (2007) consider merger with endogenous entry in a Cournot oligopoly with a homogeneous product, showing that a reduction in marginal costs after a merger always improves

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\(^1\)Nishida and Yang (2015) empirically reveal the diminished ability to retain profitability after Circle-K and Sunkus merged.


\(^3\)An extensive literature on merger assumes oligopoly without entry, such as Farrell and Shapiro (1990), Levin (1990), Froeb and Werden (1998), and Amir et al. (2009).
consumer welfare.\textsuperscript{4} Erkal and Piccinin (2010) examine the merger in differentiated oligopolies with free entry, showing that only high merger efficiencies that induce the exit of outsiders enhance consumer welfare. In contrast to this literature, our analysis demonstrates that high merger efficiencies may be anti-competitive in the market with large and small firms.\textsuperscript{5} Chen and Li (2018) demonstrate that merger efficiency may raise price with the introduction of different degrees of substitution across the insider and outsiders. We also find that merger efficiency may reduce consumer welfare, but the driving factor to our result is the endogenous product choice of the MP large insider and the asymmetric behavior between MP large firms and SP small firms. Furthermore, our results also imply that the degree of merger efficiency is crucial for the post-merger consumer welfare. Last, our model also theoretically complements to these works by taking into account the endogenous product choice of multi-product firms, which also has strong empirical implications.\textsuperscript{6}

Our modelling approach is related to several strands of literature on markets with large and small firms. The first strand is the dominant firm model (e.g., Markham, 1951; Chen, 2003; Gowrisankaran and Holmes, 2004). The dominant firm is large since it is the leader and the price-maker, while the price-taking followers are small. The second strand employs the Stackelberg model (Etro, 2004; Etro, 2006; Ino and Matsumura, 2012). In this model, the first mover is large due to the commitment power in the market. The third strand characterizes the mixed market structure by connecting oligopolistic and monopolistic competition (Shimomura and Thisse, 2012; Parenti, 2018; Pan and Hanazono, 2018). This paper is most closely linked to the third strand, but focuses on a completely

\textsuperscript{4}Spector (2003) also analyses the effects of mergers with entry in a model with homogeneous products and Cournot competition. Allowing for heterogeneity in costs across firms, he finds that without cost synergies, all profitable mergers must harm consumer welfare. In contrast, Davidson and Mukherjee (2007) assume that all firms are symmetric, so the set of mergers analyzed by Spector (2003) is empty in their model.

\textsuperscript{5}Two related papers which also analyse the effects of mergers with differentiated products and entry are Werden and Froeb (1998) and Cabral (2003). Gowrisankaran (1999) develops a dynamic Cournot game with endogenous investment, merger, entry and exit decisions. His computational analysis suggests that mergers’ anticompetitive effects are unlikely to be reversed by entry.

\textsuperscript{6}Lommerud and Sogard (1997) and Gandhi et al. (2008) also analyze post-merger product re-positioning, but they consider this issue in an oligopoly with an exogenous number of firms. In contrast, we consider a different market structure with the endogenous entry of SP small firms.
different issue.\textsuperscript{7}

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 conducts the equilibrium analysis before and after merger to examine the impact of merger efficiencies on the endogenous product choice of the insider, the mixed market structure and consumer welfare. Section 4 concludes.

2 Model

Consider a closed economy with two goods, a homogeneous good and a horizontally differentiated good. Labor is the only production factor. The homogeneous good is produced with a constant returns to scale technology, requiring one unit labor to produce one unit output. In the differentiated good market, there are a continuum of SP small firms and a few MP large firms. The large and small firms differ in the following three respects. First, each large firm has a non-negligible impact on the market and competes in an oligopolistic manner, whereas each small firm is negligible in the market and behaves as a monopolistic competitor. Second, each large firm produces a range of varieties and strategically chooses both the product range and the quantity of each variety, while each small firm produces only one variety of product. Third, the number of large firms is exogenous, while the number of small firms is endogenously determined by free entry and exit. The second and third aspects imply that the introduction of new varieties by the large firms is equivalent to the entry by small firms. \textsuperscript{8}

2.1 Preferences and Demand

The utility of the representative consumer is described by

$$U = \alpha \int_0^M q_S(i)di + \sum_{n=1}^N \int_0^{\omega_n} q^n_L(x)dx - \frac{\beta}{2} \left[ \int_0^M (q_S(i))^2di + \sum_{n=1}^N \int_0^{\omega_n} (q^n_L(x))^2dx \right]$$

$$-\frac{\gamma}{2} \left[ \int_0^M q_S(i)di + \sum_{n=1}^N \int_0^{\omega_n} q^n_L(x)dx \right]^2 + g_0,$$

\textsuperscript{7}Shimomura and Thisse (2012) and Pan and Hanazono (2018) examine the impact of a large firm’s entry, and Parenti (2018) investigates the impact of trade liberalization.

\textsuperscript{8}Our modelling strategy draws on Parenti (2018) and Pan and Hanazono (2018).
where the total mass of small firms is $M$, describing the competitive fringe, and $q_S(i)$ is the quantity of small firm $i$ with $i \in [0, M]$; the output of each small firm is of zero measure. The total number of the incumbent large firms is $N \geq 2$, which is exogenously given. Large firm $n$ ($n = 1, \ldots, N$) provides multiple varieties of products, with the product range represented by $\omega_n \geq 0$, which is chosen by large firm $n$. The quantity of variety $\omega \in [0, \omega_n]$ is $q^n_L(x)$. Here, we treat $\omega_n$ continuously. The output of the homogeneous good is $q_0$, which is treated as the numeraire. Demand parameters $\alpha$, $\beta$ and $\gamma$ are all positive. The parameters $\alpha$ and $\gamma$ capture the substitution pattern between the differentiated varieties and the numeraire: an increase in $\alpha$ or a decrease in $\gamma$ shift out the demand for the differentiated good relative to the numeraire. The parameter $\beta$ indexes the degree of differentiation between the differentiated varieties. The degree of differentiation increases with $\beta$ as a higher $\beta$ indicates stronger preferences for diversified consumption.

The representative consumer’s budget constraint is

$$\int_0^M p_S(i)q_S(i)di + \sum_{n=1}^N \int_0^{\omega_n} p^n_L(\omega)q^n_L(x)dx + q_0 = I,$$

where $p_S(i)$ and $p^n_L(\omega)$ are the prices of the small firm $i$’s and large firm $n$’s variety $\omega$, respectively. The price of the numeraire is normalized to 1. The representative consumer’s income is $I$, which is exogenously given. The inverse demand functions facing small firms and large firms are determined by the maximization of the consumer’s utility subject to the budget constraint:

The inverse demand functions for large and small firms’ products are:

$$p^n_L(\omega) = \alpha - \beta q^n_L(x) - \gamma Q \quad n = 1, \ldots, N, \quad (1)$$

$$p_S(i) = \alpha - \beta q_S(i) - \gamma Q \quad i \in [0, M]. \quad (2)$$

where $Q = \int_0^M q_S(i)di + \sum_{n=1}^N \int_0^{\omega_n} q^n_L(x)dx$ is the aggregate output quantity of the differentiated good.

\footnote{If $\beta = 0$, the varieties are perfect substitutes and consumers only care about total consumption.}
2.2 Firms

Both MP large and SP small firms incur variable costs and per-variety fixed costs. Because our focus is the post-merger cost synergy, we assume that each MP large firm incurs a constant cost $C$ and each SP small firm incurs a marginal cost $c$ before merger. The fixed cost may differ across the two types of firms. For an MP large firm, adding one more variety requires an additional fixed cost $F$. The fixed cost of an SP small firm is denoted by $f$.

2.2.1 MP Large Firm

The profit of an MP large firm is expressed by

$$\Pi^n_L = \int_0^{\omega_n} (p^m_L(\omega) - C)q^n_L(x)dx - \omega_n F.$$  

Substituting $p^m_L(\omega)$ from equation (1) into the above profit function, $\Pi^n_L(\omega_n, q^n_L(\cdot))$ can be rewritten as

$$\Pi^n_L(\omega_n, q^n_L(\cdot)) = (\alpha - C) \int_0^{\omega_n} q^n_L(x)dx - \beta \int_0^{\omega_n} (q^n_L(x))^2dx$$  

$$- \gamma \left( \int_0^{\omega_n} q^n_L(x)dx \right)^2 - \gamma Q^{-n} \int_0^{\omega_n} q^n_L(x)dx - \omega_n F.$$  

Each large firm takes as given the total output of other firms, $Q^{-n}$, defined by $Q^{-n} \equiv \int_0^M q_S(i)di + \sum_{k \neq n} \int_0^{\omega_k} q^k_L(x)dx$, and maximizes its profit with respect to both its product range, $\omega_n$, and the quantity of each variety, $q^n_L(x)$. A large firm understands that its decision has a non-negligible impact on the aggregate output $Q$.

2.2.2 Single Product Small Firm

The profit of a small firm is expressed by

$$\Pi_S(i) = (p_S(i) - c)q_S(i) - f.$$
Plugging $p_S(i)$ from equation (2) into the above profit function, $\Pi_S(i)$ can be rewritten as

$$\Pi_S(i) = (\alpha - c)q_S(i) - \beta[q_S(i)]^2 - \gamma Q q_S(i) - f.$$  

(4)

Each small firm maximizes its profit with respect to its quantity $q_S(i)$, taking as given the aggregate output $Q$. Different from a large firm, a small firm is negligible in the market, and thus, understands that its decision does not affect the aggregate output $Q$.

The free entry and exit of small firms pins the equilibrium profit of the small firm to zero:

$$\Pi_S(i) = (\alpha - c)q_S(i) - \beta[q_S(i)]^2 - \gamma Q q_S(i) - f = 0.$$  

(5)

3 The Market Outcome

3.1 Without Merger

3.1.1 Derivation of Mixed Market Equilibrium

Single Product Small Firms’ Profit Maximization and Entry A small firm only accounts for the impact of the market’s total production because its own impact on the market is negligible. Thus, it does not internalize its externality in its production. The small firm maximizes its profit given by equation (4) with respect to its output $q_S(i)$. The first-order condition of a small firm with respect to its output $q_S(i)$ is

$$\alpha - c - 2\beta q_S(i) - \gamma Q = 0,$$

which implies that

$$q_S(i) = q_S = \frac{\alpha - c - \gamma Q}{2\beta}.$$  

(6)
which is determined only by the aggregate output $Q$. All small firms choose the same level of output.

Using equation (2), the price of the small firm can be expressed by

$$p_S(i) = \frac{\alpha + c - \gamma Q}{2}.$$  (7)

Accordingly, the equilibrium price of the small firm decreases with the mass of small firms and the total output of large firms.

Entry and exit are free for small firms. Using equation (5) after plugging in (6), we can express the mass of small firms $M$ as a function of the total output of large firms $Q_L$:

$$M(Q_L) = \frac{1}{c} \left[ \sqrt{\frac{\gamma}{f}} (\alpha - c - \gamma Q_L) - 2\beta \right].$$  (8)

As shown by the above expression, the mass of small firms decreases with the total output of large firms.

Substituting (8) into (6), the optimal quantity of each small firm is

$$q^*_S = \sqrt{\frac{f}{\beta}}.$$

Owing to free entry and exit, the quantity produced by a small firm is independent of the behavior of large firms. In other words, the aggregate behavior of small firms responds to the change in the market condition only by adjusting the competitive fringe. (See also Lemma 1.)

By (6) and (7), the equilibrium price of small firms is

$$p^*_S = c + \sqrt{\beta f}.$$

**Large Firms’ Profit Maximization and Variety Choice.** Unlike small firms, large firms impose a non-negligible impact on the market. Large firm $n$ maximizes its profit given by equation (3) with respect to its output. The first-order condition of large firm $n$ with respect to the quantity of its $\omega$th variety,
\( q^n_L(x) \), is\(^{10} \)

\[
\alpha - C - 2\beta q^n_L(x) - 2\gamma \int_0^{\omega_n} q^n_L(v)dv - \gamma Q^{-n} = 0. 
\]

(9)

By symmetry, we have

\[
\int_0^{\omega_n} q^n_L(v)dv = \omega_n q^n_L.
\]

Substituting the above symmetry property into equation (9), we solve the optimal quantity of each variety \( q^n_L \) as a function of the expected aggregate output of small firms \( (Q_S) \), the expected output of other large firms \( (Q^{-n}_L) \), and its own product range \( (\omega_n) \):

\[
q^n_L = \frac{\alpha - C - \gamma Q^{-n}}{2(\beta + \gamma \omega_n)}.
\]

(10)

Everything else being equal, an increase in firm \( n \)'s product range (larger \( \omega_n \)) results in a decrease in the quantity of each variety (smaller \( q^n_L \)), implying the cannibalization effect.

Large firm \( n \) also maximizes its profit with respect to its product range. The first-order condition of large firm \( n \) with respect to its product range, \( \omega_n \), is

\[
(\alpha - C - (\beta + 2\gamma \omega_n)q^n_L - \gamma Q^{-n})q^n_L = F.
\]

(11)

From equations (10) and (11), we obtain the optimal per-variety output for the large firm:

\[
q^{n*}_L = \sqrt{\frac{F}{\beta}},
\]

which is determined only by the fixed cost of large firms and the demand parameter. This equation implies that \( q^{n*}_L = q^*_L \) for each \( n = 1, ..., N \). Thus, \( Q^{-n} = Mq^*_S + \sum_{k \neq n} \omega_k q^*_L \), and equation (11) can be expressed as

\[
2(\beta + \gamma \omega_n) = \sqrt{\frac{\beta}{F}}[\alpha - C - \gamma(Mq^*_S + \sum_{k \neq n} \omega_k q^*_L)].
\]

(12)

Substituting \( q^*_L = \sqrt{F/\beta} \) and \( q^*_S = \sqrt{f/\beta} \), the above equation can be rearranged

\(^{10}\)In Appendix A, we show that \( q^{n*}_L \) and \( \omega^*_n \) derived from (10) and (14) are locally optimal.
\[
\gamma \omega_n = \sqrt{\frac{\beta}{F}[\alpha - C - \gamma (M \sqrt{f} + \sum_{k=1}^{N} \omega_k \sqrt{F})]} - 2\beta, \quad (13)
\]

which implies that \(\omega_n = \omega\) for each \(n = 1, \ldots, N\). Hence, we can express the product range \(\omega\) as a function of the expected mass of small firms \(M\):

\[
\omega(M) = \frac{\sqrt{\beta / F}(\alpha - C - \gamma M \sqrt{f / \beta}) - 2\beta}{\gamma (N + 1)}. \quad (14)
\]

**Mixed Market Equilibrium**  The above analysis indicates that firms behave symmetrically within the group of large firms and that of small firms. Therefore, (8) can be re-expressed as

\[
M(\omega) = \frac{\sqrt{\beta / f}(\alpha - c - \gamma \omega \sqrt{F / \beta}) - 2\beta}{\gamma}. \quad (15)
\]

Equations (15) and (14) determine the equilibrium mass of small firms \(M^*\) and the equilibrium product range of a large firm \(\omega^*\). A mixed market equilibrium exists if and only if \(M^* > 0\) and \(\omega^* > 0\). Assuming this, the mass of small firms and product range of large firms in equilibrium are

\[
M^* = \sqrt{\frac{\beta \alpha - [c(N + 1) - CN] - 2\sqrt{\beta}[(N + 1)\sqrt{f} - N\sqrt{F}]}{\gamma}},
\]

\[
\omega^* = \sqrt{\frac{\beta 2\sqrt{\beta}(\sqrt{f} - \sqrt{F}) + (c - C)}{\gamma}}.
\]

Plugging \(M^*, \omega^*, q^*_S\) and \(q^*_L\) into equation (1), the price of the large firm in equilibrium is

\[
p^*_L = c + 2\sqrt{\beta f} - \sqrt{\beta F}.
\]

Substituting the equilibrium range of varieties \(\omega^*\), the output of each variety \(q^*_n\) and the equilibrium price of large firms \(p^*_L\) into equation (3), we obtain the equilibrium profit of the large firm:

\[
\Pi^*_L = \frac{[2\sqrt{\beta}(\sqrt{f} - \sqrt{F}) + (c - C)]^2}{\gamma}.
\]
The total output is

\[ Q^* = \frac{\alpha - c - 2\sqrt{\beta f}}{\gamma}, \]

which is independent of the number of MP large firms.

To ensure the coexistence of MP large and SP small firms before and after merger, we assume the following conditions throughout the paper.

**Assumption 1**

(i) \( \alpha > \max\{c(N + 1) - CN + 2\sqrt{\beta f}(N + 1)\sqrt{f} - N\sqrt{F}, cN - C(N - 2) + 2\sqrt{\beta f}|N\sqrt{f} - (N - 1)\sqrt{F}|, cN - C(N - 2) + 2\sqrt{\beta f}|N\sqrt{f} - (N - 1)\sqrt{F}| \}. \)

(ii) \( c + 2\sqrt{\beta f} > C + 2\sqrt{\beta F}. \)

The first condition, which guarantees the existence of small firm both before and after merger, requires that the market size should be sufficiently large. The second condition, which ensures that each MP large firm supplies a positive product range, i.e., \( \omega^* > 0 \), requires that the fixed cost of a single-product small firm is larger than that of a MP large firm. This condition ensures that a large firm enjoys a cost advantage over the small firm or economies of scope by supplying a range of varieties. If \( c = C \) and \( f = F \), then a large firm would rather be a single-product firm.

### 3.2 With Merger

If two MP large firms merge, there are \((N - 1)\) MP large firms, including one insider and \((N - 2)\) MP outsiders. The insider enjoys a marginal cost synergy of \((1 - \lambda)C\), hence the marginal cost is reduced to \(\lambda C\), with \(0 < \lambda < 1\).11 Furthermore, The MP insider is able to readjust its product range, which would change the total fixed cost of supplying varieties. We denote the variables after merger with the superscript “\(m\)”.

#### 3.2.1 Outsiders

**SP Small Outsiders** The bilateral merger between MP firms does not affect the behavior of each SP outsider. After merger, the equilibrium quantity of each

\[11\] We do not consider the synergies on per-variety fixed costs, which would complicate the analysis without adding more insight. We concentrate on marginal cost synergies, which allow us to generate clear results especially for the welfare consequences.
SP outsider is also determined by equations (5) and (6), which are the same conditions that pin down the equilibrium quantity of each SP outsider before merger. Therefore, \( q_{S}^{m*} = q_{S}^{*} = \sqrt{f/\beta} \).

Given \( q_{S}^{m*} \), we also obtain the equilibrium aggregate output \( Q^{m} \) by equation (7):

\[
Q^{m*} = \frac{\alpha - c - 2\sqrt{\beta f}}{\gamma},
\]

which is the same as the equilibrium aggregate output before merger \( Q^{*} \).

Given \( Q^{m*} \) and \( q_{S}^{m*} \), we can derive the equilibrium price of a small firm with the small firm’s inverse demand:

\[
p_{S}^{m*} = \sqrt{\beta f} + c,
\]

which is the same as the equilibrium price of an SP small firm before merger \( p_{S}^{*} \).

The results of the SP small firms are in line with the traditional monopolistic competition model. Here the SP small firms can be considered as monopolistic competitors. The free entry and exit of SP small firms shifts the demand curve such that there is only one equilibrium quantity at which the marginal revenue is equal to the marginal cost and the price is equal to the average cost. Furthermore, the free entry and exit of SP small firms wash out the impact of merger on the aggregate output, which is the same before and after merger.

**MP Large Outsiders**  To examine the impact of merger on MP large outsiders, first note that the per-variety quantity of each MP large outsider does not change, that is, \( q_{L}^{m*} = q_{L}^{*} = \sqrt{F/\beta} \). Since the profit maximization of each MP outsider after merger is the same as that before merger, the per-variety output is also determined by equations (9) and (11).

Moreover, the post-merger profit maximization of the MP outsider with respect to its variety range \( \omega^{m} \), which is also expressed by equation (11), can be rearranged as

\[
\gamma \omega^{m*} = \sqrt{\beta F} (\alpha - C - 2\sqrt{\beta F} - \gamma Q^{m}).
\]

The post-merger variety range of an MP outsider \( \omega^{m} \) only depends on the aggregate output \( Q^{m*} \). Because \( Q^{m*} = Q^{*} \), as shown earlier, \( \omega^{m*} = \omega^{*} \).
Because \( q^*_{Lm} = q^*_L \) and \( Q^m = Q^* \), by the inverse demand (1), we have \( p^*_{Lm} = p^*_L \), that is, the post-merger price of each MP large outsider is the same as the pre-merger one.

Therefore, the bilateral merger between MP firms does not affect the behavior of each MP large outsider.

We conclude that when large and small firms coexist, neither merger nor post-merger product choice exert any impact on the behavior of each individual (SP and MP) outsider or the aggregate output. The following lemma summarizes the results.

**Lemma 1** The merger between two MP large firms has no impact on (i) SP small firms’ behavior (that is, \( p^m_S = p^*_S \) and \( q^m_S = q^*_S \)) (ii) the aggregate output (that is, \( Q^m = Q^* \)) and (iii) the MP outsiders’ behavior (that is, \( p^m_L, \omega^m \) and \( q^m_L \)). the individual behavior of SP and MP outsiders do not change.

### 3.2.2 MP Insider

The first-order condition of the insider with respect to the quantity of its \( \omega \)th variety, \( q^m_I(\omega) \), is

\[
\alpha - \lambda C - 2\beta q^m_I(x) - 2\gamma \int_0^{\omega^m_I} q^m_I(v)dv - \gamma Q^m_{-I} = 0,
\]

which also implies that \( q^m_I(x) = q^m_I \) for each \( x \in [0, \omega^m_I] \). The variable \( Q^m_{-I} = \int_0^{\omega^m_I} q^m_S(i)di + \sum_{k=1}^{N-2} \int_0^{\omega^m_L} q^m_Lk(v)dv \) is the total output of all the outsiders.

Using the envelope theorem, the first-order condition of the insider with respect to its product range, \( \omega^m_I \), is

\[
2(\beta + \gamma \omega^m_I) = \sqrt{\frac{\beta}{F}} (\alpha - \lambda C - \gamma Q^m_{-I}).
\]

By the above two equations, we obtain the optimal output per variety for the large firm:

\[
q^*_{Im} = q^*_L = \sqrt{\frac{F}{\beta}}.
\]
The insider’s product range is changed to

$$\omega_{I}^{m^*} = \sqrt{\frac{2\sqrt{3}(\sqrt{F} - \sqrt{F}) + (c - \lambda C)}{F}}.$$

Consequently, the insider’s profit is

$$\Pi_{I}^{m^*} = \left[2\sqrt{3}(\sqrt{F} - \sqrt{F}) + (c - \lambda C)\right]^{2}.$$

A profitable merger implies $$\Pi_{I}^{m^*} - 2\Pi_{L}^{*} > 0$$, which requires

$$\lambda < 1 - \frac{(\sqrt{3} - 1)[2\sqrt{3}(\sqrt{F} - \sqrt{F}) + (c - C)]}{C} \equiv \bar{\lambda},$$

where $$\bar{\lambda}C$$ is the maximum marginal cost for the insider to avoid the merger paradox.

Comparing the change in its varieties, the insider expands its product range if $$\omega_{I}^{m^*} > 2\omega_{L}^{*}$$, which requires

$$\lambda < 1 - \frac{2\sqrt{3}(\sqrt{F} - \sqrt{F}) + (c - C)}{C} \equiv \underline{\lambda},$$

where $$\underline{\lambda}C$$ is the maximum marginal cost for the insider to expand its product range.

It is readily verified that $$\bar{\lambda} > \underline{\lambda} > 0$$. The insider’s choice on its product range depends on two effects, the efficiency effect and the inframarginal effect. The efficiency effect stems from the marginal cost synergy, which increases with a smaller $$\lambda$$. In addition, the merger expands the variety range of the insider, which strengthens the cannibalization effect among the varieties. This generates a negative inframarginal effect on the varieties, inducing a contraction of the insider’s product range. When the marginal cost synergy is moderate, i.e., $$\underline{\lambda} < \lambda < \bar{\lambda}$$, the efficiency effect would be dominated by the inframarginal effect, and hence, the insider would shrink its product range. Nevertheless, if the marginal cost synergy is drastic, i.e., $$\lambda < \underline{\lambda}$$, the efficiency effect would dominate the inframarginal effect, which invites an expansion of the insider’s product range.

The following proposition summarizes the profitability of merger with the con-
sideration of product choice.

**Proposition 1** (i) If the merger efficiency is moderate, i.e., \( \lambda < \lambda < \bar{\lambda} \), the two MP large firms would merge with a shrinkage of product range;

(ii) If the merger efficiency is high, i.e., \( \lambda < \lambda \), it is more profitable for the insider to expand its product range.

Proposition 1 indicates when the fixed cost savings from shrinking the product range are high relative to the marginal cost synergy, the insider chooses to shrink its product range because the saving on fixed cost dominates the loss of profits earned from adding more varieties. However, when the marginal cost synergy is relatively strong, the opposite occurs.

### 3.2.3 Market Performance

Having examined individual firms’ behavior, now we investigate how the market structure reacts to merger. While the number of MP large firms decreases by one after merger, it is worth studying the endogenous change in the number of SP small firms.

The pre-merger aggregate output \( Q^* \) and post-merger aggregate output \( Q^m \) are respectively expressed as

\[
Q^* = M^* q^*_S + N \omega^* q^*_L,
\]
\[
Q^m = M^{m*} q^{m*}_S + (N - 2) \omega^{m*} q^{m*}_L + \omega^{m*} I q^{m*}_I.
\]

As shown in Lemma 1, the aggregate output \( Q^{m*} \), the quantity of each SP small firm \( q^{m*}_S \), and the quantity of each MP large outsider \( \omega^{m*} q^{m*}_L \) do not change after merger. Subtracting both sides of the above two expressions and rearranging, we obtain

\[
\Delta V_S q^*_S + \Delta V_L q^*_L = 0,
\]

where \( \Delta V_S = M^{m*} - M^* \) is the change in the number of SP small firms, and \( \Delta V_L = \omega^{m*} - 2 \omega^* + (N - 2)(\omega^{m*} - \omega^*) = \omega^{m*} - 2 \omega^* \) is the change in the variety range of MP large firms. Equation (16) implies that \( \Delta V_S > 0 \) iff \( \Delta V_L < 0 \), i.e.,
the expansion of the competitive fringe must be accompanied with the shrinkage of the insider’s product range.

The intuition is as follows. When the insider chooses to shrink its product range with moderate merger efficiencies, it produces less than the sum of the two merging firms’ outputs before the merger, leaving more market for other firms. Hence, more SP small firms enter the market.

Proposition 2 summarizes the impact of merger efficiencies on the mass of SP small firms.

**Proposition 2** Moderate merger efficiencies, which induce a shrinkage of the insider’s product range, increase the mass of SP small firms. By contrast, high merger efficiencies, which induce an expansion of the insider’s product range, reduce the mass of SP small firms.

Furthermore, any profitable merger would increase the market concentration. The HHI without merger is defined by

\[
HHI = \sum_{n=1}^{N} (\omega_n q^n_L)^2.
\]

Since each SP firm produces a negligible quantity, SP firms are not accounted into the HHI. Owing to the symmetric behavior of MP firms in equilibrium, HHI without merger can be expressed by

\[
HHI^* = N(\omega^* q^*_L)^2.
\]

Similarly, substituting into the equilibrium values with merger, the HHI with merger is expressed by

\[
HHI^{m*} = (N - 2)(\omega^* q^*_L)^2 + (\omega^{m*}_I q^*_L)^2.
\]

In consequence, the difference in the HHI with and without merger is

\[
\Delta HH = HH^{m*} - HH^* = [(\omega^{m*}_I)^2 - 2(\omega^*)^2](q^*_L)^2;
\]

which is positive iff \(\omega^{m*}_I > \sqrt{2}\omega^*\). Since a profitable merger requires that \(\Pi^{m*}_I > \)
2\Pi^*_L^*, which can be expressed as \( \gamma(\omega^{m*}_L q^*_L)^2 > 2 \gamma(\omega^*_L q^*_L)^2, \omega^{m*}_L > \sqrt{2} \omega^*_L \) always holds for a profitable merger. Therefore, any profitable merger always raises the HHI. We summarize the impact on market concentration in the following proposition.

**Proposition 3** Any profitable merger is HHI increasing.

Now we consider the impact on consumer welfare. The change in consumer welfare depends on two effects, the replacement effect and the variety effect:

\[
\Delta U = \frac{\beta}{2} \Delta V_S [(q^*_S)^2 - (q^*_L)^2] + \frac{\beta}{2} \Delta V (q^*_L)^2,
\]

The first effect on consumer welfare is the replacement effect. A change in the large MP insider’s product range would result in the replacement of varieties between large MP and small SP firms, the sign of which depends on the comparison of per-variety quantity between large MP firms and small SP firms in the replaced portion of variety range. In addition, the consumer prefers diversified consumption, so a merger that changes the aggregate variety range would generate a variety effect on consumer welfare.

With a moderate marginal cost synergy, the large MP insider shrinks its variety range, then the competitive fringe expands (\( \Delta V_S > 0 \)), and thus, a portion of large MP firms’ product range is replaced by that of small firms. As shown earlier, the per-variety quantity of a small firm and that of a large firm depend on their fixed costs, \( f \) and \( F \), respectively. Thus, the sign of the replacement effect depends on the comparison between \( f \) and \( F \). If a small firm’s fixed cost is larger than a large firm’s, i.e., \( f > F \), then a small firm behaves more aggressively than a large firm, i.e., \( q^*_S > q^*_L \), and hence, the expansion of the competitive fringe generates a positive replacement effect on consumer welfare. However, equation (16) implies that the shrinkage of the insider’s variety range reduces the total product range (\( \Delta V < 0 \)), hence resulting in a negative variety effect. We show that the replacement effect dominates the variety effect, so a moderate marginal cost synergy, which leads to a shrinkage of the large insider’s variety range, actually improves consumer welfare in spite of the net decrease in the mass of varieties. This is because small firms, which are more aggressive than large ones, occupy a larger
share of the market after merger. In contrast, a large marginal cost synergy, which expands the insider's variety range but induces the exit of small firms, is harmful to consumer welfare. Taking the endogenous post-merger product choice of the large insider into account, our welfare results imply that a moderate post-merger cost synergy may improve consumer welfare owing to the presence of more aggressive small firms.

On the contrary, if a small firm’s fixed cost is no larger than a large firm’s, i.e., $f \leq F^{12}$, then a small firm behaves less aggressively than a large firm, i.e., $q^*_S \leq q^*_L$, and hence, the expansion of the competitive fringe generates a negative replacement effect on consumer welfare. In this case, when the marginal cost synergy is moderate, the replacement effect is negative. Moreover, when $f \leq F$, equation (16) implies that the shrinkage of the insider’s variety range increases the total product range ($\Delta V > 0$) and hence the variety effect is positive. The same as in the case when $f > F$, the replacement effect dominates the variety effect. Therefore, when $f \leq F$, large firms behave more aggressively than small firms, and hence, a moderate marginal cost synergy that invites more small firms is detrimental to consumer welfare, while a high marginal cost synergy is beneficial to consumer welfare.

Proposition 4 summarizes the impact of merger efficiencies on consumer welfare.

**Proposition 4** When the fixed cost of a SP small firm is higher than the per-variety fixed cost of a MP large firm, i.e., $f > F$, merger is beneficial to consumer welfare with moderate merger efficiencies, but is detrimental to consumer welfare with high merger efficiencies. In contrast, when the fixed cost of a SP small firm is lower than the per-variety fixed cost of a MP large firm, i.e., $f \leq F$, merger is detrimental to consumer welfare with moderate merger efficiencies, but is beneficial to consumer welfare with high merger efficiencies.

Propositions 3 and 4 also imply that higher market concentration may be beneficial or detrimental to consumers, depending on the comparison between the fixed cost of a SP small firm $f$ and the per-variety fixed cost of a MP large firm $F$.

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12By the second condition in Assumption 1, this is satisfied only if $c > C$, i.e. a SP small firm’s marginal cost is strictly higher than a MP large firm’s.
4 Conclusion

In this paper, we examine how a bilateral merger between MP large firms with the endogenous choice on product range affects the market structure and consumer welfare. In the presence of single product small firms, we show that low merger efficiencies in the form of a moderate marginal cost synergy for the merging firms (insider), which lead to the shrinkage of the insider’s product range, are beneficial to consumer welfare. By contrast, high merger efficiencies, which induce the expansion of the insider’s product range, are detrimental to consumer welfare. Our findings suggest that high merger efficiencies may be anti-competitive in the market with large and small firms. For future study, we would test the robustness by considering price competition. In addition, it would also be interesting to generalize the current framework by adopting the aggregative game toolkit.\(^{13}\)

References


\(^{13}\)Anderson et. al. (2016) and Nocke and Shultz (2018) provide the theoretical foundation and a feasible way.


Appendix A The second-order conditions for a large MP firm
The first-order derivatives of large firm n’s profit with respect to its per-variety quantity $q^n_L$ and product range $\bar{\omega}_n$ are given by

$$\frac{\partial \Pi^n_L}{\partial q^n_L} = (\alpha - C - 2\beta q^n_L - 2\gamma \omega_n q^n_L - \gamma Q^{-n}) \omega_n,$$

$$\frac{\partial \Pi^n_L}{\partial \omega_n} = (\alpha - C - (\beta + 2\gamma \omega_n)q^n_L - \gamma Q^{-n})q^n_L - F.$$

Therefore, the Hessian matrix is given by

$$H = \begin{pmatrix}
-2(\beta + \gamma \omega_n) & \alpha - C - 2(\beta + 2\gamma \omega_n)q^n_L - \gamma Q^{-n} \\
\alpha - C - 2(\beta + 2\gamma \omega_n)q^n_L - \gamma Q^{-n} & -2\gamma (q^n_L)^2
\end{pmatrix},$$

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with determinant

$$|H| = 4\beta\gamma(\beta + \gamma\omega_n)(q^n_L)^2 - [\alpha - C - 2(\beta + 2\gamma\omega_n)q^n_L - \gamma Q^{-n}]^2.$$ 

We can readily show that $-2(\beta + \gamma\omega_n) < 0$ and $-2\gamma(q^n_L)^2 < 0$.

Substituting the first-order conditions in (9) and (11), we have

$$\alpha - C - 2\gamma\omega_nq^n_L - \gamma Q^{-n} = 2\beta q^n_L.$$ 

Therefore,

$$|H^*| = 4\beta\gamma\omega_n(q^n_L)^2 > 0,$$

and $q^n_L$ and $\omega_n$ are locally optimal.\(^\dagger\) \hspace{1cm} Q.E.D

\(^\dagger\)The second order conditions for the MP large outsiders are also satisfied, following the same proof. As for the MP large insider, we can modify the proof by replacing $C$ with $\lambda C$ for the proof on the MP large insider.