Abstract:
We consider dynamic price-setting by firms in the presence of rating systems and asymmetric information about product quality. We provide a general framework in which the price charged determines the characteristics of purchasing consumers. The price thus has two effects on future ratings: (i) a direct \textit{price effect} on reviews if consumers evaluate a product relative to the purchase price, and (ii) an indirect \textit{selection effect} through the determination of the set of reviewing consumers. Inference in our setup is characterized by a pair of inferred quality and purchasing consumers’ tastes such that given the current price the aggregate rating is consistent and consumers’ purchase decisions are individually rational. Sufficient conditions such that this inference is uniquely determined are obtained and the impact on the myopic and dynamic pricing incentives of a strategic firm are outlined. We provide empirical evidence for the relevance of our model and estimate the sign and magnitude of the effect of price on ratings using a unique dataset matching the price at which consumers purchased video games to their reviews. Finally, we show analytically for a class of additively separable utility and review functions that inference is correct in the long run provided that ratings aggregate all past reviews.

JEL: D21, D82, L15
Keywords: Rating Systems, Dynamic Pricing, Asymmetric Information
The system will learn what reviews are most helpful to customers...and it improves over time. It’s all meant to make customer reviews more useful.

- Amazon spokeswoman Julie Law, Interview with cnet.com, 2015

1. Introduction

Online platforms such as Amazon.com, eBay or Steam put extensive effort into the design of their review and rating systems, claiming to provide consumers with additional information to ameliorate the asymmetric information problem concerning product quality. This calls into question the effectiveness of price signaling in large, anonymous online markets – if prices were perfectly informative about product quality there would be no need for an effective rating system.\(^1\) Corroborating this, the economic literature has documented ample empirical evidence that ratings have significant effects on demand (see e.g. Anderson and Magruder (2012), Chevalier and Mayzlin (2006), Luca (2011), Moreno and Terwiesch (2014) and Cabral and Hortacsu (2010)), as well as that signaling via price or advertising and informative rating systems are substitutes in terms of information provision regarding product quality (Bhargava and Feng (2015)).

In this paper, we analyze the incentives for sellers in online markets to strategically adjust their pricing to influence induced reviews and thus ratings. The main idea is as follows. Ratings reflect the experience of previous consumers and thereby shape the inference and induced demand and profits in the future. If consumers are heterogenous, sellers’ pricing affects the composition of purchasing consumers and thereby the induced (average) reviews they provide. These translate into the aggregate rating displayed to future consumers and thereby affect future profits. Sellers therefore have to trade off myopic considerations (which price maximizes the current flow profit) with dynamic ones (which price maximizes the rating to increase future profits). While the literature has addressed issues such as investment/disinvestment in quality (Board and Meyer-ter Vehn (2013)) or certification of quality (Marinovic et al. (2018)), these articles typically feature flow profits which are dependent on the current quality perception but unaffected by current period decisions of the seller. In particular, we are not aware of articles which explicitly assess the strategic use of prices to affect the composition of purchasing consumers and hence future (expected) review and rating scores and thus profits.

To illustrate the core mechanism of our framework, consider a monopolistic seller who has to decide on the current-period price of a good of fixed quality to consumers with heterogenous tastes. All else, in particular beliefs about the inherent quality of the good, equal, a higher price charged by the seller increases the threshold level of horizontal taste for the good above

\(^1\)Depending on the considered platform, product quality incorporates various considerations such as the actual quality of the considered product (Amazon, Steam) or the reliability of the seller in terms of shipping and handling (eBay). There is a substantive literature assessing settings in which prices may serve as an effective means to convey information about product quality in markets with asymmetric information: Products of different quality are sold at different prices which not only reflect differences in production costs but also allow consumers to infer (something about) the underlying quality when making their purchase decision (see e.g. Bagwell and Riordan (1991), Osborne and Shapiro (2014), Wolinsky (1983)).
which consumers choose to buy. In expectation, a higher price thus leads to a higher average gross utility of purchasing consumers. If this gross utility is (part of) the basis for consumer reviews, a high price may thus serve to increase (expected) review scores and hence future profits – potentially at the cost of current period flow profits. We thus emphasize the role of prices not as a signaling device, but as a selection device amongst potential consumers to strategically influence the induced reviews and ratings. Similarly, the price may also affect future reviews via inducing a different expectation of purchasing consumers, or directly if satisfaction is assessed relative to price.

Specifically, we provide a tractable framework in which a long-lived seller sells a good of fixed quality to short-lived consumers. Consumers value quality but do not directly observe it prior to consumption. Moreover, they differ in their taste for the product and exhibit horizontal differentiation. A consumer contemplating purchase of the good forms her belief about the product’s quality using the current price together with a summary statistic of reviews provided by consumers who purchased the good in prior periods. These reviews, crucially, may depend not only on the inherent quality of the good, but also reflect the taste for the good, and how satisfied consumers were relative to the purchase price or quality expectation upon purchase. If these characteristics matter and are not observable to future consumers, quality inference is confounded and the seller may strategically set prices to affect future inference and thus profits. This is in line with findings by De Langhe et al. (2015) who show a significant difference between aggregate ratings on Amazon.com and objective quality measures provided by Consumer Reports quality scores, even for products where vertical differentiation is likely to be more important than subjective tastes. Moreover, the assumption of imperfect observability of review- and rating-relevant information is validated by noting that even if individual reviews are reported in addition to aggregate ratings (e.g. on Amazon.com), there is no information obtainable about the price at which the transaction leading to the review occurred.

We set up our general framework in Section 2. Four components may affect the utility consumers derive from consumption, and/or the review they leave post purchase: the inherent quality of the good, the horizontal taste for the given product, the price at which the product was purchased, and the expectation about the product’s quality upon purchase. When making the purchase decision, consumers observe the current price and the aggregate rating left on average by previous consumers. Consumers do not directly infer quality from the price, but instead solve a two-dimensional fixed point problem to conduct inference: They look for the combination of product quality and set of purchasing consumers which at the current price would lead to the current aggregate rating. In Section 3, we provide conditions under which the inference is uniquely determined, and derive comparative statics regarding how the current price affects the inferred quality, cutoff taste of previously purchasing consumers, and induced reviews.

Figure 1 shows patterns that we can rationalize with our model: Panel (a) shows how price and the average review move over time for a flashlight on Amazon.com. For this product, higher prices lead to higher average monthly reviews. Note that if price would only have a direct price effect on the induced reviews, i.e. if consumers would simply evaluate a product’s quality relative to the price paid, we would expect the opposite relation: for the same product, a lower price should lead to a higher review. By contrast, if the price affects the set of purchasing consumers,
Figure 1: Price and rating paths on Amazon.com and Steam.

Panel (a) shows the price and review paths of a flashlight on Amazon.com over time. Panel (b) shows the price and review paths of video games on Steam over time.

A higher price induces only those consumers with a high idiosyncratic preference for the product to buy it, which leads to a higher average satisfaction. For the flashlight in Panel (a), the latter indirect selection effect seems to dominate over the spectrum of prices.

For video games on Steam, the relation between prices and reviews is more nuanced as displayed in Panel (b). Note that in this panel the relation is shown in terms of discount levels to take different quality levels of video games into account. Lower discounts imply higher prices and it can be seen that the relationship between prices and reviews is nonmonotonic: Within our model, this is rationalized by both a selection effect in terms of consumer tastes, and a direct price effect being relevant. For low discounts, the selection effect dominates – a lower price induces consumers who are on average less inclined towards the good to purchase it, which lowers the average review. For sufficiently large discounts, however, the direct effect that consumers judge a good relative to the price they paid for it dominates and the average review increases.

In Section 4 we provide a detailed empirical analysis of ratings on the video game sales platform Steam and show that an additively separable specification of the review function performs well in matching the empirical patterns. We use a dataset which is uniquely suitable to sign and quantify the effect of prices on induced reviews: it matches consumers’ purchases, and, crucially, the price they paid, to the review they left on the platform. The common and often deep discounts on Steam provide the price variation needed to identify the effect of price on reviews, while controlling for many potential confounders. We show that the relation between discounts and induced reviews for a given game is non-monotonic as illustrated in Figure 1: For low discount levels the selection effect of prices dominates while for high discount level the direct price effect on reviews dominates. This holds both unconditionally and when controlling for time since release, consumer, and game fixed effects.

Following the empirical analysis, we assess the pricing incentives of a strategically acting firm in Section 5 and discuss how the firm trades off myopic and future profit considerations. We then restrict attention to a class of additively separable utility and review functions to analytically show that an aggregate rating which equally weighs average reviews across periods leads to correct inference by consumers in the long run. Prices and aggregate ratings converge to a state
in which the firm charges the same price in each period, the aggregate rating no longer moves as the induced average review coincides with it, and the quality inference by consumers is correct. Notably, this is not the case for rating systems with ‘limited memory’ where reviews sufficiently far in the past no longer enter the current rating visible to consumers. This is important as it serves as a cautionary tale regarding recent changes to rating systems putting disproportionate weight on more recent reviews.\footnote{For example, Steam has recently adopted a separate display of the average score of “Recent Reviews” in addition to “All Reviews”. Similarly, Amazon.com changed the rating system by putting more weight on more recent reviews in 2015.} It is not clear that these changes, which are proclaimed to be in consumers’ interests, actually improve their inference. Section 6 concludes.

1.1. Related Literature

We relate to several strands of literature. While we focus on signaling via ratings and the indirect effect prices have on future quality inference, there is a vast literature on direct price signaling. Starting with the seminal article by Wolinsky (1983), studies such as Bagwell and Riordan (1991) have focused on identifying the conditions such that separating equilibria are obtained – a theme of this strand of literature is the required markup by a high-quality (and high-cost) firm to separate from a low-quality (and low-cost) competitor. This markup prevents the low-quality competitor from imitating as it decreases sales which is comparatively more costly for a low-quality and low-cost firm. A central prediction is that prices should decrease over time as the required markup is lowered when more information is already known to consumers, e.g. due to learning in an experience good context (Bagwell and Riordan (1991)). This mechanism also plays a role in our setup. As the information provided via ratings becomes less sensitive to new reviews over time, the value of strategically influencing reviews is lowered, which, in the context of naive consumers reporting gross utility, leads to a lower markup over the myopically optimal price. Osborne and Shapiro (2014) embed price-signaling considerations in a dynamic context where a monopolist chooses both quality and price – consumers thus dynamically learn about the relation between quality and price and the firm strategically affects this inference. A similar consideration is present in our model. However, in our setting the exogenous review and rating system in place forms the basis for the firms’ strategic actions impacting future quality inferences from price and ratings.

We also relate to articles assessing strategic firm behavior in the presence of review or reputation concerns, see e.g. Bar-Isaac et al. (2008) for an overview. Board and Meyer-ter Vehn (2013) show that a seller exhibits a reputation build-up and exploitation behavior in a setting with costly investment in quality. Crucially, the price-setting game is not explicitly assessed; firms simply extract the current willingness-to-pay determined by the quality inference. The price-path is thus implicitly predicted to follow the quality belief and, due to the build-up and exploitation strategy, exhibits a hump-shape. By contrast, we consider a fixed quality and focus purely on the strategic impact of pricing on reputation as measured by the quality inference conducted via the rating system. Marinovic et al. (2018) extend the analysis by Board and Meyer-ter Vehn (2013) by considering a framework in which quality can be certified (and certification is costly). They show that benefits of reputation may be adversely affected by excessive certification, but show
that first-best investment may arise if certification decisions are made based on the time since last certification. Cabral and Hortacsu (2010), building on an earlier working paper version, Cabral and Hortacsu (2004), consider effort decisions by sellers on eBay. They show a similar mechanism in terms of managing reputation. For certain parametrizations, a low-type seller exerts effort and thus builds up reputation by mimicking the high type only until her type is discovered. The model’s prediction about seller exit being preceded by a period of high likelihood of negative reviews, as well as the general negative impact of a negative review on sales, prices, and likelihood to remain in the market, are empirically verified.

Finally, our analysis draws from and complements the empirical literature on the impact of review and rating systems on online platforms. Ratings have been shown to substantially impact demand in a variety of contexts such as restaurants (on Yelp.com, Anderson and Magruder (2012)), books (in a comparative study of Amazon.com and Barnesandnoble.com, Chevalier and Mayzlin (2006)), and online service marketplaces (Moreno and Terwiesch (2014)). However, ratings have been shown to not simply reflect the inherent quality of a good: De Langhe et al. (2015) show a significant difference between aggregate ratings on Amazon.com and objective quality measures provided by Consumer Reports quality scores, even for products where vertical differentiation is likely to be more important than subjective tastes. More generally, Bhargava and Feng (2015) estimate the relative impact of the external information environment incorporating potential reviews, and signaling via price distortions. They show that an effective rating system and price signaling are substitutes in terms of information provision: More external information such as ratings are associated with decreased price distortion.\(^3\)

2. Model Setup

We consider a monopolistic long-lived producer of a good with privately known and fixed quality. Consumers are short-lived and exhibit horizontal differentiation in their taste for the good. A review and rating system allows for information transmission across periods.

**Time**  Time is discrete and covers $T < \infty$ periods, $t = 1, 2, 3, \ldots, T$. The economy consists of a single long-lived seller and a mass of short-lived consumers in any given period.

**Seller**  The seller (‘She’) wishes to sell a good of exogenously given quality $\theta$, where $\theta \sim F(\cdot)$ on $\Theta : [\bar{\theta}, \bar{\theta}]$ is distributed according to a known cdf $F$. The realization of $\theta$ is private information to the seller. In each period, the seller decides on the price $p_t$ at which she is willing to sell. Marginal costs of production are independent of quality and normalized to 0. The seller is risk-neutral and discounts future profits at a rate $\delta \in [0, 1)$.\(^4\)

\(^3\)However, increased information provision via external sources may increase price distortions if it affects the signaling mix between advertising and price signaling a firm employs.

\(^4\)For $\delta = 0$, the firm does not take the future into account and hence prices myopically optimal in every period.
Consumers In each period $t$, there is a mass one of risk-neutral consumers. Consumers are short-lived and only present for one period. Consumers value quality, and are horizontally differentiated with respect to their personal liking for the good which the seller offers. Each consumer $i$ has type $\omega_i \sim [\bar{\omega}, \bar{\omega}]$. The gross utility of a consumer may depend on the actual quality of the good, $\theta$, the consumer’s taste for the product $\omega$, the price paid for the product $p$ and the quality expectation $\mu$. A consumer’s net utility $u$, derived as the gross consumption utility $u(\cdot)$ net of price, is characterized by

$$u = u(\theta, \omega, p, \mu) - p. \quad (1)$$

Reviews and Rating System While consumers are short-lived, we allow for information transmission across periods via a review and rating system. The review $\psi$ left by a consumer conditional on purchase is characterized by

$$\psi = \psi(\theta, \omega, p, \mu). \quad (2)$$

Similar to the utility, the review provided by a consumer may depend not only on the actual quality $\theta$ and the taste $\omega$, but also on the belief $\mu$ about quality $\theta$ prior to purchase, and the price paid for it. We assume that both a consumer’s gross utility and the induced review are increasing in both the actual quality of the good and the individual taste for it. Moreover, we assume that a higher expectation weakly decreases both enjoyment and rating.

Assumption 1 (Gross utility & review) A consumer’s gross utility $u(\theta, \omega, p, \mu)$, as well as a consumer’s rating $\psi(\theta, \omega, p, \mu)$, are continuously differentiable in all variables, and

(i) strictly increasing in quality $\theta$: $\frac{\partial u}{\partial \theta} > 0$, $\frac{\partial \psi}{\partial \theta} > 0$

(ii) strictly increasing in taste $\omega$: $\frac{\partial u}{\partial \omega} > 0$, $\frac{\partial \psi}{\partial \omega} > 0$.

(iii) weakly decreasing in expectation $\mu$: $\frac{\partial u}{\partial \mu} \leq 0$, $\frac{\partial \psi}{\partial \mu} \leq 0$

Timing of the stage game The timing of a given period is as follows: The firm observes the current state of the market characterized by the aggregate rating $\bar{\psi}_t$ and sets the price $p_t$ at which it is willing to sell. Consumers then observe $p_t$ and $\bar{\psi}_t$ and decide whether to purchase the good or not. If consumers choose to purchase, they realize their net utility as in (1) and leave a review as in (2).

For tractability, we assume that every consumer who purchases the good leaves a review, and that only the average review in a given period is used to update the aggregate rating. The rating system is hence characterized by the mapping from current aggregate rating $\bar{\psi}_t$ and current average review $\psi_t$ into next period’s aggregate rating $\bar{\psi}_{t+1}$. We denote this mapping by $\rho$ and let it potentially depend on $t$. This allows to capture different rating systems: For example, aggregate ratings may become less sensitive to new reviews over time if the average of all ratings is reported, or may ‘forget’ reviews sufficiently far in the past and only reflect more
recent reviews. This is particularly relevant given the recent pushes by online platform such as Amazon.com to make more recent reviews matter more for the displayed aggregate rating.

\[
\tilde{\psi}_{t+1} = \rho^t \left( \tilde{\psi}_t, \psi_t \right).
\]  

**Discussion of the Setup**  There are several assumptions inherent in the setup which warrant further discussion. First, we assume that consumers only observe the current aggregate rating and the current price. Both assumptions are unlikely to be perfectly satisfied in reality. Amazon.com, for example, lists all reviews written for a particular product, while price tracking websites such as camelcamelcamel.com provide at least partial access to historical price data. Nonetheless, we believe that at least a sizable number of potential consumers is primarily guided by the aggregate rating displayed in a 0 to 5 star format on Amazon and is moreover unlikely to use price tracking websites for more than price watches (consisting of alerts when the price falls below a threshold, which is the main service these websites provide), if even that. As such, we feel comfortable with this assumption as a first step of the analysis of price-setting behavior in online markets. In particular, it reflects an important feature of rating systems in online markets – a given review cannot be matched with the price at which the transaction occurred, an issue which even price tracking websites cannot circumvent.

The assumption that every purchasing consumer rates is more restrictive. As only the average review enters the updated rating, the fact that all consumers rate and the law of large numbers ensure that a strategic firm can perfectly forecast next period’s rating, which greatly facilitates the analysis. This helps in isolating the qualitative tradeoffs faced by the firm in the presence of the strategic pricing incentives. Related to this consideration, the number of reviews in a given period has no impact in the current formulation: Only the average review in a given period affects the update process of the aggregate rating. It stands to reason that in reality, the more consumers purchase the good in a given period, the more reviews are obtained. This in turn affects the future responsiveness of the aggregate rating statistic. We plan on relaxing this assumption in the future. However, the qualitative predictions in terms of price path and rating path should not be altered by incorporating this.

**2.1. Consumer Inference**

A central requirement is to specify how consumers conduct quality inference given their observables. In principle, quality inference could be based on direct price-signaling. In Appendix B, we characterize the equilibria of the static game where consumer inference is based purely on observed prices. While both pooling and separating equilibria arise, pooling in fact weakly dominates separating equilibria in terms of firm profits. Absent ratings, any combination whereby a stage-game equilibrium (either pooling or separating) is played in every period would hence constitute an equilibrium.\(^5\) However, if ratings are present, extending the analysis with price-signaling to multiple periods is not straightforward: Given that ratings carry information, the

\(^5\)If there are no ratings, the fact that consumers are short-lived implies that no information is transmitted across periods.
implications of any deviation from a conjectured equilibrium are often ambiguous and it is – without a substantial set of restrictive assumptions on the behavior of $u(\cdot)$ and $\psi(\cdot)$ – not clear in which direction the most profitable deviation takes place. \(^6\)

Crucially, empirical evidence suggests that asymmetric information concerns prevail even in the presence of rating systems: Uncertainty about product quality is a major concern. This speaks against fully rational consumers even given the limited observables in the form of current aggregate rating and price only. \(^7\) In principle, fully rational consumers could ‘solve’ the firm’s maximization problem at any point in time for any quality $\theta$ and use this to infer the quality which leads to an aggregate rating of $\bar{\psi}_t$. \(^8\) Full rationality thus has two drawbacks: On the one hand, it imposes high computational requirements on the consumers. Given that consumers only observe the aggregate rating and the current price, they would need to compute the optimal response by the firm for all possible situations and use this to draw inference along the unobserved path of prices and reviews. On the other hand, even if these computational requirements and associated tractability issues are met, it would allow for ostensibly perfect inference, which, as previously argued, is unlikely to hold in reality.

Given the tractability issues associated with perfectly rational consumers, as well as the observation that asymmetric information concerns prevail despite the presence of informative rating systems, we abstract from direct price signaling considerations. Moreover, we assume that consumers use a heuristic: They conduct quality inference from the observables (consisting of aggregate rating $\bar{\psi}_t$ and current price $p_t$) under the hypothetical scenario that all past consumers were faced with the same price-rating-combination. Note that this still lets price have an indirect effect on quality inference; however, there is no direct informational content associated with a given price.

**Assumption 2 (Quality inference by consumers)** Consumers conduct quality inference by imposing that all past consumers faced the same aggregate rating/price combination they currently see. As such, their inference consists of a pair $(\mu^*,\omega^*)$ of inferred quality $\mu^*$ and inferred

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\(^6\)To illustrate the difficulties, consider the case of $T = 2$ and a conjectured pooling equilibrium in the first period. Under reasonable assumptions, this would lead to perfect revelation of the quality in period 2 via the observed rating. If a firm deviates from the equilibrium in the first period, it sacrifices flow profits (due to pessimistic off-path beliefs) but may induce a higher rating and hence higher profits at $t = 2$. Whether this higher rating is attained by charging a higher or lower price at $t = 1$, however, depends on the relative impact of $\theta$, $p$, $\mu$, and $\omega$ on the average review $\bar{\psi}$ and requires substantial assumptions to be evaluated. Similarly, the degree to which flow profits are sacrificed and the resolution of the tradeoff required to verify whether a conjectured equilibrium can be rationalized depends on the sensitivity of $u(\cdot)$ to these inputs. Even for relatively simple utility functions such as $u(\theta,\omega,p,\mu) = \theta + \rho \omega$, i.e.,additively separable gross utility independent of price and belief, the analysis is rendered intractable for reasonably general forms of $\psi(\cdot)$.

\(^7\)If consumers were to observe more than current aggregate rating and price, rational inference would pin down the firm’s quality in a variety of different settings. Any review which is based on a known price would immediately identify product quality $\theta$ given a specified inference, as would knowledge of the full review or price path (where the fact that the firm sets prices optimally allows for perfect inference).

\(^8\)Note that this is feasible if $t$ is known, i.e.,if consumers are aware of the number of periods which have already passed. While online platforms typically depict since when a particular product has been sold, it seems unreasonable to map this directly into the number of periods of the game. This is particularly the case once one takes into account that while the number of reviews may provide a good proxy for the number of sales, there is no information provided about the price at which a given sale took place. Thus, there is no way for consumers to infer how many reviews (purchases) occurred at one price, i.e.,in one period. The product’s online presence timestamp only applies to exactly that particular listing – if the seller sold the same product with a different listing before, or changed the specifics of a particular listing (such as selling a new combination of products), this would be hidden from consumers.
Note that inference consists not only of forming a belief about the quality of the good, $\mu^*$, but also the cutoff type of purchasing consumers $\omega^*$. This is because despite the use of a heuristic, consumers are cognizant of the fact that reviews are driven by the characteristics of purchasing past consumers and in particular their taste for the product: Inference about the quality cannot be conducted in isolation from inference about the set of purchasing consumers.

The assumption greatly improves tractability as it reduces inference to a two-dimensional fixed-point problem. If all past consumers faced exactly the same scenario as current consumers, the inferred quality-cutoff-pair must be such that the aggregate rating is consistent. Contingent upon purchase, the average review left by consumer $\omega^*(\omega^*)$ given that purchase occurred at price $p_t$ and quality is correctly believed to be $\mu^*$ must be consistent with $\bar{\psi}_t$, see (CONS). Moreover, the cutoff type must have been exactly indifferent between purchasing and not purchasing, that is, her gross utility has to be equal to the price, see (RAT). Note that since utility is weakly increasing in taste $\omega$, (RAT) implies that all purchase decisions in the hypothetical scenario were individually rational.

An alternative way of interpreting the assumption is that consumers conduct quality inference by treating the game as if it were in a stationary equilibrium: They deem the good to be of the quality $\mu^*$ such that given the induced cutoff type $\omega^*$, the average rating will be exactly equal to the current aggregate rating $\bar{\psi}_t$.

While the assumption is restrictive, we do not consider it to be too far removed from reality. As discussed previously, past prices are not directly observable on online platforms. While individual reviews typically are available, they cannot be directly linked to the price at which the good was purchased even with the use of historical price data from price-tracking websites (which by itself is cumbersome to obtain). As they are moreover noisy due to horizontal differentiation, the assumption that consumers base their quality inference only on the aggregate rating and current price seems realistic for a large set of potential consumers. Given that consumer inference is based only on these two inputs, the heuristic used by treating the posted price as part of a 'quasi-stationary' equilibrium seems a reasonable approximation in light of the difficulty of imposing full rationality – consumers are often unclear about how many periods have passed and how often the firm changed prices in the past.

We proceed by providing additional assumptions and sufficient conditions such that the inference given Assumption 2 is uniquely determined.
3. Inference: Existence, Uniqueness & Comparative Statics

3.1. Existence & uniqueness of inference

We want to show under which conditions a solution \((\mu, \omega) = (\mu^*, \omega^*)\) to the equation system

\[
\psi(\mu, \omega^*(\omega), p, \mu) = \tilde{\psi} \quad \text{(CONS)}
\]
\[
u(\mu, \omega, p, \mu) = p. \quad \text{(RAT)}
\]

exists. To do so, we proceed in several steps. We first impose an additional restriction on the utility and review functions which is that they depend more on changes in quality than on changes in expectation. This ensures that the slope of the solution pairs \((\mu^*, \omega^*)\) of (CONS) and (RAT) respectively in the \(\mu - \omega\)-space is monotone. Next, we require that the price is such that solutions to (CONS) and (RAT) exist. Finally, we show that provided the slopes of the solutions to (CONS) and (RAT) exhibit a monotonicity property, the solution is uniquely determined and the implicit function theorem (henceforth IFT) can be applied to assess the impact of price changes on inference and future reviews. Note that these conditions are not necessary but sufficient. By studying several examples we conclude that they are not overly restrictive and cover several intuitive utility and review function combinations.

Assumption 3  In addition to Assumption 1 it holds that for all \(\theta \in [\bar{\theta}, \bar{\theta}]\)

\[
\frac{\partial u}{\partial \theta} > \left| \frac{\partial u}{\partial \mu} \right| \quad \text{(4)}
\]
\[
\frac{\partial \psi}{\partial \theta} > \left| \frac{\partial \psi}{\partial \mu} \right|, \quad \text{(5)}
\]

whenever belief and quality coincide, that is, for \(\theta = \mu\).

Denote by \(\mu^*_i(\omega^*)\) for \(i \in \{C, R\}\) the solution for (CONS) and (RAT), respectively. Assumption 3 implies that both \(\mu^*(\omega^*)\) are decreasing in \(\omega^*\). To see this, note that both (CONS) and (RAT) are evaluated at \(\theta = \mu\). Under Assumption 3, a higher \(\mu^*\) hence induces a higher \(u(\cdot)\) and \(\psi(\cdot)\), respectively, necessitating a lower \(\omega^*\) to satisfy the equation.

The monotonicity of the solutions implies that if at the boundaries \(\overline{\omega}\) and \(\overline{\omega}\) the corresponding solution \(\mu^*_i(\omega), \mu^*_i(\overline{\omega}) \in [\underline{\theta}, \bar{\theta}]\), then any \(\mu^*\) in the interior. Moreover, the \(\mu^*_i(\cdot)\) are continuous due to the continuous differentiability of \(u(\cdot)\) and \(\psi(\cdot)\). To ensure that a solution for (CONS) and (RAT) exists (separately) over the entire support of \(\omega\), i.e., over \(\Omega = [\underline{\omega}, \overline{\omega}]\), it suffices that the price \(p\) observed by consumers is such that at the boundaries the corresponding \(\mu^*\) is in \([\underline{\theta}, \bar{\theta}]\).

\[\text{As discussed in more detail subsequently, this is particularly relevant to avoid the case of unbounded flow profits – inference may be such that higher prices actually decrease the threshold taste for purchase. If this is the case, the implicit bounds due to a valid inference provide upper bounds on prices and thus flow profits.}\]
Assumption 4  The current price $p$ is such that the solution $\mu^*_C(\omega), \mu^*_R(\omega)$ to each of the equations

\begin{align*}
\psi(\mu^*_C(\omega), \omega^e(\omega), p, \mu^*_C(\omega)) &= \bar{\psi} \\
u(\mu^*_R(\omega), \omega, p, \mu^*_R(\omega)) &= p
\end{align*}

and $\mu^*_C(\omega), \mu^*_R(\omega)$ to each of the equations

\begin{align*}
\psi(\mu^*_C(\bar{\omega}), \omega^e(\bar{\omega}), p, \mu^*_C(\bar{\omega})) &= \bar{\psi} \\
u(\mu^*_R(\bar{\omega}), \omega, p, \mu^*_R(\bar{\omega})) &= p
\end{align*}

are in the set of possible quality levels $[\theta, \bar{\theta}]$.

This implicitly gives bounds on $p$ such that (CONS) and (RAT) separately exhibit solutions $\mu^*(\omega^*) \in [\theta, \bar{\theta}]$ for all $\omega^* \in \Omega$. The final step is to provide a sufficient condition which allows us to conclude existence of a $(\mu^*, \omega^*)$-pair which jointly solves (CONS) and (RAT).

Assumption 5  We assume that $\mu^*_i(\omega) \geq \mu^*_j(\omega)$ and $\mu^*_i(\bar{\omega}) \leq \mu^*_j(\bar{\omega})$ for $i, j \in \{C, R\}$, $i \neq j$.

If Assumption 5 holds, existence of a joint solution to (CONS) and (RAT) immediately follows from the intermediate value theorem as $\mu^*_i(\cdot)$ are continuous on $\Omega$. This allows us to conclude the following Lemma.

Lemma 1  Suppose Assumption 3 to 5 hold. Then, the consumer-inference problem described by (CONS) and (RAT) has at least one solution.

Uniqueness  While Assumption 3 to 5 ensure that inference can be conducted, it is still possible that multiple solution pairs $(\mu^*, \omega^*)$ to (CONS) and (RAT) exist. To rule out this case, it suffices to ensure that the solution functions $\mu^*_i(\cdot)$ are such that one is steeper than the other over the entire support. This is captured by the following condition.

Assumption 6  For the entire range of $\bar{\omega} \in [\omega, \bar{\omega}]$ and $\bar{\mu} \in [\theta, \bar{\theta}]$, and for any combination $(\bar{\psi}, \bar{\mu})$ which satisfies Assumption 4, it holds that

\begin{equation}
\frac{\partial \psi}{\partial \omega} + \frac{\partial \psi}{\partial \mu} > \frac{\partial u}{\partial \omega} + \frac{\partial u}{\partial \mu} \quad \text{or} \quad \frac{\partial \psi}{\partial \omega} + \frac{\partial \psi}{\partial \mu} < \frac{\partial u}{\partial \omega} + \frac{\partial u}{\partial \mu}, \quad \text{(STP)}
\end{equation}

where partial derivatives are evaluated at $\theta = \bar{\mu}, \omega = \bar{\omega}, p = \bar{p}$ and $\mu = \bar{\mu}$.

Note that as $\psi(\cdot)$ is evaluated at $\omega^e(\omega)$ for a given cutoff $\omega$, we denote by $\frac{\partial \psi}{\partial \omega}$ not the partial derivative of $\psi$ with respect to $\omega$, but the partial derivative with respect to a change in the cutoff, i.e., formally

\begin{equation}
\frac{\partial \psi}{\partial \omega} = \frac{\partial \psi}{\partial \omega} \bigg|_{\omega = \omega^e(\bar{\omega})} \cdot \frac{\partial \omega^e}{\partial \bar{\omega}},
\end{equation}
where $\frac{\partial \psi}{\partial \omega}$ is the derivative with respect to the second input of $\psi(\theta, \omega, p, \mu)$ and $\frac{\partial \psi}{\partial \omega}$ is the derivative of the induced average taste $\omega^e$ with respect to the cutoff $\tilde{\omega} - \frac{\partial \psi}{\partial \omega}$ measures how much the average consumer’s taste changes given a change in the cutoff.

(STP) itself assesses the relative impact of a change in the cutoff $\omega$ compared to a simultaneous change in belief $\mu$ across $\psi(\cdot)$ and $u(\cdot)$. Note that a change in $\mu$ affects $\psi(\cdot)$ both via the actual belief ($\frac{\partial \psi}{\partial \mu}$) and via the quality ($\frac{\partial \psi}{\partial \theta}$) as inference is conducted treating the inferred quality as the actual one, i.e., imposing $\theta = \mu = \mu^*$. The same also applies to $u(\cdot)$. To illustrate this, suppose that

$$\frac{\partial \psi}{\partial \omega} + \frac{\partial \psi}{\partial \mu} < \frac{\partial u}{\partial \omega} + \frac{\partial u}{\partial \mu}$$

and that $\mu = \theta$ increases while $\omega$ decreases in such a fashion as to leave $\psi(\cdot)$ overall unchanged. In that case, we can expect $u(\cdot)$ – given the same changes – to decrease: It reacts comparatively more strongly to the change in the cutoff than $\psi(\cdot)$ and this negative effect thus outweighs the positive overall effect of an increase in $\mu = \theta$. The sign of (STP) at the solution pair $(\mu^*, \omega^*)$ to (CONS) and (RAT) has important implications for the comparative statics of the inference.

This is because the relative strength of these effects determines whether the utility (and hence (RAT)) or review (and hence (CONS)) function reacts more strongly to changes in the aggregate rating ($\tilde{\psi}$) and price ($p$), respectively.

To see that the steepness condition (STP) is sufficient for at most one joint solution to (CONS) and (RAT) to exist, note the following. The implicit function theorem yields that the solution functions $\mu^*_C(\cdot)$ exhibit

$$\frac{\partial \mu^*_C}{\partial \omega^*} = \frac{\partial \psi}{\partial \omega} + \frac{\partial \psi}{\partial \mu}$$

and hence Assumption 6 implies that $\mu^*_C(\cdot)$ and $\mu^*_R(\cdot)$ can intersect at most once on $\Omega$.

**Lemma 2** Suppose that Assumption 3 to 6 hold. Then, a solution to the consumer-inference problem described by (CONS) and (RAT) exists and is unique.

Given Lemma 2, we know that consumer inference according to Assumption 2 can be conducted and yields to a uniquely determined consumer behavior. Thus, the consumer problem is well-defined and can the firm’s problem can be studied.

### 3.1.1. Discussion of the assumptions

The main purpose of this section was to provide assumptions under which the inference conducted by the consumer according to the quasi-stationarity Assumption 2 is uniquely determined. We will see that it is straightforward to verify the assumptions once utility and review functions are specified and that they are in fact satisfied by a wide range of functions determining utility and review behavior.
The assumptions themselves are relatively innocuous. Assumption 3 states that consumers respond more strongly to changes in the actual quality than to changes in the expectation. Even if utility and subsequent reviews are determined relative to expectation, the actual quality of the good is required to have a stronger effect. It seems sensible to assume that actual quality is more important for a consumer than the expectation of quality. Assumption 4 is a constraint on the price charged by the firm and can easily be rationalized once the full game is assessed: Denoting the implicit bounds on \( p \) by \( \bar{p} \) and \( \bar{p} \), we can simply close the inference for the full game by imposing that no consumer is willing to purchase if the price falls outside \([\bar{p}(\tilde{\psi}_{t}), \tilde{p}(\tilde{\psi}_{t})]\). This captures the fact that consumers are not willing to purchase if rational and consistent inference according to (RAT) and (CONS) is not possible: No consumer is willing to purchase if the deal offered for the product seems off, and thereby ensures that the firm only charges prices such that valid inference is feasible. Finally, the boundary and steepness assumptions Assumption 5 and Assumption 6 can both be rationalized once one takes into account that \( \psi \) is evaluated at \( \omega^e(\tilde{\omega}) \), the expected horizontal component conditional on purchase, while \( u \) is evaluated at \( \tilde{\omega} \), which is the lowest purchasing consumer’s horizontal component. This difference leaves plenty of scope for the assumptions to be satisfied, as we will see in detail when discussing examples.

It should also be noted that even if (STP) in Assumption 6 does not hold on the entire support, it may be the case that the inference according to Assumption 2 is uniquely determined; it is a sufficient but not a necessary condition. Moreover, provided that (STP) holds locally at such a solution, the comparative statics can be assessed as in Section 3.2 as the IFT is applicable.

### 3.2. Comparative statics

Given a unique inference \((\mu^*, \omega^*)\), we are able to apply the IFT to assess the comparative statics. Note that inference is affected by the current aggregate rating \( \tilde{\psi} \), as well as the price charged, \( p \). Recall that the IFT is applicable as the determinant of the Jacobian is nonzero provided that the steepness condition (STP) in Assumption 6 is satisfied. We apply the IFT to the equation system

\[
0 = \psi(\tilde{\mu}, \omega^e(\tilde{\omega}), p, \tilde{\mu}) - \tilde{\psi}
\]

\[
0 = u(\tilde{\mu}, \tilde{\omega}, p, \tilde{\mu}) - p.
\]

around the solution \((\mu^*, \omega^*)\) which exists and is unique if Assumption 1 to 6 hold. In interpreting the effects, note that a higher cutoff is synonymous with an in-period reduction in the quantity sold. This is particularly important to bear in mind once considering the tradeoff faced by the monopolist in determining the optimal price which balances current period profit considerations with those pertaining to future profits via changes in the induced rating.
3.2.1. Effect of aggregate rating on belief and cutoff

The comparative statics of how the present rating affects the induced consumers’ belief and cutoff (for a given price $p$) can be characterized by applying the implicit function theorem around the solution to the inference problem.

**Lemma 3 (Effect of aggregate rating on belief and cutoff)** The effect of a change in the aggregate rating on the inferred belief about quality and the cutoff is given by

\[
\begin{pmatrix}
\frac{\partial \mu^*}{\partial \psi} \\
\frac{\partial \omega^*}{\partial \psi}
\end{pmatrix} = \frac{-\frac{\partial u}{\partial \omega} \left( \frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \mu} \right) \partial \psi}{\frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \mu}} \frac{\partial u}{\partial \omega} \left( \frac{\partial \psi}{\partial \theta} + \frac{\partial \psi}{\partial \mu} \right) - \frac{\partial u}{\partial \omega} \frac{\partial u}{\partial \theta} \left( \frac{\partial \psi}{\partial \theta} + \frac{\partial \psi}{\partial \mu} \right).
\]

(12)

**Proof.** See Appendix A.1. ■

Given Assumption 1, we have that $-\frac{\partial u}{\partial \omega} < 0$ and hence belief increases (decreases) in the current rating if

\[
\left( \frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \mu} \right) \frac{\partial \psi}{\partial \omega} < \left( > \right) \frac{\partial u}{\partial \theta} \left( \frac{\partial \psi}{\partial \theta} + \frac{\partial \psi}{\partial \mu} \right)
\]

\[
\Leftrightarrow \frac{\partial \omega}{\partial \psi} + \frac{\partial \psi}{\partial \mu} < \left( > \right) \frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \mu},
\]

(13)

while the cutoff increases (decreases) if the belief decreases (increases) as the numerator is positive by Assumption 3 and the denominator the same as for the belief. Note that the steepness condition in Assumption 6 thus pins down whether the inference pair $(\mu^*, \omega^*)$ increases or decreases in the current rating. Moreover, a change in $\bar{\psi}$ always has opposing effects on the induced belief $\mu^*$ and the induced cutoff $\omega^*$: Either more consumers purchase the product ($\omega^* \downarrow$) while holding a higher belief ($\mu^* \uparrow$), or less consumers purchase and hold a lower belief ($\omega^* \uparrow$, $\mu^* \downarrow$).

Suppose for example that (STP) is satisfied in that we have on the full support

\[
\frac{\partial \psi}{\partial \omega} + \frac{\partial \psi}{\partial \mu} < \frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \mu},
\]

(14)

i.e. that a cutoff change is relatively less important for $\psi(\cdot)$ than for $u(\cdot)$. This implies that the consistency of the rating (CONS) reacts less strongly to the change in $\bar{\psi}$ than the rationality condition (RAT). In that case, the denominator is negative and the quality inference increases in the aggregate rating, while the cutoff decreases.

To illustrate this, consider consumer inference given an increase in the aggregate rating $\bar{\psi}$. The inference $(\mu^*, \omega^*)$ has to adjust such that the higher rating is matched. In principle, this can be facilitated either by a higher inferred quality $\mu^*$, or a higher cutoff type $\omega^*$. However, the cutoff type still needs to be indifferent and the change in rating does not matter for (RAT). Thus, the changes in $\mu^*$ and $\omega^*$ must exactly offset each other as it regards the cutoff type’s
utility. Because of (14), the only way to facilitate this is via an increased inferred quality $\mu^*$ and decreased cutoff $\omega^*$: $\psi(\cdot)$ reacts relatively less to a change in $\omega^*$ than $\mu^*$ compared to $u(\cdot)$ and (RAT) – by keeping the cutoff type indifferent, the lower responsiveness to the decreasing $\omega^*$ than the increasing inferred quality $\mu^*$ thus allows to match the higher rating $\bar{\psi}$ keeping the cutoff type indifferent.

Intuitively, the consumer’s inference adjusts by adjusting the relatively more responsive dimension in the same direction as the change in aggregate rating: If $\bar{\psi}$ increases and reviews are more responsive to changes in $\mu^*$ than $\omega^*$, inferred quality $\mu^*$ increases and the cutoff $\omega^*$ decreases. Similarly, if $\bar{\psi}$ were to decrease, $\mu^*$ would decrease under the same condition: The higher responsiveness to $\mu^*$ allows to match the aggregate rating consistently, while the cutoff type is exactly indifferent.

3.2.2. Effect of price on belief and cutoff

Similar to the previous section, we can also assess how a price change affects the consumer’s belief about quality $\mu^*$ and the cutoff type $\omega^*$.

Lemma 4 (Effect of price on belief and cutoff) The effect of a change in the aggregate rating on the inferred belief about quality and the cutoff is given by

$$
\begin{pmatrix}
\frac{\partial \mu^*}{\partial p} \\
\frac{\partial \omega^*}{\partial p}
\end{pmatrix} = \begin{pmatrix}
\frac{\partial u}{\partial \omega} \frac{\partial \psi}{\partial p} - \frac{\partial \psi}{\partial \tilde{\omega}} \left( \frac{\partial u}{\partial p} - 1 \right) \\
\frac{\partial u}{\partial \theta} \frac{\partial \psi}{\partial p} + \frac{\partial u}{\partial \mu} \frac{\partial \psi}{\partial p}
\end{pmatrix} \left( \frac{\partial u}{\partial \omega} \frac{\partial \psi}{\partial p} - \frac{\partial \psi}{\partial \tilde{\omega}} \left( \frac{\partial u}{\partial p} - 1 \right) \right)^{-1} - \frac{\partial u}{\partial \omega} + \frac{\partial u}{\partial \mu} \frac{\partial \psi}{\partial p}
$$

(15)

Proof. See Appendix A.2.

A marginal increase in price therefore increases the belief if

$$
\frac{\partial \mu^*}{\partial p} = \frac{\partial u}{\partial \omega} \frac{\partial \psi}{\partial p} - \frac{\partial \psi}{\partial \tilde{\omega}} \left( \frac{\partial u}{\partial p} - 1 \right) \left( \frac{\partial u}{\partial \omega} + \frac{\partial u}{\partial \mu} \frac{\partial \psi}{\partial p} \right) > 0
$$

(16)

and decreases it otherwise. Similarly, the cutoff is increased by a marginal increase in the price if

$$
\frac{\partial \omega^*}{\partial p} = \frac{\partial u}{\partial \theta} \frac{\partial \psi}{\partial p} - \frac{\partial u}{\partial \mu} \frac{\partial \psi}{\partial p} \left( \frac{\partial u}{\partial \omega} + \frac{\partial u}{\partial \mu} \frac{\partial \psi}{\partial p} \right) > 0.
$$

(17)

Note that the denominator in both cases again corresponds to the steepness condition (STP). For the change in the induced belief ($\frac{\partial \mu^*}{\partial p}$), the numerator assesses relative impact of a simultaneous change in the cutoff to a simultaneous change in price across (CONS) and (RAT). For the change in the induced cutoff ($\frac{\partial \omega^*}{\partial p}$), it is the relative impact of a simultaneous change in belief
(which corresponds to $\theta$ for the inference) to a simultaneous change in price which determines the sign.

To illustrate this, consider consumer inference when faced with a price increase. Suppose again that
\[
\frac{\partial \psi}{\partial \omega} + \frac{\partial \psi}{\partial p} < \frac{\partial u}{\partial \omega} + \frac{\partial u}{\partial p},
\]
holds, i.e., that a cutoff change is relatively less important for $\psi(\cdot)$ than for $u(\cdot)$. In addition, let
\[
\frac{\partial u}{\partial \omega} \frac{\partial \psi}{\partial p} - \frac{\partial \psi}{\partial \omega} \left(\frac{\partial u}{\partial p} - 1\right) < 0 \Leftrightarrow \frac{\partial \psi}{\partial \omega} \left(\frac{\partial u}{\partial p} - 1\right) < \frac{\partial u}{\partial \omega} \frac{\partial u}{\partial p} - 1 < \frac{\partial u}{\partial \omega} \frac{\partial u}{\partial p} \left|\frac{\partial \psi}{\partial \omega} \left(\frac{\partial u}{\partial p} - 1\right)\right| < \frac{\partial \psi}{\partial \omega} \left(\frac{\partial u}{\partial p} - 1\right),
\]
that is let $\psi(\cdot)$ (and thus (RAT)) be relatively more sensitive to a change in the cutoff than to a change in price compared with $u(\cdot)$ (and thus (CONS)). The consumers’ inferred belief and cutoff have to adjust following the price increase such that (RAT) and (CONS) are satisfied. Overall, the price-increase must be offset such that $\bar{\psi}$ is consistent. But because the consistency requirement reacts less strong to a change in the cutoff than to a change in belief (given (18)) and the change in price (given (19)) than the rationality requirement, the increase in $\omega^*$ necessary to satisfy (CONS) will lead to (RAT) being violated. Hence, the only way that the consumers inference is both consistent and rational is for $\mu^*$ to increase following the price increase, and $\omega^*$ adjusting in such a fashion as to balance (RAT) and (CONS). Whether this is facilitated by an increase or decrease in $\omega^*$ depends in addition on the relative reactiveness of (RAT) and (CONS) to changes in belief and price (see the numerator of (17)).

### 3.3. Effect of price on review

A central contribution of this paper is to analyze the dynamic pricing incentives generated by the linkage of periods through the rating system. Hence, it is not only relevant to understand how the current price affects the current belief and cutoff, but also how the current price affects the
review and hence future profits via the updated rating. Overall, there are three ways in which the price affects the average review in a given period: first, price directly affects the induced average review through the effect of price on the review itself, $\frac{\partial \omega}{\partial p}$. Second, the expectation (which is affected by the price) and the price itself determine the marginal consumer who is indifferent between buying and not buying. This selection effect of the price determines who purchases the product and, via the impact of the taste on the review, the induced average review. Third, it indirectly affects the expectation of consumers and therefore their review as this may condition on the expectation.

The total effect of the price on the review is given by the following equation

$$\frac{d\psi_t}{dp_t} = \frac{\partial \psi_t}{\partial \omega_t} + \frac{\partial \psi_t}{\partial \mu_t} + \frac{d\omega^*}{dp_t} + \frac{\partial \psi_t}{\partial \mu_t} + \frac{d\mu^*}{dp_t} \geq 0 \quad (20)$$

Plugging in the previously obtained equations for $\frac{\partial \omega^*}{\partial p}$ and $\frac{d\mu^*}{dp}$ yields

$$\frac{d\psi_t}{dp_t} = \frac{\partial \psi_t}{\partial \omega_t} + \frac{\partial \psi_t}{\partial \mu_t} \left( \frac{\partial u_t}{\partial \mu_t} - 1 \right) + \frac{\partial \psi}{\partial \omega_t} \frac{\partial u_t}{\partial \mu_t} - \left( \frac{\partial u_t}{\partial \mu_t} + \frac{\partial u_t}{\partial \omega_t} \right)$$

$$= \left( \frac{\partial \psi}{\partial \omega_t} \frac{\partial u_t}{\partial \mu_t} - \frac{\partial \psi}{\partial \omega_t} \frac{\partial u_t}{\partial \omega_t} \right) = \left( \frac{\partial \psi}{\partial \omega_t} \frac{\partial u_t}{\partial \mu_t} - \frac{\partial \psi}{\partial \omega_t} \frac{\partial u_t}{\partial \omega_t} \right)$$

$$= -\frac{\partial \psi}{\partial \theta} \frac{d\mu^*}{dp}.$$  \hfill (21)

There are three points to take away from this equation: First, the firm is able to manipulate induced reviews strategically via its pricing. This is because inference is conducted using the current price as the best guess for past prices (given the quasi-stationary inference). Second, given that $\frac{\partial \psi}{\partial \theta} > 0$, the effect of a price change on the current inference $(\frac{d\mu^*}{dp})$ always goes in the opposite direction than that of a price change on the induced rating in the current period after purchase. Third, the degree to which the rating is affected depends particularly on the reactiveness of the review function $\psi$ to the actual quality $\theta$. This is notable because one might expect that it is the reactiveness of $\psi(\cdot)$ to the belief itself which affects the induced rating.

To rationalize these observations, consider the two cases where the belief decreases in price $(\frac{d\mu^*}{dp} < 0)$ and increases it $(\frac{d\mu^*}{dp} > 0)$, respectively. If the inferred belief $\mu^*$ decreases (locally) following a price increase, it immediately follows that as $(\mu^*, \omega^*)$ has to satisfy (CONS) and (RAT), $\omega^*$ has to increase significantly. This in turn induces a higher rating in the future due to the selection effect. If $\mu^*$ increases, this selection effect is either mitigated (if $\frac{d\mu}{dp} > 0$ still holds) or even reversed (if $\frac{d\omega^*}{dp} < 0$). Crucially, in both cases, a change in the belief enters the

---

12The below derivation implicitly assumes that partial derivatives are identical irrespective of whether they are evaluated at $(\theta, \omega^*(\omega^*), \mu^*)$ (for the actual review) or $(\mu^*, \omega^*(\omega^*), n, \mu^*)$ (for inference). If this is not the case, we have equality only for correct inference, i.e., $\mu^* = \theta$ and an approximation otherwise. Note that any additively separable specification where both utility and review function are linear in quality satisfy this.
quality inference both via the effect it has directly (\(\frac{\partial \psi}{\partial \mu}\) and \(\frac{\partial u}{\partial \mu}\)) and via the effect of quality on rating and utility (i.e. via \(\frac{\partial \omega}{\partial \mu}\) and \(\frac{\partial \psi}{\partial \mu}\)) – this is because inference is conducted treating the actual quality as identical to the belief. However, for the actual review it is the true quality \(\theta\) (which is exogenously fixed) which matters.

This can also be seen in the following manner: Denote \(\mu^*(p, \tilde{\psi}_t)\) and \(\omega^*(\omega^*(p, \tilde{\psi}_t)) \equiv \hat{\omega}\). Recall that \(\tilde{\psi}_t\) is a state variable and cannot be affected by the firm. As such, how the induced review \(\hat{\psi} \equiv \psi(\theta, \hat{\omega}, p, \mu^*)\) changes in response to a price change is identical to how \(\hat{\psi} - \tilde{\psi}_t\) changes. But as \(\tilde{\psi}_t = \psi(\mu^*, \hat{\omega}, p, \mu^*)\), we have

\[
\frac{\partial \hat{\psi} - \tilde{\psi}_t}{\partial p} = \frac{\partial \psi \partial \hat{\omega}}{\partial \omega \partial p} + \frac{\partial \psi \partial \mu^*}{\partial \mu \partial p} - \left(\frac{\partial \psi \partial \mu^*}{\partial \theta \partial p} + \frac{\partial \psi \partial \hat{\omega}}{\partial \hat{\omega} \partial p} + \frac{\partial \psi}{\partial p} + \frac{\partial \psi \partial \mu^*}{\partial \mu \partial p}\right) \quad \text{evaluated at } (\theta, \hat{\omega}, p, \mu^*)
\]

\[
\approx - \frac{\partial \psi}{\partial \theta} \frac{\partial \mu^*}{\partial p},
\]

where we have equality provided that \(\mu^* = \theta\), i.e., that inference is correct, or if partial derivatives are identical for the two points of evaluations. The latter is given for additively separable specifications linear in quality which we focus on in later parts of the paper.

Given that even after the price change, the same \(\tilde{\psi}_t\) has to be matched for the inference to be consistent. The only difference between \(\tilde{\psi}_t\) and the induced review \(\hat{\psi}\) is that the review function \(\psi(\cdot)\) is evaluated at the inferred quality \(\mu^*\) to match \(\tilde{\psi}_t\), while the true quality \(\theta\) enters it for the induced review. Given that the inference changes following a price change are exactly such that \(\tilde{\psi}_t\) is still matched when evaluated at inferred quality \(\mu^*\), the induced review moves in the opposite direction of the induced belief change – it is still evaluated at the unaffected true quality \(\theta\). For example, if \(\frac{\partial \mu^*}{\partial p} > 0\), a price change leads to a higher inferred quality. \(\tilde{\psi}_t\) is still matched given this positive effect on \(\psi(\mu^*, \hat{\omega}, p, \mu^*)\), which in turn implies that the effect of the change in inference and price via the induced cutoff, the price itself, and the belief itself is negative. As these are the effects which also carry over to the induced review, while the quality \(\theta\) remains unchanged, the induced review is negatively affected.

Thus, if the sensitivity of the rating to quality is large, i.e., for \(\frac{\partial \omega}{\partial \mu} \gg 0\), strategic pricing of the firm can be significantly biased because inference treats the inferred quality as the true one in looking for the fixed-point \((\mu^*, \omega^*)\). A small change in the inferred \(\mu^*\) thus leaves scope for larger corresponding changes in \(\omega^*\) and hence a larger effect on the actual review via the selection effect.

4. Empirical Evidence

In this section, we provide evidence that the mechanism identified in the previous sections is empirically relevant. In particular, we want to show that prices affect reviews through both a direct effect on reviews as well as a selection effect on the consumers reviewing the product.
4.1. Data

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>SD</th>
<th>p25</th>
<th>p50</th>
<th>p75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Price in $</td>
<td>22.2</td>
<td>17.1</td>
<td>10</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>Discount in %</td>
<td>18.9</td>
<td>27.6</td>
<td>0</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>Recommended</td>
<td>0.76</td>
<td>0.43</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Playtime at Review (in Minutes)</td>
<td>734.2</td>
<td>3 755.4</td>
<td>41.5</td>
<td>169.0</td>
<td>530.0</td>
</tr>
<tr>
<td>Reviews Written</td>
<td>63.7</td>
<td>172.8</td>
<td>12</td>
<td>22</td>
<td>46</td>
</tr>
<tr>
<td>Games in Library</td>
<td>457.8</td>
<td>783.6</td>
<td>104</td>
<td>221</td>
<td>467</td>
</tr>
<tr>
<td>Number of Ratings for Review</td>
<td>13.4</td>
<td>40.2</td>
<td>2</td>
<td>4</td>
<td>11</td>
</tr>
<tr>
<td>Fraction Helpful</td>
<td>0.70</td>
<td>0.28</td>
<td>0.5</td>
<td>0.74</td>
<td>1</td>
</tr>
<tr>
<td>Length of Review (in Words)</td>
<td>745.0</td>
<td>1 135.2</td>
<td>110</td>
<td>337</td>
<td>888</td>
</tr>
<tr>
<td>Purchased on Steam</td>
<td>0.74</td>
<td>0.44</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1: Summary Statistics. An observation corresponds to a rating-purchase combination. N = 12,083.

We use data from the video game marketplace Steam to provide empirical evidence towards the direction and magnitude of some of the effects predicted by the model. Steam is an online platform on which video game developers advertise their games and make them available for players to purchase and download. As of 2018, around 20,000 games are offered to 67 million monthly active users, giving Steam an estimated market share of the PC video game market of 50-70%. Annual revenue of the platform in 2017 is estimated at $4.3 billion. After purchasing a game on the platform (and only then), players can leave a binary rating (either ‘Recommended’ or ‘Not Recommended’) as well as a written review text. Both are visible to potential buyers on the Steam page of the game. After purchasing a game, it is part of a player’s ‘library’ and can be launched through the platform. Some players choose not to make their game libraries private, so that it is publicly visible which games they own. This empirical section uses a unique dataset, which matches individual players’ purchases of video games to the ratings they left them on the platform, as well as to player characteristics. The dataset was created by crawling through the libraries of around 50,000 players every day from February to August 2017 and registering daily changes in the libraries as game purchases. The purchase price was obtained by crawling through all game sites on Steam on a daily basis. Finally, using the players’ unique platform identification number, the ratings left by a subset of the purchasers can be matched to their purchase dates and prices, as well as some player-specific variables. The resulting dataset consists of around 12,000 player-rating-purchase price matches. Observed variables include the full purchase price and discount (if any), whether the rating was positive, how long the purchasing player played the game before writing the review, how many other games the player owns and other player-specific variables. Summary statistics for

---

13We are very grateful to Johannes Dittrich for sharing the dataset with us.
all observed variables are in Table 1.

Many games are purchased on Steam while they are on discount. Table 1 shows that the average discount for all purchases is nearly 20%. Figure 2 shows that deep discounts of 75% or more are not uncommon on the platform, providing substantial variation in the games purchase prices over time.

![Figure 2: Number of Observations by Discount Level.](image)

### 4.2. Empirical Specification and Results

We exploit this price variation in order to sign and quantify the association between price changes and the probability of receiving a positive review. However, reviews not only depend on price, but also on game quality. In particular, games of lower quality are less likely to receive a positive review, but they are also likely to be cheaper. To control for the time-invariant quality of the game, we include game fixed effects in the regression and rely on within-game price variation over time provided by discounts to identify the effect of prices on reviews. Observable characteristics of the reviewer-game match, such as for how long she played the game before writing the review, how helpful her review was to other potential buyers and how old the game was at the time of purchase, are included as control variables, as they may also have an effect on reviews. Including the squared discount allows for a non-linear relationship between the level of the discount and the review. It thus allows the sign of the effect of discounts on reviews to depend on how deep the discount is. Finally, in some specifications, we include an interaction term between the discount and the number of games in the library of a player at the time of purchase. Players
who purchased many games in the past are likely less price sensitive, making the selection effect relatively more important. The regression framework is given by:

$$y_{ig} = \lambda_g + \beta \cdot X_{ig} + \delta_1 \cdot D_{igt} + \delta_2 \cdot D^2_{igt} + \delta_3 \cdot D_{igt} \cdot \text{nr\_games}_{it} + \epsilon_{ig}$$  \hspace{1cm} (23)$$

The outcome variable $y$ is the binary rating player $i$ gave game $g$. $\lambda_g$ denotes game fixed effects, $X_{ig}$ is a vector of reviewer-game specific control variables, $D_{igt}$ is the discount level at which $i$ purchased $g$, $\text{nr\_games}_{it}$ is the number of games player $i$ owns at time $t$ and $\epsilon_{ig}$ denotes the error term. Regression results are in Table 2. Controlling for time-invariant game quality is crucial, as it is highly likely to be one of the main determinants of the reviews. Therefore, all specifications include game-fixed effects.\textsuperscript{16}

\begin{table}[h]
\centering
\begin{tabular}{lcccc}
\hline
 & (1) & (2) & (3) & (4) \\
\hline
\text{discount} & 3.09e-04 & -5.27e-04 & -0.003*** & -0.002** \\
 & (3.24e-04) & (3.28e-04) & (0.001) & (0.001) \\
\text{discount2} & . & . & 3.70e-05** & 3.59e-05** \\
 & (.)) & (.) & (1.63e-05) & (1.63e-05) \\
\text{gamesint} & . & . & . & -8.54e-07** \\
 & (.)) & (.) & (.) & (4.44e-07) \\
\text{dayint} & . & -4.15e-07 & -2.50e-07 & -2.28e-07 \\
 & (.)) & (1.02e-06) & (1.02e-06) & (1.02e-06) \\
\text{dayint2} & . & 1.49e-09 & -6.42e-10 & -1.01e-09 \\
 & (.)) & (8.74e-09) & (8.78e-09) & (8.78e-09) \\
\text{reviewplaytime} & . & 4.23e-06** & 4.19e-06** & 4.22e-06** \\
 & (.)) & (1.66e-06) & (1.66e-06) & (1.66e-06) \\
\text{numreviews} & . & 1.11e-04** & 1.09e-04** & 1.23e-04** \\
 & (.)) & (5.17e-05) & (5.17e-05) & (5.22e-05) \\
\text{numownedgames} & . & -6.94e-06 & -6.66e-06 & 9.98e-07 \\
 & (.)) & (9.57e-06) & (9.56e-06) & (1.04e-05) \\
\text{frachelpful} & . & 0.614*** & 0.615*** & 0.616*** \\
 & (.)) & (0.021) & (0.021) & (0.021) \\
\text{length} & . & -2.92e-05*** & -2.94e-05*** & -2.94e-05*** \\
 & (.)) & (4.54e-06) & (4.54e-06) & (4.54e-06) \\
\text{purchasetype} & . & 0.027* & 0.027* & 0.028* \\
 & (.)) & (0.015) & (0.015) & (0.015) \\
\text{rated} & . & -8.20e-04*** & -8.27e-04*** & -8.39e-04*** \\
 & (.)) & (1.22e-04) & (1.22e-04) & (1.22e-04) \\
\hline
\text{Game FE} & Yes & Yes & Yes & Yes \\
\text{N} & 5709 & 5709 & 5709 & 5709 \\
\text{R2} & 0.344 & 0.458 & 0.458 & 0.459 \\
\hline
\end{tabular}
\caption{Table 2: Results for OLS regressions of Equation (23). The discount variable ranges from 0-100. The control variables are as follows: dayint and dayint2 denote linear and quadratic time trends multiplied with the age of the game at the beginning of the sample period, respectively. reviewplaytime is the time (in minutes) the player had played the game for when she left the review. numreviews and numownedgames denote the number of other reviews the player had left and the number of other games in her library. rated and frachelpful are the number of times the review itself was reviewed by other potential buyers and the fraction of those that found the review helpful. length is the length of the written review in words. purchasetype is equal to one if the game was purchased on Steam and zero otherwise.}
\end{table}

\textsuperscript{16}This implies that only observations of games which were purchased by at least two reviewers remain in the sample.
Columns (1) and (2) of Table 2 show no statistically significant association between discount and review. However, imposing a linear relationship between the two masks substantial heterogeneity across discount levels. Including discount squared in the regression (column (3)) shows that for purchases made at shallow discounts, ratings are more likely to be negative, while for deep discounts the opposite is true. Finally, the coefficient on the interaction between discount and the number of previously owned games is negative and significant, implying that for a given discount level, players who have many games in their library are less likely to leave a positive review. Players who own many games are less price sensitive, so that the negative selection effect is relatively more important, inducing a higher probability of a negative review. Conversely, players who have not purchased many games in the past are more price sensitive, so that the positive price effect of the discount makes it more likely that they leave a positive review. However, the coefficient on the interaction term is small, indicating that different levels of price sensitivity are not driving the main empirical results.

Figure 3: Predicted marginal effect of discount in percent on the probability of a rating being positive, according to the parameter estimates in Table 2.

Figure 3 shows the predicted effect of discount levels on the probability of receiving a positive review which are computed based on our estimates. Moderate discounts of around 35% have the most detrimental effect, with a predicted decrease in the probability of a positive review of over 4%. Deep discounts of around 70% or more on the other hand are associated with an increase in the probability of receiving a positive review; a discount of 90%, which is not uncommon in the data, is predicted to improve the chances of receiving a recommendation by around 7%.

These findings illustrate the relation between our model and the observed paths in Figure 1.
particular, we see that for low discounts the selection effect dominates while for high discount levels the direct price effect dominates. We next provide a simple specification of our model which rationalizes this relationship.

4.3. Additively Separable Specification

We assume that consumers are uniformly distributed on $[0, 1]$ in their horizontal taste component, $\omega$. Moreover, we assume that the utility function is linear and additively separable, i.e., $u(\theta, \omega) = \alpha \theta + \beta \omega$. The review function is also additively separable and depends on the consumer’s utility and the price, i.e., $\psi(\theta, \omega, p) = \alpha \theta + \beta \omega + g(p)$ and, in particular, we assume for this subsection that $g(p) = \kappa (\bar{p} - p)^2$. We choose the formulation with distance to a reference price as it directly corresponds to the discount measure used in the empirical specification.

The derivations from the previous sections then yield the following expressions:

- The belief of the consumer for price $p$ and aggregate rating $\bar{\psi}$ is given by
  \[
  \mu(p, \bar{\psi}) = \frac{1}{\alpha} \left( 2(\bar{\psi} - \kappa (\bar{p} - p)^2) - p - \beta \right). \quad (24)
  \]

- The cutoff consumer $\tilde{\omega}$ for price $p$ and aggregate rating $\bar{\psi}$ is given by
  \[
  \tilde{\omega}(p, \bar{\psi}) = \frac{1 + \frac{2}{\beta} (\bar{\psi} - \kappa (\bar{p} - p)^2 - p)}{2}. \quad (25)
  \]

- The review left for price $p$ and aggregate rating $\bar{\psi}$ is given by
  \[
  \psi(\theta, \tilde{\omega}, p) = \alpha \theta + \beta + p - (\bar{p} - p) + 2\kappa (\bar{p} - p)^2 - \bar{\psi}. \quad (26)
  \]

- The effect of the discount on the review is given by
  \[
  \frac{d\psi}{dp}(p, \bar{\psi}) = -1 + 4\kappa (\bar{p} - p). \quad (27)
  \]

The key equation is (27). If we impose $\kappa > 0$ in line with our empirical findings, a lower price – all else equal – yields a better review. Then we directly obtain the non-monotonic relationship between discount levels and induced reviews. For low levels of discounts, marginally increasing the discount reduces the rating due to a negative selection effect. If, however, the discount is high, the effect is reversed: marginally increasing the discount increases the rating because the price effect dominates.

5. Dynamic Pricing and Long-Run Effectiveness of Rating Systems

In the previous sections we have developed a framework to study the interaction between a firm’s pricing and consumer inference through rating systems. Moreover, we have established
the importance of the price and the selection effect on reviews empirically. In this section, we
take the framework as given and discuss the firm's dynamic pricing problem and the long-run
effectiveness of rating systems.

5.1. Dynamic Pricing

Consider the pricing problem of a firm in a given period $t$: It aims to maximize the sum of the
flow profits and discounted future profits. Future profits in turn depend on two state variables:
The period, $t$, which cannot be affected by the firm, and the aggregate rating, which can be
strategically influenced. Denote the value of future profits as $V(\bar{\psi}_{t+1})$ and we obtain for the
firm's maximization problem:

$$V_t(\bar{\psi}_t) = \max_{p_t} p_t \cdot q_t(p_t) + \delta V_{t+1}(\bar{\psi}_{t+1}),$$

(28)

where

$$q_t = \frac{\bar{\omega} - \omega^*(p_t, \bar{\psi}_t)}{\bar{\omega} - \omega}$$

(29)

$$\bar{\psi}_{t+1} = \rho'(\bar{\psi}_t, \psi_t)$$

(30)

$$\psi_t = \psi(\theta, \omega^*(\omega^*(p_t, \bar{\psi}_t)), p_t, \mu^*(p_t, \bar{\psi}_t)).$$

(31)

The first order effect of a price change is hence given by

$$\frac{\bar{\omega} - \omega^*(p_t, \bar{\psi}_t)}{\bar{\omega} - \omega} + p \cdot \frac{\partial \omega^*}{\partial p} \cdot \frac{1}{\bar{\omega} - \omega} + \delta V_{t+1}'(\bar{\psi}_{t+1}) \cdot \frac{\partial \rho'}{\partial \psi_t} \cdot \frac{\partial \psi_t}{\partial p_t}.$$

(32)

We thus have three effects: First, flow profits are affected by the increase in the price. This
is captured by $\frac{\bar{\omega} - \omega^*(p_t, \bar{\psi}_t)}{\bar{\omega} - \omega}$. Second, flow profits may either decrease or increase
depending on the impact the change in price has on the induced cutoff and hence the quantity.
This is reflected by $p \cdot \left( -\frac{\partial \omega^*}{\partial p} \cdot \frac{1}{\bar{\omega} - \omega} \right) = p \cdot \frac{\partial q}{\partial p}$. It is possible that a price increase has a (locally)
unambiguously positive effect on flow profits if the price increase is associated with an increase
in the quantity due to the inference.\(^{17}\) Third, the price change impacts future profits via the
change in the induced rating. This effect in turn can be decomposed into the (discounted)
sensitivity of the future profits to the aggregate rating next period ($V_{t+1}'(\bar{\psi}_{t+1})$), the sensitivity
of the aggregate rating in the next period to the induced current review ($\partial \rho' / \partial \psi_t$), and the effect
the price change has on the current review ($\partial \psi_t / \partial p_t$).

It follows immediately that over time, the incentive to price strategically decreases as the re-
ponsiveness of the future profits to the aggregate rating decreases over time (as $t \rightarrow T$, less
periods remain in which the high aggregate rating can be milked) provided that the aggregate
rating becomes less sensitive over time. Moreover, the way the aggregate rating is computed

\(^{17}\)Recall from the previous discussion that this does not imply that flow profits are unbounded – given that
inference is required to be valid, there still remains an upper bound on myopic profits even in this case.
matters for the incentives to price away from the myopic optimum. For example, if the aggregate rating is simply the average over all past ratings, $\bar{\psi}_t = \frac{\sum_{\tau=0}^{t-1} \psi_\tau}{t}$, we have $\frac{\partial \psi_t}{\partial \psi_t} \xrightarrow{t \to \infty} 0$ and the incentive to price away from the myopic optimum decreases over time. If, however, the memory of the rating is limited, i.e., $\bar{\psi}_t = \sum_{\tau=0}^{t-m} \psi_\tau$, the incentive to affect future rating with current prices does not shrink to zero.

Finally, if incentives to deviate from myopically optimal prices obtain, they may come both in the form of incentives to increase the price (if $\frac{\partial \psi_t}{\partial p} > 0$) or decrease the price ($\frac{\partial \psi_t}{\partial p} < 0$), which in turn depends on the specific sensitivity of utility and review functions to their inputs. We illustrate different cases in an additively separable specification below.

**Illustration in a 2-period model.** To capture the strategic incentives of dynamic pricing, consider a 2-period model similar to the additively separable framework from the empirical section: We consider $u = \alpha \theta + \beta \omega$, $\psi_t = \alpha \theta + \beta \omega_t - \kappa p_t$, an initial rating $\bar{\psi}_1$ and a rating in period 2 given by $\psi_2 = \frac{(\tau-1)\psi_1 + \psi_1}{\tau}$, where $\frac{1}{\tau}$ measures the responsiveness of the rating. In period 2, the firm chooses the myopic monopoly price given by

$$p_2 = \frac{\bar{\psi}_2}{2(1 - \kappa)}$$

which yields as profits

$$\pi_2 = \frac{\psi_2^2}{2(1 - \kappa) \beta}.$$  

Hence, we observe that profits are increasing in the current rating. Moving to period 1, the profit function of the firm becomes

$$\pi_1 = p_1 (1 - \omega_1 (p_1, \bar{\psi}_1)) + \delta \pi_2 (\bar{\psi}_2 (p_1))$$

with first-order condition

$$-2 \left( 2p_1 (1 - 2\kappa) - \bar{\psi}_1 \right) + \delta \frac{\partial \pi_2 (\bar{\psi}_2 (p_1))}{\partial p_1}.$$  

To understand the firm’s pricing problem in period 1, we have to derive the effect of current prices on future profits

$$\frac{\partial \pi_2 (\bar{\psi}_2 (p_1))}{\partial p_1} = \frac{\partial \pi_2}{\partial \bar{\psi}_2} \frac{\partial \bar{\psi}_2}{\partial \psi_1} \frac{\partial \psi_1}{\partial p_1}$$

$$= \frac{\bar{\psi}_2 (p_1)}{1 - \kappa} \frac{1}{\tau} (1 - 3\kappa).$$

---

18 We use the linear specification of the price effect in this section to get simpler expressions.
Plugging this into the first-order condition yields for the optimal period-1 price

\[ p_1 = \frac{\bar{\psi}_1}{2(1-2\kappa)} + \frac{\delta}{4\tau(1-2\kappa)(1-\kappa)} \bar{\psi}_2(p_1). \]  

(39)

It is easy to see from this that the firm will change its price away from the static monopoly price if it takes the effect of current prices on future ratings into account. In particular, as profits are increasing in the rating, the firm will deviate from the static monopoly price in a way that increases future ratings. If \( \kappa > \frac{1}{2} \), the price effect is sufficiently strong and dominates the selection effect in the review. In this case, the firm will choose a lower price in the first period than the static monopoly price. By contrast, for \( \kappa < \frac{1}{2} \) the screening effect dominates the price effect in the review and the firm will choose a higher price in the first period than the static monopoly price. Clearly, the effects are stronger if the firm discounts less (high \( \delta \)) and the rating is more responsive (low \( \tau \)). An important takeaway is that the direction of the price distortion relative to the myopically optimal price depends on the relative importance of the price and selection effect.

5.2. Long-Run Effectiveness of Rating Systems

So far, we have established that firms have an incentive to affect future profits via ratings using their current prices. An important question that remains, however, is whether ratings are effective in that they allow learning of consumers over time. The stated purpose of rating systems is to eliminate the asymmetric information between firms and consumers. If firms have the ability to affect ratings via their pricing decisions, it is not obvious whether consumers’ beliefs converge to the true quality over time.

To tackle this issue, we restrict attention to a specific additively separable specification of the model. To be precise, we consider

\[
\begin{align*}
    u(\theta, \omega, p, \mu) &= \alpha\theta + \beta\omega = u(\theta, \omega) \\
    \psi(\theta, \omega, p, \mu) &= \alpha\theta + \beta\omega + g(p) = \psi(\theta, \omega, p).
\end{align*}
\]

(40) \hspace{2cm} (41)

In this formulation, consumer utility is additively separable and linear in both the quality and the taste. The review in addition allows for a direct price effect. Notably, we abstract from the belief influencing either the utility or the review. This specification is consistent with the one we used in the empirical application. Within this specification, we show the following Proposition.

**Proposition 1** Consumers’ beliefs about quality converge to the true quality if firms price strategically, the rating has full memory and the time horizon grows large, i.e., \( \lim_{T \to \infty} \mu_T = \theta \).

**Proof.** The proof is carried out in Appendix C. \( \blacksquare \)
Proposition 1 shows that rating systems are indeed able to eliminate asymmetric information and allow consumers to learn the quality of the project if they have full memory and the time horizon grows large. In shorter time frames, firms manipulate future ratings through their pricing and consumers do not learn the true quality. Moreover, the proof of the proposition shows that with limited memory, consumers will not be able to learn the true quality even if the time horizon is infinite.

Corollary 1 If the rating system has limited memory, e.g. for $\psi_t = \sum_{\tau=1}^{t-m} \psi_{t-\tau}$ for $m < \infty$, consumers’ beliefs do not converge to the true quality, i.e., $\lim_{T \to \infty} |\mu_T - \theta| > \epsilon$.

While Proposition 1 shows that rating systems have the desirable property that consumers learn the true quality in the long run if the rating has full memory, Corollary 1 shows that this property is lost if ratings are computed with finite memory. While this holds for a specific formulation of finite memory rating systems, this finding nonetheless shows that the recent changes where rating systems put sizable emphasis on more recent reviews may not be fully desirable for consumers.

6. Conclusion

This paper develops a framework for assessing strategic pricing incentives of firms in online markets. The price a firm charges serves as a selection device for purchasing consumers’ characteristics which in turn affect the reviews they write and therefore future profits. We propose an inference method whereby consumers treat the game as quasi-stationary and use the current price as the best predictor for past prices and look for a combination of purchasing consumers’ tastes and quality which matches individual rationality conditions and the aggregate rating. For this inference method, we provide sufficient conditions such that it is unique and show how the firm’s pricing is affected once it takes into account the impact on future profits via reviews. Crucially, whether a firm has an incentive to over- or underprice relative to the myopically optimal price depends on the relative strength of the direct price effect and indirect selection effect. If the direct price effect is large, that is, if a small price change has a large direct effect on the induced reviews because consumers’ reviews have a strong dependence on the price level, the firm has an incentive to underprice. By contrast, if this effect is small, the selection effect – a larger price on average leads to higher tastes of purchasing consumers and thus a higher induced review – dominates and the firm has an incentive to overprice.

We show empirically that both effects are relevant and that their interaction can explain patterns observed on online platforms such as Amazon.com and Steam. In particular, marginally lower prices can lead to worse reviews if the selection effect dominates the direct effect of prices on reviews. However, for large discounts the price effect may dominate and reviews improve when prices marginally decrease. We estimate how ratings vary with prices using a uniquely suited dataset of the video game platform Steam in which we can link reviews to prices paid. We find that there is precisely this type of non-monotonic relation: For low discount levels the selection effect dominates while for high discount levels the direct price effect dominates.
Using an additively separable specification which nests the specification estimated in the empirical section, we show that if the time horizon goes to infinity and the rating system aggregates all reviews equally, consumers learn the true quality over time. However, if limited memory ratings are applied, this learning does not occur, even when the time horizon is infinite. This shows that recent policies applied by platforms such as Amazon.com should be evaluated carefully and are not necessarily helpful in alleviating the asymmetric information problem.

References


A. Proofs

A.1. Proof of Lemma 3

Applying the IFT gives

\[
\begin{pmatrix}
\frac{\partial \mu^*}{\partial \bar{\psi}} \\
\frac{\partial \omega^*}{\partial \bar{\psi}}
\end{pmatrix} = - \begin{pmatrix}
\frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \mu} \\
\frac{\partial \psi}{\partial \theta} + \frac{\partial \psi}{\partial \mu}
\end{pmatrix}^{-1} \begin{pmatrix}
0 \\
-1
\end{pmatrix}.
\] (42)

For the inverse of the Jacobian, we have

\[
\begin{pmatrix}
\frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \mu} \\
\frac{\partial \psi}{\partial \theta} + \frac{\partial \psi}{\partial \mu}
\end{pmatrix}^{-1} = \frac{1}{\left(\frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \mu}\right)\frac{\partial \psi}{\partial \theta} + \frac{\partial \psi}{\partial \mu} - \left(\frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \mu}\right)} \begin{pmatrix}
\frac{\partial \psi}{\partial \theta} + \frac{\partial \psi}{\partial \mu} \\
\frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \mu}
\end{pmatrix}.
\] (43)

Hence, we obtain that

\[
\begin{pmatrix}
\frac{\partial \mu^*}{\partial \bar{\psi}} \\
\frac{\partial \omega^*}{\partial \bar{\psi}}
\end{pmatrix} = - \frac{1}{\left(\frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \mu}\right)\frac{\partial \psi}{\partial \theta} + \frac{\partial \psi}{\partial \mu} - \left(\frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \mu}\right)} \begin{pmatrix}
\frac{\partial \psi}{\partial \theta} + \frac{\partial \psi}{\partial \mu} \\
\frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \mu}
\end{pmatrix} \begin{pmatrix}
0 \\
-1
\end{pmatrix}.
\] (44)

\[
\begin{pmatrix}
\frac{\partial \mu^*}{\partial \bar{\psi}} \\
\frac{\partial \omega^*}{\partial \bar{\psi}}
\end{pmatrix} = - \frac{1}{\left(\frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \mu}\right)\frac{\partial \psi}{\partial \theta} + \frac{\partial \psi}{\partial \mu} - \left(\frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \mu}\right)} \begin{pmatrix}
\frac{\partial \psi}{\partial \theta} + \frac{\partial \psi}{\partial \mu} \\
\frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \mu}
\end{pmatrix} = \left(\frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \mu}\right).
\] (45)

\[
\begin{pmatrix}
\frac{\partial \mu^*}{\partial \bar{\psi}} \\
\frac{\partial \omega^*}{\partial \bar{\psi}}
\end{pmatrix} = \left(\frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \mu}\right).
\] (46)

A.2. Proof of Lemma 4

The implicit function theorem yields

\[
\begin{pmatrix}
\frac{\partial \mu^*}{\partial \bar{\psi}} \\
\frac{\partial \omega^*}{\partial \bar{\psi}}
\end{pmatrix} = - \begin{pmatrix}
\frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial \mu} \\
\frac{\partial \psi}{\partial \theta} + \frac{\partial \psi}{\partial \mu}
\end{pmatrix}^{-1} \begin{pmatrix}
\frac{\partial u}{\partial \bar{\psi}} - 1 \\
\frac{\partial \psi}{\partial \bar{\psi}}
\end{pmatrix}.
\] (47)

Using the inverse of the Jacobian from (43) this gives
\[
\begin{align*}
\left( \frac{\partial u^*}{\partial p} \right) & = -\frac{1}{\left( \frac{\partial u}{\partial p} + \frac{\partial u}{\partial \mu} \right) \frac{\partial u}{\partial \omega} - \frac{\partial u}{\partial \mu} \left( \frac{\partial u}{\partial \omega} + \frac{\partial u}{\partial p} \right)} \left( -\frac{\partial \psi}{\partial \omega} \left( \frac{\partial u}{\partial p} - 1 \right) - \frac{\partial u}{\partial \mu} \frac{\partial \psi}{\partial p} \right) \\
\left( \frac{\partial \omega^*}{\partial p} \right) & = \frac{\left( \frac{\partial u}{\partial p} \right) \left( \frac{\partial u}{\partial \mu} \right) \frac{\partial u}{\partial \omega} - \frac{\partial u}{\partial \mu} \left( \frac{\partial u}{\partial \omega} + \frac{\partial u}{\partial p} \right)}{\left( \frac{\partial u}{\partial p} + \frac{\partial u}{\partial \mu} \right) \frac{\partial u}{\partial \omega} - \frac{\partial u}{\partial \mu} \left( \frac{\partial u}{\partial \omega} + \frac{\partial u}{\partial p} \right)}.
\end{align*}
\] 

(48)

(49)

B. Static Game Equilibrium with Price-Signaling

B.1. Monopoly Price under Perfect Information

Consider the static one-period game (i.e. no ratings) and a firm of a given quality \( \theta \). We assume that in addition to Assumption 1, a consumer’s utility is weakly decreasing in \( p \) and \( \mu \), i.e., that a higher price or higher quality expectation weakly decreases the gross utility.\(^{19}\) Formally, we assume that

\[
\frac{\partial u}{\partial p} \leq 0, \quad \frac{\partial u}{\partial \mu} \leq 0.
\]

To characterize the monopoly price under perfect information, suppose that the quality \( \theta \) is publicly known. Hence, a consumer’s gross utility is \( u(\theta, \omega_i, p, \theta) \) where \( \omega_i \sim U[\bar{\omega}, \bar{\omega}] \) with \([\bar{\omega}, \bar{\omega}] \equiv \Omega \). A consumer buys iff

\[
u(\theta, \omega_i, p, \theta) \geq p \iff \omega_i \geq \tilde{\omega}(\theta, p) \quad \text{where} \quad u(\theta, \tilde{\omega}(\theta, p), p, \theta) = p
\]

implicitly defines \( \tilde{\omega} \) as a function of \( \theta \) and \( p \). Moreover, the assumptions ensure that \( \tilde{\omega}(\theta, p) \) is unique, decreasing in \( \theta \) and increasing and convex in \( p \). This induces a quantity

\[
q(\theta, p) = \frac{\tilde{\omega} - \tilde{\omega}(\theta, p)}{\tilde{\omega} - \omega}
\]

which is increasing in \( \theta \) and decreasing and concave in \( p \). As such, the monopoly price of a firm of quality \( \theta \) under symmetric information is derived from

\[
\max_p p \cdot q(\theta, p)
\]

and is unique with \( p^m(\theta) \) increasing in \( \theta \).

\(^{19}\)The idea being that a consumer is less likely to be satisfied the higher the price paid for the good, or the higher the initial quality expectation.
Throughout the subsequent analysis of the static game, we restrict attention to the most pessimistic off-path beliefs, i.e., upon observing a price $\tilde{p}$ not featured in equilibrium, consumers believe the quality to be $\tilde{\theta}$: $\tilde{\theta}(\tilde{p}) = \tilde{\theta}$.

B.2. Static Game – Pooling Equilibria

First note that in any pooling equilibrium of the static game, all firms charge the same price $p^p$ and the associated quality inference is $\tilde{\theta}(p^p) = E[\theta] \equiv \bar{\theta}$. Next, note that given that production costs are independent of $\theta$ (and normalized to 0), profits at any price $\tilde{p}$ are independent of $\theta$ and only depend on the quality inference.

It follows immediately that pooling can be sustained in equilibrium at any price $p^p$ such that all firms earn at least as much as the monopoly profits of the lowest-quality firm under symmetric information. Crucially, this allows for pooling at the monopoly price under symmetric information charged by the lowest type firm, in which case profits are strictly larger than the monopoly profits of the firm with quality $\tilde{\theta}$ under symmetric information.

Lemma 5 (Pooling Equilibria of the Static Game) Any $p^p$ such that

$$\pi(p^p) \geq p^m(\bar{\theta}) \cdot q(\bar{\theta}, p^m(\bar{\theta})) \equiv \pi^m(\bar{\theta})$$

can be sustained as a pooling equilibrium. $p^m(\bar{\theta})$ is one price at which pooling can be sustained.

Proof.

Any deviation to $\tilde{\theta} \neq p^m(\bar{\theta})$ gives quality inference $\tilde{\theta}(\tilde{p}) = \bar{\theta}$ and hence yields profits $\pi(\tilde{p}) = \tilde{p} \cdot q(\bar{\theta}, \tilde{p}) \leq p^m(\bar{\theta})$ so no firm wants to deviate. If $p^p = p^m(\bar{\theta})$, $\bar{\theta} > \tilde{\theta}$ ensures that $\tilde{\theta}(p^m(\bar{\theta})) > \bar{\theta}$ and hence that $\pi(p^m(\bar{\theta})) > \pi^m(\bar{\theta})$. ■

B.3. Static Game – Separating Equilibria

In any separating equilibrium, each firm is uniquely identified through its price. Denote by $p^s(\theta)$ the equilibrium price schedule (which depends on $\theta$) and by $\pi^s(\theta) = p^s(\theta) \cdot q(\theta, p^s(\theta))$ the equilibrium profits. Again recall that a firm’s profit only depends on the price it charges and thus the inferred $\theta$ by consumers, but not directly on its own quality. This directly implies that all firms have to make identical profits.

Given the most pessimistic off-path beliefs, it is straightforward that the lowest quality firm ($\bar{\theta}$) has to exactly charge its monopoly price: As it is identified as the lowest-quality firm in equilibrium, even most pessimistic off-path beliefs would cause it to deviate to $p^m(\bar{\theta})$ – if $p^m(\bar{\theta})$ is not part of the equilibrium pricing of any firm, this yields monopoly profits, while if $p^m(\bar{\theta})$ is associated with a quality $\tilde{\theta} > \bar{\theta}$, profits are even higher. Both cases would imply a profitable deviation.
The fact that firms’ profits have to be identical also directly pins down all other prices: It has to hold that for all \( \theta \in (\bar{\theta}, \hat{\theta}) \):

\[
p^*(\theta) \cdot q(\theta, p^*(\theta)) = \pi^m(\theta) < p^m(\theta). \tag{50}
\]

This is intuitive: Any firm with \( \theta \neq \hat{\theta} \) has to charge a price in equilibrium distorted from its monopoly price under symmetric information to prevent the lowest type firm to imitate it. Because of the previously derived properties of \( q(\theta) \) under symmetric information (which prevails in equilibrium), there are two candidate prices \( \bar{p}^s(\theta) \) and \( \hat{p}^s(\theta) \) with \( \bar{p}^s(\theta) < p^m(\theta) < \hat{p}^s(\theta) \) such that the profit condition (50) is satisfied.

**Lemma 6 (Separating Equilibria of Static Game)** *There are infinitely many separating equilibria. In each separating equilibrium, the lowest quality firm charges its monopoly price \( p^m(\theta) \), while all other firms of quality \( \theta \neq \hat{\theta} \) charge either \( \bar{p}^s(\theta) \) or \( \hat{p}^s(\theta) \).*

### B.4. Pooling vs. Separating

Note that pooling profits are bounded from below to be \( p^m(\theta) \) for each firm (and strictly higher for all but one of the equilibria), while in a separating firm each firm receives \( p^m(\theta) \).

### C. Proof of Proposition 1

**Proof.** We prove the result in two steps: First, we show that under myopic pricing, consumers will learn the quality \( (\mu_t \to \theta) \) when \( T \to \infty \). Second, we show that this happens also under strategic pricing using the convergence of myopic pricing.

**Lemma 7** *If the firm prices myopically, learning will be perfect as \( T \to \infty \).*

**Proof.** Profits under myopic pricing and with the additively separable model are given by

\[
\pi_t(\overline{\psi}_t, p_t) = \frac{2}{\beta} \left( \overline{\psi}_t - p_t - g(p_t) \right) p_t \tag{51}
\]

yielding as first-order condition

\[
\overline{\psi}_t = 2p_t + g(p_t) + p_t g'(p_t). \tag{52}
\]

Note that this first-order condition has to be satisfied in every period and hence, the current-period rating pins down the price charged by the firm.
Moreover, observe that under this price, the rating in \( t + 1 \) will be given by
\[
\bar{\psi}_{t+1} = \frac{t-1}{t} \bar{\psi}_t + \frac{1}{t} \psi_t
\]
(53)
\[
= \frac{t-1}{t} \bar{\psi}_t + \frac{1}{t} \left( \alpha \theta + \beta + p_t - \bar{\psi}_t + 2g(p_t) \right)
\]
(54)
\[
= \frac{t-2}{t} \bar{\psi}_t + \frac{1}{t} \left( \alpha \theta + \beta + p_t + 2g(p_t) \right).
\]
(55)
Making use of the first-order conditions in \( t \) and \( t + 1 \), we can rewrite this as a function of prices to get
\[
2p_{t+1} + g(p_{t+1}) + p_{t+1}g'(p_{t+1}) = \frac{1}{t} \left( \alpha \theta + \beta + 2g(p_t) + p_t \right) + \frac{t-2}{t} \left( 2p_t + g(p_t) + p_t g'(p_t) \right).
\]
(56)

It follows that as \( t \to \infty \), \(|p_{t+1} - p_t| \to 0\) and we know that prices converge to some \( p \). If prices converge, it directly follows that ratings must also converge from the first-order condition. When prices and ratings converge, it is clear that also the consumer’s inference must converge, as this is only based on these two variables. Hence, we also know that \( \mu_t \to \bar{\mu} \) as \( t \to \infty \).

It remains to show that \( \bar{\mu}, \bar{\psi}, \bar{\mu} \) are such that consumers learn the quality of the product, i.e., that \( \bar{\mu} = \theta \). To see this, assume that \( \bar{\mu} \neq \theta \). Then, the review left in period \( t \) will be \( \psi_t(\theta, \omega^e, p_t) \) which is different from the review used in the inference of the consumer that determined the consumers belief \( \psi_t(\mu_t, \omega^e, p_t) \) before learning the true quality. However, as the inference solves the following equation \( \psi_t(\mu_t, \omega^e, p_t) = \bar{\psi}_t \), the review \( \psi_t \neq \bar{\psi}_t \). This implies that, if \( \mu_t \to \bar{\mu} \neq \theta \), ratings cannot converge. A contradiction.

If firms price strategically, their first-order condition in period \( t \) is given by
\[
\bar{\psi}_t = 2p_t + g(p_t) + p_t g'(p_t) + \frac{dV_t}{dp_t}(\bar{\psi}_t, p_t)
\]
(57)
where the latter effect is the impact of the current price on continuation values. As continuation values are affected through aggregate ratings, it follows that \( \frac{dV_t}{dp_t} \to 0 \) as \( t \to \infty \). Hence, the first-order conditions for strategic and myopic pricing converge and we get that \( |p_t(\bar{\psi}_t) - p_t^m(\bar{\psi}_t)| < \epsilon \) for \( \epsilon > 0 \) and \( p_t^m \) denoting the optimal price under myopic pricing. Consequently, \( |\psi_t(p_t(\bar{\psi}_t)) - \psi_t(p_t^m(\bar{\psi}_t))| < \epsilon \) and \( |\bar{\psi}_{t+1}(\psi_t(p_t(\bar{\psi}_t))) - \bar{\psi}_{t+1}(\psi_t^m(p_t(\bar{\psi}_t)))| < \Delta(\epsilon, \epsilon) \). From Lemma 7 we know that \( |\bar{\psi}_{t+1}(\psi_t^m(p_t(\bar{\psi}_t))) - \bar{\psi}| < \gamma \) and hence that \( |\bar{\psi}_{t+1}(\psi_t(p_t(\bar{\psi}_t))) - \bar{\psi}| < \Delta(\epsilon, \epsilon) + \gamma \). As \( \epsilon, \epsilon, \gamma \to 0 \) as \( t \to \infty \), it follows together with Lemma 7 that \( \mu_t \to \theta \) under strategic pricing.