Coo-petition between platforms

*Very Preliminary*

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Abstract

In this paper we develop a model in which two platforms are substitutable on one side but can be complementary on the other side. We analyze the optimal strategies and cooperation incentives of the platforms and assess whether cooperation is socially desirable.

We find that when platforms are rather complementary on this second side, they are able to choose a price that optimizes their demand-margin trade-off, leaving some surplus to users. Moreover, cooperation incentives of platforms are aligned with the best interest of users. However, when platforms are close substitutes on this second side, they are constrained in their choice of price and choose to extract all the consumer surplus. In this case, even though firms have an incentive to cooperate, this is detrimental to users.

*Keywords*: Platforms, Competition, Cooperation, Mergers.

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1 Introduction

1.1 Motivation

Two-sided markets have raised new issues for competition authorities. Indeed, in one-sided markets – provided there is sufficient information – it is clear now for economists how to handle cooperation (such as joint-marketing or even mergers) between firms that sell complements or substitutes. Whether the one-sided results are simply extendable to two-sided markets, however, is not that obvious. As a matter of fact, one specificity of two-sided markets is that they can potentially be complements on one side and substitutes on the other side. In this paper we develop a model in which two platforms are substitutable on one side but can be complementary on the other side. We analyze the optimal strategies and cooperation incentives of the platforms and assess whether cooperation is socially desirable or not.

One particular industry in which the latter market structure may appear is that of online platforms. Indeed, firms could easily offer complementary services to a user base but compete to offer advertising spots. If cooperating on one side has an impact on the ability to compete on the other side, firms may face contradictory incentives that make their optimal strategy less clear cut than in the one-sided case. The decision for a competition authority to encourage or not this cooperation will also be harder to take.

1.2 Illustration: The case of Facebook Connect

Successive scandals in the past few years such as Rapleaf in 2010 or Cambridge Analytica in 2018 have revealed how much user data Facebook was
sharing with third parties. A lot of this data sharing is done through the social network’s API platform Facebook for Developers and through its service Facebook Connect that allows users to log into numerous websites with their Facebook account.

Despite the threat that these revelations could be for its reputation, the social network seems determined to keep data sharing via Facebook Connect and Facebook for Developers at the heart of its expansion strategy. As a matter of fact, the goal of Facebook is to encourage other companies to develop additional services that could enrich its users’ social networking experience. To attract these companies, Facebook offers them a controlled access to its member’s data. The potential complementarity of Facebook’ services with that of the other companies is doubtlessly an opportunity. However, the loss of exclusivity on its users’ data could increase competition and threaten its advertising revenue on the other side of the market. This is all the more true that 98% of the revenues generated by the social network come from advertising.

In the same way, when choosing a compatibility with Facebook, other companies benefit from social networking tools as well as from an opportunity to improve their ad targeting through the shared user data. However they need, in exchange, to give access to the data generated by their own website or application. As a consequence, they lose exclusivity on part - or

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1Facebook Annual Report 2017, SEC EDGAR Library: “We believe that our ability to compete effectively depends upon many factors both within and beyond our control, including [...] our ability to establish and maintain developer’s interest in building mobile and web applications that integrate with Facebook and our other products.”

2Facebook Annual Report 2017, SEC EDGAR library

3Facebook Platform terms of use, 2017, Things you should know: “We can analyze your app, website, content, and data for any purpose, including commercial.”

Facebook Privacy policy, 2017: “We use cookies if you have a Facebook account, use the Facebook Products, including our website and apps, or visit other websites and apps that use the Facebook Products (including the Like button or other Facebook Technologies).”
the entirety - of their databases. This could weaken these companies which revenues depend the most on advertising.

The firm Yelp for instance that publishes crowd-sourced ratings on small local businesses, and which revenues depend at 91% from ads\(^4\), offers a Facebook Connect option. This option allows the website to have more complete profiles than those created with a simple email address, and hence more reliable ratings and comments. It also enables users to view the opinion and favorites of their friend, thus recreating an almost authentic word-of-mouth system. The added value of integrating social networking functionalities on the app or on the website is hence potentially important for users. However, Facebook is explicitly considered a competitor by Yelp on the ads side\(^5\) and the imposed data sharing could jeopardize its business model. As a matter of fact, other companies which offer services to users that are closer to that of the social network are incompatible, or became incompatible with Facebook. That is the case of Snapchat, Twitter’s Vine, or Microsoft\(^6\) for instance.

We build on this example of online platforms and construct a model in which two websites serve a base of users on one side and a base of advertisers on the other side. We impose substitutability on the advertiser side but allow for different levels of differentiation between the platforms. On the user side,


\(^5\)Yelp Annual Report 2017, SEC EDGAR Library: “Competitors also include Internet search engines, such as Google and Bing, review and social media websites, such as Facebook, as well as various other online service providers. These include regional websites that may have strong positions in particular markets. [...] In particular, major Internet companies, such as Google, Facebook, Amazon and Microsoft, may be more successful than us in developing and marketing online advertising offerings directly to local businesses, and may leverage their relationships based on other products or services to gain additional share of advertising budgets.”

\(^6\)Microsoft official website: https://support.office.com/en-US/article/Facebook-Connect-is-no-longer-available-f31c8107-7b5a-4e3d-8a22-e506dacb6db6
we allow for the full spectrum of competition structures: the services offered by the two platforms can range from perfect complements to perfect substitutes. In the paper, platforms are offered an opportunity to cooperate, i.e. to make their services compatible and develop a joint-service. This joint-service includes extra features that are not present in the two independent ones. They then compete in price for users and advertisers. Cooperation increases the value to the user but requires the platforms to share a part of their data. Sharing data, however, creates a trade off. It decreases the differentiation between the platform’s datasets and thus increases the competitive pressure on the advertiser side of the market. With this model, we analyze the cooperation incentives of firms and question the social desirability of this cooperation.

We find that when services to users are rather complementary, platforms can choose a price that optimizes their demand-margin trade-off, leaving some surplus to their users. In this case, compatibility is chosen when the positive demand effect on the user side is greater than the negative differentiation effect on the advertiser side. Moreover we find that cooperation incentives of platforms are aligned with the best interest of their users. However, when platforms are close substitutes on the user side, we find that they are constrained in their choice of price and choose to extract all the consumer surplus. In this case, even though firms have an incentive to cooperate, this is detrimental to users as all the extra value created by cooperation is captured by the platforms. As a consequence, cooperation leaves users indifferent but allows less of them to access the services.
1.3 Related Literature

- What is known with substitutes/substitutes or substitutes/independent in terms of mergers and cooperation: Rochet-Tirole (2002), Armstrong (2006)
- Complementarity between platform and hosts: Hagiu-Jullien-Wright (2018)

2 Model

Let us consider a situation in which Platforms 1 and 2 offer a service to Internet users (side U). They will typically offer content or social networking services. In exchange, the two platforms collect data from users who adopt the service and use it to sell advertising spots to advertisers (side A). The services offered to users by the two platforms can range from perfect complements (when adopting only one of the two services yields almost no utility compared to adopting two) to perfect substitutes (when adopting the two services instead of just one does not yield any extra utility).

Additionally, platforms will be able to cooperate and develop or not a compatibility with the other platform. Making services compatible implies sharing with the other platform a proportion $q \in (0, 1]$ of the data collected on multi-homing users but will create a gain in utility for the latter. Having data in common will however have an impact on the market power of platforms on the advertiser side of the market.
2.1 Users

Preferences – Suppose Users have one unit of time to spend on the services offered by the two platforms. They can choose to consume one (single-homing), two (multi-homing), or none of the services offered.

Users get a utility normalized to 1 if they adopt the two services and $1 - \epsilon$ if they adopt only one. Hence $\epsilon \in (0, 1]$ is the additional benefit when adopting both of the services instead of just one. It can be seen as the degree of complementarity or substitutability between the two platforms on the user side. As a matter of fact, when $\epsilon$ tends to 0, consuming one or two of the services provides the same utility: services are perfect substitutes. When $\epsilon$ tends to 1, subscribing to only one of the two services yields a utility of 0: services are perfect complements.$^7$

Users have a heterogeneous outside option $\theta$, uniformly distributed over $[0, 1]$.

The choice of platforms to allow for compatibility or not also has an impact on the utility of users. Compatibility increases the utility of users by an amount depending on the quantity of data platforms share on multi-homing users $q \in (0, 1)$.

Indeed, if the services are compatible, the utility of users is increased by $b(q) > 0$.

This additional utility could be interpreted for the user as the difference between sharing her time-unit navigating between the two platforms and spending it fully on a partially integrated service including features from both platforms and possibly original ones.

We suppose that the function $b(.)$ is $C^2$, increasing in $q$. Additionally, we suppose that $b(0) = 0$, i.e. that not sharing any data with each other would not

\footnote{This element plays exactly the same role as its equivalent in Rey-Tirole (2018).}
allow firms to improve the quality of their joint service. As a consequence, throughout the paper, we will consider \( q = 0 \) as representing incompatibility. Furthermore, we fix \( b(1) < 1 \), such that the benefit from compatibility is always strictly lower than the maximum baseline utility of consuming both services. Finally, we assume that \( b(\cdot) \) is either strictly convex or concave.

In this paper, we study the case in which the marginal value created by platforms on the user side by increasing the compatibility of services (and thus sharing more data) is not too small. That is \( b(\cdot) \geq \frac{2}{7} \quad \forall q \in [0, 1] \).

Participation – We denote by \( n^U_i \) the number of users adopting the service of platform \( i \) and \( n^A_i \) the number of advertisers buying data from company \( i \). We suppose that there exists a cross-side externality between users and advertisers and we denote by \( \alpha^U \) the effect of the number of advertisers on users (possibly negative). Knowing this, we define the quality-adjusted price \( w^U_i = p^U_i - \alpha^U n^A_i \).

To choose whether they want to consume two, one or no services, users compare their utility in each case. It is given by:

- \( 1 + b(q) - w^U_1 - w^U_2 \) if they use two services (with \( q = 0 \) when services are incompatible),
- \( 1 - \epsilon - w^U_i \) if they use only the service of company \( i \) (compatibility does not matter),
- \( \theta \sim U_{[0,1]} \) if they use none.

\(^8\)Note that this implies that \( b'(q) > \frac{b(q)}{q} > b'(0) > \frac{2}{7} \quad \forall q \in (0, 1] \) when \( b(\cdot) \) is convex, and that \( b'(0) > \frac{b(q)}{q} > b'(q) > \frac{2}{7} \quad \forall q \in (0, 1] \) when \( b(\cdot) \) is concave.
Users are ready to adopt the two compatible services if \( 1 + b(q) - w_1^U - w_2^U \geq \theta \).

The demand for the joint service is then

\[
D(w_1^U + w_2^U, q) = 1 + b(q) - w_1^U + w_2^U
\]

And the demand for the two incompatible services is thus \( D(w_1^U + w_2^U, 0) \).

In the same way, the demand for a single service \( i \) is

\[
D(\epsilon + w_i^U, 0) = 1 - \epsilon - w_i^U
\]

2.2 Data Brokers or Advertisers

For the moment we do not define precisely the preferences and participation rules of this side of the market. We simply assume that the platform’s revenue from data is proportional to the number of users \( n_i^U \).

We also suppose that this per-user revenue depends positively on a differentiation parameter \( t(\epsilon, q) > 0 \), that represents how unique the data-bases of the two platforms are. The more unique the data of the platform is, the lower the competitive pressure, the higher the per-user revenue from advertising.

This differentiation parameter \( t(\cdot) \), varies, in turn, positively with \( \epsilon \) \( (\partial t / \partial \epsilon > 0) \) and negatively with \( q \) \( (\partial t / \partial q < 0) \). In words, we assume that data-bases are more differentiated when platforms offer complementary services on the user side than when they offer substitute services. This is tantamount to assuming that similar services tend to produce the same kind of data. In the same way, we assume that data-bases are less differentiated when platforms choose to be compatible, as being compatible implies sharing a proportion \( q \) of the

\footnote{This assumption can be motivated by pricing habits like pay-per-click. Moreover, platforms often regard their revenue per user as a Key Performance Indicator (KPI); it is thus interesting to define a per-user revenue function.}
data collected on multi-homing users. The higher $q$, the less differentiated the data-bases.

The total revenue from the data side will hence be denoted $n_i^U \pi(p_i^A, t)$.

2.3 Platforms

In the first stage, the two platforms decide whether they want to be compatible or not. If both choose compatibility, then it occurs. If at least one chooses incompatibility, then the two services stay incompatible. Being compatible implies sharing a proportion $q \in (0, 1)$ (exogenous) of the data collected on multi-homing users with the other platform, but creates an added value for consumers who use the joint service.

In the second stage, platforms simultaneously choose prices for the two sides. The total profit for platform $i$ is given by

$$\Pi_i(w_i^U, p_i^A) = n_i^U(w_i^U)[w_i^U + \alpha^U n_i^A(p_i^A) + \pi(p_i^A, t)]$$

2.4 Welfare Analysis

As it is often done by competition authorities, we will base the welfare analysis mainly on user the measure of User Surplus.

The total User Surplus is given by the addition of the surplus of users who buy the two services, the surplus of users who buy the service of platform 1, and the surplus of users who buy the service of platform 2, conditional on the existence of each of these user groups. We will denote it in the following way:

$$US(w_i^U, w_j^U, q) = us_1(w_i^U + w_j^U, q) + us_1(w_1^U + \epsilon, q) + us_2(w_2^U + \epsilon, q)$$
2.5 Game

Equilibrium concept – We look for a subgame perfect Nash equilibrium of the above-mentioned game, and solve it by backward induction.

Timeline –

Figure 1: Baseline Model

Stage 1
- Platform 1 & 2 take a stand on compatibility
- Compatibility or Incompatibility becomes effective

Stage 2
- Websites simultaneously choose prices for the two sides
- Users choose to use 2, 1 or 0 services

3 Symmetric duopoly

3.1 Equilibrium user participation

Let $\hat{w}(\epsilon, q)$ and $\hat{p}(\epsilon, q)$ be the symmetric solutions to the following problem:

$$\begin{align*}
(P) \quad \max_{w_i^U, p_i^A} D(w_i^U + w_j^U, q) & \left[ w_i^U + \alpha^U n_i^A(p_i^A) + \pi(p_i^A, t(\epsilon, q)) \right]
\end{align*}$$

For a given compatibility choice (we treat incompatibility as $q$ being null), users decide to adopt the two services instead of only service $i$ only if:

$b(q) - w_j^U \geq -\epsilon$, i.e. only if $b(q) + \epsilon \geq w_j^U$. In words, a necessary condition for users to adopt both services instead of only that of company $i$
requires the price of the other service to be below or equal to the extra utility users get by adopting two services instead of one.

When adopting only one, they decide to adopt service $i$ over service $j$ only if $1 - \epsilon - w_i^U > 1 - \epsilon - w_j^U$, i.e. only if $w_i^U < w_j^U$ (firms share the market if consumers are indifferent).

**Proposition 1.** In equilibrium, users always adopt the two services or none, whether services are compatible or not. Furthermore, the symmetric quality-adjusted price equilibrium is:

$$w^N \equiv \min\{\epsilon + b(q), \hat{w}(\epsilon, q)\}$$

**Proof.** (See more detailed version in Appendix)

Above $\epsilon + b(q)$, platforms have an incentive to undercut each other down to $\epsilon + b(q)$ in a *Bertrand*-manner to get the full market. Below $\epsilon + b(q)$, their best choice is to play their profit-maximizing equilibrium $\hat{w}(\epsilon, q)$.

In both cases, users prefer to consume two services instead of just one as $b(q) + \epsilon \geq w^N$

This equilibrium quality adjusted price deserves several comments. First, one can notice that it splits the competition spectrum in two areas. In the first one (when $\epsilon + b(q)$ is relatively low), the competition is strong enough for platforms to undercut each other in a *Bertrand* manner, down to $\epsilon + b(q)$. In the second part (when $\epsilon + b(q)$ is relatively high compared to $\hat{w}(\epsilon, q)$), competition is not as intense and platforms can pick their profit
maximizing price. This is coherent with a lower $\epsilon$ representing a higher degree of substitutability.

Second, it may seem strange that users would adopt the two services when these are perfect substitutes. However, notice that in this case $\epsilon \approx 0$ and the maximum price charged for the joint service is $b(q)$ (the additional utility users get from two compatible services) when services are compatible and 0 if they are not. It is thus normal that they adopt both.

Third, notice that the two equilibrium prices – $\epsilon + b(q)$ and $\hat{w}(\epsilon, q)$ – can be seen as the solution to the same profit-maximization program, with an upper constraint on the price chosen defined by the marginal utility for users of adopting a second service when they already use one. When this marginal utility is high, the constraint is not binding and the platform can optimize its margin-demand trade-off. When this marginal utility is low, the constraint is binding and the price chosen by the platform depends on the marginal user value and not on the demand. Hence, in the rest of this paper, we will call $\hat{w}(\epsilon, q)$ the unconstrained price and say that is demand driven. Conversely, we will call $\epsilon + b(q)$ the constrained price and say that it is value driven. Finally, notice that the constrained price area ($\epsilon + b(q) \leq \hat{w}(\epsilon, q)$) depends on the value of $q$. Compatibility choices will hence have an impact on the dynamic of competition. Compatibility may increase or decrease the range of values of $\epsilon$ for which the price will be constrained, depending on the expressions of $b(.)$ and $\hat{w}(.)$.

3.2 Platforms equilibrium pricing decisions

As seen in the previous section, the equilibrium quality-adjusted price on the user side is $w^N = \min \{\epsilon + b(q), \hat{w}(\epsilon, q)\}$. At this price, users adopt either the joint service or no service at all. On the other side of the market, Advertisers
anticipate that the user base of both platforms will be the same. However, as platforms may sell differentiated services, their data bases may be different, as well as their ability to identify relevant users for advertisers.

To represent this and detail better the compatibility choices in the first stage of the game, we introduce a functional form for $\pi$, i.e. a model for the Advertiser-side of the market.

We suppose that advertisers’s valuation for a contact with each of the multihoming users is uniformly distributed on a Hotelling line. In other words, advertisers have a preference regarding the platform through which they can get in touch with the consumer. This can be explained by the fact that services to users are differentiated for any $\epsilon > 0$. It means that the data collected is also differentiated and platforms could offer targeted advertising on different criterion. A cook book editor for instance may find it easier to segment consumers with the food taste information available on Yelp for instance than with the information available on Facebook.

We hence assume that the two platforms are standing at the two extremes of the line. We also assume that the Hotelling transportation cost depends positively on the degree of substitutability and on the quantity of exclusive data when services are compatible. We take it of the form $t(\epsilon, q) = \epsilon(1 - q)$ for compatible services, with $t(\epsilon, 0) = \epsilon$ when services are incompatible.

The average per-user demand for the platform from advertisers/data brokers is thus:

$$n_i^A(p_i^A, p_j^A, t(\epsilon, q)) = \frac{1}{2} - \frac{p_i^A - p_j^A}{2t(\epsilon, q)}$$

and the total revenue of the platform on the data side is:

$$n_i^U \pi(n_i^A, \epsilon + c) = n_i^U p_i^A \left[ \frac{1}{2} - \frac{p_i^A - p_j^A}{2t(\epsilon, q)} \right]$$
Proposition 2. \(\hat{p}(\epsilon, q)\) and \(\hat{w}(\epsilon, q)\) exist and their value is given by:

\[
\hat{w}(\epsilon, q) \equiv \frac{1}{3} \left( 1 + b(q) - \frac{1}{2} t(\epsilon, q) \right)
\]

\[
\hat{p}(\epsilon, q) \equiv t(\epsilon, q) - \alpha^U
\]

- The symmetric solutions to problem \((P)\) are its unique solutions in this extended setting.
- The optimization on \(p_i^A\) does not depend on \(w_i^U\).

Hence \(p^N(\epsilon, q) = \hat{p}(\epsilon, q)\) for any value of \(\epsilon\) and \(q\).

Given \(\hat{w}(\epsilon, q)\) and \(\hat{p}(\epsilon, q)\), the equilibrium profits of Platform 1 & 2 are now equal and given by:

\[
\hat{\Pi}(\epsilon, q) \equiv \left[ \frac{1}{3} (1 + b(q) + t(\epsilon, q)) \right]^2 \quad \text{if } w^N = \hat{w}(\epsilon, q)
\]

\[
\tilde{\Pi}(\epsilon, q) \equiv \left( 1 - b(q) - 2\epsilon \right) \left( \epsilon + b(q) + \frac{1}{2} t(\epsilon, q) \right) \quad \text{if } w^N = \epsilon + b(q)
\]

We notice that when the degree of complementarity \(\epsilon\) is sufficiently high and the equilibrium price is unconstrained, the equilibrium profit does not depend on \(\epsilon\) more than through \(t(.)\), the degree of differentiation between firms on the advertiser side. Indeed, as users always adopt both services in equilibrium, the equilibrium user value for the services does not depend on \(\epsilon\) in the unconstrained problem.
3.3 Platforms equilibrium compatibility decision

The \textit{Constraint} frontiers

We have seen when studying the users equilibrium participation that the Nash user price is defined differently when the marginal value to users of adopting a second service when they already use one ($\epsilon + b(q)$) is low as opposed to when it is not.

This implies that for each compatibility status (on or off) the space is split in two areas, one in which the \textit{constrained} price applies, and the other in which the \textit{unconstrained} one applies. We call the “\textit{Constraint} frontiers”, the lines that define these two areas in the compatibility case and in the incompatibility case.

\textbf{Lemma 1.} When goods are compatible, the Bertrand frontier is given by :

$$
\bar{\epsilon}(q) \equiv \frac{2(1-2b(q))}{7-q}
$$

When goods are incompatible, the Bertrand frontier is given by :

$$
\bar{\epsilon}^\circ \equiv \bar{\epsilon}(0) = \frac{2}{7}
$$

Furthermore, when represented in a $(q, \epsilon)$ space, $\bar{\epsilon}^\circ \geq \bar{\epsilon}(q)$ $\forall q \in [0, 1]$. Drawing the two lines in the same graph hence splits the space in three parts.
The split is illustrated in the following graph, picturing the example in which \( b(q) = \log(1 + q) \).

Figure 2: The Constraint frontiers split the space in three distinctive Areas

- **Area 1**: Compatibility is not too small \((\epsilon \geq \bar{\epsilon}\)\).
  Above \(\bar{\epsilon}(q)\) and \(\bar{\epsilon}\), the price is unconstrained – whether compatibility \((C)\) is chosen or incompatibility \((IC)\) – and the equilibrium price is the one that optimizes the demand-margin trade-off.

- **Area 2**: Services are substitutes and compatibility requires a high amount of data sharing \((\bar{\epsilon}^o \geq \epsilon > \bar{\epsilon}(q))\).
  Below \(\bar{\epsilon}^o\) the equilibrium price is constrained when services are incompatible. However, as we are above \(\bar{\epsilon}(q)\), the equilibrium price is unconstrained when services are compatible. As a consequence, in this area, changing the compatibility status does not only change the quantity of data to be shared or not, it also has an impact on the dynamic of
competition between the two platforms.

- Area 3 : Services are substitutes and compatibility requires sharing few data ($\bar{\epsilon}(q) > \epsilon$)

Below $\bar{\epsilon}(q)$ and $\bar{\epsilon}^o$, the equilibrium price is constrained – whether compatibility is chosen or not –, i.e. it is the maximum price that leave users indifferent between adopting two services or just one.

### Profit Comparison

Given the previous results and depending on the value of $\epsilon$ and the equation of $b(.)$, we can characterize the relevant equilibrium profits in each subgames in the following way:

**Lemma 2.** • In subgames where the services are compatible:

\[
\Pi^C(\epsilon, q) = \begin{cases} 
\hat{\Pi}(\epsilon, q) & \text{if } \epsilon \geq \bar{\epsilon}(q) \\
\tilde{\Pi}(\epsilon, q) & \text{if } \epsilon < \bar{\epsilon}(q)
\end{cases}
\]

• In subgames where the services are incompatible:

\[
\Pi^{IC}(\epsilon, q) = \begin{cases} 
\hat{\Pi}(\epsilon, 0) & \text{if } \epsilon \geq \bar{\epsilon}^o \\
\tilde{\Pi}(\epsilon, 0) & \text{if } \epsilon < \bar{\epsilon}^o
\end{cases}
\]

*Given this, we can observe that the Subgame Perfect Equilibrium profit is always positive.*

*Proof.* The relevant profits are a direct consequence of the previous results. For the positivity of profits, see proof in Appendix. □

To make the optimal compatibility choice, platforms will hence compare:
• In Area 1: \( \epsilon \geq \bar{\epsilon}^\circ \)
  \( \hat{\Pi}(\epsilon, q) \) and \( \hat{\Pi}(\epsilon, 0) \).

• Area 2: \( \bar{\epsilon}^\circ \geq \epsilon > \bar{\epsilon}(q) \)
  \( \hat{\Pi}(\epsilon, q) \) and \( \hat{\Pi}(\epsilon, 0) \).

• In Area 3: \( \bar{\epsilon}(q) > \epsilon \)
  \( \hat{\Pi}(\epsilon, q) \) and \( \hat{\Pi}(\epsilon, 0) \).

Area 1

**Proposition 3.** There always exists a subset of Area 1 in which incompatibility is preferred to compatibility. Incompatibility will be preferred when the degree of compatibility (\( \epsilon \)) is high enough.

*Proof.* See proof in Appendix.

The compatibility or incompatibility decision depends on the sign of \( (-\epsilon q + b(q)) \). In this expression \(-\epsilon q\) is the loss induced by lower differentiation on the advertiser side when choosing compatibility, whereas \(b(q)\) is the gain in demand on the user side when choosing compatibility. The intuition for what happens in area 1 is hence simple: if the gain in demand on the user side – induced by sharing data – is greater than the loss in profit on the advertiser side – due to the loss of exclusivity on data–, then Platform 1 will choose compatibility. Else, it will choose incompatibility.

Given the assumptions on \( b(.) \), and as \( \epsilon \) and \( q \) can go up to \( \approx 1 \), for \( \epsilon \) and \( q \) sufficiently high, i. e. for a quantity of data required sufficiently high and for a degree of complementarity sufficiently high, Platform 1 will always want incompatibility.

The link between high complementarity and desired incompatibility is not
obvious. However, remember that we assumed that the higher the degree of complementarity is, the more differentiated the collected data will be. This implies that the opportunity cost of sharing data will be higher for services that have a higher degree of complementarity. Hence for a given amount of data required to make services compatible \( q \), platforms that are more compatible have a higher chance of choosing incompatibility.

Area 2

**Proposition 4.** In Area 2, compatibility is always chosen.

*Proof.* \( \frac{\partial \hat{\Pi}}{\partial q}(\epsilon, q) \) is always positive for any \((\epsilon, q)\) such that \( \epsilon \leq b'(q) \). Here \( \epsilon \leq \tilde{\epsilon}^o = \frac{2}{7} < b'(q), \quad \forall q \in [0, 1] \).

As a consequence, \( \hat{\Pi}(\epsilon, q) \) is greater or equal to \( \hat{\Pi}(\epsilon, 0) \), which itself is greater or equal to \( \bar{\Pi}(\epsilon, 0) \), by optimality of \( \hat{\Pi}(\epsilon, q) \).

In our model, when services are relatively close substitutes and prices are unconstrained, the gain on user demand when services are made compatible is always greater than the loss of differentiation (and hence the loss in revenue) on the advertiser side.

One intuition for this is that, on the one hand, the marginal loss in differentiation on the advertiser side when improving compatibility on the user side is proportional to the degree of complementarity/substitutability \( \epsilon \). The lower this \( \epsilon \), the more substitutable the services, the more similar the data collected by the two platforms, the lower the opportunity cost of sharing a proportion of it. The marginal benefit for users of more integrated services, on the other end, does not depend on \( \epsilon \) and is, by assumption, not too small.

To sum up, we can say that in area 2, platforms always prefer to be in a situation in which they choose an unconstrained price instead of one imposed by
the marginal value to users of adopting a second service when they already use one.

**Area 3**

**Lemma 3.** In Area 3 (when $\epsilon < \bar{\epsilon}(q)$), $\Delta \tilde{\Pi}(\epsilon, q) \equiv \tilde{\Pi}(\epsilon, q) - \tilde{\Pi}(\epsilon, 0)$ is always decreasing in $\epsilon$. As a consequence, for a given $q$, there is at most one point in which $\Delta \tilde{\Pi}(\epsilon, q) = 0$. The set of all these points (for all values of $q \in [0, 1]$) forms a continuous line $\epsilon = \tilde{\delta}(q)$.

**Proof.** See proof in Appendix.

**Proposition 5.** In Area 3, for $\epsilon < \tilde{\delta}(q)$, compatibility is chosen. On the contrary, for $\epsilon > \tilde{\delta}(q)$, incompatibility is chosen.

**Proof.** Direct consequence of the previous Lemma.

As an illustration, the compatibility choice analysis would give the following results in our $b(q) = \log(1 + q)$ example.

### 3.4 Welfare analysis (User Surplus)

**Area 1**

**Proposition 6.** In Area 1, the incentives of the platforms and of the users are aligned. Platforms choose compatibility if and only if users prefer it as well.

**Proof.** To be written.

**Area 2**

**Proposition 7.** In Area 2, ...
Figure 3: Final compatibility analysis

**Area 3**

**Proposition 8.** *In Area 3, users always prefer services to be incompatible.*

*Proof.* To be written.

**Intuition:** all the extra value created by compatibility is captured by the platforms. As a consequence, the price increases but the net value to users stay the same.

### 3.5 Optimal compatibility level

Imagine now that Stage 1, Platform 1 can choose the optimal $q$ instead of just compatibility or incompatibility.
4 Sequential Entry

Are the compatibility incentives the same when Platform 1 is an incumbent and Platform 2 is contemplating entry?

5 Mergers and horizontal diversification

We will take a look at the example of merger between Whatsapp and Facebook, and at the diversification of Facebook into dating (competing against Tinder with which there was a compatibility). What’s the possible welfare impact of these decisions?
References


Appendix

Proof of Proposition 1

The possible responses of platform 1 (wlog) to platform 2’s \( w_U^2 \) has the following logic (very similar to that of Rey-Tirole (2018)), whether services are compatible or not:

1) Suppose that \( w_U^2 > \epsilon + b(q) \), then firm 1 can choose \( w_U^1 \) in four different ways:

1.1) \( w_U^2 > w_U^1 > \epsilon + b(q) \): Then platform 1 gets the full market

1.2) \( w_U^1 > w_U^2 > \epsilon + b(q) \): Then platform 2 gets the full market (strictly dominated by choice 1.1 for platform 1)

1.3) \( w_U^1 = w_U^2 > \epsilon + b(q) \): Then platform 1 & 2 share the market (strictly dominated by choice 1.1 for platform 1)

1.4) \( w_U^2 > \epsilon + b(q) > w_U^1 \): Then platform 1 gets the full market

2) Suppose that \( w_U^2 \leq \epsilon + b(q) \), then platform 1 can choose \( w_U^1 \) in two different ways:

2.1) \( w_U^1 > \epsilon + b(q) > w_U^2 \): Then platform 2 gets the full market (strictly dominated by choice 2.2 for platform 1)

2.2) \( w_U^1 \leq \epsilon + b(q) \): Then users adopt the two services.

With a symmetric reasoning and by iterated elimination of strictly dominated strategy, we see that playing above \( \epsilon + b(q) \) would induce a Bertrand-like race to the bottom and that platform 1 & 2 will never do it. We get a result very similar to that of Rey-Tirole (2018). Namely, when \( \hat{w}(\epsilon, q) \geq \epsilon + b(q) \), competition drives the quality-adjusted price down to \( \epsilon + b(q) \), as above this
value users adopt only one of the two products. When $\hat{w}^c < \epsilon + b(q)$, the equilibrium quality-adjusted price is $\hat{w}^c$. We thus have:

$$w^N = \min\{\epsilon + b(q), \hat{w}(\epsilon, q)\}$$

**Proof of Proposition 2**

Knowing that all users who adopt one service also adopt the other, and taking into account the functional form for $\pi_i$, the profit equation platform $i$ should maximize is:

$$\Pi(w^U_i, p^A_i, \epsilon, q) = D(w^U_i + w^U_j, q)\left[w^U_i + \alpha^U n^A_i(p^A_i) + \pi(p^A_i, t(\epsilon, q))\right]$$

$$= \left[1 + b(q) - w^U_i - w^U_j\right] \times \left[w^U_i + \alpha^U n^A_i(p^A_i) + n^A_i(p^A_i)\left[t(\epsilon, q)\left(\frac{1}{2} - n^A_i(p^A_i)\right) + p^A_j\right]\right]$$

Which is exactly (P).

Remember that we have $w^U_i = p^U_i - \alpha^U n^A_i(p^A_i)$. It means that for $w^U_i$ optimal, it is possible to adjust $p^U_i$ to sustain any value of $n^A_i(p^A_i)$ and thus any value of $p^A_i$ ($n^A_i$ and $p^A_i$ are in bijection). As a consequence, we can simply optimize (P) with respect to $w^U_i$ and $p^A_i$, as if $w^U_i$ was not depending on $p^A_i$.

The problem (P) is concave in $w^U_i$ and $p^A_i$, and FOCs are given by:

For platform 1 and 2, the FOC with respect to $w^U_i$ is:

$$D(w^U_i + w^U_j, q) + \left[w^U_i + \alpha^U n^A_i(p^A_i) + \pi(p^A_i, t(\epsilon, q))\right] \times D'(w^U_i + w^U_j, q) = 0$$
and the FOC with respect to $p^A_i$ is:

$$(\alpha_U + p^A_i) \times n^A_i(p^A_i) + n^A_i(p^A_i) = 0$$

Solving this four-equation linear system yields:

$$w^U_i = w^U_j = 1$$

$$p^A_i = p^A_j = t(\epsilon, q) - \alpha_U \equiv \hat{p}(\epsilon, q)$$

The symmetric solutions exist and are unique.

**Proof of lemma 1**

This is a direct consequence of the previous profit calculations.

Isolating $\epsilon$ the inequality $\hat{w}(\epsilon, q) \leq \epsilon + b(q)$, with $t(\epsilon, q) = \epsilon(1 - q)$, and with $q \in (0, 1]$ for subgames with compatibility and $q = 0$ for subgames with incompatibility yields the values of $\bar{\epsilon}(q)$ and $\bar{\epsilon}^o$. Additionally, we have:

$$\bar{\epsilon}(q) - \bar{\epsilon}^o = \frac{2}{7 - q} \left( 1 - 2b(q) \right) - \frac{2}{7}$$

$$= \frac{2}{7(7 - q)} \times [q - 14b(q)]$$

This is positive if and only if $q - 14b(q) \geq 0 \iff \frac{b(q)}{q} \leq \frac{1}{14}$. As by assumption on $\beta(.)$, $\frac{\beta(q)}{q} > \frac{2}{7}$, $\bar{\epsilon}(q)$ is never greater or equal to $\bar{\epsilon}^o$, for any $q \in (0, 1]$. 

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Proof of Lemma 2

It is easy to see that $\hat{\Pi}(\epsilon, q) \geq 0$ as it is the square of a positive function. However, $\check{\Pi}(\epsilon, q)$ is positive under the condition that $\epsilon \leq \check{z}(q) \equiv \frac{1-b(q)}{2}$.

We know from the previous result that $\check{\Pi}(\epsilon, q)$ is used as a comparison element for the compatibility decision when $\epsilon < \bar{\epsilon}(q)$. It is hence sufficient to show that $\bar{\epsilon}(q) \leq \check{z}(q)$ for $\check{\Pi}(\epsilon, q)$ to be always positive, when it is the relevant profit.

As a matter of fact we have:

$$\bar{\epsilon}(q) - \check{z}(q) = \frac{-b(q)(1 + q) - 3 + q}{2(7 - q)}$$

for which the denominator is always positive and the numerator always negative (for any value of $q \in [0, 1]$).

As a consequence, the equilibrium profits, in subgames where the services are compatible, are always positive. The subgame perfect equilibrium (SPE) profits will be at least the equilibrium profits of these subgames. Hence SPE profits will always be positive.

Proof of Proposition 3

Comparing $\hat{\Pi}(\epsilon, q)$ and $\hat{\Pi}(\epsilon, 0)$ in Area 1, we have:

$$\hat{\Pi}(\epsilon, q) - \hat{\Pi}(\epsilon, 0) = \left[\frac{1}{3}(1 + b(q) + \epsilon(1 - q))\right]^2 - \left[\frac{1}{3}(1 + \epsilon)\right]^2$$

$$= \left[\frac{1}{3}(1 + b(q) + \epsilon(1 - q)) + \frac{1}{3}(1 + \epsilon)\right]$$

$$\times \left[\frac{1}{3}(1 + b(q) + \epsilon(1 - q)) - \frac{1}{3}(1 + \epsilon)\right]$$
The first part of the equation is positive. The second part can be reduced by computation to \( \frac{1}{3}(-\epsilon q + b(q)) \). As a consequence \( \tilde{\Pi}(\epsilon, q) - \tilde{\Pi}(\epsilon, 0) \geq 0 \iff \epsilon \leq \frac{b(q)}{q} \equiv \delta^o(q) \).

As \( \delta^o(1) < 1 \) by assumption, and as \( \Delta^o(\cdot) \) continuous over \((0, 1]\), there exists an area above \( \delta^o(\cdot) \) and below 1. Hence, there is a subset of Area 1 in which incompatibility is chosen.

**Proof of Lemma 3**

We want to prove that there always exists a sub-area of Area 3 in which compatibility is not chosen, i.e. that there always exists at least one couple \((\epsilon, q)\) for which \( \tilde{\Pi}(\epsilon, q) < \tilde{\Pi}(\epsilon, 0) \).

Let us study the function \( \Delta \tilde{\Pi}(\epsilon, q) \equiv \tilde{\Pi}(\epsilon, q) - \tilde{\Pi}(\epsilon, 0) \) when \( \epsilon \leq \epsilon^o = \frac{2}{7} \) and \( q \in (0, 1) \).

\[
\Delta \tilde{\Pi}(\epsilon, q) = \left(1 - b(q) - 2\epsilon\right)\left(\epsilon + b(q) + \frac{1}{2}t(\epsilon, q)\right) - \left(1 - 2\epsilon\right)\left(\epsilon + \frac{1}{2}t(\epsilon, 0)\right)
= b(q)\left(1 - \frac{1}{2}\epsilon(7 - q) - b(q)\right) - \frac{1}{2}\epsilon q \left(1 - 2\epsilon\right)
\]

Claim 1. \( \Delta \tilde{\Pi}(\cdot) \) is decreasing in \( \epsilon \) in Area 2 and 3.

We have \( \frac{d\Delta \tilde{\Pi}}{d\epsilon}(\epsilon, q) = 2q > 0 \), meaning that \( \frac{d\Delta \tilde{\Pi}}{d\epsilon} \) is increasing in \( \epsilon \).

We also have :

\[
\frac{d\Delta \tilde{\Pi}}{d\epsilon}(\epsilon, q) = -\frac{1}{2}\left[b(q)(7 - q) + q(1 - 4\epsilon)\right]
\]

Note that \( \frac{d\Delta \tilde{\Pi}}{d\epsilon}(0, q) < 0 \). In the same way, we observe that \( \frac{d\Delta \tilde{\Pi}}{d\epsilon}(\frac{2}{7}, q) < 0 \) when \( \frac{b(q)}{q} \geq \frac{1}{(7-q)} \). As \( \frac{b(q)}{q} \geq \frac{2}{7} \) by assumption on \( b'(\cdot) \), we conclude that \( \frac{d\Delta \tilde{\Pi}}{d\epsilon}(\epsilon, q) < 0 \), \( \forall \epsilon \in \left[0, \frac{2}{7}\right] \).
As a consequence, $\Delta \tilde{\Pi}$ is strictly decreasing in $\epsilon$ in Areas 2 and 3.

\textbf{Claim 2.} There exists $\bar{q} > 0$ such that :
\[ \forall q \in [0, \bar{q}], \exists \epsilon \in \left[0, \frac{2}{7}\right] \quad \Delta \tilde{\Pi}(\epsilon, q) = 0. \]

We use the continuity of $\Delta \Pi(.)$, its strict monotonicity (established in Claim 1) and the Intermediate Value Theorem:

- **When** $\epsilon = 0$ : $\Delta \tilde{\Pi}(0, q) = b(q)(1 - b(q)) \geq 0$ as $1 > b(q) \geq 0, \forall q \in [0, 1]$.

- **When** $\epsilon = \frac{2}{7}$ : We have :
\[ \Delta \tilde{\Pi}\left(\frac{2}{7}, q\right) = \left[\frac{1}{7}\right]^2 \times q(7b(q) - 3) - [b(q)]^2 \]

As a consequence, we have :
\[ \frac{d\Delta \tilde{\Pi}}{dq}\left(\frac{2}{7}, q\right) = b'(q)\left[\frac{q}{7} - 2b(q)\right] + \left(\frac{1}{7}\right)^2 (7b(q) - 3) \]

And thus :
\[ \frac{d\Delta \tilde{\Pi}}{dq}\left(\frac{2}{7}, 0^+\right) = -3\left(\frac{1}{7}\right)^2 < 0 \]

First notice that, by definition, for $q = 0$ we have $\Delta \tilde{\Pi}(\epsilon, 0) = 0$. The previous results states that for $\epsilon = \frac{2}{7}$, the function $\Delta \tilde{\Pi}(.)$ is strictly decreasing in $q$ for $q = 0$. As the function is continuous in $q$, we have $\Delta \tilde{\Pi}\left(\frac{2}{7}, 0^+\right) < 0$. Let us call $\bar{q}$ the first value of $q$ at which the function $\Delta \tilde{\Pi}\left(\frac{2}{7}, .\right)$ becomes positive if it exists. If $\Delta \tilde{\Pi}\left(\frac{2}{7}, .\right)$ is always negative for all $q \in [0, 1]$ then we will say that $\bar{q} \equiv 1$.

Then $\forall q \in [0, \bar{q}]$ and $\epsilon = \frac{2}{7}$, $\Delta \tilde{\Pi}\left(\frac{2}{7}, q\right) \leq 0$.

As a consequence, by the Intermediate Value Theorem, and by continuity
and strict monotonicity of $\Delta \tilde{\Pi}(.)$ in $\epsilon$,
\[ \forall q \in [0, \bar{q}], \exists \epsilon \in \left[0, \frac{2}{7}\right] \quad / \quad \Delta \tilde{\Pi}(\epsilon, q) = 0. \]

Additionally, by continuity of the function in $q$ and $\text{epsilon}$, the set of points $(\epsilon, q)$ that verify $\Delta \tilde{\Pi}(\epsilon, q) = 0$ forms a continuous line that crosses the line $\epsilon = \frac{2}{7}$ in $q$. We will denote the equation of this line $\epsilon = \delta^q(q)$. 

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