Platform Contracting With Developers*

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Abstract

A monopoly integrator platform contracts with two developers and sells to a continuum of consumers. We study two contractual implementation modes between the platform and the developers under adverse selection, also permitting for a development stage. Exemplifying a new "openness-versus-control" trade-off, Bayesian implementation leads to more distortions in equilibrium. The alternative, more open contracting mode of Pareto-dominant strategy implementation, not yet studied in a platform context, enables more first-best outcomes under joint quality determination and higher expected consumer surplus. Our analysis delivers a new explanation for platform-based organizational choices.

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1 Introduction

How should a platform organize relationships with two different developers that simultaneously produce a quality-relevant input, running on the platform? With consumers present who appreciate quality, would an organizational mode in which the platform retains more control be the better choice, or should the platform use a governance structure that rests on openness, this way giving the developers more freedom to jointly determine quality?

Our leading example is the videogame industry where a platform (game console producer) hires multiple video game developers (studios). The industry classifies such contractual modes into first, second and third party contracts, mainly justifying this distinction by the diverging degree of integration. First party contracts and third party contracts are quite obvious boundary cases. Nintendo's "Mario Bros." as a fully integrated game is running in many versions on all Nintendo consoles – from the early GameBoy to the Wii. Third party contracts mark the other end of the spectrum, usually thought of the case without integration. Still, Nintendo's strategy makes also specific use of second party developers and differs quite a bit from Microsoft's practice to *fully* integrate a series of smaller studios.

A second party studio is defined as a developer who is tied to a specific console manufacturer by a contract. The difference between second party and first party developers is that the second party developer is a *separate entity* from the console manufacturer, while the first party developer is a division of the company.[1]

There is little doubt that vertical integration has led to a series of exclusive vertical arrangements that may have influenced platform competition in the video game industry(Lee, 2013). Yet, this is not our object of study. We are interested in identifying the consequences of the limited contractual options which a platform has at hand to solve an inherent incentive and control problem in second party contracts. This touches on the ensuing question why some contracts are preferred over others and goes way beyond vertical integration. Simple ownership arguments fall short of capturing the motive for a platform's contractual choice. To reduce second party contracts simply to an intermediate form of vertical integration which should fit the span between full and no integration will fail to capture the nature of these contracts, specifically of why platforms may prefer forms with tighter control over forms that permit coordination when hiring multiple outside developers.

A paper that has paved the way to our analysis is Riordan (1990), who has argued in favor of abandoning property rights theory and to model vertical relations through a form of complete contracting, what he calls "requirements contracting." We follow his idea that production contracts need to include specific arrangements and build on the distinction between two options, full control via Bayesian implementation versus a more

[1]See http://nintendo.wikia.com/wiki/Category:Second_party_developers

open or less controlled regime with Pareto dominant strategy implementation, which we find to emerge in a three-player setup with asymmetric information. That contracts are essential to platform governance has already been illustrated in Boudreau and Hagiu (2009) who have shown how platforms rule through contracts to keep control. The thrust of our paper is, however, different. We show that organizational boundaries shift together with the organizational mode chosen, and this mode is determined by the chosen contractual relationship.

Few papers have studied this aspect in greater detail. While venturing in a different direction, Hagiu and Spulber (2013) focus on first and third party content and why there is a strategic overuse of first party contracts in the presence of two-sided network effects. In our set-up without two-sided network effects across the two sides of the market, best classified as an integrator platform (Boudreau and Lakhani, 2009, according to), externalities are contractual in nature, and they arise between agents on one side of the platform due to an informational advantage that may be solved by different forms of implementation in different ways.

Our paper has more in common with Hagiu and Wright (2019) (HW thereafter). There, contractual options are studied in a double-sided moral hazard setup. Both the principal and the agent need to be incentivized to carry out investments which increase jointly created revenues. This permits the authors to distinguish two forms of contracting with different degrees of control rights, not dissimilar in spirit from our paper. In turn, we study the platform's options to choose the preferred organizational mode when two developers are contracted simultaneously. In our setup two developers deliver a joint input, which can be done by using two distinct organizational modes which emerge naturally from two specific forms of contractual implementation. This property of contract implementation remains hidden as long as contracts are studied solely under moral hazard. To venture into new grounds, we model three players under adverse selection where the principal (platform) contracts the two agents (developers) simultaneously.

The starting point for our analysis is an existing informational asymmetry between the platform on one side and the two agents on the other. Our paper therefore distinguishes a specific "control" mode from an "enabling" mode as studied in HW, but our model is much different in detail: instead of focusing on network effects, we study Bayesian and Pareto-dominant strategy implementation between platform and developers, which brings our paper close to the implementation literature (Mookherjee and Reichelstein, 1992; Laffont and Martimort, 1997, 1998; Lawarrée and Shin, 2005; Shin, 2007, 2008).

The remainder of this paper is structured as follows. Section 2 describes the market model, Section 3 describes the full-information benchmark, Section 4 introduces asymmetric information and studies a development stage under ex-ante contracting. Section 5 treats Pareto-dominant strategy implementation, Section 6 offers a comparison, Section 7 concludes. Additional proofs are given in the appendix.

2 The Model

There is a platform (principal), two developers (agents) and a continuum of consumers. The principal offers a contract to the two firms simultaneously. Consumers are differentiated by the taste parameter s which represents their willingness to pay for the quality Q of the platform service. The platform does not know the individual consumer's taste parameter s, so in addition to asymmetric information between platform and producers, there is also asymmetric information between the platform and the buyer side.

The two developers might differ in the parameter θ. Each productive agent can be of two types: low and high. A low-type (efficient) agent has a better technology to produce a given amount of quality, which determines its cost parameter θ. For simplicity, we assume that θ can only assume two values (low and high). A low-type firm has a cost parameter of $0 < \underline{\theta}$, while a high-type developer's cost type is $\bar{\theta} > \underline{\theta}$. We define $\Delta\theta \equiv \bar{\theta} - \underline{\theta}$ as the parameter difference between an inefficient and an efficient type. Quality Q offered on the platform is contractible, and it is the sum of the qualities of the application produced by the two firms, with $Q = q_1 + q_2$.

After characterizing the full information benchmarks we allow for asymmetric information. Under asymmetric information, only the agent under consideration knows potentially his own type, while the principal and the other agent, matched with the agent considered here, are ex ante uninformed. Each agent learns his type after a development stage in which he acquires information. After this stage, he shares this potentially soft information with the other agent. What is common knowledge is the distribution of types. Both agents' types are stochastically independent; a low-type agent occurs with probability ν, and a high-type agent with probability $1 - \nu$.

With three players, a clarification of "full information" versus "asymmetric information" is in order. Ex ante the developers do not know each other's type. After each developer potentially reveals his cost parameter θ via its contract choices, each produces his individual quality q. The principal values quality Q according to a concave value function $V(Q)$, while his surplus is $S(Q) = V(Q)$ *minus transfers*. Costs of producing quality q to a firm is given by a convex cost function θq^2. Each firm's reservation payoff is zero, and firms receive equal transfers according to the verifiable quality Q of the platform product. A firm which reveals to be of type $\underline{\theta}$ receives a transfer of $\underline{t}$ should the other firm be of type $\underline{\theta}$ as well, and $\hat{t}$ when the other firm reveals to be of type $\bar{\theta}$. When both firms are of type $\bar{\theta}$, their transfer is $\bar{t}$.

Moreover, when both cost types are the same, both types produce the same quality in equilibrium. We label the two qualities with both firms having the same cost type $\bar{Q}$ and $\underline{Q}$, meaning each firm contributes with either $\bar{Q}/2$ or $\underline{Q}/2$ to the quality $\bar{Q}$ or $\underline{Q}$ of the platform product. When types differ, the efficient $\underline{\theta}$-type of firm produces $\rho\hat{Q}$ and the inefficient $(1-\rho)\hat{Q}$.

Timing. The game has the following timing under *Bayesian implementation.*

• At $t = 1$, nature determines the type of each agent (quality type θ). The two developers potentially learn their own type at $t = 3$ and each other's type at $t = 5$.

• At $t = 2$, the platform principal offers a contract $(\underline{t}, \hat{t}, \bar{t}, \underline{Q}, \hat{Q}, \bar{Q}, \rho)$ to the two developers.

• At $t = 3$, the agents are in the development phase where they each can invest ψ to learn their own type.

• In $t = 4$, all agents accept or reject the offered contract.

• At $t = 5$, the two developers learn each other's type, given that each of them had invested the necessary ψ in $t = 3$ and knows his own type.

• Lastly, in $t = 6$, the productive agents deliver the joint good and receive the contracted transfer. The platform sets the market price for the joint product and sells it to the consumers.

With *Pareto-dominant strategy implementation* the timing is the same, but in $t = 5$ the two developers do not only interchange information, but potentially also side payments which allows them to coordinate their choices of Q and t from the offered contract.

Market Model. We assume a continuous mass of consumers with the measure $N = 1$ whose appreciation for the platform good's quality $s \in [0, \bar{s}]$ is uniformly distributed on the interval $[0, \bar{s}]$. They either buy one or none of the platform goods. Their surplus is

$$U(Q, p) = sQ - p, \tag{1}$$

if they consume one platform good with joint quality Q at a price p and $\underline{U} = 0$ if they do not consume a platform good. The platform cannot observe a single consumer's willingness to pay s and can therefore not differentiate the price according to s but sets a uniform price p.[2] All consumers whose appreciation exceeds

$$\tilde{s} = \frac{p}{Q}$$

[2] This type of market modelling goes back to Mussa and Rosen (1978).

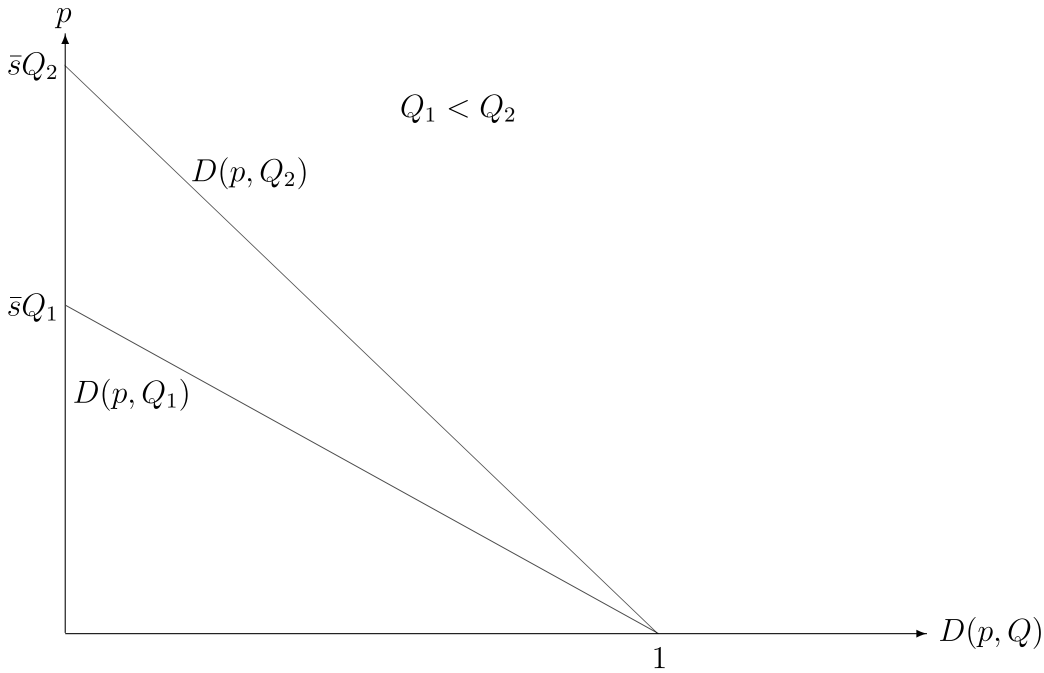

Figure 1: *Demand for the Platform Good*

are going to buy the platform good because for all of them $sQ - p \geq 0$. The demand for the platform good is

$$D(p, Q) = N \int_{\tilde{s}}^{\bar{s}} \frac{1}{\bar{s}} ds = \frac{\bar{s} - \tilde{s}}{\bar{s}} = 1 - \frac{p}{\bar{s}Q}.$$

Note that demand is increasing in the platform good's joint quality Q in the way characterized in figure 1.

As long as the quality Q of the platform good finally satisfies $Q \geq p/\bar{s}$, its monopoly profit is

$$\pi(p, Q) = p(1 - \frac{p}{\bar{s}Q})$$

and the profit maximizing monopoly price is

$$p^m = \frac{\bar{s}Q}{2}.$$

Substituting this price into the monopoly platform's monopoly profit yields the platform's surplus in terms of the platform product's quality

$$\Pi(Q) = \pi(p^m, Q) = \frac{\bar{s}Q}{4} = S(Q).$$

Note that if we follow this standard IO model the surplus function is convex (linear) but not strictly convex in Q.

Consumer surplus in the market is

$$CS(Q) = N \int_{\tilde{s}}^{\bar{s}} (sQ - p^m) \frac{1}{\bar{s}} ds = \int_{\frac{\bar{s}}{2}}^{\bar{s}} \left(sQ - \frac{\bar{s}Q}{2} \right) \frac{1}{\bar{s}} ds = \frac{\bar{s}Q}{8} \tag{2}$$

Feasible Qualities. With two agents who can be of two types, the platform's possible permutations of quality targets are the following four combinations: LL, LH, HL, and HH related to the case where both agents have a low cost of quality $\underline{\theta}$ (LL), where one has a low cost $\underline{\theta}$ and the other a high cost $\bar{\theta}$ of quality (LH or HL) or where both have a high cost of quality $\bar{\theta}$ (HH). If the platform cannot discriminate between the two developers, if they have different types, the combination of two developers which can be of two types result in three quality and transfer combinations (and thus contract realizations) where $(Q, t) = (\underline{Q}, \underline{t})$ is the contract realization for LL, $(Q, t) = (\hat{Q}, \hat{t})$ for either LH or HL and $(Q, t) = (\bar{Q}, \bar{t})$ for HH. If the platform can distinguish between the two types of agents when they report different types, the contract realization for LH is $(Q, t_L, t_H) = (\hat{Q}, \hat{\underline{t}}, \hat{\bar{t}})$.

3 The Full Information Benchmark

One can distinguish two cases where in stage $t = 3$ not only the two developers but also the platform learns the two developers' types without any of them having to invest ψ. In the first case the platform can verify this information, whereas in the second case it cannot. This means that in the first case the platform can make the paid transfers contingent on the individual developer's type and on the type of the other developer it is matched with. In the second case the platform can make the transfers only contingent on the verifiable quality level of the composite good designed by the two developers. Thus, whereas in the first case the transfers can be differentiated between the two agents, this is not possible in the second case.

3.1 Full Information with Perfect Transfer Discrimination (First best)

Here the optimal contract defines the individual transfers $\underline{t}$, $\bar{t}$,$\hat{\underline{t}}$, $\hat{\bar{t}}$ to each agent and the quality of the composite good $\underline{Q}$,$\bar{Q}$ and $\hat{Q}$ under all contingencies. In addition it defines the quality contribution of the low cost agent $\rho\hat{Q}$ and the high cost agent $(1-\rho)\hat{Q}$ in the case where both agents' types differ. If both firms have the same type their individual quality contribution is assumed to be identical, meaning either $\frac{\bar{Q}}{2}$ or $\frac{\underline{Q}}{2}$.

Thus, the principal's maximization problem is

$$\max_{\underline{Q},\hat{Q},\bar{Q},\underline{t},\hat{\bar{t}},\hat{\underline{t}},\bar{t},\rho} \underline{\nu}^2[S(\underline{Q})-2\underline{t}]+2\underline{\nu}(1-\underline{\nu})[S(\hat{Q})-\hat{\underline{t}}-\hat{\bar{t}}]+(1-\underline{\nu})^2[S(\bar{Q})-2\bar{t}]$$

$$\begin{aligned} \text{subject to} \qquad \underline{t}-\underline{\theta}\left(\frac{\underline{Q}}{2}\right)^2 &\geq 0, \\ \bar{t}-\bar{\theta}\left(\frac{\bar{Q}}{2}\right)^2 &\geq 0, \\ \hat{\underline{t}}-\underline{\theta}(\rho\hat{Q})^2 &\geq 0, \\ \hat{\bar{t}}-\bar{\theta}\left((1-\rho)\hat{Q}\right)^2 &\geq 0. \end{aligned}$$

Due to the convex cost of quality, both the efficient and the inefficient agent contribute to the quality of the joint platform good with full information if they are matched.[3] In this case the principal should also use the inefficient firm to some extent because with a sufficiently smaller contribution to the total quality the inefficient firm has the same marginal cost of quality than the efficient firm. This way the efficient firm finally contributes more to the quality of the joint platform product with $\rho^* > 1/2$. The full information contract is characterized in proposition 1 which is proven in Appendix A.1.

Proposition 1 *With full information and perfect transfer discrimination the optimal contract implies that the quality of the platform product is chosen such that the platform's marginal extra surplus of the quality is equalized with the marginal costs which both agents have when producing it with*

$$S'(\underline{Q}^*)=\underline{\theta}\underline{Q}^*; S'(\hat{Q}^*)=2\rho^*\underline{\theta}\hat{Q}^*, \textit{with } \rho^*=\frac{\bar{\theta}}{\underline{\theta}+\bar{\theta}}>\frac{1}{2} \textit{ and } S'(\bar{Q}^*)=\bar{\theta}\bar{Q}^*.$$

The transfers are determined by the agents' binding participation constraints and none of them realizes a rent.

Note that there is the usual decreasing quality schedule of $\underline{Q}^* > \hat{Q}^* > \bar{Q}^*$.

[3]This differs our analysis also from the formally very similar analysis in Shin (2007) and (2008).

3.2 Full Information with Imperfect Transfer Discrimination

If the principal cannot discriminate the transfers in case of a match between an inefficient and an efficient agent, the optimal contract defines the individual transfers to each agent only for three distinguishable cases $\underline{t}$, $\bar{t}$, and $\hat{t}$, which correspond to the three different quality outcomes of the composite good $\underline{Q}$,$\bar{Q}$ and $\hat{Q}$ which the principal demands contingent on the two agents' efficiency levels. Again the principal also defines the quality contribution of the low cost agent $\rho\hat{Q}$ and the high cost agent $(1-\rho)\hat{Q}$ in the case where both agents' types differ. Otherwise their individual quality contribution is assumed to be identical. The maximization problem is the same as before, but the platform loses one instrument and has to account for the extra constraint $\hat{\underline{t}} = \hat{\bar{t}} = \hat{t}$.

Again both, the efficient and the inefficient agent, contribute to the quality of the joint platform good if they are matched. The full information contract with incomplete transfer discrimination is characterized in proposition 2 which is proven in Appendix A.2.

Proposition 2 *With full information and imperfect transfer discrimination the optimal contract implies that the quality of the platform product is chosen such that the platform's marginal extra surplus of the quality is equalized with the platform's marginal cost of inducing the agents to produce it with*

$$S'(\underline{Q}^{**}) = \underline{\theta}\underline{Q}^{**}; S'(\bar{Q}^{**}) = \bar{\theta}\bar{Q}^{**} \, and$$

$$S'(\hat{Q}^{**}) = \frac{4\underline{\theta}\bar{\theta}}{(\sqrt{\underline{\theta}}+\sqrt{\bar{\theta}})^2}\hat{Q}^{**} = 4\sqrt{\underline{\theta}}\sqrt{\bar{\theta}}\rho^{**}(1-\rho^{**})\hat{Q}^{**}, \;\; with \; \rho^{**} = \frac{\sqrt{\bar{\theta}}}{\sqrt{\bar{\theta}}+\sqrt{\underline{\theta}}} > \frac{1}{2}.$$

The transfers are determined by the agents' binding participation constraints and none of them realizes a rent.

The quality level of the joint platform product is obviously the same as before with full information and perfect transfer discrimination, if either two efficient or two inefficient agents are matched ($\underline{Q}^* = \underline{Q}^{**}$ and $\bar{Q}^* = \bar{Q}^{**}$). However, this changes if an efficient agent is matched with an inefficient one. Due to the impossibility to discriminate transfers in this case the quality level regulated by the platform is lower and from this lower quality level a smaller share is provided by the efficient developer. These results are summarized in the following corollary which results directly from comparing the outcomes of proposition 1 and 2.

Corollary 1 *With full information, but no possible transfer discrimination, if two agents with different efficiencies in producing quality are matched, the joint platform product's quality is lower ($\hat{Q}^{**} < \hat{Q}^*$) and the more efficient firm's provides a lower share of this lower quality ($\rho^{**} < \rho^*$) than if transfer discrimination were possible in this case.*

Despite the lower quality level, in case two agents with different efficiency levels are matched, the optimal contract still implies the usual decreasing quality schedule with $\underline{Q}^{**} > \hat{Q}^{**} > \bar{Q}^{**}$.

4 Asymmetric Information and Bayesian Implementation

Here we first assume that the two developers learn their own type for free in $t = 3$. Afterwards, in subsection 4.2, we consider the situation where acquiring information about one's own cost type is costly and $\psi > 0$ holds.

4.1 Bayesian Implementation

This basic setup assumes that only the firms learn each other's type after participation, whereas their types are no longer observable for the principal. Transfers are conditioned on the quality levels of the joint platform product. This means that the platform can again not discriminate the transfers in case of two different efficiencies, meaning $\underline{\hat{t}} = \bar{\hat{t}} = \hat{t}$.

Thus, the principal's maximization problem is

$$\max_{\{\underline{Q},\hat{Q},\bar{Q},\underline{t},\hat{t},\bar{t},\rho\}} \nu^2[S(\underline{Q}) - 2\underline{t}] + 2\nu(1-\nu)[S(\hat{Q}) - 2\hat{t}] + (1-\nu)^2[S(\bar{Q}) - 2\bar{t}],$$

subject to four individual rationality (IR) constraints:

$$\underline{t} - \underline{\theta}\left(\frac{\underline{Q}}{2}\right)^2 \geq 0, \tag{IR$_{\underline{\theta}\underline{\theta}}$}$$

$$\bar{t} - \bar{\theta}\left(\frac{\bar{Q}}{2}\right)^2 \geq 0, \tag{IR$_{\bar{\theta}\bar{\theta}}$}$$

$$\hat{t} - \underline{\theta}(\rho\hat{Q})^2 \geq 0, \tag{IR$_{\underline{\theta}\bar{\theta}}$}$$

$$\hat{t} - \bar{\theta}[(1-\rho)\hat{Q}]^2 \geq 0. \tag{IR$_{\bar{\theta}\underline{\theta}}$}$$

In addition there are four incentive compatibility (IC) constraints to make sure that each type contributes according to its own and the other developer's true type:

$$\underline{t} - \underline{\theta}\left(\frac{\underline{Q}}{2}\right)^2 \geq \hat{t} - \underline{\theta}[(1-\rho)\hat{Q}]^2, \tag{IC$_{\underline{\theta}\underline{\theta}}$}$$

$$\bar{t} - \bar{\theta}\left(\frac{\bar{Q}}{2}\right)^2 \geq \hat{t} - \bar{\theta}(\rho\hat{Q})^2, \tag{IC$_{\bar{\theta}\bar{\theta}}$}$$

$$\hat{t} - \underline{\theta}(\rho\hat{Q})^2 \geq \bar{t} - \underline{\theta}\left(\frac{\bar{Q}}{2}\right)^2, \tag{IC$_{\underline{\theta}\bar{\theta}}$}$$

$$\hat{t} - \bar{\theta}[(1-\rho)\hat{Q}]^2 \geq \underline{t} - \bar{\theta}\left(\frac{\underline{Q}}{2}\right)^2. \tag{IC$_{\bar{\theta}\underline{\theta}}$}$$

As explained in Appendix B.1.1, four constraints are binding, namely $IC_{\underline{\theta}\bar{\theta}}$, $IC_{\underline{\theta}\underline{\theta}}$, $IR_{\bar{\theta}\bar{\theta}}$ and $IR_{\bar{\theta}\underline{\theta}}$. There we also derive the requested optimal quality levels of the platform product in the second-best optimal contract. They need to satisfy

$$S'(\underline{Q}^n) = \underline{\theta}\underline{Q}^n, \tag{3}$$

$$S'(\hat{Q}^n) = \left(\frac{2(1-\rho)\rho\underline{\theta}\left[2\bar{\theta} - \nu(\bar{\theta}+\underline{\theta})\right]}{(1-\nu)\left[(1-\rho)\bar{\theta}+\rho\underline{\theta}\right]}\right)\hat{Q}^n, \tag{4}$$

and

$$S'(\bar{Q}^n) = \left(\frac{(1-\rho)\bar{\theta}^2 + \left[\rho(1+\nu^2) - 2\nu\right]\bar{\theta}\underline{\theta} + \nu^2(1-\rho)\underline{\theta}^2}{(1-\nu)^2\left[(1-\rho)\bar{\theta}+\rho\underline{\theta}\right]}\right)\bar{Q}^n. \tag{5}$$

Whereas the optimal quality contribution ρ in a match with one efficient and one inefficient type is then implicitly defined by

$$[\bar{\theta}(1-\rho)^2 - \underline{\theta}\rho^2](\hat{Q}^n)^2 = (\bar{\theta}-\underline{\theta})\left(\frac{\bar{Q}^n}{2}\right)^2. \tag{6}$$

As also explained in Appendix B.1.1, one can show that $0 < \bar{Q}^n < \hat{Q}^n < \underline{Q}^n$ must hold for any $0 < \rho < 1$. However, due to these quality relationships the second best optimal ρ, defined by equation (6), must fulfill

$$\frac{1}{2} < \rho < \frac{\sqrt{\bar{\theta}}}{\sqrt{\bar{\theta}}+\sqrt{\underline{\theta}}}. \tag{7}$$

The next proposition characterizes how the quality levels compare with the first-best qualities.

Proposition 3 *Under asymmetric information, Bayesian implementation leads to an allocation that is partially distorted compared to the first-best. In case of two inefficient agents quality is distorted downward, meaning $\bar{Q}^n < \bar{Q}^*$. The quality level is also distorted downward with an efficient and an inefficient agent, since $\hat{Q}^n < \hat{Q}^*$ holds. There is no distortion at the top, because $\underline{Q}^n = \underline{Q}^*$ holds.*

Proof. For the most inefficient pair equation (5) must hold, whereas the first best quality level in this case requires $S'(\bar{Q}^*) = \bar{\theta}\bar{Q}^*$. Since $S''(Q) \leq 0$ is assumed, a sufficient condition for $\bar{Q}^n < \bar{Q}^*$ is

$$\frac{(1-\rho)\bar{\theta}^2 + [\rho(1+\nu^2) - 2\nu]\,\bar{\theta}\underline{\theta} + \nu^2(1-\rho)\underline{\theta}^2}{(1-\nu)^2\left[(1-\rho)\bar{\theta} + \rho\underline{\theta}\right]} > \bar{\theta},$$
$$\Leftrightarrow 2\bar{\theta} > \nu(\bar{\theta} + \underline{\theta}),$$

which is satisfied.

For a mixed match with one efficient and one inefficient agent the first best quality implies

$$S'(\hat{Q}^*) = \frac{2\bar{\theta}\underline{\theta}}{\underline{\theta} + \bar{\theta}}\hat{Q}^*,$$

whereas the second-best quality in this case is characterized in equation (4). Using the same argument as before, $\hat{Q}^n < \hat{Q}^*$ holds if

$$\frac{2(1-\rho)\rho\underline{\theta}\left[2\bar{\theta} - \nu(\bar{\theta} + \underline{\theta})\right]}{(1-\nu)\left[(1-\rho)\bar{\theta} + \rho\underline{\theta}\right]} > \frac{2\bar{\theta}\underline{\theta}}{\bar{\theta} + \underline{\theta}}$$

is satisfied. Given that ρ in the second best contract must be somewhere in the range, defined in condition (7), this relationship holds.

There is no distortion at the top as the quality $\underline{Q}$ here and in the first-best case satisfies

$$S'(\underline{Q}) = \underline{\theta}\underline{Q},$$

meaning $\underline{Q}^n = \underline{Q}^*$. ■

Quality schedules in the mixed-type case. We now turn to the comparison of quality proportions under first and second best in a mixed match. Note first that under full information, we have

$$\rho^* = \frac{\bar{\theta}}{\bar{\theta} + \underline{\theta}} > \frac{\sqrt{\bar{\theta}}}{\sqrt{\bar{\theta}} + \sqrt{\underline{\theta}}},$$

whereas ρ in the second best optimal contract is necessarily smaller, since it satisfies condition (7).

Rents. Because of two agents which can each be of two different types, the rent depends on the respective match. An efficient $\underline{\theta}$-agent is not only able to obtain a rent, when paired with another $\underline{\theta}$-agent, expressed as

$$\underline{t} - \underline{\theta}\left(\frac{\underline{Q}}{2}\right)^2 = \Delta\theta[(1-\rho)\hat{Q}]^2. \tag{D}$$

but also, when paired with an inefficient $\bar{\theta}$-type of agent. In the latter case he receives a rent of

$$\hat{t} - \underline{\theta}(\rho \hat{Q})^2 = \Delta\theta\Big(\frac{\bar{Q}}{2}\Big)^2.$$

Using equation (6) and that ρ satisfies condition (7), one can show that

$$[(1-\rho)\hat{Q}]^2 \geq \Big(\frac{\bar{Q}}{2}\Big)^2$$

holds. Thus, the efficient type's rent is higher, if he is matched with another efficient type than, if he is matched with an inefficient type. An inefficient type with $\theta = \bar{\theta}$ never realizes a rent, no matter whether he is matched with another efficient or inefficient type.
Obviously, the downward distortion of $\hat{Q}$ and $\bar{Q}$ reduces the rent necessary to prevent the efficient type from mimicking an inefficient type in case of a match with another efficient type or another inefficient type. The right-hand side of (D) reveals, however, that the downward distortions of the quality proportion of the efficient type, when matched with an inefficient type, increases the necessary information rent for the efficient type, when matched with another efficient type.

Corollary 2 *The quality proportion ρ $(1-\rho)$ of an efficient (inefficient) type is distorted downward (upward), compared to the first best, in a mixed match with the other type.*

We finally compare the quality levels in the second best optimal contract with those under full information, but with imperfect transfer discrimination. Since $\underline{Q}^* = \underline{Q}^{**}$ and $\bar{Q}^* = \bar{Q}^{**}$ hold (see proposition 2), we know already from proposition 3 that the contract with Bayesian implementation does not distort the contract with two efficient agents and distorts it downwards with two inefficient agents, compared to a situation with full information and only limited transfer discrimination. However, a deviation occurs, if an efficient agent is matched with an inefficient one, as we describe in the proposition below.

Proposition 4 *Bayesian implementation distorts the quality downward with two inefficient, and not at all with two efficient agents, compared to the full information case with imperfect transfer discrimination. With an efficient and an inefficient agent the direction of distortion cannot be determined without further specifying the model. However, if the efficiency difference between two different types is sufficiently large, then the quality level with two different types is always distorted downward, meaning $\hat{Q}^n < \hat{Q}^{**}$. With relatively small efficiency differences, an upward distortion with $\hat{Q}^n \geq \hat{Q}^{**}$ cannot be excluded.*

Proof. The quality level with an efficient and an inefficient agent under full information, but with limited transfer discrimination is characterized by

$$S'(\hat{Q}^{**}) = \frac{4\underline{\theta}\bar{\theta}}{(\sqrt{\underline{\theta}} + \sqrt{\bar{\theta}})^2}\hat{Q}^{**}.$$

Taking into account (4) reveals that $\hat{Q}^n < \hat{Q}^{**}$ only holds, if

$$\frac{4\underline{\theta}\bar{\theta}}{(\sqrt{\underline{\theta}}+\sqrt{\bar{\theta}})^2} < \frac{2(1-\rho)\rho\underline{\theta}\left[2\bar{\theta}-\nu(\bar{\theta}+\underline{\theta})\right]}{(1-\nu)\left[(1-\rho)\bar{\theta}+\rho\underline{\theta}\right]}.$$

One can show that this is only true for all ρ satisfying (7) if the efficiency difference is sufficiently large. ■

4.2 Development stage and ex-ante contracting

We expand on the general contracting mode under Bayesian Implementation with two real agents by assuming that the two firms do not know their marginal costs θ when offered a contract but need to undergo a development stage to learn their type. This is a realistic scenario for two developer firms that need to check before programming complementary game modules whether they are able to produce them at low or at high costs. Under this scenario, two ex-ante participation constraints need to be fulfilled as well as there is an information gathering stage. We compare two possible cases:

$$\nu(\hat{t}-\underline{\theta}(\rho\hat{Q})^2)+(1-\nu)[\bar{t}-\bar{\theta}\Big(\frac{\bar{Q}}{2}\Big)^2]-\psi \geq max\{\hat{t}-E(\theta)(\rho\hat{Q})^2, \bar{t}-E(\theta)\Big(\frac{\bar{Q}}{2}\Big)^2\} \qquad (\text{EAPC}_{\hat{Q}\bar{Q}})$$

and

$$\nu[\underline{t}-\underline{\theta}\Big(\frac{\underline{Q}}{2}\Big)^2]+(1-\nu)[\hat{t}-\bar{\theta}[(1-\rho)\hat{Q}]^2-\psi \geq max\{\underline{t}-E(\theta)\Big(\frac{\underline{Q}}{2}\Big)^2, \hat{t}-E(\theta)[(1-\rho)\hat{Q}]^2\} \qquad (\text{EAPC}_{\underline{Q}\hat{Q}})$$

• **Case 1:** $\underline{Q}, \hat{Q}$.

Instead of using the expected value, we can re-express the sides of $EAPC_{\underline{Q}\hat{Q}}$ as follows:

$$\nu\underline{U}+(1-\nu)\hat{\hat{U}}-\psi \geq max\{\underline{U}-(1-\nu)\Delta\theta\Big(\frac{\underline{Q}}{2}\Big)^2, \hat{\hat{U}}+\nu\Delta\left[(1-\rho)Q\right]^2\} \quad , \quad (\text{EAPC}_{\underline{Q}\hat{Q}'})$$

where $\underline{U}$ and $\hat{\hat{U}}$ denote each firm's information rent in case of two efficient firms and in case of an inefficient firm, matched with an efficient firm, respectively.

In the ex-ante setup, this leads to two subcases:

• **Sub-Case 1a:**

$$\hat{\hat{U}}-\underline{U} \geq \frac{\psi}{1-\nu}-\Delta\theta\Big(\frac{\underline{Q}}{2}\Big)^2,$$

and

- **Sub-Case 1b:**

$$\underline{U} - \hat{\bar{U}} \geq \frac{\psi}{\nu} + \Delta\theta \left[(1-\rho)\hat{Q}\right]^2 .$$

Similarly, we have a second case, again with two sub-cases:

- **Case 2:** $\hat{Q}, \bar{Q}$:

$$\nu\hat{\underline{U}} + (1-\nu)\bar{U} - \psi \geq max\{\hat{\underline{U}} - (1-\nu)\Delta\theta(\rho Q)^2, \bar{U} + \nu\Delta\theta\Big(\frac{\bar{Q}}{2}\Big)^2\}, \qquad (\text{EAPC}_{\hat{Q}\bar{Q}'})$$

with the two subcases of

- **Sub-Case 2a:**

$$\bar{U} - \hat{\underline{U}} \geq \frac{\psi}{1-\nu} - \Delta\theta(\rho\hat{Q})^2,$$

and

- **Sub-Case 2b:**

$$\hat{\underline{U}} - \bar{U} \geq \frac{\psi}{\nu} + \Delta\theta\Big(\frac{\bar{Q}}{2}\Big)^2,$$

We are now able to derive two implementability conditions from case 1,

$$\nu(1-\nu)\Delta\theta\Big[\Big(\frac{\underline{Q}}{2}\Big)^2 - \Big[(1-\rho)\hat{Q}\Big]^2\Big] \geq \psi, \qquad (\text{IP1})$$

and from case 2,

$$\nu(1-\nu)\Delta\theta\Big[(\rho\hat{Q})^2 - \Big(\frac{\bar{Q}}{2}\Big)^2\Big] \geq \psi. \qquad (\text{IP2})$$

We now consider the level of development costs. For low values of development costs ψ, the constraints (IP1) and (IP2) are slack. The key issue here is that when development cost ψ increase, further distortions in the optimal contract will appear, up to situations that we will discuss below, namely the case with the platform optimally not anymore incentivizing the agent to perform development.

Proposition 5 *We first assume that ψ is sufficiently low. Then, ex-ante contracting does not lead to a different solution compared to before, and we have*

(i) No distortion "at the top", when both agents are efficient,

(ii) and a downward distortion of quality, when one or both agents are inefficient.

Proof. One can show that with $\psi \to 0$ the two equilibrium implementability conditions are slack in the second-best solution, that is,

$$\nu(1-\nu)\Delta\theta\Big[\Big(\frac{\underline{Q}^n}{2}\Big)^2 - \Big[(1-\rho)\hat{Q}^n\Big]^2\Big] > \psi, \tag{IP1'}$$

and

$$\nu(1-\nu)\Delta\theta\Big[[(\rho\hat{Q}^n)^2 - \Big(\frac{\bar{Q}^n}{2}\Big)^2\Big] > \psi \tag{IP2'}$$

holds. This follows from the four incentive compatibility constraints ($\text{IC}_{\underline{\theta}\underline{\theta}}$), ($\text{IC}_{\bar{\theta}\bar{\theta}}$), ($\text{IC}_{\underline{\theta}\bar{\theta}}$) and ($\text{IC}_{\bar{\theta}\underline{\theta}}$). They can only be fulfilled at the same point in time as long as

$$\Big(\frac{\underline{Q}^n}{2}\Big)^2 - \Big[(1-\rho)\hat{Q}^n\Big]^2 > 0, \text{ and}$$

$$(\rho\hat{Q}^n)^2 - \Big(\frac{\bar{Q}^n}{2}\Big)^2 > 0$$

hold which, if ψ is close enough to 0, ensure that (IP1') and (IP2') are also satisfied. ■

Proposition 6 *With ψ increasing, the ex-ante constraints become binding and require additional distortions to the ones in the second-best case. If (IP1) is binding, the principle of no distortion at the top does no longer prevail and the quality with two efficient types is distorted upward compared to the second best optimal contracts. The quality level with two inefficient agents is always distorted further downward if (IP1) or (IP2) is binding, whereas the quality in a mixed match is distorted upward if only (IP1) is binding and downward if only (IP2) is binding. If both are binding, the effect with a mixed match depends on which of the two is more restrictive.*

Proof. See the Appendix B.1.2. ■

Note lastly that at even higher levels of ψ, it becomes worthwhile for the principal to not incentivize information gathering anymore. The platform satisfies only the ex-post participation constraint for the inefficient type $\bar{\theta}$.

5 Asymmetric information and Pareto-dominant Strategy Implementation

Here again we first consider the simple case where the developers can find out about their own type for free in $t = 3$. Afterwards, in subsection 5.2, similar to the Bayesian implementation case, we consider costly information with regard to one's own type with $\psi > 0$ in the development stage $t = 3$.

5.1 Pareto-dominant Strategy Implementation

Instead of using Bayesian implenentation through designing rents such that each type chooses its quality distribution according to its true type in the presence of another developer, we now study the borderline case where Bayesian implementation coincides with Pareto-dominant strategy implementation. We now permit that, after contract acceptance and becoming aware of their mutual types, the two firms may use side payments to coordinate each other's contract choices. This all is possible under the assumption that there is now unlimited wage discrimination. Limited transfer discrimination assumes that in case of a mixed match both firms receive the same transfer (wage) $\hat{t}$.

Consider the following matchings. In the presence of a $\bar{\theta}$-type of agent, the transfer to a $\underline{\theta}$ covers his production costs $\underline{\theta}\left(\frac{\bar{\theta}}{\underline{\theta}+\bar{\theta}}\hat{Q}^*\right)^2$ plus his information rent to induce truth telling, that is $\bar{t}-\underline{\theta}\left(\frac{\bar{Q}}{2}\right)^2$.

$$\hat{t}=\underline{\theta}\left(\frac{\bar{\theta}}{\underline{\theta}+\bar{\theta}}\hat{Q}^*\right)^2+\bar{t}-\underline{\theta}\left(\frac{\bar{Q}}{2}\right)^2. \qquad (\text{IC}_{\underline{\theta}\bar{\theta}}\text{'})$$

When transfer discrimination is limited, the agent type $\bar{\theta}$ with higher marginal costs free-rides on the more efficient agent type $\underline{\theta}$ Assume that the principal sets first-best targets of $\rho=\rho^*$ and $\hat{Q}^*$.

The contracting problem lies in the fact that the principal, not knowing the types, cannot reduce $\hat{t}$ below the level in $IC'_{\underline{\theta}\bar{\theta}}$ as this would no longer induce the $\underline{\theta}$-type to truthfully choose his quality contribution. Would this happen, the $\underline{\theta}$-type would misrepresent his type as being $\bar{\theta}$. This, in fact, would deprive the real $\bar{\theta}$-type of any rent as the contract would envisage $\bar{t}$.

However, under limited transfer discrimination there is space for side-contracting. Making use of it, the principal could require the $\bar{\theta}$-type of agent to pay

$$\hat{s}=\hat{t}-\bar{\theta}[(1-\rho)\hat{Q}]^2-\left[\bar{t}-\left(\frac{\bar{Q}}{2}\right)^2\right], \qquad (\text{IR}^s_{\bar{\theta}\underline{\theta}})$$

to so make the less efficient agent participate in paying for the IC constraint to induce truth telling. Under this contracting mode, the constraints for the mixed case change into:

$$\hat{t}-\underline{\theta}(\rho\hat{Q})^2+\hat{s}\geq 0, \qquad (\text{IR}^s_{\underline{\theta}\bar{\theta}})$$

$$\hat{t}-\bar{\theta}[(1-\rho)\hat{Q}]^2-\hat{s}\geq 0. \qquad (\text{IR}^s_{\bar{\theta}\underline{\theta}})$$

and

$$\hat{t}-\underline{\theta}(\rho\hat{Q})^2+\hat{s}\geq\bar{t}-\underline{\theta}\left(\frac{\bar{Q}}{2}\right)^2, \qquad (\text{IC}^s_{\underline{\theta}\bar{\theta}})$$

$$\hat{t} - \bar{\theta}[(1-\rho)\hat{Q}]^2 - \hat{s} \geq \underline{t} - \bar{\theta}\left(\frac{\underline{Q}}{2}\right)^2. \qquad (\mathrm{IC}^{s}_{\bar{\theta}\underline{\theta}})$$

Note that also $\mathrm{IC}_{\underline{\theta}\underline{\theta}}$ and $\mathrm{IC}_{\bar{\theta}\bar{\theta}}$ may change:

- Whenever $\hat{t} - \underline{\theta}(\rho\hat{Q})^2 + \hat{s} \geq \bar{t} - \underline{\theta}\left(\frac{\bar{Q}}{2}\right)^2$, $\mathrm{IC}_{\underline{\theta}\underline{\theta}}$ rewrites into

$$\underline{t} - \underline{\theta}\left(\frac{\underline{Q}}{2}\right)^2 \geq \hat{t} - \underline{\theta}[(1-\rho)\hat{Q}]^2.$$

- Similarly, $\mathrm{IC}_{\bar{\theta}\bar{\theta}}$ changes into

$$\bar{t} - \bar{\theta}\left(\frac{\bar{Q}}{2}\right)^2 \geq \hat{t} - \bar{\theta}(\rho\hat{Q})^2$$

  if $\hat{t} - \bar{\theta}[(1-\rho)\hat{Q}]^2 - \hat{s} \geq \underline{t} - \bar{\theta}\left(\frac{\underline{Q}}{2}\right)^2$. Clearly, whenever $\hat{s}$=0, the situation remains identical with the one characterized in the base model.

There are situations where both agents have an incentive to misrepresent their types. It is obvious that the IC constraints will not any longer induce truth telling. To induce truth-telling, the following six coalition incentive constraints must be satisfied:

$$2\underline{t} - 2\underline{\theta}\left(\frac{\underline{Q}}{2}\right)^2 \geq 2\hat{t} - \underline{\theta}\left(\rho\hat{Q}\right)^2 - \underline{\theta}[(1-\rho)\hat{Q}]^2 \qquad (\mathrm{CIC}_{\underline{\theta}\underline{\theta},\underline{\theta}\bar{\theta}})$$

$$2\underline{t} - 2\underline{\theta}\left(\frac{\underline{Q}}{2}\right)^2 \geq 2\bar{t} - 2\underline{\theta}\left(\frac{\bar{Q}}{2}\right)^2 \qquad (\mathrm{CIC}_{\underline{\theta}\underline{\theta},\bar{\theta}\bar{\theta}})$$

$$2\hat{t} - \underline{\theta}\left(\rho\hat{Q}\right)^2 - \bar{\theta}[(1-\rho)\hat{Q}]^2 \geq 2\bar{t} - \underline{\theta}\left(\frac{\bar{Q}}{2}\right)^2 - \bar{\theta}\left(\frac{\bar{Q}}{2}\right)^2 \qquad (\mathrm{CIC}_{\underline{\theta}\bar{\theta},\bar{\theta}\bar{\theta}})$$

$$2\hat{t} - \underline{\theta}\left(\rho\hat{Q}\right)^2 - \bar{\theta}[(1-\rho)\hat{Q}]^2 \geq 2\underline{t} - \underline{\theta}\left(\frac{\underline{Q}}{2}\right)^2 - \bar{\theta}\left(\frac{\underline{Q}}{2}\right)^2 \qquad (\mathrm{CIC}_{\underline{\theta}\bar{\theta},\underline{\theta}\underline{\theta}})$$

$$2\bar{t} - 2\bar{\theta}\left(\frac{\bar{Q}}{2}\right)^2 \geq 2\hat{t} - \bar{\theta}(\rho\hat{Q})^2 - \bar{\theta}[(1-\rho)\hat{Q}]^2 \qquad (\mathrm{CIC}_{\bar{\theta}\bar{\theta},\underline{\theta}\bar{\theta}})$$

$$2\bar{t} - 2\bar{\theta}\left(\frac{\bar{Q}}{2}\right)^2 \geq 2\underline{t} - 2\bar{\theta}\left(\frac{\underline{Q}}{2}\right)^2 \qquad (\mathrm{CIC}_{\bar{\theta}\bar{\theta},\underline{\theta}\underline{\theta}})$$

Maximizing again expected payoff of

$$\nu^2[S(\underline{Q}) - 2\underline{t}] + 2\nu(1-\nu)[S(\hat{Q}) - 2\hat{t}] + (1-\nu)^2[S(\bar{Q}) - 2\bar{t}] \qquad (8)$$

s.t. all IR, IC and CIC constraints permits to summarize our findings under the following proposition.

Proposition 7 *$IR_{\bar{\theta}\bar{\theta}}$, $CIC_{\underline{\theta}\bar{\theta},\bar{\theta}\bar{\theta}}$ and $CIC_{\underline{\theta}\underline{\theta},\bar{\theta}\bar{\theta}}$ are binding. The solution entails: $\underline{Q}^C = \underline{Q}^*, \hat{Q}^C = \hat{Q}^*$, and $\bar{Q}^C < \bar{Q}^*$. Furthermore, the optimal contract has optimal targets with $\rho^C = \rho^* = \frac{\bar{\theta}}{\underline{\theta}+\bar{\theta}}$, if an efficient agent is matched with an efficient one.*

Proof. See the Appendix B.2.1. ■

Consider in particular the result for the intermediate quality case $\hat{Q}$, when an efficient agent is matched with an inefficient one. Under this form of contracting, the first-best overall quality is reached in this case, and each agent contributes in the first-best proportion to this quality.

5.2 Development stage and ex-ante contracting

We now consider again ex-ante contracting. With the two constraints

$$\nu(\hat{t}-\underline{\theta}(\rho\hat{Q})^2)+(1-\nu)[\bar{t}-\bar{\theta}\Big(\frac{\bar{Q}}{2}\Big)^2]-\psi \geq max\{\hat{t}-E(\theta)(\rho\hat{Q})^2, \bar{t}-E(\theta)\Big(\frac{\bar{Q}}{2}\Big)^2\} \qquad (\mathrm{EAPC}_{\hat{Q}\bar{Q}})$$

and

$$\nu[\underline{t}-\underline{\theta}\Big(\frac{\underline{Q}}{2}\Big)^2]+(1-\nu)[\hat{t}-\bar{\theta}[(1-\rho)\hat{Q}]^2-\psi \geq max\{\underline{t}-E(\theta)\Big(\frac{\underline{Q}}{2}\Big)^2, \hat{t}-E(\theta)[(1-\rho)\hat{Q}]^2\}, \qquad (\mathrm{EAPC}_{\underline{Q}\hat{Q}})$$

For low values of ψ, our findings in Proposition 4 apply. In addition, and of particular interest, consider again the crossover case under mixed qualities. This downward distortion in $\hat{Q}$ expressed in D is

$$\underline{t}-\underline{\theta}\Big(\frac{\underline{Q}}{2}\Big)^2 = \Delta\theta[(1-\rho)\hat{Q}]^2, \qquad (\mathrm{D})$$

Proposition 8 *Implementability is easier to reach in the second mode of contracting.*

Proof. Consider again equation D. Because of its first-best values for ρ in the mixed quality case, the new contracting mode reveals that an increase in ρ will effectively *decrease* the rent of a $\underline{\theta}$ agent necessary to satisfy the constraint. As a result, this mode is more likely to induce information gathering. ■

6 Comparison of Bayesian Implementation vs. Pareto-dominant Strategy Implementation in the Market Model

Our results in proposition 7 and 3 seem to suggest that the platform owner should prefer a situation where side payments between the two developers are possible. With Pareto-dominant strategy implementation the quality level of the platform good is only inefficient if both developers have high costs. Otherwise, meaning with two efficient developers and with a mixed match, it is always efficient. With Bayesian implementation efficiency is only preserved with asymmetric information in case of two efficient developers. The problem is, however, that we do not know anything about the payments made to the developers in both cases and how the case with two inefficient developers compares with both types of implementation. To shed some light on this, we now compare our model for the specific assumptions about the market demand and compare the two types of implementation from the point of view of the platform owner, the consumers on the market and finally the two developers.

To be continued

7 Concluding Remarks

Our paper has set out to determine what drives the choice of a platform's organizational mode in the presence of two developers that are contracted simultaneously and deliver an input that jointly determines the quality running on the platform. We have reached some results that help answering the question when and why platforms would prefer to recur to tighter organizational forms with close boundaries and full control, and when they would rather prefer a more flexible organizational mode that gives developers more freedom to decide on quality. Our paper has extended, in some way, Riordan's (1990) direction of research in favor of complete contracting.

With this in mind, we have analyzed two feasible contracting modes under adverse selection: Bayesian and Pareto-dominant strategy implementation. Our general findings are that Pareto-dominant strategy implementation can be superior and can deliver a higher surplus to the platform. The aggregate consumer surplus in our model is linear in the quality provided by the platform, with

$$\int_{\tilde{s}}^{\bar{s}} U(Q, p^m) ds = \frac{Q\bar{s}^2}{4}.$$

Thus, despite higher monopoly prices, consumers always benefit from higher expected qualities and therefore would prefer Pareto dominant strategy implementation.

With our paper, we have addressed a specific question relevant for understanding platform organization in the presence of multiple developers who jointly determine the quality, running on the platform. We focussed on the specific aspect of platform governance as characterized in Boudreau (2017). Like other organizations, platforms change with their chosen organizational mode. Our results suggest that the platform might be faced with a trade-off between retaining control at the expense of distortions with regard to the quality of the platform product, vis-a-vis relinquishing control and utilizing coordination between the two developers, this way allowing potentially for higher rents on the side of the agents. Depending on the parameters of the model with regard to the possible difference in efficiency and the probability of a more efficient developer the platform's trade-off can favour one organizational mode over the other.

Appendix

A Full Information

A.1 Full Information with Perfect Transfer Discrimination

The platform's maximization problem can be solved via maximizing the Lagrangian

$$\begin{aligned}\mathcal{L} &= \nu^2[S(\underline{Q}) - 2\underline{t}] + 2\nu(1-\nu)[S(\hat{Q}) - \hat{\underline{t}} - \hat{\bar{t}}] + (1-\nu)^2[S(\bar{Q}) - 2\bar{t}] \\ &\quad + \lambda_1(\underline{t} - \underline{\theta}\left(\frac{\underline{Q}}{2}\right)^2) + \lambda_2(\bar{t} - \bar{\theta}\left(\frac{\bar{Q}}{2}\right)^2) \\ &\quad + \lambda_3(\hat{\underline{t}} - \underline{\theta}(\rho\hat{Q})^2) + \lambda_4(\hat{\bar{t}} - \bar{\theta}\left((1-\rho)\hat{Q}\right)^2)\end{aligned} \tag{9}$$

The first order conditions result in

$$\frac{\partial \mathcal{L}}{\partial \underline{t}} = -2\nu^2 + \lambda_1 = 0 \Leftrightarrow \lambda_1 = 2\nu^2$$

$$\frac{\partial \mathcal{L}}{\partial \bar{t}} = -2(1-\nu)^2 + \lambda_2 = 0 \Leftrightarrow \lambda_2 = 2(1-\nu)^2$$

$$\frac{\partial \mathcal{L}}{\partial \hat{\underline{t}}} = -2\nu(1-\nu) + \lambda_3 = 0 \Leftrightarrow \lambda_3 = 2\nu(1-\nu)$$

$$\frac{\partial \mathcal{L}}{\partial \hat{\bar{t}}} = -2\nu(1-\nu) + \lambda_4 = 0 \Leftrightarrow \lambda_4 = 2\nu(1-\nu)$$

$$\frac{\partial \mathcal{L}}{\partial \underline{Q}} = \nu^2 S'(\underline{Q}) - \frac{\lambda_1 \underline{\theta}\underline{Q}}{2} = 0 \Leftrightarrow S'(\underline{Q}) = \underline{\theta}\underline{Q}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{Q}} = (1-\nu)^2 S'(\bar{Q}) - \frac{\lambda_2 \bar{\theta}\bar{Q}}{2} = 0 \Leftrightarrow S'(\bar{Q}) = \bar{\theta}\bar{Q}$$

$$\frac{\partial \mathcal{L}}{\partial \hat{Q}} = 2\nu(1-\nu)S'(\hat{Q}) - \lambda_3 2\underline{\theta}\rho^2\hat{Q} - \lambda_4 2\bar{\theta}(1-\rho)^2\hat{Q} = 0$$

Taking into account the results for $\lambda_3 = \lambda_4 = 2\nu(1-\nu)$ yields

$$S'(\hat{Q}) = 2[\underline{\theta}\rho^2 + \bar{\theta}(1-\rho)^2]\hat{Q}.$$

$$\frac{\partial \mathcal{L}}{\partial \rho} = -\lambda_3 2\underline{\theta}\rho\hat{Q}^2 + \lambda_4 2\bar{\theta}(1-\rho)\hat{Q}^2 = 0 \Leftrightarrow \rho = \frac{\bar{\theta}}{\bar{\theta} + \underline{\theta}}$$

Substituting ρ into the equation which determines $\hat{Q}$ yields

$$S'(\hat{Q}) = \frac{2\underline{\theta}\bar{\theta}(\bar{\theta} + \underline{\theta})}{(\underline{\theta} + \bar{\theta})^2}\hat{Q} = \frac{2\underline{\theta}\bar{\theta}}{\underline{\theta} + \bar{\theta}}\hat{Q} = 2\underline{\theta}\rho\hat{Q}$$

A.2 Full Information with Imperfect Transfer Discrimination

The platform's maximization problem can be solved via maximizing the Lagrangian

$$\begin{aligned}\mathcal{L} &= \nu^2[S(\underline{Q}) - 2\underline{t}] + 2\nu(1-\nu)[S(\hat{Q}) - 2\hat{t}] + (1-\nu)^2[S(\bar{Q}) - 2\bar{t}] \\ &\quad +\lambda_1(\underline{t} - \underline{\theta}\left(\frac{\underline{Q}}{2}\right)^2) + \lambda_2(\bar{t} - \bar{\theta}\left(\frac{\bar{Q}}{2}\right)^2) \\ &\quad +\lambda_3(\hat{t} - \underline{\theta}(\rho\hat{Q})^2) + \lambda_4(\hat{t} - \bar{\theta}\left((1-\rho)\hat{Q}\right)^2)\end{aligned} \tag{10}$$

The first order conditions result in

$$\frac{\partial \mathcal{L}}{\partial \underline{t}} = -2\nu^2 + \lambda_1 = 0 \Leftrightarrow \lambda_1 = 2\nu^2$$

$$\frac{\partial \mathcal{L}}{\partial \bar{t}} = -2(1-\nu)^2 + \lambda_2 = 0 \Leftrightarrow \lambda_2 = 2(1-\nu)^2$$

$$\frac{\partial \mathcal{L}}{\partial \hat{t}} = -4\nu(1-\nu) + \lambda_3 + \lambda_4 = 0 \Leftrightarrow \lambda_3 = 4\nu(1-\nu) - \lambda_4 \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial \underline{Q}} = \nu^2 S'(\underline{Q}) - \frac{\lambda_1\underline{\theta}\underline{Q}}{2} = 0 \Leftrightarrow S'(\underline{Q}) = \underline{\theta}\underline{Q}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{Q}} = (1-\nu)^2 S'(\bar{Q}) - \frac{\lambda_2\bar{\theta}\bar{Q}}{2} = 0 \Leftrightarrow S'(\bar{Q}) = \bar{\theta}\bar{Q}$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \hat{Q}} &= 2\nu(1-\nu)S'(\hat{Q}) - \lambda_3 2\underline{\theta}\rho^2\hat{Q} - \lambda_4 2\bar{\theta}(1-\rho)^2\hat{Q} = 0 \\ &\Leftrightarrow \nu(1-\nu)S'(\hat{Q}) = \hat{Q}[\lambda_3\underline{\theta}\rho^2 + \lambda_4\bar{\theta}(1-\rho)^2],\end{aligned} \tag{12}$$

$$\frac{\partial \mathcal{L}}{\partial \rho} = -\lambda_3 2\underline{\theta}\rho\hat{Q}^2 + \lambda_4 2\bar{\theta}(1-\rho)\hat{Q}^2 = 0 \Leftrightarrow \lambda_3 = \frac{\lambda_4\bar{\theta}(1-\rho)}{\underline{\theta}\rho} \tag{13}$$

Solving the equations (11) and (13) simultaneously for λ_3 and λ_4 yields

$$\lambda_3 = \frac{4\nu(1-\nu)\bar{\theta}(1-\rho)}{\bar{\theta}(1-\rho) + \underline{\theta}\rho} \text{ and } \lambda_4 = \frac{4\nu(1-\nu)\underline{\theta}\rho}{\bar{\theta}(1-\rho) + \underline{\theta}\rho} \tag{14}$$

Note that $\lambda_3 > 0$ and $\lambda_4 > 0$ hold as long as $0 < \rho < 1$ holds, meaning that in this case both agent's participation constraints need to bind. The latter implies

$$\hat{t} - \underline{\theta}(\rho\hat{Q})^2 = \hat{t} - \bar{\theta}\left((1-\rho)\hat{Q}\right)^2 = 0,$$

$$\Leftrightarrow \underline{\theta}\rho^2 = \bar{\theta}(1-\rho)^2 \Leftrightarrow \rho = \frac{\sqrt{\bar{\theta}}}{\sqrt{\bar{\theta}} + \sqrt{\underline{\theta}}} = \rho^{**}$$

with $0 < \rho^{**} < 1$, as requested. Substituting now λ_3 and λ_4 into equation (12) yields

$$S'(\hat{Q}) = \frac{4\underline{\theta}\bar{\theta}}{(\sqrt{\underline{\theta}} + \sqrt{\bar{\theta}})^2}\hat{Q} = 4\sqrt{\underline{\theta}}\sqrt{\bar{\theta}}\rho^{**}(1-\rho^{**})\hat{Q}$$

B Asymmetric Information

B.1 Bayesian Implementation

B.1.1 Proof of Proposition 3 (based on Shin, 2007).

It is easy to show that if both $IC_{\underline{\theta}\theta}$ and $IR_{\bar{\theta}\underline{\theta}}$ hold, then $IR_{\underline{\theta}\theta}$ must be satisfied and non-binding. The same is true for $IR_{\underline{\theta}\bar{\theta}}$, if both $IC_{\underline{\theta}\bar{\theta}}$ and $IR_{\bar{\theta}\bar{\theta}}$ hold. Therefore these two IR constraints do not need to be considered in the maximization problem.

Assuming now that $IR_{\bar{\theta}\bar{\theta}}$ and $IR_{\bar{\theta}\underline{\theta}}$ are binding, yields $\bar{t} = \bar{\theta}(\bar{Q}/2)^2$ and $\hat{t} = \bar{\theta}[(1-\rho)\hat{Q}]^2$. Substituting both transfers into the IC constraints shows that $IC_{\bar{\theta}\bar{\theta}}$ is not binding if $\rho > 1/2$ holds for the optimal contract. Similarly one can show that if $\underline{t} < \bar{\theta}(\frac{\bar{Q}}{2})^2$ holds in the optimal contract, then $IC_{\bar{\theta}\underline{\theta}}$ is also not binding. Therefore for now we assume that this is the case in the optimal contract and check after solving for the optimal contract, whether these two conditions are satisfied.

Given that only $IC_{\underline{\theta}\underline{\theta}}$, $IC_{\underline{\theta}\bar{\theta}}$, $IR_{\bar{\theta}\bar{\theta}}$ and $IR_{\bar{\theta}\underline{\theta}}$ are binding constraints the optimal contract can be found by maximizing the following Lagrange function with regard to $\underline{Q}$, $\hat{Q}$, $\bar{Q}$,$\underline{t}$, $\hat{t}$, $\bar{t}$ and ρ.

$$\begin{aligned}
\mathcal{L} \;=\;& \nu^2[S(\underline{Q}) - 2\underline{t}] + 2\nu(1-\nu)[S(\hat{Q}) - 2\hat{t}] + (1-\nu)^2[S(\bar{Q}) - 2\bar{t}] \\
&+\lambda_1\left[\bar{t} - \bar{\theta}\left(\frac{\bar{Q}}{2}\right)^2\right] + \lambda_2\left\{\hat{t} - \bar{\theta}\left[(1-\rho)\hat{Q}\right]^2\right\} + \lambda_3\left\{\left[\hat{t} - \underline{\theta}(\rho\hat{Q})^2\right] - \left[\bar{t} - \underline{\theta}\left(\frac{\bar{Q}}{2}\right)^2\right]\right\} \\
&+\lambda_4\left\{\underline{t} - \underline{\theta}\left(\frac{\underline{Q}}{2}\right)^2 - \left\{\hat{t} - \underline{\theta}\left[(1-\rho)\hat{Q}\right]^2\right\}\right\}
\end{aligned}$$

We reach the following first-order conditions:

$$\frac{\partial\mathcal{L}}{\partial\underline{Q}} = \nu^2 S'(\underline{Q}) - \lambda_4\underline{\theta}\frac{\underline{Q}}{2} = 0 \Leftrightarrow \lambda_4 = \frac{2\nu^2 S'(\underline{Q})}{\underline{\theta}\underline{Q}} > 0 \tag{15}$$

$$\frac{\partial\mathcal{L}}{\partial\hat{Q}} = 2\nu(1-\nu)S'(\hat{Q}) - \left[2\lambda_2(1-\rho)^2\bar{\theta} + 2\lambda_3\rho^2\underline{\theta} - 2\lambda_4(1-\rho)^2\underline{\theta}\right]\hat{Q} = 0 \tag{16}$$

$$\frac{\partial\mathcal{L}}{\partial\bar{Q}} = (1-\nu)^2 S'(\bar{Q}) - \left[\lambda_1\bar{\theta} - \lambda_3\underline{\theta}\right]\frac{\bar{Q}}{2} = 0 \tag{17}$$

$$\frac{\partial\mathcal{L}}{\partial\underline{t}} = -2\nu^2 + \lambda_4 = 0 \Leftrightarrow \lambda_4 = 2\nu^2 > 0 \tag{18}$$

$$\frac{\partial\mathcal{L}}{\partial\hat{t}} = -4\nu(1-\nu) + \lambda_2 + \lambda_3 - \lambda_4 = 0 \Leftrightarrow \lambda_2 = \lambda_4 + 4\nu(1-\nu) - \lambda_3 \tag{19}$$

$$\frac{\partial\mathcal{L}}{\partial\bar{t}} = -2(1-\nu)^2 + \lambda_1 - \lambda_3 = 0 \Leftrightarrow \lambda_1 = \lambda_3 + 2(1-\nu)^2 \tag{20}$$

$$\frac{\partial \mathcal{L}}{\partial \rho} = \left[2\lambda_2(1-\rho)\bar{\theta} - 2\lambda_3\rho\underline{\theta} - 2\lambda_4(1-\rho)\underline{\theta}\right]\hat{Q}^2 = 0. \tag{21}$$

Substituting λ_4 from (18), λ_2 from (19) and solving for λ_3 yields

$$\lambda_3 = \frac{2\nu(1-\rho)[2\bar{\theta} - \nu(\bar{\theta}+\underline{\theta})]}{(1-\rho)\bar{\theta}+\rho\underline{\theta}} > 0 \text{ for all } 0 \leq \rho < 1 \tag{22}$$

This means that $\text{IC}_{\underline{\theta}\bar{\theta}}$ is binding for all $0 \leq \rho < 1$. Substituting λ_3 from (22) into (20) yields

$$\lambda_1 = \frac{2[(1-\rho)(\bar{\theta}-\nu^2\underline{\theta}) + (1-\nu)^2\rho\underline{\theta}]}{(1-\rho)\bar{\theta}+\rho\underline{\theta}} > 0 \tag{23}$$

Thus, $\text{IR}_{\bar{\theta}\bar{\theta}}$ is binding as assumeed. By substituting λ_3 from (22) and λ_4 from (18) into (19) we obtain

$$\lambda_2 = \frac{2\nu\underline{\theta}[\nu + 2\rho(1-\nu)]}{(1-\rho)\bar{\theta}+\rho\underline{\theta}} > 0, \tag{24}$$

which means that $\text{IR}_{\bar{\theta}\underline{\theta}}$ is also binding as assumed. From (15) and (18) we know already that $\lambda_4 > 0$ holds which ensures that $\text{IC}_{\underline{\theta}\underline{\theta}}$ is binding. Therefore we can now use the binding constraints to characterize the transfers to the agents.

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = \bar{t} - \bar{\theta}\left(\frac{\bar{Q}}{2}\right)^2 = 0 \Leftrightarrow \bar{t} = \bar{\theta}\left(\frac{\bar{Q}}{2}\right)^2, \tag{25}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = \hat{t} - \bar{\theta}\left[(1-\rho)\hat{Q}\right]^2 = 0 \Leftrightarrow \hat{t} = \bar{\theta}\left[(1-\rho)\hat{Q}\right]^2 \tag{26}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_3} = \hat{t} - \underline{\theta}(\rho\hat{Q})^2 - \left[\bar{t} - \underline{\theta}\left(\frac{\bar{Q}}{2}\right)^2\right] = 0 \Leftrightarrow \hat{t} = \underline{\theta}(\rho\hat{Q})^2 + \left[\bar{t} - \underline{\theta}\left(\frac{\bar{Q}}{2}\right)^2\right]. \tag{27}$$

Substituting $\bar{t}$ from (25) into $\hat{t}$ from (27) yields

$$\hat{t} = \underline{\theta}(\rho\hat{Q})^2 + (\bar{\theta}-\underline{\theta})\left(\frac{\bar{Q}}{2}\right)^2. \tag{28}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_4} = \underline{t} - \underline{\theta}\left(\frac{\text{Q}}{2}\right)^2 - \left\{\hat{t} - \underline{\theta}\left[(1-\rho)\hat{Q}\right]^2\right\} = 0 \Leftrightarrow \underline{t} = \underline{\theta}\left(\frac{\text{Q}}{2}\right)^2 + \left\{\hat{t} - \underline{\theta}\left[(1-\rho)\hat{Q}\right]^2\right\}. \tag{29}$$

Substituting $\hat{t}$ from (26) into $\underline{t}$ from (29) finally yields

$$\underline{t} = \underline{\theta}\left(\frac{\text{Q}}{2}\right)^2 + (\bar{\theta}-\underline{\theta})\left[(1-\rho)\hat{Q}\right]^2. \tag{30}$$

Obviously, if these transfers become part of a second best contract, then (25) and (26) imply that an inefficient agent never realizes a rent, whereas (28) and (30) show that an efficient agent realizes an information rent, no matter whether the efficient agent is matched with another efficient or inefficient agent.

Equalizing $\hat{t}$ from equation (26) and from (28) reveals that

$$\bar{\theta}\left[(1-\rho)\hat{Q}\right]^2 = \underline{\theta}(\rho\hat{Q})^2 + (\bar{\theta}-\underline{\theta})\left(\frac{\bar{Q}}{2}\right)^2,$$

$$\Leftrightarrow [\bar{\theta}(1-\rho)^2 - \underline{\theta}\rho^2]\hat{Q}^2 = (\bar{\theta}-\underline{\theta})\left(\frac{\bar{Q}}{2}\right)^2, \tag{31}$$

must hold. If $\hat{Q} > 0$, $\bar{Q} > 0$ and $\hat{Q} > \bar{Q}$ hold in the second-best optimal contract, this means that

$$\bar{\theta}(1-\rho)^2 - \underline{\theta}\rho^2 > 0$$

and, because the term on the left decreases in ρ and is equal to $1/4(\bar{\theta}-\underline{\theta})$ for $\rho = 1/2$, that ρ must satisfy

$$\frac{1}{2} < \rho < \frac{\sqrt{\bar{\theta}}}{\sqrt{\bar{\theta}}+\sqrt{\underline{\theta}}}. \tag{32}$$

The latter implies that the targeted share of the efficient firm's quality contribution in a match with an inefficient agent is distorted downward in an optimal second best contract, not only compared to the first best, but also to the full information contract with limited transfer discrimination.

By substituting λ_1, λ_2, λ_3, and λ_4 from (23), (24), (22) and (18) into the first order conditions with respect to $\underline{Q}$, (15), $\hat{Q}$, (16), and $\bar{Q}$, (17) we obtain the implicit functions for the second best optimal quality levels

$$S'(\underline{Q}) = \underline{\theta}\underline{Q}, \tag{33}$$

$$S'(\hat{Q}) = \hat{Q}\left(\frac{2(1-\rho)\rho\underline{\theta}[2\bar{\theta}-\nu(\bar{\theta}+\underline{\theta})]}{(1-\nu)[(1-\rho)\bar{\theta}+\rho\underline{\theta}]}\right). \tag{34}$$

and

$$S'(\bar{Q}) = \bar{Q}\left(\frac{(1-\rho)\bar{\theta}^2 + [\rho(1+\nu^2)-2\nu]\bar{\theta}\underline{\theta} + \nu^2(1-\rho)\underline{\theta}^2}{(1-\nu)^2[(1-\rho)\bar{\theta}+\rho\underline{\theta}]}\right), \tag{35}$$

From equation (33) it is obvious that in case of a match with two efficient agents the quality level is first best efficient, since the condition for the second best optimal contract is the same as the one derived in Appendix A.1 for the first best. Note that $\hat{Q} > 0$ and $\bar{Q} > 0$ must hold due to the assumption $S'(Q) > 0$ and to the fact that the expression in brackets on the right hand side of (34) and (35) is positive for any $\rho \in (0,1)$. In addition $\underline{Q} > \hat{Q} > \bar{Q}$ must also hold because

$$\underline{\theta} < \frac{2(1-\rho)\rho\underline{\theta}[2\bar{\theta}-\nu(\bar{\theta}+\underline{\theta})]}{(1-\nu)[(1-\rho)\bar{\theta}+\rho\underline{\theta}]} < \frac{(1-\rho)\bar{\theta}^2 + [\rho(1+\nu^2)-2\nu]\bar{\theta}\underline{\theta} + \nu^2(1-\rho)\underline{\theta}^2}{(1-\nu)^2[(1-\rho)\bar{\theta}+\rho\underline{\theta}]}$$

is satisfied for $\rho \in (0,1)$. Now, let us check on $\underline{t}$. Substituting (30) into the condition $\underline{t} \leq \bar{\theta}\left(\frac{\underline{Q}}{2}\right)^2$ yields

$$\underline{t} = \underline{\theta}\left(\frac{\underline{Q}}{2}\right)^2 + (\bar{\theta} - \underline{\theta})\left[(1-\rho)\hat{Q}\right]^2 \leq \bar{\theta}\left(\frac{\underline{Q}}{2}\right)^2,$$

$$\Leftrightarrow (1-\rho)\hat{Q} < \frac{\underline{Q}}{2}.$$

This inequality must hold, given $\hat{Q} < \underline{Q}$ and $\rho > 1/2$. Thus, the solution to the equations (33), (34), (35) and (31) characterize the second best optimal contract.

B.1.2 Proof of Proposition 5 and 6

Setting up the Lagrangian with development and Bayesian implementation leads to

$$\begin{aligned}
\mathcal{L} \;=\; & \nu^2[S(\underline{Q}) - 2\underline{t}] + 2\nu(1-\nu)[S(\hat{Q}) - 2\hat{t}] + (1-\nu)^2[S(\bar{Q}) - 2\bar{t}] \\
& +\lambda_1\left[\bar{t} - \bar{\theta}\left(\frac{\bar{Q}}{2}\right)^2\right] + \lambda_2\left\{\hat{t} - \bar{\theta}\left[(1-\rho)\hat{Q}\right]^2\right\} + \lambda_3\left\{\left[\hat{t} - \underline{\theta}(\rho\hat{Q})^2\right] - \left[\bar{t} - \underline{\theta}\left(\frac{\bar{Q}}{2}\right)^2\right]\right\} \\
& +\lambda_4\left\{\underline{t} - \underline{\theta}\left(\frac{\underline{Q}}{2}\right)^2 - \left\{\hat{t} - \underline{\theta}\left[(1-\rho)\hat{Q}\right]^2\right\}\right\} + \lambda_5\left\{\nu(1-\nu)\Delta\theta\left[\left(\frac{\underline{Q}}{2}\right)^2 - \left[(1-\rho)\hat{Q}\right]^2\right] - \psi\right\} \\
& +\lambda_6\left\{\nu(1-\nu)\Delta\theta\left[(\rho\hat{Q})^2 - \left(\frac{\bar{Q}}{2}\right)^2\right] - \psi\right\}
\end{aligned}$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial \underline{Q}} = \nu^2 S'(\underline{Q}) - \lambda_4\underline{\theta}\frac{\underline{Q}}{2} + \lambda_5\nu(1-\nu)\Delta\theta\frac{\underline{Q}}{2} = 0 \tag{36}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \hat{Q}} \;=\; & 2\nu(1-\nu)S'(\hat{Q}) - \left[2\lambda_2(1-\rho)^2\bar{\theta} + 2\lambda_3\rho^2\underline{\theta} - 2\lambda_4(1-\rho)^2\underline{\theta}\right]\hat{Q} \\
& -2\lambda_5\nu(1-\nu)\Delta\theta(1-\rho)^2\hat{Q} + 2\lambda_6\nu(1-\nu)\Delta\theta\rho^2\hat{Q} = 0
\end{aligned} \tag{37}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{Q}} = (1-\nu)^2 S'(\bar{Q}) - \left[\lambda_1\bar{\theta} - \lambda_3\underline{\theta}\right]\frac{\bar{Q}}{2} - \lambda_6\nu(1-\nu)\Delta\theta\frac{\bar{Q}}{2} = 0 \tag{38}$$

$$\frac{\partial \mathcal{L}}{\partial \underline{t}} = -2\nu^2 + \lambda_4 = 0 \Leftrightarrow \lambda_4 = 2\nu^2 > 0 \tag{39}$$

$$\frac{\partial \mathcal{L}}{\partial \hat{t}} = -4\nu(1-\nu) + \lambda_2 + \lambda_3 - \lambda_4 = 0 \Leftrightarrow \lambda_2 = \lambda_4 + 4\nu(1-\nu) - \lambda_3 \tag{40}$$

$$\frac{\partial \mathcal{L}}{\partial \bar{t}} = -2(1-\nu)^2 + \lambda_1 - \lambda_3 = 0 \Leftrightarrow \lambda_1 = \lambda_3 + 2(1-\nu)^2 > 0 \tag{41}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \rho} \; = \; & \left\{2\lambda_2(1-\rho)\bar{\theta} - 2\lambda_3\rho\underline{\theta} - 2\lambda_4(1-\rho)\underline{\theta} \right. \\ & \left. + \left[2\lambda_5\nu(1-\nu)\rho\Delta\theta + 2\lambda_6\nu(1-\nu)(1-\rho)\Delta\theta\right]\right\} \hat{Q}^2 = 0. \end{aligned} \tag{42}$$

Case 1: If ψ is low, and none of the IP constraints are binding, then $\lambda_5 = \lambda_6 = 0$ holds. In that case, the setup collapses to the one in Appendix B.1.1, of course, with the same result.

Case 2: ψ is high enough, such that either or both IP constraints become binding, which can be expressed as

$$\left(\frac{\underline{Q}}{2}\right)^2 - \left(\rho\hat{Q}\right)^2 \geq \frac{\psi}{\nu(1-\nu)\Delta\theta}, \tag{43}$$

and

$$[(1-\rho)\hat{Q}]^2 - \left(\frac{\bar{Q}}{2}\right)^2 \geq \frac{\psi}{\nu(1-\nu)\Delta\theta}. \tag{44}$$

Compared to the setup without development costs and compared to the first best solution there are now different distortions. The "no distortion at the top" property does not necessary hold anymore and the quality with two inefficient agents are more downward distorted, due to the higher ψ. The quality in a mixed match can be more upward or more downward distorted than the second best optimal quality in this case. Note also that the principal will optimally adjust ρ as well. These conclusions can be drawn from analysing the potentially binding or not binding constraints. Substituting λ_2 and λ_4 from equations (40) and (39) into equation (42) and solving for λ_3 yields

$$\lambda_3 = \frac{\nu\left\{\Delta\theta(1-\nu)[(1-\rho)\lambda_5 + \rho\lambda_6]) + 2(1-\rho)[(2-\nu)\bar{\theta} - \nu\underline{\theta}]\right\}}{(1-\rho)\bar{\theta} + \rho\underline{\theta}} > 0, \tag{45}$$

By substituting λ_3 from equation (45) into λ_2 from equation (40) we obtain

$$\lambda_2 = \frac{\nu\left\{2[\nu + 2(1-\nu)\rho]\underline{\theta} - \Delta\theta(1-\nu)[(1-\rho)\lambda_5 + \rho\lambda_6]\right\}}{(1-\rho)\bar{\theta} + \rho\underline{\theta}}. \tag{46}$$

Thus, it depends on the level of λ_5 and λ_6, whether the individual rationality constraint of an inefficient agent in a mixed match with an efficient agent is binding or not. If both are sufficiently large, $\lambda_2 = 0$ holds and the inefficient agent in a mixed match is able to realize a positive rent. Due to $\lambda_1 > 0$, concluded from equation (41), an inefficient agent matched with another inefficient agent cannot achieve a rent. The fact that $\lambda_3 > 0$ and $\lambda_4 > 0$

hold, according to equations (45) and (39), reveals that the two incentive compatibility constraints of an efficient agent are always binding.

Substituting λ_4 from (39) into (36) reveals that with a binding (IP1), meaning $\lambda_5 > 0$ and (43) holding with equality, the quality in a match with two efficient agents can be distorted upward compared to the first best and the *no distortion at the top* second best optimal contract, because

$$S'(\underline{Q}) = \left(\underline{\theta} - \frac{\lambda_5(1-\nu)\Delta\theta}{2\nu}\right)\underline{Q} < \underline{\theta}\underline{Q}$$

must hold. Substituting λ_2 from equation (40), λ_4 from equation (39) and λ_3 from (45) into equation (38) yields

$$S'(\hat{Q}) = \hat{Q}(1-\rho)\rho\frac{(1-\nu)\Delta\theta(\underline{\theta}\lambda_5 - \bar{\theta}\lambda_6) + 2\underline{\theta}[(2-\nu)\bar{\theta} - \nu\underline{\theta}]}{(1-\nu)[(1-\rho)\bar{\theta} + \rho\underline{\theta}]}.$$

Depending on whether $[\underline{\theta}\lambda_5 - \bar{\theta}\lambda_6]$ is positive or negative, $\hat{Q}$ is either distorted more downward or more upward as in the optimal second best contract. Finally substituting λ_1 from equation (41) and λ_3 from (45) into equation (38) yields

$$S'(\bar{Q}) = \bar{Q}\left(\frac{\Delta\theta(1-\nu)\nu[\bar{\theta}\lambda_6 + \Delta\theta\lambda_5] + 2(1-\rho)\bar{\theta}^2 + 2\left[\rho(1+\nu^2) - 2\nu\right]\bar{\theta}\underline{\theta} + 2\nu^2(1-\rho)\underline{\theta}^2}{2(1-\nu)^2\left[(1-\rho)\bar{\theta} + \rho\underline{\theta}\right]}\right).$$

Thus, $\bar{Q}$ is clearly distorted more downward than in the second-best optimal contract.

Case 3: Consider first the principal's surplus that he derives from his objective function when incentivizing the agent under consideration to perform a development stage:

$$\nu^2[S(\underline{Q}) - 2\underline{t}] + 2\nu(1-\nu)[S(\hat{Q}) - 2\hat{t}] + (1-\nu)^2[S(\bar{Q}) - 2\bar{t}] - \psi \geq \max_Q\{S(Q) - E(\theta)Q\},$$

where $E(\theta) = \nu\underline{\theta} + (1-\nu)\bar{\theta}$. If this condition is not fulfilled, the principal would prefer to not incentivize the agent under consideration and to satisfy ex-post participation constraints as mentioned in Proposition 5.

B.2 Dominant Strategy Implementation

B.2.1 Proof of Proposition 7

Setting up the Lagrangian leads to

$$\begin{aligned}\mathcal{L} &= \nu^2[S(\underline{Q}) - 2\underline{t}] + 2\nu(1-\nu)[S(\hat{Q}) - 2\hat{t}] + (1-\nu)^2[S(\bar{Q}) - 2\bar{t}] \\ &+\lambda_1\left[\bar{t} - \bar{\theta}\left(\frac{\overline{Q}}{2}\right)^2\right] + \\ &\lambda_2\left[2\hat{t} - \underline{\theta}\left(\rho\hat{Q}\right)^2 - \bar{\theta}[(1-\rho)\hat{Q}]^2 - 2\left(\bar{t} - \underline{\theta}\left(\frac{\bar{Q}}{2}\right)^2 - \bar{\theta}\left(\frac{\bar{Q}}{2}\right)^2\right)\right] + \\ &\lambda_3\left\{2\underline{t} - 2\underline{\theta}\left(\frac{\underline{Q}}{2}\right)^2 - \left[2\hat{t} - \underline{\theta}\left(\rho\hat{Q}\right)^2 - \underline{\theta}[(1-\rho)\hat{Q}]^2\right]\right\} + \\ &\lambda_4\left\{2\underline{t} - 2\underline{\theta}\left(\frac{\underline{Q}}{2}\right)^2 - \left[2\bar{t} - 2\underline{\theta}\left(\frac{\bar{Q}}{2}\right)^2\right]\right\}.\end{aligned}$$

$IR_{\bar{\theta}\bar{\theta}}$, $CIC_{\underline{\theta}\bar{\theta},\bar{\theta}\bar{\theta}}$ and $CIC_{\underline{\theta}\underline{\theta},\bar{\theta}\bar{\theta}}$ are binding. The solution entails: $\underline{Q}^C = \underline{Q}^*$, $\hat{Q}^C = \hat{Q}^*$, and $\bar{Q}^C < \bar{Q}^*$. Furthermore, the optimal contract has optimal targets, with $\rho^C = \rho^* = \frac{\bar{\theta}}{\underline{\theta}+\bar{\theta}}$.

We now show that in the optimal contract, only $CIC_{\underline{\theta}\bar{\theta},\bar{\theta}\bar{\theta}}$, $CIC_{\underline{\theta}\underline{\theta},\bar{\theta}\bar{\theta}}$ and $IR_{\bar{\theta}\bar{\theta}}$ are binding. The F.O.C. are

$$\frac{\partial\mathcal{L}}{\partial\underline{Q}} = \nu^2 S'(\underline{Q}) - (\lambda_3\underline{\theta} + \lambda_4\underline{\theta})\underline{Q} = 0 \tag{C1}$$

$$\frac{\partial\mathcal{L}}{\partial\hat{Q}} = 2\nu(1-\nu)S'(\hat{Q}) - \left\{2\lambda_2[\rho^2\underline{\theta} + (1-\rho)^2\bar{\theta}] - 2\lambda_3[\rho^2 + (1-\rho)^2]\underline{\theta}\right\}\hat{Q} = 0 \tag{C2}$$

$$\frac{\partial\mathcal{L}}{\partial\bar{Q}} = (1-\nu)^2 S'(\bar{Q}) + \left[\lambda_1\bar{\theta} + 2\lambda_2(\bar{\theta} + \underline{\theta}) + 2\lambda_4\underline{\theta}\right]\frac{\overline{Q}}{2} = 0 \tag{C3}$$

$$\frac{\partial\mathcal{L}}{\partial\underline{t}} = -2\nu^2 + 2\lambda_3 + 2\lambda_4 = 0 \tag{C4}$$

$$\frac{\partial\mathcal{L}}{\partial\hat{t}} = -4\nu(1-\nu) + 2\lambda_2 - 2\lambda_3 = 0 \tag{C5}$$

$$\frac{\partial\mathcal{L}}{\partial\bar{t}} = -2(1-\nu)^2 + \lambda_1 - 2\lambda_2 - 2\lambda_4 = 0 \tag{C6}$$

$$\frac{\partial\mathcal{L}}{\partial\rho} = \left\{2\lambda_2\left[(1-\rho)\bar{\theta} - \rho\underline{\theta}\right] + 2\lambda_3\left[\rho - (1-\rho)\right]\underline{\theta}\right\}\hat{Q}^2 = 0 \tag{C7}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_1} = \left[\bar{t} - \bar{\theta}\left(\frac{\bar{Q}}{2}\right)^2\right] \geq 0; \quad \lambda_1 \geq 0; \quad \lambda_1\left(\frac{\partial \mathcal{L}}{\partial \lambda_1}\right) = 0. \tag{C8}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_2} = 2\hat{t} - \underline{\theta}(\rho\hat{Q})^2 - \bar{\theta}\left[(1-\rho)\hat{Q}2\right]^2 - 2\bar{t} + (\underline{\theta} + \bar{\theta})\left(\frac{\hat{Q}}{2}\right)^2 \geq 0; \quad \lambda_2 \geq 0; \;\; \lambda_2\left(\frac{\partial \mathcal{L}}{\partial \lambda_1}\right) = 0 \tag{C9}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_3} = 2\underline{t} - 2\underline{\theta}\left(\frac{\underline{\underline{Q}}}{2}\right)^2 - 2\hat{t} + \underline{\theta}(\rho\hat{q})^2 + \underline{\theta}\left[(1-\rho)\hat{Q}\right]^2 \geq 0; \quad \lambda_3 \geq 0; \quad \lambda_3\left(\frac{\partial \mathcal{L}}{\partial \lambda_3}\right) = 0 \tag{C10}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_4} = 2\underline{t} - 2\underline{\theta}\left(\frac{\underline{\underline{Q}}}{2}\right)^2 - \left[2\bar{t} - 2\underline{\theta}\left(\frac{\bar{Q}}{2}\right)^2\right] \geq 0; \quad \lambda_4 \geq 0; \quad \lambda_4\left(\frac{\partial \mathcal{L}}{\partial \lambda_4}\right) = 0 \tag{C11}$$

Using $C4$ to $C6$ leads to $\lambda_3 + \lambda_4 = \nu^2$, $\lambda_2 - \lambda_3 = 2\nu(1-\nu)$, and $\lambda_1 = 2$. From $\lambda_2 - \lambda_3 = 2\nu(1-\nu)$ results that $\lambda_2 > 0$, plugged into $C9$ shows that $CIC_{\underline{\theta}\bar{\theta},\bar{\theta}\bar{\theta}}$ is binding. Moreover, $C6$ and $C8$ show that also $IR_{\bar{\theta}\bar{\theta}}$ is binding. This permits to determine ρ:

$$\rho^c = \frac{\lambda_2\bar{\theta} - \lambda_3\underline{\theta}}{\lambda_2\bar{\theta} - \lambda_3\underline{\theta} + 2\nu(1-\nu)\underline{\theta}}. \tag{ρ}$$

It remains to determine the sub-cases for λ_3. The only admissable value is $\lambda_3 = 0$ and thus $\lambda_4 = \nu^2$. Plugged into $C11$ reveals that $CIC_{\underline{\theta}\underline{\theta},\bar{\theta}\bar{\theta}}$ is binding. With $\lambda_3 = 0$ we determine $\lambda_2 = 2\nu(1-\nu)$, which permits to solve equation ρ. We so reach the first-best value of

$$\rho^c = \frac{\bar{\theta}}{\bar{\theta} + \underline{\theta}} = \rho^*.$$

Using the values found for the Lagrange multipliers λ_1 to λ_4 shows the requested result for the three quality values. ■

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