Subsidies and Self-Funding*

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Abstract
This paper studies how the combination of subsidies and (implicit or explicit) commitment to self-funding affects R&D and its efficiency. We develop a simple model where a funding body, which does not know the true cost of a project, can require a credit-constrained recipient to commit to a minimum amount of self-funding for the project. We demonstrate that such a commitment can help the funding body to direct funds to recipients who are genuinely in need of subsidies, and mitigate socially wasteful diversion of funds. Self-funding commitment leads to “crowding-in”, where the expected private R&D expenditure increases with the amount of subsidies, since higher subsidies induce more high-cost projects to be undertaken whilst ensuring at least some self-funding for any implemented project. Meanwhile, requiring too much self-funding discourages socially desirable projects due to the recipient’s own financing cost. We derive the optimal subsidization policy, which depends crucially on the cost of funds for both the funding body and the recipients, as well as the positive externalities of projects and potential social loss from diversion. We find that i) requiring self-funding commitment may be desirable only if the funding body’s financing cost is lower than that of the recipient; and ii) self-funding commitment is efficiency enhancing if the recipient’s cost of funding and social loss from diversion are high, and the positive externalities from the project are not very high.

Keywords: Subsidies; Self-funding; Diversion; Matching funds; R&D

JEL Classification: O38; L15

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1 Introduction

Many subsidization schemes by the government require the recipients to bear a fraction of the total cost of a proposed project. Combining subsidies and self-funding is common and such schemes are sometimes called matching funds, which are used, for example in the US, by the Department of Defense and the Small Business Administration in order to provide funds to universities and private firms for their R&D. Government funding for NGOs operating in development countries also tends to take a similar form, such as the ones by the Department for International Development in Britain, the Canadian International Development Agency, and the Swedish International Development Cooperation, to name a few. In the US, the National Endowments for the Arts uses the combination of subsidies and commitment to self-financing to provide grants for arts organizations and projects. In Japan, a major university subsidization program to promote “Global Centers of Excellence” in teaching and research involved the maximum amount of fund (500 million yen $\approx$ $4.4$ million) per university beyond which the universities had to promise to bear the rest of the costs as specified in the grant applications.

When a government agency provides funding to universities, charities, or private firms, there are at least two important economic questions, namely i) whether the proposed project generates enough value (positive externalities) for society; and ii) whether the project genuinely requires public funds to be materialized especially when the applicant has some self-funding capability. However, the presence of asymmetric information makes it difficult for funding bodies to gauge them for certain types of projects, and as a result, applicants may have incentives to request funds for projects even when their externalities are low, or when the projects can actually be materialized without subsidies in order to save directly incurred costs. The latter concern has probably become more important in recent years particularly for R&D investments, as many governments in advanced economies are under pressure to reduce their debt especially after the Great Recession and the science and technology budgets have not increased much. One natural solution for selecting desirable projects is to conduct intense assessment and monitoring, but recruiting competent and unbiased reviewers may prove difficult especially for projects that involve profit-making opportunities as well as positive externalities. The efficacy of selection may be greatly enhanced if a simple self-selection mechanism is entailed in the application process.

This paper studies how subsidies and commitment to self-financing can be used together to select projects that are socially desirable and cannot be realized without the
subsidies. We are particularly concerned with subsidies to organizations that may have some self-funding capacity, so that a proposed project might not actually need to be subsidized by the government. Our approach is distinct from, but complementary to the existing literature on R&D subsidies that has primarily focused on the selection and realization of projects with sufficiently large positive externalities. How should commitment to self-financing be used to induce self-selection of projects that are not materialized without subsidies? How does the fraction of the cost to be borne by the recipient organization change according to its budget constraints and the nature of the proposed project? Many existing matching funds have the matching ratio of 1:1, where a funding body exactly matches the amount of money financed by the recipient. When and how is this ratio socially desirable?

We address these questions by developing a simple screening model that features both the amount of subsidies and the money committed and self-funded by the recipient.

There is a vast literature supporting the rationale of the governmental support on R&D. As Bernstein and Nadiri (1988) shows, the social rate of return of R&D is larger than the private rate. Katz et al. (1990) discusses the difference between social and private incentives on R&D occurs because the profit maximizing firm does not take into account the externality of the innovation. In order to fill the gap, the government gives enough private incentive to develop the new technology by several R&D policies, for example, the intellectual property right (for example, Gallini, 1992; Gilbert and Shapiro, 1990; O’Donoghue et al., 1998; Shapiro, 1985), the prize (for example, Che and Gale, 2003; Fu, Lu, and Lu, 2012; Kremer, 1998), and the subsidization. The theoretical literature on the impact of R&D subsidies is scarce in spite of its frequent use as a policy tool. Romano (1989, 1991) consider the R&D policy that is the combination of the patent protection and the subsidies. Romano (1989, 1991) focus on the excess social costs of public funding that is because of the welfare loss from taxation to finance the subsidies and the researcher’s moral hazard problem. Romano (1989, 1991) assume that each subsidy has a social cost per dollar. His setting is attractive because of simplicity; however, it is too simple to consider the researcher’s incentive to apply the subsidies when his project does not need the governmental support. Kleer (2010) shows the R&D subsidies may be a signal for good project to private investors. In his analysis, the government can screen the projects by setting the quality requirements. In his setting, the researchers do not have enough funds to implement their research. The empirical literature on the impact of R&D mainly focuses on the governmental support causes either crowding out of private
Theoretically, the subsidies make the unprofitable project to a profitable one by reducing the private cost. Then, the subsidy may stimulate the private spending on R&D. On the other hand, there exists the possibility that the public support crowds out the private expenditure on R&D. The firm has an incentive to use the governmental grants to implement the projects that have been undertaken without the subsidy. The net effect of public support to the private R&D level is not clear from a theoretical point of view.

The rationale of the matching funds is shown by Scotchmer (2004, 2013). She shows that the government can screen out the low value innovation by requiring the researchers to make a commitment of funds to obtain the subsidies. Scotchmer (2013) discusses the subsidies to the university either crowding out the university’s R&D expenditure or crowd it in. Scotchmer (2013) shows that the matching funds may mitigate the crowding out problem since the university has to commit to the payment to obtain the governmental support. However, she does not consider the social cost of the matching funds. In this paper, we take into account the social cost of the matching funds and consider when the government should apply the matching funds system.

2 Model

Our model features a funding body and a recipient/researcher. The researcher is endowed with a project \((v, a, c)\), where \(v\) is the private (commercial) revenue of the project to the recipient, \(a \geq 0\) is the size of the positive externalities, and \(c\) denotes the cost of completing the project, which is assumed to be private information to the researcher. We assume \(v = 1/2\) for normalization and \(c\) is distributed on \([0, 1]\).

We study a linear funding scheme \((R, \beta)\) that involves the recipient’s total commitment to self-funding \(R + \beta G\) and the subsidy \((1 - \beta)G\), where \(\beta \in [0, 1)\) represents the self-funding commitment proportional to the amount of subsidy. In accordance with \(c \in [0, 1]\), we assume \(G \in [0, 1]\) since the project costs at most 1 by assumption. The recipient may also be required to contribute at least \(R \geq 0\) regardless of the size of the subsidy. Therefore, a pure subsidization scheme with no self-funding commitment is characterized by \(R = 0\) and \(\beta = 0\). A “matching fund” where the funding body exactly matches the contribution by the researcher features \(R = 0\) and \(\beta = 1/2\).

\[\text{Zúñiga-Vicente et al. (2014) offer an excellent survey about this topic.}\]
\[\text{The notation is for expositional convenience. Alternatively, we can define } G \text{ as the size of subsidy and } \xi G \text{ as the corresponding proportional self-funding commitment.}\]
We can interpret positive $R$ not only as the fixed minimum contribution from the recipient but more broadly as any self-funded expenditure on a project prior to the application to the scheme that cannot be reimbursed. For example, $R$ would be high if a subsidy requires a near completed prototype for further funding.

The researcher faces a multiplicative cost of funds $\gamma \geq 0$, which captures the interest rate and any other additional costs, so that the private cost of completing the project is $(1 + \gamma)c$. Likewise, the cost of funds for the funding body is denoted by $\gamma$, which incorporates the interest rate, opportunity costs and also represents the degree of budgetary constraint the funding body faces. Thus, giving a subsidy $(1 - \beta)G$ to the researcher costs the funding body $(1 + \gamma)(1 - \beta)G$.

Given a funding scheme $(R, \beta)$, the researcher who has learnt $c$ faces three choices, namely i) to implement the project entirely by self-funding; ii) to apply for a fund by requesting $(1 - \beta)G$ and committing to $R + \beta G$; and iii) not to implement the project. If the researcher self-funds the project, then his payoff is $1/2 - (1 + \gamma)c$, since the expected revenue is $v = 1/2$. The researcher’s payoff function when he uses the subsidy depends on the size of $c$. If the cost is larger than the amount committed by both parties $R + G$, then he needs to pay $c - (1 - \beta)G$ in total by himself, and thus the researcher’s payoff $\pi$ is given by

$$\pi = 1/2 - (1 + \rho)(c - (1 - \beta)G).$$

Meanwhile, if the cost is lower than $R + G$, he has to spend more than necessary and the payoff is given by

$$\pi = 1/2 - (1 + \rho)(\beta G + R) + (1 - \mu)(R + G - c),$$

where $\mu$ represents the researcher’s loss from diversion. If the researcher can use the residual completely freely then $\mu = 0$; but if he has to return it or spend it on the project (for no use), then $\mu = 1$. We assume $\mu \in [0, 1)$ so that the researcher spends the residual for some private benefit, not least because the funding body does not observe true $c$ nor would the researcher have any incentive to reveal it truthfully.

The funding body is concerned with both the researcher’s private payoff and the externalities denoted by $av = a/2$, where $a$ parametrizes the size of the externalities of the project with respect to the private value. The funding body also takes into account its own financing cost. Thus the expected payoff of the funding body when the project is
implemented with the subsidy is

\[ U = E[\pi] + av - (1 + \gamma)(1 - \beta)G. \]

We have \( U = E[\pi] \) if the researcher does not take up the subsidy.

The timing of the game is summarized as follows:

1. The funding body announces \((\beta, R)\).

2. The researcher learns \( c \) and chooses to i) entirely self-fund the project; ii) request \((1 - \beta)G\) by committing to \( R + \beta G \); or iii) discard the project.

3. If the researcher applies for the subsidy, the funding body supplies \((1 - \beta)G\).

4. The payoffs are realized.

We assume away any assessment (e.g. by a panel of experts) of the project and once the researcher applies, the funding body supplies \((1 - \beta)G\) as requested, conditional on the the searcher’s commitment to the pre-specified self-funding. This is to focus on the effect of a funding scheme \((\beta, R)\) on the decision to apply for the fund and the quality of the project to be funded. If we incorporated a possibly noisy assessment procedure into the model, then the model would feature additional uncertainty for the funding body and the researcher, but it would only affect their behaviour through a lower expected revenue \( v \). This would not change our results qualitatively. Also, \( R \) can be thought of as how demanding the assessment is: when \( R \) is higher the funding body requires a more developed idea or prototype, which involves a higher non-refundable investment before the subsidy is provided.

3 Researcher’s Investment and Funding Request

We start our analysis with the researcher’s behaviour given a funding scheme \((\beta, R)\). The following Lemma indicates that the amount the researcher requests can be reduced to three choices, namely i) to request the maximum subsidy available; ii) to request a subsidy so that the cost of implementing the project is exactly covered; or iii) to request no subsidy.

**Lemma 1.** If \( \beta < \frac{1-\mu}{1+\rho} \), the researcher requests the maximum subsidy available \((G = 1)\). Otherwise, the researcher requests \( G = \min\{c - R, 0\} \).
When the coefficient of self-funding $\beta$ is small with respect to his funding cost $\rho$ and he can use the residual for his private benefit (low $\mu$), the researcher requests the maximum subsidy available. Both $\beta$ and $\mu$ matter because the request for funding is motivated not only by the implementation of the project but also by the private use of the residual. Naturally, for example, when $\mu = 1$, the residual gives him no benefit, and therefore he either requests the minimum amount required to implement the project ($c - R$) or chooses not to apply for the subsidy.

Note that the researcher’s choice is two-fold. He decides whether to implement his project, and whether to use the subsidization scheme $(\beta, R)$ for implementation. That is, asking no request for a subsidy means either the project is completely self-funded, or the project is not implemented. In what follows we present the details on the researcher’s decision whether to implement the project.

**Proposition 1.** For any $\beta > 0$ and $R > 0$, the researcher’s choice for a given $c$ is characterized by at most two cutoff points $c' < c''$ such that,

- if $c \in [0, c']$ he implements the project entirely by self-funding;
- if $c \in (c', c'')$ he implements the project by the subsidy and self-funding;
- if $c \in (c'', 1]$ he does not implement the project.

Specifically, we have

\[
\begin{align*}
    c' &= R \\
    c'' &= \frac{1}{2\beta(1 + \rho)} - \frac{(1 - \beta)R}{\beta}.
\end{align*}
\]

**Proof.** See the Appendix.

The researcher’s choice as described in Proposition 1 for $0 < c' < c'' < 1$ is summarized in Figure 1. If the cost $c$ is sufficiently low, the commitment to self-funding exceeds $c$, and thus the overall cost of self-funding outweighs the benefit of receiving any subsidy. As a result, the researcher requests no subsidy to implement the project. If the cost is very
high, the project is not profitable even with the subsidy under the scheme. Therefore, the researcher resorts to the funding scheme when $c$ is in an intermediate range.

Note that $c''$ may not be in the interval $(0, 1)$, in which case the researcher’s choice is characterized only one cut-off $c'$. For example, if $R$ and $\beta$ are very small, the self-funding commitment is small, so that we have $c'' \geq 1$, since implementing the project becomes very cheap. This implies that the researcher implements the project for any $c \in [0, 1]$, either entirely by his own fund, or by requesting the subsidy if doing so is more profitable.

Next, consider as a benchmark the researcher’s behaviour when the project has to be self-funded. In this case, the researcher either self-funds the project or chooses not to implement it depending on the cost of the project $c$.

The researcher’s choice in this case is represented in Figure 2. The cutoff for $c$ decreases as the researcher’s cost of funds $\rho$ increases.

Our analysis of the researcher’s choice illustrates the simple cutoff rule with respect to the realized cost $c$, given the parameter values and the funding scheme $(\beta, R)$. The comparison between Figures 2 and 1 readily indicates that the funding scheme expands the range of $c$ for which the project is implemented. We also note that if no fixed commitment is required ($R = 0$), then the project would never be completely self-funded.

### 4 Funding Policy

In this section we study the funding body’s payoff-maximizing choice of the funding scheme $(\beta, R)$, given the researcher’s choice we discussed in the previous section. For most of our analysis of the optimal scheme we need to specify the distribution of the project implementation cost $c$. However, we can derive some results for large interest rates $\gamma$ or $\rho$ without making an assumption on the distribution.

**Proposition 2.** (i) If the funding body’s cost of funds $\gamma$ is sufficiently high, the funding body does not subsidize the project ($\beta = 1$).

(ii) If the researcher’s cost of funds $\rho$ is sufficiently high, the funding body chooses either no subsidization ($\beta = 1$), or full subsidization ($R = 0, \beta = 0$).
Both points are straightforward. When the funding body’s cost of funds $\gamma$ is large enough, any $e_0$ is too costly to subsidize the project. Meanwhile, when the researcher’s cost of funds is very high, the project should be fully subsidized as long as its benefit to the society is high enough. Otherwise the project should not be implemented.

In what follows we illustrate the trade-off the funding body faces with respect to the self-funding requirement. On one hand, requiring a larger commitment to self-funding $R$ reduces the total cost the funding body incurs. On the other hand, large $R$ leads to a higher total cost borne by the researcher and thus a higher probability that the project is not implemented, even if the realized cost is such that the funding body would like the project to be implemented. We will show that the trade-off is the key in determining the optimal funding scheme for the funding body.

In order to study how the funding body’s funding policy, for simplicity, we assume that $c$ is uniformly distributed between 0 and 1. The following represents the funding body’s expected payoff for the benchmark case where no funding scheme is offered ($R = 1$):

$$EU_B = \int_0^1 \frac{1}{2(1+\rho)} \left( \frac{1+\alpha}{2} - (1+\rho)c \right) dc$$

which is also the researcher’s ex ante expected payoff. Also in what follows, we rule out trivial the case where $\alpha$ is so large that the funding body optimally induces the researcher to implement the project for any $c \in [0, 1]$. In other words, we focus on the case where some projects should not be implemented from the funding body’s viewpoint.

4.1 First Best

Before deriving the funding body’s optimal scheme, let us consider the first best in this model, which obtains when there is no asymmetric information or the funding body is able to appropriate any residual from the researcher in case the amount of subsidy exceeds the total cost. We need to consider the two cases separately, namely where i) the researcher’s cost of funds is lower than that of the funding body ($\gamma < \rho$); and ii) the the researcher’s cost of funds is higher ($\gamma > \rho$).

\textsuperscript{3}This for example might allow it to fund projects by other researcher that generate positive externalities.
4.1.1 $\gamma < \rho$ (funding body’s cost of funds is lower)

In this case, since the funding body’s payoff function incorporates the researcher’s payoff, the funding body is better off paying the entire cost on behalf of the researcher if the project is to be implemented. The funding body effectively decides whether to implement the project, and the funding body does so if the project generates a positive payoff, that is,

$$\frac{1}{2} - c + G + \frac{\alpha}{2} - (1 + \gamma)G > 0. \tag{1}$$

Thus the project is implemented if

$$c < \min \left\{ \frac{1 + \alpha}{2(1 + \gamma)}, 1 \right\}.$$ 

As we noted earlier, we rule out the trivial case that the project should be implemented for any $c$, which is equivalent to $\frac{1 + \alpha}{2(1 + \gamma)}$ being smaller than 1. Thus under the assumption the expected utility of the funding body is given by

$$E[U_{FB1}] = \int_{0}^{\frac{1 + \alpha}{2(1 + \gamma)}} \left(\frac{1 + \alpha}{2} - (1 + \gamma)G\right) dG.$$ 

The expected gain from the first best funding, relative to the case without any funding scheme is given by

$$E[U_{FB1}] - E[U_{B}] = \frac{(\rho - \gamma)(2\alpha + 1) + \alpha^2(1 + \rho)}{8(1 + \gamma)(1 + \rho)} \tag{2}$$

4.1.2 $\rho < \gamma$ (researcher’s cost of funds is lower)

If the researcher’s cost of funds is lower, the funding body makes the researcher pay for the project as much as he can. In this case, the researcher breaks even and the private revenue from the project has to be equal to the total cost the researcher incurs, that is,

$$\frac{1}{2} = (1 + \rho)(c - G) \quad \iff \quad G = \frac{2c(\rho + 1) - 1}{2(\rho + 1)} \tag{3}$$

The funding body subsidizes the project if its implementation leads to a positive payoff
The inequality shows that, for efficiency, the project should be subsidized only when the amount of the positive externalities, $\alpha/2$, is larger than the total cost of the subsidy $(1 + \gamma)G$. From (3) and (4), we see that the funding body subsidizes the project if

\[
c \in \left( \frac{1 + \rho}{2}, \frac{1 + \alpha + \alpha \rho + \gamma}{2(1 + \rho)(1 + \gamma)} \right).
\]

Note that if $c \leq \frac{1+\rho}{2}$ the funding body lets the researcher pay for the project. The expected payoff of the funding body is given by

\[
E[U_{FB2}] = \int_0^{1/(1+\rho)} \left( \frac{1 + \alpha}{2} - (1 + \rho)c \right) dc + \int_{1/(1+\rho)}^{1+\alpha+\alpha \rho + \gamma/(1+\gamma)} \left( \frac{\alpha}{2} - (1 + \gamma) \left( c - \frac{1}{2(1 + \rho)} \right) \right) dc.
\]

Thus we can obtain the expected utility gain relative the case without any funding scheme

\[
E[U_{FB2}] - E[U_B] = \frac{\alpha^2}{8(1 + \gamma)}.
\]

The comparison of the two cases above suggests that the cost of funds is an essential part of a desirable scheme. If the funding body’s cost of funds is lower, then the project should be funded entirely by the funding body. If the researcher’s cost of fund is lower, the funding body should subsidize the project insofar as the positive externalities are large enough relative to the total cost of providing the subsidy.

### 4.2 Optimal Funding Scheme

Let us study the funding body’s choice of the funding scheme $(\beta, R)$, given the researcher’s response examined in Section 3. If both of the two cutoff points for the researcher’s choice as presented in Proposition 1 are in $(0, 1)$, the funding body’s expected payoff is given by
\[ E \mathbb{[} U_{MF} \mathbb{]} = \int_0^R \left( \frac{1 + \alpha}{2} - \frac{(1 + \rho) c}{\text{total benefit}} \right) dc \]

\[ + \int_R^{2(1+\rho)\beta} \left( \frac{1 + \alpha}{2} - \frac{(1 + \rho) (\beta c + (1 - \beta) R) - (1 + \gamma) (1 - \beta) (c - R)}{\text{total benefit}} \right)\frac{R(1-\beta)}{\beta} dc \]

\[ \text{implemented with scheme } (\beta, R) \]

(6)

The first term is the expected payoff when the project is self-funded, and the other term is the expected payoff when the project is implemented by resorting to the funding scheme \((\beta, R)\). The funding scheme that maximizes (6) is characterized as follows:

**Proposition 3.** When \( \rho < \gamma \), the funding body’s payoff is maximized by

\[ \beta^* = \frac{1 - \mu}{1 + \rho}; \quad R^* = \frac{1}{2(1 + \rho)} - \frac{(1 - \mu) \alpha}{2(1 + \rho)(\mu + \gamma)}, \]

where \( \alpha \) needs to satisfy following condition

\[ \alpha < \begin{cases} \frac{\mu + \gamma}{1 - \mu} & \text{when } \mu < 1/2 \\ \frac{(\mu + \gamma)(2\rho + 1)}{(\rho + \mu)} & \text{otherwise} \end{cases} \]

In this proposition, we pay attention to the case that the funding body’s cost \( \gamma \) is larger than the researcher’s cost \( \rho \). In this case, the funding body has a strong incentive to save his cost of funds by screening out the low R&D cost project. From proof of proposition 1, we know that the researcher’s strategies are divided into three regions. We would like to ignore the unrealistic regions that are all projects are realized by using assumption. If \( \alpha \) is larger than the limit, \( c' \) and \( c'' \) in this equilibrium are not in \((0, 1)\). In this equilibrium, the funding body tries to increases the probability of researcher’s self investment by setting positive \( R^* \). \( \beta^* \) is the smallest ratio that the researcher requires the amount he needs. In order to understand effect of the matching funds, next proposition shows the comparative statics of \((R^*, \beta^*)\) characterized by proposition 3.

**Proposition 4.** When the funding body applies the matching funds, we can obtain the following results.

\[ \frac{\partial R^*}{\partial \mu} > 0, \quad \frac{\partial R^*}{\partial \gamma} < 0, \quad \frac{\partial R^*}{\partial \rho} > 0, \quad \frac{\partial R^*}{\partial \alpha} \leq 0 \]
\[
\frac{\partial \beta^*}{\partial \mu} < 0, \quad \frac{\partial \beta^*}{\partial \rho} < 0
\]

When the externality of the innovation \( \alpha \) becomes large, the funding body’s incentive to implement that project is also large. Then, the funding body sets lower \( R^* \). If the funding body funding cost \( \gamma \) becomes large, the funding body has an incentive to save the payment. On the other hand, if the researcher’s funding cost increases, the funding body increases his payment instead of researcher. When \( \mu \) becomes large, the social loss by the researcher’s diversion. Comparing the expected payoff of the funding body with the matching funds and without subsidisation policy yields the following lemma.

**Lemma 2.** When the funding body applies the matching funds, the expected utility gain in equilibrium satisfies

\[
\frac{\partial EUG_{MF}^*}{\partial \rho} \geq 0, \quad \frac{\partial EUG_{MF}^*}{\partial \gamma} \leq 0, \quad \frac{\partial EUG_{MF}^*}{\partial \alpha} \geq 0, \quad \frac{\partial EUG_{MF}^*}{\partial \mu} \geq 0.
\]

\( EUG_{MF}^* \) means the differences between the expected utility with matching funds and that without public support. When \( \rho \) is large, the payoff without matching funds becomes small, since the complete self funding region becomes small. Then, \( EUG_{MF}^* \) is an increasing function of \( \rho \). If the funding body’s cost of fund \( \gamma \) becomes large, the expected payoff under the matching funds becomes small since the funding body hesitates to implement the project. It is clear that \( EUG_{MF}^* \) becomes an increasing function of \( \alpha \) because the socially desirable project may be implemented by public support. When \( \gamma \) is larger than \( \rho \), the funding body has a strong incentive to save his payment by screening out the project that can be implemented without public money. If \( \mu \) is large, the researcher can not enjoy the diversion of extra subsidies. In other words, the screening by the matching fund system works well.

### 4.3 Comparison

Now, we are ready to compare the first best result and the matching funds system. Figure 3 shows the researcher’s action in every scheme that we discussed in this paper. When \( \rho \) is smaller than \( \gamma \), screening is beneficial for funding body. In order to save the funding body funding cost, the funding body tries to fill the researcher’s shortage of R&D costs. If the funding body applies the matching funds, he sets high \( R \) to kick out the project that can be implemented by researcher himself. Setting high \( R \) decreases the probability
Corollary 1. When $\gamma$ is larger than $\rho$ and $\mu = 1$, the funding body can achieve the first best with the matching funds system.

If $\mu$ is equal to 1, the funding body can set $\beta = 0$ since the researcher can not benefit from the diversion. In this case, the funding body can achieve the perfect screening under the matching funds.

5 Private Investment

The empirical literature on the public support on R&D expenditure mainly consider whether the public money crowds out of the private R&D expenditure or not. Theoretically, the public supports make the unprofitable project to a profitable one by reducing the private cost. Then, the public support may stimulate the private spending on R&D. On the other hand, there exists the possibility that the public support crowds out the private expenditure on R&D since the firm has an incentive to use the funding body grants to implement the projects that have been undertaken without the subsidy. The net effect of public support to the private R&D level is not clear from a theoretical point of view. If the private R&D expenditure is smaller than the socially optimal expenditure, as past literature discussed, the crowding in seems to be socially desirable. However, we need to discuss carefully whether the crowding out is socially desirable or not because we assumed that there exists the funding cost. In this paper, we assume that if the researcher’s expected R&D payment gain compared by the benchmark case is larger than the funding body’s payment, the crowding in occurs. First of all, we try to check the relationship between the public support and the private R&D expenditure in the first best model.
Proposition 5. Relative to the benchmark: in the first best,

(i) when \( \rho > \gamma \), crowding-out occurs

(ii) when \( \rho < \gamma \), \( \rho \) is not small and \( \alpha \) is large, crowding-out occurs. Otherwise, crowding-in occurs.

The first result of this proposition is intuitive clear. If \( \rho \) is larger than \( \gamma \), the funding body has an incentive to pay the development cost on behalf of the researcher to save the funding cost. In this case, the researcher’s payment is crowded out by the public support. If \( \rho \) is smaller than \( \gamma \), the funding body tries to fill the shortage of R&D costs because his funding cost is high. In the first best case, the funding body gives money to the research whose R&D cost is in \([1/2(1 + \rho), 1/2(1 + \rho) + \alpha/2(1 + \gamma)]\). The R&D payment of all researcher who obtain the public money is equal to \(1/2(1 + \rho)\). If \( \alpha \) is large, the probability that the researcher needs public money increases. If \( \rho \) is large, the researcher’s payment with public money is small. Then, the crowding out occurs.

Next, we try to consider how the public support affect the private R&D payment under the matching funds system. In order to make it easier to understand, researcher’s project can be broken down according to whether the project can be implemented under the benchmark case or not (whether \( c \) is smaller than \( 1/2(1 + \rho) \) or not).

When we focus on the case that \( c \) is smaller than \( 1/2(1 + \rho) \), we can obtain the researcher’s payment gain as

$$
ECG_1 = \int_0^{1/2(1+\rho)} cdc - \int_0^R cdc - \int_R^{1/2(1+\rho)} (\beta c + (1 - \beta)R) dc = -\int_R^{1/2(1+\rho)} (1 - \beta) (c - R) dc.
$$

The first term of this equation means the researcher’s payment under the benchmark case. The remaining terms is equal to the researcher’s payment under the matching funds. The funding body’s payment in this region is equal to
The researcher tries to save his payment by using the public support, in other words, crowding out occurs in this region. If the funding body increases $\beta$, the degree of crowding out is mitigated since the researcher’s payment under the matching funds scheme increases and the funding body payment decreases. Similarly, if the funding body increases $R$, the researcher’s payment increases and the public payment decreases. In addition to these effects, the probability of the researcher chooses self-funding increases. Then, the funding body can mitigate the crowding out by increasing $R$ in this region.

When we focus on the case that $c$ is larger than $\frac{1}{2}(1+\rho)$, we can obtain the researcher’s payment gain as

$$ECG_2 = \int_{\frac{1}{2}(1+\rho)}^{\frac{1}{2}(1+\rho)} \frac{(1-\beta)R}{\beta} dc. \quad \text{(9)}$$

The funding body’s payment in this region is equal to

$$GP_2 = \int_{\frac{1}{2}(1+\rho)}^{\frac{1}{2}(1+\rho)} \frac{(1-\beta)R}{\beta} \left(1 - \beta \right) dc. \quad \text{(10)}$$

Whether the crowding in out occurs or not depends on the sign of the following equation

$$ECG_2 - GP_2 = \int_{\frac{1}{2}(1+\rho)}^{\frac{1}{2}(1+\rho)} \frac{(1-\beta)R}{\beta} \left(2\beta - 1 \right) dc. \quad \text{(11)}$$

The sign of equation (11) depends on $c$, $R$ and $\beta$. If the funding body increases $\beta$, the
researcher’s payment increases and the public payment decreases. In this region, if the researcher’s ratio of burden $\beta$ increases, the probability that the researcher applies the matching fund decreases. This effect stimulates the crowding out. If the funding body increases $R$, the researcher’s payment increases and the public payment decreases. In addition to that, the probability of the researcher chooses no-investment increases. Next proposition shows whether the public money stimulate private R&D expenditure or not in equilibrium.

**Proposition 6.** In the matching fund case,

(i) When $\beta$ and $R$ are large, crowding-in occurs.

(ii) Under the optimal matching funds system, the public support crowds out the private R&D effort when $\alpha$ is larger than $2(\mu + \gamma)/(2 + \rho - \mu)$. Otherwise, crowding-in occurs.

The intuition of this result is as follows. If the researcher’s development cost is smaller than $1/2(1 + \rho)$, he can implement his project without the public support. In this case, the researcher’s R&D payment is crowded by the matching funds. If the researcher’s cost is larger than the limit, the crowding in may occur. When $\alpha$ is large, the funding body sets lower $R$ to implement the project. In other words, the probability that researcher whose R&D cost is smaller than $1/2(1 + \rho)$ applies the matching funds system increases.

6 Conclusion

The aim of this paper was to discuss how requirement for commitment to self-financing can be used to induce self-selection of projects that are not materialized without subsidies. The following results were obtained. First, it was shown that the funding body can screen out the low cost research by using the matching fund system. Especially, this screening system is socially desirable when (i) the funding body’s cost of funds is higher than that of the researcher and (ii) the externalities of the project $\alpha$ is not so large. If the first condition is satisfies, the funding body has a incentive to kick out the low cost project by screening. We need the second condition to ignore the trivial cases where all projects are implemented.

Second, we discuss whether the requirement for commitment to self-financing stimulate the recipients R&D or not. In this paper, we take into account the cost of funds. Thus, it is not clear that the crowding-in is socially desirable or not. For example, we showed that the crowding-out may be occurred even if we consider the first best cases. As Scotchmer (2013) shows, the matching funds may mitigate the crowding out problem if we do not
consider the cost of funds. We show the possibility that the crowding out problem may occur if we take into account the social cost of the matching funds.

7 Appendix

7.1 Lemma 1

Proof. We consider the following two cases:

1. When $R + G \geq c$, the researcher’s profit under the matching funds is equal to $\pi = 1/2 - (1 + \rho)(\beta G + R) + (1 - \mu)(R + G - c)$. If $\beta$ is larger than $(1 - \mu)/(1 + \rho)$, the researcher’s profit is a decreasing function of $G$ since $\partial \pi / \partial G = -\beta(1 + \rho) + 1 - \mu$. Otherwise, his profit becomes an increasing function of $G$.

2. When $R + G < c$, the researcher’s profit is given by $\pi = 1/2 - (1 + \rho)(c - (1 - \beta)G)$. In this case, the profit is clearly an increasing function of $G$ since $\partial \pi / \partial G = (1 + \rho)(1 - \beta)$.

Therefore, if $\beta$ is smaller than $(1 - \mu)/(1 + \rho)$, the researcher has a strong incentive to request $G$ as high as possible since share of researcher’s payment is small. If $\beta$ is larger than $(1 - \mu)/(1 + \rho)$, the researcher sets $G$ to satisfy $R + G = c$. In other words, the researcher will set $G = \min\{c - R, 0\}$.

7.2 Proposition 1

Proof. Let us consider the researcher’s decision whether to implement the project with or without the subsidy. Note that from Lemma 1, the researcher requests $G = \min\{c - R, 0\}$. In what follows we consider two cases:

1) Suppose $\nu = 1/2 \geq (1 + \rho)c$. In this case, the researcher will implement the project even with no subsidy as the private revenue is higher than the total funding cost he incurs. Thus his choice is whether to receive the subsidy to implement the project. The researcher’s payoff if he is to use the funding scheme is $1/2 - (1 + \rho)(\beta c + (1 - \beta)R)$, since as noted earlier he requests $G = c - R$. The researcher implements the project entirely by self-funding if $1/2 - (1 + \rho)(\beta c + (1 - \beta)R) < 1/2 - (1 + \rho)c$, which reduces to $c > R$ and otherwise (i.e. if $c \leq R$) the researcher uses the funding scheme. Thus we have the first cutoff point $c' = R$ for low $c$.

2) Next, suppose $\nu = 1/2 < (1 + \rho)c$. In this case the project is too expensive to be entirely self-funded. Thus the researcher implements the project and uses the funding scheme if $1/2 - (1 + \rho)(\beta c + (1 - \beta)R) > 0$, where the right hand side denotes the zero payoff.
for not implementing the project. This condition can be rewritten as \( c'' \leq c \) whereby

\[
c'' = \frac{1}{2\beta(1 + \rho)} - \frac{(1 - \beta)R}{\beta}
\]

as stated.

From 1) and 2) we see that the researcher’s choice is characterized by the two cutoff points \( c' \) and \( c'' \). We can represent the researcher’s choice \((R, \beta)\) in Figure 6.

**Proof of Corollary 2**

First of all, we need to show the expected funding body utility function in each regions. The cumulative distribution function of \( c \) is given by \( F(c) \).

\[
EU_{Region I} = F\left(\frac{1}{2(1 + \rho)}\right) \times \frac{1 + \alpha}{2} - F\left(\frac{1}{2(1 + \rho)}\right) \times (1 + \rho)E\left(c \mid c \leq \frac{1}{2(1 + \rho)}\right)
\]

\[
EU_{Region II} = F\left(\frac{1}{2(1 + \rho)} - \frac{R(1 - \beta)}{\beta}\right) \times \frac{1 + \alpha}{2} - F\left(R\right) \times (1 + \rho)E\left(c \mid c \leq R\right)
\]

\[
- F\left(R \leq c \leq \frac{1}{2(1 + \rho)} - \frac{R(1 - \beta)}{\beta}\right) \times (1 + \rho)\left(\beta E\left(c \mid R \leq c \leq \frac{1}{2(1 + \rho)}\right) - \frac{R(1 - \beta)}{\beta}\right) +
\]

\[
- F\left(R \leq c \leq \frac{1}{2(1 + \rho)} - \frac{R(1 - \beta)}{\beta}\right) \times (1 + \gamma)(1 - \beta)\left(E\left(c \mid R \leq c \leq \frac{1}{2(1 + \rho)}\right) - \frac{R(1 - \beta)}{\beta}\right)
\]
\[ EU_{RegionIII} = \frac{1 + \alpha}{2} - F(R) \times (1 + \rho)E(c|0 \leq c \leq R) \]
\[-F(R \leq c \leq 1) \times (1 + \rho)(\beta E(c|R \leq c \leq 1) + (1 - \beta)R)\]
\[-F(R \leq c \leq 1) \times (1 + \gamma)(1 - \beta)(E(c|R \leq c \leq 1) - R)\]

It is clear that when \( \gamma \rightarrow \infty \), the funding body expected payoff becomes negative except for Region I. When \( \rho \rightarrow \infty \), the funding body expected payoff becomes negative other than Region I. If the funding body sets \((\beta, R)\) to be Regime I, his expected payoff becomes zero since the researcher does not invest his idea. The funding body has another option to set \( R = 0 \) and \( \beta = 0 \). In this case, the researcher does not have to spend financial cost.

**Proof of Lemma 2**

To focus on the realistic cases, we ignore the cases that all projects are implemented by the funding body support. In other words, we focus on Region II. When the funding body can apply the matching funds, he (or she) can control both \( \beta > 0 \) and \( R > 0 \) to maximize his expected welfare gain. We consider the optimal strategies in Region II. The expected utility gain of the funding body in this region is given by the following equation.

\[
\max_{\beta, R} EUG_{MF} = \int_0^R \left( \frac{1 + \alpha}{2} - (1 + \rho)c \right) dc + \int_R^{1 + \rho \beta} \left( \frac{1 + \alpha}{2} - (1 + \rho)(\beta c + (1 - \beta)R) \right) dc
- \int_R^{1 + \rho \beta} \left( 1 + \gamma)(1 - \beta)(c - R) \right) dc
\]

The second derivatives of \( \beta \) and \( R \) are as follows.

\[
\frac{\partial^2 EUG_{MF}}{\partial R^2} = -\frac{(1 - \beta)(1 + \gamma - \beta(1 + \rho))}{\beta^2}
\]

\[
\frac{\partial^2 EUG_{MF}}{\partial \beta^2} = -\frac{(1 - 2R - 2R\rho)((1 - 2R - 2R\rho)(2\beta + \beta\rho + \beta\gamma - 3\gamma - 3) + 2\alpha \beta(1 + \rho))}{4\beta^4(1 + \rho)^2}
\]

It is not clear whether (13) is positive or negative. Therefore, we try to compare every possible cases in order to obtain the optimal \((\beta, R)\).

Case I: \( \gamma > \rho \)
First, we consider the case that $\gamma > \rho$. In this case, (12) becomes negative.

**<Case I-1>**

If (13) is negative, we can derive optimal $(\beta, R)$ from first order conditions. From first order condition, we can obtain

$$R = \frac{1 + \gamma}{2(1 + \rho)(1 + \gamma - \beta - \beta \rho)} - \frac{\beta(1 + \alpha)}{2(1 + \gamma - \beta - \beta \rho)} \quad (14)$$

$$\beta = \frac{2(1 + \gamma)(1 - 2R - 2R \rho)}{2\alpha + 2\alpha \rho + \gamma + \rho + 2 - 2R(1 + \rho)(2 + \gamma + \rho)} \quad (15)$$

From these equations, we can obtain

$$\beta^*_I = 0, \quad R^*_I = \frac{1}{2(1 + \rho)} \quad (16).$$

However, the optimal $\beta$ violates the constraint since we consider the case $\beta$ is larger than $(1 - \mu)/(1 + \rho)$. Then, we need to pay attention to corner solutions.

In this setting, $R$ and $\beta$ need to satisfy
$$\frac{1}{2(1 + \rho)} \geq R \geq \frac{1}{2(1 + \rho)(1 - \beta)} - \frac{\beta}{1 - \beta} \quad \text{and} \quad 1 \geq \beta \geq (1 - \mu)/(1 + \rho).$$

**<Case I-2>**

We consider the case that (13) is positive. If $R$ satisfies (14) while $\beta = (1 - \mu)/(1 + \rho)$, there are two cut-off points on $c$.

We can obtain

$$\beta^*_II = \frac{1 - \mu}{1 + \rho}, \quad R^*_II = \frac{1}{2(1 + \rho)} - \frac{(1 - \mu)\alpha}{2(1 + \rho)(\mu + \gamma)} \quad (17).$$

In order to guarantee two cut-off points, $(\beta^*_II, R^*_II)$ needs to satisfy $R^*_II \geq \max 1/2(1 + \rho)(1 - \beta^*_II) - \beta^*_II/(1 + \rho)$ and $1 \geq 1/2\beta^*_II(1 + \rho) - (1 - \beta^*_II)R^*_II/\beta^*_II$. Then, $\alpha$ needs to satisfy following condition.

$$\alpha < \begin{cases} \frac{\mu + \gamma}{1 - \mu} & \text{when } \mu < 1/2 \\ \frac{(\mu + \gamma)(2\rho + 1)}{(\rho + \mu)} & \text{otherwise} \end{cases} \quad (18).$$

**<Case I-3>**

We consider the case that (13) is positive. If $R$ satisfies (14) while $\beta = 1/2(1 + \rho)(1 - R) - R/(1 - R)$, there exists only one cut-off point on $c$. All projects will conduct in this case.

We can obtain

$$\beta^*_III = \frac{1}{1 + \rho} + \frac{\gamma + 2\rho \gamma - \alpha \rho}{(1 + \rho)(1 + 2\rho - \alpha)}, \quad R^*_III = \frac{1 + 2\gamma - \alpha}{2(\gamma - \rho)} \quad (19).$$
In order to guarantee one cut-off point, \( (\beta_{III}^*, R_{III}^*) \) needs to satisfy \( (1 + \gamma)/(1 + \rho) > \beta_{III}^* > (1 - \mu)/(1 + \rho) \) and \( 1/2(1 + \rho) > R_{III}^* > 0 \). Then, we need the following conditions.

\[
\frac{(\mu + \gamma)(2\rho + 1)}{(\rho + \mu)} < \alpha < 2\gamma + 1 \\
\mu > \frac{1}{2}
\]  \hspace{1cm} (20)

If both of the two cutoff points for the researcher’s choice are in \((0, 1)\), the funding body’s expected payoff is maximized by (17). When \( \gamma > \rho, \alpha \) needs to satisfy (18).

**Case II:** \( \gamma \leq \rho \) and \( \beta < (1 + \gamma)/(1 + \rho) \)

In this case, (12) becomes negative.

**Case II-1**

If (13) is negative, we can obtain the same result as Case I-1.

**Case II-2**

We consider the case that (13) is positive. If \( R \) satisfies (14) while \( \beta = (1 - \mu)/(1 + \rho) \), there exists only one cut-off point on \( c \). In this case, we can obtain the same results of case I-2.

**Case II-3**

We consider the case that (13) is positive. If \( R \) satisfies (14) while \( \beta = 1/2(1 + \rho)(1 - R) - R/(1 - R) \), there exists only one cut-off point on \( c \). All projects will conduct in this case.

We can obtain

\[
\beta_{IV}^* = \frac{1}{1 + \rho} + \frac{\gamma + 2\rho\gamma - \alpha\rho}{(1 + \rho)(1 + 2\rho - \alpha)}, \quad R_{IV}^* = \frac{\alpha - 1 - 2\gamma}{2(\rho - \gamma)}. \]  \hspace{1cm} (21)

In order to guarantee one cut-off point, \( (\beta_{IV}^*, R_{IV}^*) \) needs to satisfy \( (1 + \gamma)/(1 + \rho) > \beta_{IV}^* > (1 - \mu)/(1 + \rho) \) and \( 1/2(1 + \rho) > R_{IV}^* > 0 \). Then, we need the following conditions.

\[
2\gamma + 1 < \alpha < \frac{(\mu + \gamma)(2\rho + 1)}{(\rho + \mu)} \\
\mu > \frac{1}{2}
\]  \hspace{1cm} (22)

When \( \mu < 1/2 \), the funding body’s payoff is maximized by \( (\beta_{II}^*, R_{II}^*) \). Otherwise, his profit is maximized by either \( (\beta_{II}^*, R_{II}^*) \) or \( (\beta_{IV}^*, R_{IV}^*) \). It is clear that \( (\beta_{IV}^*, R_{IV}^*) \) is desirable for the funding body. Then, \( \alpha \) needs to satisfy the following condition.
\[ \alpha < \begin{cases} \frac{\mu + \gamma}{1 - \mu} & \text{when } \mu < 1/2 \\ 2\gamma + 1 & \text{otherwise} \end{cases} \]

<Case III: \( \gamma \leq \rho \) and \( \beta \geq (1 + \gamma)/(1 + \rho) \)

In this case, (12) becomes positive.

<Case III-1>

We consider case that (12) becomes positive and (13) is negative. If \( \beta \) satisfies (15) while \( R = 1/2(1 + \rho)(1 - \beta) - \beta/(1 - \beta) \), there exists only one cut-off point on \( c \). All projects will conduct in this case. We can obtain

\[ \beta^*_V = \frac{2((2\rho + 1)(1 + \gamma) - \alpha(1 + \rho))}{(2\rho + 1)(\rho + \gamma + 2) - 2\alpha(1 + \rho)}, \quad R^*_V = \frac{2\alpha(1 + \rho) - 2 - 3\rho - 3\gamma - 4\rho\gamma}{2(\rho - \gamma)(1 + \rho)} \]

In order to guarantee one cut-off point, \((\beta^*_V, R^*_V)\) needs to satisfy \((1 + \gamma)/(1 + \rho) > \beta^*_V > (1 - \mu)/(1 + \rho)\) and \(1/2(1 + \rho) > R^*_V > 0\). Then, we need following conditions.

\[ \frac{3\gamma + 3\rho + 4\rho\gamma + 2}{2(1 + \rho)} < \alpha < \frac{(1 + \gamma)(2\rho + 1)}{2(1 + \rho)} \]

However, we can easily show that there does not exist \( \alpha \) such that satisfies this conditions.

<Case III-2>

We consider case that (12) becomes positive and (13) is negative. If \( \beta \) satisfies (15) while \( R = 0 \), there exists only one cut-off point on \( c \). All projects will conduct in this case. We can obtain

\[ \beta^*_V = \frac{2(1 + \gamma)}{\rho + \gamma + 2 + 2\alpha(1 + \rho)}, \quad R^*_V = 0. \]

In order to guarantee one cut-off point, \((\beta^*_V, R^*_V)\) needs to satisfy \( 1 > \beta^*_V > (1 + \gamma)/(1 + \rho) \). Then, we need following conditions.

\[ \alpha < \frac{\rho - \gamma}{2(1 + \rho)} \]

<Case III-3>

We consider case that both (12) and (13) are positive. However, we can easily show that there does not exist the combination of corner solutions that satisfy \( 1 > \beta^* > (1 + \gamma)/(1 + \rho) \) and \( 1/2(1 + \rho) > R^* > 1/2(1 + \rho)(1 - \beta) - \beta/(1 - \beta) \).
Now, we are ready to consider the funding body’s optimal strategy when $\gamma \leq \rho$. The expected utility of funding body under each cases are given by

$$EU(\beta^*_II, R^*_II) = \frac{\mu + \gamma + 2\alpha\mu + 2\alpha\gamma + \alpha^2\rho + \alpha^2\mu}{8(1 + \rho)(\mu + \gamma)}$$

$$EU(\beta^*_IV, R^*_IV) = \frac{3\alpha + 2\alpha\rho + 1}{8(1 + \rho)}$$

$$EU(\beta^*_VI, R^*_VI) = \frac{(\rho + \gamma + 2\alpha + 2\alpha\rho + 2)^2}{32(1 + \rho)(1 + \gamma)}$$

$$EU(\beta^*_II, R^*_II) - EU(\beta^*_IV, R^*_IV) = \frac{\alpha(\alpha(\rho + \mu) - (\mu + \gamma)(2\rho + 1))}{8(1 + \rho)(\mu + \gamma)}$$

The right hand side of equation (23) becomes negative if $\alpha$ is smaller than $(\mu + \gamma)(2\rho + 1)/(\rho + \mu)$. Therefore, $\alpha$ must be smaller than $2\gamma + 1$.

$$EU(\beta^*_II, R^*_II) - EU(\beta^*_VI, R^*_VI) = \frac{(\rho - \gamma)(4\alpha(\gamma + \mu - \alpha + \alpha\mu)(1 + \rho) + (\gamma + \mu)(\rho - \gamma))}{32(1 + \gamma)(\mu + \gamma)(1 + \rho)^2}$$

The right hand side of equation (24) becomes negative if $\alpha$ is smaller than $(\mu + \gamma)/(1 - \mu)$. Therefore, when $\gamma \leq \rho$ and $\mu > 1/2$, $\alpha$ must be satisfied

$$\frac{\rho - \gamma}{2(1 + \rho)} < \alpha < 2\gamma + 1$$

When $\gamma \leq \rho$ and $\mu < 1/2$, $\alpha$ must be satisfied

$$\frac{\rho - \gamma}{2(1 + \rho)} < \alpha < \frac{\mu + \gamma}{1 - \mu}$$

When $\rho$ is large enough, the region of the alpha does not exist.

**Proof of Proposition 2**

From lemma 1, we know that the funding body sets $R^* = (\mu + \gamma - \alpha + \alpha\mu)/(1 + \rho)(\mu + \gamma)$ under the matching funds policy. We can calculate as follows.

$$\frac{\partial R^*}{\partial \mu} > 0, \frac{\partial R^*}{\partial \gamma} \geq 0, \frac{\partial R^*}{\partial \rho} < 0, \frac{\partial R^*}{\partial \alpha} \leq 0$$

$$\frac{\partial R^*}{\partial \mu} = \frac{\alpha(1 + \gamma)}{2(\mu + \gamma)^2(1 + \rho)} > 0$$
Proof of Lemma 3

From lemma 1, the expected utility gain in equilibrium is given by

\[ EUG^*_{MF} = \frac{\alpha^2 (\mu + \rho)}{8 (1 + \rho) (\mu + \gamma)} \]

We can calculate as follows.

\[ \frac{\partial EUG^*}{\partial \mu} = \frac{\alpha^2 (\gamma - \rho)}{8(\mu + \gamma)^2(1 + \rho)} \]

\[ \frac{\partial EUG^*}{\partial \gamma} = -\frac{\alpha^2(\mu + \rho)}{8(\mu + \gamma)^2(1 + \rho)} \leq 0 \]

\[ \frac{\partial EUG^*}{\partial \rho} = \frac{\alpha^2 (1 - \mu)}{8(\mu + \gamma)(1 + \rho)^2} \geq 0 \]

\[ \frac{\partial EUG^*}{\partial \alpha} = \frac{\alpha (\mu + \rho)}{4 (1 + \rho) (\mu + \gamma)} \geq 0 \]

Proof of Proposition 3

(i) When \( \rho > \gamma \), the funding body will pay the development cost on behalf of the researcher in order to save the funding cost. Then, it is clear that crowding-out occurs.

(ii) When \( \rho < \gamma \), the funding body tries to save the funding cost by filling the shortage of R&D costs. We know that the funding body will subsidize when \( 1/2(1 + \rho) \leq c \leq \min \{(1 + \alpha + \alpha \rho + \gamma)/2(1 + \rho)(1 + \gamma), 1\} \). We focus on the case that \( (1 + \alpha + \alpha \rho + \gamma)/2(1 + \rho)(1 + \gamma) \) is smaller than 1 in order to ignore the unrealistic case. The researcher’s expected R&D payment gain compared by the benchmark case is given by

\[ ECG_{FB} = \int_{1/2(1 + \rho)}^{\alpha} \left( c - \left( c - \frac{1}{2(1 + \rho)} \right) \right) dc = \frac{\alpha}{4(1 + \rho)(1 + \gamma)} \]

The funding body’s payment is equal to
\[ GP_{FB} = \int_{\frac{1}{2(1+\rho)}}^{\frac{\alpha}{2(1+\rho^2)}} \left( c - \frac{1}{2(1+\rho)} \right) dc = \frac{\alpha^2}{8(1+\gamma)^2} \]

If the sign of \( ECG_{FB} - GP_{FB} \) is positive, the crowding in occurs.

\[ ECG_{FB} - GP_{FB} = \frac{\alpha (2 + 2\gamma - \alpha - \alpha\rho)}{8(1 + \rho)(1 + \gamma)^2} \tag{25} \]

From (25), crowding in occurs when \( \alpha < \frac{2(1 + \gamma)}{(1 + \rho)} \). We focus on the case that \( \frac{(1 + \alpha + \alpha\rho + \gamma)}{2(1 + \rho)(1 + \gamma)} < 1 \), in other words, \( \alpha < \frac{(2\rho + 1)(1 + \gamma)}{(1 + \rho)} \). Then, we can easily show that crowding out occurs when \( \rho \) is not small and \( \alpha \) is large.

**Proof of Proposition 4**

From lemma 1, we know that the researcher requires \( G = c - R \) when \( \beta \) is larger than \( \frac{(1 - \mu)}{(1 + \rho)} \). In this case the researcher’s expected R&D payment is given by

\[ EC_{MF} = \int_{0}^{R} c dc + \int_{R}^{\frac{1}{2(1+\rho^2)}} \frac{R(1 - \beta)}{\beta} (\beta c + (1 - \beta)R) dc \]

We also need to check the funding body’s payment \( (1 - \beta)G \) under the matching funds system. The funding body’s payment is equal to

\[ GP_{MF} = \int_{R}^{\frac{1}{2(1+\rho^2)}} \frac{R(1 - \beta)}{\beta} (1 - \beta) (c - R) dc \]

We can obtain the researcher’s expected R&D cost without public support as

\[ EB = \int_{0}^{\frac{1}{2(1+\rho^2)}} c dc. \]

Now, we are ready to check whether the matching funds system can mitigate the crowding out problem or not. If the sign of \( EC_{MF} - EB - GP_{MF} \) is positive, the crowding in occurs.
\[ EC_{MF} - EC_B - GP_{MF} = \frac{(1 - \beta)(1 - 2R - 2R\rho)(\beta (2R + 2R\rho + 1) + 2R + 2R\rho - 1)}{8\beta^2(1 + \rho)^2} \] 

(26)

The sign of (26) depends on the following equation.

\[ EC_{MF} - EC_B - GP_{MF} \geq 0 \iff \beta \geq \frac{1 - 2R - 2R\rho}{2R + 2R\rho + 1}. \] 

(27)

We can easily show that \( EC_{MF} - EC_B - GP_{MF} \) tends to be positive when \( \beta \) and \( R \) are large.

In equilibrium, we can show that

\[ EC^\ast_{MF} - EC_B - GP^\ast_{MF} = \frac{\alpha(\rho + \mu)(2\mu + 2\gamma - 2\alpha - \alpha\rho + \alpha\mu)}{8(\mu + \gamma)^2(1 + \rho)^2} \] 

(28)

It is clear that the right-hand side of equation (28) is a decreasing function of \( \alpha \). If \( \alpha \) is larger than \( 2(\mu + \gamma)/(2 + \rho - \mu) \), we can say that the public support crowds out the researcher’s R&D payment. We can easily show that there exists \( \alpha \) that is larger than the limit in the range of (18).

References


