Environmental policy instruments and strategic restraint: caps versus taxes

Daniel Cardona† and Jenny De Freitas‡ and Antoni Rubí-Barceló§

July, 2019

Abstract

In the debate about options for abatement of carbon dioxide emissions, two instruments focused the discussion: a cap-and-trade system and a tax on emissions. One of the arguments used in this debate is related to the political manipulation that interest-groups may promote under each of these two regulatory systems. This study compares the performance of caps and taxes when the choice of the policy-maker is sensitive to the pressure of lobbies.

JEL classification: Q58; H23; D72.

Keywords: environmental regulation; taxes; caps; lobbying; strategic restraint.

1 Introduction

The Paris Climate Agreement re-opened the debate on how to achieve the climate stabilization objective. Regarding the regulatory tools to drive the reduction of GHG emissions, the discussion is centered between market based instruments as a cap-and-trade system and a tax on emissions, or command-and-control or direct regulation. Environmentalists and academics have provided different arguments to this discussion supporting each of the different alternatives. In this study, we

---

* Authors acknowledge financial support from the Spanish Ministerio de Economía y Competitividad and FEDER through grant ECO2015-67901-P (MINECO/FEDER)
† University of the Balearic Islands. Department of Applied Economics. Cra. Valldemossa Km 7 07122 Palma, Spain. Email: d.cardona@uib.cat
‡ Email: jenny.defreitas@uib.es
§ Email: antoni.rubi@uib.eu
aim to introduce a new aspect to consider in this discussion: the performance of two regulatory tools, a command-and-control emission cap and a tax on emissions, under the pressure of interest-groups on the regulator.

Evidence suggests that different regulatory systems are susceptible to be affected by lobbies. As Jeffrey D. Sachs noticed: ‘Cap-and-trade emissions trading [...] can be worked out with special-interest groups in back-room negotiations’ (Yale Environment 360, 2009). Likewise, there are many examples of influences from special-interest groups on environmental tax proposals. For instance, in 1993 the tax on ‘British Thermal Units’—the centerpiece of President Bill Clinton’s budget proposal—was not finally approved after the pressure from energy intensive sectors of the economy. More recently, fossil fuel companies facing an inevitable regulation supported a ‘small carbon tax (Rosenberg, 2017). Therefore, the influence of interest-groups cannot be neglected when evaluating the effectiveness of alternative regulatory settings. In this line, we model the lobbying activity of interest groups to analyze the implications of the regulatory setting (either taxes or caps) on this activity. Specifically, we consider that lobbyists (industry and environmentalist) make two decisions in the competing process: First, they decide the target-policy they will fight for; and second, they select the resources devoted to support the implementation of that policy in the struggle. We show that the target-policy choice crucially depends on whether taxes or caps are used to regulate emissions. As a consequence, the specific regulatory system affects expected emissions, lobbyists utilities and overall welfare.

This two-stage strategic interaction was analyzed in detail by Epstein and Nitzan (2004, 2007) who showed that lobbyists may have incentives to moderate their claims to reduce the opponents’ aggressiveness in the contest. That is referred as strategic restraint\(^2\). Our results show that contestants will not moderate their claims under the tax-regulation setting but they will do it under a cap-on-emissions system. These differences trigger the rest of implications of the regulatory setting. The strategic moderation of claims under the cap system (i) lowers the level of conflict, decreasing the efforts exerted in the contest, (ii) affects the equilibrium probabilities of winning the contest and consequently (iii) alters the expected level of production/emissions. As a result, the comparison between the two alternative regulatory settings shows that: First, the cap system is more effective in reducing emissions. Second, the environmentalist lobby is better off under the cap-on-emissions setting. Third, industry also prefers the cap system. Finally, the comparison between both regulatory settings in terms of

\(^2\)Other references analyzing strategic restraint are Münster (2006), Cardona & Rubí-Barceló (2016) or Cardona, De Freitas & Rubí-Barceló (2018).
welfare is ambiguous: The cap-on-emissions system favors efficiency because lowers emissions and lobbyists’ efforts but it is harmful for consumers as the expected production is lower. The exact configuration of parameters determines the most efficient regulatory tool.

Our study contributes to the strand of the literature dealing with strategic restraint in environmental contests (Damania, 1999; Graichen et al., 2001; Liston-Heyes, 2001; Dijkstra, 2004; Friese, 2013; Haan, 2016). The most akin to us is Damania (1999) as we both develop a comparative analysis between caps and taxes within this strategic framework. Unlike Damania (1999), we consider that lobbyists are ideological, as they care about the specific policy implemented and not only about the probability of winning the contest. As a consequence, there is no full convergence of policy proposals and conflict might be alleviated in our model but not eliminated, as in Damania (1999). Additionally, our simpler set up allows us to derive some conclusions on expected emissions and perform a basic welfare analysis. In a related paper MacKenzie (2017) follows a positive approach to contribute to the debate of price vs. quantity regulation in a rent-seeking game. Nevertheless, he addresses a very different question as rent seeking is over the rents generated by the type of regulation. These are contestable by all members of society in the case of a tax and only potentially contestable by firms participating in the program in a cap-and-trade system.

Another aspect of the comparison between these regulatory settings that has been analyzed by the literature relates to the influence of the market structure. For example, Requate (2006) shows how optimal taxes and caps change when the regulated firm is a monopolist instead of a price-taker. Additionally, Heuson (2010) exploits a setting similar to Weitzman (1974) to compare taxes and caps in a Cournot oligopoly. In general, the optimality of the policy instrument will depend on how it exacerbates or mitigates the distortion created by imperfect competition. This effect would also be present in our framework. Nevertheless, as our aim is to analyze the influence of the lobbying activity on the relative performance of caps and taxes in terms of emissions and social welfare, we abstract away from these considerations by analyzing an industry constituted by a monopolist (as in Graichen et al., 2001).

In an extended version of our baseline model, we give some insights on how the capacity to invest in abatement technology influences the relative performance of the two regulatory instruments. Unlike Damania (1999), this decision is modeled as an R&D investment that reduces emission intensity and it is made prior to regulation. As pointed out by previous literature, e.g. Fredriksson (1997), Damania (2001), Fredriksson & Wollscheid (2008), this investment might be used

---

to reduce the level of conflict in the contest. Therefore, it is another channel to alter the contestants’ positions, as strategic restraint. In Section 4, we analyze how expected emissions and contestants’ utilities depend on the level of abatement and how regulatory tools compare under this extended setting. The rest of the paper is organized as follows: Section 2 presents the main setting. In Section 3, we compare equilibrium outcomes under the two alternative regulatory settings. Finally, Section 5 discusses some limitations of the model and concludes.

2 Model

Consider a market with a monopolist $M$ producing an output $x$. Production generates pollutant emissions $z = \gamma x$, where $\gamma > 0$ stands for the emission rate. In turn, these emissions cause a proportional damage internalized by an environmental activist $E$. Specifically, the utility of $E$ is given by $U_E = K - \beta z$, so that $\beta > 0$ represents the marginal damage of emissions. This monopolist faces an inverse demand given by $p = 1 - x$ and it produces at a constant marginal cost, which is normalized to zero. Thus, the utility of the monopolist can be written as $U_M = (1 - x)x$. This means that, in an unregulated market, the optimal production of the monopolist is $x^u = 1/2$, yielding utilities $U_M^u = 1/4$ and $U_E^u = K - \beta \gamma / 2$, and emissions $z^u = \gamma / 2$.

A regulator may intervene in the market to correct the externalities generated by the monopolist. We compare two alternative regulatory settings that may underlie this intervention: either a tax system or a cap system that might require the producer to purchase permits to pollute. Under the first regulatory setting, the policy-maker must select the tax level per unit of emission $\hat{t} \in [0, 1/\gamma]$ whereas under the cap system he fixes a cap on emissions $\hat{z}$. For the sake of simplicity, we assume that the price of pollution permits is fixed and exogenous; $\phi \in [0, 1/\gamma]$ denotes the price of these permits per unit of emission. A priori, the most

---

4Note that our assumption on the demand form and the marginal cost are very convenient to derive results regarding expected emissions and overall welfare, but the main results of the are robust to different demand specifications as discussed in the appendix. Basically that both regulatory regimes entails different incentives to concede by lobbyists and in general there are less incentives to concede, and therefore the conflict is more severe, under the tax regulatory setting.

5In our baseline model, the monopoly cannot adapt the abatement technology to influence $\gamma$. We discuss this possibility in Section 4.

6There are other regulatory instruments that might be relevant, as an output subsidy or a refunded tax (Fischer, 2011). Nevertheless, we decided to frame our analysis on the debate between taxes and quantities.

7The case $\phi = 1/\gamma$ is excluded because this would imply no production. Note that, in a richer model with a different market structure, (e.g., an oligopoly) one might also study the possibility of an auction for these permits, as in Fischer et al. (2003). Nevertheless, this is beyond the scope of our

4
preferred tax for \( E \) is \( 1/\gamma \) and zero for the monopolist, whereas the preferred caps on emissions for \( E \) and \( M \) are 0 and \( \gamma (1 - \gamma \phi) / 2 \), respectively. Therefore, independently of the regulatory setting, there is always a conflict of interest between the monopolist and the environmental activist.

In both settings, the policy choice of the regulator is assumed to be sensitive to the claims of \( M \) and \( E \), who can exert efforts (monetary or not) to influence that choice through lobbying. As in Epstein and Nitzan (2004)\(^8\), these claims are strategically selected before the lobbying stage and do not need to coincide with the preferred policies specified above. The lobbying process is modeled through a contest (with a standard Tullock contest success function), where contestants \( M \) and \( E \) exert costly efforts \( e_M \) and \( e_E \), respectively. Then, the regulator decides to implement the target-policy (claim) of \( M \) with probability

\[
f(e_M, e_E) = \begin{cases} 
  e_M / (e_M + e_E) & \text{if } e_M + e_E > 0 \\
  1/2, & \text{otherwise}
\end{cases}
\]

and that of \( E \) with probability \( 1 - f(e_M, e_E) \).

Agents’ utilities after the resolution of the contest depend on the target-policies previously selected by them. Let \( U_i \) denote the utility that agent \( i \in \{M, E\} \) obtains if her target-policy is implemented whereas \( \overline{U}_i \) denotes the utility of agent \( i \) when the target-policy of the opponent is selected. Thus, \( D_i = U_i - \overline{U}_i \) is the stake of agent \( i \in \{M, E\} \). Hence, assuming a constant marginal cost of effort equal to 1, the expected utilities of \( M \) and \( E \) can be written as

\[
V_M = f(e_M, e_E) D_M + \overline{U}_M - e_M \\
V_E = [1 - f(e_M, e_E)] D_E + \overline{U}_E - e_E,
\]

respectively.

Once both the regulatory instrument and target-policies have been selected, contenders choose effort levels to maximize their expected utilities \( V_M \) and \( V_E \), defined above. Therefore, any interior solution satisfies the first-order conditions\(^9\)

\[
\begin{align*}
  e_E (e_M + e_E)^2 D_M - 1 &= 0 \\
  e_M (e_M + e_E)^2 D_E - 1 &= 0
\end{align*}
\]

\[
\begin{align*}
  e_M^* &= \frac{D_E D_M^2}{(D_E + D_M)^2} \\
  e_E^* &= \frac{D_M D_E^2}{(D_E + D_M)^2}
\end{align*}
\]


\(^9\)We assume that stakes are positive. It is obvious that otherwise agents would exert zero effort. Nevertheless, as we will show, in equilibrium this is always the case.
This induces an equilibrium probability
\[ f(e^*_M, e^*_E) = \frac{D_M}{D_M + D_E}, \]  
and equilibrium expected utilities
\[ V_M^* = \frac{D^3_M}{(D_E + D_M)^2} + \bar{U}_M \quad \text{and} \quad V_E^* = \frac{D^3_E}{(D_E + D_M)^2} + \bar{U}_E. \]

3 Unitary taxes vs. caps on emissions

Having derived the expected utilities of \( E \) and \( M \) as a function of both their stakes and their losing utilities, we can now study (i) how agents set their claims in each of the two alternative regulatory settings; and (ii) how these policy instruments compare to each other with respect to expected emissions, equilibrium expected utilities and aggregate welfare. We address these issues next.

3.1 Strategic restraint

In the contest environment detailed above, the target-policies settled by contestants will determine the level of conflict (stakes); consequently, their equilibrium efforts and, as a result, their equilibrium utilities. In this subsection, we focus on analyzing how the incentives to strategically moderate their claims crucially depend on the regulatory setting—a moderation implies selecting a target-policy that is closer to the preferred policy of the opponent.

Suppose first that the regulatory tool is a unitary tax on emissions \( t \). In this setting, the utility of \( M \) is \( U_M = (1 - x) - t \gamma x \), so that \( M \) would optimally produce \( x^0(t) = \frac{(1 - t \gamma)}{2} \), yielding utilities \( U_M = \frac{(1 - t \gamma)^2}{4} \) and \( U_E = K - \beta \gamma (1 - t \gamma) / 2 \).

Anticipating the optimal choice of the monopolist, agents simultaneously set their claims \( t_M \) and \( t_E \) before the contest. This would yield utilities \( U_M = (1 - \gamma_M)^2 / 4 \), \( U_M = (1 - \gamma_E)^2 / 4 \), \( U_E = K - \beta \gamma (1 - \gamma_E) / 2 \) and \( U_E = K - \beta \gamma (1 - \gamma_E) / 2 \). Therefore, stakes are \( D_M = \gamma (t_E - t_M) (2 - \gamma (t_M + t_E)) / 4 \) and \( D_E = \beta \gamma^2 (t_E - t_M) / 2 \). Using [5], we obtain the contestants’ expected utilities as a function of their target-policies as

\[ V_M(t_E, t_M) = \frac{1}{4} \gamma \left( \frac{t_E - t_M}{\gamma (t_M + t_E)} - 2 (1 + \beta \gamma) \right)^3 + \bar{U}_M \quad \text{and} \]
\[ V_E(t_E, t_M) = 2 \beta^3 \gamma^4 \left( \frac{t_E - t_M}{\gamma (t_M + t_E)} - 2 (1 + \beta \gamma) \right)^2 + \bar{U}_E. \]
Therefore, lobbyists would select their target-policies \((t_M)\) and \((t_E)\) in order to maximize the previous functions. Differentiation of \(V_E(t_E, t_M)\) yields
\[
\frac{\partial V_E(t_E, t_M)}{\partial t_E} = -2\beta^3 \gamma^4 (2\beta \gamma - 3\gamma_M + \gamma_E + 2) \frac{(\gamma(t_M + t_E) - 2(1 + \beta \gamma))^3}{(\gamma(t_M + t_E) - 2(1 + \beta \gamma))^3},
\]
which is positive for all \(t_M \leq t_E\). Consequently, the corner solution \(t_E^* = 1/\gamma\) is attained. Moreover, for any \(\gamma > 0\),
\[
\frac{\partial V_M(t_E, t_M)}{\partial t_M} \bigg|_{t_E=1/\gamma} = \frac{1}{2} \frac{(\gamma_M - 1)^2}{(\gamma(t_M + 1/\gamma) - 2(1 + \beta \gamma))^3} < 0,
\]
implying that \(t_M^* = 0\). Therefore, the next proposition follows.

**Proposition 1.** When the environmental instrument is a unitary tax on emissions, lobbyists do not strategically moderate their claims with respect to their optimal taxes.

Let consider now a cap system in which the monopolist must buy emission permits to produce and lobbyists exert effort to influence the choice of the regulator \(\hat{\gamma}\), the cap on emissions. Given the monopolist’s and environmentalist’s claims, \(z_M\) and \(z_E\), their winning and losing utilities are 
\[
U_M = \left(1 - \frac{z_M}{\gamma}\right) \frac{z_E}{\gamma} - \phi z_M, \quad U_E = K - \beta z_E \quad \text{and} \quad \overline{U}_E = K - \beta z_M.
\]

So,
\[
D_M = \frac{1}{\gamma} (z_M - z_E) \left(\gamma - \phi \gamma^2 - z_E - z_M\right) \quad \text{and} \quad D_E = \beta (z_M - z_E).
\]

Substituting these stakes into (5) we obtain the utility of the contestants as a function of the claims \(z_M\) and \(z_E\) they will lobby for in the contest stage. Thus, both \(M\) and \(E\) would select the target-policy that maximizes their respective utilities. Partial derivatives yield
\[
\frac{\partial V_M(z_E, z_M)}{\partial z_M} = - (\phi \gamma^2 - \gamma + z_M + z_E)^2 \frac{Q(z_E, z_M)}{\gamma^2 R(z_E, z_M)} \quad \text{and}
\]
\[
\frac{\partial V_E(z_E, z_M)}{\partial z_E} = \beta^3 \gamma^4 \left(\gamma - 3z_M + z_E - \phi \gamma^2 + \beta \gamma^2\right)^2 R(z_E, z_M),
\]
where
\[
Q(z_E, z_M) \equiv \phi^2 \gamma^4 + \gamma^2 + 2z_M^2 - 3\gamma z_M - \gamma z_E + 2z_M z_E - 2\phi \gamma^3 + \beta \gamma^3 - 4\beta \gamma^2 z_M + 2\beta \gamma^2 z_E - \phi \beta \gamma^4 + 3\phi \gamma^2 z_M + \phi \gamma^2 z_E,
\]
\[
R(z_E, z_M) \equiv (z_M - \gamma + z_E + \phi \gamma^2 - \beta \gamma^2)^3.
\]

In any equilibrium, we must have \(z_E < z_M\) (otherwise, there is no conflict) and \(z_M \leq \gamma (1 - \gamma \phi) / 2\), which is the monopolist’s optimal quantity of emissions. These
restrictions imply that \(\phi \gamma^2 - \gamma + z_M + z_E < 0\) and \(R(z_E, z_M) < 0\). Thus, any interior Nash equilibrium would solve:

\[
0 = Q(z_E, z_M), \\
0 = \gamma - 3z_M + z_E - \phi \gamma^2 + \beta \gamma^2
\]

yielding\(^{10}\)

\[
z_M^* = \frac{1}{2} \gamma (1 - \phi \gamma - \beta \gamma) \quad \text{and} \quad z_E^* = \frac{1}{2} \gamma (1 - \phi \gamma - 5 \beta \gamma).
\]

This interior solution is consistent if and only if

\[
z_M^* > z_E^* \geq 0 \iff \gamma \leq \frac{1}{\phi + 5 \beta}.
\]

Therefore, when \(\gamma < \frac{1}{\phi + 5 \beta}\), \(z_E^* > 0\) and \(z_M^* < (1 - \gamma \phi) \gamma / 2\). However, when \(\gamma \geq \frac{1}{\phi + 5 \beta}\), it can be easily verified that the corner solution is

\[
z_E^* = 0 \quad \text{and} \quad z_M^* = \frac{1}{4} \gamma (4 \beta \gamma - 3 \phi \gamma + 3 - H),
\]

where

\[
H \equiv \sqrt{16 \beta \gamma (\beta \gamma + 1 - \gamma \phi) + (1 - \gamma \phi)^2}.
\]

Thus, the next result follows.

**Proposition 2.** When the environmental instrument is a cap on emissions, the lobbyist \(M\) always moderates its claim with respect to its optimal cap whereas the environmental activist \(E\) only moderates when \(\gamma < \frac{1}{\phi + 5 \beta}\).

Propositions 1 and 2 show the stark difference between both regulatory settings in terms of their implications on the lobbying contest between industry and environmentalists. Unlike the cap system, when the regulatory tool is a tax on emissions, lobbyists do not have incentives to moderate their claims in the contest. Below, we detail the causes of these differences.

Epstein and Nitzan (2004) and Cardona and Rubí-Barceló (2016) show that, in contests with a Tullock success function, strategic moderation of contestants crucially depends on the functional form of their utility function with respect to policies. When this utility is concave (Epstein and Nitzan, 2004) a target-policy moderation originates a positive effect by reducing the aggressiveness of the

\(^{10}\)The system has another solution, given by \(z_M = \frac{1}{2} \gamma (1 - \phi \gamma + \beta \gamma)\), \(z_E = \frac{1}{2} \gamma (1 - \phi \gamma + \beta \gamma)\), which does not satisfy \(z_E \leq z_M \leq \frac{\gamma^{1-\phi}}{2}\).
opposing contestant that offsets the reduction of the winning utility (see also Epstein and Nitzan, 2007). This is because concavity implies that a slight concession only decreases the winning utility marginally but decreases the stake of the opponent non-negligibly, so that his equilibrium effort is reduced and that increases the winning probability of the conceding contestant. On the contrary, when contestants’ utility functions are linear (Cardona and Rubí-Barceló, 2016) or convex, moderating the target-policy decreases the utility from winning the contest non-marginally and reduces the incentives of all contestants (not just the opponents) to lobby for their own target-policy, so that the effects of moderation on the winning probability might not be favorable.

As illustrated by Figure 1, the shape of the contestants’ utility functions varies across regulatory settings. Under the cap system, the monopolist’s utility function with respect to \( z \) is concave, with the maximum at \( z = \frac{(1 - \phi \gamma)}{\gamma} \). From this point, moderating the target cap on emissions, i.e. choosing a \( z_M < \frac{(1 - \phi \gamma)}{\gamma} \), is relatively cheap: Lowering emissions does not alter the marginal cost of production and, when \( \phi > 0 \), it also involves a saving on emission permits. However, under the tax system, the monopolist’s utility function with respect to \( t \) is convex: A monopolist’s moderation of the target tax from \( t_M = 0 \) to \( t_M > 0 \) is more expensive because it increases the marginal cost of production. Consequently, the tax system discourages the monopolist to moderate its claim and the environmentalist responds accordingly. This intensifies the conflict between contestants, affecting their equilibrium utilities, the expected emissions and the social welfare, as we show next.

\[11\] These functional forms are not altered when \( \phi > 0 \).

Figure 1: \( U_E \) (green) and \( U_M \) (red) for \( K = 1/3, \beta = 1, \phi = 0 \) and \( \gamma = 2/3 \).
3.2 Comparative analysis

The different effects that the environmental policy instrument has on the incentives of lobbies to strategically moderate their claims underlie all the comparative analysis we perform next.

3.2.1 Expected emissions

Given \( \{4\} \), the monopolist’s equilibrium winning probability under the tax system is

\[
f_t^* = \frac{1}{2\beta\gamma + 1}
\]

and under the cap system is

\[
f_c^* = \begin{cases} \frac{3}{4} & \text{if } \gamma \leq \frac{1}{\phi + 5\beta} \\ 1 - \frac{4\beta\gamma}{1 + H - \gamma\phi} & \text{otherwise.} \end{cases}
\]

Using the equilibrium claims of the lobbyists derived in the previous section, we obtain the expected emissions under both the tax and the cap systems as

\[
\bar{z}_t = f_t^* x^o(t_M^*) + (1 - f_t^*) x^o(t_E^*)
\]

and

\[
\bar{z}_c = f_c^* x_M^* + (1 - f_c^*) x_E^*,
\]

respectively. Comparing the two expressions above, we obtain the following proposition. Hereinafter, all proofs are relegated to the Appendix.

**Proposition 3.** Expected emissions are larger under the tax regulatory setting.

In principle, strategic restraint generates two opposing effects on emissions: The moderation of the monopolist claim under the cap system would reduce the expected emissions whereas the environmentalist’s moderation would increase them. Additionally, the equilibrium winning probabilities would also be affected. The previous proposition reflects the result of the interaction among all these effects.

3.2.2 Lobbyists’ expected utilities

Using \( \{5\} \), we can obtain the agents’ expected utilities under the tax system,

\[
V_M(t_E^*, t_M^*) = \frac{1}{4(2\beta\gamma + 1)^2} \quad \text{and} \quad V_E(t_E^*, t_M^*) = K - \frac{1}{2} \frac{\beta\gamma}{(2\beta\gamma + 1)^2} (4\beta\gamma + 1);
\]
and under the cap setting,

\[ V_M(z^*_E, z^*_M) = \begin{cases} 
\frac{1}{8}(2 - 23\beta^2\gamma^2 + 2\gamma \phi (\gamma \phi - 2)) & \text{if } \gamma \leq \frac{1}{\phi + 5\beta} \\
\frac{H+1-\gamma\phi-4\beta\gamma}{16(H+1-\gamma\phi)^3} (H - 4\beta \gamma + 3\gamma \phi - 3) & \text{otherwise}; 
\end{cases} \]

\[ V_E(z^*_E, z^*_M) = \begin{cases} 
K - \frac{1}{8}\beta \gamma (4 - 5\beta \gamma - 4\gamma \phi) & \text{if } \gamma \leq \frac{1}{\phi + 5\beta} \\
K - \frac{3\beta \gamma (1 - \gamma \phi)^2}{5 - 5\gamma \phi + 4\beta \gamma + H} & \text{otherwise.} 
\end{cases} \]

A direct comparison between those utilities generates the following result.

**Proposition 4.** When \( \phi = 0 \), both lobbyists obtain a larger expected utility under the cap system. When \( \phi > 0 \), the environmentalist always prefers the cap setting. However, the monopolist prefers the tax system under some parameter configurations: always for a sufficiently small \( \beta \) and possibly for other parameter regions.

When pollution permits are grandfathered (\( \phi = 0 \)), the comparison between the expected utilities under the two alternative settings is unambiguous: The cap system generates better outcomes because the strategic moderation of lobbyists’ claims reduces the level of conflict and that diminishes their costly efforts in the contest stage. The environmentalist also prefers the cap system for any positive \( \phi \). However, when the pollution permits are costly (\( \phi > 0 \)) the comparative advantage of the cap system for the monopolist is diluted. Figure 2 illustrates how the preferred regulatory setting of the monopolist may vary with the parameter specification. We observe that when \( \beta \) is sufficiently small the monopolist always prefers the tax system. Moreover, taxes might also be preferred under other parameter configurations, as shown in Figures 2c and 2d.

### 3.2.3 Welfare analysis

In addition to the expected utilities of the contestants, we also include the consumer surplus and the rents received by the regulator as part of the aggregate welfare. Since lobby rents and tax revenues are transferred from the contestants to the regulator, they cancel out, and welfare is defined as the sum of gross profits, environmentalist’s utility and consumer surplus. Therefore,

\[ W(z) \equiv \left( 1 - \frac{z}{\gamma} \right) \frac{z}{\gamma} + K - \beta z + \frac{z^2}{2\gamma^2}. \]

According to this measure, the efficient quantity of emissions is \( z^{se} = \gamma (1 - \beta \gamma) \). Notice that in this setting, there are two market failures at place,
imperfect competition and a negative externality. So, depending on which market failure dominates, the unregulated market generates a quantity of emissions $z^u = \gamma/2$ that might be either below or above this efficient level. Specifically, if $\beta\gamma > 1/2$, i.e. the marginal damage of production is large, then $z^e < z^u$ because the effects of negative externalities offset the other market failure.

Following the spirit of our analysis, we next compare the expected welfare attained in both systems. Under the tax setting,

$$\tilde{W}_t = f_t^*W(z^e_M(t_M)) + (1 - f_t^*)W(z^e_E(t_E))$$

and under the cap system

$$\tilde{W}_c = f_c^*W(z^c_M) + (1 - f_c^*)W(z^c_M).$$
As we have already seen from comparing the contestants’ utilities, when emission permits are costly (\( \phi > 0 \)), the monopolist might either prefer taxes or caps, depending on the parameter specification. Alternatively, when emission permits are grandfathered (\( \phi = 0 \)), both contestants prefer the cap system. However, Figure 2 illustrates that in all those cases the consumer surplus is (weakly) higher under the tax system, as the expected production is higher in this case (see Appendix C for a general comparison). These contrasting interests imply that, generally, there would not be a regulatory setting that unambiguously leads to higher expected welfare. In order to illustrate this claim as cleanly as possible, we next focus on the grandfathering case (\( \phi = 0 \)). For this case, Figure 3 depicts the difference between expected welfares under both regulatory settings with respect to \( \alpha \equiv \beta \gamma \), a parameter that can be interpreted as the marginal environmental damage of production.

There are three factors that underlie this relative difference. First, the advantage of the cap system is that it induces lobbyists to strategically moderate their claims. This reduces the conflict and decreases the equilibrium efforts of the contestants, which benefits both of them. Second, expected emissions are lower under the cap system, as shown in Proposition 3. This benefits the environmentalist. Finally, and related to the previous point, the expected output is lower under the cap system. This hurts the monopolist and also reduces the consumer surplus. From the interaction among these three factors, it turns out that the cap system generates a higher welfare when \( \alpha \) is either sufficiently high or sufficiently low, as shown in Figure 3. The intuitive explanations for these differences depend on the parameter specification: When \( \alpha \), i.e. the marginal environmental damage of production, is high then the lower expected emissions
under the cap system are crucial for generating a higher social welfare. Instead, when $\alpha$ is sufficiently low, emissions are not so hurtful, so the key factor in favor of the cap system is the lower level of conflict it generates. For intermediate levels of $\alpha$, the tax mechanism induces a higher social welfare.

4 Endogenous abatement

Our main model considers an exogenous and fixed emission rate $\gamma$. Nevertheless, firms might have the option to choose the abatement technology that determines this rate. We next consider this possibility. In particular, we allow the monopolist to affect the emission rate $\gamma$ by investing on abatement. This investment, denoted by $a$, reduces the emission rate from $\gamma_0$ to $\gamma = \gamma_0 / (1 + a)$ at some cost $c(a)$, with $c(0) = 0$, $c'(a) > 0$ and $c''(a) \geq 0$ for all $a > 0$. Thus, $a = 0$ means no abatement whereas the complete elimination of emissions can never be attained—this might be the case if the producer buys a Selective Catalytic Reactor that reduces but does not eliminate the emissions of a series of pollutants. We assume that investments are decided before contestants select their policy claims. The timing of investment in abatement technology is crucial. As pointed out by previous literature (Damania, 2001; Fredriksson, 1997; Fredriksson & Wollscheid, 2008; Friehe, 2013), agents can strategically select the level of investment in abatement to affect the subsequent stages in the regulation process. In our model, this investment can be seen as a channel to alleviate the level of conflict in the contest, as the strategic restraint. In this setting, we next study (1) how contestants’ expected utilities and expected emissions depend on the level of abatement and (2) how the two regulatory settings under study compare to each other when abatement is endogenously decided by the monopolist.

In order to address the influence of the abatement technology, we analyze the two regulatory settings in turn. Under the tax system, we showed that there is no strategic restraint so that the equilibrium target-policies are $t_M^* = 0$ and $t_E^* = 1 / \gamma$. At first glance, it seems that investing on abatement would harm the monopolist as this would increase the tax claim of the environmental lobby. However, there is also an indirect effect to consider: reducing $\gamma$ implies that emissions would be lower in case that the monopolist target-policy is implemented. This reduces the stake of the environmentalist and, consequently, his effort in the contest. Thus, from the it can be deduced that the winning probability of the monopolist is increased, as

\footnote{Other references as Damania (1999), Dijkstra (2007) or Fischer et al. (2003) model investment decisions that are more compatible with an end-of-pipe abatement.}

\footnote{We refer to Amir et al. (2018), André et al. (2009), or Montero (2002) for a more detailed discussion about the incentives to invest on environmental quality under different regulatory settings.}
\[ \frac{\partial f_i^*}{\partial a} = \frac{\partial \left( \frac{1}{2(\beta \gamma + 1)} \right)}{\partial \gamma} \frac{\partial \gamma}{\partial a} = -\frac{2\beta}{2(\beta \gamma + 1)^2} \frac{\partial \gamma}{\partial a} < 0. \]

Moreover, investing on abatement would reduce expected emissions, since

\[ \frac{\partial z_t}{\partial a} = \frac{\partial \left( \frac{\gamma}{4(\beta \gamma + 2)} \right)}{\partial \gamma} \frac{\partial \gamma}{\partial a} = -\frac{2}{(4\beta \gamma + 2)^2} \frac{\partial \gamma}{\partial a} < 0. \]

As a consequence of all these effects, investing on abatement increases the (gross) expected utility of both the monopolist and the environmental group\(^{14}\). That is,

\[ \frac{\partial V_M (t_E^*, t_M^*)}{\partial a} = -\beta \frac{\partial \gamma}{\partial a} > 0 \text{ and } \frac{\partial V_E (t_E^*, t_M^*)}{\partial a} = -\beta \frac{6\beta \gamma + 1}{2(2\beta \gamma + 1)^3} \frac{\partial \gamma}{\partial a} > 0. \]

Under the cap regulatory setting, the effects of abatement are more ambiguous. For the sake of exposition, we only consider that \( \gamma \leq 1/(\phi + 5\beta) \). As argued previously, the investment on abatement reduces the level of conflict and this benefits the monopolist as

\[ \frac{\partial V_M (z_M^*, z_E^*)}{\partial a} = -\left\{ \frac{23}{4} \gamma^2 + \frac{1}{2} \frac{\phi (1 - \gamma)}{\phi} \right\} \frac{\partial \gamma}{\partial a} > 0. \quad (9) \]

However, the effects of \( a \) on both expected emissions and the environmentalist’s expected utility depend on the parameter configuration. When \( \gamma < 1/ (\phi + 5\beta) \) such dependence can be easily derived and it is summarized in Table\(^{15}\).

It is worth to note that, counter-intuitively, a higher investment on abatement might increase both expected emissions and the environmentalist’s expected utility. This may happen because lowering the emissions per unit of output will reduce the level of conflict in the contest, lowering the equilibrium effort exerted by the contestants in the confrontation. As a consequence, expected emissions might go up but the environmentalist can be better off because of the effort saving in the contest.

Regarding the comparison between the two regulatory settings when \( \gamma \) is endogenous, there is not a clear pattern, as illustrated by the numerical examples

\[^{14}\text{At this point we do not include the abatement costs associated to a marginal reduction of the emission rate.}\]

\[^{15}\text{For the threshold values, derivatives are zero when there is a change in the sign. Note that this table does not specify any equilibrium relationship.}\]
\[ \frac{\partial E}{\partial a} \frac{\partial z}{\partial a} \frac{\partial \gamma}{\partial a} \frac{\partial V}{\partial a} \]

\[
\begin{array}{c}
0, \frac{1}{2(\phi + 5\beta)} \\
\frac{1}{2(\phi + 5\beta)} \cdot \frac{1}{2(\phi + 2\beta)} \\
\frac{1}{2(\phi + 2\beta)} \cdot \frac{1}{2(\phi + 1.25\beta)} \\
\frac{1}{2(\phi + 1.25\beta)} \cdot \frac{1}{2(\phi + \beta)} \\
\frac{1}{2(\phi + \beta)}, \infty
\end{array}
\]

\[ - - - + \]
\[ + - - + \]
\[ + + - + \]
\[ + + + - \]

Table 1: Effects of abatement on the outcomes under the cap system.

displayed in Table 2, where the abatement cost function is assumed to be quadratic; i.e., \( c(a) = a^2 \).

\[
\begin{array}{cccccc}
(\beta, \phi, \gamma_0) & a_c & a_t & \gamma_c & \gamma_t & z_c & z_t \\
\hline
(1,1,0.25) & 0.04585 & 0.0278 & 0.0956 & 0.0973 & 0.0341 & 0.0407 \\
(1,3,0.25) & 0.0705 & 0.0278 & 0.0934 & 0.0973 & 0.0249 & 0.0407 \\
(1,0,0.2) & 0.0890 & 0.0350 & 0.1836 & 0.1932 & 0.0581 & 0.0697 \\
(0.1,0,0.2) & 0.0011 & 0.0087 & 0.1998 & 0.1983 & 0.0959 & 0.0953 \\
(0.1,1/3,2/3) & 0.0507 & 0.0221 & 0.6345 & 0.6523 & 0.2099 & 0.2885 \\
(0.2,1/3,2/3) & 0.0786 & 0.0314 & 0.6181 & 0.6463 & 0.1690 & 0.2568 \\
(0.7,1/3,0.25) & 0.0852 & 0.0341 & 0.2304 & 0.2417 & 0.0692 & 0.0903 \\
\end{array}
\]

Table 2: Numerical examples

Under both regulatory settings, investment in abatement increases with the marginal damage \( \beta \), and consequently expected emissions decrease with \( \beta \). In most cases, the endogenous rate of emissions is larger under the tax setting—since this investment can be seen as a form of strategic restraint. This is similar to the pattern obtained previously; that is, the incentives to moderate claims are higher under the cap system. Consequently, expected emissions are generally lower under the cap system (as in our baseline model, see Proposition 3). However, the contrary may also happen. For example, when \( \beta = 0.1, \phi = 0, \gamma_0 = 0 \),
and $\gamma_0 = 0.2$ we obtain $z_c = 0.0959 > 0.0953 = z_t$.

5 Final Remarks

This study compares the performance of caps and taxes under the pressure of interest-groups (environmentalist and industry) on a regulator. We consider their existence as given and assume that these groups make two choices in the lobbying process: First, they decide the policy to lobby for, and second, they exert an effort to persuade the regulator to implement that policy. We show that the regulatory setting (caps or taxes) has an influence on the first choice that triggers a series of consequences on expected emissions and social welfare. Specifically, we show that interest-groups moderate their target-policies under the cap system and this reduces the level of conflict between them. This moderation does not occur under taxes. This affects the equilibrium efforts in the lobbying stage and the winning probabilities. As a result of the interaction between all these effects: (i) the cap regulation is more effective in limiting emissions, (ii) consumers prefer taxes over caps, (iii) the environmentalist prefers caps over taxes and (iv) the industry might prefer one or the other depending on the parameter configuration (although caps are always preferred when emission permits are grandfathered). If the producer has the possibility to improve the abatement technology, we show that generally the cap system gives more incentives to invest on abatement. As a consequence, caps are generally more effective to reduce emissions, as in our baseline model.

Our model aims to focus on the presence of pressure groups as a new aspect to consider when comparing taxes and caps as regulatory environmental instruments. In order to present their influence as cleanly as possible, we opted for not including other features or variables that might also be present in the framework under analysis. One of our simplifications relates to the market structure of the industry. By considering a monopoly, we abstracted away from analyzing the effects of the regulatory tool on the competitive structure of the industry. Considering alternative market structures would have implications on both expected emissions and overall welfare that our model is not able to capture. Future contributions considering an scenario with multiple firms will need to model how the target-policy of the industry is collectively selected and how the lobbying effort of individual producers is decided and aggregated to conform the total industry effort. A second simplification, directly linked to the previous discussion, relates to the market of permits. Including it would require to model how emission permits are initially allocated and traded afterwards. Obviously, with a unique producer, our model cannot reproduce such a market. Another feature of our model that could be enriched relates to the role played by the
policy-maker. One might consider a regulator with policy preferences or other interests (benevolent or opportunist). All these other aspects fall beyond the scope of our study and are left for further research.

References


[31] Yale Environment 360 (2009). Putting a Price on Carbon: An Emissions Cap or a Tax? (May 7). Retrieved from [http://e360.yale.edu/features/putting_a_price_on_carbon_an_emissions_cap_or_a_tax](http://e360.yale.edu/features/putting_a_price_on_carbon_an_emissions_cap_or_a_tax)
A General demand

Here we explore the necessary conditions to have similar results as in Proposition 1 and 2 to the case of a more general demand function \( p = A - bX \). We assume there is a constant marginal cost of production \( \phi > 0 \). Two cases are considered.

The case with \( A, b > 0 \) and \( r = 1 \) corresponds to the linear demand case. Where \( A - \phi > 0 \) and under the tax regime \( t < (A - \phi)/\gamma \). It can be easily shown that the profit of the monopolist under the tax regime equals \( \frac{z}{\gamma} (A - b\frac{r}{\gamma} - \phi - t\gamma) \).

It can be shown that the above expression is strictly positive for all \( x_E < x_M \), \( b > 0 \), \( \beta \gamma > 0 \) and \( -1/3 < r < 0 \).

Therefore the monopolist never concedes under the tax regulatory regime, choosing \( t_M = 0 \).

B Proofs

Proof of Proposition 3

We show next that expected emissions are larger under the tax policy for all
\[ \beta, \gamma, \phi \geq 0 \] as long as \( \phi \gamma < 1 \). These emissions are:

\[ z_t = \gamma \frac{1}{2(2\beta \gamma + 1)} \] \hspace{1cm} (10)

\[ z_c = \begin{cases} \frac{\gamma}{2} (1 - \phi \gamma - 2\beta \gamma), & \text{for } \gamma \leq \frac{1}{\phi + 3 \beta} \\ \frac{\gamma}{2} (2 - \phi \gamma + 2\beta \gamma - H), & \text{otherwise}, \end{cases} \] \hspace{1cm} (11)

where

\[ H = \sqrt{16\beta \gamma (\beta \gamma + 1 - \gamma \phi) + (1 - \gamma \phi)^2} \]

Computing \( z_t - z_c \) at the interior solution (\( \gamma \leq 1 (\phi + 3 \beta) \)), we obtain:

\[ z_t - z_c^{\text{int}} = \gamma \frac{1}{2(2\beta \gamma + 1)} (1 - (1 - \phi \gamma - 2\beta \gamma)(2\beta \gamma + 1)) \] \hspace{1cm} (12)

\[ = \frac{\gamma^2 (\phi + 2\phi \beta \gamma + 4\beta^2 \gamma)}{2(1 + 2\beta \gamma)} > 0. \] \hspace{1cm} (13)

At the corner solution (\( \gamma > 1 (\phi + 3 \beta) \)) the difference in expected emissions \( z_t - z_c \) is

\[ z_t - z_c^{\text{cor}} = \gamma \frac{1}{2(2\beta \gamma + 1)} - 2(2\beta \gamma + y) + \sqrt{16\beta \gamma (\beta \gamma + y)} + y^2 \]. \hspace{1cm} (14)

where Let \( 1 - \phi \gamma = y \in [0, 1] \).

The derivative of (14) with respect to \( y \) is

\[ -2 + \left( \frac{(y + 8\beta \gamma)}{\sqrt{16\beta \gamma (\beta \gamma + y) + y^2}} \right) < 0, \]

because

\[ \sqrt{16\beta \gamma (\beta \gamma + y) + y^2} > y + 4\beta \gamma. \]

As \( z_t - z_c^{\text{cor}} \) is decreasing in \( y \), it attains its lowest value at \( y = 1 \). Evaluating (14) at \( y = 1 \) yields

\[ \frac{\gamma}{2} \left( \frac{1}{2\beta \gamma + 1} - 2(2\beta \gamma + 1) + \sqrt{16\beta \gamma (\beta \gamma + 1)} + 1 \right) \] \hspace{1cm} (15)

It can be easily checked that the above expression is strictly positive for all \( \beta \gamma > 0 \). Hence, the claim follows.

Proof of Proposition [4] We next compare the contestants’ equilibrium utilities under the two regulatory settings.
(a) **Monopolist.** We distinguish two cases:

(i) $\phi = 0$. In this case, the monopolist indirect utility at the equilibrium is always larger under the cap-on-emissions mechanism. These utilities are

\[
V_M(t^*_E, t^*_M) = \frac{1}{4} \frac{1}{(1 + 2\beta \gamma)^2} \tag{16}
\]

\[
V_M(z^*_E, z^*_M) = \begin{cases} 
\frac{1}{8} \left( 2 - 23(\beta \gamma)^2 \right), & \text{for } \beta \gamma \leq \frac{1}{5} \\
\frac{(3+4\beta \gamma-H)(1-4\beta \gamma+H)^3}{16(1+H)^2}, & \text{otherwise.} 
\end{cases} \tag{17}
\]

where,

\[
H = \sqrt{16\beta \gamma(\beta \gamma+1)+1}
\]

Let $\alpha \equiv \beta \gamma$.

For $\alpha \leq 1/5$,

\[
V_M(t^*_E, t^*_M) - V_M(z^*_E, z^*_M) = -\frac{\alpha}{8(1+2\alpha)^2} \left( -8 + 15(1/5) + 92(1/5)^2 + 92(1/5)^3 \right) \geq -\frac{73\alpha}{8(125)(1+2\alpha)^2} > 0. \tag{18}
\]

For $\alpha > 1/5$,

\[
V_M(t^*_E, t^*_M) - V_M(z^*_E, z^*_M) = -\frac{(H + 1 - 4\alpha)^3}{16(H + 1)^2} (H - 4\alpha - 3) - \frac{1}{4 (2\alpha + 1)^2} \tag{19}
\]

\[
-\frac{(H + 1 - 4\alpha)^3}{16(H + 1)^2} (H - 4\alpha - 3) = \frac{1}{4} + 2\alpha \left( \frac{H + H^2 + 1}{H + 1} - 2 - 4\alpha \right) \tag{20}
\]
\[
V_M(t_E^*, t_M^*) - V_M(z_E^*, z_M^*) = \frac{1}{4} + 2\alpha \left( \frac{H + H^2 + 1}{H + 1} - 2 - 4\alpha \right) - \frac{1}{4(2\alpha + 1)^2} \\
= -\frac{\alpha (-8H^2\alpha^2 - 8H^2\alpha - 2H^2 + 32H\alpha^3 + 40H\alpha^2 + 15H\alpha + H + 32\alpha^3 + 40\alpha^2 + 15\alpha + 1)}{(2\alpha + 1)^2(H + 1)} \\
= \frac{\alpha (8\alpha^2 + 8\alpha + 2)(16\alpha (\alpha + 1) + 1) - (15\alpha + 40\alpha^2 + 32\alpha^3 + 1)(H + 1)}{(2\alpha + 1)^2(H + 1)} \\
= \frac{25\alpha - H - 40H\alpha^2 - 32H\alpha^3 - 15H\alpha + 128\alpha^2 + 224\alpha^3 + 128\alpha^4 + 1}{(2\alpha + 1)^2(H + 1)} \\
= \frac{128\alpha^4 + 224\alpha^3 + 128\alpha^2 + 25\alpha + 1}{(2\alpha + 1)^2(H + 1)} - \frac{(32\alpha^3 + 40\alpha^2 + 15\alpha + 1)H}{(2\alpha + 1)^2(H + 1)} .
\]

The sign of the above expression is determined by the sign of the numerator.

\[
(128\alpha^4 + 224\alpha^3 + 128\alpha^2 + 25\alpha + 1) > (32\alpha^3 + 40\alpha^2 + 15\alpha + 1)H \\
(128\alpha^4 + 224\alpha^3 + 128\alpha^2 + 25\alpha + 1)^2 > (32\alpha^3 + 40\alpha^2 + 15\alpha + 1)^2(16\alpha (\alpha + 1) + 1) \\
(128\alpha^4 + 224\alpha^3 + 128\alpha^2 + 25\alpha + 1)^2 - (32\alpha^3 + 40\alpha^2 + 15\alpha + 1)^2(16\alpha (\alpha + 1) + 1) \\
= 4\alpha(44\alpha^3 + 56\alpha^2 + 20\alpha + 1) > 0.
\]

(ii) \( \phi > 0 \). The expected indirect utility under the cap-on-emissions policy becomes,

\[
V_M(z_E^*, z_M^*) = \begin{cases} 
\frac{1}{8}(2 - 23\beta^2\gamma^2 + 2\gamma\phi (\gamma\phi - 2)) & \text{if } \gamma \leq \frac{1}{\phi + 5\beta} \\
\frac{16(H + 1 - \gamma\phi - 4\beta\gamma)^3}{16(H + 1 - \gamma\phi)^3} (H - 4\beta\gamma + 3\gamma\phi - 3) & \text{otherwise}
\end{cases}
\]

where

\[
H = \sqrt{16\beta\gamma(\beta\gamma + 1 - \gamma\phi) + (1 - \gamma\phi)^2}.
\]

We showed previously that when the quota is grandfathered (\( \phi = 0 \)) both \( E \) and \( M \) prefer emission caps over taxes, that is \( V_E(z_E^*, z_M^*) \geq V_E(t_E^*, t_M^*) \) and \( V_M(z_E^*, z_M^*) \geq V_M(t_E^*, t_M^*) \). At \( \phi > 0 \), the additional cost \( \phi \zeta \) may change agents expected utilities. \( V_M(t_E^*, t_M^*) - V_M(z_E^*, z_M^*) \) becomes

\[
V_M(t_E^*, t_M^*) - V_M(z_E^*, z_M^*) = \frac{1}{4(2\beta\gamma + 1)^2} - \frac{1}{4} - \frac{\gamma}{8}(2\phi^2\gamma - 23\beta^2\gamma - 4\phi) \\
= \frac{1}{4}(2\beta\gamma + 1)^2 - \frac{1}{4} - \frac{\gamma}{8}(2\phi^2\gamma - 23\beta^2\gamma - 4\phi) \\
= \frac{1}{4}(2\beta\gamma + 1)^2 - \frac{1}{4} - \frac{\gamma}{8}(2\phi^2\gamma - 23\beta^2\gamma - 4\phi) (21)
\]
Evaluating $V_M(t^*_E, t^*_M) - V_M(z^*_E, z^*_M)$ at $\beta = 0$, we find that the preference of $M$ preference between taxes and caps is reversed. That is,

$$
(V_M(t^*_E, t^*_M) - V_M(z^*_E, z^*_M)) |_{\beta=0} = \frac{\phi \gamma}{4} (2 - \phi \gamma) > 0. \tag{22}
$$

This means that, using a continuity argument, there exists $\beta (\gamma, \phi)$ such that, for all $\beta < \beta (\gamma, \phi)$ the tax instrument is preferred by $M$ to the cap system.

(b) Environmentalist. Expected indirect utilities of $E$ are

$$
V_E(t^*_E, t^*_M) = K - \frac{\beta \gamma (1 + 4 \beta \gamma)}{2 (2 \beta \gamma + 1)^2} \tag{23}
$$

$$
V_E(z^*_E, z^*_M) = \begin{cases} 
K - \frac{1}{8} \beta \gamma (4 - 5 \beta \gamma - 4 \gamma \phi) & \text{if } \gamma \leq \frac{1}{\phi + 5 \beta} \\
K - \frac{3 \beta \gamma (1 - \gamma \phi)^2}{5 - 5 \gamma \phi + 4 \beta \gamma + H} & \text{otherwise.} 
\end{cases} \tag{24}
$$

where, $H = \sqrt{16 \beta \gamma (\beta \gamma + 1 - \gamma \phi) + (1 - \gamma \phi)^2}$.

For $\gamma \leq \frac{1}{\phi + 5 \beta}$, we obtain that

$$
K - \frac{1}{8} \beta \gamma (4 - 5 \beta \gamma - 4 \gamma \phi) - \left( K - \frac{1}{2} \beta \gamma \frac{(4 \beta \gamma + 1)}{(2 \beta \gamma + 1)^2} \right) \\
= \frac{1}{8} \beta \gamma \frac{(20 \beta^3 \gamma^2 + 16 \phi \beta^2 \gamma^2 + 4 \beta^2 \gamma \phi + 16 \phi \beta \gamma + 5 \beta + 4 \phi)}{(2 \beta \gamma + 1)^2} > 0.
$$

For $\gamma > \frac{1}{\phi + 5 \beta}$,

$$
K - \frac{3 \beta \gamma (1 - \gamma \phi)^2}{5 - 5 \gamma \phi + 4 \beta \gamma + H} - \left( K - \frac{1}{2} \beta \gamma \frac{\gamma}{(2 \beta \gamma + 1)^2} (4 \beta \gamma + 1) \right) \\
= \frac{1}{2} \beta \gamma \frac{(-24 \beta^2 \gamma^3 \phi^2 + 48 \beta^2 \gamma^3 \phi - 8 \beta^2 \gamma^2 - 24 \beta \gamma^2 \phi^2 + 28 \beta \gamma^2 \phi + 4H \beta \gamma - 6 \gamma^2 \phi^2 + 7 \gamma \phi + H - 1)}{(2 \beta \gamma + 1)^2 (H + 4 \beta \gamma - 5 \gamma \phi + 5)} \\
> \frac{\beta \gamma^2 2 \beta \gamma (2 \beta + \phi + 6 \beta \gamma \phi) + 3 \phi (2 \beta \gamma + 1)^2 (1 - \gamma \phi)}{(2 \beta \gamma + 1)^2 (H + 4 \beta \gamma - 5 \gamma \phi + 5)} > 0.
$$

where $H > 4 \beta \gamma$ and $H > (1 - \gamma \phi)$ has been used in the last step, as

$$
H = \sqrt{16 \beta \gamma (\beta \gamma + 1 - \gamma \phi) + (1 - \gamma \phi)^2} > \max \{(1 - \gamma \phi), 4 \beta \gamma\}.
$$

Therefore, expected indirect utility of the environmentalist is larger under the cap-on-emissions policy for all $\phi \in [0, 1/\gamma]$. 

\[ \square \]
C Expected Consumer Surplus

In equilibrium, the expected Consumer Surplus under different policy alternatives (when $\phi = 0$) is

$$CS(t^*_E, t^*_M) = \frac{1}{8(2\beta \gamma + 1)}, \text{ or } (25)$$

$$CS(z^*_E, z^*_M) = \begin{cases} \frac{1}{8} \left(1 - 4\beta \gamma + 7(\beta \gamma)^2\right), & \text{for } \gamma \leq \frac{1}{5} \\ \frac{1}{16} (7 + 4\beta \gamma (9 + 8\beta \gamma) - (5 + 8\beta \gamma) H) & \text{otherwise.} \end{cases} \quad (26)$$

where

$$H = \sqrt{16\beta \gamma (\beta \gamma + 1) + 1}$$

We next show that the expected consumer surplus is larger under the tax system.

Let $\alpha = \beta \gamma$. At the interior solution ($\alpha \leq 1/5$),

$$CS(t^*_E, t^*_M) - CS(z^*_E, z^*_M) = \frac{1}{8(2\alpha + 1)} \left(2 + \alpha - 14\alpha^2\right), \quad (27)$$

which is positive for all $\alpha \in (0, 1/5]$. At the corner solution ($\alpha > 1/5$),

$$CS(t^*_E, t^*_M) - CS(z^*_E, z^*_M) = \frac{1}{8(2\alpha + 1)} \left[1 - \frac{1}{2} (1 + 2\alpha) (7 + 4\alpha (9 + 8\alpha) - (5 + 8\alpha) H)\right]$$

The sign of the above expression is determined by the sign of the expression in brackets, which is increasing in $\alpha$ and attains its minimal value at $\alpha = 1/5$. Then, $CS(t^*_E, t^*_M) - CS(z^*_E, z^*_M) \geq CS(t^*_E, t^*_M) - CS(z^*_E, z^*_M) |_{\alpha=1/5} = 41/1400 > 0$. 

26