Private versus public inventory information in oligopoly

Joao Montez†  Nicolas Schutz‡

March 2019 (first draft)
Preliminary and incomplete. Please do not circulate.

Abstract

In industries characterized by production in advance, firms may or not observe how much inventories their rivals have sourced before choosing prices (respectively public and private inventories information). Public information on inventories softens oligopoly competition and increases industry profits. However, and contrary to conventional wisdom, we find that public information on inventories can increase both social welfare and consumer surplus in static oligopoly, and also make it harder for firms to collusively obtain the industry monopoly profit in a repeated oligopoly.

Keywords: Oligopoly, production in advance, horizontal information, collusion.

† We thank Simon Anderson, Patrick Rey, Yossi Spiegel for helpful comments and suggestions.
‡ Faculty of Business and Economics, University of Lausanne. Email: joao.montez@unil.ch.
§ University of Mannheim. Email: schutz@uni-mannheim.de.
1 Introduction

The theory of homogeneous-goods oligopolies lies at the foundations of industrial economics. In its simplest version, with constant marginal cost and no entry costs, if firms choose prices (and quantities adjust to clear the market), then firms price at cost and there are no distortions – the Bertrand outcome. In a similar situation, if instead firms choose quantities (and it is prices that adjust to clear the market), then firms charge positive mark-ups and, even if all inventory is sold, there is underproduction relative to the first best, with an associated deadweight loss – the Cournot outcome. In an arguably more realistic setting, firms would choose both quantities (or inventory) and prices. In that case, if inventory choices are observed before price competition then, under some assumptions on demand, the unique subgame-perfect equilibrium of the game does coincide with the Cournot outcome (Kreps and Scheinkman, 1983). Perhaps surprisingly their result holds regardless of how recoverable or sunk inventory costs are. Indeed, (7) found that even in situations where almost all the unit cost “is incurred subsequent to the realization of demand (situations that will look very Bertrand-like) will still give the Cournot outcome.’

The conventional interpretation of this result is that a Cournot analysis is appropriate when firms need to build up inventories before demand is realized (production in advance), whereas Bertrand should be preferred to study industries where firms choose prices, but only produce or source their products once consumers have made their purchase decisions (production to order) (see, e.g., Belleflamme and Peitz, 2010, pp. 66–67).

However in many industries characterized by production in advance it is the case that firms are unlikely to know how much inventories their rivals have built up when choosing their prices (inventory information is private). For example, unlike the size of a plant, retail inventories are hard to observe and, given their transient nature, historical data are insufficient to provide a reliable estimate of current inventory holdings. This concern is shared by a large operations research literature, where newsvendor settings typically assume that retailers choose prices and inventories without knowing their rivals’ choices.

In a companion paper, Montez and Schutz (2018), we characterized oligopoly market outcomes in situations where similar to the model studied by Kreps and Scheinkman (1983) inventory build-up precedes price competition but instead inventory information is private. We have found that in equilibrium, each store randomizes its price, ordering a low inventory when it sets a high price, and a high inventory when it holds a sale.
Because each store holds enough inventory to serve all its targeted demand, the aggregate inventory level often exceeds total demand, resulting in unsold inventories. As the fraction of the inventory cost that can be recovered tends to one, the equilibrium distribution of prices converges to an equilibrium of the associated Bertrand game, in which stores only choose prices and produce to order (i.e., source inventories to meet demand only after consumers made purchase decisions). The equilibrium is thus said to be Bertrand convergent. Away from that limit, our closed-form equilibrium characterization thus generalizes the Bertrand-type equilibrium to production-in-advance industries where the value of unsold inventories falls short of their acquisition value.

Our analysis thus suggests a more nuanced model selection for production-in-advance settings, which explicitly takes into account whether inventory choices are observed by rivals or not. In the former case, the Cournot outcome remains a reasonable benchmark. In the latter case, the Kreps-Scheinkman mechanism fails as a low inventory choice can no longer provide a commitment to soften price competition. Then, our Bertrand convergence result suggests that a Bertrand approach is better justified if most of the unit cost can be salvaged, and our analysis characterizes the equilibrium behavior for lower salvage values.

As whether or not inventories are observable affects industry conduct and performance in a fundamental way in markets with production in advance, the role of such information needs to be better understood. The present paper provides a first step in this direction by studying a benchmark case where firms are symmetric and goods are homogeneous.

With public inventory information, we are back in the Kreps and Scheinkman (1983) framework, and firms earn the Cournot profit. On the other hand under private information firms make zero profit, i.e., their Bertrand profits. It is therefore clear that the industry benefits from public information. The consumer surplus and social welfare effects of such information sharing are a priori unclear. We find that public information also benefits consumers if the costs are high and mostly sunk, and market demand is not too ‘curved’. Moreover it is more likely to raise social welfare if inventory costs are high and mostly sunk, and demand is more ‘curved’.

One may however worry that public information could also facilitate collusion. To investigate whether this is the case, we embed our static game into a repeated-games framework, and study collusive agreements enforced by trigger strategies. We show that, contrary to conventional wisdom, public information actually makes it harder for firms to collusively obtain the industry monopoly profit. However, inventory information always helps to sustain collusion if such information can be made available conditional
on past cooperation, i.e., if firms can credibly threaten to make such information private after a deviation occurs.

Our work contributes to multiple literatures (from which we name but a few contributions): simultaneous price-quantity setting games (Levitan and Shubik, 1978; Tasnadi, 2004, 2006), price competition with endogenous or exogenous capacity constraints (Kreps and Scheinkman, 1983; Davidson and Deneckere, 1986; Deneckere and Kovenock, 1996) and information sharing in oligopoly (Novshek and Sonnenschein, 1982; Clarke, 1983; Raith, 1996; Vives, 2002; Einy, Moreno, and Shitovitz, 2002).

2 The Model

Consider a homogeneous-products industry with two symmetric firms. Industry demand is given by the function \( D : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \). \( D \) is continuous, strictly positive, and non-increasing on \([0, p^0)\), and identically equal to zero on \([p^0, \infty)\), with \( p^0 > 0 \).

The two firms have the same constant returns-to-scale production technology. We assume that, for each unit produced, part of the unit cost is incurred even if that unit does not end up being sold (e.g., capacity or sourcing costs), while another part is incurred only if that unit is actually sold (e.g., sales, and post-sales services). In our model this is captured by the firms having a common constant cost per unit that is sold \( c_i = c \in (0, p^0) \), but for each unit that remains unsold a fraction \( \alpha \in [0, 1) \) of the cost \( c \) is recovered. The parameter \( \alpha \) thus captures the extent to which inventory costs are sunk or remain variable. For example, it has been widely documented in a variety of contexts and for a multitude of products that manufacturers often let retailers return unsold goods for a fraction of the wholesale price. Thus, the cost to a retailer of unsold inventory is generally less than the cost of sold inventory. We will later present a dynamic extension of our static game. In that context, the parameter \( \alpha \) will also reflect the extent to which unsold inventories are perishable, the time value of money, and the amount of time that elapses between periods of competition.

If firm \( i \) sets a price of \( p_i \), chooses an inventory level of \( q_i \), and ends up selling \( s_i \) units, then its profit is given by

\[
(p_i - \alpha c)s_i - (1 - \alpha)cq_i.
\]

The number of units that end up being sold, \( s_i \), is determined by the vector of prices and quantities, the demand function, and the rationing rule. If firm \( i \) sets a price below

\(^1 D \) is therefore right-continuous at \( p^0 \).
its rival’s \( (p_i < p_j) \), then it faces the entire industry demand \( D(p_i) \), and therefore sells \( s_i = \min(q_i, D(p_i)) \). If instead \( p_i > p_j \), then firm \( i \) only faces a residual demand from those consumers who were unable to buy from firm \( j \) because of a stock-out, which arises when \( q_j < D(p_j) \). That residual demand is pinned down by the rationing rule, which (like in Kreps and Scheinkman (1983)) is assumed to be efficient.\(^2\) Finally, we assume that the monopoly profit function \( p \in (c, p^0) \mapsto (p - c)D(p) \) is strictly quasi-concave.

We are interested in comparing the outcomes of two games. In the private inventory information game firms 1 and 2 compete by choosing price-quantity pairs \( (p_i, q_i) \in \mathbb{R}_+^2 \) \( (i = 1, 2) \) simultaneously. Note that this game is equivalently to one in which firms first choose quantities (or inventories, or capacities), and then prices. The equilibrium concept for such a game is simultaneous-moves price-quantity game is Nash equilibrium. In the public inventory information game firms 1 and 2 first simultaneously choose quantities, and only after observing the inventory choices do they simultaneously choose prices. The equilibrium concept for such a sequential complete information game is subgame perfect Nash equilibrium.

The latter coincides with the game studied by Kreps and Scheinkman (1983). They showed that the Cournot outcome emerges as the unique subgame-perfect equilibrium. An interesting but often overlooked aspect of that result is that it holds for every \( c \), regardless of \( \alpha \), i.e., regardless of how much of the cost is committed before price competition. Quoting from their work, “thus, situations where ‘most’ of the cost is incurred subsequent to the realization of demand (situations that will ‘look’ very Bertrand-like) will still give the same (Cournot) outcome.” Accordingly, in their setting the extent to which inventory costs are sunk or remain variable have no impact on market competition.

Their result is explained by the commitment that is made available to firms when inventory levels are observable: By choosing a low inventory level a firm shows its rival that it will not find it optimal to engage in price cutting to gain in volume. This mechanism allows firms to soften price competition but is clearly independent of \( \alpha \). This commitment effect and the implied incentives to choose low inventory levels is of course absent in our framework with unobservable inventories. In that case, as we have seen above, market outcomes change fundamentally with \( \alpha \).

The private inventory information game has been studied in a companion paper (see Montez and Schutz, 2018). Note that the Bertrand outcome \( (p_1 = p_2 = c, q_1 = q_2 = D(c)/2) \) cannot be an equilibrium: Firm \( i \) would have incentives to raise its price to

\(^2\)These rationing rules are defined formally in the Appendix. See Davidson and Deneckere (1986) for a discussion on rationing rules in pricing games with capacity constraints.
c + \varepsilon$, and supply the (strictly positive) residual demand at that price. The Cournot outcome cannot be an equilibrium either: Firm $i$ would have incentives to undercut the Cournot price and supply the entire industry demand at that slightly lower price. The same line of reasoning shows that any Nash equilibrium must involve non-degenerate mixing. Indeed we found that in the private inventory information game there exists a unique Nash equilibrium $(\sigma^*, \sigma^*)$. In that equilibrium, both firms make zero profit. Each firm draws its price from the cumulative distribution function

$$F^*(p) = \begin{cases} 0 & \text{if } p < c, \\ \frac{p-c}{p-\alpha c} & \text{if } p \in [c, p^0), \\ 1 & \text{if } p \geq p^0. \end{cases}$$

Conditional on setting the price $p_i \in [c, p^0]$, firm $i$ supplies the entire industry demand at that price $(q_i = D(p_i))^3$.

It is simple to check that the strategy profile $(\sigma^*, \sigma^*)$ is indeed a Nash equilibrium. Suppose that firm $j$ randomizes according to $\sigma^*$. If firm $i$ chooses a price $p_i \in [c, p^0)$ and a quantity $q_i \in [0, D(p_i)]$, then it makes a profit of

$$\pi_i = (p_i (1 - F^*(p_i)) - c) q_i + \alpha c F^*(p_i) q_i,$$

which is equal to zero by definition of $F^*$. Given firm $j$’s randomizing behavior, firm $i$ is therefore indifferent between all the price-quantity pairs that lie below the graph of $D$. It follows that taking a random draw from $\sigma^*$ is indeed a best response to $\sigma^*$. Intuitively, if firm $i$ raises its price, then it earns a higher margin on each unit it sells, but the probability that it ends up with (costly) unsold units increases as well. The cumulative distribution function $F^*$ is chosen in such a way that these two effects exactly offset each other.

Showing that $(\sigma^*, \sigma^*)$ is the unique equilibrium is significantly more involved. Earlier attempts at establishing equilibrium uniqueness have appeared in Levitan and Shubik (1978) and Gertner (1986). Unfortunately their proofs either omit important non-trivial steps (Levitan and Shubik, 1978) or contain several important inaccuracies (Gertner, 1986). The proof we provided addressed these shortcomings (our setting is also more

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3A remark on what we mean by uniqueness is in order here. Note that, no matter what firm $j$ does, all the pure strategies in the set $\mathbb{R}_+ \times \{0\}$ (i.e., firm $i$ produces 0 and sets some non-negative price) give rise to the same profit levels for both firms and to the same amount of surplus for consumers. To get around this source of (irrelevant) equilibrium multiplicity, we therefore define a mixed-strategy as a probability distribution over $(\mathbb{R}_+ \times \mathbb{R}_+) \cup \{(p^0, 0)\}$.
general in terms of demand structure, rationing possibilities, and in allowing for only part of the inventory costs to be sunk).

The mixed-strategy equilibrium shown above has three interesting features. First, firms randomize over prices, so the model generates price dispersion as in Varian (1980). Second, both firms set a price equal to the choke price with probability 

\[ 1 - F(p^0) = \frac{(1-\alpha)c}{\alpha c} > 0. \]

Third, with strictly positive probability, a firm ends up with unsold inventories: If firm \( i \) plays \((p_i, D(p_i))\), then, with probability \( F^*(p_i) \), firm \( j \) sets \( p_j < p_i \), and firm \( i \) does not sell its output. Thus, despite the economic environment being deterministic, the coordination failures inherent to mixed strategy equilibria result in wasted inventories in equilibrium.

Comparative statics on \( \alpha \) and \( c \) can be easily derived. An increase in \( \alpha \) (meaning that production costs become less sunk) or a decrease in \( c \) gives rise to a first-order stochastic dominance shift in the equilibrium mixed strategies towards lower prices and higher inventory levels, thereby unambiguously raising consumer surplus and social welfare. Intuitively, when the cost of holding unsold inventories becomes lower, it becomes more attractive for firm \( i \) to choose low prices and high inventory levels. In order to keep that firm indifferent between high and low prices, firm \( j \) has to become more aggressive. In the limit, as \( \alpha \) tends to 1 (resp., as \( c \) tends to 0), the cumulative distribution function of prices converges weakly to a mass point at \( p = c \) (resp. \( p = 0 \)), which is the Bertrand outcome.

Suppose that each firm is playing the equilibrium of the private inventory game, which consists in randomizing over prices according to \( F \) and supplying the entire industry demand conditional on the price that has been drawn. Then social welfare, which is equal to expected consumer gross utility minus expected production costs plus expected recovered costs, is given by:

\[
W(F) = \int_{[0,p^0]} \left( \int_p^{p^0} D(t)dt + pD(p) \right) dG(p) - 2c \int_{[0,p^0]} D(p)dF(p) + \alpha c \int_{[0,p^0]} D(p)dH(p),
\]

where \( G = 1 - (1 - F)^2 \) and \( H = F^2 \) are the cumulative distribution functions of the minimum price and the maximum price, respectively.

3 The static effects of inventory information

We next use these results discussed above to study the effects of information about inventory levels. When inventories are observed the firms make the Cournot profit,
whereas they make zero profit if inventories are not observable. It is therefore clear that the industry has collective incentives to share information. The consumer surplus and social welfare effects of such information sharing are *a priori* less clear, and will be analyzed below.

We first focus on the consumer surplus effects of information sharing. The Cournot outcome obtained under public information suffers from the standard deadweight distortion associated with a non competitive prices. The outcome under private information also suffers from this distortion, but it is unclear whether the price offered (in expectation) will be above or below the Cournot one. As seen above, consumer surplus under unobservability is given by

\[
CS_u = \int_{[0,p^0]} \left( \int_p^{p^0} D(t)dt + pD(p) \right) 2(1 - F(p))dF(p).
\]

Consumer surplus under observability (i.e., under Cournot competition) is denoted by \(CS^o\).

Recall that an increase in \(\alpha\) (for given \(c\)) or a decrease in \(c\) (for given \(\alpha\)) imply a first-order stochastic dominance shift to the left of the the equilibrium price distribution, and that the price distribution converges weakly to cost as \(\alpha \to 1\) or \(c \to 0\). On the other hand, when inventory levels are observable, the equilibrium price is the Cournot price, which is bounded away from cost. It follows that consumers are better off if firms do not share information on inventories if \(\alpha\) is sufficiently high and/or \(c\) is sufficiently low, reflecting the fact that the Bertrand limit dominates the Cournot outcome from the consumers’ point of view.

To establish a more complete comparison of the two situations, and to understand the effect of demand curvature, we impose some structure on the demand function \(D\). We consider a family of demand functions given by:

\[
D(p) = 1 - p^{\frac{1}{x}} \text{ with } x \in (0,1],
\]

which have a common choke price at 1. \(x\) is a demand curvature parameter. Demand becomes rectangular as \(x \to 0\), and linear as \(x \to 1\).

We obtain the following proposition:

**Proposition 1.** For every \((x, \alpha) \in (0,1] \times [0,1]\), there exists a cost threshold \(\tilde{c}(x, \alpha) \in \) 

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4We leave the question of whether firms have unilateral incentives to disclose inventories to future research.
Figure 1: Consumer surplus effect when $\alpha = 0$. Purple area: Consumers are worse off under information sharing.

$(0, 1)$ such that $CS^u > CS^o$ if and only if $c < \hat{c}(x, \alpha)$.

Moreover, $\hat{c}(\cdot, \cdot)$ is decreasing in $x$ and increasing in $\alpha$, and

$$\lim_{x \to 0} \hat{c}(x, \alpha) = \lim_{\alpha \to 1} \hat{c}(x, \alpha) = 1.$$ 

Proof. See Appendix.

Thus, public information on inventory levels benefits consumers when costs are high and mostly sunk, and when market demand is not too curved. Figure 1 provides a graphical illustration of Proposition 1 in the worst case for private information, namely, when $\alpha = 0$. In the purple area, consumers are worse off under information sharing. As $\alpha$ increases, the linear that separates the red and purple regions rotates counterclockwise around the point $(1, 0)$. 

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Next, we turn our attention to social welfare. In addition to the changes in consumer surplus studied above, we also need to consider profits. The Cournot outcome obtained under public information leaves firms with strictly positive profits $\pi^o$, the industry Cournot profit. Social welfare welfare under observability is therefore given by $W^o = CS^o + \pi^o$. On the other hand, there is no producer surplus under private information since firms make their Bertrand profits of zero. Thus, welfare with unobservable inventories is simply $W^u = CS^u$.

Since $CS^o > CS^u$ implies $W^o > W^u$, it follows from Proposition 1 that public information raises social welfare when $c$ and $x$ are high and $\alpha$ is low. Another case of particular interest is the limiting case where demand becomes inelastic (i.e., $x \to 0$). In that case $CS^o = 0$, since, in the Cournot equilibrium, firms charge a price equal to consumers’ valuations. However, since demand is inelastic, this means that the firms are extracting the first-best surplus. Hence, it is also the case that public information increases welfare when $x$ is sufficiently close to zero.

The following proposition characterizes the set of parameters under which public information on inventories raises (resp. lowers) social welfare:

**Proposition 2.** For every $(x, \alpha) \in (0, 1] \times [0, 1)$, there exists a cost threshold $\tilde{c}(x, \alpha) \in (0, 1)$ such that $W^u > W^o$ if and only if $c < \tilde{c}(x, \alpha)$. Moreover, $\tilde{c}(\cdot, \cdot)$ increases with $x$ and $\alpha$, $\lim_{x \to 0} \tilde{c}(x, \alpha) = 0$, and $\lim_{\alpha \to 1} \tilde{c}(x, \alpha) = 1$.

Proof. See Appendix.

Thus, public information tends to increase social welfare when the costs are high and mostly sunk, and when demand is more curved. Figure 2 provides a graphical illustration of Propositions 1 and 2 for an intermediate cost level ($c = 0.6$), when inventory costs are fully sunk ($\alpha = 0$). It plots, as a function of the curvature parameter $x$, the ratio of $W^u = CS^u$ (in blue), $W^o$ (in green), and $CS^o$ (in red) to first-best social welfare. Changes in $c$ or $\alpha$ have little impact on Cournot distortions, and therefore do not affect the green and red curves significantly. On the other hand, as $c$ increases from 0 to 1, the blue line shifts downward, thereby making it more likely that public information on inventories improves market performance.

### 4 Dynamics and collusion

A natural concern that arises is that better-informed firms may also be better able to coordinate pricing, i.e., public information on inventories may facilitate collusion as well in a repeated version of the game.
We focus on collusive agreements enforced by “trigger strategies”, whereby firms revert back to static Nash behavior as soon as a deviation takes place (see Friedman, 1971). Moreover we focus on collusion at the industry monopoly price. As a first step, we also look at collusive agreements in which, in addition to setting the monopoly price, the firms do not overaccumulate capacities, and therefore obtain the industry monopoly profit. In general collusion is possible if, for each firm $i$, the following incentive compatibility constraint is satisfied

$$\frac{1}{1-\delta}\pi^c_i > \pi^d_i + \frac{\delta}{1-\delta}\pi^n_i,$$

where $\pi^c_i$ is the collusive profit, $\pi^d_i$ the single period deviating profit, and $\pi^n_i$ the static Nash profit.

Under private information we have that $\pi^n_i = 0$. Moreover, the optimal deviation for firm $i$ is clearly to undercut the monopoly price and supply the entire industry demand. Thus, thus $\pi^d_i = \pi^m_i$. If the two firms share the market equally when they collude, which is the situation in which both firms’ incentive compatibility constraints are simultaneously satisfied for the lowest $\delta$, then each firm $i$ makes $\pi^c_i = \pi^m_i/2$. Hence, regardless of $c$ or $\alpha$, the collusive, deviation and punishment profits all coincide with those obtained under Bertrand competition. Collusion is therefore sustainable if and
only if $\delta > \delta^u \equiv 1/2$.

Under public information, most demand functions do not allow for closed-form solutions, so we focus on the case where demand is linear, i.e., $x = 1$. We first focus on collusive agreements that can support the industry monopoly profit in equilibrium, and thus, given the static Nash equivalence with Cournot discussed above, we have that

$$\pi^c_i = \frac{\pi^m}{2} = \frac{(1 - c)^2}{8}, \text{ and } \pi^n_i = \frac{\pi^C}{2} = \frac{(1 - c)^2}{9}.$$  

We still need to consider firm $i$’s optimal deviation from the collusive agreement. Clearly, deviating at the pricing stage without deviating at the quantity-setting stage cannot be profitable. Suppose that, in the quantity-setting stage, firm $j$ chooses $q^m/2$ as prescribed by the collusive agreement, and that firm $i$ deviates to some $q_i$. Firm $j$ observes $q_i$ before choosing its price $p_j$. Therefore firm $i$ must find the optimal $q_i$, anticipating a Bertrand-Edgeworth price competition game in the next stage. Fortunately, while the equilibrium of that game might involve mixed strategies in prices, we know that the firm with the larger inventory always receives its minmax revenues in equilibrium (e.g., Kreps and Scheinkman, 1983; Benoit and Krishna, 1987). Anticipating this, firm $i$’s optimal deviation maximizes its current-period profit when firm $j$’s price is equal to zero. With a linear demand and efficient rationing the solution to this program is equivalent to

$$\max_p \left[(1 - p - \frac{q^m}{2})(p - c)\right] = \frac{9(1 - c)^2}{64} = \pi^d_i$$

Thus, the optimal deviation when prices are chosen endogenously corresponds to the optimal deviation where the price would instead clear the market, as in the case of Cournot competition. All three payoffs are therefore equivalent to those under Cournot, and thus the critical discount factor under observability is the same as in Cournot, i.e., $\delta^o \equiv \frac{9}{17} > \frac{1}{2}$. We can therefore conclude that, with linear demand, information sharing makes it harder to sustain collusion that supports the industry monopoly profit in equilibrium.

Under private information, as under Bertrand competition, the critical discount factor $\delta^u = 1/2$ is independent of the price at which the firms are trying to collude. On the other hand, under public information, given the outlined equivalence to Cournot payoffs established above, collusion with profit levels below monopoly can also be sustained with discount factors below $\delta^o$. Information sharing does however, unlike Cournot, also
allow for collusion with prices and sales at the monopoly level, but profits below the monopoly level to be sustained for discount factors below $δ^o$. We now focus on this case.

Under private information a firm can only start punishing at $t + 1$ after a deviation at $t$. On the other hand, with public information it is possible for firm $j$ to punish immediately at the pricing stage of period $t$ following an inventory deviation by firm $i$. However, if in equilibrium each firm holds only $q^m/2$ units of inventories, the inventory necessary to achieve the industry monopoly profit, then the extent of this punishment will be limited. This punishment would be more dissuasive if both firms were to hold larger inventories to punish deviations (but which are intended to remain unused if no firm deviates). Suppose then that each firm $i$ is supposed in equilibrium to play $(p^m, q^*)$ in every period, with $q^* > q^m/2$, and that any choice of an inventory $q \neq q^*$ at some $t$ triggers firms to immediately revert to Markovian play, with firms playing a Bertrand-Edgeworth game $t$, and the one-shot Nash equilibrium thereafter. Then, with efficient rationing, the period $t$ payoff of a firm that deviates to some $q > q^*$ is its minmax revenue minus cost, which coincides with the Cournot best response to $q^*$. Due to strategic substitutability, the deviating inventory $q$ and profit are both decreasing in $q^*$ and $q > q^*$, provided that $q^* < q^C$. The latter condition also implies that deviations to some $q < q^*$ are not optimal.\(^5\) With linear demand, the period $t$ payoff obtained with the optimal inventory deviation is therefore equivalent to

$$\max_p [(1 - p - q^*)(p - c)] = \left(\frac{1}{2} - \frac{1}{2}c - \frac{1}{2}q^*\right)^2.$$  

In addition to deviations at the inventory stage, we must also consider deviations at the pricing stage, i.e., it must also be the case that, after having chosen $q^*$, firm $i$ does not wish to deviate by slightly undercutting its rival from the monopoly price level to sell all its inventory $q^*$. With a linear demand, its deviation payoff would then be

$$(p^m - c)q^* = \left(\frac{1}{2} - \frac{1}{2}c\right)q^*.$$  

For $q^m/2 < q^* < q^C$, the payoff obtained from a price deviation exceeds that from an inventory deviation if and only if $q^* > (2 - \sqrt{3})(1 - c)$.\(^6\) Thus, the collusive, deviation,

\(^5\)If $q^* < q^C$, if firm $i$ deviates to some $q < q^*$ then the outcome of the pricing game is such that both firms choose the market clearing price. Therefore the optimal inventory deviation corresponds to the Cournot best response to $q^*$, which due to strategic substitutability must exceed $q^*$ if $q^* \leq q^C$.

\(^6\)In the relevant range $q^m/2 = (1 - c)/4 < q^* < (1 - c)/3 = q^C$.  

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and punishment profits are given by:

\[
\pi_i^c = \frac{\pi_m}{2} - c(q^* - \frac{q_m}{2}), \pi_i^d = \max \left\{ \left( \frac{1}{2} - \frac{1}{2}c \right)q^*, \left( \frac{1}{2} - \frac{1}{2}c - \frac{1}{2}q^* \right)^2 \right\} \text{ and } \pi_i^n = \frac{\pi_C}{2}.
\]

It can be further shown that, if the cost is sufficiently low then there exist a range of values \(q^m/2 < q^* < q^C\) relaxing the collusion incentive constraint, and in those cases, outcomes where both firms charge the monopoly price and sell the monopoly quantity but hold excess inventory (which remains unsold in equilibrium) can be sustained even with discount factors below \(\delta^u\).\(^7\) Because of this enforcement cost, the firms fail to achieve the industry monopoly profit. The result is different from, but reminiscent of Benoit and Krishna (1987) — there, firms make long a one-time long-term capacity choice. The following proposition summarizes our findings discussed:

**Proposition 3.** If demand is linear, collusive outcomes where firms appropriate the industry monopoly profit are easier to sustain under private inventory information, i.e., \(\delta^u < \delta^o\). However, under public information, when \(c\) or \(\alpha\) are sufficiently low, there also exist discount factors \(\delta < \delta^u\), under which collusion at the monopoly price with profits below the monopoly level can be sustained. In those equilibria, firms accumulate excess capacity to deter deviations.

*Proof.* See Appendix. \(\Box\)

The result is interesting for two reasons. First, contrary to conventional wisdom, it shows that public information on inventory levels actually makes it harder for firms to collusively obtain the industry monopoly profit in a repeated version of the game. Second, while the waisted inventory distortion disappears when firms collude under private inventory information, Proposition 3 shows that this distortion might come back under public information. The reason is that, under public information, holding excess inventories allows firms to inflict a harsher punishment on deviators. We can therefore conclude that if collusion at the monopoly price does take place with both private and public information, then social welfare is (weakly) under private information since no excess inventories are sourced in that regime.

To counterbalance the previous result, note that public inventory information will however always help to facilitate collusion if the availability of that information can be made conditional on past cooperation, that is if firms can commit to make make inventory information public if and only if no deviation occurs. We have:

\(^7\)Specifically, as low as 0.39 for \(c = 0\).
Proposition 4. Collusion with monopoly profits is easiest to sustain (in the sense that the critical discount factor is lower) with conditional information than under private or public inventory information alone.

Proof. Immediate.

This result is intuitive: To the benefit of immediate punishment available under public information, conditionality adds the benefit of a severe punishment available under private information where profits drop to zero.

5 Conclusion

While public information on inventories unambiguously raises industry profit, its consumer surplus and social welfare effects depend on the cost structure and on demand curvature. We have also shown that, contrary to conventional wisdom, public inventory information actually makes it harder for firms to obtain the industry monopoly profit in a collusive equilibrium. However, such information does facilitate collusion if its availability can be made conditional on past cooperation.

A Appendix

To be completed.
References


