Davids and Goliath: Spatial competition of niche and general products

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Abstract

We offer a spatial competition model to study markets in which general market-wide products coexist with specific niche products, e.g. local producers competing with a large online distributor. Our rake model is an adaptation of the spokes model of [Chen and Riordan, 2007], the market-wide product is located at the top of the handle and specific products are at the edges of tines. We solve the monopolist problem and find equilibria of monopolistic competition. We show that the general product can be sold even if it is a poor substitute to niche products, and, in particular, when all consumers prefer some niche product at the same price. When the products are sufficiently valuable, the general product is overproduced by the monopolist and even more so in oligopolistic competition. So, in particular, the monopolistically competitive market realizes lower welfare than the multi-product monopolist does.

Keywords: online vs offline markets competition, rake model, spatial competition, niche vs general products, product variety.

JEL Codes: C72, D42, D43, L12, L13.

1 Introduction

Consumers are diverse in their needs and tastes. This is the main reason why we observe such a great variety of products. Some of the products clearly cater to specific tastes while some are general purpose. For instance, sporting shoes companies offer specific-activity shoes for a variety of activities, such as running, football, volleyball, tennis etc, and sell general purpose cross-trainers that can be attractive for the whole market but are not ideal for any sport in particular. Variation comes not only in product characteristics but also in advertising and

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distribution channels. Local producers and stores compete with big brands, and physical (offline) stores compete with online distributors.

In this paper we focus on such asymmetric markets, where local, specific, or niche products compete with general market-wide ones. We introduce a spatial competition model, the rake model, to study these markets. The model is an adaptation of the spokes model of 
[Chen and Riordan, 2007], the market-wide product is located at the top of the handle and specific products are at the edges of tines. The length of the handle represents how well the general product substitutes for a specific one: the longer is the handle the less attractive (a poorer substitute) the general product is. The model has several important features: each niche product competes with the general product but not with other niche products; consumers are located along the tines and differ in the degree of their preferences toward their ideal product; and entry/ exits of new specific products can be easily accommodated into the analysis.

We address the following questions. What determines the coexistence of the general and specific products in the market and what is the relative pricing of these goods? Under which conditions will specific goods push the general good out of the market and vice versa? How does the composition of products offered, consumer surplus, welfare change when we switch from monopolistic to oligopolistic environment? How and whether these markets have to be regulated?

We start with the analysis of monopolistic environment under three scenarios. First, we suppose that all specific goods are available and that the monopolist is restricted to set prices for all the products. Second, we consider settings in which some specific products are not available. Finally, we lift all restrictions on the monopolist’s choice of the selling scheme and, specifically, allow for the monopolist to offer lotteries or opaque products. The monopolist sells the general good more aggressively than it is socially optimal. It may offer the general product even if all of the consumers prefer some specific goods to the general one at the same price. In this case, the monopolist uses the general product as a “damaged good” to price discriminate consumers.

If some of the specific products are unavailable, the general good is priced even lower to capture more of the demand on those segments, and it can be offered under wider set of conditions. With fixed consumer preferences and fixed potential market segments, the marginal revenue of introducing one additional specific product increases in the number of available products. This is due to the effect that the monopolist wants to be less aggressive with pricing of the general product the more specific goods are available. Thus, the optimal number of specific products on the market for the monopolist is either zero or maximum available number of specific segments, assuming that all segments have an a priori identical potential market demand. If the monopolist is unrestricted in the choice of the selling scheme, she may introduce lotteries mixing specific and general products. There is a wider set of conditions under which the monopolist will sell the general product via lotteries relative to pure price setting schemes.

Next, we analyze monopolistic competition. First we consider an oligopolistic market in which each product is produced by one firm, and then turn to mixed competition between
one multi-product firm and several single-product firms producing niche goods. Naturally, all the products are typically priced lower under competition than by the monopolist. In competition, the general good producer will price the general product more aggressively than the monopolist and so have larger production than the monopolist and offer the general product under wider set of parameters. As a result, the overall surplus (welfare) in oligopolistic market will be even lower than in monopolistic one. Under some circumstances, in mixed competition, it may happen that the price of niche products produced by local firms will be even lower than the price of the general product; and that in those local niche markets the local product may completely push out the general one out of the local market. Since the pricing of the general good is more aggressive in the presence of local competition, the marginal revenue from new niche product is lower if offered by local competitor, which may result in a lower number of specific products available in oligopolistic markets.

1.1 Literature review

(A sketch. Literature review is incomplete.) Our paper contributes to the literature on markets with general and niche products as well as to the literature studying coexistence of local and global selling channels in the same market, such as local stores vs. outlet malls or online vs. offline shops.

There is an extensive literature studying online-offline channel substitution. Most theoretical papers use spatial competition approach based on either [Hotelling, 1929] linear city ([Chun and Kim, 2005], [Liu et al., 2006]) or [Salop, 1979] circular city ([Balasubramanian, 1998], [Viswanathan, 2005], [Jeffers and Nault, 2011]). These works assume that consumers differ in transportation costs they incur when buying from offline stores, which can include traveling costs or opportunity costs of time, but have identical disutility costs of shopping online, associated with inability to assess product fit or quality, inconvenience of possible return, a lack of immediate gratification and so on. [Goolsbee, 2001] finds evidence of the intense competition between online and conventional computer stores with high cross-price elasticity. [Forman et al., 2009] studies online-offline book sales patterns in the United States and offers empirical support to importance of both physical distance to the local retail bookstore and disutility costs of shopping online.

[Coughlan and Soberman, 2005] present a model of competition between two manufacturers located at the endpoints of the Hotelling line, who decide on whether to distribute through exclusive high service retailers located at these endpoints or through the low service outlet malls located equidistant from all consumers, which are heterogenous not only in their location on the line but also in their price and service sensitivity. They find that manufacturers choose to distribute through both channels when price sensitivity is the main dimension of consumers heterogeneity, i.e. they price discriminate consumers by encouraging those who are more price sensitive and less service sensitive to purchase from the outlet malls. Although in our model we do not specify consumers sensitivity towards price and service explicitly, differences in their location can be interpreted as heterogeneity not only in physical distance to purchasing channels but also in preferences toward service, resulting in a similar price-discriminating role of outlet malls.
As in the mentioned literature, we use spatial competition approach. In essence, we represent the competition between the general and niche products as a collection of Hotelling models, which can also be represented as an adaptation of the spokes model of [Chen and Riordan, 2007]. This is a model of nonlocalised spatial competition, which allows to vary consumers’ preferences not only with respect to offline local stores but to online channel or outlet malls as well. This is reasonable because people not only have different traveling or time opportunity costs, but also some may value highly an opportunity to examine goods physically or to consume them immediately, also some consumers can distrust online channels. We are able to study markets with an arbitrary number of segments, including segments where local stores are not present.

Next stream of relevant literature is on the markets with general purpose and specific products, which includes papers on spatial competition and multiproduct firms.

The models of spatial competition are typically based on the circle model of [Salop, 1979]. For example, [von Ungern-Sternberg, 1988] and [Hendel and de Figueiredo, 1997] consider n single-product firms which first make entry decisions and then choose transportation costs and prices. These papers endogenze transportation costs in order to let firms define a degree of product specificity: higher transportation costs mean the product is more specific and more narrowly targeted, lower transportation costs mean the product is more general. Two peculiarities of these models are that, first, they do not capture the tradeoff between a higher number of potential consumers and lower degree of satisfaction with the general product. In particular, higher transportation costs, i.e. higher specificity, do not increase fit for any consumers. Second, a change in the price of the general product does not affect directly all market segments, but only those buying two neighboring products. On the contrary, the rake model assumes that general good satisfies consumer needs worse than any of the specific goods on average, although it allows for a part of the market to be better satisfied with it. Also, there is no localized competition in our model, segments are connected to each other only through the general good, which is reasonable in some cases when, for example, local stores are located in distant districts or cities or even countries, or when specific goods are not close substitutes as in the example of sport shoes, where, for instance, football shoes are not really useful for basketball.

[Doraszelski and Draganska, 2006] study multiproduct duopoly where firms choose whether to produce the general purpose product or specific products in the market of two segments. They introduce the following trade-off: relative to the general purpose product, the specific product has an increased fit for targeted segment and increased misfit for the other segment. They found that whether firms produce only general purpose goods, only specific goods or have a full line at the equilibrium depends on the degree of fit and misfit of specific goods, the intensity of competition modeled by idiosyncratic preferences of consumers toward the firms, and the fixed cost of offering an additional product. Our model differs in our spatial competition approach, which allows us to vary consumer preferences toward general and specific goods in each segment. Although we don’t allow each product variety to be produced by more than one firm we allow an arbitrary number of segments, and a firm is not required to charge the same price for all its products. Also, the rake model is useful for analysis of
both monopolistic and competitive environment.

[Balestrieri et al., 2015a] offer a general solution to the unrestricted problem of the two-product monopolist in the Hotelling model. They show that the monopolist in addition to pricing pure products may offer lotteries of the products.

The rest of this paper is organized as follows. Section 2 describes the model setup. Section 3 finds the optimal selling scheme for the multi-product monopolist. In Section 4 we analyze the monopolistic competition between producers of the specific and general products. Section 5 concludes. All proofs are in the Appendix.

2 The Model

There are \( N + 1 \) products potentially available for production and sale at the market. Products indexed 1 to \( N \) are specific products (also local or niche), demanded only by a subset of customers (a market segment). Product 0 is the general product, a substitute for any of the specific products. Customers in market segment \( i \) differ in their preferences toward the specific product relative to the general one. Such a market can be conveniently represented as several Hotelling lines ([Hotelling, 1929]), emanating from a point representing the general product and with the specific products located at the outer edges, see Figure 1a. Consumers in each segment are distributed along the corresponding line, closer to the edge (shown as a bold segment), which captures the idea that the general product might not be a perfect substitute for any specific one.

In the general model, the lengths of rays or tines, the sizes of segments, and consumer’s distributions and preferences can differ. For the most of the paper, as we are interested in general patterns of competition in such asymmetric markets, we would be considering a model with symmetric specific goods. Such a model can be conveniently represented by a variant of the spokes model of [Chen and Riordan, 2007], see Figure 1b. Here, consumers belong to \( N \) equal market segments, represented as tines of a rake, coming from the center \( C \). A consumer’s location is determined by a pair \((i, x)\), where \( i \in \{1, \ldots, N\} \) is the market segment (or locality) she belongs to, and \( x \in [0, 1] \) is the distance from the edge, i.e. location of the specific product. Consumer \( ix = (i, x) \) demands at most one unit of either product \( i \) or the general product 0, and receives no value from any other product.

The utility consumer \( ix \) obtains from purchasing product \( j \in \{0, i\} \) at price \( p_j \) is given by:

\[
U_{ix}(0, p_0) = v - t(1 + a - x) - p_0, \\
U_{ix}(i, p_i) = v - tx - p_i,
\]

where \( v > 0 \) is the base consumption value from any variety of the product, \( t > 0 \) can be interpreted as the transportation cost the consumer has to travel to receive the respective product or as the preference cost from consuming a product with the specific characteristic by a consumer whose ideal product would have had characteristic \( ix \). Parameter \( a \geq 0 \) represents the additional loss of value (measured as an extra distance) from consuming the
general product. We suppose that consumers are uniformly distributed on $N$ tins and the mass of consumers on each tine is normalized to 1. Without loss of generality, variable costs of production are normalized to zero.

A consumer’s location is determined by a vector $(i, x)$, where $i$ is the number of the market segment and $x \in [0, 1]$ is the distance from the center. A consumer’s location is her private information. The consumer purchases at most one unit of the product and can buy either the general product or the specific product of her segment. Specific products of the other segments are not included in the choice set. Transportation costs are equal to $td$, where $t > 0$ is the unit transportation cost and $d$ is the distance equal to $a + x$ if the consumer buys the general product and $1 - x$ if she buys the specific product.

![Rake model with N specific goods](image)

Figure 1: Rake model with $N$ specific goods

### 3 Multiproduct Monopoly

As a benchmark, we first solve for the optimal strategy for the multi-product monopolist. We do so under three different scenarios. First, we suppose that all specific goods are available and that the monopolist is restricted to set prices for all the products. Second, we consider settings in which some specific products are not available. Finally, we lift all restrictions on the monopolist’s choice of the selling scheme and, specifically, allow for the monopolist to offer lotteries or opaque products.

Previewing some results, the optimal strategy of the monopolist would change depending on how good the general product is as a substitute to specific products. We will have three regions, depending on the value of parameter $a$. We say that for $0 \leq a \leq 2$ the general
product is a *good substitute* for specific products; for $2 < a \leq 3$, it is a *fair substitute*; and for $a > 3$, it is a *poor substitute*.

### 3.1 Optimal product pricing by a monopolist

Here we suppose that the monopolist is restricted to selling pure specific and general products only.

**Proposition 1.** *If the general product is a fair or poor substitute, i.e. for $a > 2$, the multiproduct monopolist sells only specific goods. The product sales and prices are as follows:*

$$
\begin{align*}
q_i^* &= 1, \quad p_i^* = v - 1, \quad \text{for } v \geq 2t, \\
q_i^* &= \frac{v}{2t}, \quad p_i^* = \frac{v}{2}, \quad \text{for } v < 2t.
\end{align*}
$$

**Proof.** Straightforward.

Thus, we have that for a high enough base consumption value, $v \geq 2t$, all consumers buy their preferred specific products and the market is *fully covered*. For a low base consumption value, $v < 2t$, there are consumers that buy nothing. The general product is not offered since its distance penalty is too high.

**Proposition 2.** *If the general product is a good substitute, i.e. for $a \leq 2$, the monopolists’ optimal pricing and product sales are as follows:*

$$
\begin{align*}
q_i^* &= \frac{1}{2} + \frac{a}{4}, \quad p_i^* = v - \frac{t}{2} - \frac{at}{4}, \quad \text{for } v \geq (1 + \frac{a}{2})t, \\
q_0^* &= N \left( \frac{1}{2} - \frac{a}{4} \right), \quad p_0^* = v - \frac{t}{2} - \frac{3at}{4}, \\
q_i^* &= \frac{v}{2t}, \quad p_i^* = \frac{v}{2}, \quad \text{for } at \leq v < (1 + \frac{a}{2})t, \\
q_0^* &= N \left( \frac{v}{2t} - \frac{a}{2} \right), \quad p_0^* = \frac{v}{2} - \frac{at}{2}, \\
q_i^* &= \frac{v}{2t}, \quad p_i^* = \frac{v}{2}, \quad q_0^* = 0, \quad p_0^* \geq 0, \quad \text{for } v < at.
\end{align*}
$$

**Proof.** The proof of this and the other propositions is in Appendix.

Thus, depending on the value of $v$, we have three separate regimes. For a high value, $v \geq (1 + \frac{a}{2})t$, the market is fully covered, the products are price so that the customers closer to the center buy the cheaper general product. For a low value, $v < at$, only the specific products are sold and only to customers close to the edges of tines. For other intermediate values, both the specific products and general products are sold, but they do not compete with each other. The market is not fully covered in this case, and there are customers who buy nothing.
3.2 Missing specific products

Here we suppose that the monopoly can produce the general product and \( n < N \) specific ones.

We consider only cases: \( v \geq t + at/2 \) for \( a \leq 2 \), and \( v > at \) for \( a > 2 \). In all other cases the monopolist can optimize independently over the specific and general product sales as described in Propositions 1 and 2.

Intuitively, when some specific products are unavailable, the monopoly will try to expand sales of the general product by lowering its price in order to serve more consumers in segments with unavailable specific goods, and the lower the number of available specific goods the more aggressive the expansion will be.

In describing the optimal strategy we use constant \( \Psi = \frac{N}{N-n} \), and denote \( q_{0+}^* \) and \( q_{0-}^* \) to be the sales of the general good on the segments with available and not available specific goods, respectively; \( q_0^* = nq_{0+}^* + (N-n)q_{0-}^* \).

**Proposition 3.** Suppose the monopolist can sell only \( n < N \) specific products and the general product is a good substitute, \( a \leq 2 \). Then, there are three regions for optimal pricing and sales.

**Region I.** When \( v < \frac{1+a}{2}t + \Psi t \), specific and general product sales are fully separated!!! not separated, just \( q_{0+}^* = q_{0-}^* \)-specific and general product sales are fully separated, the market is not fully covered.

\[
\begin{align*}
q_i^* &= \frac{N(2+a)t-(N-n)v}{2(N+n)t}, \\
q_{0+}^* &= q_{0-}^* = \frac{(N-n)v+2nt-Nat}{2(N+n)t}, \\
p_i^* &= \frac{(3N+n)v-N(2+a)t}{2(N+n)}, \\
p_0^* &= \frac{(N+3n)v-Nat-2n(1+a)}{2(N+n)}.
\end{align*}
\]

**Region II.** When \( \frac{1+a}{2}t + \Psi t \leq v < (1+a)t + \Psi t \), the segments with offered specific products are fully covered, the other segments are not.

\[
\begin{align*}
q_i^* &= \frac{1+a}{4}, \\
q_{0+}^* &= \frac{3-a}{4}, \\
q_{0-}^* &= \frac{1}{2} \left( \frac{v}{t} + 1 - a - \Psi \right), \\
p_i^* &= \frac{1}{2} (v + \Psi t), \\
p_0^* &= \frac{1}{2} (v - (1+a)t + \Psi t), \\
p_0^* &= \frac{1}{2} \left( \frac{v}{t} + 1 - a - \Psi \right).
\end{align*}
\]

**Region III.** When \( v \geq (1+a)t + \Psi t \), the market is fully covered, the general product is sold to all consumers on segments with unavailable specific products.

\[
\begin{align*}
q_i^* &= \frac{1+a}{4}, \\
q_{0+}^* &= \frac{3-a}{4}, \\
q_{0-}^* &= 1, \\
p_i^* &= v - \frac{(1+a)t}{4}, \\
p_0^* &= v - (1+a)t,
\end{align*}
\]

Figure 2 displays segment shares of the general and specific goods for three regions of \( v \), and Figure 3 shows the functional relationship of the general product segment shares \( q_{0-}^* \).
and $q_{0+}^*$ and $v$. Recall that when $n = N 1/2 - a/4$ of the market buy the general product. When $n < N$ the general product share may reach $3/4 - a/4$ in segments with both types of products and $1$ in segments with the general product only. Higher share of the general product relative to the case of full variety of specific goods is not surprising, because the monopoly tries to increase sales of the general product in segments where the consumers don’t have an alternative. As a side-effect, the part of the consumers in the other segments also switch to the general product because of the attractive price. With higher $v$ the general and specific products become less differentiated, which even more encourages consumers to switch to the general product. See in Figure 3 how the general product segment shares behave depending on $v$. In region I they increase in all segments, in regions II and III they remain constant in segments with both products available (since the prices of the general and specific products increase with the same speed). In segments without alternatives the share of the general product continues to grow in region II and in region III it is equal to one.

![Figure 2: General and specific product sales with $N = 5$ segments and $n = 3$ available specific goods, $0 \leq a \leq 2$](image)

(a) region I  
(b) region II  
(c) region III

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Figure 2: General and specific product sales with $N = 5$ segments and $n = 3$ available specific goods, $0 \leq a \leq 2$
From Figure 3 it is also clear that the share of the general product is (non-strictly) decreasing in $n$ in both types of segments. $\Psi$ is an increasing function of $n$. That means region II having fixed width $(1 + a)t/2$ moves to the right whenever $n$ increases. That makes $q^*_0$ and $q^*_0^{+}$ non-strictly decrease in $n$. Thus, removing one variety of the specific good encourages the monopoly to expand sales of the general product.

At the first glance it may look surprising that in the case of large $N$ removing one specific good from the full variety of specific products can lead to quite a dramatic increase of $1/4$ (from $1/2 - a/4$ to $3/4 - a/4$) in the share of the general product in segments with both product types. But note, that this only happens for $v$ in regions II and III, which cover only very large values of $v$ in case of $n = N - 1$ ($\Psi = N$ in this case) and large $N$. The general and specific products become very close substitutes and it is not surprising that a slight decrease in the price of one of the products leads to a significant increase in its sales.

Figure 4 displays behavior of prices depending on $v$ in comparison with the case of $n = N$. As expected, the monopoly sets a lower price for the general product in case of $n < N$ in order to increase sales in the segments without specific products. More interesting is what happens with the price of specific goods. On the one hand, the monopoly wants to retain consumers buying specific products, because they are willing to pay higher price for them, and therefore should decrease price $p_i$. On the other hand, the monopoly wants to compensate profit losses from the lower price of the general product and set higher prices for specific products. It turns out that the second effect dominates for lower $v$ and the monopoly sets the price $p_i$ that is higher than that when $n = N$. For higher $v$ the first effect dominates and the monopoly lowers the price of specific goods relative to the case of $n = N$.

Another interesting observation is that indifferent consumers in segments where both products are available do not necessarily receive zero utility, as it was in the case with the full variety of specific products. While for $v$ in region I their utility is zero, in regions II and
III they obtain strictly positive utility. This is due to the two effects mentioned earlier: the monopoly sets lower price $p_0$ but doesn’t increase $p_i$ by the same amount or even decrease it to retain customers buying specific products. Thus, the indifferent customers are left with positive surplus but more consumers buy more expensive products.

How does the monopoly adjust prices when the number of available specific products changes? This can be studied from Figure 4. For $v$ in region I the price of the general product $p_0$ is increasing and the price of specific products $p_i$ is decreasing in $n$. In region II both $p_0$ and $p_i$ are increasing in $n$. In region III $p_0$ and $p_i$ do not react to changes in $n$ unless increase in $n$ moves $v$ to region II, in which case both $p_i$ and $p_0$ increase.

Thus, introducing a new specific product generally leads to increase in the price of the general product or doesn’t affect it for higher $v$. By raising $p_0$ the monopoly encourages the customers to switch to more expensive specific products. For lower $v$ the prices of specific products decrease when a new specific product is introduced, encouraging even more consumers to switch to specific products. $p_i$ increases in $n$ for intermediate $v$, when the monopoly prefers to make more profits from each switching customer rather than to increase their number aggressively, and $p_i$ doesn’t change for very large $v$, when the general and specific products are close substitutes.

**Proposition 4.** Suppose the monopolist can sell only $n < N$ specific products and the general product is a poor substitute, $a \geq 3$. Then, there are three regions for optimal pricing and
sales.

Region I. When \( v \leq (a - 1)t + \Psi t \), the general product is not offered. Segments without specific products are not served at all. For each available specific product \( i \), the whole segment is served, \( q^*_i = 1 \), \( p^*_i = v - t \).

Region II. When \( (a - 1)t + \Psi t < v < (a + 1)t + \Psi t \), the monopoly sells the general product only in segments in which specific products are unavailable and only to some consumers.

\[
\begin{align*}
q^*_i & = 1, & p^*_i & = \frac{1}{2} (v + (a - 3)t + \Psi t), \\
q^*_0^+ & = 0, & p^*_0^+ & = \frac{1}{2} (v - (1 + a)t + \Psi t), \\
q^*_0^- & = \frac{1}{2} \left( \frac{v}{t} + 1 - a - \Psi \right).
\end{align*}
\]

Region III. When \( v \geq (a + 1)t + \Psi t \), the monopoly sells the general product in segments without specific products and only specific products in segments where they are available. The market is fully covered. The prices are:

\[
p^*_0 = v - (a + 1)t, \quad p^*_i = v - t.
\]

Recall that in the case of the full variety of specific products the monopoly does not produce the general good when \( a \geq 2 \). In the present case when \( a \geq 3 \) the general product is so poor substitute that the consumers may buy it only if they don’t have an alternative choice. Figure 5 shows the shares of the general and specific goods for three regions of \( v \). When the monopoly produces the general good it has to set a very low price for it but it doesn’t want consumers from segments with specific goods to switch from buying more expensive products, therefore it has to reduce prices of specific products as well and lose some revenues from sales of specific goods. When \( v \) is small (region I) the losses are too high relative to the revenues from the sales of the general product and the monopoly decides not to sell the general product. Higher \( v \) reduces product differentiation and it becomes profitable to introduce the general product but only in segments without specific products. For very high \( v \) (region III) the monopoly serves the whole market.
Figure 5: General and specific product shares with $N = 5$ segments and $n = 3$ available specific goods when $a \geq 3$

$2 \leq a \leq 3$ (the general product is a fair substitute). Proposition 5 summarizes the optimal strategy of the monopoly for this case.

**Proposition 5.** In the market with $N$ segments if the monopoly is able to produce the general product and $n < N$ specific products and the general product is a fair substitute the optimal strategy is:

Region I. For $a \leq v \leq 2t + (a-2)\Psi t$ the monopoly does not sell the general product in any segment. Segments without specific products are not served at all. Sales of specific products in segments where they are available are equal to one. The price is $p_i^* = v - t$.

The market is not fully covered.

Region II. For $2t + (a-2)\Psi t < v < \frac{(1+a)t}{2} + \Psi t$ the prices and the market shares are the same as in Proposition 3 case 1.

The market is not fully covered.

Region III. For $\frac{(1+a)t}{2} + \Psi \leq v < (1+a)t + \Psi$ the prices and the market shares are the same as in Proposition 3 case 2.

The market is not fully covered.

Region IV. For $v \geq (1+a)t + \Psi$ the prices and the market shares are the same as in Proposition 3 case 3.

The market is fully covered.

This is an intermediate case when the monopoly treats the general product as either “good” or “poor” substitute depending on $v$. When $v$ is low (region I) this is the case of a poor substitute - the monopoly does not sell the general product and the market structure is the same as in Figure 5(a). For $v$ in regions II, III and IV the market structure and the behavior of the shares of the general product and prices are similar to those depicted in Figures 2, 3 and 4, meaning that in these cases the general product is a good substitute.
3.3 Selling Lotteries

Here, we allow the monopolist to suggest any selling scheme to consumers. [Balestrieri et al., 2015a] show that in a horizontally differentiated market with two products located at the edges of the Hotelling line, for a high enough base consumption value \( v \) the monopolist would offer lotteries of the base products in addition to products themselves. For linear transportation costs, a specific lottery will be offered, namely selling each of the two goods with equal probabilities at a lower price.

We follow [Balestrieri et al., 2015b] approach to derive the monopoly’s optimal selling scheme. We are looking for a direct mechanism, where each consumer finds it optimal to report truthfully her location \( ix \).

Any direct mechanism can be represented as \( \mu = (q_0, q_i, p) \), where \( q_0(ix) \) and \( q_i(ix) \) are probabilities of obtaining the general good and specific good \( i \), respectively,\(^1\) and \( p(ix) \) is the required payment for the consumer reporting location \( ix \). For truth-telling to be an equilibrium consumers’ individual rationality (IR) and incentive compatibility (IC) constraints have to be satisfied.

In addition, since consumers have unit demand, feasible allocations are such that

\[
\forall i = 1 \ldots N, \; \forall x \in [0, 1], \; q_0(ix) \geq 0, \; q_i(ix) \geq 0, \; q_0(ix) + q_i(ix) \leq 1. \tag{F}
\]

Probabilities \( q_0(ix) \) and \( q_i(ix) \) can be viewed as parameters of lottery \( l(ix) \) with the two goods as prizes. Lottery \( l(ix) \) is sure prize if \( q_0(ix) + q_i(ix) = 1 \).

Denote \( U(iy|ix) \) the utility of a consumer located at \( ix \) reporting \( iy \) and \( U(ix) = U(ix|ix) \). Then,

\[
U(iy|ix) = q_i(iy)(v - tx) + q_0(iy))(v - t(1 + a - x)) - p(iy).
\]

Individual rationality condition requires that the utility of reporting the true type is higher than buying nothing:

\[
\forall i = 1 \ldots N, \; \forall x \in [0, 1], \; U(ix) \geq 0. \tag{IR}
\]

Incentive compatibility condition requires that reporting the truth brings the (non-strictly) highest utility given that all other consumers also report the truth.

\[
\forall i = 1 \ldots N, \; \forall x, y \in [0, 1], \; U(ix) \geq U(iy|ix). \tag{IC}
\]

Given that all the market segments are identical denote

\[
U(ix) = U(x), \; q_0(ix) = q_0(x), \; q_i(ix) = q_i(x), \; p(ix) = p(x).
\]

The monopoly’s problem is then:

\[
\max_{q_0,q_i,p} \int_0^1 p(x)dx. \tag{MP}
\]

\(^1\)Without loss of generality we can omit specification of \( q_j(ix) \), immediately assuming that good \( j \) would never be allocated to consumer who reports that she is on tine \( i \).
To solve the revenue maximization problem we proceed as in [Balestrieri et al., 2015b], where the optimal mechanism is derived for the monopolist in the standard Hotelling setting with two goods located at the endpoints of segment [0,1]. Essentially, our setting is different in that the linear city has an unpopulated segment of the length $a$ and, hence, consumers are not distributed symmetrically between the two goods at the endpoints.

We consider three regions for $a$ separately: $a < 1, 1 < a < 3$ and $a > 3$.

**Proposition 6.** For $a < 1$ the optimal selling mechanism is

1. for $0 < v < at$ only specific products are sold with prices and sales

   $$ p_i^* = \frac{v}{2}, \quad q_i^* = \frac{v}{2t}. $$

2. for $at < v < \frac{t+at}{2}$ the general good and specific goods are sold with prices and sales

   $$ p_0^* = \frac{v-at}{2}, \quad p_i^* = \frac{v}{2}, $$

   $$ q_0^* = \frac{v}{2t} - \frac{a}{2}, \quad q_i^* = \frac{v}{2t}. $$

3. for $v > \frac{t+at}{2}$ the general good, specific goods and lottery $l = \left(\frac{1}{2}, \frac{1}{2}\right)$ are sold with prices and sales

   $$ p_0^* = v - \frac{(1+3a)t}{4}, \quad p_i^* = v - \frac{(1+a)t}{4}, \quad p_l^* = v - \frac{(1+a)t}{2} $$

   $$ q_0^* = \frac{1-a}{4} (x > x_2), \quad q_i^* = \frac{1+a}{4} (x < x_1), \quad q_l^* = \frac{1}{2} (x \in (x_1, x_2)). $$

Note that lottery $l = \left(\frac{1}{2}, \frac{1}{2}\right)$ brings the same utility for all the consumers in the market. Given that, for $v > \frac{t+at}{2}$ the prices are determined by the IR constraint for consumers $ix$ and by IC constraint for the threshold consumers located at $x_1$ and $x_2$. For $v < \frac{t+at}{2}$ the prices and the segment shares are taken from Proposition 3.

Next we consider case $1 < a < 3$. Note that in this case $x_2 > 1$, so the monopoly sells the general good only through the lottery.

**Proposition 7.** For $1 < a < 3$ the optimal selling mechanism is

1. for $0 < v < \frac{t+at}{2}$ only specific products are sold with prices and segment shares

   $$ p_i^* = \frac{v}{2}, \quad q_i^* = \frac{v}{2t}. $$
2. for \( v > \frac{t+at}{2} \) specific goods and lottery \( l = (\frac{1}{2}, \frac{1}{2}) \) are sold with prices

\[
P_i^* = v - \frac{(1 + a)t}{4}, \quad p_l^* = v - \frac{(1 + a)t}{2}
\]

and segment shares

\[
q_i^* = \frac{1 + a}{4}, \quad q_l^* = \frac{3 - a}{4}.
\]

Finally, when \( a > 3 \) we have \( x_1 > 1 \) which implies that only pure specific goods are sold in the market. The general good is a too bad substitute for specific goods and cannot be sold even through lotteries.

The natural question that arises is how does the market change relative to the case where the monopoly is not able to sell lotteries? First, we see that for \( a < 3 \) the market becomes fully covered for lower \( v \) when selling lotteries is possible \((\frac{t+at}{2} < t + \frac{at}{2} \text{ for any } a > 0 \text{ and } \frac{t+at}{2} < at \text{ for } a > 1)\).

Second, for \( a < 3 \) the monopoly charges higher prices for pure goods, both general and specific, when they are supplemented with lotteries. Thus, the monopoly enjoys higher profits by charging higher prices from consumers located closer to specific goods and to the general good (when \( a < 1 \)) and offering the lottery, which price is lower than that for pure goods, to those located in the middle.

Recall that without lotteries the general good is not produced for \( a > 2 \). When lotteries are possible, the pure general good is not sold when \( a > 1 \) but is produced as a lottery prize for \( a < 3 \). Previously we noticed that the general good, being produced even when the whole market prefers specific goods \((1 < a < 2)\), plays a role of a “damaged” good. Through the lottery it stays in the market even for higher \( a \), meaning that the lottery now becomes a “damaged” good. Indeed, for \( a > 1 \) consumers have a choice: to buy a specific good that they value higher than the general good for a high price or to buy a cheaper lottery where they get the same specific good with probability 1/2 and the less valuable general good otherwise. Essentially, the lottery represents a “damaged in expectation” specific good.

While selling lotteries brings more profits to the monopoly, the effect on consumer surplus is clearly negative. Although market coverage increases for low \( v \), those who buy the lottery gain zero utility in expectation and those who buy pure goods pay higher prices.

3.4 Fixed Costs and Optimal Variety

to be written

4 Competition

In this section we study peculiarities of the market in competitive environment and make comparative analysis with the monopolistic one. First, we assume that each good is produced by a separate firm and then proceed with competition of the multiproduct firm producing the general good and several specific goods and single-product firms selling specific goods in separate segments.
4.1 Single-product Firms

Assume that there are \( N + 1 \) firms: firm 0 produces the general product and firm \( i \), where \( i = 1, 2, \ldots, N \), produces specific product \( i \). Fixed and variable costs are zero for all the firms.

We also assume that a firm stays in the market even if it produces nothing. That means it sets some price for its product and is ready to sell it whenever the competitors raise their prices sufficiently high.

Given that all market segments are identical we focus on Bertrand-Nash (pure strategy) equilibria in which firm 0 sets the price \( p^*_0 \) and all firms producing specific products set identical prices \( p^*_i \) with identical sales \( q^*_i \) and earn profits \( \pi^*_i = p^*_i q^*_i \).

Note that when \( v \leq t + at/2 \) and \( a < 2 \) or when \( v \leq at \) and \( a \geq 2 \) the firms are local monopolists and do not compete with each other. For the case \( v \leq t + at/2 \) and \( a < 2 \) the prices and segment shares can be found in Proposition 3; when \( v \leq at \) and \( a \geq 2 \) the general product is not produced, the prices and segment shares of specific products are as stated in Proposition 1. So, I will focus on the rest of the cases and consider separately two regions of \( a \): \( 0 \leq a < 3 \) and \( a \geq 3 \), and compare the results to those of the monopoly producing the full variety of the products.

\( 0 \leq a < 3 \). Proposition 8 describes competitive equilibria in this case.

**Proposition 8.** Assume that \( v > t + at/2 \) if \( a < 2 \) and \( v > at \) if \( 2 \leq a < 3 \). Then:

1. if \( v < \frac{3t}{2} + \frac{at}{2} \) then there is a **continuum of equilibria**, in which the price of specific products satisfies the following conditions:

   (a) if \( 0 \leq a \leq 2 \) then:
   
   \[
   p^*_i \in \left[ \frac{v}{2} \frac{3v}{2} - t - \frac{at}{2} \right] \quad \text{for} \quad t + \frac{at}{2} < v \leq \frac{6t}{5} + \frac{2at}{5};
   \]
   
   \[
   p^*_i \in \left[ \frac{4v}{3} - t - \frac{at}{3}; \frac{3v}{2} - t - \frac{at}{2} \right] \quad \text{for} \quad \frac{6t}{5} + \frac{2at}{5} < v \leq \frac{6t}{5} + \frac{3at}{5};
   \]
   
   \[
   p^*_i \in \left[ \frac{4v}{3} - t - \frac{at}{3}; \frac{2v}{3} \right] \quad \text{for} \quad \frac{6t}{5} + \frac{3at}{5} < v < \frac{3t}{2} + \frac{at}{2}.
   \]

   (b) if \( 2 < a < 3 \) then:
   
   \[
   p^*_i \in \left[ \frac{4v}{3} - t - \frac{at}{3}; \frac{3v}{2} - t - \frac{at}{2} \right] \quad \text{for} \quad at < v \leq \frac{6t}{5} + \frac{3at}{5};
   \]
   
   \[
   p^*_i \in \left[ \frac{4v}{3} - t - \frac{at}{3}; \frac{2v}{3} \right] \quad \text{for} \quad \frac{6t}{5} + \frac{3at}{5} < v < \frac{3t}{2} + \frac{at}{2}.
   \]
The price of the general product is equal to:

\[ p_0^* = 2v - (1 + a)t - p_i^*, \]

and the sales are:

\[ q_0^* = N \left( 1 - \frac{v - p_i^*}{t} \right), \quad q_i^* = \frac{v - p_i^*}{t}. \]

2. if \( v \geq \frac{3t}{2} + \frac{at}{2} \) then there is a unique equilibrium in which the prices are:

\[ p_0^* = t - \frac{at}{3}, \quad p_i^* = t + \frac{at}{3}, \]

and the sales are:

\[ q_0^* = N \left( \frac{1}{2} - \frac{a}{6} \right), \quad q_i^* = \frac{1}{2} + \frac{a}{6}. \]

First, recall from Proposition 2 that the monopoly sells the general product only if it is a “good” substitute for specific products, i.e. only when \( a < 2 \). In case of competition the general product is produced even if it is a “fair” substitute, i.e. when \( a < 3 \). That means, transition from monopoly to competitive market may lead to introduction of a new product - a general good, which is a “fair” substitute for specific products.

Second, consider the case when \( v \geq \frac{3t}{2} + \frac{at}{2} \). In this case there is a continuum of equilibria, in which consumers indifferent between buying general or specific products receive zero utility, as they did in the case of monopoly. When \( a < 2 \) segment shares of the general product can be higher as well as lower than in case of monopoly. When the share of the general good is higher (lower) than in the case of monopoly then the price of the general good is lower (higher) and the price of specific goods is higher (lower) than those set by the monopoly.

When \( 2 \leq a < 3 \) the general good is produced in the market and the price of specific products is higher than the monopolistic price. In other words, for these values of \( a \) the competitive market implements price discrimination by introducing a “damaged” good, although the monopoly finds it optimal not to price-discriminate.

It is also interesting that when \( a < 1/3 \), i.e. when the general product is a very good substitute for specific goods, there are equilibria in which the price of the general product is higher than the price of specific goods. This is quite unusual because consumers’ valuation of specific goods is higher than that of the general good on average and in the monopoly case the price of the general product is always lower than the price of specific goods.

When \( v > \frac{3t}{2} + \frac{at}{2} \) there is a unique equilibrium that is quite intuitive: the prices for all goods in the market are lower than corresponding monopolistic prices. Also, there is more general product in the market. Recall, that in the monopolistic outcome segment shares of the general product are equal to \( \frac{1}{2} - \frac{a}{4} \) while in the competitive environment they are equal to \( \frac{1}{2} - \frac{a}{6} \).

\( a \geq 3 \). Proposition 9 describes the competitive equilibrium in this case.
Proposition 9. Assume that \( v > at \) and \( a \geq 3 \). Then there is a unique equilibrium in which the prices are:

\[
p_0^* = 0, \quad p_i^* = (a - 1)t
\]

and the sales are:

\[
q_0^* = 0, \quad q_i^* = 1.
\]

The general good is not produced.

When \( a \geq 3 \), i.e. the general good is a poor substitute, competitive and monopolistic outcomes are similar in that there is no general product in the market. But, firms producing specific products have to keep prices lower than the monopoly in order to prevent entry of the firm that produces the general good. This situation can be slightly modified by introducing small entry costs. Then, firm 0, anticipating zero profits, does not enter the market and firms producing specific products are able to set monopolistic prices \( p_i^{*m} = v - t \) and monopolistic and competitive prices and product shares become identical.

4.2 Multiproduct and Single-product Firms

Now assume that firm 0 is a multiproduct firm, which produces the general good and \( n \) specific goods, where \( 0 < n < N \). There are also \( N - n \) single-product firms, producing the rest of specific goods. Without loss of generality, we can rearrange specific goods and denote by \( i = 1, \ldots, n \) specific goods produced by firm 0 and by \( j = n + 1, \ldots, N \) those produced by single-product firms.

We will be looking for a unique Bertrand-Nash equilibrium, in which firm 0 sets prices \( p_0^* \) for the general good and \( p_i^* \) for the specific goods, single-product firms set identical prices \( p_j^* \) for their products. We again focus on the situation when the market is fully covered, i.e. \( v \geq t + at/2 \) if \( a < 2 \) and \( v \geq at \) if \( a \geq 2 \).

First, assume that the shares of the general and specific goods are positive in each segment. Then the location of the indifferent consumer in segment \( i \), served entirely by firm 0 is:

\[
\bar{x}_i = \frac{1 + a}{2} - \frac{p_i - p_0}{2t},
\]

and the location of the indifferent consumer in segment \( j \), where firm 0 and firm \( j \) compete, is:

\[
\bar{x}_j = \frac{1 + a}{2} - \frac{p_j - p_0}{2t}.
\]

Consider firm 0. The demand for the general good is equal to:

\[
q_{00} = n(1 - \bar{x}_i) + (N - n)(1 - \bar{x}_j) = \frac{1 - a}{2} + \frac{1}{2t} (np_i + (N - n)p_j - p_0).
\]

The demand for specific good \( i \) produced by firm 0 is:

\[
q_{0i} = \bar{x}_i = \frac{1 + a}{2} - \frac{p_i - p_0}{2t}.
\]
Similarly, firm j’s demand is:

\[ q_{0j} = \bar{x}_j = \frac{1 + a}{2} - \frac{p_j - p_0}{2t}. \]

Thus, profit functions for firms 0 and j are:

\[
\pi_0 = p_0q_{00} + np_0q_{0i} = \frac{1}{2t}((1 - a)Np_0t + (1 + a)np_it + (N - n)p_0p_j + 2np_0p_i - Np_0^2 - np_i^2),
\]
\[
\pi_j = p_jq_j = \frac{1}{2t}p_j((1 + a)t - p_j + p_0).
\]

**Proposition 10.** Let \( v \geq \frac{(21N - 5nt)}{12(N - n)} + \frac{5at}{12} \), \( n/N < 3/5 \) and \( a < 3 - \frac{2n}{N - n} \). Then there is a unique equilibrium in which the prices are:

\[
p_0^* = \frac{(3N + n)t}{3(N - n)} - \frac{at}{3}, \quad p_1^* = \frac{(9N - n)t}{6(N - n)} + \frac{at}{6}, \quad p_j^* = \frac{(3N - n)t}{3(N - n)} + \frac{at}{3}.
\]

Sales of the general good in segments served by firm 0 \((q_{0i}^*)\) and in competitive segments \((q_{0j}^*)\) are:

\[
q_{0i}^* = n\frac{3 - a}{4}, \quad q_{0j}^* = \frac{3N - 5n}{6} - \frac{a}{6}(N - n)
\]

and sales of specific goods in segments served by firm 0 \((q_i^*)\) and in competitive segments \((q_j^*)\) are:

\[
q_i^* = \frac{1 + a}{4}, \quad s_j^* = \frac{3N - n}{6(N - n)} + \frac{a}{6}.
\]

Three interesting observations can be made from the described equilibrium.

First, specific goods are priced lower in competitive segments, than in the segments entirely served by firm 0. This is quite intuitive since while firm 0 maximizes profits from selling both general and specific goods, the other firms earn their revenues only from the single specific product. Single-product firms compete with firm 0 trying to increase sales by lowering their prices.

Second, firm 0 prices the general good lower than its specific goods, i.e. \( p_1^* > p_0^* \). This is consistent with the monopolistic outcomes, in which the price of the general good was also lower than the prices of specific goods because the general good is less valued by consumers on average.

Third, in competitive segments specific goods can be priced lower than the general good. This is again the consequence of competition. Direct comparison of expressions for \( p_0^* \) and \( p_j^* \) gives the following condition for this to happen: \( a < n/(N - n) \), i.e when \( a \) is sufficiently small, which means general and specific goods are sufficiently close substitutes. Note that \( p_0^* \) positively depends on \( n \), i.e. the higher the number of specific goods produced by firm 0, the less it is engaged in competition and the less the incentives to lower the price of the general good to increase shares in competitive segments. Thus, for larger \( n \) the upper bound
for values of $a$, for which specific goods in competitive segments are priced lower than the general good, will be higher.

Note that the described equilibrium holds only when the fraction of specific goods produced by firm 0 is not very high, $n/N < 3/5$. What happens when firm 0 produces more than $3/5$ of the overall assortment of specific goods? In this case firm 0 does not find it profitable to compete with single-product firms by lowering the price for the general good since it loses more money in non-competitive segments, that gains in competitive trying to increase segment shares. The same situation occurs when the fraction of specific goods produced by firm 0 is less than $3/5$, but $a \geq 3 - \frac{2n}{N-n}$. The following proposition summarizes all the cases when firm 0 finds it unprofitable to compete.

**Proposition 11.** If $v \geq \max \left[ \frac{21N-5n}{12(N-n)} + \frac{5at}{12}, at \right]$ and $a \geq \max \left[ 3 - \frac{2n}{N-n}, 0 \right]$ then firm 0 acts like a monopoly and sets prices as stated in Proposition 1 or 2. The general good is not sold in competitive segments and single-product firms set their prices such that consumers at the center in the corresponding segments are indifferent between buying specific goods and the general good (if produced by firm 0) or not buying at all (if the general good is not produced in the market). More specifically:

1. if $n/N \leq 1/3$ and $a > 3 - \frac{2n}{N-n}$ then firm 0 does not produce the general good and $p^*_j = v - t$.

2. if $1/3 < n/N < 3/5$ then for $3 - \frac{2n}{N-n} \leq a < 2$ firm 0 produces the general good and $p^*_j = v - 3t/2 + at/4$ and for $a \geq 2$ firm 0 does not produce the general good and $p^*_j = v - t$.

3. if $n/N \geq 3/5$ then for $a < 2$ firm 0 produces the general good and $p^*_j = v - 3t/2 + at/4$ and for $a \geq 2$ firm 0 does not produce the general good and $p^*_j = v - t$.

As before, if firm 0 produces the general good the price of specific good is lower in competitive segments than in non-competitive. Specific goods in competitive segments can be priced lower than the general good if $a$ is sufficiently low ($a < 1$) and $n/N > 1/2$.

### 5 Conclusion

to be written

### 6 Extensions

#### 6.1 Social Welfare

- Under monopolistic environment the general good is overproduced relative to the socially optimal production level ($1/2-a/4$ under monopoly vs. $(1-a)/2$ - optimal market share).
• Under oligopolistic competition the general good is overproduced even more \((1/2-a/6)\). Social welfare is lower than in the monopolistic case. Although, consumers are better off on average.

6.2 Cone

Customers are distributed on the cone s.t. their concentration is the higher the closer to the vertex and uniform along a generatrix. \(N\) specific goods are uniformly distributed along the directrix with distance \(Y\) between two neighboring specific goods.

A customer’s location is defined by \((i, x, y)\), where \(i\) is a market segment between two directrices corresponding to neighboring specific goods \(i\) and \(i+1\), \(x\) is a customer’s vertical coordinate along a generatrix and \(y\) is a horizontal coordinate along the circumference to the generatrix corresponding to specific good \(i\). Customers travel only first to the closest directrix along the circumference and then along the directrix to the required good.

As before, denote \(p_0\) and \(p_i\) the prices of the general good and specific good \(i\) respectively. Then indifferent consumer in segment \(i\) has the following coordinates:

\[
\bar{x} = \frac{1 + a}{2} + \frac{p_0 - p_i}{2t} ;
\]
\[
\bar{y} = \frac{Y(1 - x)}{2} + \frac{p_{i+1} - p_i}{2t} .
\]

In this setup consider the monopolist maximizing profits:

\[
\pi = p_0 (1 - \bar{x}) + p_i \bar{x} .
\]

The resulting prices are:

\[
p_0 = v - \frac{3at}{4} - \frac{t}{2} + \frac{Yt}{8} \left( \frac{Y}{2} + a - 1 \right) ;
\]
\[
p_i = v - \frac{at}{4} - \frac{t}{2} + \frac{Yt}{8} \left( \frac{Y}{2} + a - 3 \right) .
\]

Market share of specific goods is:

\[
\bar{x} = 1/2 + a/4 + Y/8 ,
\]

which positively depends on \(Y\) - distance between specific goods.

6.3 Negative \(a\)

All the results hold true for \(a > -2/7\) (for Monopoly) or \(a > -3/11\) (for Oligopoly). Otherwise in some cases IR condition for consumers close to the center is binding.
6.4 One more general good producer

If we allow one more firm to produce the same general good then we will come to the Bertrand outcome with general good sold at zero price. Alternatively, the model can be modified as shown in the picture: two general goods located at the opposite ends with specific goods in the middle.

6.5 Transportation costs

Monopoly. When all specific goods are available a decrease in transportation costs leads to higher prices and doesn’t change product sales.

With missing specific goods the prices of general and specific products can decrease or increase depending on $v$ and $a$. General good sales either increase or don’t change when $t$ drops, specific good sales either decrease or don’t change.

Oligopoly. In oligopolistic environment, however, prices decrease when transportation costs go down. Sales don’t depend on $t$ when the equilibrium is unique.

6.6 Location of specific goods

Now assume that specific good producers can choose the location of specific products on the tines, while the location of the general good is fixed.

Monopoly. It turns out that the monopolist will choose to move specific products slightly towards the center. The optimal distance from the tines ends to specific goods is
equal to:
\[ l_i = \frac{a + 2}{7} \]

The prices rise and are equal to:
\[ p_0 = v - \frac{5at - 3t}{7} \]
\[ p_i = v - \frac{at - 2t}{7} \]

Sales of specific goods also rise and are equal to:
\[ s_i = \frac{4 + 2a}{7} \]

Note that the general good in this case will be sold only for \( a < \frac{3}{2} \).

**Oligopoly.** The analysis is quite complicated because linear costs lead to discontinuous demand and corner solutions but it looks like specific good producers try to move their products as close to the center as possible conditional on IR constraints of indifferent consumers and those located at the tines ends.

### 6.7 Other notes

- Doubling the number of customers in one segment is equivalent to adding one more specific good (as long as fixed costs are zero)
- In oligopolistic case it doesn’t matter whether single product firms or just one firm produces specific goods.

**References**


Appendix. Proofs of Propositions

Proof of Propositions 1 and 2. First, assume that the monopolist sells only specific goods. Denote the location of the consumer from segment $i$ indifferent between purchasing and not purchasing specific product $i$ as $(i, \bar{x}_i)$. Then the indifference condition is $v - t\bar{x}_i - p_i = 0$ or $\bar{x}_i = (v - p_i)/t$. The demand and profits for segment $i$ are:

$$q_i = \bar{x}_i = \frac{v - p_i}{t},$$

$$\pi_i = p_i q_i = \frac{p_i(v - p_i)}{t}.$$

Total profits are:

$$\pi = N p_i q_i = N \frac{p_i(v - p_i)}{t}.$$

Maximizing profits under constraint $0 \leq \bar{x}_i \leq 1$ gives $p_i^* = v/2$ and $q_i^* = N^2 v^{2t}$ for $0 < v \leq 2t$.

If $v \geq 2t$ the market is fully covered and the monopoly sets such a price that the consumer at the center receives zero utility. Thus, the price in this case is $p_i^* = v - t$.

Second, assume that the monopolist sells only the general good. The indifferent consumer $(i, \bar{x}_0)$ satisfies the condition $v - t(a + 1 - \bar{x}_0) - p_0 = 0$ or $\bar{x}_0 = 1 + a - (v - p_0)/t, \bar{x}_0 \in [0, 1]$. The demands in all segments are identical and the total demand and profits are equal to:

$$q_0 = N(1 - \bar{x}_0) = N \left(\frac{v - p_0}{t} - a\right),$$

$$\pi_0 = p_0 q_0 = N p_0 \left(\frac{v - p_0}{t} - a\right).$$

Maximizing profits under constraint $0 \leq \bar{x}_0 \leq 1$ gives $p_0^* = v/2 - at/2$ and $q_0^* = N(v/(2t) - a/2)$ for $at < v \leq (a + 2)t$.

If $v \geq (a + 2)t$ the market is fully covered and the monopoly sets such a price that the consumers at the spokes terminals receive zero utility. This corresponds to the price $p_0^* = v - (1 + a)t$.

Finally, assume that the monopolist is able to sell both the general and specific goods. First, let $a < 2$. The proof for cases 1 and 2 follows from Propositions 1 and 2 and from what is discussed just above Proposition 3. Consider case 3 when $v \geq t + at/2$. In this case the market is fully covered. Assuming that the monopoly produces both the general product and specific products the indifferent consumer is defined from the equation:

$$v - t(a + 1 - \bar{x}) - p_0 = v - t\bar{x} - p_i,$$

$$\bar{x} = \frac{1 + a}{2} - \frac{p_i - p_0}{2t},$$

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where \( x \in [0, 1] \). The demands for the general product and specific products are:

\[
q_0 = N(1 - \bar{x}) = N \left( \frac{1 - a}{2} + \frac{p_i - p_0}{2t} \right),
\]

\[
q = N\bar{x} = N \left( \frac{1 + a}{2} - \frac{p_i - p_0}{2t} \right).
\]

Profits are:

\[
\pi = p_0q_0 + p_iq = Np_0 \left( \frac{1 - a}{2} + \frac{p_i - p_0}{2t} \right) + Np_i \left( \frac{1 + a}{2} - \frac{p_i - p_0}{2t} \right).
\]

The indifferent consumers receive the lowest utility in the market and in order to extract more profits the monopoly will drive their utility to zero. Thus, the profits are maximized under condition \( U_0(i, \bar{x}) = v - t(a + 1 - \bar{x}) - p_0 = 0 \) or:

\[
p_0 = 2v - p_i - (1 + a)t.
\]

Plugging this equation to profits function and taking first order condition w.r.t. \( p_i \) gives exactly the results for prices and market shares stated in the proposition.

For \( a \geq 2 \) and \( v \leq at \) nobody is willing to buy the general good for a nonnegative price. Thus, the solution is the same as in Proposition 1 with only specific goods produced.

When \( a \geq 2 \) and \( v > at \) the market is fully covered. Assume that consumers buy both types of the good, i.e. \( q_0^* = N(1 - \bar{x}) > 0 \). After same calculations as for the case of \( a < 2 \) and \( v > t + at/2 \) we get \( q_0^* = 1/2 - a/4 \), which is nonpositive for \( a \geq 2 \). Hence the monopoly does not produce the general good when \( a \geq 2 \).

**Proof of Propositions 3-5.** First, calculate demand functions for two types of market segments: those, where specific product is available and those where only the general product is available. If only the general product is available then the location of the indifferent consumer is the same and the demand function is:

\[
\bar{x}_0 = 1 + a - \frac{v - p_0}{t},
\]

replace \( N \) segments with \( N - n \) and obtain:

\[
q_{00} = \frac{N - n}{t}(v - p_0 - at).
\]

For segments where both product types are available the indifference condition is the same and demand functions are:

\[
\bar{x} = \frac{1 + a}{2} - \frac{p_i - p_0}{2t},
\]

Demand for the general product:

\[
q_{0i} = \frac{n}{2t}((1 - a)t + p_i - p_0),
\]
demand for all specific products:
\[ q = \frac{n}{2t} ((1 + a)t - p_i + p_0). \]

Profit function is:
\[ \pi = p_0(q_{00} + q_{0i}) + p_i q = \frac{1}{2t} [-p_0^2(2N - n) - np_i^2 + 2np_i p_0 + p_i n (1 + a)t + p_0(2(N - n)v - 2Nat + nat + nt)]. \]

This function is maximized under five constraints:

1. Utilities of the indifferent agents in segments with both product types should be non-negative:
   \[ U_i(i, \bar{x}) = v - \frac{1}{2}(p_i + p_0 + (1 + a)t) \geq 0. \]

2. \( \bar{x}_0 \geq 0 \) or equivalently \( p_0 + (a + 1)t - v \geq 0. \)

3. \( \bar{x}_0 \leq 1 \) or \( v - p_0 - at \geq 0. \)

4. \( \bar{x} \geq 0 \), which is not binding according to Lemma 1.

5. \( \bar{x} \leq 1 \) or \( (1 - a)t + p_i - p_0 \geq 0. \)

Further I consider all possible combinations of binding and non-binding constraints (16 in total) and find the solution of the monopolist’s problem for the corresponding ranges of the parameters of the model.

1. Assume that only constraint 1 is binding, i.e. \( U_i(i, \bar{x}) = 0 \). That means indifferent consumers receive zero utility, segments without specific products are not fully covered and the general product is present in all the segments. Maximizing profits under this constraint gives the solution for regions 1 in Proposition 3 and 2 in Proposition 4.

2. Assume that no constraint is binding, i.e. indifferent consumers in segments with specific products receive positive utility and, as in the previous case, segments without specific products are not fully covered and the general product is present in all the segments. Maximizing profits gives the solution for regions 2 in Proposition 3 and 3 in Proposition 4.

3. Assume that \( \bar{x}_0 = 0 \) and the other constraints are not binding. The market is fully covered and the general product is present in all the segments. Note that in this case constraint 1 cannot bind. Segments without specific products are fully covered, which means all the consumers except those located at the terminals receive positive utility. Hence, in segments with both types of the product indifferent consumers, which are located at the interior of the segments, receive positive utility. This rules out one more case and gives the solution for regions 3 in Proposition 3 and 4 in Proposition 4.
4. Now assume that the monopoly does not produce the general product at all, i.e. \( \bar{x}_0 = 1 \) and \( \bar{x} = 1 \). Note that the latter follows from the former. If the general product is not consumed in segments without alternative there is no chance that it is consumed in segments where there is an alternative good. Also, it automatically follows that \( \bar{x}_0 > 0 \) and \( U_i(i, \bar{x}) = 0 \) - indifferent consumers in segments with specific goods choose between buying and not buying. Thus, here we cover 8 combinations of binding and non-binding constraints and find the solution for regions 1 in Proposition 4 and 1 in Proposition 5.

5. Now let the monopoly produce the general product but sell it only in segments where specific goods are unavailable. Also, assume that these segments are not fully covered. Thus, \( \bar{x}_i = 1 \) and other constraints are not binding. Note, that from \( \bar{x}_i = 1 \) and \( \bar{x}_0 < 1 \) it follows that \( U_i(i, \bar{x}) > 0 \). Consumers at the center obtain positive utility if they buy the general good. In order to make them buy specific products (if available) the utility of buying a specific product must be positive as well. So, two cases are considered here, which gives the solution for region 2 in Proposition 5.

6. Finally, consider the same case as before but let the market be fully covered. So, two constraints are binding: \( \bar{x}_i = 1 \) and \( \bar{x}_0 = 0 \). Maximizing profits under these constraints gives the solution for region 3 of Proposition 4.

\[ v - t\bar{x} - p_i = v - t(a + 1 - \bar{x}) - p_0 \quad \iff \quad \bar{x} = \frac{1 + a}{2} - \frac{p_i - p_0}{2t}. \]

The demand of firm 0 is then equal to:

\[ q_0 = N(1 - \bar{x}) = N \left( \frac{1 - a}{2} + \frac{p_i - p_0}{2t} \right) \]

and the demand of firm \( i \) is:

\[ q_i = \bar{x} = \frac{1 + a}{2} - \frac{p_i - p_0}{2t} \]

The profits of firm 0 and firm \( i \) are:

\[ \pi_0 = p_0 q_0 = NP_0 \left( \frac{1 - a}{2} + \frac{p_i^* - p_0^*}{2t} \right), \]

\[ \pi_i = p_i \left( \frac{1 + a}{2} - \frac{p_i - p_0^*}{2t} \right), \]

where \( p_0^* \) and \( p_i^* \) are equilibrium prices.

The firms maximize their profits under two constraints:

\[ \square \]
1. $\bar{x} \leq 1$ or $(1 - a)t + p_i^* - p_0^* \geq 0$.

2. The utility of the indifferent consumer is nonnegative, i.e.

$$2v - (1 + a)t - p_0^* - p_i^* \geq 0.$$ 

First, assume that the indifferent consumer receives zero utility, i.e. the second constraint is binding, and $\bar{x} \leq 1$, i.e. the first constraint is not binding. That means

$$p_0^* = 2v - (1 + a)t - p_i^*. \quad (1)$$

Any equilibrium must satisfy two conditions:

1. Firms don’t have incentives to decrease prices. That means $\frac{\partial \pi_0}{\partial p_0} \geq 0$ and $\frac{\partial \pi_i}{\partial p_i} \geq 0$ or:

$$\begin{align*}
(1 - a)t + p_i^* - 2p_0^* &\geq 0, \quad (2) \\
(1 + a)t + p_0^* - 2p_i^* &\geq 0. \quad (3)
\end{align*}$$

2. Firms don’t have incentives to increase prices. When one of the firms increase its price the market becomes not fully covered and the firm’s profit is described by the profit function of the monopoly ($\pi_0^m$ for firm 0, $\pi_i^m$ for firm $i$). The condition is satisfied when $\frac{\partial \pi_0^m}{\partial p_0} \leq 0$ and $\frac{\partial \pi_i^m}{\partial p_i} \leq 0$.

From the proof of Propositions 1 and refPm2 $\pi_i^m = \frac{1}{2}p_i(v - p_i)$ for $p_i \geq v - t$. When $p_i < v - t$ the derivative of $\pi_i^m$ is always positive. Thus, the condition $\frac{\partial \pi_i^m}{\partial p_i} \leq 0$ is equivalent to:

$$\begin{align*}
\begin{cases}
p_i^* \geq v/2, \\
p_i^* \geq v - t.
\end{cases} \quad (4)
\end{align*}$$

Also, $\pi_0^m = Np_0\left(\frac{v - p_0}{t} - a\right)$ for $p_0 \geq v - (1 + a)t$. When $p_0 < v - (1 + a)t$ the derivative of $\pi_0^m$ is always positive. Thus, the condition $\frac{\partial \pi_0^m}{\partial p_0} \leq 0$ is equivalent to:

$$\begin{align*}
\begin{cases}
p_0^* \geq (v - at)/2, \\
p_0^* \geq v - (1 + a)t.
\end{cases} \quad (5)
\end{align*}$$

Solving together equation (1) and inequations (2) - (5) gives a continuum of equilibria for $0 \leq a < 3$ and $v < \frac{3t}{2} + \frac{at}{2}$.

Next, assume that both constraints are not binding, i.e. general and specific goods are produced and indifferent consumers receive nonnegative utility. First-order conditions for firm’s 0 and firm’s $i$ problems are:

$$\begin{align*}
\frac{\partial \pi_0}{\partial p_0} &= (1 - a)t + p_i^* - 2p_0^* = 0, \\
\frac{\partial \pi_i}{\partial p_i} &= (1 + a)t + p_0^* - 2p_i^* = 0,
\end{align*}$$

from which:

$$p_0^* = t - \frac{at}{3}, \quad p_i^* = t + \frac{at}{3}.$$
Sales are:

\[ q^*_0 = N(1 - \bar{x}) = N(\frac{1}{2} - \frac{a}{6}), \quad q^*_i = \bar{x} = \frac{1}{2} + \frac{a}{6}. \]

It is straightforward to verify that the constraints are satisfied if and only if \(0 \leq a < 3\) and \(v \geq \frac{3t}{2} + \frac{at}{2}\).

Next, assume that \(\bar{x} = 1\) and constraint 2 is not binding. The general product is not produced in this case but firm 0 is in the market and is ready to sell its product for the price \(p^*_0\). Given these conditions the only possible value of \(p^*_0\) at the equilibrium is zero. If \(p^*_0 = 0\) firm 0 cannot decrease its price and doesn’t have incentives to raise it because it already produces nothing. \(p^*_0 > 0\) cannot be an equilibrium because firm 0 can lower the price and start to sell a positive amount of the general product and increase its profits.

Given that \(p^*_0 = 0\) from the condition \(\bar{x} = 1\) it can be found that

\[ p^*_i = (a - 1)t. \]

We need to check that firm \(i\) does not have incentives to deviate. Given that firm \(i\) serves the whole segment it doesn’t have incentives to lower the price. It also does not have incentives to increase the price when:

\[ \partial \pi_i / \partial p_i = (1 + a)t - 2p^*_i \leq 0. \]

Substitute \(p^*_i = (a - 1)t\) and get \(a > 3\) - the condition when \(p^*_i = (a - 1)t\) and \(p^*_0 = 0\) constitute a unique equilibrium.

Finally, assume that both constraints are binding. That means:

\[ (1 - a)t + p^*_i - p^*_0 = 0, \]
\[ 2v - (1 + a)t - p^*_i - p^*_0 = 0, \]

from which it follows that \(p^*_0 = v - at\), which, as it was discussed previously, must be equal to zero. That means \(v = at\), which refers to the case of firms being local monopolists and is not considered in this proposition.

\[ \square \]

**Proof of Proposition 10.** Firm 0 maximizes profits w.r.t. \(p_0\) and \(p_i\). Corresponding first order conditions are:

\[ \frac{\partial \pi_0}{\partial p_0} = \frac{1}{2t} \left( (1 - a)Nt + (N - n)p^*_j + 2np^*_i - 2Np^*_0 \right) = 0, \]
\[ \frac{\partial \pi_0}{\partial p_i} = \frac{1}{2t} \left( (1 + a)nt + 2np^*_0 - 2np^*_i \right) = 0 \]

FOC for firm \(j\):

\[ \frac{\partial \pi_j}{\partial p_j} = \frac{1}{2t} \left( (1 + a)t + p^*_0 - 2p^*_j \right) = 0. \]
From the two latter equations it follows that:

$$p_i^* = p_0^* + \frac{(1 + a)t}{2}, \quad p_j^* = \frac{p_0^*}{2} + \frac{(1 + a)t}{2},$$

which means that specific goods in competitive segments are priced lower than in segments served by firm 0. Solving the system of these three FOCs gives the equations for equilibrium prices stated in the proposition.

Segment shares are derived from the locations of indifferent consumers $\bar{x}_i$ and $\bar{x}_j$ by substituting the equations for equilibrium prices. Then, we need to check two conditions. First condition - general product segment shares should be positive, and second - IR condition of indifferent consumers. Since we established that $p_i^* > p_j^*$, then $\bar{x}_i < \bar{x}_j$ and $u(\bar{x}_i) < u(\bar{x}_j)$. Thus, we need to check that $\bar{x}_j < 1$ and $u(\bar{x}_i) \geq 0$.

$\bar{x}_j < 1$ is equivalent to $\frac{3N-5n}{6(N-n)} - \frac{a}{6} > 0$, which in its turn is equivalent to condition $a < 3 - \frac{2n}{N-n}$. From $u(\bar{x}_i) \geq 0$ we derive the condition for $v$: $v \geq \frac{(21N-5n)t}{12(N-n)} + \frac{5nt}{12}$.

Proof of Proposition 11. From the proof of Proposition 10 we have the condition for positive sales of the general good in competitive segments ($\bar{x}_j < 1$): $a < 3 - \frac{2n}{N-n}$ and $n/N < 3/5$. That means that if $a \geq 3 - \frac{2n}{N-n}$ or $n/N \geq 3/5$ then in the competitive equilibrium firm 0 does not sell the general good in competitive segments. Then firm 0 should not care about these segments and maximize profits only in the segments that it serves entirely. That means it sets monopolistic prices for the general and specific goods. Single-product firms in their turn set prices as high as possible such that consumers at the center in their segments are indifferent between buying the general and specific good, if firm 0 produces the former, or between buying specific good and not buying at all, if firm 0 does not produce the general good. When $a \geq max[2, 3 - \frac{2n}{N-n}]$ firm 0 does not produce the general good. $3 - \frac{2n}{N-n} = 2$ when $n/N = 1/3$. Thus, when $n/N > 1/3$ and $max[0, 3 - \frac{2n}{N-n}] \leq a < 2$ firm 0 produces the general good and when $a \geq 2$ it doesn’t.

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