Endogenous Prominence and Maximal Obfuscation - Preliminary Draft

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March 14, 2019

Abstract

This paper studies the relationship between endogenous prominence and obfuscation in the form of product or price framing in imperfectly competitive markets. It analyses a sequential move framework where either frame differentiation or frame complexity is the main source of consumer confusion. For given prominence levels, the firms’ frame choices in equilibrium depend on the relative symmetry of the firms. When firms make investments that affect their relative salience, in the sequential equilibrium prominence obtains endogenously and, under a wide range of parameters, there is maximal obfuscation. Our model can be used to explore the effectiveness and implications of consumer protection policy.

JEL Classification: D18; D43; D91; L13

Keywords: Price frames, Price complexity, Endogenous prominence

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1 Introduction

In many retail markets, the sellers use differentiated or complex price or product presentation formats. Discounts for grocery products or household supplies are often framed differently, for instance, as percentage reductions, reference pricing, volume offers, and free offers. Prices for the same type of fruit or vegetable are expressed in different units of measurement (e.g. “price per kilogram” and “price per unit”). Prices of restaurant meals, hotel rooms, or online products are presented in “all-inclusive” and “partitioned” formats (the latter with VAT, table service, parking charges, or shipping fees quoted separately). Close substitutes are differentiated spuriously by labels, package size, or pressure selling. For instance, ready meals can be framed as healthy options, locally sourced, family-friendly, or extra large. In retail energy or financial and banking markets, the firms often use different disclosure methods and, even when the firms use common price formats, these may be complex multi-part tariffs or may involve technical language.

Both the differentiation and the complexity of price or product presentation formats may limit the comparability of competing offers, resulting in consumer confusion. In nearly homogeneous product markets, the firms may strategically use these practices to exploit consumer bounded rationality and soften competition.

In some markets where firms frame their prices or products, confused consumers’ decisions are affected by firm prominence. For instance, confused consumers may rely on information provided by intermediaries, who steer them towards a salient product, or on persuasive advertising which increases a product’s or brand’s relative prominence. So, biases in favour of a brand created by investments in prominence may be easier to exploit when consumers are confused by price or product framing.¹

This paper aims to shed some light on the interplay between endogenous firm prominence and strategic obfuscation in the form of price or product

framing in imperfectly competitive markets. A recent investigation by the UK Competition and Markets Authority (CMA) of practices used by some of the biggest online hotel booking sites illustrates the coexistence of these practices. Although not focusing specifically on the interplay between prominence investments and framing, the CMA expressed concerns regarding misleading discount claims, pressure selling, and the effect of commissions on the order of hotel listing on the platforms.\footnote{On February 6, 2019, Expedia, Booking.com, Agoda, Hotels.com, ebookers, and trivago formally agreed to change their practices in response to the CMA’s enforcement action. See ?}

In this paper, we consider a sequential game where two firms sell a homogeneous product to a unit mass of consumers. In the first stage, the firms make investments which affect their relative prominence (e.g., in persuasive advertising or preferential product location). In the second stage, the firms choose the presentation formats. There are two formats (or frames) available and consumers may get confused by offers in different frames or by offers presented in a common, complex frame. In the final stage, the firms choose prices.

Consumers who get confused cannot compare the firms’ prices and their random choices reflect the firms’ relative prominence levels. If the share of consumers who get confused is largest when the two firms use different frames, the “frame differentiation” is the main source of confusion. In contrast, if the share of confused consumers is largest when the two firms choose a common but complex frame, then “frame complexity” is the main source of confusion.

Recognizing that it takes time for a firm to gain consumer recognition, we model the advertising investments which determine the firms’ relative prominence as long-term decisions. The last two stages of our model are an asymmetric and sequential version of the symmetric and simultaneous move duopoly model in Chioveanu and Zhou (2013). As in our model, the firms commit to frames before competing in prices, these can be interpreted as either price or product presentation formats.

In the final stage, the firms’ pricing strategies depend on the frame profile. If both firms choose a common simple frame, all consumers can identify the

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best offer and the firms compete a la Bertrand. However, when the firms choose a common, complex frame or when they choose different frames, there are two types of consumers, fully rational and confused. In these cases, which are closely related to Narasimhan (1988), the firms face a conflict between extracting surplus from the confused and competing for the rational consumers. As a result, in the unique equilibrium, the firms choose prices randomly from a common interval.

The analysis of the framing competition stage leads to a "relative symmetry" condition, which has a bearing on the nature of the firms’ equilibrium strategies. When frame differentiation is the main source of confusion, this condition is satisfied for low levels of prominence or when the share of consumers confused by frame complexity is low enough. For instance, the condition is always satisfied if both frames are simple. When frame complexity is the main source of confusion, the condition is satisfied if the share of consumers confused by frame differentiation is high enough and prominence low enough. Intuitively, the condition holds when the market is relatively more symmetric (hence the name).

Consider first the exogenous prominence game where frame differentiation is the main source of confusion. Then, in the reduced form framing game, if the relative symmetry condition is satisfied there are two pure strategy equilibria where the firms choose different frames, while if the condition does not hold there is a unique pure strategy equilibrium where the prominent firm chooses the simple frame and the rival chooses the complex frame. The latter result is related to the non-monotonicity of the less prominent firm’s profit in the share of confused consumers.

In the exogenous prominence game where frame complexity is the main source of confusion, there is a unique pure strategy equilibrium where the prominent firm chooses the complex frame. The rival chooses the simple frame if the relative symmetry condition is satisfied, whereas it chooses the complex frame if the condition does not hold.

Our analysis of the endogenous prominence game shows that unless confusion is extremely efficient (i.e., there are no rational consumers in
equilibrium), the firms’ salience spending levels are asymmetric in any subgame perfect equilibrium so that one firm is more prominent than the other. Furthermore, for a very wide range of parameters, there is maximal obfuscation in equilibrium. We use our framework to assess the impact of consumer protection policies, such as consumer literacy or awareness programs or format standardization. In particular, we focus on the effect of intervention on firms’ prominence investment levels and, through this channel, on consumer surplus and welfare.

In a modified model we analyze a market where prominence is itself a source of confusion. There, the firms’ investments in prominence affect also the total share of confused consumers, not only their allocation between firms.

2 Related Literature

Our study is related to a recent stream of literature which examines the impact of strategic obfuscation in imperfectly competitive markets. Part of the research investigating obfuscation focuses on the interaction between strategic sellers and bounded rational consumers.\(^3\)

Developing the model of information frictions proposed by Varian (1980), Carlin (2009) analyses a homogeneous product oligopoly market where the firms compete by choosing both prices and the complexity of their price offers. While some consumers are able to identify the lowest price and behave fully rationally, others cannot assess the firms’ prices due to complexity and buy from a random firm. In Carlin’s model, the share of confused depends on the aggregate complexity level.

In a duopoly model where the firms’ price format choices affect the comparability of their offers, Piccione and Spiegler (2012) examine the relationship between a general format structure and the equilibrium patterns. Chioveanu

\(^3\)Other analyses explore strategic obfuscation in sequential search models with fully rational consumers. See, for instance, Wilson (2010) for an early analysis and Spiegler (2016) for related discussion.
and Zhou (2013) provide an oligopoly analysis, showing that in markets where firms compete in prices and price formats an increase in the number of firms may harm consumers. Both these analyses show that the nature of the equilibrium depends on the relative effectiveness of price format differentiation and price format complexity as sources of consumer confusion, and focus on markets with identical firms.

Gu and Wenzel (2014) examine a duopoly model where firms first choose complexity levels and then compete in prices to examine the effectiveness of consumer protection policies which promote market transparency. They employ an asymmetric framework, which highlights the interplay between firm prominence (e.g., brand recognition) and strategic obfuscation, and show that policies designed to reduce the scope for obfuscation may backfire and lead to an increase in the obfuscation level of the less prominent rival.

In an asymmetric duopoly framework where firms choose prices and price complexity levels simultaneously, Chioveanu (2019) shows that consumer surplus is not monotonic in the degree of prominence. For instance, if an entrant in a retail energy market succeeds in raising its profile compared to a salient incumbent, this may harm consumers. Both Gu and Wenzel (2014) and Chioveanu (2019) analyze the interplay between price complexity and exogenous firm prominence.

Our work complements these studies by endogenising prominence in markets with consumer confusion and exploring the impact of framing competition on firms’ salience investments. By considering a unified framework where either frame differentiation or frame complexity can be the main source of confusion, we investigate the effectiveness of alternative consumer policy interventions.

The next section introduces our sequential model. Section 4 analyses the exogenous prominence game where ex-ante asymmetric firms choose frames and then compete in prices and discusses some policy implications. In section 5 we analyze the firms’ prominence investments in the reduced form investment game and explore the obfuscation levels in the subgame perfect equilibrium of the endogenous prominence game. Section 4 presents our
conclusions. All proofs missing from the text are relegated to appendices.

3 The Model

Consider a market where two sellers supply a homogeneous product to a unit mass of consumers. Each consumer demands at most one unit of the product and is willing to pay at most $v = 1$. Firms’ marginal costs of production are constant and normalized to zero. We explore endogenous prominence in a setting where firms compete in both prices and price frames, by analyzing the following three-stage model.

In the first stage, the firms simultaneously and independently choose investment levels, $\gamma_1$ and $\gamma_2$, which determine their relative prominence. These may be, for instance, investments in advertising or marketing activities that increase the salience of a firm’s product (or brand recognition). In the second stage, after observing the prominence levels, the firms simultaneously and independently select price presentation formats, $z_1$ and $z_2$. Each firm chooses one of two available formats (or frames), so $z_i \in A, B$. In the final stage, after observing the rival’s price format, the firms simultaneously and independently set prices, $p_1$ and $p_2$, where $p_i \in [0, 1]$.

Frame $A$ is a simple format (e.g., an all-inclusive price) and frame $B$ is a different and possibly more complex frame (e.g., a multi-dimensional price). A frame is simple if two offers presented in this common frame are perfectly comparable. A frame is complex if not all consumers are able to perfectly compare two offers presented in this common frame. Frame $B$ may also be a simple frame but different from $A$. This is the case, for example, when $A$ is ”price per unit” and $B$ is ”price per kilogram”.

We focus on markets where investments that affect prominence are long-term decisions, where modifications of price frames take time, for instance, because they require a re-design of the contract or the tariff, and where prices can be changed relatively easily.

In our setting, price framing is a source of consumer confusion. If firms choose different frames, a fraction $\alpha(A, B) = \alpha_1 > 0$ of consumers are
confused and unable to compare the two prices, while the remaining $1 - \alpha_1$ share of consumers are fully rational and purchase the cheapest product that provides a positive net surplus. We refer to the fully rational consumers as ‘informed’. If both firms choose frame $A$, then all consumers are informed and buy from the lowest price seller. In this case, the share of confused consumers is $\alpha(A, A) = \alpha_0 = 0$. If both firms choose frame $B$, a fraction $\alpha(B, B) = \alpha_2 \geq 0$ of consumers are confused, and hence are unable to compare the two prices. The remaining $1 - \alpha_2$ choose the cheapest product. If $\alpha_2 = 0$, all consumers are informed and can identify the best deal.

While (simple) frame $A$ can cause confusion only when it is combined with a different frame $B$, while if $\alpha_2 > 0$ frame $B$ is confusing in itself and can obfuscate price comparisons even if both firms adopt it. Therefore, this model accommodates two sources of consumer confusion, frame differentiation and frame complexity. If $\alpha_1 > \alpha_2 \geq 0$, frame differentiation is more confusing than frame complexity, whereas if $\alpha_2 > \alpha_1 > 0$, frame complexity is more confusing than frame differentiation.

Firm prominence, which is determined by the firms’ first stage investments, has an impact on product choice. Consumers who are confused by price framing, or informed consumers who are indifferent between the two products, favour the more prominent firm in the sense that they are more likely to purchase its product.\footnote{In their oligopoly analysis, Chioveanu and Zhou (2013) explore an alternative setting where prominence does not play a role, but confused consumers may favour the product presented in a simple price frame.}

Given a profile of first stage investments $\gamma = (\gamma_1, \gamma_2)$, firm $i$’s prominence is $\sigma_i(\gamma) \in [0, 1]$, for $i = 1, 2$, where $\sigma_1(\gamma) + \sigma_2(\gamma) = 1$. Firm $i$’s prominence level ($\sigma_i$) increases in the firm’s own spending ($\partial \sigma_i / \partial \gamma_i > 0$) and decreases in the rival’s spending ($\partial \sigma_i / \partial \gamma_j \leq 0$, with strict inequality if $\gamma_i > 0$). If only one firm invests in prominence, it attracts all confused or indifferent consumers, that is $\sigma_i(\gamma_i, 0) = 1$ if $\gamma_i > 0$ and $\sigma_i(0, \gamma_j) = 0$ if $\gamma_j > 0$. If neither seller invests in prominence, due to the ex-ante symmetry of the firms, we assume that confused and indifferent consumers shop at random, that is $\sigma_i(0, 0) = 1/2$. Furthermore, $\sigma_i$ is a symmetric function $\sigma_i(\gamma_i, \gamma_j) = \sigma_j(\gamma_j, \gamma_i)$, for $i, j = 1, 2$. 
and \( i \neq j \). Contest success functions that satisfy these assumptions can be found, for instance, in Skaperdas (1996) and Baik (1998). We use some of them in later numerical examples.

Taking prominence as given, we first characterize the subgame perfect Nash equilibrium of the two-stage game where firms first choose price frames and then prices, and where one firm may be more prominent than its rival. We refer to this as the ‘exogenous prominence game’. Then, we analyze the reduced-form investment game and characterize the subgame perfect equilibrium of the three stage game where firms prominence is determined endogenously.

In our setting, confused consumers do not pay more than their reservation price. Arguably, if price framing prevents some consumers from comparing competing offers, it may also prevent them from accurately comparing framed prices to their willingness to pay. In this case, one way to justify our assumption is that consumers can figure out at checkout (or after purchase) if a product’s price exceeds their valuation and can decline to buy it (or return it). Given this ex-post participation constraint, the firms have no incentive to charge prices above the reservation price \( 1 \). For tractability, we also assume that confused consumers’ choices are affected by prominence, but independent of firms’ prices. This captures the idea that confusion in price comparisons reduces consumers’ price sensitivity and weakens price competition.

### 4 The Exogenous Prominence Game

In this section, we assume that firms’ prominence levels are exogenous. Without loss of generality, let firm 1 be the more prominent firm, i.e., let \( \sigma_1 = \sigma \in [1/2, 1) \) and \( \sigma_2 = 1 - \sigma \). We first analyze the final stage where firms compete by choosing prices and then the second stage where the firms choose price frames.
4.1 Price Competition

In the pricing stage, the firms’ prominence levels and their price frame choices \((z_i, z_j)\) are taken as given. Focusing on weakly undominated prices \(p_i \in [0, 1]\), firm \(i\)'s profit can be written as

\[
\pi_i(p_i, p_j, z_i, z_j) = \begin{cases} 
    p_i \cdot [\sigma_i \alpha(z_i, z_j) + (1 - \alpha(z_i, z_j))] & \text{if } p_i < p_j \\
    p_i \cdot [\sigma_i \alpha(z_i, z_j) + \sigma_i(1 - \alpha(z_i, z_j))] & \text{if } p_i = p_j \\
    p_i \cdot [\sigma_i \alpha(z_i, z_j)] & \text{if } p_i > p_j
\end{cases}
\]

where \(i, j \in \{1, 2\}, i \neq j\), \(\sigma_i\) is firm \(i\)'s level of prominence and \(\alpha(z_i, z_j)\) the share of confused consumers in the market.

Standard observations lead to the following result.

**Lemma 1.** If \((z_1, z_2) = (A, A)\), or \((z_1, z_2) = (B, B)\) and \(\alpha_2 = 0\), in the unique equilibrium of the pricing subgame, both firms choose prices equal to the marginal cost and make zero profits.

If both firms choose frame \(B\) but \(\alpha_2 > 0\), then there are \(1 - \alpha_2\) informed consumers and \(\alpha_2\) confused consumers. Firms 1 and firm 2's shares of confused consumers are given, respectively, by \(\sigma \alpha_2\) and \((1 - \sigma) \alpha_2\).\(^5\) If one firm chooses frame \(A\) and the rival chooses frame \(B\), there are \(1 - \alpha_1\) informed consumers and \(\alpha_1\) confused consumers. Then, firm 1 and firm 2's shares of confused consumers are given, respectively, by \(\sigma \alpha_1\) and \((1 - \sigma) \alpha_1\). In these cases, there is a conflict between the incentives to fully exploit confused consumers and to vigorously compete for informed consumers which leads to the non-existence of pure strategy price equilibria. The proof of next result is standard and therefore omitted. (see, e.g., Narasimhan, 1988)

**Lemma 2.** If \((z_1, z_2) = (B, B)\) and \(\alpha_2 > 0\), or \((z_1, z_2) = (A, B)\), in the unique equilibrium of the pricing subgame, both firms randomize on prices on a common support \([p_0, 1]\) with \(p_0 = \frac{\sigma \alpha(z_i, z_j)}{1 - (1 - \sigma) \alpha(z_i, z_j)}\), according to the following

\(^5\)In the terminology of Rosenthal (1980), \(\sigma \alpha_2\) of the confused consumers are captive to firm 1 and \((1 - \sigma) \alpha_2\) to firm 2 because of their respective prominence levels.
cumulative distribution functions

\[ F_1(p) = 1 + \frac{(1 - \sigma)\alpha(z_i, z_j)}{1 - \alpha(z_i, z_j)} - \frac{\sigma\alpha(z_i, z_j)(1 - \sigma\alpha(z_i, z_j))}{[1 - (1 - \sigma)\alpha(z_i, z_j)](1 - \alpha(z_i, z_j))p}; \]

\[ F_2(p) = 1 + \frac{\sigma\alpha(z_i, z_j)}{1 - \alpha(z_i, z_j)} - \frac{\sigma\alpha(z_i, z_j)}{(1 - \alpha(z_i, z_j))p}. \]

Firm 1’s price c. d. f. is continuous on \([p_0, 1)\) but has a mass point at the upper bound, while firm 2’s c. d. f. is continuous everywhere. There is no equilibrium where both firms use pure price strategies. Firms’ expected profits as functions of \(\sigma\) and \(\alpha\) are, respectively,

\[ \pi_1(\sigma, \alpha) = \sigma\alpha(z_i, z_j) \] and \[ \pi_2(\sigma, \alpha) = \frac{\sigma\alpha(z_i, z_j)(1 - \sigma\alpha(z_i, z_j))}{1 - (1 - \sigma)\alpha(z_i, z_j)}. \]

Firm 1’s and the industry’s expected profits are strictly increasing in the share of confused consumers and, for \(\alpha(z_i, z_j) < 1\), in the degree of prominence. However, firm 2’s expected profit may be non-monotonic in the share of confused consumers and in the degree of prominence.

\[ \frac{d\pi_2}{d\alpha} = \frac{\sigma[1 - \sigma\alpha(z_i, z_j)(2 - \alpha(z_i, z_j) + \sigma\alpha(z_i, z_j))]}{[1 - (1 - \sigma)\alpha(z_i, z_j)]^2}. \]

For example, when \(\sigma \to 1\), \(d\pi_2/d\alpha = 1 - 2\alpha(z_i, z_j) \geq 0\) if \(\alpha(z_i, z_j) \leq 1/2\).

\[ \frac{d\pi_2}{d\sigma} = -\alpha(z_i, z_j) + \frac{\alpha(z_i, z_j)(2 - \alpha(z_i, z_j))\alpha(z_i, z_j)}{[1 - (1 - \sigma)\alpha(z_i, z_j)]^2}. \]

For example, when \(\alpha(z_i, z_j) = 1/2\), \(d\pi_2/d\sigma > 0\) for \(\sigma \in (0.5, 0.73)\) and \(d\pi_2/d\sigma < 0\) for \(\sigma \in (0.73, 1)\).

4.2 Frame Competition

The firms’ profits in the pricing stage are presented in Table 1 for all possible frame profiles.

Using backward induction, we now analyze the firms’ frame choices. We distinguish between two cases, depending on which source of confusion
Table 1: Payoffs for all possible frame profiles
dominates. In both cases, the following condition plays a role.

**Condition 1. Relative Symmetry.**

\[(\alpha_1 - \alpha_2) [1 - \sigma \alpha_1 - \sigma \alpha_2 + \sigma (1 - \sigma) \alpha_1 \alpha_2] > 0\]

\[\iff \text{sign} [1 - \sigma \alpha_1 - \sigma \alpha_2 + \sigma (1 - \sigma) \alpha_1 \alpha_2] = \text{sign}(\alpha_1 - \alpha_2).\]

When \(\alpha_2 > \alpha_1\), a necessary condition for the inequality above is \((\alpha_1 + \alpha_2) > 1\).

### 4.2.1 Frame Complexity is More Confusing than Frame Differentiation

Consider first the case where \(\alpha_2 > \alpha_1 > 0\). Table 1 indicates that choosing frame \(B\) is a strictly dominating strategy for the prominent firm. If firm 1 chooses frame \(A\), the less prominent rival is better off choosing frame \(B\). If firm 1 chooses frame \(B\), firm 2 is better off choosing frame \(A\) iff Condition 1 is satisfied. These results are summarized below.

**Lemma 3.** For \(\alpha_2 > \alpha_1 > 0\), in the frame competition stage, there is a unique pure strategy equilibrium in which the prominent firm chooses \(z_1 = B\). If Condition 1 holds, firm 2 chooses \(z_2 = A\) and the firms profits are

\[\pi_1^* = \sigma \alpha_1 \quad \text{and} \quad \pi_2^* = \frac{\sigma \alpha_1 (1 - \sigma \alpha_1)}{1 - (1 - \sigma) \alpha_1}.\]  

(1)

If Condition 1 does not hold, firm 2 chooses \(z_2 = B\) and the firms profits are

\[\pi_1^* = \sigma \alpha_2 \quad \text{and} \quad \pi_2^* = \frac{\sigma \alpha_2 (1 - \sigma \alpha_2)}{1 - (1 - \sigma) \alpha_2}.\]  

(2)
The following examples explore the impact of (a) consumer protection policy which reduces the share of confused consumers (such as an educational program) and (b) reductions in firm 1’s level of prominence. They show that, when frame complexity dominates frame differentiation as a source of consumer confusion, both (a) and (b) may harm consumers by inducing the less prominent firm to employ the more complex frame. See also Gu and Wenzel (2014) and Chioveanu (2019).

**Example 1.** Suppose $\sigma = 0.8$. For $\alpha_2 > \alpha_1$, Condition 1 holds iff

$$[1 - 0.8\alpha_1 - 0.8\alpha_2 + 0.8(1 - 0.8)\alpha_1\alpha_2] < 0.$$ 

It can be checked that this is the case if $\alpha_1 = 0.6$ and $\alpha_2 = 0.8$. Then the firms choose $(z_1, z_2) = (B, A)$ and, by (1), industry profits are 0.76 ($\pi^*_1 = 0.48$ and $\pi^*_2 = 0.28$).

A consumer protection policy which reduces $\alpha$ 1 to 0.5 and $\alpha$ 2 to 0.7, no longer satisfies the condition. Then firms choose $(z_1, z_2) = (B, B)$ and industry profits, using (2), become 0.85.

**Example 2.** Suppose $\alpha_2 = 0.9 > \alpha_1 = 0.5$. When $\sigma = 0.8$, Condition 1 holds, firms choose $(z_1, z_2) = (B, A)$ and industry profits are 0.67. However, if firm 1’s level of prominence decreases to $\sigma = 0.7$, then Condition 1 no longer holds, firms choose $(z_1, z_2) = (B, B)$, and industry profits, using (2), are 0.95.

### 4.2.2 Frame Differentiation is More Confusing than Frame Complexity

Consider next the case where $\alpha_1 > \alpha_2$. Table 1 shows that, if firm 2 chooses frame $A$ (frame $B$), it is a best response for firm 1 to choose frame $B$ (frame $A$). If firm 1 chooses frame $A$, firm 2 is better off choosing frame $B$. Moreover, if firm 1 chooses frame $B$, firm 2 is better off choosing frame $A$ iff Condition 1 is satisfied. If $\alpha_1 > \alpha_2 = 0$, this condition is trivially satisfied (as $1 - \sigma\alpha_1 > 0$).
However, if firm 1’s prominence level is relatively large (e.g., $\sigma \rightarrow 1$) and the share of confused in the market is sufficiently large, the condition does not hold (e.g., when $\alpha_1 = 1$, it requires $\alpha_2 < 0$).

**Lemma 4.** Suppose $\alpha_1 > \alpha_2$. (i) If Condition 1 holds, in the price framing stage, there are two pure strategy equilibria where the firms choose different price frames. (ii) If Condition 1 does not hold, in the reduced form price framing stage, there is a unique pure strategy equilibrium where the prominent firm chooses $z_1 = A$ and the less salient firm chooses $z_2 = B$. In any of the equilibria in (i) and (ii), the firms’ profits are

$$
\pi_1^* = \sigma \alpha_1 \quad \text{and} \quad \pi_2^* = \frac{\sigma \alpha_1 (1 - \sigma \alpha_1)}{1 - (1 - \sigma) \alpha_1}.
$$

Condition 1 holds when $\alpha_2$ is small enough (i.e., either both frames are simple or the share of confused consumers when both firms use the common, more complex frame B is low) or if the prominence level ($\sigma$) is low. Intuitively, the second part of Lemma 4 is related to the fact that the less prominent firm’s profit is not monotonic in the share of confused, as shown in the previous subsection.

It is easy to see that, regardless of whether Condition 1 holds or not, the firms choose different frames in equilibrium and total industry profit is increasing in both the share of confused consumers and the degree of prominence ($\sigma$). Therefore, in markets where frame differentiation is the main source of confusion, policy intervention which reduces consumer confusion or reduces prominence improve consumer welfare.

If Condition 1 holds, there is also a mixed strategy equilibrium. However, as the mixed strategy equilibrium is Pareto dominated by the pure strategy equilibria in Lemma 4, in the remainder of the analysis we focus on the latter and relegate the former to the appendix.
4.3 Equilibrium in the Exogenous Prominence Game

Let

\[ U = \begin{cases} 
\alpha_1 & \text{if } \alpha_1 > \alpha_2 \text{ or if } \alpha_2 > \alpha_1 \text{ and Condition 1 holds} \\
\alpha_2 & \text{if } \alpha_2 > \alpha_1 \text{ and Condition 1 does not hold}
\end{cases}. \quad (3) \]

Proposition 1. The price frame choices in Lemmata 3 and 4, together with the pricing strategies in Lemma 2, give the subgame perfect Nash equilibria of the exogenous prominence game. Firms’ equilibrium profits are

\[ \pi_1^* = \sigma U \text{ and } \pi_2^* = \sigma U \frac{1 - \sigma U}{1 - (1 - \sigma)U}, \]

where the share of confused consumers is given in equation (3).

Corollary 1. Consider the SPNE in Proposition 1. (i) The profit of the prominent firm and total industry profit increase in both \( \sigma \) and \( U \). (ii) The profit of the less prominent firm is not monotonic in \( \sigma \) or \( U \). (iii) The lower
bound of the pricing support \((p_0)\) increases in both \(\sigma\) and \(U\). (iv) ADD
Average prices?

5 Endogenous Prominence

This section examines the firms’ prominence-related spending (e.g. advertising or marketing-related). Building on the results from Section 4, we focus on the the reduced-form prominence spending game and the relative ranking of the firms’ equilibrium investment levels. In the reduced-form game, the firms simultaneously and independently invest in prominence. Profits gross of prominence spending and the expected share of confused \(U\) are presented in Proposition 1. Then, firm 1 and 2’s profits in the reduced form prominence game are

\[
\Pi_1 = \sigma_1(\gamma)U - \gamma_1 \quad \text{and} \quad \Pi_2 = \sigma_1(\gamma)U \frac{1 - \sigma_1(\gamma)U}{1 - \sigma_2(\gamma)U} - \gamma_2,
\]

where \(\gamma = (\gamma_1, \gamma_2)\) and \(\sigma_1(\gamma) + \sigma_2(\gamma) = 1\). If \(\gamma_1 = \gamma_2\), then \(\sigma_1 = \sigma_2 = 1/2\) and \(\Pi_1 = \Pi_2\).

The f.o.c.s of the profit maximization problems with respect to \(\gamma_1\) and \(\gamma_2\), respectively, are given by

\[
\frac{\partial \Pi_1}{\partial \gamma_1} = 0 \Leftrightarrow \frac{\partial \sigma_1(\gamma)}{\partial \gamma_1} U - 1 = 0; \quad (4)
\]

\[
\frac{\partial \Pi_2}{\partial \gamma_2} = 0 \Leftrightarrow \frac{\partial \sigma_1(\gamma)}{\partial \gamma_2} U \frac{1 - \sigma_1(\gamma)U}{1 - \sigma_2(\gamma)U} + \frac{\partial \sigma_2(\gamma)}{\partial \gamma_2} U (1 - \sigma_1(\gamma)U) - 1 = 0. \quad (5)
\]

Assumption 1.

\[
\frac{\partial^2 \sigma_1(\gamma)}{\partial \gamma_1^2} < 0, \quad \frac{\partial^2 \sigma_1(\gamma)}{\partial \gamma_2^2} \left( \sigma_1(\gamma) U \frac{1 - \sigma_1(\gamma)U}{1 - \sigma_2(\gamma)U} \right) < 0, \quad \text{and} \quad \frac{\partial \sigma_1}{\partial \gamma_1}(0,0) > \frac{1}{U}.
\]

The first two inequalities in Assumption 1 guarantee that the firms’ profits in the reduced-form game are concave in the region where \(\alpha_1 \geq \alpha_2\). This
guarantees the existence of a unique interior candidate equilibrium in this region, \((\gamma^*_1, \gamma^*_2)\) which satisfies (4) and (5). The last inequality in Assumption 1 guarantees there is scope for investment in prominence in this market.

For this candidate equilibrium to be symmetric, firm 1 should not have incentives to increase its spending from \(\gamma^*_1 = \gamma^*\), while firm 2 should not have incentives to decrease its spending from \(\gamma^*_2 = \gamma^*\), that is,

\[
\frac{\partial \Pi_1}{\partial \gamma_1}(\gamma^*, \gamma^*) \leq 0 \quad \text{and} \quad \frac{\partial \Pi_2}{\partial \gamma_2}(\gamma^*, \gamma^*) \geq 0.
\]

We summarize our findings on the candidate equilibrium below and relegate the proof of this result to the appendix.

**Proposition 2.** Suppose Assumption 1 holds. If \(U < 1\), where \(U\) is defined in (3), in any pure strategy Nash equilibrium of the reduced form prominence spending game, the firms’ prominence spending levels satisfy \(\gamma^*_1 > \gamma^*_2\) - and so \(\sigma_1 = \sigma(\gamma^*_1, \gamma^*_2) > 1/2\). If \(U = 1\), in the unique equilibrium of the reduced form game, \(\gamma^*_1 = \gamma^*_2\) and so \(\sigma_1 = \sigma(\gamma^*_1, \gamma^*_2) = 1/2\). For any \(U\), the values of \(\gamma^*_1\) and \(\gamma^*_2\) are implicitly defined by f.o.c.s (4) and (5).

When \(U = 1\), it is easy to see that the firms’ profits in the reduced form game are symmetric as \(\Pi_i = \sigma_i(\gamma) - \gamma_i\) for \(i = 1, 2\) and so under Assumption 1 they are globally concave. In this case, Assumption 1 is sufficient to guarantee existence of a unique equilibrium in the reduced form game. Recall from (expression for \(U\)) that \(U = 1\) either if \(\alpha_2 = 1\) but Condition 1 does not hold or if \(\alpha_1 = 1\).

When \(U < 1\), for the candidate prominence spending levels \(\gamma^*_1 > \gamma^*_2\) to be an equilibrium in the reduced-form game, the firms should not have incentives to leapfrog each other. That is, firm 1 should not have incentives to choose a lower spending than firm 2 and firm 2 should not have an incentive to choose a higher spending than firm 1. After introducing a sufficient condition, we present our next result.
Assumption 2.

\[
\frac{\partial^2 \sigma_1(\gamma)}{\partial \gamma_1 \partial \gamma_2} \leq 0 \quad \text{and} \quad \frac{\partial^2}{\partial \gamma_1 \partial \gamma_2} \left( \frac{\sigma_1(\gamma)U}{1 - \sigma_1(\gamma)U} \right) \leq 0.
\]

Under Assumption 2, the firms’ profits are weakly submodular.

**Proposition 3.** Suppose \( U < 1 \) and Assumptions 1 and 2 hold. The prominence investments \( \gamma^*_1 > \gamma^*_2 \) which satisfy (4) and (5) are a Nash Equilibrium of the reduced form game prominence game. In this case, one firm is more prominent than the rival \((\sigma > 1/2)\).

The difference-form contest success function, for instance, satisfies the first part of Assumption 2 with equality:

\[
\sigma_1(\gamma_1, \gamma_2) = \frac{\gamma_1}{\gamma_1 + \gamma_2} = 1 - \sigma_2(\gamma_1, \gamma_2).
\]

for some \( f \in C^1 \) with \( f' > 0 \) (e.g. \( f(\gamma_i) = \gamma_i^a \) for \( a > 0 \)) and \( b > 0 \).

However, Assumption 2 is a sufficient but not necessary condition for the existence of an asymmetric equilibrium in the reduced form game where \( U < 1 \). Below we present an example which does not satisfy Assumption 2 (i.e., it is easy to check that the profit function of firm 1 is supermodular whenever \( \gamma_1 > \gamma_2 \)) but where the prominence spending profile defined by (4) and (5) are a Nash equilibrium.

Let

\[
\sigma_1(\gamma_1, \gamma_2) = \frac{\gamma_1}{\gamma_1 + \gamma_2} = 1 - \sigma_2(\gamma_1, \gamma_2).
\]

Then, firm 1 and firm 2’s profits in the reduced form prominence game are

\[
\Pi_1^1 = \frac{\gamma_1}{\gamma_1 + \gamma_2} U - \gamma_1;
\]

\[
\Pi_2^2 = \frac{\gamma_1}{\gamma_1 + \gamma_2} U \frac{\gamma_1 + \gamma_2 - U \gamma_1}{\gamma_1 + \gamma_2 - U \gamma_2} - \gamma_2.
\]
Let $U = 0.78$. Then, $(\gamma_1^*, \gamma_2^*) = (0.174, 0.088)$ solves the system of f.o.c.s

$$
\begin{align*}
\frac{\partial \Pi_1}{\partial \gamma_1} &= 0 \iff \frac{\gamma_2}{\gamma_1 + \gamma_2}(0.78) - 1 = 0 \\
\frac{\partial \Pi_2}{\partial \gamma_2} &= 0 \iff -\frac{\gamma_1}{(\gamma_1 + \gamma_2)^2}(0.78)\gamma_2 + \frac{\gamma_1}{(0.951)\gamma_1} - \frac{(\gamma_1 + \gamma_2)^2}{\gamma_1 + \gamma_2} = 0 \\
\frac{\gamma_1}{\gamma_1 + \gamma_2}(0.78)\gamma_2 - \frac{(\gamma_1 + \gamma_2)^2}{(\gamma_1 + \gamma_2)} - 1 &= 0
\end{align*}
$$

The associated profits in this candidate equilibrium are

$$
\Pi_1^* = \Pi_1(\gamma_1^*, \gamma_2^*) = 0.343 \text{ and } \Pi_2^* = \Pi_2(\gamma_1^*, \gamma_2^*) = 0.25.
$$

Below we check that neither firm has an incentive to leapfrog the rival.

Consider firm 2’s profit given that firm 1 chooses $\gamma_1^* = 0.174$:

$$
\Pi_2 = \left\{\begin{array}{ll}
0.174 & \text{if } \gamma_2 < 0.174 \\
0.174 + \gamma_2 - (0.78)(0.174) & \text{if } \gamma_2 = 0.174 \\
0.174 + \gamma_2 & \text{if } \gamma_2 > 0.174
\end{array}\right.
$$

Clearly for $\gamma_2 < 0.174$, $\Pi_2$ is maximized at $\gamma_2^* = 0.088$.

For $\gamma_2 > 0.174$, $\Pi_2$ is maximized at $\gamma_2 = 0.19$ and the associated profit is $\Pi_2 = 0.2217 < 0.25$.

So firm 2 has no incentive to leapfrog firm 1.

Consider firm 1’s profit given that firm 2 chooses $\gamma_2^* = 0.088$:

$$
\Pi_1 = \left\{\begin{array}{ll}
\frac{\gamma_1}{\gamma_1 + 0.088} & \text{if } \gamma_1 > 0.088 \\
\frac{0.088}{\gamma_1 + 0.088} & \text{if } \gamma_1 < 0.088
\end{array}\right.
$$

Clearly for $\gamma_1 > 0.088$, $\Pi_1$ is maximized at $\gamma_1^* = 0.174$.

For $\gamma_1 < 0.088$, $\Pi_1$ is maximized at $\gamma_1 = 0.07$ and the associated profit is $\Pi_1 = 0.305 < 0.0343$. 

5.1 Maximal Obfuscation

The next result focuses on the share of confused consumers in the subgame perfect equilibrium of the endogenous prominence game. We find sufficient conditions, which cover a wide range of parameter values, under which the equilibrium obfuscation level is maximal. Examples suggest that equilibrium obfuscation may be maximal when these conditions do not hold.

Proposition 4. In the SPNE of the prominence game, there is maximal obfuscation (i.e., the expected share of confused is largest) if either of the following conditions holds \( \alpha_1 > \alpha_2 \) or \( \alpha_1 + \alpha_2 \leq 1 \) or \( \alpha_1(2 - \alpha_1) < (\alpha_1 + \alpha_2)/2 \).

Proof of Proposition 4: (1) Consider \( \alpha_1 > \alpha_2 \). The result follows directly from Proposition 1, as \( U = \alpha_1 \).

(2) Consider \( \alpha_2 > \alpha_1 \). (i) If \( (\alpha_1 + \alpha_2) < 1 \) then Condition 1 does not hold. The result follows directly from Proposition 1, as \( U = \alpha_2 \). (ii) Suppose now that Condition 1 holds. A necessary condition is \( (\alpha_1 + \alpha_2) > 1 \), so in this case it must hold that

\[
\alpha_2 > \max\{\alpha_1, 1 - \alpha_1\},
\]

which requires \( \alpha_2 > 1/2 \). Then firm 2’s equilibrium in the reduced form investment game is

\[
\Pi_2^*(\gamma^*) = \frac{\sigma(\gamma^*)\alpha_1(1 - \sigma(\gamma^*))\alpha_1}{1 - (1 - \sigma(\gamma^*))\alpha_1} - \gamma_2^*,
\]

where \( \gamma^* = (\gamma_1^*, \gamma_2^*) \) solve (4) and (5) for \( U = \alpha_1 \). If firm 2 deviates to \( \gamma_2^d = \gamma_1^* \) and \( z_2 = B \), as \( \sigma(\gamma_1^*, \gamma_2^d) = 1/2 \), Condition 1 does not hold, and so the deviation is profitable if

\[
\Pi_2^d(\gamma_1^*, \gamma_2^d) > \Pi_2^*(\gamma^*) \iff \gamma_1^* - \gamma_2^* < \frac{\alpha_2}{2} - \frac{\sigma(\gamma^*)\alpha_1(1 - \sigma(\gamma^*))\alpha_1}{1 - (1 - \sigma(\gamma^*))\alpha_1} \equiv \rho_1.
\]
Furthermore, as $\gamma_1^*$ is a best response to $\gamma_2^*$, it must be that

$$\Pi_1^*(\gamma^*) = \sigma(\gamma^*)\alpha_1 - \gamma_1^* > \Pi_1^*(\gamma_2^*, \gamma_2^*) = \frac{\alpha_1}{2} - \gamma_2^* \iff \frac{\gamma_1^* - \gamma_2^*}{2} \leq \frac{(2\sigma(\gamma^*) - 1)\alpha_1}{2} \equiv \rho_2.$$ 

Hence, a sufficient condition for firm 2’s deviation to $\gamma_2^* = \gamma_1^*$ and $z_2 = B$ to be profitable is

$$\rho_2 < \rho_1 \iff \rho_3 \equiv \frac{\sigma(\gamma^*)\alpha_1(2 - \alpha_1)}{1 - (1 - \sigma(\gamma^*))\alpha_1} < \frac{\alpha_1 + \alpha_2}{2}.$$ 

But $\rho_3$ increases in $\sigma$ for $\sigma \in [1/2, 1]$ and, as $\rho_3 \leq \alpha_1(2 - \alpha_1)$, a (stronger) sufficient condition for the deviation to be profitable is

$$\alpha_1(2 - \alpha_1) < \frac{\alpha_1 + \alpha_2}{2}. \quad (7)$$

The pairs $(\alpha_1, \alpha_2)$ which satisfy (7) are presented in red in Figure 1. The pairs where maximal obfuscation may not apply - that is, which satisfy (6) - are presented in blue. The next example illustrates that maximal obfuscation may be part of the equilibrium also in the region where (7) does not hold, but where (6) holds.

**Example 3.** Let $\alpha_2 > \alpha_1 = 0.4$ and

$$\sigma(\gamma_1, \gamma_2) = \frac{\gamma_1^{1/2}}{\gamma_1^{1/2} + \gamma_2^{1/2}}.$$ 

If $\alpha_2 > 0.88$ (7) holds, so in the SPNE, both firms choose price frame $B$ and the share of confused is maximal.

Suppose that $\alpha_2 \in (0.4, 0.88)$. If firm 2 chooses $A$ in the second stage, then in the reduced form investment game $\gamma_1^* = 0.009$ and $\gamma_2^* = 0.00002$ - so that $\sigma(\gamma_1^*, \gamma_2^*) = 0.953$ - and the firms’ profits are $\Pi_1^*(\gamma_1^*, \gamma_2^*) = 0.372$ and $\Pi_2(\gamma_1^*, \gamma_2^*) = 0.240$. Consistency requires Condition 1 to hold, that is $\alpha_2 > 0.662$. 

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(i) If \( \alpha_2 \in (0.4, 0.662) \), in any SPNE, both firms choose price frame \( B \) (as Condition 1 does not hold) and the share of confused is maximal.

(ii) If \( \alpha_2 \in (0.662, 0.88) \), consider a unilateral deviation by firm 2 to \( z_2 = B \) and \( \gamma_2 = \gamma_1^* \), where \( \sigma(\gamma_1^*, \gamma_1^*) = 1/2 \) and Condition 1 does not hold. Firm 2’s deviation profit is \( \Pi_2^d = \alpha_2/2 - 0.009 \) and \( \Pi_2^d > 0.240 \) whenever \( \alpha_2 > 0.498 \) and so for all \( \alpha_2 \)’s considered here.

6 Conclusions

This paper studies the relationship between endogenous prominence and obfuscation in the form of product or price framing in imperfectly competitive markets. It analyses a sequential move framework where either frame differentiation or frame complexity is the main source of consumer confusion. For given prominence levels, the firms’ frame choices in equilibrium depend on the relative symmetry of the firms. When firms make investments that affect their relative salience, in the sequential equilibrium prominence obtains endogenously and, under a wide range of parameters, there is maximal obfuscation. Our model can be used to explore the effectiveness and implications of consumer protection policy.

7 Appendix

7.1 Competition in Price Frames: \( \alpha_1 > \alpha_2 \)

Mixed Strategy Equilibria

**Lemma 5. Mixed Strategy Equilibrium.** If \( \alpha_1 > \alpha_2 \) and Condition 1 holds, there exists a (unique) mixed strategy equilibrium in which firm \( i \) choose frame \( A \) with probability \( \lambda_i \) and frame \( B \) with probability \( (1 - \lambda_i) \) with

\[
\lambda_1 = \frac{\pi_2(\sigma, \alpha_1) - \pi_2(\sigma, \alpha_2)}{2\pi_2(\sigma, \alpha_1) - \pi_2(\sigma, \alpha_2)} \quad \text{and} \quad \lambda_2 = \frac{\alpha_1 - \alpha_2}{2\alpha_1 - \alpha_2}.
\]
Firms’ expected profits are

\[ \pi^*_1 = \sigma \frac{\alpha_1^2}{2\alpha_1 - \alpha_2} \quad \text{and} \quad \pi^*_2 = \frac{\pi_2^2(\sigma, \alpha_1)}{2\pi_2(\sigma, \alpha_1) - \pi_2(\sigma, \alpha_2)}. \]

7.2 The Prominence Spending Game

Proof of Proposition 2: Suppose there exists a symmetric equilibrium \( \gamma^* = (\gamma^*_1, \gamma^*_2) \) with \( \gamma^*_1 > 0 \). Note that \( \sigma(\gamma^*) = 1/2 \). Then,

\[ \frac{\partial \Pi_2}{\partial \gamma_2}(\gamma^*) = 0 \iff \frac{\partial \sigma_1}{\partial \gamma_2}(\gamma^*) U + \frac{U/2}{(1 - U/2)} \left[ -\frac{\partial \sigma_1}{\partial \gamma_2}(\gamma^*) U + \frac{\partial \sigma_2}{\partial \gamma_2}(\gamma^*) U \right] - 1 = 0. \]

As symmetry implies that \( \frac{\partial \sigma_1}{\partial \gamma_1}(\gamma^*) = \frac{\partial \sigma_2}{\partial \gamma_2}(\gamma^*) \), the difference between firms’ marginal profits at \( (\gamma^*) \) is

\[ \left( \frac{\partial \Pi_1}{\partial \gamma_1}(\gamma^*) - \frac{\partial \Pi_2}{\partial \gamma_2}(\gamma^*) \right) \frac{1}{U} = \left( \frac{\partial \sigma_1}{\partial \gamma_1}(\gamma^*) - \frac{\partial \sigma_1}{\partial \gamma_2}(\gamma^*) \right) \frac{2(1 - U)}{(2 - U)} \equiv \Delta. \]

As \( \gamma^* > 0 \), \( \frac{\partial \sigma_1}{\partial \gamma_1}(\gamma^*) > 0 \) and \( \frac{\partial \sigma_1}{\partial \gamma_2}(\gamma^*) < 0 \). Then if \( U < 1 \), \( \Delta > 0 \) and so \( \gamma^*_1 > \gamma^*_2 \). But if \( U = 1 \), then \( \Delta = 0 \) and \( \gamma^*_1 = \gamma^*_2 \).

Suppose there exists a symmetric equilibrium \( \gamma^* = (\gamma^*_1, \gamma^*_2) \) with \( \gamma^* = 0 \). Then it must be that \( \frac{\partial \Pi_1}{\partial \gamma_1}(0, 0) = \frac{\partial \sigma_1}{\partial \gamma_1}(0, 0) U - 1 \leq 0 \). A contradiction as \( \frac{\partial \sigma_1}{\partial \gamma_1}(0, 0) > 1/U \) by Assumption 1.

Proof of Proposition 3: For the candidate spending levels in Proposition 2, \( \gamma^*_1 > \gamma^*_2 \) to be part of a SPNE, firm 1 (2) must not have incentives to reduce (increase) its spending.

Suppose firms 1 and 2 choose spending levels \( \gamma^*_1 \) and \( \gamma_2 \), respectively. Then,
firm 2’s profit is

\[
\Pi_2(\gamma_1, \gamma_2) = \begin{cases} 
\sigma_1(\gamma_1^*, \gamma_2)U \frac{1 - \sigma_1(\gamma_1^*, \gamma_2)U}{1 - \sigma_2(\gamma_1^*, \gamma_2)U} - \gamma_2 \equiv \Pi_2^L(\gamma_1^*, \gamma_2) \text{ if } \gamma_2 \leq \gamma_1^* \\
\sigma_2(\gamma_1^*, \gamma_2)U - \gamma_2 \equiv \Pi_2^H(\gamma_1^*, \gamma_2) \text{ if } \gamma_2 > \gamma_1^* 
\end{cases} 
\]

Consider \( \gamma_2 \leq \gamma_1^* \). By Assumption 1, \( \Pi_2^L(\gamma_1^*, \gamma_2) \) is strictly concave and so maximized at \( \gamma_2 = \gamma_2^* \).

Consider \( \gamma_2 > \gamma_1^* \). By Assumption 2, \( \frac{\partial \Pi_2^H}{\partial \gamma_2}(\gamma_1^*, \gamma_2) \leq \frac{\partial \Pi_2^H}{\partial \gamma_2}(\gamma_2^*, \gamma_2) \text{ as } \gamma_2^* < \gamma_1^* \).

But \( \frac{\partial \Pi_2^H}{\partial \gamma_2}(\gamma_2^*, \gamma_2) < \frac{\partial \Pi_2^H}{\partial \gamma_2}(\gamma_2^*, \gamma_1^*) = 0 \) as, by Assumption 1, \( \Pi_2^H \) is strictly concave and maximized at \( \gamma_2 = \gamma_1^* \) and \( \gamma_1 = \gamma_2^* \). Then, as \( \frac{\partial \Pi_2^H}{\partial \gamma_2}(\gamma_1^*, \gamma_2) < 0 \), firm 2 has incentives to decrease its spending.

Suppose firms 1 and 2 choose spending levels \( \gamma_1 \) and \( \gamma_2^* \), respectively. Then, firm 1’s profit is

\[
\Pi_1(\gamma_1, \gamma_2^*) = \begin{cases} 
\sigma_2(\gamma_1, \gamma_2^*)U \frac{1 - \sigma_2(\gamma_1, \gamma_2^*)U}{1 - \sigma_1(\gamma_1, \gamma_2^*)U} - \gamma_1 \equiv \Pi_1^L(\gamma_1, \gamma_2^*) \text{ if } \gamma_1 < \gamma_2^* \\
\sigma_1(\gamma_1, \gamma_2^*)U - \gamma_1 \equiv \Pi_1^H(\gamma_1, \gamma_2^*) \text{ if } \gamma_1 \geq \gamma_2^* 
\end{cases} 
\]

Consider \( \gamma_1 \geq \gamma_2^* \). By Assumption 1, \( \Pi_1^H(\gamma_1, \gamma_2^*) \) is strictly concave and so maximized at \( \gamma_1 = \gamma_1^* \).

Consider \( \gamma_1 < \gamma_2^* \). By Assumption 2, \( \frac{\partial \Pi_1^L}{\partial \gamma_1}(\gamma_1, \gamma_2^*) \geq \frac{\partial \Pi_1^L}{\partial \gamma_1}(\gamma_1^*, \gamma_2^*) \text{ as } \gamma_1^* > \gamma_2^* \).

But \( \frac{\partial \Pi_1^L}{\partial \gamma_1}(\gamma_1^*, \gamma_2^*) > \frac{\partial \Pi_1^L}{\partial \gamma_1}(\gamma_2^*, \gamma_1^*) = 0 \) as, by Assumption 1, \( \Pi_1^L \) is strictly concave and maximized at \( \gamma_1 = \gamma_2^* \) and \( \gamma_2 = \gamma_1^* \). Then, as \( \frac{\partial \Pi_1^L}{\partial \gamma_1}(\gamma_1, \gamma_2^*) > 0 \), firm 1 has incentives to increase its spending.

References


