Entry Regulations, Product Variety, and Productivity in Retail

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Abstract

This paper examines the relationship between entry regulations, product variety, and store productivity in retail using a dynamic framework that allows for a multiproduct service technology and controls how consumers choose of stores. We estimate store productivity and demand shocks related to consumers’ quality of the shopping experience using detailed data on stores, products, and local entry regulations in Swedish retail. The findings show that a less restrictive entry regulation improves store productivity and product variety. Changes in product variety due to entry regulations are higher in small markets than in large markets. A more liberal regulation decreases the cost with inventories. Counterfactual policy experiments show that consumer surplus increases more from a more liberal entry regulation than from a reduction in the tax on sales. The trade-off between store productivity and demand shocks plays a crucial role in the observed heterogeneity in product variety across markets with different stringency of regulation.

Keywords: Retail markets; entry regulation; inventories; productivity; competition.

JEL Classification: L11, L13, L81.

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1 Introduction

Entry regulations affect competition by changing the supply of stores and the consequences are frequently debated.\(^1\) Stores react to changes in competition by aiming to improve productivity and altering their product variety. Whether there is too much or too little product variety is an empirical question. Consumers benefit from large product variety and low prices, and stores aim to allocate resources to their best. Information on determinants of consumers’ choices of stores and products inside the stores can be used by stores to reposition and to improve their productivity to manage a greater product variety. This paper evaluates the impact of local entry regulations on store productivity and product variety in Swedish retailing using a dynamic model with multiproduct service technology that incorporates how consumers choose stores.

Entry regulations affect the relationship between product variety and store productivity and understanding the mechanisms and the trade-offs play a key role in welfare. Economies of scale and scope explain the multiproduct nature of retailers. The economies of scope come from factors that make it cheaper to sell many products together than selling them separate (Panzar and Willig, 1981). These factors can be related to business process such as cross-selling products using the same employees and systems (machinery and equipment) or business sharing centralized functions such as finance and marketing.\(^2\) While multiproduct is part of retail and services, this fact is omitted in the productivity literature (i.e., when estimating productivity) which is problematic when evaluating the overall impact of entry regulations that affect stores’ product variety and productivity.

We propose a transcendental multiproduct service technology and a demand system to identify store productivity and demand shocks that affect how consumers choose stores controlling for the effect of the stringency of local entry regulation.\(^3\) Then, we analyze

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\(^1\)See, for example, European Competition Network (2011), European Commission (2012). Pozzi and Schivardi (2016) survey the literature on the impact of regulation on retail development.

\(^2\)In general, economies of scope can appear from two sources: local complementarities and fixed costs (Gorman, 1985). Local complementarities imply that a higher level of output for one product reduces the marginal cost of other outputs. Fixed costs can ensure economies of scope in the absence of local complementarities.

\(^3\)Mundlak (1964) provides an early discussion on the identification of multiproduct transcendental production functions.
their relationship with product variety at the store and market level. We estimate the model using detailed information on stores and product categories in retail sale of new goods in specialized stores and local entry regulations in Sweden for the period 2003-2009.\textsuperscript{4} Store productivity and the demand shocks associated with consumers’ quality of the shopping quality experience determine sales and are important factors when stores make investments and inventory decisions. They also affect the number of products offered by stores because a store has limited resources to manage product variety and consumers choose stores with a set of available products.

Entry regulations are common in Organization for Economic Cooperation and Development (OECD) countries, but the implementation differ across countries.\textsuperscript{5} All stores are subject to regulation in Sweden, and each municipality has power to make land use decisions. In general, local authorities require each store to fill a formal application when seeking entry. The application is approved or rejected after the evaluation of the potential consequences of entry on various factors as, for example, market share and concentration (rarely all applications are approved in Sweden). Entry regulations affect store costs (e.g., location and size of buildings), which affect the intensity of competition (Bertrand and Kramarz, 2002; Pozzi and Schivardi, 2016; Suzuki, 2013; Turner et al., 2014; Maican and Orth, 2018a). Entry regulations impact competition and stores react by repositioning their product variety and inventory level. Following the previous literature on land use and entry regulations, our main measure of the stringency of regulation in local markets is the number of approved applications divided by the population density.

A more liberal entry regulation can lead to store productivity improvements through different channels.\textsuperscript{6} First, stores have the opportunity to learn the best business practices

\textsuperscript{4}The majority of the stores in our sample are single establishment in a local market, we treat each store as a decision maker. We use product categories as a proxy for product variety at the store level. First, the number of product categories provides a direct information on variety inside a store. Second, the sales of a product category include information on product lines (range) inside a product category.

\textsuperscript{5}Countries like the United States has more flexible zoning laws, while the United Kingdom and France explicitly regulate large entrants.

\textsuperscript{6}Empirical literature on productivity and competition often find positive effects of the increasing competition on productivity due to firms’ external factors such as trade liberalization, less restrictive regulation (i.e., passive effects), e.g., Syverson (2011), De Loecker (2011), De Loecker (2013), Maican and Orth (2015), and Maican and Orth (2017).
from the new entrants (i.e., spillover effect). Second, the selection effect due to entry regulations affects store productivity through the exit of low-productivity stores. In this paper, we also recognize that a store engages in active efforts to improve productivity based on the information from demand shocks related to the quality of the shopping quality experience inside store. In other words, we allow stores to learn from their information on demand. In our model, productivity follows an endogenous process, that is, current external and internal factors such as stringency of local entry regulation and shopping quality impact future productivity.

The empirical model allows for an endogenous product selection and builds on an industry dynamics framework (e.g., Jovanovic, 1982; Hopenhayn, 1992; Ericson and Pakes, 1995), where a store chooses the product variety based on its productivity, consumers’ perceived quality of the shopping experience, and characteristics of local environment. Stores achieve economies of scale by improving their productivity as response to the increasing competition due to a less restrictive entry regulation. As result, stores increase their profits by selling more. How an increase in store’s scale impacts product variety is an empirical question since product repositioning helps to increase differentiation. The change in product variety also depends on the demand side, where consumers choose stores based on their available products and the quality of the shopping experience inside store. Therefore, the change in product variety depends on both store productivity and shopping quality, where the trade-off between these variables drives the dynamics of product variety and welfare.

This paper contributes to several strands of literature. First, our work on retail contributes to a recent avenue of research on multiproduct and productivity using similar data for manufacturing that use sales and volume (physical quantity) data to infer a price measure (De Loecker et al., 2016; Dhyne et al., 2016; Valmari, 2016). De Loecker et al.’s (2016) study at the product level uses information from production technology of single-

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7There is growing theoretical literature on the competition between multiproduct firms where the findings can depend on the choice of the demand system, e.g., Brander and Eaton (1984), Anderson and De Palma (1992), Anderson and De Palma (2006), Nocke and Schutz (2018). Nocke and Schutz (2018) provide a comprehensive review of the literature and propose a general oligopolistic competition model with multiproduct firms that can be applied empirically.

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product firms to investigate the outcomes of multiproduct firms. Our paper is closely related to Dhyne et al. (2016) that estimate separate technology functions for each product while accounting for the quantity of other goods produced by the firm using output measured by physical quantity. In a multiproduct setting, Valmari (2016) estimates input allocations and productivity for each product based on Olley and Pakes (1996) and using separability and aggregation assumptions without taking into account the strong link between product selection, firms’ profit maximization, and production technology. In the multiproduct case, a service technology function consistent with profit maximization implies aggregation over physical products, and this is restrictive for many data sets due to large heterogeneity (especially in retailing). The service sector is characterized only by multiproduct and, in many cases, it is also difficult to measure the physical product.

We contribute to this multiproduct literature by proposing a framework based on a multiproduct transcendental technology and a simple CES demand system that is transparent over the aggregation across products and rate of substitution between products. We show that the proposed multiproduct technology is consistent with store’s profit maximization behavior, and we use this information on the identification of the model. We transform service technology using the CES demand system to allow the use of product-level sales and to benefit from the multiproduct technology when measuring store productivity. The recovered productivity measure is still revenue total factor productivity (TFPR), which is not restrictive in the service industries where it is difficult to measure physical products and aggregate without distortions. However, we separate from the productivity measure the main demand shocks that affect store’s market share.

Second, we contribute to the literature on the impact of entry regulations on store performance surveyed in Pozzi and Schivardi (2016). This paper extends the previous

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8Separating input allocations per product can be difficult in service industries. For examples, different machinery and equipment is used to carry or to store different product categories in the same time to increase efficiency. In addition, separation of all inputs is not consistent with economies of scale and scope.

work on the impact of entry regulations on store productivity in different directions (e.g., Maican and Orth, 2015). By using a multiproduct technology to measure store productivity, we allow for economics of scale and scope when evaluating the impact of regulation. We separately identify the impact of entry regulations and the feedback from store’s demand shocks on productivity. In this paper, the impact of changes in local competition on productivity due to regulation is modeled in a passive way. Store productivity also depends on the product competition for shelf space inside the store (e.g., sales of a product are affected by the sales of other products inside a store). In addition, we also estimate the structure of the cost of inventory and the impact of entry regulations on this cost by solving store’s dynamic optimization problem.

Third, we use recent advances in structural estimation of production functions following Olley and Pakes (1996) to incorporate information at product level (Olley and Pakes, 1996[OP]; Levinsohn and Petrin, 2003[LP]; Doraszelski and Jaumandreu, 2013; Ackerberg et al., 2015). We contribute to this literature by recovering two shocks and allowing for a multiproduct technology. In particular, we investigate the importance of demand shocks related to consumer’s shopping experience for productivity improvements separate from the impact of entry regulation. By recovering two unobserved store-level shocks, we relate to the literature endogenous productivity that studies the role of demand shocks for productivity measure (Klette and Griliches, 1996). We recover store productivity and demand shocks using store’s labor demand and inventory (Doraszelski and Jaumandreu, 2013; Maican and Orth, 2017). We discuss in details the identification of multiproduct technology and also provide Monte-Carlo simulations of simplified versions of our model.

Fourth, this paper contributes to the growing empirical literature that finds a positive relationship between competition and product variety (linked to inventory) (Olivares and Cachon, 2010; Watson, 2009; Ren et al., 2011).10 By modeling the multiproduct retailing. They show how entry costs of store formats affect local competition in short- and long-run.

10Using data on U.S. General Motors dealerships, Olivares and Cachon (2010) find that dealers hold larger inventories in local markets with more intense competition. Watson (2009) find that eyewear stores hold more inventory when rivals are located nearby in the U.S. Using data for all Best Buy and Circuit City stores in the U.S., Ren et al. (2011) find that store product variety increases with the presence of rival stores in the market (increasing competition), but that product variety decreases when stores are co-located (i.e., stores differentiate and select product variety to not overlap that of rivals).
technology on the supply-side together with the demand shocks, this paper complements the literature on the importance of variety in a multiproduct setting has recently been studied using rich data from the demand side (for example, Berry and Waldfogel (2001); Sweeting, 2010; Sweeting, 2013; Eizenberg, 2014; and Berry et al., 2016). By modeling the effect of competition on the relationship between multiproduct and productivity, we also add to the growing empirical literature in trade that focuses on the impact of trade liberalization on product variety (e.g., Bernard et al., 2010; Bernard et al., 2011; Dhingra, 2013; Nocke and Yeaple, 2014).

Lastly, we add to the scarce literature on trying to estimate productivity in services industries. Heterogeneity in productivity across stores and over time in narrowly defined industries is a well-documented fact (Syverson, 2011). Despite the rapidly growing importance of services and retail for economic activity, there is little work on how to obtain accurate measures of productivity in these sectors that are challenged by questions on measuring output. In retail trade, productivity improvements are due to the adoption of various information technologies, inventory and price management, wholesale and distribution networks, economies of scale and density, intangible assets and skills (e.g., brand recognition, reputation, and human capital), vertical contracts and integration, and external factors (e.g., competition and regulation).\footnote{See Holmes (2001), Schivardi and Viviano (2011), Basker (2012), Maican and Orth (2017). Braginskiy et al. (2015) and Maican and Orth (2018b) highlight the link between inventories, productivity and profitability. The adoption of new technology increases product variety. Holmes (2001) finds complementarities between information technology and frequency delivery. He provides a theoretical framework that shows how inventories affect frequency delivery and optimal store size. His model can explain why introduction of the bar codes has increased frequency delivery and product variety (store size).}

We find that that median labor and inventory productivity are larger in markets after the acceptance of new applications. The consumers benefit from more product variety in markets with a more liberal regulation. The estimation results show that a more liberal entry regulation has a positive impact on productivity and the impact is decreasing in productivity. Stores with high demand shocks (i.e., high shopping quality) have high future productivity, and the impact is increasing in productivity. An increase in store productivity yields an increase in the number of product categories at the store level and
the impact is decreasing in productivity levels and is larger in liberal than in restrictive markets. Stores with high shopping quality experience have fewer product categories. Our findings show that increasing investments in technology in restrictive markets has a positive effect on the number of product categories. We find that a more liberal regulation decreases the cost with inventories, which highlights the importance of entry regulation for optimal inventory.

Counterfactual policy experiments show that consumer surplus increases more from a more liberal entry regulation than from a reduction in the tax on sales. The trade-off between store productivity and demand shocks plays a crucial role in the observed heterogeneity in product variety across markets with different stringency of regulation. Changes in shopping quality bring changes in labor inside the store.

The next section presents the model and Section 3 discusses the identification and estimation of the empirical model. Section 4 presents the data. Section 5 presents the empirical results and robustness checks, and Section 6 counterfactual policy experiments. In several places, we refer to an online Appendix containing various analyses that are not discussed in detail in the paper.

2 Multi-product service technology

The most common characteristic of retailers is to offer multiple products and services to consumers. The multiproduct characteristic creates difficulties in aggregating the service output when there is not a single value function because the composite service output of a store depends on other things including prices. Moreover, the productivity of resources in a product or a specific service is not independent of the level of services in other products, which implies that we need to understand the restrictions on the parameters of the service generating function that are consistent with static profit maximization behavior at the store level.
The multiproduct service generating function for a store can be written as an implicit function, which can be described by a transcendental function that generalizes Cobb-Douglas:

\[ F(Q, V) = G(Q) - H(V) = 0 \]  

where \( G(Q) = Q_1^{\tilde{\alpha}_1} \times \cdots \times Q_d^{\tilde{\alpha}_d} \exp(\tilde{\gamma}_1 Q_1 + \cdots + \tilde{\gamma}_d Q_d) \); \( H(V) = V_1^{\tilde{\beta}_1} \times \cdots \times V_m^{\tilde{\beta}_m} \exp(\tilde{\omega}) \); \( Q \) is the vector of service output; \( Q_i \) is the \( i \)-th service output of the store, \((i = 1, d)\); \( V_e \) is the \( e \)-th service input of the store (e.g., labor, capital, inventories), \((e = 1, m)\); \( \tilde{\omega} \) is retailer’s productivity (Mundlak, 1964). In what follows, we use the \( i \) to index the service outputs and \( e \) to index the inputs.

In the empirical applications, the theoretical results of multi-output service function related to profit maximization play a crucial role in identification. For example, in general, productivity is defined as aggregate output over aggregate inputs, that is, the output and input coefficients \( \tilde{\alpha}_i \) and \( \tilde{\beta}_j \) affect productivity measure. For simplicity of exposition of multi-output technologies, we assume that the prices are given and focus on no adjustment cost in inputs. We relax this assumption in the empirical setting, which allows for dynamic inputs such as capital stock and inventories. The static profit maximization problem at the store level is given by

\[ \max V \Pi = P'Q - W'V F(Q, V) = 0 \]  

where \( P \) and \( W \) are the vectors of output and input prices, respectively. We provide a general result on the restrictions of the coefficients of transcendental multiproduct functions that are required to satisfy the static profit maximization conditions.

**Theorem 1:** Consider a general service generating function \( F(Q, V) = G(Q) - H(V) = 0 \), where \( G(Q) = Q_1^{\tilde{\alpha}_1} \times \cdots \times Q_d^{\tilde{\alpha}_d} \exp(\tilde{\gamma}_1 Q_1 + \cdots + \tilde{\gamma}_d Q_d) \); \( H(V) = V_1^{\tilde{\beta}_1} \times \cdots \times V_m^{\tilde{\beta}_m} \exp(\tilde{\omega}) \). If the parameters satisfy the following conditions: (a) \( \tilde{\alpha}_i < 0 \) and \( \tilde{\gamma}_i > 0 \) for all \( i = 1, d \); (b) \( \tilde{\beta}_e > 0 \) for all \( e = 1, m \), then the conditions for profit maximization are satisfied.

**Proof:** The main idea of the proof is that the sign of determinant of the bordered Hessian matrix of the optimization problem (2) should satisfy the second order requirement
for profit maximization. We refer the reader interested in the technical details to online Appendix A for the proof and additional discussion.

The introduction of the $\tilde{\gamma}_i$ parameters plays a key role for understanding the properties of the service output function and their empirical implications. Under certain values of $\tilde{\gamma}_i$, the service output sold at the minimum cost and the inputs yield minimum revenues. In the multiproduct case, we want to avoid these situations (saddle points).\textsuperscript{12} Proposition 1 describes these cases.

**Proposition 1:** If the service function is simple Cobb-Douglas in outputs ($\tilde{\gamma}_i = 0$ for all $i$) and inputs and the first-order conditions are satisfied, then optimal service quantity $Q^*$ is sold at the minimum cost and any inputs $V^*$ yields minimum revenues. The profit $\pi(Q^*,V^*)$ at the point $(Q^*,V^*)$ is a saddle point, i.e., $\pi(Q^*,V) \leq \pi(Q^*,V^*) \leq \pi(Q,V^*)$.

**PROOF:** The proof uses the sign of the determinant of the Hessian matrix. For the full proof and an additional discussion, we refer the reader interested in the technical details to online Appendix A.\textsuperscript{11}

A direct consequence of Proposition 1 is that when the inputs $V$ produce minimum revenues and the first-order conditions are satisfied then the profit can be maximized by a selection of products, i.e., a corner solution. This problem does not exist in the case of a single product. The condition $\tilde{\alpha}_i < 0$ and $\tilde{\gamma}_i > 0$ for all $i$ is not the only second-order condition for profit maximization.\textsuperscript{13}

Another key aspect of a multiproduct technology is that the sign of the parameters $\tilde{\gamma}_i$ determines the sign of the product (factor) substitution. Figure 1 shows the marginal rate of the product (factor) substitution. The dotted line in Figure 1c shows the marginal rate of substitution for $\tilde{\gamma}_i = 0$, which implies that product-product marginal rate of substitution is a convex function. This function is concave when $\tilde{\gamma}_i > 0$.

**Multi-product technology and role of sales.** To write the service generating function at the product level, we need to normalize one parameter to one, say the $i$-th output,

\textsuperscript{12}See Mundlak (1964) for a discussion in a case of two inputs and two outputs.

\textsuperscript{13}It is important to note that the result in Theorem 1 holds when some $\tilde{\alpha}_i$ are positive (not all) and, in this case, the corresponding $\tilde{\gamma}_i$ can be set to zero, which can be useful to reduce the number of parameters.
which can be done by raising the service function at the $-\tilde{\alpha}_i$ power. In this case, the resulting parameters of other products than $i$ will have a reverse sign when $\tilde{\alpha}_i$ is negative. Furthermore, the number of parameters can be reduced observing sales at the store level, e.g., we consider $\tilde{\gamma}_i = \tilde{\alpha}_y P_i$, where $P_i$ is the output price of product $i$. In this case, $\sum_{i=1}^{d} \tilde{\gamma}_i Q_i = \tilde{\alpha}_y \sum_{i=1}^{d} P_i Q_i = \tilde{\alpha}_y Y$, which is the total sales at the store $Y$ multiplied by $\tilde{\alpha}_y$, and it has a meaningful interpretation.

In summary, store’s total sales plays a key role in the relationship between inputs and outputs for multiproduct service generating function because it drives the substitution between the products. We use this result from the transcendental production functions to write a product sales generating function that accounts for the role of sales of other products. While service function (1) describes the multiproduct service technology inside the store, to measure sales of each product we need to understand how consumers choose products inside a store. By aggregating consumers’ shopping experience for each product inside the store, we obtain a good proxy for consumer’s shopping experience inside the store, which is a key determinant for store’s market share. In the next section, we provide an empirical model for the impact of local regulations on store productivity and product variety based on a multiproduct service function (1) and how consumers choose stores and products inside the store.

3 Empirical framework

Entry regulations affect the supply and the size of stores in local markets and have a direct effect on stores’ cost structure. Changes in competition and store’s cost structure
due to local regulations have an impact on store productivity and product variety. We start the empirical framework by modeling store’s multiproduct sales generating function accounting for the impact of local regulations. The theoretical results in Section 2 are used to construct a sales generation function at the product level that emphasizes the impact of sales of other products inside the store. In our model, store productivity and demand shocks affect sales and they are not observed by the researcher but they are observed by stores when making decisions. How consumers choose products inside the store provides useful information to recover the demand shocks related to the quality of the shopping experience and to construct sales of each product at the store level.

**ASSUMPTION 1:** All stores use the same service technology to sell their product categories and this technology does not depend on the product category.

Based on the transcendental technology (1), the multiproduct service generation function for the store \( j \) in logs is given by

\[
\sum_{i=1}^{d} \tilde{\alpha}_i q_{ijt} + \tilde{\alpha}_y Y_{jt} = \tilde{\beta}_l l_{jt} + \tilde{\beta}_k k_{jt} + \tilde{\beta}_a a_{jt} + \tilde{\omega}_{jt} + \tilde{u}_p^j, \tag{3}
\]

where \( q_{ijt} \) is the log of quantity of product \( i \) sold by store \( j \) in period \( t \); \( Y_{jt} \) are total sales of store \( j \) in period \( t \); \( l_{jt} \) is log of the number employees; \( k_{jt} \) is log of capital stock; \( a_{jt} \) is log of the sum between the inventory level in the beginning of period period \( t \), \( n_{jt} \), and the products bought during the period \( t \); and \( \tilde{u}_p^j \) are the remaining service output shocks.\(^{14}\) Assumption 1 allows us to reduce the number of parameters to be estimated in empirical applications, and it can be relaxed, i.e., have separate technologies for each product. Section 7 discusses the estimation when allowing different technology for each product.

In most of the service industries, it might be difficult to measure service output in physical units, and therefore, most data sets record sales of the product/service, that is,

\(^{14}\)We follow the common notation of capital letters for levels and small letters for logs. While the sales function (3) is written at the product level, we can write a total sales generating function using Cobb-Douglas technology (in logs): \( y_{jt} = \tilde{\beta}_l l_{jt} + \tilde{\beta}_k k_{jt} + \tilde{\beta}_a a_{jt} + \tilde{\omega}_{jt} + \tilde{\mu}_{jt} + u_{jt} \), where \( \tilde{\mu}_{jt} \) are exogenous shocks that are not controlled by the store. Online Appendix C uses a simple demand and supply model to show the intuition behind the derivation of this aggregate service function equation.
\( y_{ijt} = q_{ijt} + p_{ijt} \), where \( p_{ijt} \) is the log of the price of product \( i \) in store \( j \) in period \( t \). In addition, as discussed in Section 2, store’s sales play a key role in product substitution inside a store, which is another reason to rewrite the service function (3) in terms of sales.

**Consumers’ product choices inside a store.** How much stores sell of different products depends on consumer preferences across the product range inside a store. In our setting, consumers are homogeneous and have CES preferences over the differentiate products and services \( i \in \{1, \cdots, d\} \) of the store \( j \). We use the CES demand function to obtain the log of price of product \( i \) in store \( j \), \( p_{ijt} \)

\[
p_{ijt} = -\frac{1}{\sigma}(q_{ijt} - q_{0t}) + x'_{ijt} \beta_x + \frac{\sigma_a}{\sigma} a_{jt} + \frac{1}{\sigma} \tilde{\mu}_{ijt},
\]

where \( x_{ijt} \) are the observed determinants of the intensive and extensive margins of the utility function when the consumers buy the product \( i \) from store \( j \); \( \sigma \) is the elasticity of substitution; \( \tilde{\mu}_{ijt} \) are unobserved product characteristics at the store level, for example, the quality of the shopping experience attached to product \( i \) in store \( j \); and \( q_{0t} \) is the outside option. Multiplying (4) by the output weights (elasticities) \( \tilde{\alpha}_i \), summing up over the number of products, and using the result in (3), we obtain the sales generating function at the store level that is used to obtain the sales for product \( i \), \( y_{ijt} \)

\[
y_{ijt} = -\alpha_y y_{-ijt} + \beta_x l_{jt} + \beta_k k_{jt} + \beta_a a_{jt} + \frac{1}{\sigma} y_{mt} + x'_{jt} \beta_x + \omega_{jt} + \mu_{jt} + u_{ijt},
\]

where \( y_{-ijt} \) is the log of sales of other products than \( i \) and \( y_{mt} \) measures sales of outside option.\(^{17}\) The observed and unobserved product characteristics are aggregated at the store level using \( \tilde{\alpha}_i \) as weights. For example, \( \mu_{jt} \equiv (1/\sigma) \sum_{i=1}^d \tilde{\alpha}_i \mu_{ijt} \) measures the quality of the shopping experience at the store level. The vector \( x_{jt} \) includes observed store or local market characteristics (for example, population, population density, and income).

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\(^{15}\)Hortacsu and Joonhwi (2015) provides an extensive discussion on the link between CES and discrete choice demand approaches. Online Appendix B provides an extensive discussion on modeling demand choice.

\(^{16}\)\( \sigma \) is globally identified for the set of products with positive individual choice probabilities because this system satisfies the connected substitutes condition provided by Berry et al. (2013), i.e., it is invertible.

\(^{17}\)In the empirical implementation, the outside option is captured by the sales of products in a local market that do not belong to the five-digit sub-sector.
Consumers prefer stores with products in stock to minimize the search cost. We capture this effect by adding inventories in the demand equation. However, we will not be able to separately identify the effect on inventory on supply and demand, that is, we identify the net effect through $\beta_a$.

The multiproduct sales generation function (5) differs from a single product function by controlling for the impact of sales of other products on sales of a product inside a store, i.e., the impact of “competition” inside the store. By using sales of different products in equation (5), we are able to reduce the number of parameters to be estimated. Therefore, we estimate only the coefficient of sales of other products than product $i$ at the store $j$, i.e. $\alpha_y$, and not all coefficients $\alpha_i$, $i = 1, d$.\footnote{To obtain equation (5), we denote $\tilde{\alpha} i_yj_{jt} + \tilde{\alpha} i_yY_{ijt} \equiv \alpha i_yj_{jt}$ and $\tilde{\alpha} i_yy_{ijt} + \tilde{\alpha} i_yY_{ijt} \equiv \alpha y_yy_{ijt}$ and normalize $\alpha_i = 1$. The coefficients $\beta_c = \tilde{\beta}_c(1 - 1/\sigma)$ where $c \in \{l, k, x\}$.} The coefficient $\alpha_y$ plays a key role for both the persistence and productivity level. Estimating only one coefficient for the products (i.e., $\alpha_y$) when controlling for unobserved prices has a cost, that is, we cannot get a clean measure of technical productivity $\bar{\omega}_i$ because the coefficients of labor, capital and inventories include demand shocks even if we control for the elasticity of substitution. Therefore, the variable $\omega_{jt}$ ($\omega_{jt} \equiv (1 - 1/\sigma)\tilde{\omega}_{jt}$) measures the revenue total factor productivity (TFPR). The productivity measure $\omega_{jt}$ might include sales shocks due to approximations in (5), but all these demand shocks are other than the demand shocks $\mu_{jt}$ that affect consumer’s preferences for a store. In other words, we are able to separate productivity shocks $\omega_{jt}$ from the demand shocks that are associated with store’s quality of the shopping experience $\mu_{jt}$ (which is part of the demand system).

**Consumer’s store choice.** The store-level shocks $\omega_{jt}$ and $\mu_{jt}$ affect sales of each product according to equation (5). We can obtain information about the demand shocks $\mu_{jt}$ using a market share equation from a simple discrete choice demand system, which provides information about how consumers choose stores under the assumption that each consumer buys a product basket from a store. The number of products at the store level is an important determinant of store choice. Based on consumer choices, we can write the
store’s market share equation as (Berry, 1994):

\[
\ln(ms_{jt}) - \ln(ms_{0t}) = \rho_{np}np_{jt} + \rho_{inc,1}inc_{mt} + \rho_{inc,2}inc_{mt}^2 + \mu_{jt} + \nu_{jt},
\]

(6)

where \(ms_{0t}\) is the market share of the outside option, \(np_{jt}\) is the number of products (categories) at the store level, \(inc_{mt}\) is the log of average income in local market \(m\), and \(\nu_{jt}\) is mean independent of all store’s inputs and local market characteristics. The variable \(\mu_{jt}\) contains unobserved store-level demand shocks to service that are valuable for consumers when choosing stores. In other words, sales depend on both the demand shocks \(\mu_{jt}\) and productivity \(\omega_{jt}\), whereas market shares depend only on \(\mu_{jt}\) after controlling for store product variety and income.\(^{19}\)

The multiproduct setting in Section 2 emphasizes the importance of having a positive \(\alpha_y\) for static profit maximization to hold. In a dynamic setting, this condition also holds because a policy function (input choices) should be optimal in each period in a dynamic setting.\(^{20}\) Therefore, the next step is to discuss the store’s decisions that are used to recover \(\omega_{jt}\) and \(\mu_{jt}\).

Store’s decisions. In our setting, stores compete in the product market and collect their payoffs. At the beginning of each time period, incumbents decide whether to exit or continue to operate in the local market. Stores are assumed to know their scrap value received upon exit \(\delta\) prior to making exit and investment decisions. If the store continues, it chooses the optimal levels of labor \(l\) (the number of employees), investment \(i\), products bought from the wholesaler and to adjust their inventory \(a\). Store \(j\) maximizes the discounted expected value of future net cash flows using the Bellman equation (Olley and Pakes, 1996):

\[
V(s_{jt}) = \max \left\{ \delta, \sup_{a_{jt},i_{jt},l_{jt}} \left[ \pi(s_{jt}) - c_i(i_{jt},k_{jt}) - c_n(a_{jt}) + \beta E[V(s_{jt+1})|F_{jt}] \right] \right\},
\]

(7)

\(^{19}\)To account for the store’s size of the store, we can include capital in both sales and market share equations. However, this implies additional assumptions to identify the capital coefficient.

\(^{20}\)Most important, the sign conditions on first and second derivatives that are used to prove Theorem 1 and Proposition 1 remain the same in a dynamic setting.
where $s_{jt} = (\omega_{jt}, \mu_{jt}, k_{jt}, n_{jt}, w_{jt}, x_{mt}, r_{mt})$; $\pi(s_{jt})$ is the profit function that is increasing in $\omega_{jt}$, $\mu_{jt}$, and $k_{jt}$; $c_i(i_{jt}, k_{jt})$ is the investment cost of new capital (equipment), which is increasing in investment choice $i_{jt}$ and decreasing in current capital stock $k_{jt}$ for each fixed $i_{jt}$ (Pakes, 1994); $c_n(a_{jt})$ is the inventory wage at the store $j$; $x_{mt}$ is a vector of log of local market characteristics in local market $m$ (i.e., population, population density, and income); $r_{mt}$ measures entry regulation in local market $m$ in period $t$; $\beta$ is a store’s discount factor; and $F_{jt}$ represents the information available at time $t$. Labor, inventory holdings, and investments in technology might have dynamic implications due to adjustment costs, where both $\omega_{jt}$ and $\mu_{jt}$ are important for such adjustments.

**Assumption 2:** The store information set $F_{jt}$ includes only current and past information on productivity, demand shocks, input prices, and local market characteristics (not future values), for example, \( \{\omega_{jt}, \mu_{jt}, w_{jt}, x_{mt}, r_{mt}\}_{t=0}^T \). The remaining output shocks $u^p_{ijt}$ satisfies $E[u^p_{ijt}|F_{jt}] = 0$.

**Assumption 3:** Store productivity and exogenous shocks follow two first order Markov processes: (i) an endogenous process: $P_{\omega}(\omega_{jt} | \omega_{jt-1}, \mu_{jt-1}, r_{mt-1})$, where $r_{mt-1}$ measures regulation in local market $m$ in period $t-1$, (ii) an exogenous process: $P_{\mu}(\mu_{jt} | \mu_{jt-1})$, and (iii) the distributions $P_{\omega}($·$)$ and $P_{\mu}($·$)$ are stochastically increasing in $\omega$, $\mu$, and $r$, and they are known to the stores.

Assumption 2 says that stores know their productivity $\omega_{jt}$, demand shocks $\mu_{jt}$, and local market conditions when they make their inputs, inventory, investments, and exit decisions. Assumption 3 states that the demand shocks $\mu_{jt}$ are correlated over time according to a first-order Markov process

$$
\mu_{jt} = h^\mu(\mu_{jt-1}; \gamma^\mu) + \eta_{jt},
$$

where $h^\mu(\cdot)$ is an approximation of the conditional expectation, and $\eta_{jt}$ are shocks that are mean-independent of all information known at $t - 1$.

---

21 In the empirical implementation, there are no additional observed variables at the store level than those already introduced. Therefore, the vector $x_{jt}$ includes only local market variables, and we can write $x_{mt}$ instead of $x_{jt}$.
Store productivity $\omega_{jt}$ follows an endogenous first-order Markov process where productivity, previous demand shocks, and entry regulation affect future productivity:

$$
\omega_{jt} = h^{\omega}(\omega_{jt-1}, \mu_{jt-1}, r_{mt-1}; \gamma^{\omega}) + \xi_{jt},
$$

where $h^{\omega}(\cdot)$ is an approximation of the conditional expectation and $\xi_{jt}$ are shocks to productivity that are mean-independent of all information known at $t - 1$. In our setting, stores can learn to improve productivity using information on demand shocks and increasing competition due to a less restrictive entry regulation. A less restrictive regulation increases entry and, therefore, local competition. To survive on the market, stores improve productivity by learning successful practices from entrants (that is, external learning). Stores also use information from the previous quality of the shopping experience, that is a key factor for consumers to choose a store, to improve productivity. For example, re-arranging the products on the shelves such that consumers have faster access. As explained above, the productivity $\omega_{jt}$ is a sum of technical productivity and other demand shocks that are not related to shopping experience shocks $\mu_{jt}$.22

ASSUMPTION 4: Capital stock is a dynamic input that accumulates according to

$$
K_{jt+1} = (1 - \delta)K_{jt} + I_{jt},
$$

where $\delta$ is the depreciation rate. The investment $I_{jt}$ is chosen in period $t$ and affects the firm in period $t + 1$. Inventory level in period $t + 1$ evolves according to

$$
N_{jt+1} = A_{jt} - Y_{jt},
$$

where $A_{jt}$ is adjusted inventory, i.e., the inventories in the beginning of the period $N_{jt}$ adjusted by the products bought in period $t$.

Inventory affects store service output because high inventory is costly to keep in stock and low inventory reduces consumers’ choices. Products bought from wholesalers is an input that together with inventory at the beginning of period $t$ lead to inventory levels in the beginning of period $t + 1$ after realization of sales in period $t$. Stores with high $\mu_{jt}$ increase their products bought from wholesalers. However, this also leads to a drop in inventories at the beginning of next year because of the unexpected increase in sales.

---

22It is straightforward to control for selection as in Olley and Pakes’ (1996) framework, i.e., by adding $P_{jt}$ as an additional variable of $h^{\omega}(\cdot)$ function, where $P_{jt}$ are predicted survival probabilities of being in the data in period $t$, conditional on the information in $t - 1$, $P_{jt} = Pr(\chi_{jt} = 1|\mathcal{F}_{jt-1})$. The Markov process (9) implies that store productivity should shift, and stores that cannot improve productivity have to exit.
In other words, there is a distinction between how \( \mu_{jt} \) affects current inventories and products bought from the wholesaler, and the realization of inventories at the end of the year/start of next year.\(^{23}\)

The solution to a store’s maximization problem (7) yields optimal policy functions for the sum of store’s inventories at the beginning of period and cost of products bought

\[
a_{jt} = \tilde{a}_{jt}(s_{jt}), \quad \text{and} \quad \chi_{jt+1} = \tilde{\chi}_{jt}(s_{jt}). \]

\(^{24}\) We assume that labor \( l_{jt} = \tilde{l}_{jt}(s_{jt}) \), which is part of profits \( \pi(\cdot) \), is chosen to maximize short-run profits (Levinsohn and Petrin, 2003; Doraszelski and Jaumandreu, 2013; Maican and Orth, 2015; Maican and Orth, 2017).\(^{25}\)

The next step is to understand the assumptions on the policy functions (i.e., input demand functions) that are required to recover the shocks \( \omega_{jt} \) and \( \mu_{jt} \).

**ASSUMPTION 5:** The labor demand function \( l_{jt} = \tilde{l}_{jt}(s_{jt}) \) is strictly increasing in \( \omega_{jt} \). The store’s input products function \( a_{jt} = \tilde{a}_{jt}(s_{jt}) \) is strictly increasing in demand shocks \( \mu_{jt} \). The store productivity \( \omega_{jt} \) and demand shocks \( \mu_{jt} \) are part of the state space, i.e. \( \omega_{jt}, \mu_{jt} \in s_{jt} \), and the multivariate function \( (\tilde{l}_{jt}, \tilde{a}_{jt}) \) is a bijection onto \( (\omega_{jt}, \mu_{jt}) \).

The assumptions on the policy functions are not restrictive and most likely hold in many data sets. The most important assumptions that a policy function should satisfy to be consistent with the Bellman equation is to be strictly monotonic in the state variables. First, that productivity is increasing in labor can be shown when using Cobb-Douglas technology (Doraszelski and Jaumandreu, 2013; Maican and Orth, 2015; Maican and Orth, 2017). This characteristic implies that more productive stores do not have disproportionately higher markups than less productive stores. In addition, that the inventory demand function is increasing in demand shocks linked to consumers’ quality of the shopping experience inside the store is valid in retail markets. Maican and Orth

\(^{23}\)Cachon and Olivares (2010) argue that differences in inventories at the store level can arise because of differences in demand and competition. Lower margins decrease inventories, while a large choice set for consumers raises inventories. In addition, service production inside the store can also drive differences in inventories across stores.

\(^{24}\)The exit rule \( \chi_{jt} \) depends on the threshold productivity \( \omega_{nt} \), which is a function of all state variables except store productivity (Olley and Pakes, 1996). A store continues \( (\chi_{jt} = 1) \) if its productivity is larger than the local market threshold \( (\omega_{jt} > \omega_{nt}) \).

\(^{25}\)If labor has dynamic implications (e.g., in the case of labor adjustment costs), then labor in the previous period is part of the state space, and the optimal policy function for labor \( l_{jt} = \tilde{l}_{jt}(s_{jt}) \) is derived from solving the dynamic optimization problem (7).
(2017) show that the input demand function is strictly increasing in productivity under imperfect competition when input marginal product is increasing in productivity, which is fully consistent with store profit maximization behavior. Second, in our case with two unobservables, we discuss the invertibility of a system of nonlinear equations. A key condition for invertibility is that the determinant of the Jacobian is not zero. This condition is satisfied when productivity and demand shocks have different impacts on labor and inventory and the relative impact is not the same \(\frac{\partial \tilde{l}/\partial \omega}{\partial \tilde{l}/\partial \mu} \neq \frac{\partial \tilde{a}/\partial \omega}{\partial \tilde{a}/\partial \mu}\), which can be empirically tested. Online Appendix E discusses in greater detail the invertibility of the system of equations in our model.

3.1 Identification and estimation

This subsection discusses the identification and estimation of the service-generating function (3), which includes the estimation of the Markov processes for \(\omega_{jt}\) and \(\mu_{jt}\). In other words, we estimate \(\beta_k, \beta_a, \alpha_y, \sigma, \rho_{np}, \rho_{inc,1}, \rho_{inc,2}, \gamma^\omega, \) and \(\gamma^\mu\) together using a OP two-step estimator (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg et al., 2015). Compared to Olley and Pakes (1996), we have two unobservables to recover and apart from differences in the Markov processes, we show how the additional output equation (i.e., market share equation) helps to recover demand shocks \(\mu_{jt}\) separate from productivity \(\omega_{jt}\) and ensures the identification of the model (see also Maican et al., 2018).\(^{26}\) The dynamics of future store productivity is more complex in this paper since productivity is affected by both the actual shopping quality shock and local entry regulations. However, the core of identification comes from having a system of two equations with two unobserved variables where one variable is part of only one equation. In addition, two control functions based on the store’s optimal policy functions are used to proxy for \(\omega_{jt}\) and \(\mu_{jt}\).

\(^{26}\)Maican et al. (2018) estimate impact of R&D investments on shocks to domestic and foreign sales in Sweden. Using a single product function setting and a system of two equations (domestic and export sales), they recover the two shocks from investment demand and number of export destinations functions. See also Ackerberg et al. (2007) for an extensive discussion on identification of production functions.
Recovering productivity and demand shocks. The general labor demand and inventory functions that arise from the stores’ optimization problem (7) are

\[ l_{jt} = \tilde{l}_t(\omega_{jt}, \mu_{jt}, k_{jt}, n_{jt}, w_{jt}, x_{mt}, r_{mt}) \]

\[ a_{jt} = \tilde{a}_t(\omega_{jt}, \mu_{jt}, k_{jt}, n_{jt}, w_{jt}, x_{mt}, r_{mt}) \]

To back out \( \omega_{jt} \) and \( \mu_{jt} \), the assumption 5 must hold, i.e., the functions \( \tilde{l}_t(\cdot) \) and \( \tilde{a}_t(\cdot) \) must be strictly monotonic in \( \omega_{jt} \) and \( \mu_{jt} \), which holds under mild regularity conditions on the dynamic programming problem (7).\(^{27}\) A higher \( \mu_{jt} \) implies an increase in the quality of the shopping experience inside the store, and stores react by increasing inventories without changing product variety or increase the product variety (i.e., higher love-for-variety), which implies an increase in inventories. Technological advances inside the store can benefit the existing number of products through faster product lines and a higher frequency of turnover (Holmes, 2001). However, high store productivity creates incentives for stores to increase their product variety and store size. By inverting these policy functions to solve for \( \omega \) and \( \mu \) we obtain

\[ \omega_{jt} = f^1_t(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{mt}, x_{mt}, r_{mt}) \]

\[ \mu_{jt} = f^2_t(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{mt}, x_{mt}, r_{mt}) \]

(10)

i.e., the productivity and exogenous shocks are nonparametric functions of the observed variables in the state space and the controls.

In our setting, the estimation of the service generating function (5) and the market share equation (6) is done in two steps. In the first step, we construct measures of productivity \( \omega_{jt} \) and the demand shocks \( \mu_{jt} \) as functions of the structural parameters that do not include the sales and market shares shocks (i.e., \( u^p_{ijt} \) and \( \nu_{jt} \)). To do this, we use the equations (5) and (6) and the solution of the system of nonparametric policy functions given by (10).

By substituting the nonparametric inversion \( f^2_t(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{mt}, x_{mt}, r_{mt}) \) for \( \mu_{jt} \) in (6) and considering that the number of products \( np_{jt} \) is also a function of the store

\(^{27}\)See Online Appendix E, Pakes (1994) and Maican (2016b).
state variables (i.e., it is a policy function of the store optimization problem), the market share equation can be written as \( \ln(ms_{jt}) - \ln(ms_{0jt}) = b_t(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{mt}, x_{mt}, r_{mt}) + \nu_{jt} \), which can be estimated using OLS and a polynomial expansion of order 3 in \( l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{mt}, x_{mt}, r_{mt} \) to approximate function \( b_t(\cdot) \). Therefore, we obtain an estimate of \( b_t(\cdot) \), i.e., \( \hat{b}_t \), which is the predicted \( \ln(ms_{jt}) - \ln(ms_{0jt}) \). This allows us to write demand shocks \( \mu_{jt} \) as a parametric function, i.e., \( \mu_{jt} = \hat{b}_jt - \rho_{np}np_{jt} - \rho_{inc,1}inc_{mt} - \rho_{inc,2}inc_{mt}^2 \), which will be treated as an input in the multi-output service-generating function (5).

In the second step, by substituting \( \mu_{jt} \) (predicted) and \( \omega_{jt} \) into (3), the service-generating function becomes

\[
y_{ijt} = -\alpha_y y_{-ijt} + \phi_t(l_{jt}, k_{jt}, n_{jt}, w_{jt}, a_{jt}, y_{mt}, x_{mt}, r_{mt}) + \nu_{ijt}'p,
\]

where \( \phi_t(\cdot) = \beta_l l_{jt} + \beta_k k_{jt} + \beta_a a_{jt} + \frac{1}{\sigma} y_{mt} + x_{jt}' \beta_x + \omega_{jt} + \mu_{jt} \). The function \( \phi_t(\cdot) \) can be approximated using a polynomial expansion of order 3 in its arguments. The estimation of (11) yields an estimate of service output without service output shocks \( \nu_{ijt}'p \), which gives us \( \hat{\phi}_t \), that is used to obtain store productivity \( \omega_{jt} \) as a function of the parameters, \( \omega_{jt} = \hat{\phi}_jt - \beta_l l_{jt} - \beta_k k_{jt} - \beta_a a_{jt} - \frac{1}{\sigma} y_{mt} - x_{jt}' \beta_x - \hat{b}_jt - \rho_{np}np_{jt} + \rho_{inc,1}inc_{mt} + \rho_{inc,2}inc_{mt}^2 \).

The next step is to rewrite the sales and market share equations using parametric forms of productivity \( \omega_{jt} \) and demand shocks \( \mu_{jt} \) and Markov processes

\[
y_{ijt} = -\alpha_y y_{-ijt} + \beta_l l_{jt} + \beta_k k_{jt} + \beta_a a_{jt} + \frac{1}{\sigma} y_{mt} + x_{jt}' \beta_x + \hat{b}_jt - \rho_{np}np_{jt} - \rho_{inc,1}inc_{mt} - \rho_{inc,2}inc_{mt}^2 - \rho_{inc,1}inc_{mt} - \rho_{inc,2}inc_{mt}^2 - \rho_{inc,1}inc_{mt} - \rho_{inc,2}inc_{mt}^2
\]

\[
\hat{b}_jt + \rho_{np}np_{jt} + \rho_{inc,1}inc_{mt} + \rho_{inc,2}inc_{mt}^2 + \hat{b}_jt - 1 + \rho_{np}np_{jt-1} + \rho_{inc,1}inc_{mt} + \rho_{inc,2}inc_{mt}^2
\]

\[
- \rho_{np}np_{jt-1} - \rho_{inc,1}inc_{mt} - \rho_{inc,2}inc_{mt}^2 + \xi_{jt} + u_{jt}'p
\]

\[
ln(ms_{jt}) - ln(ms_{0jt}) = \rho_{np}np_{jt} + \rho_{inc,1}inc_{mt} + \rho_{inc,2}inc_{mt}^2 + \rho_{r}r_{mt} + h^\mu(\hat{b}_jt - 1)
\]

\[
- \rho_{np}np_{jt-1} - \rho_{inc,1}inc_{mt-1} - \rho_{inc,2}inc_{mt-1}^2 + \eta_{jt} + \nu_{jt}
\]

\[
\xi_{jt} + u_{ijt}'p
\]

The parameters of the multiproduct sales function (12) and market share equation (13) are identified using moment conditions on the remaining shocks in these equations, \( \xi_{jt} + u_{ijt}'p \).

\[28\] Other approximations can be used, such as b-splines, for example.
and \( \eta_{jt} + \nu_{jt} \).

**Estimation.** In the empirical implementation, we approximate the functions \( h(\cdot) \) and \( h(\cdot) \) in the Markov processes of \( \omega_{jt} \) and \( \mu_{jt} \) by polynomials. Therefore, the estimated Markov processes are:

\[
\omega_{jt} = \gamma_0^\omega + \gamma_1^\omega \omega_{jt-1} + \gamma_2^\omega (\omega_{jt-1})^2 + \gamma_3^\omega (\omega_{jt-1})^3 + \gamma_4^\omega \mu_{jt-1} + \gamma_5^\omega r_{mt-1} + \gamma_6^\omega \omega_{jt-1} \times \mu_{jt-1} + \gamma_7^\omega \omega_{jt-1} \times \mu_{jt-1} + \xi_{jt}
\]

(14)

\[
\mu_{jt} = \gamma_0^\mu + \gamma_1^\mu \mu_{jt-1} + \gamma_2^\mu (\mu_{jt-1})^2 + \gamma_3^\mu (\mu_{jt-1})^3 + \eta_{jt}
\]

(15)

Because the \( \omega_{jt} \) and \( \mu_{jt} \) are functions of the parameters of sales and market share equations, we can identify these parameters using moment conditions on residuals in the Markov processes, \( \xi_{jt} \) and \( \eta_{jt} \).  

The parameters \( \theta = (\beta_1, \beta_2, \beta_3, \sigma, \theta_x, \rho_{np}, \rho_{inc}, \gamma^\omega, \gamma^\mu) \) are estimated by minimizing the Generalized Method of Moments (GMM) objective function

\[
\min_{\beta} Q_N = \left[ \frac{1}{N} W' v(\theta) \right]' A \left[ \frac{1}{N} W' v(\theta) \right],
\]

(16)

where \( v_{jt} = (u_{ijt} + \xi_{jt}, \nu_{jt} + \eta_{jt})' \), \( W \) is the matrix of instruments, and \( A \) is the weighting matrix defined as \( A = \left[ \frac{1}{N} W' v(\beta) v(\beta) W \right]^{-1} \). Standard errors are computed according to Ackerberg et al. (2012).  

The current shocks are conditionally independent from information in \( t-1, F_{jt-1} \), which implies that the variables in the period \( t-1 \) are good as instruments, i.e., \( y_{-i,jt-1}, l_{jt-1}, k_{jt-1}, a_{jt-1}, y_{mt-1}, x_{mt-1}, np_{jt-1} \ inc_{mt-1}, inc^2_{mt-1} \). The Markov processes parameters \( \gamma^\omega \) and \( \gamma^\mu \) are identified using the corresponding polynomial terms in the equations (14) and (15) as instruments.

---

29As Ackerberg et al. (2015) discuss in Section IV(i), there are many ways to estimate an Olley and Pakes’ framework based on second step moments. Most important, stronger assumptions can lead to more precise estimates.

30Bootstrapping might not be the best choice when the underlying model is more complicated.
3.2 Estimation of the impact of regulation on inventory cost

Entry regulations affect stores’ operating cost (Joskow and Rose, 1989; Maican and Orth, 2018a). A more restrictive regulation can increase stores’ operating cost due to an increase in the fixed cost (e.g., expensive location or to build). Second, there are few stores in a market and as result the cost with logistics can increase and product differentiation decreases. The decrease in product differentiation can be associated with lower time consumer spend in a store, i.e., consumers travel more and spend less time in a store when there are with fewer stores in the market. Therefore, the impact of entry regulations on the store’s cost affects optimal inventory.

We assume that stores have a quadratic adjustment cost in inventories (millions SEK), i.e.,

\[ c_n(A_{jt}, r_{mt}; \varphi) = \varphi_0 + \varphi_1 A_{jt} + \varphi_2 A_{jt}^2 + \varphi_3 A_{jt} r_{mt}. \] (17)

The cost specification (17) allows for the impact of stringency of regulation on store’s inventory cost. The marginal effect of a more liberal regulation (i.e., an increase in \( r_{mt} \)) on cost of inventory depends on the size of store’s inventory \( A_{jt} \). A change in entry regulation affects the store’s cost and its productivity and, therefore, the optimal inventory policy. Thus, the sales of different product categories are affected. To compute total sales at the store level from using our model, we need to solve a system of nonlinear equations at the store level.\(^{31}\)

The store’s value function is given by the following Bellman equation

\[ V(s_{jt}) = E_{\zeta_{jt}} \left[ \max_{A_{jt}, L_{jt}} [Y_{jt}(s_{jt}) - c_l(L_{jt}) - c_n(A_{jt}, r_{mt}; \varphi) + \beta E[V(s_{jt+1}, \zeta_{jt+1}) | F_{jt}]] \right], \] (18)

where \( Y_{jt}(s_{jt}) \) are total sales, \( c_l(L_{jt}) = W_{jt} L_{jt} \) is the variable cost of labor, and \( \zeta_{jt} \sim N(0, 1) \) are private shocks to net profits.\(^{32}\) Therefore, \( \varphi = (\varphi_0, \varphi_1, \varphi_2, \varphi_3) \) are the parameters to be estimated in the dynamic stage using value function approximation and

\(^{31}\)The system of equations have a unique solution and is solved using fixed-point iteration (see also Maican and Orth, 2019).

\(^{32}\)An alternative specification is to allow private shocks to marginal cost, i.e., \( \zeta_{jt} \sim N(0, \sigma^2_\zeta) \) is part of the cost function: \( c_n(A_{jt}, r_{mt}, \zeta_{jt}; \varphi) = \varphi_0 + \varphi_1 A_{jt} + \varphi_2 A_{jt}^2 + \varphi_3 A_{jt} r_{mt} + \varphi_4 \zeta_{jt} A_{jt} \). This specification is computationally more demanding (see the discussion in Bajari et al. (2007) and Ackerberg et al. (2007)) and also affects the identification of the productivity and demand shock.
simulation (Ryan, 2012; Maican, 2016a).

We approximate the value function \( V \) using b-splines, i.e., \( V(s_{jt}; \kappa) = \text{bs}(s_{jt}; \kappa) \), where \( \text{bs}(\cdot) \) are the basis functions. This allow us to rewrite the Bellman equation as

\[
V(s_{jt}; \kappa) = \mathbb{E}_{\zeta_{jt}} \left[ \max_{A_{jt}, L_{jt}} \left[ Y_{jt}(s_{jt}) - c_l(L_{jt}) - c_n(A_{jt}, \varphi) + \beta \mathbb{E}[V(s_{jt+1}, \zeta_{jt+1}; \kappa) | F_{jt}] \right] \right].
\]

(19)

For each set of cost parameters \( \varphi \), we use the linearity property of of the value function approximation to find the parameters approximation parameters \( \kappa \) such that Bellman equation holds. We use b-spline approximation of to find the optimal policies, which help to construct the state variables in \( t + 1 \).

The cost parameters are estimated using the inequality estimator that uses alternative policy, i.e, \( a'(s_{jt}, \zeta_{jt}) = \hat{a}(s_{jt}, \zeta_{jt}) + \psi \), where \( \psi \sim N(0, 1) \) (see Bajari et al., 2007). Let \( d \) be any combination of \( (s_{jt}, a'_{jt}) \), and define

\[
m(d; \kappa, \varphi) = V(s_{jt}; \kappa, \varphi, a_{jt}) - V(s_{jt}; \kappa, \varphi, a'_{jt}).
\]

(20)

We denote by \( \hat{m}_{Ns}(d; \kappa, \varphi) \) a simulator of \( m(d; \kappa, \varphi) \) evaluated at the estimated policy functions, where \( N_s \) is the number of simulations. The inequality estimator minimizes

\[
\min_{\varphi} J_{NI} = \frac{1}{N_I} \sum_{k=1}^{N_I} 1 \{ \hat{m}_{Ns}(d; \kappa, \varphi) < 0 \} \hat{m}_{Ns}(d; \kappa, \varphi)^2,
\]

(21)

where \( N_I \) is the number of inequalities. The standard errors are computed using sub-sampling. Alternative estimators that can be used are the minimum distance or indirect inference estimator, which are not only more efficient estimators but also more computationally demanding since they require to solve directly for the optimal policies in our case (Pakes et al., 2007; Maican and Orth, 2018a; Maican et al., 2018).

4 Entry regulations and data

Entry regulations in Sweden. The goal of policymakers is that all individuals in the society should access a wide variety of products and stores within reasonable geographic
distances to low prices. To reach this goal, local governments have the power to decide over the entry of new stores. The Swedish Plan and Building Act (PBL) regulates the use of land, water and buildings. The regulation contains a comprehensive plan that covers and guides the use of the entire municipality and detailed development plans that cover only a fraction of the municipality. The detailed development plans divide municipalities into smaller areas for which limits on use and design are set, i.e., construction rights for real estate and whether areas can be used for work places, housing, schools, parks etc. Entering a new store requires that the PBL admits operations of retail activities in the geographic area where the store wants to enter. A formal application needs to be sent to the municipal government that is supposed to evaluate consequences on prices, accessibility of store types and products for different consumer groups, traffic and broader environmental issues etc. The local government can accept or reject an application. Our application to Sweden is relevant and has broad implications as the Swedish regulation is similar to most other European countries (Pozzi and Schivardi, 2016). Appendix F provides an extensive discussion on PBL in Sweden.

Data. We use three data sets provided by Statistics Sweden (SCB) and the Swedish Mapping, Cadastral and Land Registration Authority. The first data set covers detailed annual information about all retail firms in Sweden (census) during the period 2000 to 2010. The data contain financial statistics of input and output measures, i.e., sales, value-added, number of employees, capital stock, inventories, cost of products bought, investment, etc. Inventories capture the value of products held in stock in the end of each year and are taken from book values (accounting data). The cost of products bought measures store’s cost of buying products from the wholesaler. The cost of products bought and inventories both rely on the input prices of goods, i.e., they are based on what stores pay to the wholesaler. In other words, sales and value-added are measured in output prices, whereas cost of products bought and inventories are measured in input prices. Because of difficulties in measuring quantity units in retailing arising from the nature and complexity of the product assortments, quantity measures of output and inventories are not available.
Our second data set covers store-level information on all product categories and their yearly sales. To the best of our knowledge, such detailed data on the number of products (product categories) in services industries across local markets have not been used in the literature before. Unique identification codes allow us to perfectly match products to stores.\textsuperscript{33}

The third data set contains data on the number of the applications approved by local authorities for each municipality and year (Swedish Mapping, Cadastral and Land Registration Authority). This includes applications to change land-use plans and the total number of existing land-use plans.\textsuperscript{34}

The empirical application focuses on the three-digit industry Retail sale of new goods in specialized stores (Swedish National Industry (SNI) code 524). This retail sector includes the following sub-sectors at the five-digit SNI: clothing; furniture and lighting equipment; electrical household appliances and radio and television goods; hardware, paints and glass; books, newspapers and stationery; and other specialized stores. In the empirical implementation, we estimate the sales generating function at the product category level for three-digit industry and add controls at the five-digit level.

**Descriptive statistics and stylized facts.** Table 1 shows that there is an aggregate increase in sales, value added, average number of product categories, investments, and labor over time. From 2005 to 2009, sales increased by 36 percent, investments by 53 percent and number of employees by 21 percent. An average store has about 4 product categories. The number of product categories varies between 1 and 17 in our sample, which includes about 1,100 stores per year. Our regulation measure, i.e., the average number of approved applications over population density has increased from 0.23 to 0.29 in 2008. That is an increase from 23 to 29 approved applications per 100 squared kilometers. The increased number of approved applications per population density is associated with the increase in competition due to a more liberal regulation, and this pattern is confirmed by the negative correlations over time between the store sales per product category

\textsuperscript{33}The product data set follows a similar classification system to the one used for the sample data collected on prices and quantities in manufacturing.

\textsuperscript{34}Municipalities with a non-socialist majority approve more PBL applications. The correlation between non-socialist seats and the number of approved PBL applications in local markets is 0.6.
and our regulation measure.

Figure 1 shows the distributions of different store performance measures before and after the acceptance of new PBL applications using box-plot charts. We measure store performance by labor productivity (log of sales per employee), market share, inventory performance (log of sales per average inventory and log of cost of good sold over average inventory). The figures show that median labor and inventory productivity is larger in markets after the acceptance of new applications. In addition, the median market share decreases after accepting new PBL applications. All these findings suggest a positive relationship between increasing competition due to a more liberal regulation and store performance, which confirms the recent empirical results of the positive impact of competition on firm productivity discussed in Section 1. The changes in basic store performance distributions due to a more liberal regulation show clear evidence that we have to control for changes in entry regulation when developing more sophisticated measures of store performance such as TFP.

The box-plots in Figure 2 show that the median store has more product categories and higher sales per product category after the acceptance of new PBL applications in the local market. First, these results show that consumers benefit from more product variety in markets with a more liberal regulation. Second, stores also benefit since sales per product category increase from a more liberal regulation. However, the drivers of these patterns are unclear without a structural approach. The reason is that stores might improve their productivity as a reaction to the more intense competition, and they reposition their product portfolio to create additional demand. Therefore, it is important to control for demand when measuring productivity even if it is difficult to remove all demand shocks from productivity in service industries in absence of detailed prices. The benefit is that we better understand the role of demand for productivity dynamics.

5 Results

This section presents the estimation results of the model. We first discuss the results of the estimated multiproduct sales generating function and the evolution of store pro-
ductivity and demand shocks. Second, using estimated productivity and demand shocks we analyze their impact on the number of product categories at store and market level. Third, we estimate the structure of the adjustment cost in inventory, which is a function of entry regulation.

**Service generating function estimates.** Table 2 shows the estimates of the multi-product service generating function (equation 5) by the Ordinary Least Squares (OLS) estimator and the nonparametric two-step estimator presented in Section 3.\(^{35}\) The two-step estimator controls for the endogeneity of store input choices and allows to separately identify two shocks that affect store service generating function (productivity \(\omega_{jt}\)) and market shares (demand shock \(\mu_{jt}\)). The recovered \(\omega_{jt}\) measures revenue total factor productivity (TFPR), and it might include other demand shocks than those that affect stores’ market share (i.e., \(\mu_{jt}\)), population, population density, and income.

The estimated coefficients of labor and inventories decrease from 0.786 (OLS) to 0.571 and from 1.037 (OLS) to 0.411, respectively, using the two-step estimator. The coefficient of capital increases, i.e., it is 0.061 (OLS) and 0.289 (the two-step estimator). These changes in the estimated coefficients are in line with the production function literature following Olley and Pakes (1996), which suggests an upper bias for the coefficients of labor and inventories when omitting to control for the correlation between inputs and productivity.

The estimated elasticity of demand for product substitution \((1/\sigma)\) is 3.480. The estimates are consistent with profit maximization behavior of multiproduct firms since sales of a product category decreases when sales of other products increases (Mundlak, 1964).\(^{36}\) The magnitude of the coefficient of the other product categories plays a key role for the productivity measure since it influences the input coefficients (labor, capital, inventories).

The estimated results also give the impact of demand shifters on stores’ sales and market shares. Stores in large markets and markets with high population density sell more of each product category. The number of product categories and income have a

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\(^{35}\) To allow for comparisons across specifications, the two-step estimated coefficients are adjusted for elasticity of substitution \(\sigma\) and the coefficient of other product categories \(\alpha_y\).

\(^{36}\) On average, one percent increase in sales of other products decrease sales of a product category by 0.865 percent, which suggests a relatively high intensity of competition for sales space inside the store.
positive impact consumer’s utility function and, therefore, on store’s market share. This suggests that consumers benefit from a larger product variety (number of product categories), which is consistent with the previous literature, i.e., love-for-variety. On average, a store with a 30 percent market share gains 5 percent market share by adding one more product category.

**Productivity and demand shocks dynamics.** Table 3 shows the estimates of the processes for productivity $\omega_{jt}$ and demand shocks $\mu_{jt}$, i.e., equations (14) and (15). The estimation of both processes is done together with the estimation of the multiproduct service generating function (Section 3). In our model, entry regulation and demand shocks can affect store productivity and the size of the effects depend on the level of previous productivity, demand shocks, and stringency of regulation. The results in Table 3 show that we reject the null hypothesis that the coefficients of demand shocks $\mu_{jt}$ into the productivity process are equal to zero ($p$-value=0.000). A more liberal entry regulation has a positive impact on productivity, i.e., one more approval increases productivity by on average 0.120 percent.\(^3\) However, the impact of entry regulation on productivity is decreasing in productivity.

The demand shocks also have a positive impact on productivity, i.e., a one percent increase in $\mu_{jt}$ raises productivity by on average 0.018 percent (see Maican and Orth, 2018b). Therefore, stores with high demand shocks (i.e., high shopping quality) have high future productivity, and the impact is increasing in productivity. In other words, stores learn from managing current demand to improve their future productivity. This is not surprising in services industries where demand shocks affect inventory management, the choice of inputs, and product variety, which in turn affect the future store productivity.

A key factor that drives the dynamics in productivity and demand shocks is the persistence. The findings show that the persistence of the productivity process (0.869) is lower

\(^3\)The average is computed based on the observed population density, where the largest marginal effect is about 9 percent (the standard deviation is 0.943). Based on an earlier study period (1996-2002) and no information on products and inventories, Maican and Orth (2015) also find a positive effect of a more liberal entry regulations in different retail industries (with marginal effects up to 10 percent).
than the persistence of the store’s demand shocks (0.943). The size of the persistence in productivity is similar to the findings in other studies in the productivity literature (e.g., Doraszelski and Jaumandreu, 2013; Maican and Orth, 2017).

The distributions of productivity and demand shocks are recovered from estimating the multiproduct sales generating function and market share equation. Figure 3 presents the empirical distributions of the productivity and demand shocks for stores in different quartiles of product categories in restrictive and liberal markets. The box-plots show the following patterns. First, the median productivity ($\omega_{jt}$) and shopping quality ($\mu_{jt}$) are higher in liberal than in restrictive markets. Second, stores with a large number of product categories have higher productivity. Third, the interquartile range in the shopping quality is lower in the liberal than in restrictive markets for stores below the 75th percentile of in the product category. Correlated with the productivity findings, the results show that consumers in restrictive markets might have access to low shopping quality products/services in some markets even if the stores offer few product categories. Therefore, it is important to analyze the determinants of product categories at the store level in order to understand the heterogeneity across stores and markets.

**Determinants of product variety.** The next step is to analyze the impact of stores productivity and demand shocks on the number of product categories and competition between product categories for sales space inside stores. Table 4 shows reduced-form evidence of the impact of productivity, demand shocks, investments, and capital on the product Herfindahl-Hirschman Index (HHI) and on the number of product categories inside the store.\(^{38}\) The OLS estimator is used for the HHI specification, and a quasi-Poisson estimator is used for the number of product categories. Both specifications include additional store controls and fixed effects for local market, five-digit industries, and year.

The findings show that increasing productivity results in higher competition for product space inside store (i.e, lower HHI). The negative impact of productivity on HHI is decreasing (increasing) with productivity and demand shocks level in restrictive (liberal) markets. The impact of productivity on the product competition inside store is larger.

\(^{38}\)Sales at the product category are used to compute HHI inside store.
in liberal than in restrictive markets. Stores with high shopping quality ($\mu_{jt}$) have low competition between product categories (high HHI), and there is no major difference between liberal and restrictive markets. However, the impact is increasing (decreasing) in productivity in restrictive (liberal) markets.

An increase in store productivity yields an increase in the number of product categories at the store level and the impact is decreasing in productivity levels and is larger in liberal than in restrictive markets. Stores with high shopping quality experience have a smaller number of product categories. The findings show that increasing investments in technology in restrictive markets has a positive effect on the number of product categories. Summarizing, the findings from Table 4 show that there are differences in the impact of productivity and demand shocks between the restrictive and liberal markets. To understand what drives the difference between restrictive and liberal markets is useful in designing various counterfactual policies that aim to reduce these observed differences.

Table 5 presents the determinants of the unique number of product categories at the local market. In other words, we aggregate the information from store to market level to analyze what drives product variety across markets. We find that markets with high median productivity have a larger unique number of product categories. As for the store-level results, the positive impact of productivity is larger in liberal than in restrictive markets. Markets with high shopping quality have few number of product categories. In addition, restrictive markets with large investments in technology have a large number of product categories.

Table 6 shows that the estimated coefficients of cost with inventory. The average fixed cost with inventory is about 91,000 SEK. The results highlight that marginal cost decreases with the level of inventory (i.e., coefficient of $A_{jt}^2$ is negative). Most important, a more liberal entry regulation decreases the cost of inventory, which emphasizes the importance of entry regulation for optimal inventory.
6 Policy experiments

We use the estimated model to conduct counterfactual policy experiments, that is, we compare the store choices and outcomes as well as aggregate local market outcomes before and after a hypothetical change in entry regulation, shocks to productivity and/or demand, and store’s cost of inventories. To simulate outcomes of hypothetical changes, we use the underlying primitives of the dynamic model and the estimated evolution of the state variables (i.e., productivity, shopping quality, and market characteristics) to predict stores’ decisions and changes in inputs, investment, and the number of product categories. This allows us to compute changes in welfare measures (e.g., consumer surplus).

The proposed policy experiments aim to highlight the impact of entry regulation on product variety through different channels. A change in entry regulation will affect local competition, which make stores to reposition in different directions. Changes in stringency of entry regulation affect store’s productivity and its cost of inventory which drives a repositioning in product variety.

6.1 The trade-off between promoting competition through new applications and raising stores’ shopping quality without changes in store’s cost of inventory

The first set of counterfactual policy experiments are consistent with the current policy function estimates because the cost structure does not change. This implies that the new equilibrium is consistent with the observed equilibrium in the data, which allow us to use the estimated policy functions in the simulations, i.e., there is no need to re-estimate the policy functions (Bajari et al., 2007). For each store, we compute average outcomes over 100 simulations over five years starting with the last year in the data. The base simulations use estimated means and standard deviations of the remaining productivity and demand shocks (i.e., $\xi_{jt}$ and $\nu_{jt}$). To compute store market shares and consumer surplus in the policy experiments, we use the market share equation in Section 3 and assume a logit demand model where local market characteristics such as population and income
evolve as AR(1) processes. Using the logit demand system to compute market shares for the years in the data, we accurately predict the observed store market shares. This suggests that the logit demand is not a restrictive assumption in our application since we are not interested in cross price elasticities (Berry, 1994).

This subsection discusses the effects in short- and long-run of two policy experiments that trade-off the improvements in productivity through changes in entry regulation and store shopping quality experience. The first two counterfactuals \( CF_1 \) and \( CF_2 \) show how different channels to improve productivity affect stores’ choices and outcomes in the restrictive markets. The counterfactual \( CF_1 \) aims to evaluate the effects of increasing productivity in the restrictive markets by increasing the number of approved applications (i.e., a change in the external environment for stores). The increase in productivity results from the estimated productivity process (i.e., the positive impact of a more liberal entry regulation on future productivity). Because productivity in restrictive markets will improve, this affects stores’ choices in labor, inventories, investments and the number of product categories. Consumer surplus is affected through the change in the number of product categories.

The counterfactual \( CF_2 \) studies the effects of the changes in productivity due to changes in the shopping quality in restrictive markets. This type of policy can be implemented, for example, if entry costs are too high and there are few application approved. In this case, stores aim to improve the quality of shopping experience \( \mu_{jt} \) to not loose the customers to alternative options such as online shopping platforms, for example. \( CF_2 \) is implemented by shifting the mean of the i.i.d. \( \nu_{jt} \) shocks of the demand shocks process. The estimated mean of these shocks is close to zero, and in \( CF_2 \) the mean is set to 0.02 such that the average productivity growth is the same in \( CF_1 \) and \( CF_2 \) (that is, equivalent policy experiments). In \( CF_2 \), both productivity and demand processes are affected, and, therefore, the changes in both \( \omega \) and \( \mu \) impact the number of product categories. Stores market share is affected by changes in the number of product categories and shopping

\[39\] The outside option includes stores that are not in the product-level sample of stores. Because we access input and output measures for all stores even if we do not have information about products, and we can compute store’s market shares using sales.
quality $\mu$.  

Table 7 presents the mean and interquartile range for changes in consumer surplus, the number of products, inventory level before and after sales, the number of employees, and the investments in technology from $CF_1$ and $CF_2$. The figures for the $CF_1$ show that consumer surplus increases due to a more liberal regulation, and this increase is driven by the increase in the number of products (a relatively small increase). There is a decrease in the value of the inventory before sales, but an increase in the end of year inventory. First, a more liberal regulation increases competition and stores can benefit by signing better contracts with the wholesalers, which can explain the drop in the value of the inventory before sales. Second, because of the increasing competition, stores reposition and alter their product line (range) in a product category and aim to keep more inventories on in the storage to avoid the stock-outs. The increase in competition create incentives to reduce the labor and invest in technology in the long-run.

The increase in the store’s shopping quality yields a larger increase in the consumer surplus in $CF_2$ than in $CF_1$, which can be explained by the increase in both shopping quality and the number of product categories that are part of consumer surplus. The changes in shopping quality bring changes in labor inside the store. While there is a small decrease in labor on average, there is a large interquartile range which is driven by the large positive increase in the number of employees for stores in the 75th-90th percentiles. First, a large increase in labor is explained by the increase in the number of product categories inside stores. Second, in retail, labor plays a key role in supporting an increase shopping quality.

6.2 The trade-off between subsidizing stores and a reduction in the impact of regulation on the cost of inventory

The second set of the policy experiments studies trade-off between subsidizing stores ($CF_3$) and a reduction of the marginal cost of the impact of entry regulation on the cost with inventory ($CF_4$) in the restrictive markets. The subsidy policy is implemented through decreasing the marginal tax on sales, i.e., a reduction by 10 percent. The av-
verage monetary equivalent of the change in the marginal impact of the regulation is a
decrease in the coefficient of $\varphi_3$ in the cost function from -0.0148 to -0.0310. Because of
the changes store’s primitives such as net profits and cost structure, the policy functions
need to be re-estimated. To do this, we need to solve the store’s dynamic optimization
problem to find optimal inventory before sales $a_{jt}$, optimal number of employees $l_{jt}$, and
optimal investment $i_{jt}$.

In the $CF_3$, our policy intervention does not create direct incentives for stores to do
structural changes in productivity and cost with inventory. However, there are indirect
mechanisms that will affect store performance through changes input demand. On the
other hand, $CF_4$ should be seen as allowing for innovations the store level that help to
reduce the impact of a more restrictive regulation. A direct impact of policy $CF_4$ is on
the optimal level of inventory. The cost reduction allows stores to hold more inventories,
and, therefore, product variety repositioning can appear.

Table 8 shows the results from the counterfactuals $CF_3$ and $CF_4$.\footnote{These are preliminary estimates.}\footnote{These are preliminary estimates.} First, the figures
show a larger increase in consumer surplus, inventory before sales, and in the long-run
profits after decreasing the marginal cost of inventory that is associated with entry regula-
tion. Second, cutting cost with inventories yields a larger increase in the long-run profits
than in the consumer surplus.
References


Table 1: Descriptive statistics

<table>
<thead>
<tr>
<th>Year</th>
<th>No. of stores</th>
<th>Sales</th>
<th>Value added</th>
<th>Investment</th>
<th>No. of employees</th>
<th>Mean no. of product categories at store level</th>
<th>Mean of no. of PBL applications per population density</th>
<th>Corr. sales per product and no. of PBL applications per population density</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>587</td>
<td>80,454</td>
<td>17,518</td>
<td>1.286</td>
<td>31,424</td>
<td>3.101</td>
<td>0.228</td>
<td>-0.020</td>
</tr>
<tr>
<td>2005</td>
<td>1,139</td>
<td>97,144</td>
<td>22,358</td>
<td>1.531</td>
<td>39,468</td>
<td>4.514</td>
<td>0.263</td>
<td>-0.005</td>
</tr>
<tr>
<td>2006</td>
<td>1,006</td>
<td>103,116</td>
<td>23,448</td>
<td>1.796</td>
<td>38,640</td>
<td>4.151</td>
<td>0.253</td>
<td>-0.004</td>
</tr>
<tr>
<td>2007</td>
<td>1,137</td>
<td>147,852</td>
<td>30,497</td>
<td>2.466</td>
<td>47,104</td>
<td>4.399</td>
<td>0.289</td>
<td>-0.020</td>
</tr>
<tr>
<td>2008</td>
<td>1,180</td>
<td>130,613</td>
<td>26,427</td>
<td>2.528</td>
<td>49,130</td>
<td>4.185</td>
<td>0.265</td>
<td>-0.040</td>
</tr>
<tr>
<td>2009</td>
<td>1,055</td>
<td>131,826</td>
<td>27,123</td>
<td>2.335</td>
<td>47,940</td>
<td>4.223</td>
<td>0.234</td>
<td>-0.019</td>
</tr>
</tbody>
</table>

NOTE: Sales (excl. VAT), value added, inventories (includes products bought), investment are measured in billions of 2000 SEK (1 USD= 7.3 SEK, 1 EUR= 9.3 SEK). Number of employees is measured in thousands. Sales per product category are computed at store level.

Figure 2: Store performance distributions before and after acceptance of new PBL applications
Figure 3: Multi-product store’s indicators before and after acceptance of new PBL applications
Table 2: Estimation of multiproduct service generating function

<table>
<thead>
<tr>
<th></th>
<th>OLS Estimate</th>
<th>Std.</th>
<th>Two-step estimation</th>
<th>Std.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log no. of employees</td>
<td>0.784</td>
<td>0.035</td>
<td>0.571</td>
<td>0.033</td>
</tr>
<tr>
<td>Log of capital</td>
<td>0.061</td>
<td>0.029</td>
<td>0.289</td>
<td>0.036</td>
</tr>
<tr>
<td>Log of inventory</td>
<td>1.037</td>
<td>0.021</td>
<td>0.411</td>
<td>0.054</td>
</tr>
<tr>
<td>Log of sales of other products</td>
<td>-0.896</td>
<td>0.009</td>
<td>-0.857</td>
<td>0.061</td>
</tr>
<tr>
<td>Log of sales outside option</td>
<td>-0.005</td>
<td>0.006</td>
<td>0.287</td>
<td>0.043</td>
</tr>
<tr>
<td>Log of population</td>
<td>0.014</td>
<td>0.022</td>
<td>0.176</td>
<td>0.032</td>
</tr>
<tr>
<td>Log of pop. density</td>
<td>0.018</td>
<td>0.016</td>
<td>0.697</td>
<td>0.032</td>
</tr>
<tr>
<td>Coef. of no. of products ($\rho_{np}$)</td>
<td></td>
<td></td>
<td>0.213</td>
<td>0.096</td>
</tr>
<tr>
<td>Log of income</td>
<td>38.120</td>
<td>13.360</td>
<td>0.289</td>
<td>0.058</td>
</tr>
<tr>
<td>Log of income squared</td>
<td>-3.620</td>
<td>1.257</td>
<td>0.043</td>
<td>0.058</td>
</tr>
<tr>
<td>Elasticity of substitution</td>
<td></td>
<td></td>
<td>3.480</td>
<td></td>
</tr>
<tr>
<td>Year fixed-effect</td>
<td>Yes</td>
<td></td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Subsector fixed-effect</td>
<td>Yes</td>
<td></td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.558</td>
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<tr>
<td>No. of obs.</td>
<td>16,759</td>
<td></td>
<td>16,759</td>
<td></td>
</tr>
</tbody>
</table>

NOTE: The dependent variable is the log of sales of a product category at the store level. Labor is measured as the number of full-time adjusted employees. Sales of other product categories are measured at the store level. Sales of outside option measures total sales of the other products of all other five-digit SNI codes at the local market. OLS regression controls for the current impact of entry regulation. OLS refers to ordinary least squares regression. Two-step estimation refers to the extended Olley and Pakes (1996) estimation method presented in Section 2. Reported standard errors (in parentheses) are computed using Ackerberg et al. (2012).
Table 3: Estimation of structural parameters: Productivity and demand shocks processes

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Std.</th>
<th></th>
<th>Estimate</th>
<th>Std.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productivity ($\omega_{t-1}$)</td>
<td>0.846</td>
<td>0.013</td>
<td>Demand shock ($\mu_{t-1}$)</td>
<td>0.987</td>
<td>0.018</td>
</tr>
<tr>
<td>Productivity squared ($\omega_{t-1}^2$)</td>
<td>0.025</td>
<td>0.006</td>
<td>Demand shock squared ($\mu_{t-1}^2$)</td>
<td>-0.012</td>
<td>0.004</td>
</tr>
<tr>
<td>Productivity cubic ($\omega_{t-1}^3$)</td>
<td>-0.002</td>
<td>0.001</td>
<td>Demand shock cubic ($\mu_{t-1}^3$)</td>
<td>-0.0006</td>
<td>0.0002</td>
</tr>
<tr>
<td>Demand shock ($\mu_{t-1}$)</td>
<td>0.025</td>
<td>0.004</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prod.*Demand. shock ($\omega_{t-1} \times \mu_{t-1}$)</td>
<td>0.011</td>
<td>0.002</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Entry regulation ($r_{t-1}$)</td>
<td>0.122</td>
<td>0.036</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prod.*Entry reg. ($\omega_{t-1} \times r_{t-1}$)</td>
<td>-0.026</td>
<td>0.011</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dem. sh.*Entry reg. ($\mu_{t-1} \times r_{t-1}$)</td>
<td>-0.028</td>
<td>0.006</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Year fixed-effects       Yes  | Year fixed-effects     Yes  | Adjusted R-squared         0.873 | Adjusted R-squared  0.686 |
Sub-sector fixed-effects  Yes  | Sub-sector fixed-effects Yes  |                            |                  |
Adjusted R-squared        0.873 | Adjusted R-squared        0.686 |

Coefficients of $\omega_{t-1}$ terms are zero F-test p-value 1749.183 0.000
Coefficients of $\mu_{t-1}$ terms are zero F-test p-value 23.601 0.000
Coefficients of $r_{t-1}$ terms are zero F-test p-value 7.599 0.000

Persistence ($d\omega_{t}/d\omega_{t-1}$) 0.869
Effect of demand shock ($d\omega_{t}/d\mu_{t-1}$) 0.025
Effect of entry regulation ($d\omega_{t}/dr_{t-1}$) 0.077

Persistence ($d\mu_{t}/d\mu_{t-1}$) 0.943

NOTE: Productivity is estimated using the two-step estimation method in Section 2. The mean values are presented for the marginal effects.

Figure 4: The relationship between the number of product categories, productivity, and demand shocks
Table 4: Determinants of product categories at the store level

<table>
<thead>
<tr>
<th>HHI product categories</th>
<th>No. of product categories</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Restrictive</td>
</tr>
<tr>
<td>Productivity ($\omega_t$)</td>
<td>-0.1131</td>
</tr>
<tr>
<td>Productivity squared ($\omega_t^2$)</td>
<td>0.0039</td>
</tr>
<tr>
<td>Demand shocks ($\mu_t$)</td>
<td>0.1024</td>
</tr>
<tr>
<td>Demand shocks squared ($\mu_t^2$)</td>
<td>-0.0041</td>
</tr>
<tr>
<td>Prod. x Demand sh. ($\omega_t \times \mu_t$)</td>
<td>0.0081</td>
</tr>
<tr>
<td>Log of capital stock ($k_{t-1}$)</td>
<td>-0.0404</td>
</tr>
<tr>
<td>Log of investments ($i_{t-1}$)</td>
<td>-0.0043</td>
</tr>
</tbody>
</table>

Other store/market controls | Yes | Yes | Yes | Yes | Yes | Yes |
Sector fixed-effects | Yes | Yes | Yes | Yes | Yes | Yes |
Year fixed-effects | Yes | Yes | Yes | Yes | Yes | Yes |
Market fixed-effects | Yes | Yes | Yes | Yes | Yes | Yes |
Adj. $R^2$ | 0.5404 | 0.6040 |

NOTE: All regressions include an intercept. OLS estimator is used for HHI regressions, where the dependent variable, i.e., HHI, is computed based on sales product categories. Quasi-Poisson estimator is used for the number of product categories regressions. Additional store and market controls include: inventories, wages, population, population density, income. Standard errors are clustered at sector level (see Cameron and Miller, 2015; Imbens and Kolesar, 2015).

Table 5: Determinants of the number of unique product categories in local markets

<table>
<thead>
<tr>
<th></th>
<th>Restrictive markets</th>
<th>Liberal markets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productivity ($\omega_t$)</td>
<td>0.2212</td>
<td>0.0447</td>
</tr>
<tr>
<td>Productivity squared ($\omega_t^2$)</td>
<td>-0.0164</td>
<td>0.0097</td>
</tr>
<tr>
<td>Demand shocks ($\mu_t$)</td>
<td>-0.2298</td>
<td>0.0288</td>
</tr>
<tr>
<td>Demand shocks squared ($\mu_t^2$)</td>
<td>0.0114</td>
<td>0.0018</td>
</tr>
<tr>
<td>Log of capital stock ($k_{t-1}$)</td>
<td>0.0809</td>
<td>0.0164</td>
</tr>
<tr>
<td>Log of investments ($i_{t-1}$)</td>
<td>0.0386</td>
<td>0.0127</td>
</tr>
</tbody>
</table>

Sector fixed-effects | Yes | Yes |
Year fixed-effects | Yes | Yes |
Market fixed-effects | Yes | Yes |
Adj. $R^2$ | 0.5554 | 0.4024 |

NOTE: Dependent variable is the log of unique number of product categories at local market and sector level. Controls include median values at the local market, sector and year level. OLS estimator is used. All regressions include an intercept, and additional market controls (median values), i.e., population, population density, and income. Standard errors are clustered at market level (see Cameron and Miller, 2015; Imbens and Kolesar, 2015).
### Table 6: Estimation of dynamic parameters

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Std.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept ($\phi_0$)</td>
<td>0.0911</td>
<td>0.0201</td>
</tr>
<tr>
<td>Coefficient of $A_{jt}$ ($\phi_1$)</td>
<td>0.3621</td>
<td>0.0349</td>
</tr>
<tr>
<td>Coefficient of $A_{jt}r_m$ ($\phi_2$)</td>
<td>-0.0023</td>
<td>0.0005</td>
</tr>
<tr>
<td>Coefficient of $A_{jt}r_m$ ($\phi_3$)</td>
<td>-0.0148</td>
<td>0.0020</td>
</tr>
</tbody>
</table>

NOTE: Standard errors are computed using subsampling. Preliminary.

### Table 7: Counterfactual experiments: The trade-off between a more liberal regulation and improvements store’s shopping quality in restrictive markets

<table>
<thead>
<tr>
<th></th>
<th>After 1 year</th>
<th>Mean</th>
<th>IQR</th>
<th>After 3 years</th>
<th>Mean</th>
<th>IQR</th>
<th>After 5 years</th>
<th>Mean</th>
<th>IQR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CP1:</strong> The impact of a more liberal entry regulation</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in consumer surplus</td>
<td>0.047</td>
<td>0.039</td>
<td>0.143</td>
<td>0.445</td>
<td>0.203</td>
<td>0.731</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in number of products</td>
<td>0.217</td>
<td>0.000</td>
<td>0.192</td>
<td>0.000</td>
<td>0.361</td>
<td>0.935</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in inventory before sales</td>
<td>-0.142</td>
<td>0.398</td>
<td>-0.125</td>
<td>0.591</td>
<td>0.160</td>
<td>1.975</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in inventory end of year</td>
<td>0.435</td>
<td>0.979</td>
<td>0.605</td>
<td>3.010</td>
<td>0.918</td>
<td>3.943</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in number of employees</td>
<td>-0.350</td>
<td>1.192</td>
<td>-0.414</td>
<td>1.913</td>
<td>-0.215</td>
<td>1.743</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in investment in technology</td>
<td>-3.051</td>
<td>8.023</td>
<td>1.088</td>
<td>3.720</td>
<td>1.106</td>
<td>3.165</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>CP2:</strong> The impact of positive shocks to shopping quality</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in consumer surplus</td>
<td>2.152</td>
<td>5.432</td>
<td>3.788</td>
<td>15.326</td>
<td>4.355</td>
<td>13.048</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in number of products</td>
<td>8.522</td>
<td>20.544</td>
<td>3.743</td>
<td>8.633</td>
<td>10.378</td>
<td>80.955</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in inventory before sales</td>
<td>-2.804</td>
<td>16.245</td>
<td>-0.356</td>
<td>13.153</td>
<td>-2.207</td>
<td>41.097</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in inventory end of year</td>
<td>-1.521</td>
<td>12.906</td>
<td>0.408</td>
<td>33.971</td>
<td>-1.069</td>
<td>54.887</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in number of employees</td>
<td>-2.613</td>
<td>20.499</td>
<td>0.041</td>
<td>27.695</td>
<td>-1.657</td>
<td>30.756</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in investment in technology</td>
<td>-0.450</td>
<td>7.621</td>
<td>2.809</td>
<td>10.465</td>
<td>-1.870</td>
<td>28.961</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NOTE: All numbers are in percentages. The mean and interquartile range ($IQR = Q_{90} - Q_{10}$) of changes are computed based on the simulated data using the last year in the data as the starting value, and the estimated policy functions and Markov processes. Preliminary.

### Table 8: Counterfactual experiments: The trade-off between subsidizing stores and a marginal reduction in the impact of regulation on cost with inventory

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>IQR</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>CP3:</strong> Subsidy: A decrease in sales tax by 10 percent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in consumer surplus</td>
<td>3.256</td>
<td>2.864</td>
</tr>
<tr>
<td>Change in inventory before sales</td>
<td>2.201</td>
<td>4.381</td>
</tr>
<tr>
<td>Change in the long-run profits</td>
<td>5.691</td>
<td>8.234</td>
</tr>
<tr>
<td><strong>CP4:</strong> A marginal reduction in the impact of regulation on cost with inventory</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Change in consumer surplus</td>
<td>4.826</td>
<td>3.424</td>
</tr>
<tr>
<td>Change in inventory before sales</td>
<td>4.105</td>
<td>3.325</td>
</tr>
<tr>
<td>Change in the long-run profits</td>
<td>7.415</td>
<td>4.645</td>
</tr>
</tbody>
</table>

NOTE: All numbers are in percentages. The mean and interquartile range ($IQR = Q_{90} - Q_{10}$). The figures are computed solving for optimal $a_{jt}$ using value function approximation. Preliminary.
Appendix A: General properties of the multiproduct service function

To simplify the notation, we omit the index of the firm and period and denote the group the store service inputs into the vector $V$. For example, in our empirical implementation $V = (L, K, A)$. We consider the general service generating function, i.e.,

$$F(Q, V) = G(Q) - H(V) = 0$$  \hspace{1cm} (22)

where $G(Q) = Q_{i1} \times \cdots \times Q_{id} \exp(\tilde{\gamma}_1 Q_1 + \cdots + \tilde{\gamma}_d Q_d)$; $H(V) = V_{j1} \times \cdots \times V_{jm} \exp(\tilde{\omega})$; $Q$ is the vector of service output; $Q_i$ is the $i$-th service output of the store, $(i = 1, d)$; and $V_j$ is the $j$-th service input of the store, $(j = 1, m)$. In what follows, we use the $i$ to index the service outputs and $j$ to index the inputs.

Assuming that the prices are given, the Lagrangean function of the profit maximization of at the store level is given by

$$\max_V \mathcal{L} = P'Q - W'V - \lambda F(Q, V),$$  \hspace{1cm} (23)
where \( \mathbf{P} \) and \( \mathbf{W} \) are the vectors of output and input prices, respectively. The first-order conditions (FOC) under competition are

\[
P_i - \lambda F_i = 0, \quad i = 1, d
\]
\[
W_j + \lambda F_j = 0, \quad j = 1, m
\]

where \( F_i = \partial F / \partial Q_i \) and \( F_j = \partial F / \partial V_j \). The FOC (24) conditions imply that \( \text{sign}(\lambda) = \text{sign}(F_i) \) and \( \text{sign}(\lambda) = -\text{sign}(F_j) \). The derivatives of the implicit function to respect to inputs and outputs, i.e. \( F_i \) and \( F_j \) are

\[
F_i = G(Q) \left( \frac{\partial \alpha_i}{\partial Q_i} + \gamma_i \right), \quad i = 1, d
\]
\[
F_j = -H(V) \frac{\partial \beta_j}{\partial V_j}, \quad j = 1, m
\]

The cross derivatives of the Lagrangean are the following:

\[
\partial^2 \mathcal{L} / \partial \lambda \partial \lambda = 0; \quad \partial^2 \mathcal{L} / \partial \lambda \partial Q_i = -F_i; \quad \partial^2 \mathcal{L} / \partial \lambda \partial V_j = -F_j; \quad \partial^2 \mathcal{L} / \partial Q_i \partial Q_i = -\lambda F_{ii}; \quad \partial^2 \mathcal{L} / \partial V_j \partial V_j = -\lambda F_{jj}; \quad \text{and} \partial^2 \mathcal{L} / \partial Q_i \partial V_j = -\lambda F_{ij}.
\]

The determinant of the bordered Hessian matrix \( D_{\mathcal{L}} \) is given by

\[
D_{\mathcal{L}} = \begin{vmatrix}
\frac{\partial^2 \mathcal{L}}{\partial \lambda \partial \lambda} & \frac{\partial^2 \mathcal{L}}{\partial \lambda \partial Q_i} & \frac{\partial^2 \mathcal{L}}{\partial \lambda \partial V_j} & 0 & -F_i & -F_j \\
\frac{\partial^2 \mathcal{L}}{\partial Q_i \partial \lambda} & \frac{\partial^2 \mathcal{L}}{\partial Q_i \partial Q_i} & \frac{\partial^2 \mathcal{L}}{\partial Q_i \partial V_j} & -F_i & -\lambda F_{ii'} & -\lambda F_{ij'} \\
\frac{\partial^2 \mathcal{L}}{\partial V_j \partial \lambda} & \frac{\partial^2 \mathcal{L}}{\partial V_j \partial Q_i} & \frac{\partial^2 \mathcal{L}}{\partial V_j \partial V_j} & -F_j & -\lambda F_{jj'} & -\lambda F_{jj''} \\
\end{vmatrix}
\]

where the cross derivatives of elements of the block matrices of the determinant of the hessian matrix are the following

Product-product: \( F_{ii} = \frac{F_i^2}{G(Q)} - G(Q) \frac{\partial \alpha_i}{\partial Q_i}, \quad i = 1, d \)

Product-product: \( F_{ii'} = \frac{F_i F_{i'}}{G(Q)}, \quad i \neq i', \quad i, i' = 1, d \)

Input-input: \( F_{jj} = -\frac{F_j^2}{H(V)} + H(V) \frac{\partial \beta_j}{\partial V_j}, \quad j = 1, m \)

Input-input: \( F_{jj'} = -\frac{F_j F_{j'}}{H(V)}, \quad j \neq j', \quad j, j' = 1, m \)

Product-input: \( F_{ij} = 0, \quad i = 1, d, \quad j = 1, m. \)

The second-order condition of the profit maximization requires the sign of the determinant of the bordered Hessian matrix \( D_{\mathcal{L}} \) is \((-1)^{d+m}\). To proof this, we rewrite the determinant
\( D_\mathcal{L} \) as
\[
D_\mathcal{L} = \begin{vmatrix}
A & B \\
C & D
\end{vmatrix},
\]
(28)

where \( A = 0 \) (1 \( \times \) 1 matrix); \( B = [-F_i, -F_j]^T \) (1 \( \times \) \((d+m))\); \( C = [-F_i, -F_j] \) ((\( d+m \) \( \times \) 1); and
\[
D = \begin{bmatrix}
-\lambda F_{ii'} & 0 \\
0 & -\lambda F_{jj'}
\end{bmatrix}
\]

Using Schur complement decomposition, we have that
\[
D_\mathcal{L} = det(D)det(A - BD^{-1}C). 
\]
(29)

Because the matrix \( D \) is diagonal, its inverse is given by
\[
D^{-1} = (-\lambda)^{-1} \begin{bmatrix}
F_{ii'}^{-1} & 0 \\
0 & F_{jj'}^{-1}
\end{bmatrix}, 
\]
(30)

and the determinant of \( D \) is
\[
det(D) = (-1)^{-(d+m)}(\lambda)^{-(d+m)}det(F_{ii'})det(F_{jj'}).
\]
(31)

The product \( BD^{-1}C \) can be rewritten as
\[
BD^{-1}C = -\lambda^{-1}[F_{ii'}^{-1}F_i + F_{jj'}^{-1}F_j]. 
\]
(32)

Therefore,
\[
det(A - BD^{-1}C) = (\lambda)^{-1}[F_{ii'}^{-1}F_i + F_{jj'}^{-1}F_j].
\]
(33)

Thus, the determinant of the bordered Hessian matrix is given by
\[
D_\mathcal{L} = (-1)^{-(d+m)}(\lambda)^{-(d+m+1)}det(F_{ii'})det(F_{jj'})[F_{ii'}^{-1}F_i + F_{jj'}^{-1}F_j].
\]
(34)
The block matrices $F_{ii'}$ and $F_{jj'}$ have important properties that can be used to compute their inverse and the determinant. The matrix $F_{ii'}$ and $F_{jj'}$ can be written as

$$F_{ii'} = \begin{bmatrix} \frac{F_1 F_1}{G(Q)} & \cdots & \frac{F_1 F_d}{G(Q)} \\ \vdots & \ddots & \vdots \\ \frac{F_d F_1}{G(Q)} & \cdots & \frac{F_d F_d}{G(Q)} \end{bmatrix} + \begin{bmatrix} -\frac{\delta_1}{Q_1} G(Q) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -\frac{\delta_d}{Q_d} G(Q) \end{bmatrix}$$

and

$$F_{jj'} = \begin{bmatrix} -\frac{F_1 F_1}{H(V)} & \cdots & -\frac{F_1 F_m}{H(V)} \\ \vdots & \ddots & \vdots \\ -\frac{F_m F_1}{H(V)} & \cdots & -\frac{F_m F_m}{H(V)} \end{bmatrix} + \begin{bmatrix} \frac{\beta_1}{V_1} H(V) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{\beta_m}{V_m} H(V) \end{bmatrix}.$$ 

We introduce new notations for the vectors of derivatives in outputs and inputs, i.e.,

$$u^T_i = \begin{bmatrix} -\frac{F_1}{G(Q)^{\frac{d_1}{2}}} & \cdots & -\frac{F_d}{G(Q)^{\frac{d_d}{2}}} \end{bmatrix},$$

$$u^T_j = \begin{bmatrix} -\frac{F_1}{H(V)^{\frac{d_1}{2}}} & \cdots & -\frac{F_m}{H(V)^{\frac{d_m}{2}}} \end{bmatrix},$$

$$v^T_j = \begin{bmatrix} \frac{F_1}{H(V)^{\frac{d_1}{2}}} & \cdots & \frac{F_m}{H(V)^{\frac{d_m}{2}}} \end{bmatrix},$$

$$\tilde{F}_{ii'} = \begin{bmatrix} -\frac{\delta_1}{Q_1} G(Q) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & -\frac{\delta_d}{Q_d} G(Q) \end{bmatrix}$$

and

$$\tilde{F}_{jj'} = \begin{bmatrix} \frac{\beta_1}{V_1} H(V) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{\beta_m}{V_m} H(V) \end{bmatrix}.$$
The cross-derivative matrices $F_{ii'}$ and $F_{jj'}$ can decompose as

$$F_{ii'} = \tilde{F}_{ii'} + u_iu_i^T$$
$$F_{jj'} = \tilde{F}_{jj'} + u_jv_j^T.$$  \hspace{1cm} (35)

Based on these decompositions, we can compute the inverses and the determinants of $F_{ii'}$ and $F_{jj'}$ using Sherman-Morrison formula, i.e.,

$$(\tilde{F}_{ii'} + u_iu_i^T)^{-1} = \tilde{F}_{ii'}^{-1} - \frac{\tilde{F}_{ii'}^{-1}u_iu_i^T\tilde{F}_{ii'}^{-1}}{1 + u_i^T\tilde{F}_{ii'}^{-1}u_i}$$ \hspace{1cm} (36)

$$(\tilde{F}_{jj'} + u_jv_j^T)^{-1} = \tilde{F}_{jj'}^{-1} - \frac{\tilde{F}_{jj'}^{-1}v_jv_j^T\tilde{F}_{jj'}^{-1}}{1 + v_j^T\tilde{F}_{jj'}^{-1}u_j}$$ \hspace{1cm} (37)

$$det(\tilde{F}_{ii'} + u_iu_i^T) = (1 + u_i^T\tilde{F}_{ii'}^{-1}u_i)det(\tilde{F}_{ii'})$$ \hspace{1cm} (38)

$$det(\tilde{F}_{jj'} + u_jv_j^T) = (1 + u_j^T\tilde{F}_{jj'}^{-1}u_j)det(\tilde{F}_{jj'})$$ \hspace{1cm} (39)

The inverse of the diagonal matrices $\tilde{F}_{ii'}$ and $\tilde{F}_{jj'}$ are given by

$$\tilde{F}_{ii'}^{-1} = \begin{bmatrix} -Q_i^2 \frac{1}{\beta_1 H(Q)} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & -Q_i^2 \frac{1}{\beta_m H(Q)} \end{bmatrix}$$ \hspace{1cm} (40)

$$\tilde{F}_{jj'}^{-1} = \begin{bmatrix} V_j^2 \frac{1}{\beta_1 H(V)} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & V_j^2 \frac{1}{\beta_m H(V)} \end{bmatrix}$$ \hspace{1cm} (41)

We use Sherman-Morrison formula to evaluate the terms $F_i^T\tilde{F}_{ii'}^{-1}F_i$ and $F_j^T\tilde{F}_{jj'}^{-1}F_j$, i.e.,

$$F_i^T\tilde{F}_{ii'}^{-1}F_i = G(Q)\frac{1}{2}u_i^T \left(\tilde{F}_{ii'}^{-1} - \frac{\tilde{F}_{ii'}^{-1}u_iu_i^T\tilde{F}_{ii'}^{-1}}{1 + u_i^T\tilde{F}_{ii'}^{-1}u_i}\right)u_iG(Q)\frac{1}{2}$$

$$= G(Q)\frac{u_i^T\tilde{F}_{ii'}^{-1}u_i}{1 + u_i^T\tilde{F}_{ii'}^{-1}u_i}$$ \hspace{1cm} (42)
\[ F_j^T F_{jj'} F_j = -H(V)^{\frac{1}{2}} v_j^T \left( F_j^{-1} - \frac{F_j^{-1} u_j v_j^T F_j^{-1}}{1 + v_j^T F_j^{-1} u_j} \right) u_j H(V)^{\frac{1}{2}} \]

The terms \( u_i^T \tilde{F}_{ii'}^{-1} u_i \) and \( v_j^T \tilde{F}_{jj'}^{-1} u_j \) can be computed as follows:

\[
u_j^T \tilde{F}_{jj'}^{-1} u_j = \begin{bmatrix} \frac{F_i}{H(V)^{\frac{1}{2}}} & \cdots & \frac{F_m}{H(V)^{\frac{1}{2}}} \end{bmatrix} \begin{bmatrix} \frac{V^2}{\beta_i H(V)} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{V^2}{\beta_d H(V)} \end{bmatrix} \begin{bmatrix} -\frac{E_i}{G(Q)^{\frac{1}{2}}} \\ \vdots \\ -\frac{E_d}{G(Q)^{\frac{1}{2}}} \end{bmatrix} = -\sum_{j=1}^{m} \frac{F_j^2 V_j^2}{H(V)^{\frac{1}{2}}}.
\]

Therefore, we have

\[ F_i^T \tilde{F}_{ii'}^{-1} F_i = G(Q) \frac{-\sum_{i=1}^{d} \frac{F_i^2 Q_i^2}{G(Q)^{\frac{1}{2}}}}{1 - \sum_{i=1}^{d} \frac{F_i^2 Q_i^2}{G(Q)^{\frac{1}{2}}}} \]

\[ F_j^T \tilde{F}_{jj'}^{-1} F_j = H(V) \frac{-\sum_{j=1}^{m} \frac{F_j^2 V_j^2}{H(V)^{\frac{1}{2}}}}{1 - \sum_{j=1}^{m} \frac{F_j^2 V_j^2}{H(V)^{\frac{1}{2}}}} \]

The next step is to compute the determinants of \( F_{ii'} \) and \( F_{jj'} \), i.e.,

\[
det(F_{ii'}) = (1 + u_i^T \tilde{F}_{ii'}^{-1} u_i) \det(\tilde{F}_{ii'})
= \left(1 - \sum_{i=1}^{d} \frac{F_i^2 Q_i^2}{G(Q)^{\frac{1}{2}}} \right) \prod_{i=1}^{d} \frac{\delta_i}{Q_i} G(Q)
\]

\[
det(F_{jj'}) = (1 + v_j^T \tilde{F}_{jj'}^{-1} u_j) \det(\tilde{F}_{ii'})
= \left(1 - \sum_{j=1}^{m} \frac{F_j^2 V_j^2}{H(V)^{\frac{1}{2}}} \right) \prod_{j=1}^{m} \frac{\delta_j}{V_j} H(V)
\]
Replacing expressions (31), (33), (44), (45), and (46) in (29), we have

\[ D_L = \lambda(-\lambda)^{-d+m+2} \left( 1 - \sum_{i=1}^{d} \frac{F_i^2 Q_i^2}{G(Q_i)^2} \right) \left( \prod_{i=1}^{d} \frac{\tilde{\alpha}_i}{Q_i} G(Q_i) \right) \times \left( 1 - \sum_{j=1}^{m} \frac{F_j^2 V_j^2}{H(V_j)^2} \right) \left( \prod_{j=1}^{m} \frac{\tilde{\beta}_j}{V_j} H(V_j) \right) \]

\times \left[ G(Q) - \sum_{i=1}^{d} \frac{F_i^2 Q_i^2}{G(Q_i)^2} - \sum_{j=1}^{m} \frac{F_j^2 V_j^2}{H(V_j)^2} \right], \tag{47} \]

where \( \frac{F_i^2 Q_i^2}{G(Q_i)^2} = (\tilde{\alpha}_i + \tilde{\gamma}_i Q_i)^2 \) and \( \frac{F_j^2 V_j^2}{H(V_j)^2} = \tilde{\beta}_j^2 \). We simplify the expression of \( D_L \) by introducing new notations for each term, i.e.,

\[
\begin{align*}
T_1 &= \left( 1 - \sum_{i=1}^{d} \frac{1}{\tilde{\alpha}_i G(Q_i)^2} \right) \\
T_2 &= \left( \prod_{i=1}^{d} \frac{\tilde{\alpha}_i}{Q_i} G(Q) \right) \\
T_3 &= \left( 1 - \sum_{j=1}^{m} \frac{1}{\tilde{\beta}_j H(V_j)^2} \right) \\
T_4 &= \left( \prod_{j=1}^{m} \frac{\tilde{\beta}_j}{V_j} H(V) \right) \\
T_5 &= \left[ G(Q) - \sum_{i=1}^{d} \frac{1}{Q_i} \frac{F_i^2 Q_i^2}{G(Q_i)^2} - \sum_{j=1}^{m} \frac{1}{V_j} \frac{F_j^2 V_j^2}{H(V_j)^2} \right] \\
&= \left[ -\frac{G(Q)}{1-\sum_{i=1}^{d} \frac{1}{\tilde{\alpha}_i G(Q_i)^2}} + \frac{H(V)}{1-\sum_{j=1}^{m} \frac{1}{\tilde{\beta}_j H(V_j)^2}} \right]
\end{align*}
\]

**Lemma 1:** In general case of transcendental service production function \( d \) outputs and \( m \) inputs, the determinant of the bordered Hessian matrix of the profit maximization problem is given by

\[ D_L = (-1)^{(d+m)} \lambda^{-(d+m+1)} T_1 T_2 T_3 T_4 T_5. \tag{48} \]

**PROOF:**

This finding results directly from equation (47). □

In what follows, we provide a general result on the restrictions of the coefficients of transcendental multiproduct functions that are required to satisfy the profit maximization conditions. This result is a generalization of Mundlak’s (1964) result in the case of two outputs – two factor inputs.
Theorem 1: Consider a general service generating function

\[ F(Q, V) = G(Q) - H(V) = 0 \]  \hspace{1cm} (49)

where \( G(Q) = Q_1^{\tilde{\alpha}_1} \times \cdots \times Q_d^{\tilde{\alpha}_d} \exp(\tilde{\gamma}_1 Q_1 + \cdots + \tilde{\gamma}_d Q_d); \) \( H(V) = V_1^{\tilde{\beta}_1} \times \cdots \times V_m^{\tilde{\beta}_m} \exp(\tilde{\omega}); \)

\( Q_i \) is the \( i \)-th service output of the store, \( (i = 1, d); \) \( V_j \) is the \( j \)-th service input of the store, \( (j = 1, m). \) If the parameters satisfy the following conditions

(a) \( \tilde{\alpha}_i < 0 \) for all \( i = 1, d; \)

(b) \( \tilde{\beta}_j > 0 \) for all \( j = 1, m, \)

then the condition for profit maximization are satisfied.

PROOF:

We consider \( \lambda > 0 \) and the increasing return to scales industry \( \sum_{i=1}^{d} \tilde{\beta}_j \geq 1. \) We assume \( \lambda > 0 \) and, the first-order conditions for maximizing profit imply that \( F_i > 0 \) and \( F_j < 0, \) i.e.,

\[ \left( \frac{\tilde{\alpha}_i}{Q_i} + \tilde{\gamma}_i \right) > 0, \quad i = 1, d \] \hspace{1cm} (50)

\[ \frac{\tilde{\beta}_j}{V_j} > 0, \quad j = 1, m. \] \hspace{1cm} (51)

In other words, we need to have

\[ \tilde{\gamma}_i > \left| \frac{\tilde{\alpha}_i}{Q_i} \right|, \quad i = 1, d. \] \hspace{1cm} (52)

\[ \tilde{\beta}_j > 0, \quad j = 1, m. \] \hspace{1cm} (53)

The first order condition (52) excludes the possibility that \( \tilde{\gamma}_i = 0 \) for all \( i. \) This implies that \( T_1 > 0, T_2 > 0, T_4 > 0. \) The term \( T_5 < 0, \) because \( T_3 < 0 \) and \( T_2 > 0, \) i.e., \( T_5 \) is a sum of two negative numbers. Therefore, the sign of determinant of the bordered Hessian

\[ \text{det} \]
Proposition 1: If the service function is simple Cobb-Douglas in outputs ($\tilde{\gamma}_i = 0$ for all $i$) and inputs and the first-order conditions are satisfied, then optimal service quantity $Q^*$ is sold at the minimum cost and any inputs $V^*$ yields minimum revenues. The profit $\pi(Q^*, V^*)$ at the point $(Q^*, V^*)$ is a saddle point

$$\pi(Q^*, V) \leq \pi(Q^*, V^*) \leq \pi(Q, V^*)$$

PROOF: If $\tilde{\gamma}_i = 0$ for all $i$, then from the first-order condition (50) we have that $\tilde{\alpha}_i > 0$ for all $i$. In this case, sign($T_2$) = $(-1)^d$, and the sign($D_L$) is different from $(-1)^{d+m}$ (condition for minimum) and $(-1)^{d+m}$ (condition for maximum).

A direct consequence of the Proposition 1 is that when the inputs $V$ produce minimum revenues and the first-order conditions are satisfied then the profit can be maximized by a selection of products, i.e., a corner solution. This problem does not exists in the case of single product.

Proposition 2: The condition $\tilde{\alpha}_i < 0$ and $\tilde{\gamma}_i > 0$ for all $i$ is not the only one second order condition for profit maximization.

PROOF: This result is also a direct consequence of Theorem 1. It is important to note that the result in Theorem 1 holds some $\tilde{\alpha}_i$ can be positive and, in this case, the corresponding $\tilde{\gamma}_i$ can be setted to zero, which can be useful to reduce the number of parameters.

Product (factor) substitution. Using the total differentiation of the service general
function, we can obtain the marginal rate of product (factor) substitution, i.e.,

Product-factor: \( \frac{dQ}{dV_e} = -\frac{F_e}{F_i} > 0 \)

Factor-factor: \( \frac{dV_e'}{dV_e} = -\frac{F_e}{F_{e'j}} < 0 \) \hspace{1cm} (54)

Product-Product: \( \frac{dQ_i}{dQ_j'} = -\frac{F_i'}{F_{ii}} < 0 \).

To evaluate convexity of the different marginal rate of substitution, we compute the second derivatives, i.e.,

Product-factor: \( \frac{d^2Q}{dV_e^2} = -\frac{F_{ee}}{F_i} \)

Factor-factor: \( \frac{d^2V_e'}{dV_e^2} = -\frac{F_{ee}'}{F_{e'j}} + \frac{F_{e'e}}{F_{e'j}} \) \hspace{1cm} (55)

Product-Product: \( \frac{d^2Q_i}{dQ_j'^2} = -\frac{F_{i'j}'}{F_{ii}} + \frac{F_{ii'}'}{F_{i'j}} \),

where

\[
F_{jj} = \frac{H(V)}{V_j} \tilde{\beta}_j (1 - \tilde{\beta}_j) \\
F_i = G(Q) \left( \frac{\tilde{\alpha}_i}{Q_i} + \tilde{\gamma}_i \right) \\
F_{ii} = \frac{V_{ii'}}{V_j} \frac{\tilde{\beta}_j}{\beta_j} (\tilde{\beta}_j - 1) \\
F_{i'j} = \frac{\tilde{\beta}_j^2}{V_j} \frac{1}{\beta_j} \\
F_{i'j'} = G(Q) \frac{\tilde{\alpha}_i'}{Q_i'} \left( \frac{1}{\tilde{\alpha}_i' + \tilde{\gamma}_i} \right).
\]

In the case of Cobb-Douglas in inputs \( 0 < \tilde{\beta}_j < 1 \), which implies that

Product-factor: \( \frac{d^2Q}{dV_e^2} < 0 \)

Factor-factor: \( \frac{d^2V_e'}{dV_e^2} > 0 \), \hspace{1cm} (57)

which implies that product-factor rate of substitution is a concave function (Figure 2.a), factor-factor rate of substitution is convex (Figure 2.b). The properties of the product-product rate of substitution depends on \( \tilde{\gamma}_i \), i.e.,

\[
\frac{d^2Q_i}{dQ_j'^2} = G(Q) \frac{\tilde{\alpha}_i'}{Q_i'} \left( \frac{1}{\tilde{\alpha}_i' + \tilde{\gamma}_i} \right).
\]

If \( \tilde{\gamma}_i = 0 \) then from the first order condition we have \( \tilde{\alpha}_i > 0 \), which yields \( \frac{d^2Q_i}{dQ_j'^2} > 0 \).
Therefore, \( \tilde{\gamma}_i = 0 \) implies that product-product rate of substitution is convex function (Figure 2.c). If \( \tilde{\gamma}_i > 0 \) then from the first order condition we have \( \tilde{\alpha}_i < 0 \), which yields \( d^2 Q_{ij}/dQ_{ij}^2 < 0 \). In this case, product-product rate of substitution is concave function (AB curve in Figure 2.c).

**Reduction in the number of parameters.** In an empirical application, we need to normalize one parameter to one, say \( k \)-th output, which can be done by raising the service function at the \( -\tilde{\alpha}_k \) power. In this case, the other output parameters \( \tilde{\alpha}_{-k} \) will have a reverse sign when \( \tilde{\alpha}_k \) is negative.

The number of \( \tilde{\gamma} \) parameters can be reduced to one by assuming \( \tilde{\gamma}_1 = \cdots = \tilde{\gamma}_i = \cdots = \tilde{\gamma}_d \). To have a meaningful interpretation of the sum \( \sum_{i=1}^{d} \tilde{\gamma}_i X_i \), in the estimation we can use the sales, i.e., \( \sum_{i=1}^{d} \tilde{\gamma}_i P_i X_i \) (Mundlak, 1964).

**Appendix B: Derivation of the sales generating function**

**Consumer choices.** Consumers are homogeneous and have CES preferences over the differentiate products and services \( i \in \{1, \cdots, d\} \) of store \( j \), where the utility function given by

\[
U(\{Q_{ijt}, x_{ijt}, \xi_{ijt}, \eta_{ijt}\}_{i=1}^{d}) := \left( \sum_{i=1}^{d} \kappa(x_{ijt}, \xi_{ijt}, z_{ijt}, \eta_{ijt}) Q_{ijt} \right)^{\frac{1}{\sigma}}, \quad (59)
\]

where the \( \kappa(x_{ijt}, \xi_{ijt}, z_{ijt}, \eta_{ijt}) \) is the kernel quality function (Hortacsu and Joonhwi, 2015). \( x_{ijt} \) and \( z_{ijt} \) are the observed determinants of the intensive and extensive margins of the utility function when the consumers by the product \( i \). They might include the common variables, and \( z_{ijt} \) includes at least one component that is not part of \( x_{ijt} \). \( \xi_{ijt} \) and \( \eta_{ijt} \) are determinants of the intensive and extensive margins of the utility and are unobserved to the researcher. The quality function \( \kappa(\cdot) \) allows to separate intensive and extensive margin and to accommodate for zero market share (Hortacsu and Joonhwi, 2015).
The optimization problem for the representative consumer is given by

$$\max_{Q_{ijt}, \sigma} \left( \sum_{i=1}^{d} \kappa(x_{ijt}, \xi_{ijt}, z_{ijt}, \eta_{ijt}) \frac{x_{ijt}^{\sigma-1}}{b_t} \right)^{\frac{\sigma}{\sigma-1}}$$

s.t. \( \sum_{i=1}^{d} P_{ijt}Q_{ijt} = b_t \)

(60)

The solution of this optimization problem gives us the demand function \((Q_{ijt})\) and the individual choice probability \((\pi_{ijt})\), which is the CES demand system with observed and unobserved product characteristics, i.e.,

$$Q_{ijt} = \frac{\kappa(x_{ijt}, \xi_{ijt}, z_{ijt}, \eta_{ijt})P_{ijt}^{1-\sigma} b_t}{\sum_{h=1}^{d} \kappa(x_{hjt}, \xi_{hjt}, z_{hjt}, \eta_{hjt})P_{hjt}^{1-\sigma}}$$

$$\pi_{ijt} = \frac{\kappa(x_{ijt}, \xi_{ijt}, z_{ijt}, \eta_{ijt})P_{ijt}^{1-\sigma}}{\sum_{h=1}^{d} \kappa(x_{hjt}, \xi_{hjt}, z_{hjt}, \eta_{hjt})P_{hjt}^{1-\sigma}}$$

(61)

The elasticity of substitution \(\sigma\) is globally identified for the set of products with positive individual choice probabilities, i.e. \(\pi_{ijt} > 0\). This is because the system \{\(\pi_{ijt}\)\} satisfy the connected substitutes condition provided by Berry et al. (2013), i.e., it is invertible.

The choice of the exponential kernel quality having has key implications for the identification of the demand system. For \(x_{ijt} = z_{ijt}\) and \(z_{ijt}\) exogenous for all \(i, \xi_{ijt} = \eta_{ijt}\), \(\pi_{ijt} > 0\) for all \(i\), we do not need any exclusion restriction to identify the demand system. In this case, the log of the ratio between individual choice probabilities of product \(j\) and the outside option (or numeraire) when normalize \(x_{0jt} = 0, \xi_{0jt} = 0\), and \(\kappa(x_{ijt}, \xi_{ijt}) = \exp(x'_{ijt} \beta_x + \xi_{ijt})\) is given by

$$\ln(\pi_{ijt}) - \ln(\pi_{0jt}) = -\sigma\ln(P_{ijt}) + x'_{ijt} \beta_x + \xi_{ijt}.$$  

(62)

The equation (62) is logit demand system for homogenous consumers, and it does not accommodate for zero market share.

We consider the case \(x_{ijt} \neq z_{ijt}\) and denote \(p_{ijt} = \ln(P_{ijt})\) then the individual choice

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\(^{43}\)This is because, \(\ln(\pi_{ijt}) - \ln(\pi_{0jt}) = -\sigma\ln(P_{ijt}) + \ln(\kappa(x_{ijt}, \xi_{ijt})) - \ln(\kappa(x_{0jt}, \xi_{0jt})).\)
probability for the CES demand system is given by

$$\pi_{ijt} = \frac{\kappa(x_{ijt}, \xi_{ijt}, z_{ijt}, \eta_{ijt}) \exp(-\sigma p_{ijt})}{\sum_{h=1}^{d} \kappa(x_{hjt}, \xi_{hjt}, z_{hjt}, \eta_{hjt}) \exp(-\sigma p_{hjt})}. \quad (63)$$

The market share of product $i$ is zero when $\kappa(x_{ijt}, \xi_{ijt}, z_{ijt}, \eta_{ijt}) = 0$ for some values of $z_{ijt}$ and $\eta_{ijt}$. To accommodate for zero market shares, we follow Hortacsu and Joonhwi (2015) and choose the following specification for the kernel quality:

$$\kappa(x_{ijt}, \xi_{ijt}, z_{ijt}, \eta_{ijt}) = 1_{\{\phi + z'_{ijt} \beta_x + \eta_{ijt} > 0\}} \exp(x_{ijt} \beta_x + \xi_{ijt}), \quad (64)$$

where the consumer decides to buy based on the threshold $\phi$. In this case, demand system is given by

$$\frac{\pi_{ijt}}{\pi_{0jt}} = 1_{\{\phi + z'_{ijt} \beta_x + \eta_{ijt} > 0\}} \exp(-\sigma p_{ijt} + x'_{ijt} \beta_x + \xi_{ijt}). \quad (65)$$

Because of endogeneity problem $E[\xi_{ijt} | p_{ijt}] = 0$ does not hold and we consider $c_{ijt}$ the instruments used to identify $\sigma$ using the moment condition $E[\xi_{ijt} | c_{ijt}] = 0$. Even if the moment condition $E[\xi_{ijt} | c_{ijt}] = 0$ holds it does not imply that $E[\xi_{ijt} | c_{ijt}, \pi_{ijt} > 0] = 0$ holds. This is because the consumers select the products with high $\eta_{ijt}$ and it is more likely that $\eta_{ijt}$ and $\xi_{ijt}$ are positively correlated, which is a selection problem. Thus, we have

$$E[\ln \left( \frac{\pi_{ijt}}{\pi_{0jt}} \right) | c_{ijt}, \xi_{ijt}, \pi_{ijt} > 0] = -\sigma p_{ijt} + x'_{ijt} \beta_x + E[\xi_{ijt} | c_{ijt}, z'_{ijt}, \pi_{ijt} > 0]. \quad (66)$$

The estimation is in two stages. In the first stage, we estimate $\beta_x$ using the exogeneity of $z_{ijt}$ and the exclusion restriction on $z_{ijt}$ (i.e., $z_{ijt}$ includes an element that is not part of $x_{ijt}$). Then, the parameters $(\sigma, \beta_x)$ are estimated in the second stage

$$E[\ln \left( \frac{\pi_{ijt}}{\pi_{0jt}} \right) | c_{ijt}, \xi_{ijt}, \pi_{ijt} > 0] = -\sigma p_{ijt} + x'_{ijt} \beta_x + f(1 - G(\tau_{ijt} \beta_x)), \quad (67)$$

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where $f(\cdot)$ is an unknown function. Because $\ln \left( \frac{\pi_{ijt}}{\pi_{0jt}} \right) = q_{ijt} - q_{0jt}$, we can rewrite the previous expression as

$$q_{ijt} - q_{0jt} = -\sigma p_{ijt} + x'_{ijt}\beta_x + f(1 - G_{ijt}(\hat{z}_x)),$$

(68)

### Appendix C: Single product service generation function – Input vs Output Inventories

This appendix shows technical details that are used to derive the equation (1) in Section 2. For simplicity of the exposition, we focus on and homogeneous product and pay more attention to the interaction between demand and supply. The role of input vs output in retail is somewhat ambiguous. However, this does not affect our results. First, we consider inventory as an input into the store service generating function. Second, we consider inventory as output that affects consumer’s choices.

**Input inventory.** The supply of the products (in logs) at the store level is given by

$$q_{jt} = \alpha_l l_{jt} + \alpha_k k_{jt} + \alpha_a a_{jt} + \tilde{\omega}_{jt} + u_{s_{jt}},$$

(69)

where $l_{jt}$ is the log of the number of employees; $k_{jt}$ is the log the capital stock; $a_{jt}$ is the sum between the inventory at the beginning of period $t$ and the products acquired from the wholesaler during the whole period $t$; $\tilde{\omega}_{jt}$ measures store productivity and it is correlated over time; and $u_{s_{jt}}$ are remaining shocks to supply that satisfy $E[u_{s_{jt}}|F_{jt}]$.

The demand for store products (in logs) is given by

$$q_{jt} = -\eta(p_{jt} - p_t) + \tilde{\mu}_{jt} + u_{d_{jt}},$$

(70)

where $p_{jt}$ is the log price at the store level; $p_t$ is the log price at the industry level; $\tilde{\mu}_{jt}$ are shocks to demand that are not under store control and are correlated over time; and $u_{d_{jt}}$ are remaining shocks to demand that satisfy $E[u_{d_{jt}}|F_{jt}]$. The simple demand equation (70) is similar to the specifications used in the macro literature on inventories, e.g., Klette.
The log of sales at the store level can be written as the sum between equilibrium quantity and price, i.e., \( y_{jt} = q_{jt} + p_{jt} \). Then the supply and demand equations can be re-written in terms of sales, i.e.,

\[
y_{jt} = \alpha_l l_{jt} + \alpha_k k_{jt} + \alpha_a a_{jt} + \tilde{\omega}_{jt} + p_{jt} + u^s_{jt},
\]

\[
y_{jt} = (1 - \eta)p_{jt} - \eta p_t - \tilde{\mu}_{jt} + u^d_{jt}.
\]

From equation (79), the store price is given by

\[
p_{jt} = \frac{1}{1 - \eta} y_{jt} - \frac{\eta}{1 - \eta} p_t - \frac{1}{1 - \eta} \tilde{\mu}_{jt} - \frac{1}{1 - \eta} u^d_{jt}.
\]

By replacing the price in equation (78) with the one in equation (80), we obtain the sales generating function at the store level

\[
y_{jt} = \frac{\eta - 1}{\eta} \alpha_l l_{jt} + \frac{\eta - 1}{\eta} \alpha_k k_{jt} + \frac{\eta - 1}{\eta} \alpha_a a_{jt} + \frac{\eta - 1}{\eta} \tilde{\omega}_{jt} + \frac{1}{\eta} \tilde{\mu}_{jt} + p_t + \frac{\eta - 1}{\eta} u^s_{jt} + \frac{1}{\eta} u^d_{jt}.
\]

This is fact the sales generating function in the paper, i.e.,

\[
y_{jt} = \beta_l l_{jt} + \beta_k k_{jt} + \beta_a a_{jt} + \omega_{jt} + \mu_{jt} + u_{jt},
\]

where the \( \beta \)'s coefficients, \( \omega, \mu \), and remaining residuals are adjusted by the elasticity of demand. Because it is not the aim of this paper to model demand for products, we focus on the estimation of the aggregate equation (1). The main reason is that additional data are required for more sophisticated demand and inventory model.

### Output inventory

In this case, the supply of the products (in logs) at the store level is given by

\[
q_{jt} = \alpha_l l_{jt} + \alpha_k k_{jt} + \tilde{\omega}_{jt} + u^s_{jt}.
\]
The demand for store products (in logs) is given by

\[ q_{jt} = -\eta(p_{jt} - p_t) + \alpha_a a_{jt} + \tilde{\mu}_{jt} + u^d_{jt}. \]  \hfill (77)

The simple demand equation (77) is similar to the specifications used in the macro literature on inventories, e.g., Coen-Pirani (2004). The coefficient \( \alpha_a \) measures how shocks to goods that are under store control contribute to generate higher sales at a given price.

The log of sales at the store level can be written as the sum between equilibrium quantity and price, i.e., \( y_{jt} = q_{jt} + p_{jt} \). Then the supply and demand equations can be re-written in terms of sales, i.e.,

\[ y_{jt} = \alpha_l l_{jt} + \alpha_k k_{jt} + \omega_{jt} + p_{jt} + u^s_{jt}, \]  \hfill (78)

\[ y_{jt} = (1 - \eta)p_{jt} - \eta p_t + \alpha_a a_{jt} + \tilde{\mu}_{jt} + u^d_{jt}. \]  \hfill (79)

From equation (79), the store price is given by

\[ p_{jt} = \frac{1}{1 - \eta}y_{jt} - \frac{\eta}{1 - \eta}p_t - \frac{\alpha_a}{1 - \eta}a_{jt} - \frac{1}{1 - \eta}\tilde{\mu}_{jt} - \frac{1}{1 - \eta}u^d_{jt}. \]  \hfill (80)

By replacing the price in equation (78) with the one in equation (80), we obtain the sales generating function at the store level

\[ y_{jt} = \eta - \frac{1}{\eta}\alpha_l l_{jt} + \frac{\eta - 1}{\eta}\alpha_k k_{jt} + \frac{1}{\eta}\alpha_a a_{jt} + \frac{\eta - 1}{\eta}\tilde{\omega}_{jt} + \frac{1}{\eta}\tilde{\mu}_{jt} + p_t + \frac{\eta - 1}{\eta}u^s_{jt} + \frac{1}{\eta}u^d_{jt}. \]  \hfill (81)

This is fact the sales generating function in the paper, i.e.,

\[ y_{jt} = \beta_l l_{jt} + \beta_k k_{jt} + \beta_a a_{jt} + \omega_{jt} + \mu_{jt} + u_{jt}, \]  \hfill (82)

where the \( \beta \)'s coefficients, \( \omega, \mu, \) and remaining residuals are adjusted by the elasticity of demand, which is equivalent with equation (74).
Appendix D: Selection

Store decisions to exit in period $t$ depend directly on $\omega_{jt}$ and $\mu_{jt}$; therefore, the decision is correlated with the productivity shock $\xi_{jt}$ (Olley and Pakes, 1996). Selection can affect retail markets because large stores (large $k_{jt}$) are more likely to survive larger negative productivity shocks than are small stores. Even if stores have low productivity, there might be other reasons for stores to stay active, such as favorable market conditions, logistics support by the firm, and a good location. To control for selection when estimating the service-generating function, we use predicted survival probabilities $\mathcal{P}_{jt}$ (Olley and Pakes, 1996). The probability of being in the data in period $t$ conditional on the information in $t-1$ is given by $Pr(\chi_{jt} = 1|\omega_{mt}, \mathcal{F}_{t-1}) = Pr(\omega_{jt} \geq \omega_{mt}, |\omega_{mt}, \mathcal{F}_{t-1}) = P_{jt}(l_{jt-1}, k_{jt-1}, n_{jt-1}, a_{jt-1}) = \mathcal{P}_{jt}$, where the second equality follows from the inverse of the investment and inventory functions. In the estimation, selection affects only the productivity process, i.e., $\omega_{jt} = h_\omega(\omega_{jt-1}, \mu_{jt-1}, \mathcal{P}_{jt}) + \xi_{jt}$.

Appendix E: Invertibility conditions with two unobservables

The general labor demand and inventory functions that arise from the stores’ dynamic optimization problem are

$$l_{jt} = \tilde{l}_t(\omega_{jt}, \mu_{jt}, k_{jt}, n_{jt}, w_{jt})$$

$$a_{jt} = \tilde{a}_t(\omega_{jt}, \mu_{jt}, k_{jt}, n_{jt}, w_{jt}).$$

The main aim is to recover $\omega_{jt}$ and $\mu_{jt}$ using this system of equations. The conditions required for identification can be grouped as follows: (i) general conditions that the policy functions of dynamic programming problem have to satisfy; (ii) conditions that the system of equations should satisfy to have a unique solution. In what follows, we discuss these conditions.

First, to back out $\omega_{jt}$ and $\mu_{jt}$, the assumption 5 must hold, i.e., the policy functions $\tilde{l}_t(\cdot)$ and $\tilde{a}_t(\cdot)$ must be strictly monotonic in $\omega_{jt}$ and $\mu_{jt}$, which holds under mild regularity.
conditions on the dynamic programming problem (Pakes, 1994; Maican, 2016b). The static profits are assumed to be strictly increasing in $\omega_{jt}$, $\mu_{jt}$, and $k_{jt}$ and continuously differentiable in these variables. Another condition is super-modularity of the static profits with respect to $\omega_{jt}$ and $\mu_{jt}$, i.e., the impact of productivity on profits is increasing in $\mu_{jt}$. In other words, stores with large demand shocks experience a larger increase in profits due to productivity. This assumption is not restrictive since stores that experience large demand shocks invest to increase their productivity to satisfy the demand. Furthermore, static profits are assumed to be supermodular with respect to $\omega_{jt}$ and $\mu_{jt}$, i.e., marginal product of capital is increasing in productivity and demand shocks. This condition can be also interpreted as, stores with larger capital stock have higher profits due to an increase in productivity or demand shocks. All these conditions (strict monotonicity and supermodularity) on static profits yield that value and policy functions are strictly increasing in $\omega_{jt}$, $\mu_{jt}$, and $k_{jt}$ (Pakes, 1994; Maican, 2016b).

Second, we discuss the general properties that must be satisfied by labor demand ($\tilde{l}()$) and inventory ($\tilde{a}()$) functions such that the system (83) has a unique solution. This system can be solved for $\omega_{jt}$ and $\mu_{jt}$ in terms of $k_{jt}$, $n_{jt}$, $l_{jt}$, $w_{jt}$, and $a_{jt}$ when certain partial derivatives are continuous and the $2 \times 2$ Jacobian determinant $\partial(\tilde{l}, \tilde{a})/\partial(\omega, \mu)$ is not zero. In other words, the ratios between the impact of $\omega$ and $\mu$ on the investment and inventories should not be the same, i.e., $(\partial \tilde{l}/\partial \omega)/(\partial \tilde{l}/\partial \mu) \neq (\partial \tilde{a}/\partial \omega)/(\partial \tilde{a}/\partial \mu)$. Therefore, this condition requires that productivity and demand shocks have a different impact on investment and inventory, and the relative impact is not the same.

We apply implicit function theorem to prove the invertibility of the system (83). In our case, points in $(2 + 5)$- dimensional space $\mathbb{R}^{2+5}$ can be written in the form of $(x; b)$, where $x = (\omega, \mu)$ and $b = (k, n, l, a, w)$. We can rewrite the system as $f_1(x; b) = 0$ and $f_2(x; b) = 0$ or simply as an equation $F(x; b) = 0$. To understand the invertibility of the policy functions, we need to know when the relation $F(x; b) = 0$ is a also a function. In other words, what are the conditions such that $F(x; b) = 0$ can be solved explicitly for $b$ in terms of $x$ obtaining a unique solution. The Theorem E.1 (implicit function theorem) provides the conditions that for a given point $(x_0, b_0)$ such that $F(x_0, b_0) = 0$ there exists
a neighborhood of \((x_0, b_0)\) where the relation \(F(x; b) = 0\) is a function.

**Theorem E.1.** Let \(f = (f_1, f_2)\) is a vector values function defined on the open set \(S\) in \(\mathbb{R}^{2+5}\) with values in \(\mathbb{R}^2\). Suppose \(f \in C'\) on \(S\). Let \((x_0; b_0)\) be a point in \(S\) for which \(f(x_0, b_0) = 0\) and for which the \(2 \times 2\) Jacobian determinant \(\partial(f_1, f_2)/\partial(\omega, \mu)\) is not zero at \((x_0, b_0)\). Then there exists a 5-dimensional open set \(B_0\) that includes \(b_0\) and one, and only one vector based functions \(g\) defined on \(B_0\) and having values in \(\mathbb{R}^2\) such that

(i) \(g \in C'\) on \(B_0\)

(ii) \(g(b_0) = x_0\)

(iii) \(f(g(b); b) = 0\) for every \(b\) in \(B_0\).

**PROOF:** This theorem is, in fact, the implicit function theorem applied on our case. The general proof of the theorem can found in Apostol (1974).

**Appendix F: Entry regulation: Plan and Building Act (PBL)**

On July 1, 1987, a new regulation was imposed in Sweden, the Plan and Building Act (PBL). Compared to the previous legislation, the decision process for market entry become decentralized, giving local governments power over entry in their municipality and citizens a right to appeal the decisions. Since 1987, only minor changes have been made to the PBL. From April 1, 1992, to December 31, 1996, the PBL was slightly different, prohibiting the use of buildings from counteracting efficient competition. Since 1997, the PBL has been more or less the same as it was prior to 1992. Long time lags in the planning process make it impossible to directly evaluate the impact of decisions. In practice, differences due to policy changes seem small (Swedish Competition Authority, 2001:4). Nevertheless, the PBL is considered to be one of the major entry barriers, resulting in different outcomes, e.g., price levels, across municipalities (Swedish Competition Authority, 2001:4; Swedish Competition Authority, 2004:2). Municipalities are then, through the PBL, able to put pressure on prices. Those that constrain entry have less sales per capita, while those where large and discount stores have a higher market share also have
lower prices.

The majority of OECD countries have entry regulations that empower local authorities to decide on store entry. However, the regulations differ substantially across countries (Boylaud and Nicoletti, 2001; Griffith and Harmgart, 2005; Schivardi and Viviano, 2011). While some countries strictly regulate large entrants, more flexible zoning laws exist, for instance, in the U.S. (Pilat, 1997).

The Swedish Plan and Building Act (PBL) regulates the use of land and water and buildings. The PBL consists of the planning requirements for land and water areas as well as buildings. The ultimate goal of PBL is to promote equal and adequate living conditions and a lasting sustainable environment for today and future generations. The regulation contains two documents/plans: (i) the comprehensive plan and (ii) the detailed development plan. Municipalities are required to have a comprehensive plan that covers the entire municipality and that guides decisions regarding the use of land, water areas and the built environment. The comprehensive plan records public interests and national interests. Municipalities also have to provide detailed development plans that cover only a fraction of the municipality. Municipalities are divided into smaller areas. These plans indicate and set limits on the use and design of public spaces, land and water areas.

The purpose of the comprehensive plan is to provide an attractive public environment that is sustainable. It is the basis for decisions regarding the use of land, water and the development and preservation of buildings. It reflects the public interest and addresses important environmental and risk factors that must be taken into account in the planning of any endeavor. Necessary features include the housing needs of the municipal inhabitants, the protection of valuable natural and cultural environments, and providing inhabitants with access to services.

The detailed development plan consists of a map with text that indicates what, where and how one is allowed to build, as well as appropriate uses for the area. For instance, it indicates the appropriate design and use of housing, nature and water areas. Other examples include construction rights for real estate including the size and form of structures, the possibility of opening a restaurant, work places and businesses, housing, hotels,
housing (villa or apartments), pre-schools, elementary schools, health care, energy- and water services, parks, streets, squares, etc.

The detailed development plan indicates whether retail stores are allowed. The right to open and operate a retail food store is addressed in the detailed development plan. Each store seeking to enter the market is required to file a formal application with the local government. For the entry to occur, the municipality can accept a new detailed development plan or make changes in an existing one. First, in the application, the store must state the purpose of the activity: retail, housing, offices, manufacturing, or other. Second, the store must describe the main purpose of its activity and what it is to contain, e.g., a new building of a certain size, wholesale provision with trucks, parking places and is obligated to be as detailed as possible. Before the new detailed development plan is approved, it must be made publicly available. Inhabitants of the municipality are allowed to express their opinions and views on the proposed changes. If some do not agree with the proposed plan, they can appeal. The municipality must then perform a new evaluation and look for alternative solutions to the question at hand.

When a retail store seeks to enter a local market, the municipality evaluates the consequences for exit, prices, local employment, availability of store types and product assortments for different types of consumers, purchasing patterns and purchasing trips, consumer travel behavior, traffic (e.g., generated traffic per square meter of the new sales space) including the effect it has on noise and air pollution for nearby consumers, as well as the number of individuals who will be affected - probable health effects, risk evaluations, broader environmental issues, increased distance to the store, parking, water, energy supply, etc.

In addition, the municipal council must evaluate the positive and negative consequences of the new entrant for different inhabitants, the environment, traffic, public transport, safety, etc. The municipality must consider whether new bus lines are necessary, as well as walking and biking paths. This is to ensure each consumer in the municipality has access to different types of stores, a broad product assortment and reasonable prices. A store entrant is prohibited from hindering real estate developments that
will be useful for the public interest, i.e., housing, places of work, traffic infrastructure and leisure environments. The municipal council evaluates and gives an overall assessment of the trade-offs between the public interest and private retail interests. This assessment is based on contingency analysis, an investigation of alternative solutions and developments, and strategic judgement. It is important to evaluate the effects that accepting a new detailed development plan and changing an existing one on the public interest.

All stores are regulated by the PBL in Sweden, in contrast with, for example, the U.K., which explicitly focuses on regulating large stores (Maican and Orth, 2015; Sadun, 2015). The PBL is considered one of the major barriers to entry and is the cause of a diverse array of outcomes, e.g., price levels, across municipalities (Swedish Competition Authority, 2001:4). Several reports stress the need to better analyze how entry regulation affects market outcomes (Pilat, 1997; Swedish Competition Authority, 2001:4; Swedish Competition Authority, 2004:2).