Wholesale Price Discrimination with Asymmetric Vertical Integration

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Abstract

This paper analyzes the welfare effects of price discrimination in intermediate good markets when downstream firms have asymmetric degrees of vertical integration. A monopolistic upstream firm optimally discriminates between downstream firms as asymmetric vertical integration leads to asymmetric agency costs. I show that price discrimination due to asymmetric vertical integration increases total welfare under mild conditions. I relate this result to the welfare effects of price discrimination in a model where downstream firms have different costs due to different production technologies. Under the same conditions that ensure price discrimination to be welfare-increasing with asymmetric vertical integration, price discrimination reduces welfare with asymmetric production technologies. Thus, the source of cost differences may crucially influence the welfare implications of price discrimination.

Keywords: Price discrimination, Intermediate good markets, Agency cost, Asymmetric vertical integration

JEL Classification: L12, L23, L42
1 Introduction

Competition authorities in both the United States and the European Union consider price discrimination a potential abuse of a dominant market position. On this basis, an upstream firm can be restrained from offering different tariffs to different downstream firms. The extant literature on price discrimination in intermediate good markets analyzes the welfare effects of a ban on price discrimination (Katz, 1987; DeGraba, 1990; Yoshida, 2000; Inderst and Shaffer, 2009; Inderst and Valletti, 2009; Villas-Boas, 2009; Herweg and Müller, 2014). Thereby, the literature focusses on the case in which downstream firms differ with respect to production costs, i.e., some firms use more efficient production technologies than others. However, different production technologies are only one reason for cost differences between firms. If firms have different degrees of vertical integration, cost differences may result from agency costs that arise due to conflicting interests of the stakeholders in a vertically separated organization. Whereas firms may view production costs and agency costs as equivalent, these two types of costs are very different from a welfare perspective. Production costs reflect opportunity costs and matter for total welfare. By contrast, agency costs are about the distribution of profits within an organization and are therefore neutral from a welfare perspective.

In this paper, I study the welfare effects of price discrimination in intermediate good markets in which downstream firms have different degrees of vertical integration. Asymmetric vertical integration is a feature of many industries. The German grocery market is a prominent example. The largest retailer Edeka – with a market share of around 25% – consists of regional wholesalers that exclusively serve independently run supermarkets. By contrast, the second largest retailer Rewe – with a market share of around 20% – is a centralized company that owns both wholesalers and supermarkets. Both retailers buy their products from powerful food companies such as Nestlé. Following this example, I consider a model with a monopolistic upstream firm that sells an intermediate good to two downstream firms. The first downstream firm is vertically integrated and sells the final good to consumers itself. The second downstream firm is vertically separated and sells the final good through a subcontractor. The integrated downstream firm and the subcontractor have private information about their costs of serving the consumer market. The upstream firm can offer the downstream firms arbitrary nonlinear tariffs.

First, I show that asymmetric vertical integration can create incentives for price discrimination. The upstream firm typically finds it optimal to favor the vertically separated firm. Due to the informational advantage of the subcontractor, the separated downstream firm has to give

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1. The Patman-Robinson Act regulates price discrimination in the United States. In the EU, Article 82c) of the CE treaty restricts the use of price discrimination.

2. Buehler and Schmutzler (2005) report the coexistence of vertically integrated and separated firms in the US petroleum refining industry (Bindemann, 1999), the UK package holiday industry (Buehler and Schmutzler, 2005), the UK beer industry (Slade, 1998a), gasoline retailing in Canada (Slade, 1998b), the US cable television industry (Waterman and Weiss, 1996; Chipty, 2001), and the Mexican footwear industry (Woodruff, 2002).

3. Equivalently, they could have private information about demand.

4. Price discrimination refers to third degree price discrimination, i.e., the upstream firm offers the downstream firms different tariffs. This approach is in line with the regulatory policy of the EU that does not consider quantity discounts as an abuse of a dominant market position, c.f. Herweg and Müller (2012).
an information rent to its subcontractor. This rent is an agency cost which leads to a cost disadvantage for the vertically separated downstream firm. Due to this cost disadvantage, the vertically separated downstream firm reacts more sensitively to price increases. The upstream firm therefore offers the vertically separated firm a lower tariff.

As the main result of this paper, I show that price discrimination due to asymmetric vertical integration increases total welfare under mild conditions. This result is implied by the following reasoning. Due to agency costs, the vertically separated firm sells less of the final good than its competitor even if the real costs of serving the final good markets are identical. Thus, price discrimination partially offsets the cost disadvantage of vertical separation and thereby reduces the distortion within the vertically separated organization. Under mild conditions on demand, the total sales of the downstream firms are higher under price discrimination than under uniform pricing. Together with the marginal effect described above, this implies that price discrimination increases total welfare.

I relate this result to the welfare effects of price discrimination in an observationally equivalent model with two vertically integrated downstream firms with asymmetric production costs. For this model, I show that price discrimination reduces welfare under the conditions on demand that make price discrimination beneficial in the model with asymmetric agency costs. In the model with asymmetric production costs, price discrimination implies that the downstream firms sell different quantities of the final good if they have the same costs of serving consumers. With uniform pricing, both downstream firms produce the same quantity if they have the same production costs. Thus, a ban on price discrimination reduces the difference in sales between the downstream firms. If both final good markets are covered, the same conditions as before imply that price discrimination reduces the total production of the downstream firms for identical costs. Together with the marginal effect described above, this implies that price discrimination reduces welfare in the model with asymmetric production costs.

These results imply that the source of cost differences between downstream firms crucially influences the welfare effects of price discrimination in intermediate good markets. A competition authority may be unable to assess the welfare effects of price discrimination based on observations of total costs alone. Rather, the competition authority may be obliged to inquire whether cost differences are the consequence of differences in production technologies or in organizational features which influence agency costs.

**Related literature** There is a long-standing interest in the welfare effects of price discrimination. Robinson (1933) provides the first formal analysis of the welfare effects of price discrimination in final good markets. Aguirre, Cowan and Vickers (2010) and Cowan (2012) extend and generalize her insights to more general demand functions.

The literature on price discrimination in intermediate good markets starts with Katz (1987) who shows that price discrimination can be detrimental for welfare if larger downstream firms have the possibility to engage in inefficient backward integration by producing an input instead of buying it. In a model with linear demand curves, DeGraba (1990) shows that price discrimination reduces welfare in the short and in the long run. Yoshida (2000) analyzes a model
with linear production technologies and shows that price discrimination often reduces welfare. Inderst and Valletti (2009) analyze the short and long run welfare effects of price discrimination when downstream firms have the opportunity to change their supplier. They show that price discrimination reduces consumer surplus in the short run but can increase consumer surplus in the long run. Arya and Mittendorf (2010) demonstrate that price discrimination can increase social welfare in a model where downstream firms serve several final good markets and lower demand markets are less competitive.

In contrast to the previous papers, and as in the current paper, O’Brien and Shaffer (1994), Inderst and Shaffer (2009), and Herweg and Müller (2014) analyze the case where the upstream firm sets non-linear tariffs. O’Brien and Shaffer (1994) and Inderst and Shaffer (2009) analyze settings under complete information where the upstream firm offers two-part tariffs to the downstream firms. Both papers show that price discrimination is usually welfare increasing. Herweg and Müller (2014) is closest to this paper. They analyze a model where an upstream firm sells an input to two downstream firms which have private information about their production costs. Herweg and Müller show that price discrimination is welfare-reducing as long as markets are covered under uniform pricing. The model with asymmetric production costs studied in the this paper is a version of their model.

In the first empirical analysis of the welfare effects of price discrimination in intermediate good markets, Villas-Boas (2009) analyses the German grocery market for coffee. Using structural estimations of cost and demand functions, the author simulates a ban on price discrimination and finds a positive welfare effect. However, as most of the theoretical literature, estimated costs are assumed to reflect differences in the efficiency of serving the consumer market. Given the asymmetric forms of vertical organization in the German grocery market, the results described above show that this assumption is not innocuous and may bias the results.

In contrast to the extant literature on price discrimination, the current paper studies asymmetric vertical integration as the underlying reason for price discrimination. Thus, this paper connects the literature on price discrimination in intermediate good markets with the literature on asymmetric vertical integration (Buehler and Schmutzler, 2005, 2008).

In the next section, I introduce the model. Section 3 provides the equilibrium analysis. In Section 4, the welfare effects of price discrimination are analyzed under asymmetric agency costs. Section 5 presents a comparison to the welfare result under asymmetric production costs. Section 6 concludes.

2 A model with asymmetric vertical integration

An upstream firm $U$ produces an intermediate good at zero marginal cost for two downstream firms which serve two independent markets. The first downstream firm $D_1$ serves market 1 and the second downstream firm $D_2$ serves market 2. The inverse demand function is identical on both markets and given by $P(q_i)$ where $q_i$ is the quantity which is sold on market $i$. The inverse demand function $P(q_i)$ is strictly decreasing in the quantity $q_i$. Downstream firm $D_1$ produces the final good with a one-to-one technology which transforms $q_1$ units of the intermediate good
into $q_1$ units of the final good at a cost $c_1 q_1$. The marginal cost $c_1$ is private information to $D_1$ and drawn from the interval $[c, \bar{c}] \subset \mathbb{R}_+$ according to the cumulative distribution function $F(\cdot)$ which has a twice differentiable density $f(\cdot)$. $D_2$ buys the input from $U$ and mandates the subcontractor $S$ to produce the final good using a one-to-one production technology. $S$ produces the quantity $q_2$ of the final good at production cost $c_2 q_2$. The marginal costs $c_2$ are private information to $S$ and drawn from the interval $[c, \bar{c}]$ according to the same cumulative distribution function $F(\cdot)$. I refer to $D_1$ as the vertically integrated downstream firm and to $D_2$ and $S$ as the vertically separated downstream firm. The structure of the industry is illustrated in Figure 1.

![Figure 1: Industry structure with different degree of vertical integration](image)

The upstream firm $U$ sells an input to the vertically integrated downstream firm $D_1$ and the vertically separated downstream $D_2$ that delegates production to the subcontractor $S$. $D_1$ and $D_2$ serve independent markets.

**Profit Functions** Let $t_i$ be a transfer from $D_i$ to $U$ with $i \in \{1, 2\}$, $t_s$ be a payment from $S$ to $D_2$, and $q_i$ be the quantity sold on market $i$. $U$’s profit is $t_1 + t_2$, $D_1$’s profit is $q_1 P(q_1) - c_1 q_1 - t_1$, $D_2$ receives a profit of $t_s - t_2$, and $S$ makes a profit of $q_2 P(q_2) - c_2 q_2 - t_s$.

**The contracting game** I study the following contracting game between $U$, $D_1$, $D_2$, and $S$. At the beginning of the game, marginal costs $c_1$ and $c_2$ are drawn. The parameter $c_1$ is only observed by $D_1$ and $c_2$ is only observed by $S$. Next, $U$ offers a tariff $T_1(q_1)$ to $D_1$ and a tariff $T_2(q_2)$ to $D_2$. These tariffs imply that $D_i$ can order a quantity $q_i$ of the intermediate good at a total payment of $T_i(q_i)$. Then, $D_2$ offers a tariff $T_s(q_2)$ to $S$. With this tariff, $D_2$ commits to deliver a quantity $q_2$ of the intermediate good at a payment $T_s(q_2)$. $S$ can accept or reject this offer. If $S$ accepts and orders a positive quantity $q_2$, $D_2$ is committed to accept the tariff $T_2(q_2)$. After $S$ has made the participation decision, $D_1$ chooses whether to accept or reject $T_1(q_1)$. Finally, production takes place and payoffs realize. I impose no restrictions – such as linearity – on the tariffs $T_1(q_1)$, $T_2(q_2)$, and $T_s(q_2)$. I analyze $U$’s most preferred Perfect Bayesian Equilibrium of this game.
Assumptions I make the following two assumptions about demand and the distribution of marginal costs that are maintained through the analysis. I first impose two mild conditions on the demand function.

**Assumption 1.** The inverse demand function $P(q)$ satisfies the following conditions:

1. Revenue $R(q) \equiv qP(q)$ is strictly concave with finite maximizer;
2. $P(0) > c$, i.e., production is socially beneficial with positive probability.

The second part of the assumption ensures that production is both profitable and socially beneficial if the cost parameter is sufficiently low.

Next, I state an assumption on the distribution of marginal costs. To this purpose, define the virtual marginal cost $h(c) \equiv c + \frac{F(c)}{f(c)}$. The virtual marginal cost $h(c_i)$ is distributed according to the cumulative distribution function $G(c) \equiv \Pr(h(c_i) \leq c) = F(h^{-1}(c))$ with the density $g(c) = G'(c)$.

**Assumption 2.** The distribution function $F(\cdot)$ satisfies the following conditions:

1. $F(c)/f(c)$ is increasing and convex;
2. $g(c)/f(c)$ is weakly increasing;
3. $c + \frac{F(c) + G(c)}{f(c) + g(c)}$ is weakly increasing.

Assumption 2 is satisfied for a large class of standard distribution functions such as the uniform, normal, logistic, beta, gamma, Gumbel, Fréchet, Weibull, chi squared, and chi distributions.

Part 1. of the assumption implies that $h'(\cdot) \geq 1$ and $h''(\cdot) \geq 0$. This assumption corresponds to conditions on pass-through rates of demand functions when demand is interpreted as a distribution function in the tradition of Bulow and Pfleiderer (1983). An increasing hazard rate is equivalent to a pass-through rate greater than one, and a convex hazard rate is equivalent to an increasing pass-through rate. Weyl and Fabinger (2009) show that these two assumptions are satisfied for a large class of standard distribution functions. The first part of the assumption also ensures that the inverse hazard rate of the virtual marginal costs given by $\frac{G(c)}{g(c)} = \frac{F(h^{-1}(c))}{f(h^{-1}(c))} h'(h^{-1}(c))$ is increasing.

The second part of the assumption demands that the distribution of virtual marginal cost dominates the distribution of real marginal cost in the likelihood ratio order. The last part of the assumption ensures that the virtual marginal costs under a ban on price discrimination are weakly increasing.\(^5\)

\(^5\)The uniform distribution satisfies Assumption 2. The distribution function $F(c) = e^{-1/a(c-b)^2}$ on $[b, \infty)$ has a strictly increasing and convex virtual marginal cost function. In Appendix B, I show that it also satisfies the other parts of Assumption 2.
Industry optimum  If there is no conflict of interest within the industry, the four players maximize the total industry profit. For marginal costs \( c_1 \) and \( c_2 \), the total profit is given by \( \sum_{i=1}^{2} R(q_i) - c_i q_i \). Assumption 1 implies that there is a unique pair of optimal quantities \((q_1^*(c_1), q_2^*(c_2))\) that satisfies \( q_1^*(c) = q_2^*(c) = q^*(c) \).

Public information  Private information is an essential ingredient of the model. With public information, the difference in the vertical organization of the downstream firms is irrelevant because the seller can charge nonlinear tariffs. To see this, suppose that marginal costs \( c_1 \) and \( c_2 \) are public information. In this case, \( U \) can extract the whole industry profit. \( U \) optimally offers \( D_1 \) and \( D_2 \) the menu of two-part-tariffs \( \{w(c), \Phi(c)\}_{c \in [c_1, c_2]} \) where the pair of piece-rate \( w(c) \) and fixed payment \( \Phi(c) \) is chosen by \( U \) after the realization of marginal costs. \( U \) optimally sets \( w(c) = 0 \) and extracts the whole profit with the fixed payment \( \Phi(c) = R(q^*(c)) - c q^*(c) \). \( D_1 \) cannot do better than to accept the tariff and to choose the quantity \( q_1(c_1) = q^*(c_1) \). For \( D_2 \), it is optimal to pass on the contractual terms to \( S \). \( S \) is then in the same situation as \( D_1 \) and produces the quantity \( q_2(c_2) = q^*(c_2) \). \( U \) discriminates between \( D_1 \) and \( D_2 \) only on the basis of their realized marginal costs, not on the basis of their organizational form.\(^6\) The analysis in this paper shows that asymmetric information about cost parameters renders discrimination on the basis of the organizational form profitable even if the upstream firm can offer arbitrary tariffs.

Price discrimination  I formally define price discrimination as follows.

**Definition 1.** \( U \) price discriminates between \( D_1 \) and \( D_2 \) if and only if \( T_1(\cdot) \neq T_2(\cdot) \). \( U \) favors \( D_i \) if \( T_i(\cdot) \leq T_j(\cdot) \) and there exists some \( q \) with \( T_i(q) < T_j(q) \) for \( i, j \in \{1, 2\} \) and \( i \neq j \).

Due to the presence of private information, \( U \) may choose tariffs which reflect two different forms of price discrimination. First, \( U \) may wish to offer \( D_1 \) and \( D_2 \) different contracts based on the observable difference in their vertical organization. This would be an instance of third degree price discrimination. Second, \( U \) may find it optimal to offer nonlinear contracts to the downstream firms in order to screen their private information in the most profitable way, i.e., to engage in second degree price discrimination. Here, I follow the literature on price discrimination with nonlinear tariffs (Inderst and Shaffer, 2009; Herweg and Müller, 2014) and focus on third degree price discrimination and its welfare effects. In the following, the term price discrimination always refers to third degree price discrimination.

Private information on demand  The model can be reinterpreted as a situation with private information about demand. In particular, let the demand on final good market \( i \) be given by \( \hat{P}(q, c_i) = P(q) - c_i q \), suppose that \( D_1 \) is privately informed about \( c_1 \) and the price on final good market 1 and \( S \) is privately informed about \( c_2 \) and the price on final good market 2, and assume that both \( D_1 \) and \( S \) have zero marginal costs of transforming one unit of the intermediate into one unit of the final good. The analysis in the following sections carries over to this model.

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\(^6\)The analysis of this model under public information is a special case in Inderst and Shaffer (2009) who analyze price discrimination under symmetric information with two part tariffs.
3 Optimal tariffs

In this section, I provide the equilibrium analysis of the game for the cases of price discrimination and uniform pricing.

Optimal tariffs under price discrimination

I first analyze the optimal wholesale tariffs that $U$ offers to the downstream firms $D_1$ and $D_2$ in case $U$ can discriminate between the downstream firms. In a second step, I compare these tariffs to determine whether $U$ discriminates between the downstream firms.

Optimal wholesale tariff for $D_1$ $U$ chooses the tariff $T_1(q_1)$ in order to maximize its expected profit. The optimal tariff needs to satisfy the incentive constraint that $D_1$ orders its preferred quantity for any value of the cost parameter, and the participation constraint that $D_1$ accepts the tariff $T_1(q_1)$. $U$’s optimal tariff is therefore a solution to the following maximization problem.

$$P_d^1: \max_{T_1(\cdot)} \int T_1(q_1(c_1))dF(c_1)$$

s.t. 
$$q_1(c_1) \in \arg \max_q R(q) - c_1q - T_1(q), \quad (IC_1)$$

$$\max_q R(q) - c_1q - T_1(q) \geq 0 \text{ for all } c_1 \in [\underline{c}, \bar{c}]. \quad (PC_1)$$

This problem can be solved using standard techniques as provided in the clear exposition by Martimort and Stole (2009). The solution to this problem is presented in the following Lemma.

**Lemma 1.** If price discrimination is permitted, $U$ offers the tariff $T_1^d(q_1)$ to $D_1$, $D_1$ accepts the offer, and orders a quantity $q_1^d(c_1)$. $T_1^d(q_1)$ and $q_1^d(c_1)$ are given by

$$T_1^d(q) = R(q) - \int_0^q (q_1^d)^{-1}(x)dx, \quad (1)$$

$$q_1^d(c_1) = \arg \max_q R(q) - h(c_1)q, \quad (2)$$

where $(q_1^d)^{-1}(\cdot)$ is the inverse of $q_1^d(c_1)$ with $(q_1^d)^{-1}(0) = \bar{c}_1 \equiv \inf \{c \in [\underline{c}, \bar{c}] : q_1^d(c) = 0\}$.

$U$ optimally screens by offering a tariff under which $D_1$ orders a quantity that maximizes the virtual industry profit on market 1. This is reflected by the definition of the quantity $q_1^d(c_1)$ in equation (2). The optimal tariff $T_1^d(\cdot)$ induces $D_1$ to choose this quantity.

Optimal wholesale tariff for $D_2$ I now turn to the analysis of the optimal tariff for downstream firm $D_2$. For a given tariff $T_2(q_2)$, $D_2$ offers a tariff $T_s(q_2)$ to $S$. The optimal offer of $U$ to $D_2$ depends on the tariff which $D_2$ offers to $S$ in response.

The game can be solved backwards. I first analyze the optimal offer of $D_2$ to $S$ for any given tariff $T_2(q_2)$. $D_2$ optimally offers a tariff $T_s(q_2)$ which maximizes its expected net payments, ensuring that the anticipated equilibrium quantities $q_2(c_2)$ are indeed optimal for $S$ and
guaranteeing $S$ a non-negative profit. For the given tariff $T_2(q_2)$, $D_2$ thus offers a tariff $T_s(q_2)$ which is a solution to the following problem.

$$
\mathcal{P}^d_s : \max_{T_s(\cdot)} \int (T_s(q_2(c_2)) - T_2(q_2(c_2)))dF(c_2)
$$

s.t. \quad q_2(c_2) \in \arg \max_q R(q) - c_2q - T_s(q), \quad (IC_s)

max \quad R(q) - c_2q - T_s(q) \geq 0 \quad \text{for all } c_2 \in [\underline{c}, \overline{c}]. \quad (PC_s)

$D_2$’s optimal response to the offered tariff $T_2(q_2)$ satisfies the following conditions.

**Lemma 2.** Given any tariff $T_2(q_2)$ offered by $U$, $D_2$ optimally offers a tariff $T_s(q_2)$ under which $S$ orders the quantity $q_2(c_2)$ with

$$
q_2(c_2) = \arg \max_q R(q) - h(c_2)q - T_2(q) \quad (3)
$$

and $D_2$ makes a non-negative profit for any $c_2 \in [\underline{c}, \overline{c}]$, i.e.

$$
\max_q R(q) - h(c_2)q - T_2(q) \geq 0. \quad (4)
$$

$D_2$ offers a tariff which induces $S$ to order a quantity that maximizes the virtual joint profit of $D_2$ and $S$. This is reflected in equation (3). Furthermore, $D_2$ can guarantee itself a non-negative profit. To see this, suppose $D_2$’s profit is negative for some realization of the cost parameter $c_2'$. By the envelope theorem, $D_2$’s profit for cost parameter $c_2$, given by \[ \max_q R(q) - h(c_2)q - T_2(q) \] is decreasing in $c_2$. Thus $D_2$’s profit is negative for all cost parameters higher than $c_2'$. $D_2$ can then offer a different tariff under which $S$ orders zero quantity for all values of the cost parameter above $c_2'$, $D_2$ rejects $U$’s offer for these values of $c_2$, and ordered quantities remain the same as before for cost parameters below $c_2'$. The new quantity schedule remains non-increasing and therefore there exists a tariff which makes these quantities optimal for $S$. Furthermore, this tariff increases $D_2$’s expected profit.

$U$ optimally offers a tariff $T_2(q_2)$ to $D_2$ which anticipates the quantity that $S$ orders under an optimal response tariff $T_s(q_2)$, and which guarantees $D_2$ a non-negative profit for any realization of the marginal cost $c_2$. Formally, the optimal offer $T_2(q_2)$ solves the following optimization problem.

$$
\mathcal{P}^d_2 : \max_{T_2(\cdot)} \int T_2(q_2(c_2))dF(c_2)
$$

s.t. \quad q_2(c_2) \in \arg \max_q R(q) - h(c_2)q - T_2(q), \quad (IC_2)

max \quad R(q) - h(c_2)q - T_2(q) \geq 0 \quad \text{for all } c_2 \in [\underline{c}, \overline{c}]. \quad (PC_2)

The solution to this optimization problem completes the characterization of equilibrium behavior under price discrimination.
Lemma 3. If price discrimination is permitted, $U$ offers the tariff $T^d_2(q_2)$ to $D_2$, $D_2$ offers the tariff $T^d_2(q_2)$ to $S$, $S$ and $D_2$ accept the offers, and $S$ orders the quantity $q^d_2(c_2)$. $T^d_2(q_2)$, $T^d_s(q_2)$, and $q^d_2(c_2)$ are given by

\[
T^d_2(q) = R(q) - \int_0^q h((q^d_2)^{-1}(x)) dx, \tag{5}
\]

\[
T^d_s(q) = R(q) - \int_0^q (q^d_2)^{-1}(x) dx, \tag{6}
\]

\[
q^d_2(c_2) = \arg \max_q R(q) - (h(c_2) + h'(c_2)(h(c_2) - c_2)) q. \tag{7}
\]

where $(q^d_2)^{-1}(\cdot)$ is the inverse of $q_2(c_2)$ with $(q^d_2)^{-1}(0) = \tau_2 \equiv \inf \{ c \in [\underline{c}, \overline{c}] : q_2(c) = 0 \}$.

In equilibrium, the quantities $q_2(c_2)$ maximize the virtual joint profit of the industry on market 2. Equation (7) reflects an informational double marginalization problem that arises in the three-tier hierarchy as $D_2$ has to elicit the private information about marginal costs from $S$ and $U$ has to elicit this information from $D_2$. As a consequence, $S$ receives a margin of $h(c_2) - c_2$ and $D_2$ guarantees himself a margin of $h'(c_2)(h(c_2) - c_2)$ for the marginal cost $c_2$.

Price discrimination between $D_1$ and $D_2$. Next, I compare the tariffs $T^d_1(q_1)$ and $T^d_2(q_2)$ offered to the two downstream firms to analyze whether the upstream firm uses price discrimination and which form of vertical organization – if any – it favors. The following observation facilitates the comparison between the tariffs.

Lemma 4. $U$ price discriminates between $D_1$ and $D_2$ if and only if $q^d_1(h(\cdot)) \neq q^d_2(\cdot)$.

Proof. From the definition of the tariffs $T^d_1(q_1)$ and $T^d_2(q_2)$ in equations (1) and (5), it follows that $T^d_1(\cdot) = T^d_2(\cdot) \iff (q^d_1)^{-1}(\cdot) = h((q^d_2)^{-1}(\cdot))$. The result follows from

\[
(q^d_1)^{-1}(\cdot) = h((q^d_2)^{-1}(\cdot)) \iff h^{-1}((q^d_1)^{-1}(\cdot)) = (q^d_2)^{-1}(\cdot) \iff (q^d_1 \circ h)(\cdot) = q^d_2(\cdot).
\]

$U$’s optimization problems $\mathcal{P}^d_1$ and $\mathcal{P}^d_2$ are equivalent apart from $D_1$ having a cost function of $c_1 q$ whereas $D_2$ has costs $h(c_2) q$. If $D_1$ and $D_2$ face the same tariff $T(q)$, both choose the same quantity whenever marginal costs satisfy $c_1 = h(c_2)$. Whether $U$ discriminates between the downstream firms can now be simply tested by comparing the quantities which $D_1$ and $D_2$ buy for $c_1 = h(c_2)$.

Proposition 1. $U$ discriminates between $D_1$ and $D_2$ by favoring $D_2$ if and only if $h(\cdot)$ is strictly convex for some values of the cost parameter which satisfy $q^d_1(c) > 0$. If $h(\cdot)$ is linear, $U$ offers $D_1$ and $D_2$ the same tariff.

Proof. From equations (2) and (7), it follows that $q^d_1(h(c)) = q^d_2(c)$ if and only if $h(h(c)) = h(c) + h'(c)(h(c) - c)$. This is satisfied for all $c \in [\underline{c}, \overline{c}]$ if and only if $h(\cdot)$ is linear. In contrast, if $h(\cdot)$ is strictly convex for some $c'$, $h(h(c)) > h(c) + h'(c)(h(c) - c)$ holds for $c$ close to $c'$, and
\( q_2(c) > q_1(h(c)) \). As \( h(\cdot) \) is assumed to be convex, \( q_2(c) \geq q_1(h(c)) \) for all \( c \in [\underline{c}, \overline{c}] \) and from the definitions of \( T_1^d(q_1) \) and \( T_2^d(q_2) \) in equations (1) and (5) it follows that \( D_2 \) is favored by price discrimination.

Price discrimination arises if the virtual marginal cost function \( h(\cdot) \) is curved. The curvature of \( h(\cdot) \) influences how the vertically separated firm \( D_2 \) changes the tariff for the subcontractor \( T_s(\cdot) \) when \( U \) changes the tariff \( T_2(\cdot) \). If \( h(\cdot) \) is linear, an increase in the marginal price charged by \( U \) to \( D_2 \) changes the induced demand on market 2 in the same way it changes the demand on market 1. If \( h(\cdot) \) is convex, then \( D_2 \) increases the tariffs to \( S \) more strongly such that the induced demand on market 2 reduces more than the induced demand on market 1 for the same price change. The curvature of the function \( h(\cdot) \) therefore plays a similar role as demand elasticity in models of price discrimination with linear tariffs.

**Optimal uniform tariff**

I now analyze the equilibrium behavior in the game when price discrimination is banned. If price discrimination is banned, \( U \) has to offer the same tariff to the two downstream firms. The optimal tariff needs to satisfy incentive and participation constraints for both downstream firms. These constraints are the same as in the case when price discrimination is allowed. In particular, Lemma 2 still describes the optimal tariff which \( D_2 \) offers to \( S \) in response to the offer from \( U \). The only additional constraint for \( U \) is the ban on discrimination. Formally, \( U \)'s optimization problem is

\[
\mathcal{P}^n : \max_{T(\cdot)} \int T(q_1(c_1))dF(c_1) + \int T(q_2(c_2))dF(c_2) \\
\text{s.t. } q_1(c_1) \in \arg \max_q R(q) - c_1 q - T(q), \quad (IC^n_1) \\
q_2(c_2) \in \arg \max_q R(q) - h(c_2) q - T(q), \quad (IC^n_2) \\
\max_q R(q) - c_1 q - T(q) \geq 0 \quad \text{for all } c_1 \in [\underline{c}, \overline{c}], \quad (PC^n_1) \\
\max_q R(q) - h(c_2) q - T(q) \geq 0 \quad \text{for all } c_2 \in [\underline{c}, \overline{c}]. \quad (PC^n_2)
\]

The problem \( \mathcal{P}^n \) is technically equivalent to the problems \( \mathcal{P}^d_1 \) and \( \mathcal{P}^d_2 \) with the additional non-discrimination constraint \( T_1(\cdot) = T_2(\cdot) \). This constraint connects the two otherwise independent problems. The equilibrium behavior under a ban on price discrimination is then given in the following lemma.

**Lemma 5.** If price discrimination is banned, \( U \) offers the tariff \( T^n(q) \) to \( D_1 \) and \( D_2 \), \( D_2 \) offers the tariff \( T^n_s(q) \) to \( S \); \( D_1 \) and \( S \) accept the offers, \( D_1 \) orders the quantity \( q^n_1(c_1) = q^n(h^{-1}(c_1)) \),
and $S$ orders the quantity $q^2_2(c_2) = q^n(c_2)$. $T^n(q)$, $T^n_s(q)$, and $q^n(c)$ are given by

\begin{align*}
T^n(q) &= R(q) - \int_0^q h((q^n)^{-1}(x)) dx, \\
T^n_s(q) &= R(q) - \int_0^q (q^n)^{-1}(x) dx, \\
q^n(c) &= \arg \max_q R(q) - \left( h(c) + \frac{F(h(c)) + G(h(c))}{f(h(c)) + g(h(c))} \right) q,
\end{align*}

where $(q^n)^{-1}(\cdot)$ is the inverse of $q^n(c)$ with $(q^n)^{-1}(0) = \bar{c}^n \equiv \inf \{ c \in [\underline{c}, \bar{c}] : q^n(c) = 0 \}.$

If price discrimination is banned, the additional constraint $T_1(\cdot) = T_2(\cdot)$ implies that the virtual marginal costs are a weighted average of the virtual marginal costs with price discrimination. If the vertically separated firm has marginal costs $c_2 = c$ and the vertically integrated firm has marginal costs $c_1 = h(c)$, the virtual marginal costs with price discrimination are $h(h(c)) = h(c) + \frac{F(h(c))}{f(h(c))} = h(c) + \frac{G(h(c))}{g(h(c))}$ for $D_1$ and $h(c) + h'(c)/(h(c) - c) = h(c) + \frac{G(h(c))}{g(h(c))}$ for $D_2$. Note that the virtual marginal costs in equation (10) can be written as a weighted average of these two terms with weights $\frac{f(h(c))}{f(h(c)) + g(h(c))}$ and $\frac{g(h(c))}{f(h(c)) + g(h(c))}$.

4 Welfare effects under asymmetric vertical integration

In this section, I analyze the welfare effects of price discrimination due to asymmetric vertical integration. For this purpose, I first determine the effects of a ban on price discrimination on consumers and downstream firms on the two markets. I show that the vertically separated firm $D_2$ and the consumers on market 2 are harmed by a ban on price discrimination whereas the vertically integrated firm $D_1$ and consumers on market 1 benefit. Next, I compare total welfare arising under the optimal uniform tariff with total welfare in the case where $U$ sets the optimal discriminating tariffs. I show that price discrimination increases total welfare if marginal revenue is concave. Intuitively, price discrimination is beneficial to welfare as it partially offsets the inefficiency which arises through double marginalization on market 2. Price discrimination shifts production from market 1 to market 2. As total production is smaller on market 2, the marginal consumer on market 1 has a lower valuation for the good than the marginal consumer on market 2. The shift in production from market 1 to market 2 is therefore welfare increasing.

Welfare functions The welfare on market $i$ for the marginal cost $c_i$ and the quantity $q_i$ is

$$w_i(c_i, q_i) = \int_0^{q_i} P(x) dx - c_i q_i.$$ 

This is the sum of the consumer surplus $\int_0^{q_i} P(x) dx - P(q_i) q_i$ and producer surplus $P(q_i) q_i - c_i q_i$. Given the equilibrium quantity schedule $q_i(c_i)$, the expected welfare on market $i$ is

$$W_i(q_i(\cdot)) = \int_{\underline{c}}^{\bar{c}} w_i(c_i, q_i(c_i)) dF(c_i).$$
Equivalently, one can define the expected consumer surplus on the two markets. The total expected welfare is $W(q_1(\cdot), q_2(\cdot)) \equiv W_1(q_1(\cdot)) + W_2(q_2(\cdot))$.

**The effects of price discrimination on the two markets** If price discrimination is permitted, $U$ favors the separated downstream firm $D_1$ over the integrated firm $D_2$. With a ban on price discrimination, $U$ then has to treat $D_1$ and $D_2$ the same. This implies that the quantity on market 1 increases if price discrimination is banned whereas the quantity on market 2 decreases.

**Proposition 2.** Price discrimination decreases the quantity produced by the vertically integrated downstream firm $D_1$, increases the quantity produced by the vertically separated downstream firm $D_2$, and the vertically integrated downstream firm produces a higher quantity with and without price discrimination, i.e., $q_2^R(c) \leq q_2^D(c) < q_1^D(c) \leq q_1^R(c)$.

As a consequence of the effect on market quantities, a ban on price discrimination increases the expected profit of $D_1$ and the expected consumer surplus on market 1. It decreases the expected profits of $D_2$ and $S$ and decreases the expected consumer surplus on market 2.

If price discrimination is banned, $D_2$ faces a tariff which is everywhere steeper than the tariff under price discrimination. Thus, $D_2$ increases the tariff in the subcontract and the quantity ordered by $S$ decreases for any realization of the cost parameter. It follows that not only $D_2$ and $S$ are worse off, but also the consumers on market 2 who face a higher price. The converse holds true on market 1. After a ban on price discrimination, $D_1$ faces a flatter tariff than before. $D_1$ orders a higher quantity of the input and the price on market 1 decreases to the benefit of consumers.

**Effects on total welfare** A ban on price discrimination increases welfare on market 1 but decreases welfare on market 2. Thus, price discrimination increases total welfare if the welfare gain on market 2 outweighs the welfare loss on market 1. The following main result of this paper states that this is the case whenever marginal revenue is concave.

**Proposition 3.** If marginal revenue $R'(q)$ is concave, price discrimination increases welfare with asymmetric vertical integration.

A ban on price discrimination shifts production from high value marginal consumers on market 2 to low value marginal consumers on market 1. With as well as without price discrimination, the quantity on market 1 is higher than the quantity on market 2 if marginal costs are the same for $D_1$ and $S$. This implies that the price on market 1 is lower than the price on market 2. Thus, the marginal consumer on market 1 has a lower marginal value for the good than the marginal consumer on market 2. This suggests that price discrimination should increase total welfare since it shifts the good from low to high valuation consumers.

However, the total quantity of production may be different with and without price discrimination. If total quantity remains constant in both situations, or is higher under price discrimination, then price discrimination should be allowed.
Figure 2: Effect of price discrimination

The inverse demand functions net of marginal costs are depicted for both markets and identical marginal costs $c_1 = c_2 = c$. The quantity $q^d_i$ is sold on market $i$ if price discrimination is allowed, $q^n_i$ is sold if price discrimination is banned. A ban on price discrimination increases the quantity on market 1 and reduces the quantity on market 2. The dark shaded area is the resulting welfare gain on market 1, the light shaded area represents the welfare loss on market 2.

Figure 2 illustrates this argument. In the figure, the inverse demand function net of marginal costs is depicted for both markets and identical marginal costs $c_1 = c_2 = c$. The quantities with price discrimination are given by $q^d_1$ and $q^d_2$. Due to the double marginalization problem on market 2, $q^d_1 > q^d_2$ even though marginal costs are identical. This implies that the marginal consumer on market 1 has a lower marginal value for the good than the marginal consumer on market 2. If price discrimination is banned, the quantity on market 1 increases to $q^n_1$ and the dark shaded area represents the welfare gain on market 1. However, the quantity on market 2 decreases to $q^n_2$ and the light shaded area represents the reduction in welfare on market 2. If the total quantity on both markets is similar with and without price discrimination, i.e. $q^n_1 - q^d_1 \simeq q^d_2 - q^n_2$, then the light shaded area is clearly larger than the dark shaded area and a ban on price discrimination reduces welfare.

The concavity of marginal revenue is sufficient but not necessary for price discrimination to be beneficial for welfare. Indeed, concavity of marginal revenue is a sufficient condition for higher total sales under price discrimination given identical marginal costs $c_1 = c_2$. Higher total sales are a sufficient condition for higher welfare under price discrimination. However, as welfare $w(c_i, q_i)$ is concave in the quantity $q_i$, price discrimination remains beneficial for welfare even if the total quantity on the two markets is slightly lower with price discrimination.

5 Comparison to asymmetric production costs

In this section, I relate the welfare results with asymmetric vertical integration to the welfare implications of price discrimination in a setting with asymmetric production costs. I consider a variation of the model in which downstream firms sell the final good themselves but have
The upstream firm $U$ sells to two downstream firms that are vertically integrated and serve independent markets.

different distributions of costs to serve the market. The cost difference between downstream firms is then not caused by different forms of vertical organization but by different production technologies. Thus, I compare the welfare effects of price discrimination in a model with \textit{asymmetric agency cost} to the welfare effects in a model with \textit{asymmetric production costs}.

In the literature on price discrimination in intermediate good markets, it is typically implicitly assumed that downstream firms use asymmetric production technologies. The model studied in this section is a variant of the model in Herweg and Müller (2014). I specify this model to be observationally equivalent to the model in the previous sections, i.e., the upstream firm offers the same tariffs to the downstream firms and the ordered quantities are identically distributed.

I show that price discrimination has a more positive effect on welfare in the model with asymmetric vertical integration. Furthermore, under the assumption of sufficient market coverage, price discrimination is detrimental to welfare with asymmetric production costs whenever it is beneficial to welfare in the model with asymmetric vertical integration.

\section*{A model with asymmetric production costs}

The industry structure of the model with asymmetric production costs is given in Figure 3. In contrast to the model with asymmetric vertical integration, $D_2$ produces the final good itself using a one-to-one technology. $D_2$’s cost function is given by $c_2 q_2$. Whereas the marginal cost of $D_1$ is distributed according to the distribution function $F(\cdot)$, the marginal cost $c_2$ is now distributed according to the distribution function $G(\cdot) = F(h^{-1}(\cdot))$. This gives rise to the following equilibrium.

\textbf{Lemma 6.} \textit{In the equilibrium of the model with asymmetric production costs, $U$ sets the tariffs $T_1^d(q)$ and $T_2^d(q)$ if price discrimination is permitted, and $T^u(q)$ if price discrimination is banned. The downstream firms order the quantities $q_1^D(c_1) = q_1^d(c_1)$ and $q_2^D(c_2) = q_2^d(h^{-1}(c_2))$ if price}
discrimination is permitted, and \( q^N(c) = q^*_1(c) = q^*_2(h^{-1}(c)) \) if price discrimination is banned.

A ban on price discrimination increases the quantity produced by \( D_1 \) and decreases the quantity produced by \( D_2 \), i.e., \( q^N(c) \in [q^D_1(c), q^D_2(c)] \).

The situation of the upstream firm \( U \) with asymmetric production technologies is equivalent to the case with asymmetric vertical integration. It does not matter for the incentives of \( D_2 \) whether production is delegated to the subcontractor \( S \) at virtual marginal costs \( h(c_2) \) where \( c_2 \) is distributed according to \( F(\cdot) \) or whether \( D_2 \) produces itself at real marginal costs distributed according to \( G(\cdot) = F(h^{-1}(\cdot)) \). As \( D_1 \)'s situation is obviously the same in both models, \( U \) optimally offers the same tariffs independently of the source of cost differences.

However, price discrimination in the two models has very different effects on the mapping from real marginal costs to quantities. With asymmetric production costs, a ban on price discrimination lowers the production of the more efficient downstream firm \( D_1 \) and increases the quantity of the less efficient firm \( D_2 \). With asymmetric vertical integration, \( D_1 \) produces more and \( D_2 \) produces less under uniform pricing (Proposition 2). Thus, a ban on price discrimination reduces production differences for identical marginal costs \( c \) in the model with asymmetric production costs and amplifies production differences in the model with asymmetric vertical integration.

**A comparison of the welfare effects of price discrimination**

In this section, I compare the welfare effects of price discrimination in the models with asymmetric agency costs and asymmetric production costs. I first show that price discrimination has always more favorable welfare effects in the model with asymmetric agency costs. The welfare effect of price discrimination under asymmetric production costs is given by

\[
\Delta W_P = \int_\xi [w(q^D_1(c), c) - w(q^N(c), c)] dF(c) + \int_\xi [w(q^D_2(c), c) - w(q^N(c), c)] dG(c).
\]

The welfare effect of price discrimination under asymmetric vertical integration is

\[
\Delta W_A = \int_\xi [w(q^d_1(c), c) - w(q^n_1(c), c)] dF(c) + \int_\xi [w(q^d_2(c), c) - w(q^n_2(c), c) + (c - h^{-1}(c))(q^d_2(c) - q^n_2(c))] dF(c).
\]

These welfare effects can be unambiguously ranked.

**Proposition 4.** The welfare gain from price discrimination is higher in the model with asymmetric vertical integration, i.e., \( \Delta W_A > \Delta W_P \).

The welfare effect of price discrimination with asymmetric vertical integration can be expressed using the equilibrium quantities in the model with asymmetric production costs. This
gives

$$\Delta W_A = \int c \left[ w(q^D_1(c), c) - w(q^N(c), c) \right] dF(c)$$

$$+ \int h'(c) \left[ w(q^D_2(c), c) - w(q^N(c), c) + (c - h^{-1}(c))(q^D_2(c) - q^N(c)) \right] dG(c).$$

Thus, the difference between the welfare effects in the two models is given by

$$\Delta W_A - \Delta W_P = \int h'(c) \left[ (c - h^{-1}(c))(q^D_2(c) - q^N(c)) \right] dG(c). \quad (11)$$

This expression reflects the fundamental difference between production costs and agency costs with respect to welfare. If the downstream firms differ with respect to production costs, their cost functions are directly represented in the expression of total welfare. If the downstream firms have asymmetric agency costs due to asymmetric vertical integration, their cost differences include rent payments to subcontractors. These rent payments are a private cost to the delegating downstream firm but not a social cost since they are the profit of the subcontractor.

The difference between the welfare effects is the additional profit the subcontractor receives under price discrimination in the model with asymmetric vertical integration. If the downstream firm $D_2$ has production costs of $c \cdot q$, the social costs of production are $c \cdot q$ in the model with asymmetric production costs. In the model with asymmetric agency costs, the social costs are $h^{-1}(c) \cdot q$. The difference of these terms, $(c - h^{-1}(c))q$, is the profit of the subcontractor in the model with asymmetric vertical integration and a social cost in the model with asymmetric production technologies.

It remains the question whether the source of cost differences between downstream firms has a normative implication for competition policy, i.e., whether the sign of the welfare effects $\Delta W_A$ and $\Delta W_P$ may be different? I show that this is the case under mild conditions.

**Proposition 5.** Suppose marginal revenue $R'(q)$ is concave. Price discrimination decreases welfare with asymmetric production costs if the exclusion of the more efficient downstream firm $D_1$ is unlikely, i.e., $\exists Q \in (0, 1)$ s.t. $\Pr(q^D_1(c_1) = 0) \leq Q \Rightarrow \Delta W_P < 0$.

A ban on price discrimination shifts production from low value marginal consumers on market 2 to high value marginal consumers on market 1. With price discrimination, the quantity on market 1 is lower than the quantity on market 2 for identical marginal costs. Thus, the price on market 1 is higher than the price on market 2. With uniform pricing, the quantities and the prices on the two markets are the same if marginal costs are the same. Uniform pricing therefore shifts production from the low value marginal consumer on market 2 to the high value marginal consumer on market 1. This reasoning suggests that price discrimination should be detrimental to welfare. However, a ban on price discrimination may also have effects on the total quantity that is sold on the two markets.

With concave marginal revenue, I show that the total quantity sold under price discrimination is lower than the total quantity without price discrimination. As price discrimination
reduces total production and makes the distribution of production over the two markets more unequal, total welfare is lower with price discrimination.

Taken together, Propositions 3 and 5 show that the source of cost differences between downstream firms crucially influences the welfare effects of price discrimination in intermediate good markets. Under mild conditions, price discrimination has opposite welfare effects in the models with asymmetric vertical integration and with asymmetric production costs. The optimal competition policy therefore requires information about the source of cost differences.

6 Conclusion

This paper provides an analysis of price discrimination in intermediate good markets when downstream firms have different degrees of vertical integration. I first show that asymmetric vertical integration leads to asymmetric agency costs within downstream firms which motivates monopolistic upstream firms to use price discrimination. Next, I show that price discrimination between vertically integrated and separated firms increases social welfare under mild conditions on demand. I compare this result with the welfare effects of price discrimination in a model where cost differences arise from the use of different production technologies. I demonstrate that price discrimination has more positive effects on welfare in the model with asymmetric agency costs. Furthermore, I show that – under mild conditions – price discrimination has opposite welfare effects in the two models. Thus, the source of cost differences may have important implications for competition policy.

Appendix

A Omitted proofs

Proof of Lemma 1

Define the variable \( \Pi_1(c_1) = \max_q R(q) - c_1 q - T_1(q) \). By standard arguments the incentive compatibility constraint \( IC_1 \) is equivalent to \( \Pi_1'(c_1) = -q_1(c_1) \) and \( q_1(c_1) \) being non-increasing. Using integration by parts, the problem \( \mathcal{P}_1^d \) can be restated as

\[
\max_{q_1(\cdot), \bar{c}_1, \Pi_1(\bar{c}_1)} \int_{\bar{c}_1}^{c_1} \left[ R(q_1(c_1)) - h(c_1)q_1(c_1) \right] dF(c_1) - \Pi_1(\bar{c}_1)
\]

subject to \( \Pi_1(\bar{c}_1) \geq 0 \) and \( q_1(c_1) \) non-decreasing in \( c_1 \). By standard arguments (Martimort and Stole, 2009), the solution to this problem is given by \( \Pi_1(\bar{c}_1) = 0 \), \( q_1^d(\cdot) \) with \( q_1^d(c_1) = \arg \max_q R(q) - h(c_1)q \), and \( \bar{c}_1 = \inf \{ c \in [\bar{c}_1, \bar{c}] : \arg \max_q R(q) - h(c)q = 0 \} \). \( T_1^d(\cdot) \) can be computed using the condition \( R'(q_1^d(c)) - c = (T_1^d)'(q_1^d(c)) \) which implies \( R(q) - (q_1^d)^{-1}(q) = (T_1^d)'(q) \) where \( (q_1^d)^{-1}(x) \) is the inverse function of \( q_1^d(c) \), precisely defined in the Lemma. Using this and \( T_1^d(0) = 0 \) gives the result.

\[ \square \]
Proof of Lemma 2

Define \( \Pi_s(c_2) \equiv \max_q R(q) - c_2 q - T_s(q) \). By standard arguments the incentive compatibility constraint \( IC_s \) is equivalent to \( \Pi'_s(c_2) = -q_2(c_2) \) and \( q_2(c_2) \) being non-increasing. Using integration by parts, the problem \( \mathcal{P}^d_s \) can be expressed as

\[
\max_{q_2(\cdot), \tau_2, \Pi_s(\tau_2)} \int_0^{\tau_2} \left[ R(q_2(c_2)) - h(c_2)q_2(c_2) - T_2(q_2(c_2)) \right] dF(c_2) - \Pi_s(\tau_2)
\]

subject to \( \Pi_s(\tau_2) \geq 0 \) and \( q_2(c_2) \) non-decreasing in \( c_2 \). The quantity schedule \( q_2(c_2) = \text{arg max}_q R(q) - h(c_2)q - T_2(q) \) is non-decreasing in \( c_2 \) and therefore optimal. Next, I show that \( D_2 \) can guarantee himself a positive payoff. Note first that \( \max_q R(q) - h(c_2)q - T_2(q) \) is non-increasing in \( c_2 \). Consider a BPE in which \( D_2 \) incurs a negative profit for some \( c_2 \). Then it also incurs a negative profit for all \( c' > c_2 \). Define \( \hat{c}_2 = \text{inf}\{c \in [\underline{c}, \bar{c}] : \max_q R(q) - h(c_2)k(q) - T_2(q) < 0\} \). For any \( c_2 \geq \hat{c}_2, D_2 \) could commit vis-à-vis \( S \) not to accept \( U \)'s offer. As the quantity is then zero for \( c_2 \geq \hat{c}_2 \), and positive and non-decreasing for \( c_2 < \hat{c}_2 \), the new quantity schedule is still implementable but can be implemented at a higher profit. \( D_2 \) therefore has a profitable deviation. \( \square \)

Proof of Lemma 3

The proof follows the same steps the proof of Lemma 1 with the only difference that \( c_1 \) is replaced by \( h(c_2) \). Define the variable \( \Pi_2(c_2) = \max_q R(q) - h(c_2)q - T_2(q) \). By standard arguments the incentive compatibility constraint \( IC_2 \) is equivalent to \( \Pi'_2(c_2) = -h'(c_2)q_2(c_2) \) and \( q_2(c_2) \) being non-increasing. Using integration by parts, the problem \( \mathcal{P}^d_2 \) can be restated as

\[
\max_{q_2(\cdot), \tau_2, \Pi_2(\tau_2)} \int_0^{\tau_2} \left[ R(q_2(c_2)) - (h(c_2) + h'(c)(h(c_2) - c))q_2(c_2) \right] dF(c_2) - \Pi_2(\tau_2)
\]

subject to \( \Pi_2(\tau_2) \geq 0 \) and \( q_2(c_2) \) non-decreasing in \( c_2 \). Note that \( q^d_2(c_2) \equiv \text{arg max}_q R(q) - (h(c_2) + h'(c)(h(c_2) - c))q \) is non-increasing in \( c_2 \) as \( (h(c_2) + h'(c)(h(c_2) - c)) \) is non-decreasing in \( c_2 \) due to the convexity of \( h(\cdot) \). The solution to the problem is therefore given by \( \Pi_2(\tau_2) = 0, q^d_2(\cdot), \) and \( \tau_2 = \min\{c \in [\underline{c}, \bar{c}] : \max_q R(q) - (h(c) + h'(c)(h(c) - c))q = 0\} \). \( T^d_2(\cdot) \) can be computed using the condition \( R'(q^d_2(c)) - h(c) = (T^d_2)'(q^d_2(c)) \) which implies \( R'(q) - h(q^{-1}(q)) = (T^d_2)'(q) \) where \( (q^d_2)^{-1}(\cdot) \) is the inverse function of \( q^d_2(\cdot) \), precisely defined in the Lemma. Using this and \( T^d_2(0) = 0 \) gives \( T^d_s(\cdot) \). To compute \( T^d_s(q) \), note that \( q^d_s(c) = \text{arg max}_q R(q) - dq - T_s(q) = \text{arg max}_q R(q) - h(c)q - T_2(q) \). This implies that \( T^d_2(q) - T^d_s(q) = h((q^d_2)^{-1}(q)) - (q^d_s)^{-1}(q) \) and gives \( T^d_s(q) = \int_0^q (R'(x) - (q^d_2)^{-1}(x)) dx \). \( \square \)

Proof of Lemma 5

Note at first that the results of Lemma 2 still hold if price discrimination is banned. This gives us \( \mathcal{P}^n \) as \( U \)'s optimization problem. Recall from the previous lemmas the definitions of \( \Pi_1(c_1) \) and \( \Pi_2(c_2) \) and the facts that \( (IC^u_1) \) is equivalent to \( \Pi'_1(c_1) = -q_1(c_1) \) and \( q_1(c_1) \) non-increasing.
I show at first that the payoff can be expressed as
\[
\begin{align*}
\int_{\xi}^{\bar{c}} \left[ R(q_2(c)) - (h(c) + h'(c)(h(c) - c))q_2(c) \right] dF(c)
\end{align*}
\]
subject to \( q_1(c) \) non-increasing for \( i \in \{1, 2\} \), \( \Pi_1(\bar{c}) \geq 0 \), and \( q_1(h(c)) = q_2(c) \) for all \( c \in [\xi, \bar{c}] \).

It is optimal to set \( \Pi_1(\bar{c}) = 0 \). From a change of variable from \( c_2 \) to \( z = h(c_2) \) and the definition of the quantity schedule \( \bar{q}_2(c) \equiv q_2(h^{-1}(c)) \), it follows that
\[
\int_{\xi}^{\bar{c}} \left[ R(q_2(c)) - (h(c) + h'(c)(h(c) - c))q_2(c) \right] dF(c)
\]
\[
= \int_{\xi}^{\bar{c}} \left[ R(\bar{q}_2(z)) - (z + h'(h^{-1}(z))(z - h^{-1}(z)))\bar{q}_2(z) \right] \frac{f(h^{-1}(z))}{h'(h^{-1}(z))} dz
\]

Incorporating the constraint \( q_1(c) = \bar{q}_2(c) = q(c) \) and using the definitions of \( G(c) \) and \( g(c) \), \( U \)'s payoff can be expressed as
\[
\int_{\xi}^{\bar{c}} \left[ R(q(c)) - h(c)q(c) \right] f(c) + \left[ R(q(c)) - \left( c + \frac{G(c)}{g(c)} \right) q(c) \right] g(c) \right] dc - 2\Pi_1(\bar{c})
\]
It is optimal to set \( \Pi_1(\bar{c}) = 0 \) and \( q^*(c) = \arg\max_q R(q) - \left( c + \frac{F(c) + G(c)}{f(c) + g(c)} \right) \). With \( q^*(c) = q(h^{-1}(c)) \), the optimal quantity schedules given by \( q_1^*(c_1) = q^*(c_1) \) and \( q_2^*(c_2) = q^*(c_2) \) are increasing due to Assumption 2. The tariffs \( T^a(q) \) and \( T^a_s(q) \) can be computed exactly as in the proofs of Lemmas 2 and 3. \( \square \)

**Proof of Proposition 2**

I show at first that \( q^a_1(c) \geq q^d_1(c_1) \). Using the definitions of \( q^d_1(\cdot) \) and \( q^a_1(\cdot) \), this holds as
\[
c + \frac{F(c) + h'(h^{-1}(c))F(h^{-1}(c))}{f(c) + f(h^{-1}(c))} \leq c + \frac{F(c)}{f(c)} \iff h'(h^{-1}(c))(c - h^{-1}(c)) \leq h(c) - c
\]
where the second statement is satisfied due to the convexity of \( h(\cdot) \). Next, I show that \( q^a_2(c) \leq q^d_2(c_2) \). Using the definitions of \( q^d_2(\cdot) \) and \( q^a_2(\cdot) \), this holds as
\[
h(c) + \frac{F(h(c)) + h'(h(c))F(c)}{f(h(c)) + f(c)} \geq h(c) + h'(h(c)(h(c) - c) \iff h(h(c)) - h(c) \geq (h(c) - c)h'(c)
\]
where the second statement again follows from convexity of \( h(\cdot) \). Finally, \( q^a_1(c) > q^d_2(c) \) follows from \( h(c) < h(c) + h'(c)(h(c) - c) \). \( \square \)
Proof of Proposition 3

The welfare effect of price discrimination is given by

$$\int_{\xi} \left( w(c, q_1^d(c)) + w(c, q_2^d(c)) - w(c, q_1^n(c)) - w(c, q_2^n(c)) \right) dF(c).$$

Due to the concavity of the welfare function, total welfare is higher under price discrimination if (i) \(|q_1^d(c) - q_2^d(c)| \leq |q_1^n(c) - q_2^n(c)|\) and (ii) \(q_2^d(c) \geq q_1^d(c) + q_2^n(c)\) for all \(c\). (i) holds due to Proposition 2. To show that (ii) holds, define \(\phi(x) \equiv R^{-1}(x)\). For all \(c\) such that all quantities are positive, (ii) can be written as

$$\phi(h(c)) + \phi(h(c) + h'(c)(h(c) - c)) \geq \phi \left( c + \frac{F(c) + G(c)}{f(c) + g(c)} \right) + \phi \left( h(c) + \frac{F(h(c)) + G(h(c))}{f(h(c)) + g(h(c))} \right).$$

If \(R'(q)\) is decreasing and concave, \(\phi(x)\) is decreasing and concave and (ii) holds if

$$h(c) + h(c) + h'(c)(h(c) - c) \leq c + \frac{F(c) + G(c)}{f(c) + g(c)} + h(c) + \frac{F(h(c)) + G(h(c))}{f(h(c)) + g(h(c))}. \quad (12)$$

This condition can be rewritten as

$$\frac{F(c)g(c) - G(c)f(c)}{f(c) + g(c)} \leq h'(c) \frac{F(h(c))g(h(c)) - G(h(c))f(h(c))}{f(h(c)) + g(h(c))}. $$

By Part 1. of Assumption 2, \(h'(c) \geq 1\). Thus, the condition above holds if

$$\frac{\partial}{\partial c} \left( \frac{F(c)g(c) - f(c)G(c)}{f(c) + g(c)} \right) = \frac{(F(c) + G(c))(f(c)g'(c) - f'(c)g(c))}{(f(c) + g(c))^2} \geq 0.$$

This is satisfied as \(f(c)g'(c) - f'(c)g(c) \geq 0\) follows from \(g(c)/f(c)\) being increasing due to part 3. of Assumption 2.

In the argument above, I made the assumption that both markets are always covered. However, note that market 2 is more likely to be not served if price discrimination is banned. In that case, the calculations made above are a lower bound on the welfare gain from price discrimination. As welfare is strictly concave in \(q\), a continuity argument implies that the result holds as long as \(R'(q)\) – and thus \(\phi(x)\) – is not too convex. \(\square\)

Proof of Lemma 6

The results regarding optimal tariffs and the resulting quantities follow from the observation – which is discussed in the main text – that \(D_2\)’s incentives are the same in both models. It remains to show that \(q^N(c) \in [q_1^D(c), q_2^D(c)]\). \(q^N(c) \geq q_1^D(c)\) follows from \(q^N(c) = q_1^N(c)\), \(q_1^D(c) = q_2^D(c)\), and Proposition 2. \(q^N(c) \leq q_2^D(c)\) follows from \(q^N(c) = q_1^N(c)\), \(q_2^D(c) = q_2^D(h^{-1}(c))\), and the
Proof of Proposition 4

The second expression for $\Delta \mathcal{W}_A$ can be derived as follows

$$\Delta \mathcal{W}_A = \int_{\mathcal{E}} \left[ w(c_1, q^D_1(c_1)) - w(c_1, q^N(c_1)) \right] dF(c_1) + \int_{\mathcal{E}} \left[ w(c_2, q^D_2(c_2)) - w(c_2, q^N(c_2)) \right] dF(c_2)$$

$$= \int_{\mathcal{E}} \left[ w(c_1, q^D_1(c_1)) - w(c_1, q^N(c_1)) \right] dF(c_1)$$

$$+ \int_{\mathcal{E}} \left[ w(c_2, q^D_2(h(c_2))) - w(c_2, q^N(h(c_2))) \right] dF(c_2)$$

$$= \int_{\mathcal{E}} \left[ w(c_1, q^D_1(c_1)) - w(c_1, q^N(c_1)) \right] dF(c_1)$$

$$+ \int_{\mathcal{E}} \int_{h^{-1}(z_2)} \left[ P(x) dx - h^{-1}(z_2)(q^D_2(z_2) - q^N(z_2)) \right] dG(z_2)$$

$$= \int_{\mathcal{E}} \left[ w(c_1, q^D_1(c_1)) - w(c_1, q^N(c_1)) \right] dF(c_1)$$

$$+ \int_{\mathcal{E}} \left[ w(z_2, q^D_2(z_2)) - w(z_2, q^N(z_2)) + (z_2 - h^{-1}(z_2))(q^D_2(z_2) - q^N(z_2)) \right] dG(z_2),$$

where the second equality follows from a change of variable from $c_2$ to $z_2 = h(c_2)$ with $G(c_2) = F(h^{-1}(c_2))$. For $z_2 = c_2$, this expression is equal to the expression in the main text. $\Delta \mathcal{W}_A > \Delta \mathcal{W}_P$ follows directly from Lemma 6.

Proof of Proposition 5

Suppose that $R'(q)$ is concave and define $\phi(x) \equiv R'^{-1}(x)$. The welfare effect of price discrimination is

$$\Delta \mathcal{W}_P = \int_{\mathcal{E}} \left( f(c)w(c, q^D_1(c)) + g(c)w(c, q^D_2(c)) - (f(c) + g(c))w(c, q^N(c)) \right) dc.$$
which is satisfied as \( \phi(x) \) is concave. If the probability \( \Pr(g_1^D(c_1) = 0) \) is sufficiently small, the welfare effect of price discrimination is negative, i.e., \( \Delta W_P < 0 \).

\[ \square \]

B Example for distribution satisfying Assumption 2

**Lemma 7.** The distribution function \( F(c) = e^{-1/a(c-b)} \) satisfies Assumption 2.

**Proof.** As \( f(c) = e^{-1/a(c-b)} \cdot \frac{1}{a(c-b)^2} \), the inverse hazard rate is \( F(c)/f(c) = a(c-b)^2 \) which is increasing and convex. Part 2. of the Assumption is satisfied if \( g(c) = \frac{f(h^{-1}(c))/f(h)}{h'(c) f(h(c))} \) is weakly increasing. As \( h(c) \) is increasing, this is the case if \( \frac{f(c)}{h'(c) f(h(c))} \) is increasing. It can be shown that

\[
\frac{f(c)}{h'(c) f(h(c))} = e^{-\frac{1}{a(c-b)}} \cdot \frac{1}{a(c-b)^2} \cdot \frac{1+2a(c-b)}{1+2a(c-b)^2} = e^{-\frac{1}{1+a(c-b)}} \cdot \frac{(1+a(c-b))^2}{1+2a(c-b)^2}.
\]

This term is increasing. It remains to check that 3. of Assumption 2 is satisfied. Note that

\[
e + \frac{F(c) + G(c)}{f(c) + g(c)} = e + \frac{F(c) + F(h^{-1}(c))}{f(h(c) + f(h^{-1}(c)) / h'(h^{-1}(c))}
\]

is increasing if \( h(c) + \frac{F(h(c)) + F(c)}{h'(h(c)) f(h(c))} = h(c) + h'(c) \frac{F(h(c)) + F(c)}{f(h(c)) h'(h(c)} \) is increasing. After some tedious calculations, it can be shown that

\[
h(c) + h'(c) \frac{F(h(c)) + F(c)}{f(h(c)) h'(h(c)} + f(c)
\]

\[
= c + a(c-b)^2 + (1 + 2a(c-b)) e^{-\frac{1}{a(c-b)}} + e^{-\frac{1}{a(c-b)}}
\]

\[
= c + a(c-b)^2 + (1 + 2a(c-b)) a(c-b)^2 \frac{1 + e^{-\frac{1}{a(c-b)}}}{1 + e^{-\frac{1}{a(c-b)}}} \frac{1+2a(c-b)}{1+a(c-b)}
\]

This expression is increasing if \( \frac{1+e^{-\frac{1}{a(c-b)}}}{1+e^{-\frac{1}{a(c-b)}}} \frac{1+2a(c-b)}{1+a(c-b)^2} \) is increasing. This is the case as \( \frac{1+2a(c-b)}{1+a(c-b)^2} \) is decreasing.

\[ \square \]

**References**


