Search and market structure with heterogeneous consumers

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Abstract

Consumers search for prices and horizontally differentiated product designs among monopolistically competitive firms. We study firms’ endogenous product design choices when consumers have ex ante differentiated tastes for designs, and look at redistributional consequences of changes in search costs among different consumer types. As search costs fall, firms change from pooling, offering mainstream product designs only, towards separation into targeting either niche or mainstream consumers. Prices may rise, with niche type consumers being better off as a result of better product match, while mainstream type consumers can be worse off in expectation when firms separate into specialized designs.

Keywords: Consumer search, product differentiation, endogenous product design

JEL classification: D43, D83, L11

1 Introduction

Reductions in consumer search costs typically go hand in hand with lower prices. If competition is just “one click away”, sellers whose products are not perfect matches to consumer tastes will need to offer more favourable terms in order to persuade consumers to buy at their shops. Consumers’ improving outside options to continue searching for better matches then forces down all sellers’ prices.

Apart from adapting pricing to falling search costs, firms may also adapt the design of their products to changes in search technology. Indeed, a recent literature, following the seminal contribution by Bar-Isaac, Caruana and Cuñat (2012), has explored how in equilibrium non-price characteristics of products will change. This stream of literature has shown how, with falling search costs, firms will increasingly offer niche products that have greater appeal to some fraction of consumers, and on the other hand is a worse match for others.

When product design is endogenous, price changes as search costs fall will also be affected by changes in designs. Indeed, as equilibrium product designs get more dispersed with falling search costs, there is a

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counteracting effect, in that consumers will be more willing to pay for a product that more closely matches their preferences for product characteristics. Bar-Iaac, Caruana and Cuñat (2012) indeed find that prices may not change monotonously as firms adapt their products’ designs.

A drawback of this literature is that it typically assumes that consumers are ex ante homogeneous. The dominant modelling tool in this literature of endogenous product design is based on demand rotations (as introduced in Johnson and Myatt, 2006), that increase the variance of idiosyncratic preference shocks for all searching consumers alike. For a given firm visited, consumers will be more differentiated in their preferences for more niche design choices. Ex ante, consumers are homogeneous, however, so that each consumer’s expected utility from an additional search is identical.

When instead consumers are persistently heterogeneous in their preferences for product varieties, such consumer heterogeneity will also be reflected by heterogeneity in the various consumers’ options to continue search. Niche consumers, who persistently prefer niche products over mainstream product offerings, will face differing incentives for searching for better matches than those consumers that have tastes that are closer to the average.

A focus on consumers inherent differences in tastes allows us to address some questions that require a more microscopic view of the demand side itself. How do equilibrium changes in product design and search costs affect consumers who are inherently more interested in particular niche products? Are all consumers better off as search costs fall, or is there redistribution between consumers with middle-of-the-road preferences and those with more eccentric tastes?

In this paper, we explore the redistributonal consequences of changing industry price and design choices, in response to changes in search costs, on consumers that have ex ante heterogeneous and persistent tastes. We consider consumers who have utilities for products determined by the match of their horizontal type and selling firms’ horizontal designs, plus a uniformly distributed shock. We explore firms’ equilibrium price and design choices for differing values of search costs.

For high search costs, industry equilibrium involves all, ex ante homogeneous, firms pooling in offering the same design that appeals to the average consumer type. As search costs fall, firms instead separate in equilibrium, targeting their designs more closely to consumers of specific types. While product match gets better with better targeted designs, price instead rises. The reason is that for any given consumer type, only part of the selling firms target their type, while other firms focus on different consumer types and provide poor matches to non-targeted consumers. As a result, though absolute search costs fall, effective search costs rise as part of the searches are wasted on firms selling unsuitable designs. When consumers do find a suitable firm, those with more extreme preferences get better product matches, which increases their expected surplus. Average consumers, who were served well under pooling, might actually be worse off, facing higher prices and designs more poorly aligned with their preferences.

We first explore equilibria with two symmetric consumer types, and then expand to three types: a main-
stream, average type, and two symmetrical eccentric, niche types. In the latter case, as search costs fall, middle
types may get worse off with niche types faring better as equilibrium regimes change with falling search costs.

Our model is situated in the strand of literature of sequential consumer search with horizontal differentia-
tion, as in Wolinsky (1986) and Anderson and Renault (1999). We focus on the monopolistic competition
limit of such models. Bar-Isaac, Caruana and Cuñat (2012) introduced product design as a strategic variable
apart from pricing, modelling product design choices as demand rotations as in Johnson and Myatt (2006).
A related contribution focusing on market entry, in a Salop-circle model, is González-Maestre and Granero
(2018). In these models, consumers are ex ante homogeneous, product positioning occurs through changes in
the idiosyncratic shocks that are independently drawn for each new consumer-firm pair. In our work, shocks
consist of a consumer-persistent type in addition to a random shock, which allows us to examine effects on
ex-ante different consumer classes (niche versus mainstream).

Other papers have focused on various classes of consumers. For instance, Chen and Zhang (2011) consider
pricing when some consumers do, and others do not search, and Moraga-González, Sándor and Wildenbeest
(2017) consider markets with more general heterogeneity in search costs, with some consumers participating, and
others staying out of the market, and Obradovits (2017) looks at markets with differences among willingness-
to-pay. Our contribution adds product design as a strategic choice, and introduces differences in preferences
among consumer types.

Product differentiation in addition to price setting with ex ante heterogeneous consumers was studied in
Fershtman, Fishman and Zhou (2018), who allow firms to differentiate both horizontally and vertically, but
include directed search through information transmission on product categories. In contrast, in our setting,
consumers have no prior information on a firm’s products, and need to visit the firm to find out both price and
design. Finally, Fabra and Montero (2018) look at vertically differentiated product designs, with consumers
differing in taste for quality, in a Varian-type search setting.

2 Model

We consider a unit mass of firms, as well as a unit mass of consumers. Each firm produces a single product, and
each consumer will buy at most one product. Each consumer $i$ is characterized by a constant taste parameter,
$\theta_i$. We will first explore a situation in which consumers can have either left or right taste $\theta_L$ or $\theta_R > \theta_L$. In
section 4 we will add a consumer group with middle taste $\theta_M = \frac{\theta_L + \theta_U}{2}$. These taste parameters reflect that
different consumers have different ex ante tastes. These might be viewed as consumer locations (a consumer
living on one side of the city may have preferences tilted towards shops closer to that part of the city), or might
reflect preferences for product characteristics (some might prefer action movies while other prefer comedies).

All firms are ex ante identical. Firm $j$ producing its single product, at per unit cost $c$, chooses not only a
price $p_j$, but also a product design $y_j$ to match (some) consumers’ preferences. A consumer $i$ buying product $j$ will get utility

$$U_{ij} = v - p_j - (\theta_i - y_j)^2 + \epsilon_{ij}$$

(1)

from that purchase. Here, $\epsilon_{ij}$ is a random match value that is independently and uniformly distributed on $[0, 1]$. A firm that tailors its product $y_j$ design to a particular consumer group’s tastes will generate more consumption utility for that group, and may alienate other consumers from its product. On top of that, $\epsilon_{ij}$ is a random match value that is independently and uniformly distributed on $[0, 1]$, and reflects idiosyncratic noise around the consumers’ types.

Consumers will engage in sequential search, visiting firms (e.g. visiting a retail outlet, clicking on a website) in random order. Consumers have no prior information about either price $p$ or product design $y$ of any firm, and will only learn about this by incurring a search cost $s$. After having learnt these product characteristics, consumers can buy at that firm, or keep on searching for better offers. We will assume that $v$ is large enough to make sure that all consumers will search at least once.

The optimal search strategy in equilibrium is of a threshold type: consumers have a reservation utility $w_i$, that will be dependent on their type $\theta_i$, and will compare the match utility $U_{ij}$ with that reservation utility. An equilibrium in this model will correspond to firms’ choices that optimize profits given the reservation utilities $w_i$ of consumers, and reservation utilities that are consistent with the prices and designs on offer in the market.

3 Equilibria with two types

We will first explore a symmetric two-type model, in which half of all consumers are located left (L), and the other half right (R).

3.1 Pooling

To find a potential pooling equilibrium in which all firms choose the same price $p$ and design $y$, we proceed in two steps. First, we solve for the firms’ profit optimization, finding a price and design solving the firms’ first-order conditions for given reservation utilities $w_L$ and $w_R$ for either consumer type. Then we use each consumer type’s Bellman equation to determine these reservation utilities, given offers by the firms. The candidate pooling equilibrium will be at the joint solutions to those two sets of equations.

In a pooling equilibrium, all firms offer the same prices and designs. Some consumers of both types, namely those visitors with sufficiently good idiosyncratic shock $\epsilon$, will therefore buy from a given firm. Let us denote the expected number of visitors from type $L$ visiting any given firm by $\psi_L$. These will consist of first-time searchers, but also of consumers who previously searched but found unattractive offers, due to low realizations
of the idiosyncratic shock. Likewise we define $\psi_R$ for the type $R$ consumers.

Given outside utilities $w_i$, a consumer of type $\theta_i$ will buy from a firm offering price $p$ and design $y$ will purchase the good provided total utility $U_{ij} > w_i$, or in other words,

$$\epsilon_{ij} \geq w_i - u_i$$

with

$$u_i = v - p - (\theta_i - y)^2.$$ 

With uniform $\epsilon_{ij}$, a firm’s profits then take the form

$$\pi = (\psi_L(1 - w_L + u_L) + \psi_R(1 - w_R + u_R))(p - c).$$

It follows that a firm that supplies both $L$ and $R$ types of consumer then chooses average design,

$$y = \frac{\psi_L \theta_L + \psi_R \theta_R}{\psi_L + \psi_R}.$$ 

The profit maximizing price, in turn, equals the monopoly price given both types’ reservation utilities $w_L$ and $w_R$, and is given implicitly by

$$p - c = \frac{\psi_L(1 - w_L + u_L) + \psi_R(1 - w_R + u_R)}{\psi_L + \psi_R}.$$

Next, we analyze consumer search behaviour, under consumers’ belief that all firms offer the same product at price $p$ and design $y$. A consumer of type $\theta_L$ has an expected value of continuing his search, $w_L$, that is governed by the following Bellman equation,

$$w_L = -s + \int_{w_L - u_L}^{1} (u_L + \epsilon) d\epsilon + \int_{0}^{w_L - u_L} w_L d\epsilon,$$

where the first term is the search cost of one more search, the second reflects the expected value of buying the good if its utility $u_L + \epsilon$ exceeds the reservation utility $w_L$, while the third term reflects the fact that the consumer will again search further if $\epsilon$ is so low that the product does not give sufficient utility. Rewriting gives the well known condition that

$$w_L - u_L = x,$$

with $s = \int_{x}^{1} (\epsilon - x) d\epsilon$,

or in other words

$$1 - x = \sqrt{2s}.$$
Exactly the same analysis holds for $R$-type consumers, so that in a potential pooling equilibrium also $w_R - u_R = x$. It then remains to compute the number of visitors $\psi_{L,R}$ from either type that will visit any given firm. These consist of all first-time visitors (half of each type), plus those that visited $n$ other firms but received an idiosyncratic draw $\epsilon < x$, with probability $x$ on each occasion, on each previous firm visit. Summing up all those groups of visitors,

$$\psi_L = \psi_R = \frac{1}{2(1-x)}.$$

Summarizing the analysis so far, we can now characterize the pooling solution as follows.

**Proposition 1.** A pooling solution with two consumer types, located at $\theta_L$ and $\theta_R$, involves all firms offering average design

$$y = \bar{\theta} = \frac{\theta_L + \theta_R}{2},$$

and setting price as

$$p - c = 1 - x.$$

Firm profits in this situation equal $\pi = 1 - x$.

In this pooling solution, all firms offer the same contract and consumers of either type will buy in equal intensity from each firm. The first-order conditions satisfied by firms’ price-design offers guarantee that a best-response by an individual firm that likewise sells to all consumer types will have the same price and design choice. There may however be deviating offers that will only sell to one of the two types of consumers. To check whether the pooling solution identified here will be an equilibrium, we will have to analyse such deviations. Likewise, there may be other equilibria that do not involve all firms making positive expected sales to each type of consumer. We will now proceed by analysing such a separating solution, in which some firms only target $L$-type consumers, while other serve only $R$-type ones. After that we will explore when either pooling or separating solutions will be an equilibrium.

### 3.2 Separation

We now explore a candidate equilibrium in which firms separate, some targeting only $L$-type consumers, others making offers directed only at $R$-type ones. Again, to analyse such a separating solution, we solve for price and design choices given consumer reservation wages $w_L$ and $w_R$, and then look at consumers’ optimal search strategies, determining $w_{L,R}$ for given offers by firms. We then ask these two conditions to be consistent.

Consider then a firm selling exclusively to $L$-type consumers. Its profit function will take the form

$$\pi_L = \psi_L(1 - w_L + u_L)(p_L - c),$$
where again $\psi_L$ denotes the expected number of $L$ consumers visiting the firm. The firm’s first-order conditions on price $p_L$ and design $y_L$ give

$$y_L = \theta_L, \quad p_L - c = 1 - w_L + u_L.$$

Furthermore, we will need to verify later that in the candidate equilibrium, indeed any $R$ consumer visiting a separating firm with this $L$-offer will never buy, not even for maximum idiosyncratic shock $\epsilon = 1$. This will depend on the $R$ consumers’ reservation wage $w_R$ from continuing search until a suitable $R$-product offer is found.

The analysis for a firm choosing to supply only to $R$ consumers is analogous, with $y_R = \theta_R$ and $p_R - c = 1 - w_R + u_R$.

We can then turn to the consumers’ optimal search problem, finding their reservation wages $w_{L,R}$ for given product offers. That includes specifying the fraction of firms in the candidate equilibrium choosing to supply $L$ and $R$ products. We will denote these fractions by $\alpha$ and $1 - \alpha$ (anticipating however that in this symmetric set-up, firms will split evenly with $\alpha = \frac{1}{2}$).

An $L$-type consumer, in this environment, will search further (for an expected utility $w_L$), either if he finds himself at a firm with an $L$ product offer (which happens with probability $\alpha$), but low idiosyncratic utility $c$; or if he turns out to have found an $R$ product firm (probability $1 - \alpha$), which will by assumption not be suitable for any idiosyncratic shock. The Bellman equation for this consumer then reads

$$w_L = -s + \alpha \left( \int_{w_L - u_L}^{1} (\epsilon + u_L) d\epsilon + \int_{0}^{w_L - u_L} w_L d\epsilon \right) + (1 - \alpha)w_L,$$

so that

$$w_L - u_L = x_\alpha, \text{ with } \frac{s}{\alpha} = \int_{x_\alpha}^{1} (\epsilon - x_\alpha) d\epsilon,$$

or $1 - x_\alpha = \sqrt{\frac{2s}{\alpha}}$. The analysis for an $R$-consumer is again analogous, with $1 - \alpha$ taking the role of $\alpha$.

Combining firms’ and consumers’ optimal strategies, it remains to solve for $\alpha$. This can be fixed by demanding that in a candidate equilibrium, any firm must be indifferent between supplying the $L$ or the $R$ market, or in other words, profits must be identical. To determine a firm’s profits, we need to compute the number of visitors $\psi_{L,R}$ visiting any given firm. For $L$ consumers, the probability of failing to find a good match is $x_\alpha$ if a firm offering the $L$-targeted good is visited, and $1$ if the firm happens to supply the $R$ good. Weighting with the firms’ fractions in the total population of firms, $\alpha$ and $1 - \alpha$, and summing up generations of consumers having searched once, twice and more times, we obtain

$$\psi_L = \frac{1}{2\alpha(1 - x_\alpha)}, \quad \psi_R = \frac{1}{2(1 - \alpha)(1 - x_\alpha)}.$$
Using that to compute firm profits, and equating those, in this symmetric situation, that determines \( \alpha = \frac{1}{2} \).

Summing up, we have

**Proposition 2.** A separating candidate equilibrium solution with two consumer types has

\[
y_{L,R} = \theta_{L,R},
\]

and price

\[
p - c = \sqrt{4s}
\]

Total firm profits in this situation equal \( \pi = \sqrt{4s} \).

Comparing with the pooling solution, we see that with separation, products are a better match with consumer preferences, but the separation into two types of firms also means that prices, as well as firm profits, are higher for a given value of search costs \( s \). The intuition here is that consumers are less likely to find a matching firm. Only half of all firms cater to any given consumer’s preferences, with the other half providing a product that targets a different consumer type. Since any given search successfully leads a consumer to a suitable firm with probability a half, with the other half of searches wasted, effective search costs are twice as high in the separating solution. As a result, firms have larger market power over consumers that finds them, and prices, as well as profits are higher under separation.

We next need to explore for which values of search cost \( s \) these two candidate equilibria will in fact be an equilibrium. To that end, we determine whether in either case, a firm may increase its profits by deviating to a different offer.

### 3.3 The two-type equilibrium

We will first explore whether the pooling solution can be an equilibrium. Consider a firm making a deviating offer to increase its profits. We can immediately see that such an offer can never be attractive to both consumer types: in that case, the firm’s optimal offer satisfies the same first-order condition as analysed in the construction of the pooling solution, and in fact the pooling offer is the profit-maximizing offer conditional on being attractive to both consumer types. A better deviating offer must therefore be attractive only to one of the two types, \( L \) or \( R \).

Due to symmetry we can focus on a deviating offer attractive only to \( L \)-type consumers. A deviating firm’s profits will take the form

\[
\tilde{\pi}_L = \psi_L(1 - w_L + \bar{u}_L)(\bar{p} - c)
\]

with \( \psi_L \) the pooling solution’s low-type visitor rate, and \( w_L \), the low-type consumers’ reservation utility in the
pooling equilibrium; these are given by

\[ \psi_L = \frac{1}{2(1-x)} \quad w_L = x + u_L = -\bar{\theta} \bar{\theta}_L^2 - (1 - x) - c + x, \]

with \( 1 - x = \sqrt{2s} \). The optimal deviating offer then has

\[ \tilde{\theta} = \theta_L, \quad \tilde{\theta} - c = \frac{1}{2} (\bar{\theta} - \theta_L)^2 + (1 - x) \]

and

\[ \tilde{\pi}_L = \frac{1}{2(1-x)} (\tilde{\theta} - c)^2. \]

Likewise, we can identify the optimal response to a separating solution: a deviation to attract both consumer types, rather than only \( L \) or \( R \). Comparing profits from such deviations to firm profits in the pooling and separating solutions, respectively, we find the following characterisation of the equilibrium.

**Proposition 3.** All firms pooling, offering single design \( y = \bar{\theta} \) will be an equilibrium for \( s > \bar{s} \), where

\[ \sqrt{2\bar{s}(1 - \frac{1}{2}\sqrt{2})} = \frac{1}{4}\sqrt{2(\bar{\theta} - \theta_L)^2} \]

Conversely, for low search costs, \( s < \bar{s} \) firms separate with half targeting \( L \)-type consumers, the other half targeting \( R \)-consumers.

We can next turn to the comparative statics in search costs, both for firm’s profits and the two consumer types’ surplus, represented by their reservation utilities \( w_{L,R} \).

**Proposition 4.** As search costs \( s \) fall from \( s > \bar{s} \) to \( s < \bar{s} \), and market structure changes from pooling equilibrium to separating equilibrium,

- firms’ profits take a discrete upward jump at \( \bar{s} \),
- consumer surplus increases continuously

The repositioning of firms’ offers increases total surplus, as a result of better match with consumers. This match surplus increase is completely compensated by an increase in pricing margins, as firms get larger market power. The latter occurs because, as market structure changes towards pooling, all consumers’ effective search costs double: half the firm visits will carry a design targeted at the wrong type.
4 Three types and redistribution

With two consumer types, L and R, that are equally distributed across the consumer population, consumers are symmetric and both types will have equal continuation values $w_{L,R}$. We now turn to the qualitatively different situation with three types: two niche types, L and R as before, and a middle-of-the-road central type M positioned in between the two extremal types, at $\theta_M = \frac{1}{2}(\theta_L + \theta_R)$. We assume a distribution of consumers over these three types of $0 < \phi < \frac{1}{2}$ for each of the niche types, and $1 - 2\phi$ for the $\theta_M$ type. We next again explore various potential equilibria: a pooling solution, with all firms targeting the middle design, and niche solutions in which firms separate, targeting either a mix of central and extremal types, or focusing exclusively on each single type (which will be the equilibrium for sufficiently low search costs $s$).

4.1 The pooling solution

If all firms choose the same contract, supplying all consumer types, they will choose price $p$ and design $y$ maximizing expected profits,

$$\pi_{pool} = (\psi_L(1 - w_L + u_L) + \psi_M(1 - w_M + u_M) + \psi_R(1 - w_R + u_R))(p - c).$$

taking as given the fractions of visitors of each type, $\psi_{L,M,R}$, as well as these consumers’ reservation utilities. Consumers’ utilities (excluding the idiosyncratic uniform shock $\epsilon$) are given by $u_i = v - p - (\theta_i - y)^2$.

Conversely, consumers again solve a similar search problem as before, under the belief that all firms offer the same price and design pair $p, y$, leading as before to $w_i - u_i = x$ as defined in the two-type case. Hence, also,

$$\psi_L = \psi_R = \frac{\phi}{1 - x}, \quad \psi_M = \frac{1 - 2\phi}{1 - x}.$$

Combining firm and consumer optimizations, we find the joint solution,

**Proposition 5.** With three types, $\theta_{L,M,R}$, we have a pooling solution in which

- $p - c = 1 - x$
- $y = \theta_M$
- $\pi_{pool} = 1 - x$

In the pooling solution, all firms target their design at the average type $\theta_M$. As a result, the two symmetric niche types $\theta_{L,R}$ get lower expected utility than the mainstream type $\theta_M$, $w_L = w_R = w_M - (\theta_L - \theta_M)^2$. As search costs fall, $1 - x = \sqrt{2s}$ falls and prices as well as profits at all sellers decrease.
4.2 Separation: two designs

As before, we expect (and will later verify) that the pooling solution is an equilibrium for search costs sufficiently high, but as search costs fall, catering exclusively to left or right parts of the market may be a profitable deviation, and pooling will no longer be an equilibrium. We therefore next check for a symmetric separating equilibrium, first focusing on a potential equilibrium in which firms mix between two strategies: either serving left and middle consumers, or, symmetrically, a combination of right and middle.

A firm focusing on two types, $L$ and $M$, will optimize profits

$$\pi = (\psi_L(1 - w_L + u_L) + \psi_M(1 - w_M + u_M))(p - c)$$

over price $p$ and design $y$. (A similar expression holds for the $R$ and $M$ targeting firms.) We will have to check that the optimum we will find will be consistent with $R$ consumers not taking up this offer but preferring to continue search for a combined $R$ and $M$ offer.

Turning to consumers’ optimization, there is now a difference in consumers’ search problems between $L$ and $M$ types: with a fraction of $\alpha$ firms offering the $L$-$M$ product, and $1 - \alpha$ making the other offer instead, $L$ types will only buy from one firm type, while $M$ type consumers will buy from either firm type. As a result, writing their respective Bellman equations, we find, in a symmetric equilibrium with $\alpha = \frac{1}{2}$ (guaranteeing that either choice will yield identical profits),

$$w_L - u_L = x \frac{1}{2}, \quad w_M - u_M = x$$

where again $1 - x = \sqrt{2}s$, and $1 - x_{\alpha} = \sqrt{2s/\alpha}$.

These differences in cut-off values for acceptance of contract will then be reflected in different numbers of consumers $\psi_{L,M,R}$ visiting each firm,

$$\psi_L = \psi_R = \frac{2\phi}{1 - x \frac{1}{2}}, \quad \psi_M = \frac{1 - 2\phi}{1 - x}.$$

Combining the optimization problems for firm and consumers, we find the following two-way separating solution.

**Proposition 6.** A potential hybrid equilibrium with half of the firms ($L$) supplying left and middle, the other half ($R$) right and middle types will satisfy

- $y_L = \frac{\psi_L \theta_L + \psi_M \theta_M}{\psi_L + \psi_M}$, $y_R = \frac{\psi_R \theta_R + \psi_M \theta_M}{\psi_R + \psi_M}$,

- $p - c = \frac{\psi_L}{\psi_M + \psi_L} (1 - x) + \frac{\psi_M}{\psi_M + \psi_L} \left( 1 - \frac{x}{2} \right)$

with

$$\psi_L = \psi_R = \frac{2\phi}{1 - x \frac{1}{2}}, \quad \psi_M = \frac{1 - 2\phi}{1 - x}.$$

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Since in this solution, middle types are served by firms choosing either design, left or right, while niche types find their product only at half of the firms they visit, middle types have better reservation utilities and get a better deal: if \( \phi = 1/3 \), and each type is equally prevalent in the population, \( \psi_M > \psi_{L,R} \), and as a result, design \( y \) for either choice is closer to the population middle than to the niche targeted side. However, middle types get worse design offers than in the pooling case.

### 4.3 Separation into three designs

Finally, let us look at a potential equilibrium with complete separation, expected to be an equilibrium for low \( s \). Going through similar computations as previous, we have

\[
\pi_L = \psi_L(1 - w_L + u_L)(p - c),
\]

and similarly for \( M \) and \( H \). Obviously, \( y_L = \theta_L \). Prices satisfy

\[
p - c = 1 - w_L + u_L.
\]

For \( \phi = 1/3 \), firms split symmetrically, and the Bellman equation then gives \( w_L - u_L = x_{1/3} \), with \( 1 - x_{1/3} = \sqrt{6}s \), so we have \( p - c = 1 - x_{1/3} \), for each type, and \( w_{L,M,H} = x_{1/3} = p \), or \( 1 - w_{L,M,H} - c = 2(1 - x_{1/3}) \). Finally, in that symmetric case, \( \psi_L = \frac{1}{1 - x_{1/3}} \), and the same for the other types, so that \( \pi_L = \pi_M = \pi_H = 1 - x_{1/3} \).

For different distributions, \( \phi, 1 - 2\phi, \phi \), we get a share \( \alpha \) for \( L \) and \( H \) type firms, and \( 1 - 2\alpha \) for \( M \) type firms, with

\[
\frac{\alpha}{1 - 2\alpha} = \left( \frac{\phi}{1 - 2\phi} \right)^{2/3},
\]

which is derived from the requirement that profits across different firm choices should be equal. In that case, the solution to players’ optimization is as follows.

**Proposition 7.** A potential three type separating equilibrium will satisfy

- \( y_L = \theta_L \), \( y_M = \theta_M \), \( y_R = \theta_R \).
- \( p_L - c = 1 - x_\alpha = p_R - c \), \( p_M - c = 1 - x_{1-2\alpha} \)

with

\[
\frac{\alpha}{1 - 2\alpha} = \left( \frac{\phi}{1 - 2\phi} \right)^{2/3},
\]

Profits are the same for each design choice, \( \pi = \frac{\phi}{\alpha}(1 - x_\alpha) \)
4.4 The equilibrium

After having identified three solutions to the joint firms’ and consumers’ optimization problems—pooling, hybrid with two designs, and complete separation into three distinct designs targeted at individual consumer types—we next need to work out for what values of search costs $s$ they will be an equilibrium. For this, we evaluate firms’ deviating strategies in each potential solution.

**Proposition 8.**

- The pooling solution is an equilibrium for sufficiently large search costs $s$.
- There is an intermediate range of search costs $s$ for which two-design hybrid solution constitutes an equilibrium.
- For low enough $s$, complete separation is an equilibrium.

The intuition, like in the two-type case, is that for firms, when consumers have low outside options as a consequence of high search costs, it is advantageous to be able to extract rents from all consumers that visit. When search costs drop, outside options get better and firms need to make better offers: they can do this for some consumers, by targeting their preferences, at the cost of mismatching others. The lower search costs, the higher consumers reservation utilities, and firms increasingly specialize into well-defined consumer categories.

While mainstream $M$ consumers are in fact well targeted by firms in the pooling equilibrium, niche players get poor matches. They get better off as firms specialize more. The downside of firm specialization is that firms’ pricing power goes up: as before, when firms separate, effective search costs increase, as some firm visits are wasted from consumers’ point of view.

In the transition from pooling to hybrid equilibrium, $M$ consumers are hit both by firms having designs separating to left and right directions, and by the concomitant increase in prices. As a result these consumers
actually lose out as $s$ falls below the point where pooling is an equilibrium. This is depicted in figure 1 (right-hand panel). On the other hand, niche consumers ($L$ and $R$ alike) gain more from the better specialisation in designs, than they lose from higher prices. In the right-hand panel, we see that as in the two type case, firm profits jump upwards as search costs fall below the threshold.

References


