Consumer Search and the Uncertainty Effect

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Abstract

We consider a model of Bertrand competition where consumers are uncertain about the value of the firms’ products. Consumers are expectation-based loss-averse and can inspect all products at zero cost. A loss-averse consumer may prefer a certain option to inspecting and choosing an uncertain alternative even if the worst realization of the uncertain alternative is better than the certain option. The firms’ strategic behavior can exacerbate the scope for this “uncertainty effect.” Thus, consumers with modest degrees of loss-aversion (λ < 2) may make first-order stochastically dominated choices, and firms selling inferior products may make positive profits despite Bertrand competition.

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1 Introduction

In many markets, consumers frequently do not select the product or contract that would maximize their payoff. Examples in the recent literature are markets for health insurance (e.g., Abaluck and Gruber 2011, Handel 2013, and Heiss et al. 2016), electricity (e.g., Hortaçsu et al. 2017), or mobile phone contracts (e.g., Grubb and Osborne 2015, and Genakos et al. 2018). If consumers are unwilling or unable to identify the best deal for them, firms may exploit this by charging prices that are too high or selling inferior products, thereby reducing total welfare. Typical explanations for consumers’ failure to choose optimally are search costs or cognitive costs of digesting complex product information. However, Genakos et al. (2018) show that more than 60 percent of consumers do not choose optimally even when provided with credible information about the best product for them, so that search costs and cognitive costs cannot be a cause.

In this paper, we examine a simple model of Bertrand competition to illustrate a mechanism that may render search and information-sensitive choice unattractive for consumers, even when physical search or cognitive costs are negligible. This mechanism is the “uncertainty effect.” It has first been demonstrated experimentally by Gneezy et al. (2006). It shows that a decision maker may prefer a safe option to a lottery even when the lottery’s worst outcome is weakly better than the safe option. In the context of search, this could imply that a decision maker may prefer a product she knows to an uncertain alternative that is never worse, and with positive probability strictly better. In the main experiment from Gneezy et al. (2006), subjects in one condition state their willingness to pay for a 50 USD gift certificate; in another condition, they state their willingness to pay for a gift certificate of either 50 USD or 100 USD, each with equal probability. The experimental results indicate that the willingness to pay is 26 USD in the former condition, but only 16 USD in the latter. Thus, the appeal of the deal in the second condition seems to be impaired by the associated uncertainty. This result has been replicated in several subsequent studies, including ones that consider within-subject comparisons.\(^1\)

To formally capture the uncertainty effect, we assume that consumers exhibit expectation-based loss-aversion preferences as proposed by Kőszegi and Rabin (2006, 2007). It is known\(^2\) that these preferences can generate an uncertainty effect if we assume sufficiently large degrees of loss-aversion \(\lambda\). To illustrate, consider the setting from Gneezy et al. (2006) where in the first condition the decision maker receives a certain value of \(\Gamma\) (equal to 50 USD), while in the second condition she receives a value of either \(\Gamma\) or \(2\Gamma\), with probability \(\frac{1}{2}\) each. Under the canonical specification of expectation-based loss-aversion preferences (which we explain

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\(^1\)We provide a detailed summary below.

\(^2\)Kőszegi and Rabin (2007, page 1060) briefly refer to it.
in detail below), the decision maker’s expected utility from the lottery is

\[
\frac{3}{2} \Gamma - (\lambda - 1) \times \frac{1}{2} \frac{1}{2} \Gamma.
\]

The first term, \(\frac{3}{2} \Gamma\), is the decision maker’s consumption utility, which is just the expected payoff from the lottery. The second term, \(-(\lambda - 1) \frac{1}{2} \frac{1}{2} \Gamma\), is her expected net loss from gain-loss utility. It originates from the weighted sum of gain-loss sensations, that is, the joy of obtaining \(2\Gamma\) instead of \(\Gamma\), and the disappointment from receiving only \(\Gamma\) instead of \(2\Gamma\). Note that the uncertainty effect occurs, i.e., the decision maker strictly prefers the certain outcome of \(\Gamma\) to the lottery, if \(\lambda > 3\). This value is not unreasonable; empirical examinations of risk-preferences preferences find that a significant share of individuals exhibits such degrees of loss-aversion, e.g., von Gaudecker et al. (2011). However, it is much larger than typically assumed (and accepted) in applications of expectation-based loss-aversion preferences.

We show that the interaction between firms’ strategic behavior and consumers’ loss-aversion preferences generates an uncertainty effect for modest degrees of loss-aversion (\(\lambda < 2\)). A key driver of this result is a loss-averse consumer’s separate treatment of the price and product dimension when anticipating gain-loss sensations. This is a core feature of Kőszegi and Rabin’s (2006, 2007) framework of expectation-based loss-aversion. In our model, firms offer products of differing consumption value and have uniform marginal costs. Consumers have homogeneous tastes and face uncertainty about the exact value of each product. They randomly encounter one product and then make a plan whether to inspect other products, as well as which product to purchase for a given realization of product values and prices. They also experience firm-consumer specific utility shocks of size 0 or \(\Delta\). In this framework, if consumers were loss-neutral, then the firm with the best product would price all other firms out of the market and serve all consumers, so that all gains from trade would be realized. Moreover, this would also be the outcome if loss-neutral consumers incur search costs, given that they are sufficiently small relative to \(\Delta\).

A different outcome is possible when consumers are loss-averse. Suppose that firms charge prices that are at most \(\Delta\) away from the product value, and that product values differ by multiples of \(\Gamma\) between different products. When the consumer plans to inspect all products and to purchase different products with differing probabilities, her expected utility from this plan can be written as

\[
\alpha_0 \Delta - (\lambda - 1) \times (\alpha_1 \Gamma + \alpha_2 \Delta),
\]

where \(\alpha_0, \alpha_1, \alpha_2\) are positive constants that depend on the consumer’s plan and the firms’ con-
duct. Note that the expression in (2) becomes negative if the maximal size of the utility shock $\Delta$ is small enough relative to the minimal value distance between product values $\Gamma$. Thus, even for modest degrees of loss-aversion $\lambda$, a loss-averse consumer may choose to purchase a known product that offers no surplus to her rather than to inspect other products and to purchase them with positive probability. In other words, such a consumer is willing to forgo a (certain) gain $\Delta$ if its realization implies that she has to purchase a high-value product at a high price or a low-value product at a low price, both with positive probability. The implied uncertainty in the product and price dimension renders this plan unattractive. Given this behavior, it is optimal for firms to charge prices that are at most $\Delta$ away from the product value.

We generalize this result to the case when there are both loss-averse and loss-neutral consumers. Again, if $\Delta$ is sufficiently small, an equilibrium exists where loss-averse consumers do not inspect other products and pick the product they first encountered; in contrast, loss-neutral consumers inspect all products and then end up trading with the firm that offers the highest product value; to serve all inspecting consumers, this firm offers the highest surplus. Moreover, we discuss how such an equilibrium can also emerge in a model with horizontal product differentiation.

These results generate several important implications. First, they show that even in Bertrand competition inferior firms may remain active and generate positive profits. They only have to make sure that they get some loss-averse consumers’ attention before these consumers make a plan. Second, this may imply that firms primarily compete for attention, and not for offering the product at the lowest price; this is especially true for firms with inferior products (we examine an extension where firms can invest in getting attention). Third, since our argument does not rely on physical search costs, the effects described may occur in any context, i.e., for shopping in brick-and-mortar stores and for online shopping; even when it is simple to compare prices and product information or informational nudges are provided, too few consumers may search for a better deal. Fourth, the model generates the empirical prediction that among those consumers who inspect other products the degree of loss-aversion should be smaller than among those consumers who do not search for alternative deals. Fifth, even if there are great deals in the market, loss-averse consumers may not take advantage of them if they have the (possibly wrong) belief that price and product quality are always very close to each other.

The rest of the paper is organized as follows. In Section 2, we introduce the model and the equilibrium concept. In Section 3, we first consider several benchmark cases with loss-neutral consumers and then analyze our framework with homogeneous as well as heterogeneous consumers. In Section 4, we discuss a number of extensions and further results. In Section 5, we relate our results to the previous literature. Section 6 concludes. All proofs and mathematical details are relegated to the appendix.
2 The Model

We consider the competition of \( n \geq 3 \) firms \( i = 1, \ldots, n \) for a single consumer. Her payoff from firm \( i \)'s product is \( u_i = v_i + \xi_i \), where \( v_i \) is firm \( i \)'s product value and \( \xi_i \) is a consumer-firm-specific taste shock. Ex-ante, the consumer is uncertain about both product value and taste shock: with probability \( \frac{1}{2} \) the product value is low, \( v_i = v_{i,l} \), and with probability \( \frac{1}{2} \) it is high, \( v_i = v_{i,h} \); the taste shock equals 0 or \( \Delta > 0 \), each with probability \( \frac{1}{2} \). Firms are vertically differentiated: Firm \( i \)'s product value is larger than firm \( i-1 \)'s product value. For convenience, we use the parametrization \( v_i.h = v_i.l + \Gamma \) and \( v_{i+1,l} = v_{i,h} + \Gamma \) for all \( i \), where \( \Gamma > 0 \) and \( v_{i,l} \geq \Gamma \).

Let \( p_i \) be the price that firm \( i \) charges for its product. If the consumer trades with firm \( i \), her consumption utility is \( u_i - p_i \) and firm \( i \)'s profit is \( p_i \), while the profit of the other firms is zero. If the consumer does not trade at all, firms’ profits are zero.

**Consumer Loss-Aversion.** We follow Kőszegi and Rabin (2006, 2007) to model the consumer’s expectation-based loss-aversion. Her total utility consists of consumption utility and gain-loss utility from comparisons of the actual outcome to a reference-point given by her expectations. Below we make precise when and how these expectations are formed. Suppose that the consumer expects to get payoff \( \tilde{u} \) and to pay the price \( \tilde{p} \) with certainty. If she trades with firm \( i \), her total utility equals

\[
U(u_i, p_i \mid \tilde{u}, \tilde{p}) = u_i - p_i + \mu(u_i - \tilde{u}) + \mu(-p_i + \tilde{p}).
\]  

(3)

The function \( \mu \) captures gain-loss utility. We assume that \( \mu \) is piecewise linear with slope 1 for gains and slope \( \lambda \geq 1 \) for losses. Thus, \( \lambda \) is the degree of loss-aversion.

The consumer may have stochastic expectations about the realization of payoff \( u \) and price \( p \). The reference point reflects this uncertainty. Let the distribution function \( G^u \) be her expectation regarding the outcome in the product dimension and \( G^p \) her expectation regarding the outcome in the price dimension. The consumer’s total utility from trading with firm \( i \) is then

\[
U(u_i, p_i \mid G^u, G^p) = u_i - p_i + \int \mu(u_i - \tilde{u})dG^u(\tilde{u}) + \int \mu(-p_i + \tilde{p})dG^p(\tilde{p}).
\]  

(4)

Thus, gains and losses are weighted by the probability with which the consumer expects them to occur. This preference model captures the following intuition. If the consumer expects to win either 0 or 10 units in some dimension, each with probability 50 percent, then an outcome of 6 units feels like a gain of 6 units weighted with 50 percent probability, and a loss of 4 units also weighted with 50 percent probability.

**Pricing and Inspection.** There are three stages. In Stage 1 – the “pricing stage” – firms
observe the realization of product values $V = (v_1, ..., v_n)$, but not the realization of the taste shocks. Firms 1 to $n - 1$ then choose their prices. Firm $n$ observes these prices and chooses $p_n$. While this assumption is non-standard, it captures the fact that firm $n$ is dominant so that it can repulse any attempt to price out its product. Crucially for us, this assumption allows for pure-strategy equilibria so that the model remains tractable. In Stage 2 – the “planning stage” – the consumer is randomly assigned to one firm $i^*$ and observes its product value $v_{i^*}$, utility shock $\xi_{i^*}$, and price $p_{i^*}$. Each firm’s assignment is equally likely. The consumer then makes a plan whether to inspect also the other products, and what product (if any) to buy once the inspection choice has been executed. Inspection is a binary decision $a \in \{0, 1\}$: if the consumer inspects all products, $a = 1$, she observes all product values, utility shocks, and prices; if she does not inspect all products, $a = 0$, she only observes the product value, utility shock, and price of the assigned firm $i^*$. Given her plan, the consumer forms expectations $G^u$, $G^p$ about the outcome in the utility and price dimension, respectively. Finally, in Stage 3 – the “market stage” – the consumer executes this plan and payoffs are realized. Figure 1 illustrates the timeline.

Strategies and Equilibrium. We formally define the game. Let $\mathcal{V}$ be the set of all possible realizations $V$. For $i = 1, ..., n - 1$, firm $i$’s strategy $\sigma_i$ maps the realization $V$ into a price $p_i$, $\sigma_i : \mathcal{V} \rightarrow \mathbb{R}_+$. Firm $n$’s strategy $\sigma_n$ maps the realization $V$ and the price vector $(p_1, ..., p_{n-1})$ into a price $p_n$, $\sigma_n : \mathcal{V} \times \mathbb{R}_{n-1} \rightarrow \mathbb{R}_+$. Let $\sigma_f = (\sigma_1, ..., \sigma_n)$ be the firms’ strategy profile. The consumer derives beliefs $\mu(V \mid i^*, v_{i^*}, \xi_{i^*}, p_{i^*})$ about the distribution of $V$ from the identity of the assigned firm, its product value, and price. The consumer’s strategy $\sigma_\lambda$ consists of two parts: An inspection strategy $\sigma_\lambda^{[1]}$, which maps the identity of the observed firm $i^*$, product value $v_{i^*}$, utility shock $\xi_{i^*}$, and price $p_{i^*}$ into an inspection decision $a \in \{0, 1\}$,

$$\sigma_\lambda^{[1]} : \{1, ..., n\} \times \mathbb{R}_+ \times \{0, \Delta\} \times \mathbb{R}_+ \rightarrow \{0, 1\},$$

and a purchase strategy $\sigma_\lambda^{[2]}$, which maps the identity of the observed firm $i^*$, realization $V$, utility shocks $(\xi_1, ..., \xi_n)$, prices $(p_1, ..., p_n)$, and inspection decision $a$ into a purchase decision.

In Section 4, we endognize the probability of assignment.
any possible observed firm \( i \) and strategy and beliefs, by 
\[
\sigma, \lambda \in \mathcal{E}, \xi, \mu \}
\]
the consumer’s expected payoff from strategy \( \lambda \) after observing firm \( i^* \)’s offer. We now can define the consumer’s personal equilibrium as well as the equilibrium of the complete game.

**Definition 1.** Let the firms’ strategy \( \sigma_f \) and the consumer’s beliefs \( \mu \) be given. The consumer’s strategy \( \sigma_\lambda \) is a personal equilibrium if at any possible observed firm \( i^* \), realization \( V \), utility shock \( (\xi_1, ..., \xi_n) \), and prices \( (p_1, ..., p_n) \), we have \( \sigma_\lambda^{[2]} \in \arg \max_{\sigma \in \mathcal{X}} U(u_i, p_i | G^*, G^p) \), where \( X \) is the set of choices that are available after inspection decision \( \sigma_\lambda^{[1]} \).

**Definition 2.** Let the firms’ strategy \( \sigma_f \) and the consumer’s beliefs \( \mu \) be given. The consumer’s strategy \( \sigma_\lambda \) is a preferred personal equilibrium if
\[
\mathbb{E}_{\sigma_\lambda}[U(u_i, p_i | G^*, G^p) | i^*, v_i, \xi_i, p_i] \geq \mathbb{E}_{\sigma_\lambda}[U(u_i, p_i | \hat{G}^*, \hat{G}^p) | i^*, v_i, \xi_i, p_i]
\]
for any possible observed firm \( i^* \), product value \( v_i \), utility shock \( \xi_i \), price \( p_i \), and any alternative personal equilibrium \( \hat{\sigma}_\lambda \).

**Definition 3.** The triple \( \sigma = (\sigma_f, \sigma_\lambda, \mu) \) is a perfect Bayesian equilibrium if \( \sigma_f \) implies that each firm maximizes its expected payoff given \( \sigma_\lambda \), and strategy \( \sigma_\lambda \) is a preferred personal equilibrium for given \( \sigma_f \) and \( \mu \).

To emphasize competition between firms, we make the following assumptions: If firms are indifferent between different prices, they charge the smallest one among them; consumers inspect all products if a preferred personal equilibrium exists that involves trade with firms other than the assigned firm \( i^* \); if consumers are indifferent between two or more firms, they...
choose the firm with the highest product value among them; and when consumers are indifferent between trading or not trading, they choose the former option.

3 The Market Equilibrium

In this section, we study the market equilibrium in our framework. In particular, we are interested in whether there exists an equilibrium in which firms charge prices that are close or equal to monopoly prices. We proceed in three steps. In Subsection 3.1, we consider the benchmark case where consumers are loss-neutral. In Subsection 3.2, we examine the framework with loss-averse consumers. Finally, in Subsection 3.3, we generalize our findings to a setting with heterogeneously loss-averse consumers.

3.1 Benchmark Cases

We examine three useful benchmark cases that will illustrate how expectation-based loss-aversion affects the market outcome. First, we consider our basic model when consumers are loss-neutral. Then we introduce small inspection costs (similar to search costs) to our basic model and study how these influence the market equilibrium.

The Bertrand Equilibrium. Assume first that consumers are loss-neutral. They inspect the firms’ products and then purchase the good $i$ that maximizes the surplus $u_i - p_i$. Since firm $n$ offers the highest product value, it has a competitive advantage and can price the other firms out of the market. Indeed, this strategy is optimal if the consumers’ utility shock $\Delta$ is sufficiently small relative to smallest possible product value difference $\Gamma$. In this case, firm $n$ sets the price $p_n$ so that it serves a consumer even if her utility shock at firm $n$ is zero, the utility shock at firm $n-1$ is $\Delta$, and firm $n-1$ charges a price of zero. This characterizes the unique equilibrium outcome in this market. Throughout, we will call it the “Bertrand equilibrium.” All gains from trade are realized in this equilibrium.

Lemma 1. Suppose that consumers are loss-neutral ($\lambda = 1$). If $\Delta$ is small enough relative to $\Gamma$, then in any equilibrium firm $n$ serves all consumers at price $p_n = v_n - v_{n-1} - \Delta$.

Inspection Costs. Next, we consider the market equilibrium when there are small inspection costs $c > 0$ that the consumer has to pay if she decides to inspect all products. If she abstains from inspection, these costs are zero. Assume again that consumers are loss-neutral.

To begin with, assume that there are no utility shocks, $\Delta = 0$. The Bertrand outcome is then no longer the unique equilibrium outcome. Consumers are willing to bear the inspection costs
only if they expect to get, with positive probability, a better deal than the deal offered by the assigned firm \( i^* \). Suppose that each firm \( i \) charges its monopoly price \( p_i = v_i \). Then consumers cannot gain from inspecting all products and strictly prefer to trade with their assigned firm \( i^* \). This behavior in turn makes it optimal for each firm to charge its monopoly price. Thus, we obtain a version of the Diamond Paradox: Arbitrary small inspection costs can turn a competitive market equilibrium into an equilibrium with monopoly pricing.

**Lemma 2 (Diamond Paradox).** Suppose that consumers are loss-neutral \((\lambda = 1)\), and that they have to pay \( c > 0 \) to inspect all products. If \( \Delta = 0 \), then there is an equilibrium in which each firm \( i \) serves its assigned consumers at the monopoly price \( p_i = v_i \).

Now let there be positive utility shocks, \( \Delta > 0 \). The Diamond Paradox may then no longer occur. Consider a consumer who experiences no positive utility shock at her assigned firm \( i^* \), \( \xi_{i^*} = 0 \). There is a positive probability that she experiences a positive utility shock at another firm’s product. Thus, if each firm \( i \) charges its monopoly price \( p_i = v_i \), this consumer will inspect all products if the inspection costs \( c \) are small enough relative to the utility shock \( \Delta \). Consequently, there is a share of consumers who inspect all products. To serve these consumers, firm \( n \) can price its competitors out of the market. Again, this strategy is optimal if \( \Delta \) is sufficiently small relative to \( \Gamma \). We then again obtain the Bertrand equilibrium outcome.

**Lemma 3.** Suppose that consumers are loss-neutral \((\lambda = 1)\), and that they have to pay \( c > 0 \) to inspect all products. If \( \Delta \) is small enough relative to \( \Gamma \) and \( c \) is small enough relative to \( \Delta \), then in any equilibrium firm \( n \) serves all consumers at price \( p_n = v_n - v_{n-1} - \Delta \).

### 3.2 The Market Equilibrium with Loss-Averse Consumers

We next examine how the market equilibrium is affected when consumers are loss-averse. There is an important difference between loss-neutral and loss-averse consumers when it comes to finding and exploiting advantageous deals. For loss-neutral consumers only the difference between utility from the product and its price, \( u_i - p_i \), matter for the purchase decision. It is irrelevant for them whether they get a high product value \( v_i \) at a high price \( p_i \) or a low product value \( v_j \) at a low price \( p_j \), as long as \( u_i - p_i = u_j - p_j \).

This is not the case for loss-averse consumers. Changes in the product and price dimension create experiences of loses and gains. When the outcome of a transaction is uncertain, then, by loss-aversion, the expected payoff from these gain-loss sensations is negative. To illustrate, suppose that trading with the firms \( i \) and \( j \) does not create a surplus in consumption value, \( u_i - p_i = u_j - p_j = 0 \), but that firm \( j \)’s product offers higher utility, so that \( u_j - u_i = \Gamma \) and
Consider the plan \( p_j - p_i = \Gamma \) “trade with each firm with 50 percent probability.” The expected payoff from this plan for loss-neutral consumers is 0, while for loss-averse consumers it is \(-(\lambda - 1)\frac{1}{2}\Gamma\). Thus, for loss averse-consumers, exploiting advantageous deals after inspection create costs in terms of gain-loss sensations.

Consumers’ loss-aversion significantly changes the competitive position of firms with inferior products. In the Bertrand equilibria from Lemma 1 and Lemma 3, a firm \( i \neq n \) was unable to make a profit since firm \( n \) had a superior product and could price it out of the market. Now, when a loss-averse consumer is assigned to a firm \( i^* \neq n \), this firm’s advantage with this consumer is that it can offer her a certain and (weakly) positive payoff \( u_i - p_{i^*} \). For any other firm \( i \), the consumer does not know the exact realization of the product value \( v_i \) and utility shock \( \xi_i \). Thus, any consumer plan \( \sigma^{[2]}_{\lambda} \) that contains purchasing other products with positive probability implies uncertainty, which, as seen above, reduces the expected utility from this plan. Whether or not the consumer adopts such a plan or trades with firm \( i^* \) with certainty then depends on the other firms’ equilibrium conduct.

We check under what circumstances an equilibrium exits in which each firm \( i \) charges its monopoly price \( p_i = v_i \), as in Lemma 2. As a preliminary step, we consider the consumers’ behavior under a small variation of this pricing strategy profile. Suppose that each firms \( i \neq n \) charges \( p_i = v_i \), while firm \( n \) charges \( p_n = v_n - \Delta \). A consumer assigned to a firm \( i \neq n \) then can realize a certain payoff of \( \Delta \) in case of a positive utility shock, and one of 0 in case of no positive utility shock. The consumption utility from trading with firm \( n \) with certainty is \( 2\Delta \) or \( \Delta \), each with equal probability. Loss-neutral consumers therefore would always inspect all products and trade with firm \( n \).

We examine which plan is the preferred personal equilibrium for loss-averse consumers. In general, this could be a tedious task since the mapping between plan and expected utility is complex (in the Appendix, we write down the expected utility for a generic plan). Fortunately, the problem has enough structure so that a few comparisons suffice to identify the preferred personal equilibrium. Suppose that a loss-averse consumer assigned to a firm \( i^* \neq n \) adopts the plan “always trade with firm \( n \).” Denote this plan by \( \hat{\sigma}_{\lambda}^{[2]} \). At the planning stage, the expected utility from this plan is

\[
E[U(\hat{\sigma}_{\lambda}^{[2]})] = \frac{3}{2}\Delta - (\lambda - 1)\frac{1}{2}\Gamma - (\lambda - 1)\frac{1}{8}\Delta.
\]

The first term \( \frac{3}{2}\Delta \) is the consumption utility out of this plan; the second term \(-(\lambda - 1)\frac{1}{2}\Gamma\) is the expected gain-loss utility that originates from the fact that firm \( n \)’s product value \( v_n \) and therefore also its price \( p_n \) is uncertain and varies by \( \Gamma \); the third term \(-(\lambda - 1)\frac{1}{8}\Delta \) is the

\footnote{This is of course not a fully specified strategy \( \sigma_{\lambda} \). For convenience, we use this reduced description of a strategy when it is not essential to specify further details of the complete strategy.}
expected gain-loss utility from uncertainty about whether the consumer experiences a positive utility shock at firm \( n \) or not.

Does the loss-averse consumer execute this plan, instead of trading with the assigned firm \( i^* \)? Note that both “always trade with firm \( i^* \)” and “always trade with firm \( n \)” are personal equilibria. For the former plan this is true because the consumer can choose not to inspect all products \( (a = 0) \) and then avoids any potential temptation to buy another product. For the latter plan, we show this in the proof of Lemma 4 below. If the consumer experiences a positive utility shock at firm \( i^* \), she prefers trading with \( i^* \) with certainty instead of trading with firm \( n \) if \( \mathbb{E}[U(\hat{\sigma}_{i^*}^{[2]})] < \Delta \); if the consumer experiences no positive utility shock at firm \( i^* \), she prefers trading with firm \( i^* \) with certainty if \( \mathbb{E}[U(\hat{\sigma}_{i^*}^{[2]})] < 0 \). The next result shows that this last comparison actually suffices to state a condition under which “always trade with firm \( i^* \)” is the preferred personal equilibrium for all loss-averse consumers.

**Lemma 4.** Suppose that each firm \( i \neq n \) charges \( p_i = v_i \), while firm \( n \) charges \( p_n = v_n - \Delta \). Consider a loss-averse consumer assigned to a firm \( i^* \neq n \). If the expected payoff from the plan “always trade with firm \( n \)” weakly exceeds the expected utility of trading with firm \( i^* \), then this plan is the consumer’s preferred personal equilibrium. Otherwise, the plan “always trade with firm \( i^* \)” is the consumer’s preferred personal equilibrium.

From this result and \( \frac{1}{2}\Gamma \geq \Delta \) we get that the plan “always trade with firm \( i^* \)” is the unique preferred personal equilibrium for loss-averse consumers if

\[
(\lambda - 1)\frac{3}{8}\Gamma \geq \Delta, \tag{9}
\]

provided that the firms’ pricing strategy is as stated above. We then obtain an uncertainty effect: Loss-averse consumers prefer a certain option to an uncertain alternative even though the worst outcome of this alternative is – in terms of consumption utility – weakly better than the certain option. Observe that for any degree of loss-aversion \( \lambda' > 1 \) we can find \( \Delta \) small enough such that the inequality in (9) holds. Thus, in our framework, the uncertainty effect can occur for modest degrees of loss-aversion.

We can now easily see when an equilibrium exists in which all firms charge monopoly prices. Suppose that each firm \( i \) charges \( p_i = v_i \). One then can show that any consumer plan generates a weakly lower expected utility in the planning stage as under the pricing strategy profile stated above. Thus, the plan “always trade with the firm \( i^* \)” is a preferred personal equilibrium if (9) holds. Given that consumers execute this plan and do not inspect all products, charging monopoly prices is optimal for all firms. We therefore get the following result.

**Proposition 1.** Suppose that consumers are loss-averse \( (\lambda > 1) \). If \( \Delta \) is small enough relative
to $\Gamma$, so that $(\lambda - 1)\frac{3}{8}\Gamma \geq \Delta$, then there is an equilibrium in which (i) consumers do not inspect all products, and (ii) each firm $i$ serves its assigned consumers at price $p_i = v_i$.

In this equilibrium, half of the consumers earn no consumer surplus; they forgo the possibility to realize a positive surplus $\Delta$ by finding a product that better suits them than the assigned firm’s product. The reason for this is that any plan that involves inspecting all products and purchasing a product with uncertain value and price generates negative expected utility through the uncertainty effect in the planning stage.

While technically simple, the result in Proposition 1 has some interesting properties. First, it shows that a Diamond Paradox-like outcome can be obtained even though there are consumer-firm specific taste shocks and there are no physical search or cognitive costs. All “search costs” are created in the consumers’ mind through loss-aversion. Thus, the context in which transactions take place is not important, i.e., it could be online trade or shopping in brick-and-mortar stores; the minimal hurdle to search and information-sensitive trade would always be determined by the consumers’ degree of loss-aversion. For regulation this implies that the market outcome may not become competitive even if all barriers to trade are removed, as long as firms offer differentiated products.

Second, since the implicit “search costs” originate from the consumers’ loss-aversion, they scale up in the size of the transaction. This is different from physical search costs that originate from time or effort costs and do usually not vary a lot across different products in the same product category. To illustrate, consider a consumer who searches online for a regular dress. By visiting another online shop, the consumer could gain a value of 2 Euros. If the time costs from visiting this other shop are 3 Euros, search does not pay off and the consumer chooses the default. Next, the same consumer wishes to purchase a fancy dress. The potential gains from search are around 20 Euros. Keeping time costs (and the arrangement of online shops) fixed, this implies that search costs are now not large enough to deter search. Such considerations are irrelevant for “search costs” that are generated through loss-aversion since they are just scaled up by the size of the transaction.

Third, the equilibrium is robust in the sense that consumers may stick to their choice even if – unexpectedly – they discover a product that would generate higher consumption utility than the assigned firm $i^*$’s product. Consider a consumer who did not experience a positive utility shock with the product of the assigned firm $i^*$, and who chose not to inspect all products. In the market phase, she discovers unexpectedly that the product of firm $i < i^*$ yields a positive utility shock. If this consumer deviates from her original plan and trades with firm $i$, her payoff is $\Delta + z\Gamma - \lambda(z\Gamma - \Delta)$, where $z \in \mathbb{N}_+$ indicates the distance in product value between firm $i$ and $i^*$. For $i > i^*$ this value would be $\Delta + (z\Gamma + \Delta) - \lambda z\Gamma$. Thus, if $\Delta$ is small enough relative to $\Gamma$,
so that
\[
\frac{(\lambda - 1)}{(\lambda + 1)} \Gamma \geq \Delta,
\] (10)
then the consumer sticks to her original plan and does not trade with firm \(i\). This is again different from physical search costs. Suppose consumers are loss-neutral, but face large search costs so that they trade with the assigned firm \(i^*\). If such a consumer unexpectedly finds a better product, she strictly prefers to purchase it and to give back the original product (if possible).

### 3.3 The Market Equilibrium with Heterogeneous Consumers

So far, we assumed that all consumers are loss-averse and exhibit a common degree of loss-aversion \(\lambda\). In this section, we examine to what extent our findings generalize when consumers are heterogeneous in their loss-aversion preferences.\(^5\) We assume that a share \(\beta \in (0, 1)\) of consumers is loss-neutral and exhibits \(\lambda = 1\), while the share \(1 - \beta\) is loss-averse with the degree of loss-aversion \(\lambda^* > 1\). Given some firm pricing strategy profile, these two consumer groups may choose different strategies \(\sigma_0\) and \(\sigma_{\lambda^*}\), respectively. The equilibrium concept in Definition 3 extends to this case in straightforward manner. We again ask to what extent there can be an equilibrium in which firms charge monopoly prices.

Suppose that each firm \(i\) charges the monopoly price \(p_i = v_i\). Loss-neutral consumers who do not experience a positive utility shock at the assigned firm will then inspect all products in the hope to find a product \(i\) that yields them \(\xi_i = \Delta\). Thus, there will be a positive share of consumers who inspect all products so that they can choose between firms. Any firm potentially faces the trade-off between charging a high price to serve only its assigned non-inspecting consumers and charging a lower price to serve assigned non-inspecting as well as inspecting consumers. Since firm \(n\) has a competitive advantage, it is in the best position to take advantage of this new setting. Given that each firm \(i \neq n\) charges the monopoly price \(p_i = v_i\), firm \(n\) can serve all loss-neutral consumers who inspect all products by charging \(p_n = v_n - \Delta\) (so that a loss-neutral consumer with \(\xi_i = \Delta\) for some \(i \neq n\) and \(\xi_n = 0\) still trades with firm \(n\)). Using Lemma 4, we can show that this pricing strategy can indeed be part of an equilibrium.

**Proposition 2.** Suppose that there are loss-averse and loss-neutral consumers. If \(\Delta\) is small enough for given parameters \(\lambda^*, \Gamma\) and \(\beta\), then there is an equilibrium in which (i) each firm \(i \neq n\) serves its assigned loss-averse consumers at price \(p_i = v_i\), (ii) firm \(n\) serves its assigned loss-averse consumers and all loss-neutral consumers at price \(p_n = v_n - \Delta\), and (iii) loss-averse consumers do not inspect all products, while loss-neutral consumers do.

\(^5\)Empirical work on risk-preferences finds a large degree of heterogeneity; see, for example, von Gaudecker et al. (2011).
When there are consumers who inspect all products, each firm $i \neq n$ could be tempted to reduce the price in order to serve some of them, e.g., those who experience a positive utility shock at firm $i$, $\xi_i = \Delta$, and no positive utility shock at firm $n$, $\xi_n = 0$. However, if $\Delta$ is sufficiently small, firm $n$ would always cut the price so as to keep all inspecting consumers, rendering firm $i$’s deviation non-profitable. This also implies that loss-averse consumers who observe a deviation at an assigned firm $i^* \neq n$ are typically not motivated to inspect all products.

Proposition 2 makes a clear empirical prediction. Suppose we analyze a market in which consumers can strictly benefit from inspecting all products and inspection is cheap. At the same time, only few consumers inspect all products and many consumers purchase goods that are inferior for their needs (given the consumers’ observable characteristics). Proposition 2 then suggest the following predictions: First, the average degree of loss-aversion should be lower among those consumers who inspect other products than among those who do not. And second, the average degree of loss-aversion should be higher among those consumers who purchase (given their observable characteristics) inferior goods than among those who do not.

4 Extensions

In this section, we briefly outline several potential extensions of our framework.

Pessimistic Beliefs. Whether a loss-averse consumer inspects all products depends on the size of the expected surplus relative to the size of gain-loss sensations that have to be accepted in order to realize this surplus. If this surplus is small enough in relative terms, the consumer prefers not to inspect all products and to stick to her (potentially inferior) default option. This implies that consumers’ pessimistic beliefs about the potential surplus that could be realized through inspection may limit the extent of consumer search, independent of the firms’ conduct. Suppose that a firm introduces, unexpectedly, an exceptional good deal with large surplus $v_{new} - p_{new}$. If loss-averse consumers have sufficiently pessimistic beliefs, they do not engage in inspection, so that they miss out this deal. Since this happens in an environment without search or cognitive costs, it appears as if these consumers have large “switching costs.” However, they just do not expect that such a deal is possible, which renders inspection non-profitable. Note that this belief is particularly plausible in an environment where firms have a reputation for exploiting most gains from trade.

Paying for Prominence. We assumed that every firm $i$ gets a share $\frac{1}{n}$ of consumers who then make a plan whether to inspect other products. Alternatively, we can allow firms to invest into “prominence” so that higher investments result in a higher share of consumers who know their product before making a plan. We briefly discuss such an extension of our framework.
Denote by $s_i$ the share of consumers that firm $i$ receives. Firm $i$ can affect this share by investing $b_i$. Denote by $b = (b_1, ..., b_n)$ the investments of all firms. The share $s_i$ is then given by a function $f_i(b)$, which we assume to be continuously differentiable, concave in all entries, and $f_i(b) \geq \tilde{f} > 0$ for all $i$ and $b$; moreover, we assume that it is always optimal to invest a small amount, i.e., $\frac{\partial f_i(b)}{\partial b_i} > 1$ at all $b$. Investments have to be carried out before $V$ realizes (so that consumers cannot make inferences about $V$ from their assignment).

Suppose that the continuation equilibrium in the pricing stage is that outlined in Proposition 2. We then can specify optimal investments. The profit functions are $\mathbb{E}(v_i)(1 - \beta)f_i(b) - b_i$ for all firms $i \neq n$ and $(\mathbb{E}(v_n) - \Delta)(1 - \beta)f_n(b) + \beta - b_n$ for firm $n$. Standard arguments show that an equilibrium level of investments $b^*$ exists. At such an equilibrium, the following first-order conditions must be satisfied:

$$\mathbb{E}(v_i)(1 - \beta)\frac{\partial f_i(b^*)}{\partial b_i} - 1 = 0 \quad \text{for all firms } i \neq n,$$

$$\mathbb{E}(v_n) - \Delta(1 - \beta)\frac{\partial f_n(b^*)}{\partial b_n} - 1 = 0 \quad \text{for firm } n.$$

Since all shares $s_i$ are strictly positive, Proposition 2 continues to hold (the proof essentially remains the same). So $b^*$ captures investment levels that can occur in an equilibrium of the complete game. We can derive two conclusions from the first-order conditions. First, firms with higher product values also invest more into prominence. Thus, more loss-averse consumers end up purchasing high-value products. Second, the share of loss-averse consumers $1 - \beta$ influences investments into prominence. The higher is this share, the higher are investments.

**Horizontal Product Differentiation.** We can rewrite our model by assuming horizontal instead of vertical product differentiation. Consumers still may have the same preferences with respect to different product features. However, to realize a certain surplus, consumers may then have to accept that they purchase different combinations of product features with positive probability, which renders inspection and information-sensitive trade costly for loss-averse consumers. Note that the meaning of product differentiation in such a framework is not the satisfaction of different consumer needs (as in a Hotelling model), but the creation of psychological inspection costs. To get the outcome of Proposition 2 – where loss-neutral consumers inspect all product, while loss-averse do not – we additionally have to assume that firms exhibit varying marginal costs, so that there is again a dominant firm that can price the other firms out of the market. In equilibrium, this firm will serve all assigned non-inspecting and all inspecting consumers, while each other firm serves its assigned non-inspecting consumers.
5 Related Literature

The Uncertainty Effect. So far, only the experimental literature discusses the uncertainty effect. Gneezy et al. (2006) were the first to show that some individuals may value a lottery less than its worst outcome. They applied a between-subject design and obtained the same result for different types of goods, elicitation methods, and implementation. Unsurprisingly, this provocative result triggered a sequence of papers that study its robustness. Sonsino (2008) finds in auctions for single gifts and binary lotteries on these gifts that 27 percent of subjects sometimes submit higher bids for the single gift than for the lottery even though the lottery’s worst outcome is the gift. Simonsohn (2009) conducts several within-subject variations of the experiment by Gneezy et al. (2006) and finds that 62 percent of subjects exhibit the uncertainty effect. Yang et al. (2013) show that a pronounced uncertainty effect occurs if the certain outcome is framed as a “gift certificate” while the lottery is framed as “lottery ticket” (or coin flip, gamble, raffle). In this condition, 34 to 58 percent exhibit the uncertainty effect. Most recently, Mislavsky and Simonsohn (2018) find the uncertainty effect when subjects perceive the certain outcome as more natural transaction than the lottery. They interpret the lottery as a transaction that has an unexplained feature.

This is the first paper that studies the implications of the uncertainty effect for competitive markets. Importantly, we show that the firms’ strategic behavior may increase the scope for the uncertainty effect when consumers exhibit expectation-based loss-aversion preferences. Thus, it can occur even for modest degree of loss-aversion.

Expectation-Based Loss Aversion and Markets. This paper contributes to a literature in behavioral industrial organization that analyzes the implications of expectation-based loss-aversion for trade between consumers and firms. Heidhues and Kőszegi (2008) and Karle and Peitz (2014) study imperfect competition with expectation-based loss-averse consumers. In Heidhues and Kőszegi (2008), consumers initially are uncertain about both prices and match values. Consumer loss-aversion then may eliminate price variations in the market even if firms exhibit varying production costs. Karle and Peitz (2014) consider a similar setup in which firms post their prices upfront and consumers are either informed or uninformed about their

6Moreover, in a post-experimental survey, many participants indicate “aversion to lotteries” as their explanation for such behavior.

7Andreoni and Sprenger (2011) also find the uncertainty effect in their experimental data. Dal Bó et al. (2013) find in a survey among job applicants that 39 percent prefer a certain payment of 2.5 million pesos to a lottery that pays with equal probability 2.5 or 5 million pesos. Some studies demonstrate that the uncertainty effect does not show up under certain conditions; see Rydval et al. (2009) and Wang et al. (2013).

8See Heidhues and Kőszegi (2018) for a recent overview of the behavioral industrial organization literature.

9Relatively, Courty and Nasiry (2018) show that it can be optimal for a monopolist to charge the same price for products of varying qualities.
match value. If firms differ in their production costs, the presence of uninformed loss-averse consumers leads to more competition and lower prices. Karle and Müller (2019) examine the competitive effects with loss-averse consumers in an advance purchase setting. These papers are the closest to ours in the sense that they consider competitive settings.

Heidhues and Kőszegi (2014), Rosato (2016), and Karle and Schumacher (2017) study a monopolist’s optimal pricing and marketing strategies when consumers are expectations-based loss-averse. A monopolist can exploit these preferences by creating attachment through a sophisticated pricing strategy (as in Heidhues and Kőszegi 2014 or Rosato 2016) or through the revelation of partial match value information (as in Karle and Schumacher 2017). Attachment will not play a role in our setting as loss-averse consumers avoid any gain-loss sensations in our Bertrand equilibrium.

Very few papers analyze the implications of heterogeneity in expectations-based loss-aversion preferences for trade. Herweg and Mierendorff (2013) show that the optimal two-part tariff for loss-averse consumers frequently is a flat-rate tariff. Consumers prefer such a tariff to a measured tariff under which they would pay less in expectation. In an extension, they show that the monopolist can screen between consumers by offering a flat-rate and a measured tariff. The relatively more loss-averse consumers then choose the flat-rate tariff, while those with a lower degree of loss-aversion choose the measured tariff.10

To the best of our knowledge this paper is the first to study the implications of the uncertainty effect – instead of attachment – on competition. We explicitly allow consumers to stop information gathering if the expected payoff from information-sensitive purchase plans is below the utility of the certain option. Since the uncertainty effect occurs for modest degrees of loss-aversion, firms value the opportunity of having access to consumer at an early stage of the decision-making process. This pushes prices upwards and changes the equilibrium outcome significantly.

Search Costs, Directed Search, and Prominence. A large literature examines the market equilibrium when consumers have to invest into costly search to gather information about the available options.11 Diamond (1971) showed that firms selling a homogeneous good charge monopoly prices if consumers have to pay arbitrary small search costs. A more recent strand of the search literature examines directed search, i.e., the consumer does not encounter other options randomly, but can search particular firm. Arbatskaya (2007) analyzes ordered search and finds that prices and profits decline in the firms’ position. Such findings naturally raise the

10 Other applications of expectation-based loss-aversion include Carbajal and Ely (2016) and Hahn et al. (2018) on monopolistic screening; Herweg et al. (2010) on principal-agent contracts; Lange and Ratan (2010) and Dato et al. (forthcoming) on auctions and tournaments; Dato et al. (2017) on strategic interaction in finite games; and Daido and Murooka (2016) on team incentives.

11 See Baye et al. (2007) for a good survey of this literature.
question which firms are most willing to pay for “prominence,” that is being in a position where they are sampled first by consumers. Armstrong et al. (2009) study the effect of prominence in search markets and find that the firm with the highest quality has the highest willingness to pay for prominence. Similarly, Athey and Ellison (2011) and Chen and He (2011) find that in auctions for prominent positions on websites high-quality submit higher bids, which increases welfare.

The consumer search literature is not very specific about what mechanism creates search costs. Frequently, they are referred to as time costs or the resources needed to gather information. In our model, we use a psychological mechanism – expectations-based loss-aversion – that creates search costs endogenously, and that interacts with the firms’ strategic behavior. We add three insights to the search cost literature. First, our proposed mechanism does not depend on the specific application. It can be relevant both for internet search and for search in geographically structured markets. Second, there is a substantial amount on empirical evidence on loss-aversion (e.g., Kahneman and Tversky 1979, Tversky and Kahneman 1992), expectation-based reference-points (Abeler et al. 2011, Crawford and Meng 2011, Ericson and Fuster 2011, Pope and Schweitzer 2011, Gill and Prowse 2012, Karle et al. 2015), and on how the degree of loss-aversion varies in the population (e.g., von Gaudecker et al. 2011). Third, the scale of the search friction is tied to the scale of the transaction. For example, consider search for products that cost around 10 USD (books) and search for products that cost 1000 USD (insurance). In our model, if the uncertainty about quality and price is the same for both types of products (e.g., the price of the book is either 8 or 12 USD, and for insurance either 800 or 1200 USD), also the extent of the uncertainty effect is the same.

6 Conclusion

In this paper, we presented a new argument why consumers invest too little into finding the utility maximizing deal for them. We considered loss-averse consumers who can inspect vertically differentiated products. The highest possible consumer surplus may be small relative to the variance in product values/prices that characterizes a purchase-plan realizing this surplus. In this case, a loss-averse consumer may prefer to purchase a known product that only offers a small or no surplus. This uncertainty effect can occur for modest degrees of loss aversion. Our argument is independent of physical search or cognitive costs, and builds on the empirically tested model of expectation-based loss-aversion by Kőszegi and Rabin (2006, 2007). It naturally generalizes the prediction that consumers with lower degrees of loss-aversion are more likely to inspect their available options, and to end up with the optimal deal for them. Testing this prediction could be part of important future research.
References


Appendix

Proof of Lemma 3. The proof proceeds by steps. Step 1. We show that if \( c \) is small enough relative to \( \Delta \), then the following holds: In any equilibrium, for \( n - 1 \) firms it must be the case that those consumers who at the planning stage experience no positive utility shock at their assigned firm \( i \) (\( \xi_i = 0 \)) choose to inspect all other products. Assume by contradiction that there are two firms \( i, j \) so that consumers assigned to firm \( i \) do not inspect all products when \( v_i = v_{ik_i} \) and \( \xi_i = 0 \), while consumers assigned to firm \( j \) do not inspect all products when \( v_j = v_{jk_j} \) and \( \xi_j = 0 \). Without loss of generality we assume \( v_i \geq p_i \) for each firm \( i \). Denote by \( p_{ik_i} \) (\( p_{jk_j} \)) the price firm \( i \) (\( j \)) charges if \( v_i = v_{ik_i} \) (\( v_j = v_{jk_j} \)). Consider the following alternative plan for consumers assigned for firm \( i \) when \( v_i = v_{ik_i} \) and \( \xi_i = 0 \): Inspect all products, trade with firm \( j \) if \( v_j = v_{jk_j} \) and \( \xi_j = \Delta \); otherwise, trade with firm \( i \). This plan is weakly worse than the original plan only if

\[
v_{ik_i} - p_{ik_i} \geq \frac{3}{4}[v_{ik_i} - p_{ik_i}] + \frac{1}{4}[v_{jk_j} - p_{jk_j} + \Delta] - c.
\]

Accordingly, we must have

\[
v_{jk_j} - p_{jk_j} \geq \frac{3}{4}[v_{jk_j} - p_{jk_j}] + \frac{1}{4}[v_{ik_i} - p_{ik_i} + \Delta] - c.
\]  

(13)

(14)

These two inequalities taken together imply \( c \geq \frac{1}{4}\Delta \), a contradiction if \( c \) is small enough relative to \( \Delta \). Step 2. Since \( n \geq 3 \), the result from Step 1 implies that the share \( x \) of consumers who inspect all products in equilibrium is at least \( \frac{1}{3} \) when \( c \) is small enough relative to \( \Delta \). We study which price firm \( n \) charges in equilibrium. If \( c \) is small enough relative to \( \Delta \), we get the following result: If \( p_n > v_n - \max_{i \in [1,...,n-1]}(v_i - p_i) + \Delta \), the share of consumers firm \( n \) serves is zero; if \( p_n = v_n - \max_{i \in [1,...,n-1]}(v_i - p_i) + \varepsilon, \varepsilon \in (0,\Delta] \), the share of consumers firm \( n \) serves is at most \( \frac{5}{4} + \frac{3}{4}x \); if \( p_n = v_n - \max_{i \in [1,...,n-1]}(v_i - p_i) - \varepsilon, \varepsilon \in [0,\Delta) \), the share of consumers firm \( n \) serves is at most \( \frac{1}{n} + \frac{3}{4}x \); and if \( p_n = v_n - \max_{i \in [1,...,n-1]}(v_i - p_i) - \Delta \), the share of consumers firm \( n \) serves is \( \frac{1}{n} + x \). We use this to determine the optimal price \( p_n \). Note that \( v_n - \max_{i \in [1,...,n-1]}(v_i - p_i) \geq \Gamma \). If \( \frac{1}{8}\Gamma > \Delta \), the unique optimal price for firm \( n \) is \( p_n = v_n - \max_{i \in [1,...,n-1]}(v_i - p_i) - \Delta \), so that it prices all firms out of the market. By the assumption on firm 1 to \( n - 1 \)’s pricing strategy, we get \( p_n = v_n - v_{n-1} - \Delta \), which completes the proof.

Expected utility from a generic plan. Suppose the consumer adopts plan \( \sigma^A_\lambda \). Define by \( \pi(v + \xi) \) the corresponding probability that the consumer realizes utility \( v + \xi \). Note that the probability that the consumer pays the price \( p = v \) is then given by \( \pi(v) + \pi(v + \xi) \). The consumers
expected utility in the planning phase from this plan is then given by

\[
\mathbb{E}[\bar{U}(\sigma_{\lambda}^{[2]})] = \sum_{i=1}^{n-1} \left[ \pi(v_{i,l} + \Delta) + \pi(v_{i,h} + \Delta) \right] \Delta + \left[ \pi(v_{n,l}) + \pi(v_{n,h}) \right] \Delta \\
+ \left[ \pi(v_{n,l} + \Delta) + \pi(v_{n,h} + \Delta) \right] 2\Delta - (\lambda - 1) \tilde{A}(\sigma_{\lambda}^{[2]}) - (\lambda - 1) \tilde{B}(\sigma_{\lambda}^{[2]}),
\]

(15)

where \( \tilde{A}(\sigma_{\lambda}^{[2]}) \) captures gain-loss sensations in the product dimension,

\[
\tilde{A}(\sigma_{\lambda}^{[2]}) = \sum_{i=1}^{n-1} \pi(v_{i,l}) \left[ \sum_{j=i+1}^{n} \left( \pi(v_{j,l}) + \pi(v_{j,l} + \Delta) \right) \right] \\
+ \sum_{i=1}^{n-1} \pi(v_{i,l} + \Delta) \left[ \sum_{j=i+1}^{n} \left( \pi(v_{j,l} - v_{i,l}) + \pi(v_{j,l} - v_{i,l} + \Delta) \right) \right] \\
+ \sum_{i=1}^{n-1} \pi(v_{i,h}) \left[ \sum_{j=i+1}^{n} \left( \pi(v_{j,l} - v_{i,l}) + \pi(v_{j,l} + \Delta) \right) \right] \\
+ \sum_{i=1}^{n-1} \pi(v_{i,h} + \Delta) \left[ \sum_{j=i+1}^{n} \left( \pi(v_{j,l} - v_{i,l} - \Delta) + \pi(v_{j,l} - v_{i,l}) \right) \right] \\
+ \sum_{i=1}^{n-1} \pi(v_{i,l}) \left[ \sum_{j=i+1}^{n} \left( \pi(v_{j,l} + \Delta) + \pi(v_{j,l} + \Delta - v_{i,l}) \right) \right] \\
+ \sum_{i=1}^{n-1} \pi(v_{i,h}) \left[ \sum_{j=i+1}^{n} \left( \pi(v_{j,l} - v_{i,l} - \Delta) + \pi(v_{j,l} - v_{i,l}) \right) \right] \\
+ \sum_{i=1}^{n-1} \pi(v_{i,h} + \Delta) \left[ \sum_{j=i+1}^{n} \left( \pi(v_{j,l} - v_{i,l}) + \pi(v_{j,l} - v_{i,l} + \Delta) \right) \right] \\
+ \pi(v_{n,l}) \left[ \pi(v_{n,h} + \Delta) \right] \Delta + \pi(v_{n,h}) \Gamma + \pi(v_{n,h} + \Delta) \Gamma \\
+ \pi(v_{n,l} + \Delta) \left[ \pi(v_{n,h}) (\Gamma - \Delta) + \pi(v_{n,h} + \Delta) \Gamma \right] - (\lambda - 1) \pi(v_{n,h}) \pi(v_{n,h} + \Delta) \Delta,
\]

(16)
while $\tilde{B}(\sigma^{(2)}_d)$ captures gain-loss sensations in the price dimension,

$$\tilde{B}(\sigma^{(2)}_d) = \sum_{i=1}^{n-1} [\pi(v_{i.i}) + \pi(v_{i,i} + \Delta)] \times$$

$$\times \left[ \sum_{j=1}^{n-1} [\pi(v_{j,i}) + \pi(v_{j.i} + \Delta)](v_{j,i} - v_{i,i}) + [\pi(v_{i,i}) + \pi(v_{i,i} + \Delta)](v_{i,i} - v_{i,i} - \Delta) + \sum_{j=1}^{n-1} [\pi(v_{j,i}) + \pi(v_{j,i} + \Delta)](v_{j,i} - v_{i,i}) + [\pi(v_{i,i}) + \pi(v_{i,i} + \Delta)](v_{i,i} - v_{i,i} - \Delta) \right]$$

$$\times \left[ \sum_{j=1}^{n-1} [\pi(v_{j,i}) + \pi(v_{j,i} + \Delta)](v_{j,i} - v_{i,i}) + [\pi(v_{i,i}) + \pi(v_{i,i} + \Delta)](v_{i,i} - v_{i,i} - \Delta) \right]$$

$$+ \sum_{i=1}^{n-1} [\pi(v_{i,i}) + \pi(v_{i,i} + \Delta)] \times$$

$$\times \left[ \sum_{j=1}^{n-1} [\pi(v_{j,i}) + \pi(v_{j,i} + \Delta)](v_{j,i} - v_{i,i}) + [\pi(v_{i,i}) + \pi(v_{i,i} + \Delta)](v_{i,i} - v_{i,i} - \Delta) \right]$$

$$+ [\pi(v_{i,i}) + \pi(v_{i,i} + \Delta)] [\pi(v_{i,i}) + \pi(v_{i,i} + \Delta)] \Gamma. \quad (17)$$

We use this expression implicitly in the subsequent proofs.

\[ \square \]

**Proof of Lemma 4.** The proof proceeds by steps. In Step 1, we show the statement when the consumer has no positive utility shock at the assigned firm $i^*$. In Step 2, we show the statement when there is a positive utility shock. **Step 1.** Consider first the case when the consumer experiences no positive utility shock at the assigned firm, $\xi_{i^*} = 0$. Suppose the consumer inspects all products and adopts plan $\sigma^{(2)}_d$. Define by $\pi(v + \xi)$ the corresponding probability that the consumer realizes utility $v$. We find an upper bound on the expected utility form this plan. Define for $\pi = (\pi_0, ..., \pi_6)$

$$\mathbb{E}[U(\pi)] = [\pi_1 + \pi_2 + \pi_3 + \pi_5] \Delta + [\pi_4 + \pi_6] 2\Delta$$

$$- (\lambda - 1)[\pi_0 \pi_1 + \pi_0 \pi_2 + \pi_1 \pi_2] (2\Gamma - \Delta)$$

$$- (\lambda - 1)[\pi_0 + \pi_1 + \pi_2][\pi_3 (4\Gamma - \Delta) + \pi_4 4\Gamma + \pi_5 (6\Gamma - \Delta) + \pi_6 6\Gamma]$$

$$- (\lambda - 1) \pi_3 [\pi_4 \Delta + \pi_5 2\Gamma + \pi_6 (2\Gamma + \Delta)]$$

$$- (\lambda - 1) \pi_4 [\pi_5 (2\Gamma - \Delta) + \pi_6 2\Gamma] - (\lambda - 1) \pi_5 \pi_6 \Delta. \quad (18)$$

Note that for any plan $\sigma^{(2)}_d$ we can find numbers $\pi_1, \pi_2 \in [0, \frac{1}{2}]$ and some $i \in \{1, ..., n\} \setminus \{i^*\}$ so that $\mathbb{E}[U(\pi)]$ weakly exceeds the expected utility from plan $\sigma^{(2)}_d$ when $\pi_0 = \pi(v_{i,i})$, $\pi_3 = \pi(v_{i,i})$, $\pi_4 = \pi(v_{i,i} + \Delta)$, $\pi_5 = \pi(v_{i,i})$, $\pi_6 = \pi(v_{i,i} + \Delta)$. Moreover, we can then find numbers $\pi_i^j, \pi_i^j, \pi_i^j$ for $i \in \{1, 2\}$ and $j \in \{3, 4, 5, 6\}$ so that $\pi_i^j \in [0, \frac{1}{2}]$ and $\pi_i^j \in [0, \frac{1}{8}]$ for all $i, j$, as well as
Suppose the consumer inspects all products and adopts plan
the case when the consumer experiences a positive utility shock at the assigned firm, \( \xi \), and loses at least \( u \) and chooses any other option. Compared to any possible utility-price pair
plan “always trade with firm \( n \)”, since
Using a similar argument, we can show that at a global maximum we must have, for all
that has the property mentioned above. Consider the following variation in
\( \pi = \pi_0 + \epsilon_1 \), \( \pi_1 = \pi'_1 - \epsilon_1 \), and \( \pi_2 = \pi'_2 - \epsilon_2 \). We then get
Thus, at a global maximum we must have either \( \pi_0 = 0 \), or \( \pi_1 = 0 \), or \( \pi_2 = 0 \), or \( \pi_1 = \pi_2 = 0 \).
Using a similar argument, we can show that at a global maximum we must have, for all \( j \in \{3, ..., 6\} \), that \( \pi^j_1 = \frac{1}{8} \) or \( \pi^j_2 = 0 \). Thus, we obtain a finite set of “plans” \( \pi \) that we can compare to each other in terms of \( E[U(\pi)] \). From this comparison, we get the following result. For \( \pi^{(1)} = (0, 0, 0, 0, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}) \) we get

\[
E[U(\pi^{(1)})] = \frac{3}{2} \Delta - (\lambda - 1) \left( \frac{1}{2} \Gamma - (\lambda - 1) \frac{1}{8} \Delta \right).
\]

This is also the expected payoff from the plan “always trade with firm \( n \).” Since \( \Gamma > 2\Delta \), this value exceeds \( E[U(\pi')] \) for all alternative admissible values \( \pi' \), regardless of \( \lambda > 1, \Gamma > 0 \), and \( \Delta \), except for \( \pi^{(2)} = (\frac{7}{8}, 0, 0, 0, \frac{1}{8}, 0, 0) \) with

\[
E[U(\pi^{(2)})] = \frac{1}{4} \Delta - (\lambda - 1) \frac{7}{16} \Gamma;
\]

and for \( \pi^{(3)} = (\frac{7}{8}, 0, 0, \frac{1}{8}, 0, 0, 0, 0) \) with

\[
E[U(\pi^{(3)})] = \frac{1}{8} \Delta - (\lambda - 1) \frac{7}{64} \Gamma + (\lambda - 1) \frac{7}{64} \Delta.
\]

Note that if \( E[U(\pi^{(1)})] \geq 0 \), then \( E[U(\pi^{(1)})] > E[U(\pi^{(2)})] \) and \( E[U(\pi^{(1)})] > E[U(\pi^{(3)})] \), and that \( E[U(\pi^{(2)})] < 0 \) and \( E[U(\pi^{(3)})] < 0 \) when \( E[U(\pi^{(1)})] < 0 \). It remains to show that the plan “always trade with firm \( n \)” is a personal equilibrium. Suppose that the consumer deviates and chooses any other option. Compared to any possible utility-price pair \( u_n, p_n \), the consumer then loses at least \( z \Gamma - \Delta \), \( z \in \mathbb{N}_+ \), in the product dimension, and gains \( z \Gamma - \Delta \) in the price dimension. Since \( \lambda > 1 \) and there are no gains from deviation in terms of consumption utility, the plan “always trade with firm \( n \)” is a personal equilibrium. By the argument above it is also the consumer’s preferred personal equilibrium if \( E[U(\pi^{(1)})] \geq 0 \). **Step 2.** Consider next the case when the consumer experiences a positive utility shock at the assigned firm, \( \xi'_n = \Delta \). Suppose the consumer inspects all products and adopts plan \( \sigma^{(2)}_A \). Again, define by \( \pi(v + \xi) \) the
corresponding probability that the consumer realizes utility $v + \xi$. We find an upper bound on the expected utility form this plan. Define for $\pi = (\pi_0, ..., \pi_4)$

$$\mathbb{E}[U(\pi)] = [\pi_0 + \pi_1 + \pi_3] \Delta + [\pi_2 + \pi_4] 2\Delta$$

$$- (\lambda - 1)\pi_0[\pi_1(2\Gamma - \Delta) + \pi_2 2\Gamma + \pi_3(4\Gamma - \Delta) + \pi_4 4\Gamma]$$

$$- (\lambda - 1)\pi_1[\pi_2 \Delta + \pi_3 2\Gamma + \pi_4(2\Gamma + \Delta)]$$

$$- (\lambda - 1)\pi_2[\pi_3(2\Gamma - \Delta) + \pi_4 2\Gamma] - (\lambda - 1)\pi_3 \pi_4 \Delta$$

Note that, for any plan $\sigma_A^{[2]}$, $\mathbb{E}[U(\pi)]$ weakly exceeds the expected utility from plan $\sigma_A^{[2]}$ when $\pi_1 = \pi(v_{i,l})$, $\pi_2 = \pi(v_{i,l} + \Delta)$, $\pi_3 = \pi(v_{i,h})$, $\pi_4 = \pi(v_{i,h} + \Delta)$, and $\pi_0 = 1 - \sum_{i=1,4} \pi_i$. We find a global maximum of $\mathbb{E}[U(\pi)]$ subject to the constraint that $\pi_i \in [0, \frac{1}{4}]$ for $i = 1, ..., 4$ and $\pi_0 = 1 - \sum_{i=1,4} \pi_i$. With a similar argument as used in Step 1, we can show that at a global maximum we must have, for all $i = 1, ..., 4$, that $\pi_i = \frac{1}{4}$ or $\pi_i = 0$. Thus, we obtain a finite set of $\pi$’s that we can compare to each other in terms of $\mathbb{E}[U(\pi)]$. From this comparison we get the following result. For $\pi^{[1]} = (0, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ we get

$$\mathbb{E}[U(\pi^{[1]})] = \frac{3}{2} \Delta - (\lambda - 1) \frac{1}{2} \Gamma - (\lambda - 1) \frac{1}{8} \Delta. \quad (24)$$

This is the expected payoff from the plan “always trade with firm $n$.” Since $\Gamma > 2\Delta$, this value exceeds $\mathbb{E}[U(\pi')]$ for all alternative admissible values $\pi'$, regardless of $\lambda > 1$, $\Gamma > 0$, and $\Delta$, except for $\pi^{[2]} = (\frac{1}{4}, 0, \frac{1}{4}, 0, 0)$ with

$$\mathbb{E}[U(\pi^{[2]})] = \frac{5}{4} \Delta - (\lambda - 1) \frac{3}{8} \Gamma. \quad (25)$$

This is the expected payoff from the plan “trade with firm $n$ if $u_n = v_{n, l} + \Delta$ and with firm $i'$ otherwise” when $i' = n - 1$ and $v_{n-1} = v_{n-1,h}$. Note that if $\mathbb{E}[U(\pi^{[1]}))] \geq \Delta$ then $\mathbb{E}[U(\pi^{[1]}))] \geq \mathbb{E}[U(\pi^{[2]}))$, and that $\mathbb{E}[U(\pi^{[2]}))] < \Delta$ when $\mathbb{E}[U(\pi^{[1]}))] < \Delta$. As above, we can show that in this case the plan “always trade with firm $n$” is a personal equilibrium, and the preferred personal equilibrium when $\mathbb{E}[U(\pi^{[1]}))] \geq \Delta$. This completes the proof. \hfill \Box

**Proof of Proposition 1.** The proof proceeds in steps. **Step 1.** We take the firms’ pricing strategy $p_i = v_i$ for each firm $i$ as given and consider the consumer. Suppose she adopts the plan $\sigma_A^{[2]}$. Using the notation introduced above, the consumer’s expected utility in the planning phase from this plan is given by

$$\mathbb{E}[U(\sigma_A^{[2]}))] = \sum_{i=1}^{n} [\pi(v_{i,l} + \Delta) + \pi(v_{i,h} + \Delta)] \Delta - (\lambda - 1) A(\sigma_A^{[2]})) - (\lambda - 1) B(\sigma_A^{[2]}), \quad (26)$$
where \( A(\sigma^{[2]}_A) = \bar{A}(\sigma^{[2]}_A) \); \( B(\sigma^{[2]}_A) \) is identical to \( \bar{B}(\sigma^{[2]}_A) \) except that \( p_n - p_i = v_n - v_i \) instead of \( p_n - p_i = v_n - v_i - \Delta \), for all \( i \neq n \), so that \( B(\sigma^{[2]}_A) \geq \bar{B}(\sigma^{[2]}_A) \). We therefore have \( \mathbb{E}[\bar{U}(\sigma^{[2]}_A)] \geq \mathbb{E}[U(\sigma^{[2]}_A)] \) for any plan \( \sigma^{[2]}_A \). Suppose that a consumer assigned to firm \( i^* \) experiences a positive utility shock. For this consumer it is clearly optimal not to inspect all products and to trade with firm \( i^* \). Next, suppose that a consumer assigned to firm \( i^* \) experiences no positive utility shock. Recall the plan “always trade with firm \( n^* \)” for a consumer assigned to a firm \( i^* \neq n \), denoted by \( \hat{\sigma}^{[2]}_A \). From the proof of Lemma 4 we know that \( \mathbb{E}[\hat{U}(\hat{\sigma}^{[2]}_A)] \geq \mathbb{E}[U(\sigma^{[2]}_A)] \) for all plans \( \sigma^{[2]}_A \) if \( \mathbb{E}[\hat{U}(\hat{\sigma}^{[2]}_A)] \geq 0 \). From the statement above it thus follows that if (9) holds, it is optimal for the consumer not to inspect all products and to trade with firm \( i^* \). Step 2. Consider any firm \( i \) and take the consumer’s strategy as given. We specify that the consumer’s beliefs about products \( j \neq i^* \) are independent of \( p_r \). Firm \( i^* \)’s profit from charging \( p_i = v_i \) equals \( \frac{1}{n} v_i \); its profit from charging a price \( p_i > v_i \) is at most \( \frac{1}{n} \left( v_i + \Delta \right) \). Since \( v_i \geq \Gamma \) and \( \frac{1}{2} \Gamma \geq \Delta \), this deviation is not profitable. Clearly, it also does not pay off to charge a price \( p_i < v_i \), which completes the proof.

**Proof of Proposition 2.** We specify that each firm \( i \neq n \) charges \( p_i = v_i \) and firm \( n \) charges \( p_n = v_n - \Delta - \max_{i \neq n} (v_i - p_i) \). Lemma 4 implies that for all loss-averse consumers it is optimal to trade with their assigned firm \( i^* \) if \( p_r = v_r \) and \( \Delta \) is sufficiently small relative to \( \Gamma \); loss-neutral consumers always inspect all products and trade with a firm \( i \) if firm \( i^* \)’s product maximizes consumption utility (in case of a tie, they trade with the firm that offers the highest product value). Consider a loss-averse consumer who observes that her assigned firm \( i^* \) deviates from the equilibrium strategy and charges \( p_i = v_i + \varepsilon \) for some \( \varepsilon > 0 \). Given firm \( n^* \)’s response to this deviation, the trade-off between the plans “always trade with firm \( i^* \)” and “always trade with firm \( n^* \)” is unaffected. Thus, if \( \Delta \) is sufficiently small for given \( \lambda^* \) and \( \Gamma \), loss-averse consumers assigned to firm \( i^* \) adopt the former plan. It remains to show that no firm has an incentive to deviate from the prescribed pricing strategy, given the consumers’ behavior. Suppose that firm \( i \neq n \) unilaterally deviates and charges \( p_i = v_i - \varepsilon \). If firm \( n \) then charges \( p_n = v_n \), the share of consumers it serves is \( (1 - \beta)^\frac{1}{n} \); if it charges \( p_n = v_n + \varepsilon - \Delta \), this share is \( (1 - \beta)^\frac{1}{n} + \frac{1}{n} \beta \); if it charges \( p_n = v_n - \varepsilon \), this share is \( (1 - \beta)^\frac{1}{n} + \frac{3}{4} \beta \); and if it charges \( p_n = v_n - \varepsilon + \Delta \), this share is \( (1 - \beta)^\frac{1}{n} + \beta \). All other prices are not optimal for firm \( n \). We show that if \( \Delta \) is small enough for given \( \beta \) and \( \Gamma \), it is optimal for firm \( n \) to charge \( p_n = v_n - \varepsilon - \Delta \) (so that firm \( i \) does not benefit from its deviation) or it is not optimal for firm \( i \) to charge \( p_i = v_i - \varepsilon \). This is done in steps.

**Step 1.** Assume by contradiction that charging \( p_n = v_n \) is optimal for firm \( n \). We then must have

\[
(v_n - \varepsilon - \Delta) \left( (1 - \beta)^\frac{1}{n} + \beta \right) \leq v_n (1 - \beta)^\frac{1}{n}.
\]  

(27)
If the deviation is profitable for firm $i$, we must have

$$(v_i - \varepsilon)\left(1 - \beta\frac{1}{n} + \beta\right) > v_i(1 - \beta)\frac{1}{n}. \quad (28)$$

The first inequality is equivalent to $v_n\beta \leq (\varepsilon + \Delta)((1 - \beta)\frac{1}{n} + \beta)$, the second inequality is equivalent to $v_i\beta > (\varepsilon + \Delta)((1 - \beta)\frac{1}{n} + \beta)$. Note that $v_n \geq v_i + \Gamma$. Thus, if $\Delta$ is small enough for given $\beta$ and $\Gamma$, we obtain a contradiction. **Step 2.** Assume by contradiction that charging $p_n = v_n - \varepsilon + \Delta$ is optimal for firm $n$. We then must have

$$(v_n - \varepsilon - \Delta)\left(1 - \beta\frac{1}{n} + \beta\right) \leq (v_n - \varepsilon + \Delta)\left(1 - \beta\frac{1}{n} + \frac{1}{4}\beta\right). \quad (29)$$

We can rewrite this inequality as

$$(v_n - \varepsilon)\frac{3}{4}\beta \leq 2\Delta(1 - \beta)\frac{1}{n} + \frac{5}{4}\Delta\beta. \quad (30)$$

If the deviation is profitable for firm $i$, we must have $\varepsilon \leq v_i \leq v_n - \Gamma$. Hence, the inequality in (30) is violated if $\Delta$ is small enough for given $\beta$ and $\Gamma$, which implies a contradiction. Using a similar argument, we can show the same for the price $p_n = v_n - \varepsilon$, which completes the proof. \hfill \Box