Balanced scorecards: a relational contract approach

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Abstract

Reward systems based on balanced scorecards typically connect pay to an index, i.e. a weighted sum of multiple performance measures. However, there is no formal incentive model that actually describe this kind of index contracts as an optimal solution. In this paper, we show that an index contract may indeed be optimal if performance measures are non-verifiable so that the contracting parties must rely on self-enforcement. The optimal self-enforcing (relational) contract between a principal and one multitasking agent turns out to be an index contract where the agent gets a bonus if a 'weighted sum' of performance outcomes on the various tasks (the index) exceeds a hurdle. For a parametric (multinormal) specification, the efficiency of the index contract depends on the correlation between the tasks, where negative correlation improves incentives, since it reduces the variance of the performance measure. For a similar reason, the principal may also want to include verifiable tasks in the relational index contract in order to improve incentives.

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1 Introduction

Very few jobs can be measured along one single dimension; employees usually multitask. This creates challenges for incentive providers: If the firm only rewards a subset of dimensions or tasks, employees will have incentives to exert efforts only on those tasks that are rewarded, and ignore the others. A solution for the firm is to add more metrics to the compensation scheme, but this usually imply some form of measurement problems, leading either to more noise or distortions, or to the use of non-verifiable (subjective) performance measures.

The latter is often implemented by the use of a balanced scorecard (BSC). Kaplan and Norton’s (1992, 1996) highly influential concept began with a premise that exclusive reliance on verifiable financial performance measures was not sufficient, as it could distort behavior and promote effort that is not compatible with long-term value creation. Their main ideas were indebted to the canonical multitasking models of Holmström and Milgrom (1991) and Baker (1992). However, their approach was more practical, guiding firms in how to design performance measurement systems that focus not only short-term financial objectives, but also on long-term strategic goals (Kaplan and Norton, 2001).

While measuring performance is one thing, the question of how to reward performance is a different one. As noted by Budde (2007), there is a general understanding that efficient incentives must be based on multiple performance measures. Still, the implementation is a matter of controversy. Reward systems based on BSC typically connect pay to an index, i.e. a weighted sum of multiple performance measures. However, there is no formal incentive model that actually describe this kind of index contracts as an optimal solution. In fact, Kaplan and Norton (1996) were sceptical to compensation formulas that calculated incentive compensation directly to a sum of weighted metrics. Rather they proposed to establish different bonuses for a whole set of critical performance measures, more in line with the original ideas of Holmström and Milgrom (1991) and Feltham and Xie (1994).

Despite the huge literature following the introduction of BSC (see Hoque, 2014, for a review), and the massive use of the scorecard in practice, the
index contracts that BSC-firms often prescribes, lacks a formal contract theoretic justification. ¹ This paper aims to fill this gap. Our starting point is that the performance measures are non-verifiable. This means that the incentive contract cannot be enforced by a third party and thus needs to be self-enforcing - or what is commonly termed “relational”. In the now large literature on self-enforcing relational contracts, it is mostly assumed that agents exert effort along one dimension. Only a few papers have considered relational contracts with multitasking agents (Baker, Gibbons and Murphy, 2002; Budde, 2007, Schottner, 2008; Mukerjee and Vasconcelos, 2011; and Ishihara, 2016). We generalize and extend existing literature to the case of n tasks, and allow performances on each task to correlate. To make the model tractable, we focus on multinormal distribution.²

We first show that the optimal relational contract between a principal and one multitasking agent turns out to be an "index contract", or what one may call a balanced scorecard. That is, the agent gets a bonus if a 'weighted sum' of performance outcomes on the various tasks (an index) exceeds a hurdle. This in contrast to Holmström and Milgrom (1991) where the agent gets a bonus on each task. The important difference from Holmström and Milgrom is that we consider a relational contracting setting where the size of the bonus is limited by the principal’s temptation to renege (rather than risk considerations). In such a setting the marginal incentives to exert effort on each task is higher with index contracts than with bonuses on each task.

A common feature of the BSC is that the performance measures within a scorecard can be very different from each other. This implies that the measures can be exposed to different sets of noisy signals and thus potentially be negatively correlated with each other. We show that this can be efficiency enhancing: The index contract works even better if the measures are negatively correlated. The reason is that negative correlation reduces

¹According to Hoque (2014), among the more than 100 papers published on BCS theory, only a handful have used principal agent theory to analyze BSC. See also Hesford et al (2009) for a review.

²Our paper is indebted to the seminal literature on relational contracts. The concept of relational contracts was first defined and explored by legal scholars (Macaulay, 1963, Macneil, 1978), while the formal literature started with Klein and Leffler (1981). MacLeod and Malcomson (1989) provides a general treatment of the symmetric information case, while Levin (2003) generalizes the case of asymmetric information. The relevance of the relational contract approach to management accounting and performance measurement is discussed in Glover (2012) and Baldenius et al. (2016).
the variance of the performance measure. This is beneficial not because a
more precise measure reduces risk (since agent is risk neutral), but because
it strengthens, for any given bonus level, the incentives for the agent to
provide effort.\textsuperscript{3}

We also consider the case where some measures are verifiable, and some are
not. We show that the principal will include verifiable measures in the rela-
tional index contract in order to strengthen incentives. This resembles the
balance scorecard often seen in practice, including both verifiable measures
such as sales, and non-verifyable measures such as supervisors’ quality as-
sessments (see e.g. Kaplan and Norton, 2001). By including a verifiable task
in the relational contract, the variance of the performance index may be re-
duced, which again strengthens incentives. We also show that performance
on the verifiable task is taken into the index as a benchmark, to which the
other performances are compared. Moreover, the principal will still offer an
explicit bonus contract on the verifiable task, but this bonus is a function
of the optimal relational index contract.\textsuperscript{4}

Finally, we extend the model to analyze multiple agents. Like in the sin-
gle agent case, we show that the optimal contract is an index contract,
where the bonus is based on a weighted sum of performance outcomes on
the various tasks. However, only one agent is paid a bonus, given that the
performance index exceeds a hurdle. Hence, the optimal contract is a tour-
nament scheme based on these indexes, and is thus similar to Levin’s (2002)
optimal relational contract in the single-task case.

The rest of the paper is organized as follows: In section 2 we present the
model and the main result. In Section 3 we extend the model to include
both verifiable and non-verifiable performance measures, while in Section 4
we extend to multiple agents. Section 5 concludes.

\textsuperscript{3}Similar positive effects of negative correlation are shown in Kvaløy and Olsen (2018),
who analyzes relational contracts and correlated performances in a model with multiple
agents, but single tasks.

\textsuperscript{4}Our model thus complement the influential papers by Baker, Gibbons and Murphy
(1994) and Schmidt and Schnitzer (1995) on the interaction between relational and explicit
contracts. While their results are driven by differences in fallback options created by the
explicit contracts, our results stems from correlation between the tasks.
2 Model

First we present the basic model between a principal and one multitasking agent. Consider an ongoing economic relationship between a risk neutral principal and a risk neutral agent who performs $n$ tasks with $n$ efforts, generating a joint distribution of outputs $f(x_1...x_n; e_1...e_n)$. Given observable (but not verifiable) outputs, the agent is promised a bonus $\beta(x_1...x_n)$ from the principal. Let here $v(e_1...e_n)$ denote the gross value of efforts for the principal, e.g. $v(e_1...e_n) = E(\Sigma_i x_i | e_1...e_n)$ The agent has private cost $c(e)$ that is strictly increasing and convex in effort. The agent’s IC condition for efforts $e = (e_1...e_n)$ is then

$$e \in \arg\max_{e'} E(\beta(x) | e') - c(e')$$

with first-order conditions

$$0 = \frac{\partial}{\partial e_i} E(\beta(x)|e) - c_i(e) = \int \beta(x) f_{e_i}(x,e) - c_i(e), \quad i = 1,...n$$

The total surplus (per period) in the relationship is now $v(e) - c(e)$.

We assume trigger strategies and stationary contracts. The parties honor the contract only if both parties honored the contract in the previous period, and the dynamic enforceability condition is then

$$0 \leq \beta(x) \leq \frac{\delta}{1 - \delta} (v(e) - c(e)).$$

The optimal relational contract maximizes the surplus $v(e) - c(e)$ subject to this constraint and IC.

A standard approach to solve this kind of problem is to replace the global incentive constraint for the agent with the local first-order conditions. Assuming that this first order approach (FOA) is valid, we have an optimization problem that is linear in the bonuses $\beta(x)$. The optimal bonuses will then have a bang-bang structure, and hence be either maximal or minimal, depending on the outcome $x$. Introducing the likelihood ratios

$$l_{e_i}(x, e) = f_{e_i}(x, e)/f(x, e),$$
we obtain the following:

**Lemma 1** There is a vector of multipliers $\mu$ such that the optimal bonus is maximal for those outcomes $x$ where $\Sigma_i \mu_i l_{e_i}(x, e) > 0$, and it is zero otherwise, i.e.

$$\beta(x) = \frac{\delta}{1 - \delta} (v(e) - c(e)) \quad \text{if} \quad \Sigma_i \mu_i l_{e_i}(x, e) > 0,$$

and $\beta(x) = 0$ if $\Sigma_i \mu_i l_{e_i}(x, e) < 0$.

The lemma says that there is an index $y(x) = \Sigma_i \mu_i l_{e_i}(x, e)$ such that the agent should be paid a bonus if and only if this index is positive, and the bonus should then be maximal. This index, which takes the form of a weighted sum of the likelihood ratios for the various tasks, is in this sense an optimal performance measure for the agent.

In most of the paper we will assume a *multinormal distribution*, so that the output vector $x$ is multinormal with expectation (vector) $e$ and covariance matrix $\Sigma = [s_{ij}]$ (i.e. $x \sim N(e, \Sigma)$). Then we find (due to the likelihood ratio for this distribution being linear) that $\Sigma_i \mu_i l_{e_i}(x, e) = \Sigma_j \tau_j (x_j - e_j)$. So we have:

**Proposition 1** In the multinormal case, there is a vector $\tau$ and a performance index $y = \Sigma_j \tau_j x_j$ such that the agent is optimally paid a bonus if and only if the index exceeds a hurdle ($y_0$). The hurdle is given by the agent’s expected performance in this setting ($y_0 = \Sigma_j \tau_j e_j$), and the bonus, when paid, is maximal: $\beta(x) = \frac{\delta}{1 - \delta} (v(e) - c(e))$.

This result parallels Levin’s (2003) characterization of the single-task case. However, in the multitask case, the principal offers an index $y = \Sigma_j \tau_j x_j$, i.e. a ‘weighted sum’ of performance outcomes on the various tasks, such that the agent gets a bonus if and only if this index exceeds a hurdle $y_0$. The optimal hurdle is defined as the similar weighted sum of optimal efforts. Hence, performance $x_i$ is compared to expected performance, given (equilibrium) efforts. If the weighted sum of performances exceeds what is expected, then the agent obtain the bonus.
Consider now efforts. Given the index \( y \) with hurdle \( y_0 \), and the bonus \( \beta = b \) being paid for \( y > y_0 \), the agent’s (performance related) payoff is

\[
u(b, e) = b \Pr(y > y_0 | e) - c(e) = b \Pr(\sum_j \tau_j x_j > y_0 | e) - c(e)\]

Using the normal distribution we find that the agent’s first order conditions for efforts at their equilibrium levels then satisfy

\[
b \phi_0 \frac{\tau_i}{\sigma} - c_i(e) = 0, \quad i = 1 \ldots n, \tag{1}\]

where \( \phi_0 = 1/\sqrt{2\pi} \) is a parameter of the distribution, and \( \sigma \) is the standard deviation of the performance index:

\[
\sigma = SD(y) = (\tau^\prime \Sigma \tau)^{1/2}.\]

The optimal bonus paid for qualifying performance is the maximal one, so

\[
b = \frac{\delta}{1 - \delta} (v(e) - c(e))\]

Some straightforward algebraic manipulations now show that these equations imply the following

\[
((\nabla c(e))^\prime \Sigma (\nabla c(e)))^{1/2} / \phi_0 = b = \frac{\delta}{1 - \delta} (v(e) - c(e)) \tag{2}\]

Optimal effort \( e \) must thus satisfy (2). It is clear that among the effort vectors \( e \) that satisfy this relation, the optimal one yields highest surplus \( v(e) - c(e) \). The optimal effort vector must therefore solve \( \max(v(e) - c(e)) \) s.t. (2). In fact, since the last equality in (2) reflects the dynamic enforcement constraint, we can replace it by weak inequality, and thus state the following result

**Proposition 2** In the multinormal case, the optimal solution is given by the following:

\[
\max(v(e) - c(e))
\]

s.t.

\[
\frac{\delta \phi_0}{1 - \delta} (v(e) - c(e)) \geq ((\nabla c(e))^\prime \Sigma (\nabla c(e)))^{1/2}
\]
Consider now how the variances and covariances (in $\Sigma$) affect the solution. The expression on the RHS of the constraint represents the standard deviation ($\tilde{\sigma}$) of a variable $\tilde{y} = \tilde{\tau}'x$, with $\tilde{\tau} = \nabla c(e)$. If can be written as $\Sigma_i\Sigma_j s_{ij}c_i(e)c_j(e)$, where $s_{ij} = \text{cov}(x_i, x_j)$. It is quite clear that any variation in $\Sigma$ that increases this expression will tighten the constraint, and hence reduce the total surplus. In particular, any increase of a variance or a covariance in $\Sigma$ will have this effect, and hence reduce the surplus.

These effects are illustrated in the figures below (for the case of two tasks). The figures depict curves that represent the constraint and iso-surplus combinations, respectively.

The three cases illustrated are all with costs $\frac{1}{2}(e_1^2 + e_2^2)$ and value $v(e) = p_1e_1 + p_2e_2$. In case 1 (Fig. 1) the correlation between the tasks are zero. Effort is lower on task 2 due to higher variance, and thus lower marginal incentives on that task. In Fig. 2 we vary the correlation. Case 2 has positive correlation, while case 3 has negative correlation. Effort is much lower in the former case. In the latter, first best is achieved. 5

**Remark.** It is of some interest to compare the result above to the Holmstrom-Milgrom (1991) multitask model for verifiable outputs. In that model the agent is offered a linear incentive scheme $b'x + \alpha$, and the IC constraint takes the form $b = \nabla c(e)$. The total surplus (in certainty equivalents) is

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5CASE 1: $p_1 = 1$, $p_2 = 1$, $s_1 = 1$, $s_2 = 2$, $\rho = 0$. $d \equiv \frac{\delta(0)}{1-\delta} = \frac{3}{2}$ i.e $\delta = 0.79$.

Solution: $e_1 = .8$, $e_2 = .45$.

CASE 2: $\rho = 0.5$, $p_1 = 1$, $p_2 = 1$, $s_1 = 1$, $s_2 = 1$, $\rho = 0.5$. $d = 1$ i.e. $\delta = 0.71$.

Solution: $e_1 = e_2 = .27$.

CASE 3: $\rho = -0.5$, $p_1 = 1$, $p_2 = 1$, $s_1 = 1$, $s_2 = 1$, $\rho = -0.5$, $d = 1$ i.e. $\delta = 0.71$. FB ($e_1 = e_2 = 1$) can be attained.
then \( v(e) - c(e) - \frac{\gamma}{2}(\nabla c(e))'\Sigma(\nabla c(e)) \), where the last term captures risk costs, given by \( \frac{\gamma}{2} \text{var}(b'x) \). There is thus some similarity between this model and the relational contract model outlined here. In fact, the Lagrangian for the maximization problem in the last proposition can be written as

\[
(v(e) - c(e))(1 + \lambda) - \frac{\lambda - \delta}{\delta \delta_0} ((\nabla c(e))'\Sigma(\nabla c(e)))^{1/2},
\]

where \( \lambda \) is the shadow price on the constraint, hence the optimal solution maximizes \( v(e) - c(e) - \psi ((\nabla c(e))'\Sigma(\nabla c(e)))^{1/2} \), where \( \psi = \frac{\lambda - \delta}{1 + \lambda \delta \delta_0} \) can be seen as an (endogenous) cost factor.

### 3 Multitasking with non-verifiable and verifiable tasks

Consider now a situation where the agent has one verifiable and \( n = 2 \) non-verifiable tasks, with outputs/signals \( x_0 \) and \( x_1, x_2 \), respectively. The parties can then take advantage of a court-enforced contract on the verifiable task. We consider a case where only short term such explicit contracts are feasible.\(^6\)

The agent is now given a verifiable bonus \( b_0x_0 \) on task 0, and a discretionary bonus \( \beta(x_0, x_1, x_2) \) depending on the outcome vector \( x \).

Assuming as before that FOA holds, the agent’s first-order conditions are then

\[
\int \beta(x)f_{e_i}(x, e) - c_i(e) = 0 \quad i = 1, 2
\]

\[
\int \beta(x)f_{e_0}(x, e) + b_0 - c_i(e) = 0 \quad i = 0
\]

The principal maximizes the total surplus \( v(e) - c(e) \) subject to these constraints, and the dynamic enforcement constraint. We assume as before that the parties separate if the relational contract is broken. The enforcement constraint is then

\[
0 \leq \beta(x) \leq \frac{\delta}{1 - \delta}(v(e) - c(e))
\]

\(^6\)Miller, Olsen and Watson (2018) analyse long term renegotiable explicit contracts, and show that it may be optimal to renegotiate these contracts each period when in combination with relational contracts. See also Miller and Watson (2013) on alternative strategies and "disagreement play" in repeated games.
Suppose the signals are multinormally distributed with $Ex_i = c_i$, $var x_i = s^2$, and correlations

$$\rho = corr(x_1, x_2), \quad \rho_0 = corr(x_0, x_1) = corr(x_0, x_2)$$

From the same principles as before it follows that the agent should be given the discretionary bonus if and only if an index exceeds a hurdle, and from the normal distribution it follows that this index is linear in the signals; $y = \Sigma_i \tau_i x_i$, and the hurdle is $y_0 = \Sigma_i \tau_i e_i$, where $e$ is the equilibrium effort vector. If the magnitude of the bonus is $b$, this leads now to the following first-order conditions for the agent:

$$b \phi_0 \frac{\tau_i}{\sigma} - c_i(e) = 0, \quad i = 1, 2$$
$$b \phi_0 \frac{\tau_i}{\sigma} + b_0 - c_i(e) = 0, \quad i = 0$$

where

$$\sigma = SD(y) = (\tau' \Sigma \tau)^{1/2},$$

and the bonus level $b$ must respect the enforcement constraint;

$$b \leq \frac{\sigma}{1 - \rho} (v(e) - c(e))$$

It is clear that, for any vector $\tau$, the principal can control incentives on the verifiable task by adjusting $b_0$ (which can here be chosen freely). Since a lower variance in the performance index sharpens incentives on the non-verifiable tasks, the weight $\tau_0$ should therefore be chosen to minimize this variance; i.e. to minimize

$$\sigma^2 = var(\Sigma_i \tau_i x_i) = s^2(\Sigma \tau_i^2 + 2\rho \tau_1 \tau_2 + 2\rho_0(\tau_0 \tau_1 + \tau_0 \tau_2))$$

Hence

$$\tau_0 = -\rho_0(\tau_1 + \tau_2),$$

and the minimized variance is straightforwardly seen to be

$$\sigma^2 = s^2((1 - \rho_0^2)(\tau_1 + \tau_2)^2 - 2\tau_1 \tau_2(1 - \rho))$$

Substituting for $\frac{\tau_i}{\sigma}$, $i = 1, 2$ from the FOCs in this equation, solving for the bonus level $b$, and making use of the enforcement constraint, we see that
optimal efforts and the bonus must satisfy the following relation

\[
\frac{\delta}{1 - \delta} (v(e) - c(e)) \geq b
\]

\[
= \frac{s}{\phi_0} \left[ (1 - \rho_0^2) (c_1(e) + c_2(e))^2 - 2(1 - \rho)c_1(e)c_2(e) \right] \frac{\delta^2}{2}
\]

The principal now maximizes the surplus \( v(e) - c(e) \) subject to this constraint and the FOC for the verifiable task, where \( b_0 \) can be chosen freely. Writing \( b'_0 = b_0 + b\phi_0 \frac{\tau_0}{\phi} \), the latter constraint can be written as

\[
b'_0 - c_0(e) = 0
\]

In a case where the surplus is separable across tasks, and thus can be written as \( s(e_1, e_2) + s^0(e_0) \), the bonus \( b'_0 \) would be chosen to implement the first-best effort on the verifiable task, and then \( (e_1, e_2) \) would be chosen to maximize \( s(e_1, e_2) \), subject to the enforcement constraint (3).

Given the optimal efforts, the appropriate weights \( \tau_1, \tau_2 \) in the performance index can be found from the FOCs, and the weight \( \tau_0 \) associated with the verifiable task should then be \( \tau_0 = -\rho_0(\tau_1 + \tau_2) \).

It is worth noting that the performance index takes the form

\[
y = \sum_i \tau_i x_i = \tau_1(x_1 - \rho_0 x_0) + \tau_2(x_2 - \rho_0 x_0)
\]

This illustrates that for \( \rho_0 \neq 0 \) performance on the verifiable task is taken into the index as a benchmark, to which the other performances are compared.

The hurdle for \( y \) has a similar form:

\[
y_0 = \tau_1(e_1 - \rho_0 e_0) + \tau_2(e_2 - \rho_0 e_0) = \tau_1 E(x_1 | x_0, e_1, e_0) + \tau_2 E(x_2 | x_0, e_2, e_0),
\]

where the last equality follows from expressions for conditional expectations.

The condition for the agent to get the discretionary bonus is thus

\[
y - y_0 = \sum_{i=1}^2 \tau_i (x_i - e_i - \rho_0(x_0 - e_0)) = \sum_{i=1}^2 \tau_i (x_i - E(x_i | x_0, e_i, e_0)) > 0
\]

Performance \( x_i \) is compared to expected performance, given (equilibrium) efforts and the outcome on the verifiable task. If the performance exceeds what is expected, given this information, then it contributes positively to making the index exceed the hurdle, and thus to obtaining the bonus.
As before, we see that a higher standard deviation $s = SD(x_i)$ tightens the enforcement constraint (3) and thus lowers the optimal surplus. The effect of a higher correlation $\rho$ between the two non-verifiable tasks is detrimental for the total surplus, since the enforcement constraint is tightened by higher $\rho$. On the other hand, a stronger (positive or negative) correlation $\rho_0$ between the verifiable task and each of the non-verifiable ones will relax the constraint and lead to a higher surplus. The latter effect illustrates the value of the information obtained by benchmarking the agent’s performance on the non-verifiable tasks to her performance on the verifiable one.

4 Multiple multitasking agents

Consider now several (two) multitasking agents, in a setting with only non-verifiable information. The agents are symmetric, and each agent exerts two types of efforts, denoted $(e_i, h_i)$, $i = 1, 2$. In general there are $n \geq 2$ signals $x = (x_1...x_n)$ that are informative about these efforts. The joint distribution of signals is $f(x; e, h)$, with likelihood ratios

$$l_e(x; e, h) = \frac{f_e(x; e, h)}{f(x; e, h)}, \quad l_h(x; e, h) = \frac{f_h(x; e, h)}{f(x; e, h)}, \quad i = 1, 2.$$

Agent $i$ is paid a discretionary bonus $\beta_i(x)$ from the principal, and the agent’s IC is then

$$e_i, h_i \in \arg \max E(\beta_i(x) | e_i', h_i', e_{-i}, h_{-i}) - c(e_i', h_i')$$

with first-order conditions (FOCs)

$$\frac{\partial}{\partial e_i} E(\beta_i(x) | e, h) - c_{e_i}(e_i, h_i) = 0, \quad \frac{\partial}{\partial h_i} E(\beta_i(x) | e, h) - c_{h_i}(e_i, h_i) = 0$$

The total surplus is $v(e, h) - \Sigma_i c(e_i, h_i)$, and the enforceability constraint (EC) is

$$\beta_1(x) + \beta_2(x) \leq \frac{\delta}{1-\delta}(v(e, h) - \Sigma_i c(e_i, h_i)), \quad \beta_1(x), \beta_2(x) \geq 0$$

The optimal contract maximizes the surplus subject to these constraints. As before we assume FOA to be valid, and thus consider only the FOCs for incentive compatibility.

From the Lagrangian we see that there are indexes $\mu_i l_e(x; e, h) + \eta_i l_h(x; e, h)$, one for each agent, such that the the agent who realizes the highest index
value will be awarded a bonus, provided her index value is positive. If equal index values is a zero probability event, then at most one agent will be awarded a bonus, and this occurs when at least one index is positive. The bonus scheme is then a form of a tournament, where only a "winner" is awarded a bonus, but the bonus is also conditional on the winner’s performance index exceeding a hurdle (here zero). Since the index may in principle depend on all signals, the hurdle requirement may entail a form of benchmarking vis-a-vis the other agent. If equal and positive index values can occur with positive probability, then both agents will be awarded a bonus in such an event. We have the following result.

**Proposition 3** There are multipliers such that agent $i$ is optimally paid a bonus ($\beta_i(x) > 0$) if and only if

$$\mu_i l_{e_i}(x; e, h) + \eta_i l_{h_i}(x; e, h) > 0$$

and

$$\mu_i l_{e_i}(x; e, h) + \eta_i l_{h_i}(x; e, h) \geq \mu_j l_{e_j}(x; e, h) + \eta_j l_{h_j}(x; e, h), \quad j \neq i.$$  

Both agents are paid bonuses only if the last inequality holds with equality. In any case, if at least one bonus is positive, then: $\Sigma_i \beta_i(x) = \frac{\delta}{1-\sigma}(v(e, h) - \Sigma_i c(e_i, h_i))$.

Thus there is an index $y_i(x) = \mu_i l_{e_i}(x; e, h) + \eta_i l_{h_i}(x; e, h)$, i.e. a weighted sum of the likelihood ratios for the efforts for agent $i$, such that the agent is paid a bonus iff this index is (a) positive and (b) is maximal among the two agents. The index can be interpreted as a (optimal) performance index, and agents are then rewarded in a tournament-like scheme based on these indexes. Hence, the optimality of balanced scorecard contracts also applies to multiagents setting where agents compete for bonuses.

This characterization of the optimal bonus scheme is similar to that given by Levin (2002) for the single-task multi-agent case with uncorrelated signals. In his case the equality condition for the indexes is a zero probability event. In such cases only one agent is given a bonus, namely the agent with the highest "performance index", provided that this index in addition exceeds a
hurdle (zero). The scheme outlined here is of the same type as Levin’s, but covers multi-dimensional effort and stochastic dependencies.

5 Conclusion

Workers are often evaluated along many dimensions. The performances on each dimension will typically be correlated, and at least some of the performance measures will typically be non-verifiable to a third party. The aim of this paper is to study this environment: Optimal incentives for multitasking agents whose performance measures are non-verifiable and potentially correlated. The multitask problem is extensively analyzed in the literature, but mostly under the assumption that (at least some) performance measures are verifiable. The verifiability problem is extensively analyzed by the use of self-enforcing relational contracts, but - with a few exceptions - mostly under the assumption that agents do not multitask. We generalize and extend existing literature on multitasking and relational contracts to the case of $n$ tasks, and allow outputs on each tasks to correlate.

Our main result is that the optimal relational contract is an "index contract". That is, the agent gets a bonus if a 'weighted sum' of performance outcomes on the various tasks (an index) exceeds a hurdle. If some of the performance measures are verifiable and thus enforceable by a third party, the optimizing principal will include performance on the verifiable task in the relational index contract as a benchmark, to which the other performances are compared. The efficiency of the relational index contract depends on the correlation between the tasks. Negative correlation improves incentives, since it reduces the variance of the performance measure. This is beneficial not because a more precise measure reduces risk, but because it strengthens, for any given bonus level, the incentives for the agent to provide effort. The model is then extended to analyze multiple agents, and we show that the optimal contract is still an index contract, where the bonus is based on a weighted sum of performance outcomes on the various tasks.

The index contracts that turn out to be optimal in our model bear resemblance to key features of the performance measurement system known as balanced scorecard. Reward systems based on BSC typically connect pay to
an index, but to the best of our knowledge there is no formal incentive model that actually describe this kind of index contracts as an optimal solution. In that sense, our paper provides at contract theoretic rationale for the way BSC schemes are implemented. However, while the scheme we present is a bonus contract with just one threshold (or 'hurdle'), scorecards in practise often have many thresholds and bonus levels, where the size of the bonus depends on the score. Future research can extend the model we present to incorporate e.g. risk aversion or limited liability, in order to study under which broader conditions the index contract is optimal, and what kind of index contracts that are optimal under various model specifications.

References


APPENDIX

Proof of Lemma 1. This follows directly from the Lagrangian for the problem, which takes the form

\[ L = v(e) - c(e) + \sum_i \mu_i \left( \int \beta(x)f_{ei}(x,e) - c_i(e) \right) + \int \lambda(x)(\frac{\partial}{\partial x} (v(e) - c(e)) - \beta(x)) \]

The optimal bonus satisfies

\[ \frac{\partial L}{\partial \lambda(x)} = \sum_i \mu_i f_{ei}(x,e) - \lambda(x) = 0 \text{ if } \beta(x) > 0, \quad \leq 0 \text{ if } \beta(x) = 0 \]

Hence we have

If \( \sum_i \mu_i f_{ei}(x,e) > 0 \) then \( \lambda(x) > 0 \) and hence \( \beta(x) = \frac{\partial}{\partial x} (v(e) - c(e)) \).

If \( \sum_i \mu_i f_{ei}(x,e) < 0 \) then \( \frac{\partial L}{\partial \lambda(x)} < 0 \) and hence \( \beta(x) = 0 \) (implying \( \lambda(x) = 0 \)).

Proof of Proposition 1. For \( x \sim N(e, \Sigma) \) the likelihood ratios are linear:

\[ \frac{f_{ei}(x,e)}{f(x,e)} = \sum_j k_{ij}(x_j - e_j), \]
where \( k_{ij} \) are constants. (In fact we have \([k_{ij}] = \Sigma^{-1}\).) Hence
\[
\sum_i \mu_i f_i(x,e) = \sum_j \tau_j (x_j - e_j), \quad \tau_j = \sum_i \mu_i k_{ij}
\]

The statement in Lemma 2 then follows from Lemma 1.

**Verification of equations (1) and (2).**

Given the index \( y = \sum_j \tau_j x_j \) with hurdle \( y_0 \), and the bonus \( \beta = b \) being paid for \( y > y_0 \), the agent’s (performance related) payoff is
\[
u(b,e) = b \Pr(\sum_j \tau_j (x_j - e_j) > \sum_j \tau_j (e_j^* - e_j) | e) - c(e)
\]

where \( H() \) is the CDF for \( \sum_j \tau_j (x_j - e_j) \), which is \( N(0,\sigma) \), where
\[
\sigma^2 = \text{var}(y) = \text{var}(\sum_j \tau_j x_j) = \Sigma_i \Sigma_j s_{ij} \tau_i \tau_j = \tau^T \Sigma \tau
\]

So we can write
\[
u(b,e) = b(1 - \Phi(\frac{\sum_j \tau_j (e_j^* - e_j)}{\sigma})) - c(e)
\]

with FOCs for effort
\[
u_e(b,e) = b\phi(\frac{\sum_j \tau_j (e_j^* - e_j)}{\sigma}) \frac{\tau_j}{\sigma} - c_i(e) = 0,
\]

where \( \Phi() \) is the standard normal CDF and \( \phi() \) its density. For \( e = e^* \) to be the agent’s optimal choice, we thus must have
\[
b\phi(0) \frac{\tau_j}{\sigma} - c_i(e) = 0 \quad \text{for} \quad e = e^*
\]

This verifies (1), since \( \phi(0) = 1/\sqrt{2\pi} \).

We finally note that the first equality in (2) follows from the FOCs \((b\phi_0 \frac{1}{\sigma} \tau = \nabla c(e))\), since they imply imply \( \nabla e^T \Sigma \nabla c = (b\phi_0 \frac{1}{\sigma})^2 \tau^T \Sigma \tau = (b\phi_0)^2 \) by the definition of \( \sigma \).

**Proof of Proposition 3.** The Lagrangean for the problem is
\[ L = v(e, h) - \Sigma_i c(e_i, h_i) + \Sigma_i \mu_i (\int \beta_i(x) f_{e_i}(x, e, h) - c_{e_i}(e_i, h_i)) \]
\[ + \Sigma_i \eta_i (\int \beta_i(x) f_{h_i}(x, e) - c_{h_i}(e_i, h_i)) \]
\[ + \int \lambda(x) \left( \frac{\delta}{1-\delta} (v(e, h) - \Sigma_i c(e_i, h_i)) - \Sigma_i \beta_i(x) \right) \]

Optimal bonuses entail, for all \( x \):
\[ \frac{\partial L}{\partial \beta_j(x)} = \mu_j f_{e_j}(x, e, h) + \eta_j f_{h_j}(x, e, h) - \lambda(x) \leq 0, \quad \beta_j(x) \geq 0, \]
with complementary slackness. Moreover, given that the first-best allocation cannot be attained, the EC constraint will bind and \( \lambda(x) > 0 \). Hence: if both bonuses are positive, then
\[ \mu_j f_{e_j}(x, e, h) + \eta_j f_{h_j}(x, e, h) = \lambda(x), \quad j = 1, 2 \]

This outcome (for the \( x' \)'s) may or may not be a positive probability event. If it is, then in this event of positive probability, both agents will be awarded a bonus. Otherwise at most one agent will be awarded a bonus. The bonus will then optimally be assigned to that agent for which the expression \( \mu_j f_{e_j}(x, e, h) + \eta_j f_{h_j}(x, e, h) \) is (a) positive and (b) largest among the two agents. These conditions can equivalently be expressed in terms of the likelihood ratios, i.e. in terms of the expressions \( \mu_j l_{e_j}(x, e, h) + \eta_j l_{h_j}(x, e, h) \), as exhibited in the proposition.