Buyer Power in Highly Competitive Industries *

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Abstract

This paper analyzes the competitive effects of buyer power in a vertically related industry in which upstream and downstream firms sell homogeneous products and compete in prices. We identify a new rationale why buyer power is anti-competitive: Buyers compete in the upstream market by overbidding to exclude their rivals from the downstream market. This leads to final-consumer prices above marginal costs, and to positive profits for firms at both layers, despite the fact that they sell homogeneous products. By contrast, with seller power, the equilibrium involves marginal cost pricing. Our results contrast with the long-standing wisdom, developed by Galbraith (1952), that incentives of downstream firms and final consumers with respect to the wholesale prices are aligned.

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1 Introduction

The effects of buyer power on final consumer prices and social surplus have attracted much attention in the theoretical industrial organization literature over the recent years. A main motivation for this surge of interested was certainly the increasing importance of powerful downstream firms in many industries. The resulting welfare implications have repeatedly been a concern for competition authorities. However, this squares with one of the key insights of the seminal work by Galbraith (1952) that interests of downstream firms and final consumers with respect to the price at the wholesale stage are aligned because both parties prefer lower wholesale prices. As buyer power leads to the increased ability of downstream firms to obtain discounts from upstream firms, and competition at the downstream level then leads to a (partial) pass-through of these savings onto final consumers, competition authorities should not need to worry about buyer power.

This view was challenged by recent literature (e.g., Dobson and Waterson (1997), Chipty and Snyder (1999), Inderst and Shaffer (2007), among others). These papers point out that larger and more powerful buyers can be the source of an increase in wholesale prices. The reasons for this are diverse—e.g., smaller buyers may be harmed as their bargaining power falls which leads to larger prices for them, or that a larger retailer has the incentive to carry less variety. The pros and cons of buyer power are also prominently featured in recent antitrust cases.\(^1\)

In this literature, the effects of buyer power are considered in conjunction with an increase in the market concentration in the downstream market. However, a direct competitive comparison between buyer power and supplier power without a change in the market structure has not been studied so far, probably because Galbraith’s (1952) argument that buyer power leads to lower wholesale prices seems to be the main effect at work. In this paper, we show that such a conclusion is incorrect and point out a novel anti-competitive effect of buyer power: When being endowed with bargaining power, buyers compete for contract acceptance of suppliers. Buyer power refers to bargaining power being assigned to downstream firms, which simply implies that they possess the power to propose wholesale contracts. Such competition for contract acceptance drives up wholesale prices because suppliers prefer to accept a contract which guarantees a high margin. Hence, although a buyer faces higher inputs costs, its competitors may be driven out of the market because suppliers may only accept the contract that provides them with the largest margin; indeed, accepting contracts from buyers offering lower wholesale prices implies that these buyers will sell more in the

\(^{1}\)For example, in the UK, the Competition Commission conducted several studies on the grocery industry after a merger wave. Buyer power was also an important issue in the GE/Honeywell merger—a merger in the upstream market—where Boeing and Airbus were powerful buyers.
downstream market and suppliers obtain less profits from them. By contrast, if suppliers are endowed with bargaining power, competition for downstream firms will result in low wholesale prices.

As a consequence, buyer power leads to higher wholesale prices than seller power, and, due to the pass-through effect, also to higher final-consumer prices. This effect substantially lowers consumer surplus.\textsuperscript{2} In addition, profits at both layers of the industry increase. In particular, suppliers may benefit more from buyer bargaining power than buyers themselves. In fact, whereas, in the extreme with perfect competition, they obtain no profits when having bargaining power, they obtain sizable profits with buyer power. This implies that suppliers benefit from less bargaining power. In sum, whereas it is well-known that competition between downstream firms for final consumers improves welfare, a similar conclusion does not hold when downstream firms compete for wholesale contract acceptance.

To expose this effect in the simplest possible way, we consider a situation with two upstream and downstream firms. The two firms at each layer produce a homogeneous good and compete in Bertrand fashion. In this very simple framework, we compare supplier power with buyer power by considering the two extreme situations, in which either suppliers or buyers have full bargaining power at the wholesale stage. The firms with bargaining power make take-it-or-leave-it offers consisting of a linear wholesale price.

If suppliers have bargaining power, Bertrand competition squeezes wholesale prices to marginal costs. Given this outcome in the upstream stage, and given that downstream firms also engage in price competition, the resulting equilibrium involves marginal-cost pricing in both the upstream and the downstream market.

Addressing the question whether the same equilibrium will emerge if downstream firms have bargaining power yields that the unambiguous answer is No! Given one downstream firm offers a wholesale price equal to marginal cost, its rival has the incentive to slightly increase its offer. Suppliers are then induced to accept only the contract with the higher wholesale price. The reason is that accepting the other contract as well implies that only this firm will sell in the final-consumer market as it faces lower input costs and suppliers end up with a margin equal to zero. Consequently, marginal-cost pricing does not constitute an equilibrium and buyer power necessarily involves wholesale prices above marginal cost.

Characterizing the equilibrium with buyer power is more involved. When setting its wholesale price, a downstream firm faces the following trade-off: Increasing its offer yields a higher margin for suppliers but also reduces the quantity that the firm will sell in the downstream market (due to its higher cost). If its downstream rival sets a low wholesale

\textsuperscript{2}In a simple framework with linear demand, we show that consumer surplus falls by around 40% with buyer power compared to seller power.
price, the first effect is dominating, as suppliers will then reject the rival’s offer and the downstream firm has a monopoly position when serving final consumers. By contrast, if the rival sets a high wholesale price, the second effect is dominating and the firm optimally offers a low wholesale price. Suppliers will accept both offers, which implies that downstream firms compete and the sold quantity is high because the firm with the lower wholesale price is constrained in its downstream pricing decision by the rival’s wholesale price offer. In this case, suppliers are willing to accept a lower margin as the quantity sold to consumers is high. As a consequence, the reaction function of a downstream firm with respect to the wholesale price is non-monotone and involves a kink. We nevertheless show that under relatively general conditions on demand, there exists a unique equilibrium in mixed-strategies, which entails a mass point at the kink of the reaction function.

It follows from the equilibrium characterization that (expected) wholesale prices are above marginal costs. Therefore, suppliers obtain a strictly positive profit despite the fact that they sell a homogeneous good. In addition, their profit is only positive when bargaining power is assigned to buyers and not to themselves—a counter-intuitive result. In addition, the equilibrium in mixed strategies involves downstream firms obtaining strictly positive (expected) profits: they engage either in asymmetric Bertrand competition or may even monopolize the downstream market. Therefore, buyer power allows perfectly competitive firms to reap positive profits at both layers of the vertical structure, implying a substantial fall in consumer welfare.

To determine whether this effect is sizable, we use a linear demand framework. We obtain that whereas welfare falls by less than 10%, consumer surplus falls by 40% when moving from supplier power to buyer power.

In the literature on buyer power, the perhaps most prominent work is Galbraith’s (1952) countervailing-power hypothesis arguing that large buyers can extract price concessions from suppliers, which then leads to a partial pass-through of these savings onto consumers in form of lower prices. The argument was formalized by Chen (2003) in a model in which a supplier sells a larger quantity through fringe retailers if large retailers have more bargaining power. The quantity increase then leads to a fall in final-consumer prices.

Motivated by the increasing downstream concentration levels in many industries over the past years, several papers analyzed the perils of such concentration when explicitly taking the wholesale stage into account, thereby challenging the countervailing hypothesis. For example, von Ungern-Sternberg (1996) and Dobson and Waterson (1997) analyze retail mergers and show that they reduce a supplier’s threat to sell to competing retailers, which may trigger a reduction in the wholesale price. However, as market power in the final-consumer market increases as well, retail prices nevertheless often increase. Chippy and
Snyder (1999) and Inderst and Wey (2007) show that larger retailers can negotiate lower prices if suppliers have convex costs. The reason is that smaller retailers negotiate more ‘on the margin’, where average unit costs are higher, implying that smaller retailers suffer from retail mergers. Inderst and Shaffer (2007) find that retail mergers may lead to delisting of suppliers’ products in case retailers have bargaining power, and therefore reduce product variety. Finally, Inderst and Montez (2016) analyze negotiations between multiple buyers and suppliers. They show that the size of a party creates mutual dependency, which implies that large buyers pay a lower price only if their bilateral bargaining power is high enough. In contrast to our analysis, these papers mainly focus on buyer power arising from downstream mergers (or firm size) but they do not compare supplier with buyer power directly. In addition, the effect that buyers compete for suppliers’ products through overbidding has not been identified in these papers.

Another strand of literature, pioneered by Marx and Shaffer (2007), analyzes the direct effects of buyer power by comparing buyer power with supplier power. In contrast to our paper, their focus is on the contractual terms that retailers are able to offer. Specifically, they consider the effects of a three-part tariff (a slotting allowance in addition to a two-part tariff) on retailer exclusion. They find that slotting allowances may induce the supplier to refuse to trade with smaller retailers. Marx and Shaffer (2007) in their extension also show that, without a slotting allowance, no pure-strategy equilibrium exists which is in line with our finding.

Finally, our paper also relates to the literature on competition between intermediaries. For example, Yanelle (1989) points out verbally that the outcome of intermediaries competing in prices for buyers and sellers is not equivalent to the (perfectly competitive) Walrasian outcome. We indeed prove the existence of a mixed-strategy equilibrium thereby formalizing this guess. Stahl (1988) considers a scenario in which intermediaries order input quantities before competing on the product market. This implies that downstream competition plays out as in Kreps and Scheinkman (1983). The upstream market in his model is characterized by an increasing supply function. He provides conditions for the final-consumer price being

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3Inderst and Wey (2011) analyze the impact of buyer power on a supplier’s incentive to become more cost efficient, and find that although suppliers may obtain a lower profit with buyer power, its investment incentive can be larger.

4Marx and Shaffer (2007) in their extension also show that, without a slotting allowance, no pure-strategy equilibrium exists which is in line with our finding.

5Our paper also relates to the literature showing the failure of existence of pure-strategy equilibria when firms engage in price competition with homogeneous goods (see e.g., Sharkey and Sibley, 1993, and Marquez, 1997). In contrast to our model, these papers consider a situation in which firms simultaneously decide whether to enter the market (at a fixed cost) and if so, which consumer prices to set. This implies that competition is different to classic Bertrand competition and with some probability only one firm enters.
equal to marginal costs (i.e., the Walrasian outcome occurs) but also shows that often no equilibrium exists. By contrast, we find that with price competition downstream, a mixed strategy equilibrium always exists, resulting in prices above the Walrasian outcome.\[^{6}\]

The rest of the paper is organized as follows: Section 2 sets out the model. Section 3 briefly solves the model in case of seller power. Section 4 considers buyer power, characterizes the equilibrium, and provides a comparison of the two cases. Section 5 concludes. Some proofs are relegated to an Appendix.

## 2 The Model

We consider two symmetric suppliers $S_k, k \in \{A, B\}$, producing a homogenous good at constant marginal costs which, for simplicity, are assumed to be zero. They sell to two undifferentiated retailers $R_i, i \in \{1, 2\}$. Retailers then sell the good to final consumers at no further cost and compete in Bertrand style. The production technology is one-to-one, that is, retailers need one unit of input to sell one unit of output.

The downstream demand function $D(p)$ is strictly downward-sloping, that is, $D'(\cdot) < 0$, and characterized by $D(0)$ being finite. In addition, there exists a choke-off price $p$ such that $D(p \geq p) = 0$. The demand function is well-behaved, i.e., $2D'(\cdot) + D''(\cdot)p < 0$, which guarantees that profits are strictly concave.

We consider both the scenario in which retailers make offers (buyer power) and in which suppliers make offers (seller power). Wholesale contracts consist of a simple linear price specifying the amount a retailer has to pay for one unit of the input good. The firms with bargaining power make take-it-or-leave-it offers, and offers of a firm must be non-discriminatory. That is, if suppliers have bargaining power, $S_k, k \in \{A, B\}$, makes an offer $w_k$ to both retailers, whereas if retailers have bargaining power, $R_i, i \in \{1, 2\}$, makes an offer $w_i$ to both suppliers. If a retailer is indifferent between buying from $S_A$ or $S_B$ (e.g., because both suppliers accepted the retailer’s offer), the retailer orders half of its quantity from either supplier. Finally, we assume for ease of exposition that each retailer observes its rival’s wholesale price prior to downstream competition.

The timing of events is as follows:

**Stage 1.** Each retailer (supplier) offers a contract $w_i$ ($w_k$) to both suppliers (retailers).

\[^{6}\]Eső, Nocke, and White (2010) and Burguet and Sákovics (2017) study models in which imperfectly competitive downstream firms buy an input in a perfectly competitive upstream market, which is characterized by an increasing supply function. Both papers show in different frameworks that this market structure leads to higher downstream prices than one in which the upstream market is ignored. By contrast, in our paper downstream firms compete in Bertrand style and the upstream market is characterized by a discrete number of firms with constant marginal costs.
Stage 2. Recipients of the offers (simultaneously) decide on acceptance or rejection.

Stage 3. After observing the outcome of stages 1 and 2, each retailer sets a downstream price $p_i$. Then, final demand realizes and retailers order quantities to satisfy demand.

Our solution concept is Subgame Perfect Equilibrium, with the refinement that firms select the payoff-dominant equilibrium in case multiple equilibria exist at a stage.\(^7\)

3 Seller power

As a benchmark case, we first consider the scenario in which $S_A$ and $S_B$ have bargaining power. We obtain the standard result for the case in which firms sell homogeneous products.

Lemma 1 If suppliers make take-it-or-leave-it offers to retailers, retail and wholesale prices equal zero in equilibrium.

The proof of this result is straightforward. In the third stage, $R_i$’s profit function, given that $w_k \leq w_l$, $k, l \in \{A, B\}$, and $R_i$ accepted $S_k$’s contract, is given by

$$
\Pi_{R_i} = \begin{cases} 
D(p_i)(p_i - w_k) & \text{if } p_i < p_j \\
D(p_i)(p_i - w_k)/2 & \text{if } p_i = p_j \\
0 & \text{if } p_i > p_j,
\end{cases} \tag{1}
$$

with $i, j \in \{1, 2\}$ and $j \neq i$. If instead $R_i$ only accepted the offer of $S_l$, its profit is the same as in (1) with $w_k$ replaced by $w_l$. Following the standard solution technique for price setting with homogenous goods, we obtain that in equilibrium both retailers set $p_i = p_j = w_k$ in case both accepted $S_k$’s offer and $p_i = p_j = w_l$ in case both accepted only $S_l$’s offer. In each case, they obtain zero profits. Instead, if $R_i$ accepted $S_k$’s offer whereas $R_j$ accepted only $S_l$’s offer, $R_i$ sets a price slightly below $w_l$ (i.e., $p_i = \lim_{\epsilon \to 0}(w_l - \epsilon)$), $R_j$ sets $p_j = w_l$, which leads to profits of $\Pi_{R_j} = 0$ and $\Pi_{R_i} = \lim_{\epsilon \to 0} D(w_l - \epsilon)(w_l - \epsilon - w_k)$.

Turing to the second stage, it then follows that retailers buy from the supplier offering the lowest wholesale price. Therefore, $S_k$’s profit is $w_k D(w_k)/m$ if $w_l \geq w_k$ where $m = \#\{l \in \{A, B\} : w_l = w_k\}$ and 0 if $w_l < w_k$. It follows that the same logic outlined for the retail market applies to the upstream market just with the two retailers as buyers instead of final consumers. Thus, in the unique (symmetric) Bertrand equilibrium, $w_k = w_l = 0$, and retail

\(^7\)As will become evident below, in case retailers have bargaining power, a coordination problem between suppliers regarding their acceptance decisions may occur in Stage 2.
prices are equal to wholesale prices. Both suppliers trade with both retailers and the market demand \( D(0) \) is equally split so that each retailer buys \( D(0)/4 \) from either supplier.

## 4 Buyer power

Consider now the situation in which retailers make offers to suppliers. Retailers then decide about the retail price and the wholesale price offered to suppliers whereas a supplier’s only decision is whether to accept or reject the respective contract offers. We first analyze whether the equilibrium of the benchmark case with supplier power still occurs in the case with buyer power.

Suppose that \( w_i = w_j = 0 \) for \( i \neq j \in \{1, 2\} \). Consider now a deviation of \( R_i \) in the first stage by proposing a small but positive wholesale price, that is, \( w_i^D = \delta \) with \( \delta > 0 \). In the second stage, both suppliers are then strictly better off when accepting only \( R_i \)'s contract offer. The reason is that, although \( R_i \) monopolizes the retail market, each supplier obtains strictly positive profits of \( \delta D(\text{p}_M(\delta))/2 \), where \( \text{p}_M(\delta) \) denotes the monopoly retail price with marginal cost equal to \( \delta \). Similarly, \( R_i \) benefits from this deviation as its resulting profit \( (\text{p}_M(\delta) - \delta)D(\text{p}_M(\delta)) \) is also strictly positive. We note that if \( w_i = \delta \) and \( w_j = 0 \), there also exists an equilibrium at stage 2, in which both suppliers accept \( R_j \)'s offer.\(^8\) If suppliers play this equilibrium, then a deviation by \( R_i \) from \( w_i = 0 \) to \( w_i = \delta \) is not profitable. However, this equilibrium is payoff-dominated for suppliers by the one in which they both accept only \( R_i \)'s offer, and will therefore not selected by our refinement. As a consequence, the equilibrium obtained under supplier power fails to exist if bargaining power is in the hand of retailers under the refinement of payoff dominance.

**Lemma 2.** If retailers make take-it-or-leave-it offers to suppliers, retail and wholesale prices are not equal to zero in a payoff-dominant equilibrium.

The intuition behind the result is rooted in the effect that retailers compete in two opposing ways. They want to attract consumers in the final-good market and compete for contract acceptance in their offers made to suppliers. Whereas the former effect drives retail prices down, the latter drives wholesale prices up. In fact, each retailer has the incentive to induce both suppliers to accept exclusively the own offer, which may be achieved by offering the higher wholesale price. If suppliers receive two different offers they face the following trade-off in their decision to accept solely the offer with the higher wholesale price. On the one hand, they obtain a higher margin by doing so; on the other hand, the quantity

\(^8\)In fact, if \( S_k \) accepts \( R_j \)'s offer, then \( R_j \) will win downstream competition, regardless of which offer(s) are accepted by \( S_{-k} \). It follows that also accepting \( R_j \)'s offer is among the best responses of \( S_{-k} \).
sold will accordingly be small because the retailer will monopolize the final-consumer market. Therefore, suppliers may be better off accepting both offers, which will constraint the retailer with the lower wholesale in its setting of the retail price. However, if the lower wholesale price equals zero, the margin effect dominates and suppliers will only accept the contract with the higher wholesale price. As a consequence, the equilibrium with retail and wholesale prices being equal to zero breaks down.

A direct implication that can already be stated here without having determined the resulting equilibrium is that buyer power will induce higher retail prices and deteriorate social welfare. In addition, suppliers will obtain positive profits only in case they have no bargaining power.

In what follows, we will solve for the equilibrium with buyer power and determine if the welfare loss and supplier’s profits are sizable.

Stage 3. In the retail pricing game, we need to distinguish between two scenarios with regard to stage two. Suppose first that both suppliers have accepted solely $R_i$’s offer irrespective of the wholesale prices proposed in stage one. Since this implies that $R_j$ cannot sell in the retail market, $R_i$ will set the monopoly price which is given by

$$p^m(w_i) \in \arg\max_p \{(p - w_i)D(p)\}.$$  

(2)

Suppose second that in the first stage, contract offers are such that $w_i \leq w_j$ for $i \neq j \in \{1, 2\}$ and that $w_j$ is below $p^m(w_i)$ defined by (2). Suppose further that at least one supplier has accepted both contract offers. Consequently, $R_i$ is able to prevent $R_j$ from selling a positive quantity, but cannot set its retail price according to (2) even if $w_i < w_j$. Thus, $R_i$’s best response in stage three is setting $p_i$ equal to $w_j$ to obtain a profit given by $\pi_i = (w_j - w_i)D(w_j)$ if $w_i < w_j$ and 0 otherwise.

Stage 2. We proceed with the second stage of the game where suppliers decide whether to accept or reject the contract offers made in stage one. Let us again first focus on the scenario described above where contract offers are such that $w_i \leq w_j$ for $i \neq j \in \{1, 2\}$ and $w_j < p^m(w_i)$. In stage two, each supplier chooses among three actions where the resulting profits are represented in the following strategic form.9

9We ignore here the action of rejecting both offers, which yields a profit of zero, as accepting at least one offer must lead to a weakly larger profit.
select the equilibrium where both suppliers accept only their offers of a relatively low demand at their margins. Accepting both offers results in asymmetric retail competition where suppliers are better off when selling a large quantity at their retail price, as the offer with the lower wholesale price, as accepting only one supplier’s offer would lead to zero profits. In this equilibrium, suppliers strategically limit R’s retail price, as R will charge \( p_i = w_j < p^m(w_i) \). The intuition is that with a small difference between the offered wholesale prices, suppliers are better off when selling a large quantity at a relatively low margin. Accepting both offers results in asymmetric retail competition where \( R_j \)'s wholesale price offer constrains R’s retail pricing decision. This type of equilibrium always exists in the case \( w_i \leq w_j < p^m(w_i) \). There are three different payoff-equivalent Nash equilibria, namely (accept both, accept both), (accept both, accept R), and (accept R, accept both), invoking this equilibrium type. As the payoff in all three equilibrium is not only the same for suppliers but also for retailers (i.e., \( R_i \)'s payoff is \( (w_j - w_i)D(w_j) \) and \( R_j \)'s payoff is zero), we do not need to select which of three is chosen by suppliers.

The second equilibrium type, which occurs if both suppliers accept only \( R_j \)'s offer, only exists if \( w_iD(w_j) \leq w_jD(p^m(w_j))/2 \) or \( w_i \leq w_jD(p^m(w_j))/(2D(w_j)) \). In this case, the suppliers’ margin with only \( R_j \) being active is large enough to outweigh the negative effect on their profits of a relatively low demand at \( p^m(w_j) \). Hence, neither supplier has an incentive to accept \( R_i \)'s offer and only \( R_j \) will sell in equilibrium.\(^{10}\)

It follows from the preceding discussion that both types of equilibria co-exist if \( w_i \leq w_jD(p^m(w_j))/(2D(w_j)) \). However, applying the refinement of payoff-dominance allows to select the equilibrium where both suppliers accept only \( R_j \). From above, we know that

\(^{10}\)Note that both suppliers accepting only \( R_i \)'s offer cannot be an equilibrium because \( D(w_j) > D(p^m(w_i)) \) always holds.

### Table 1: Strategic form representation of the second stage game.

<table>
<thead>
<tr>
<th>( S_i )</th>
<th>accept both</th>
<th>accept ( R_j )</th>
<th>accept ( R_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>accept both</td>
<td>( \frac{w}{2}D(w_j), \frac{w}{2}D(w_j) )</td>
<td>( w_jD(w_j), 0 )</td>
<td>( \frac{w}{2}D(w_j), \frac{w}{2}D(w_j) )</td>
</tr>
<tr>
<td>accept ( R_j )</td>
<td>( 0, w_jD(w_j) )</td>
<td>( \frac{w}{2}D(p^m(w_j)), \frac{w}{2}D(p^m(w_j)) )</td>
<td>( 0, w_iD(w_j) )</td>
</tr>
<tr>
<td>accept ( R_i )</td>
<td>( \frac{w}{2}D(w_j), \frac{w}{2}D(w_j) )</td>
<td>( w_iD(w_j), 0 )</td>
<td>( \frac{w}{2}D(p^m(w_i)), \frac{w}{2}D(p^m(w_i)) )</td>
</tr>
</tbody>
</table>
existence of this equilibrium requires that a supplier’s profit must exceed \( w_i D(w_j) \), which is larger than \( w_i D(w_j)/2 \)—the profit obtained in the first equilibrium type. Hence, payoff dominance singles out a unique equilibrium in the second stage.

For the sake of completeness consider finally that \( w_i < w_j \) and \( w_j \geq \tilde{w} = p^m(w_i) \). That is, \( R_j \)’s offered wholesale price is so large that \( R_i \) can charge the monopoly retail price even if at least one supplier also accepted \( R_j \)’s offer. Since the suppliers’ decisions follow similar patterns to those of the preceding case, we abstain from writing down the resulting profits in strategic form representation. In this case if \( w_i > w_j D(p^m(w_j))/2D(p^m(w_i)) \), in the unique equilibrium outcome both suppliers accept (at least) the offer from \( R_i \), and obtain a profit of \( w_i D(p^m(w_i))/2 \). If instead \( w_i \leq w_j D(p^m(w_j))/2D(p^m(w_i)) \), both suppliers accepting only \( R_j \)’s offer constitutes the unique payoff-dominant Nash equilibrium, leading to a profit of \( w_j D(p^m(w_j))/2 \) for each supplier.

**Stage 1.** In the first stage, retailers decide on their contract offer. Taking into account the suppliers’ actions in stage two and given the rival’s offer \( w_j \), \( R_i \) chooses among two actions. First, \( R_i \) can *overbid* \( R_j \) by offering \( w_i^o(w_j) \geq w_j \) to induce both suppliers to reject \( R_j \)’s offer. In this case, \( R_i \) will be the only seller in the retail market and set the monopoly price. Second, \( R_i \) can *underbid* \( R_j \) by offering \( w_i^u(w_j) < w_j \) just large enough to induce at least one supplier to accept. Given that \( w_j < p^m(w_i) \), retail competition takes place so that \( R_i \) will optimally set \( p_i = w_j \). Let us denote the offer of \( R_j \) at which \( R_i \) is indifferent between either strategy by \( \hat{w} \), which will be defined below.

We have now enough information to pin down \( R_i \)’s best response set \( w_i(w_j) \).

If \( w_j < \hat{w} \), \( R_i \) optimally chooses to *overbid* \( R_j \) to monopolize the retail market. Its best response \( w_i(w_j) \) is given in implicit form by

\[
 w_j = w_i D(p^m(w_i))/2D(w_i). \tag{3}
\]

If \( \hat{w} \leq w_j \leq \tilde{w} \), \( R_i \) optimally chooses to *underbid* \( R_j \) by offering

\[
 w_i = w_j D(p^m(w_j))/2D(w_j). \tag{4}
\]

Similarly, if \( w_j > \tilde{w} \), \( R_i \) optimally *underbids* \( R_j \) but without being constrained in its retail

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\(^{11}\) In what follows we denote by \( \tilde{w} \) the wholesale price of \( R_j \) that corresponds to the \( R_i \)’s monopoly retail price given \( w_j \) if \( w_i < w_j \).

\(^{12}\) In what follows, we show that \( w_j \geq p^m(w_i) \) cannot occur in equilibrium.

\(^{13}\) The tie-breaking rule is that suppliers accept the offer from the retailer with the lower wholesale price when being indifferent between both types of equilibria in stage two.
pricing decision yielding a best response of

\[ w_i = w_j D(p^m(w_j))/2D(p^m(w_i)). \tag{5} \]

We can now define \( \hat{w} \). It is an implicit solution to

\[ \pi^u_i(w^u_i, \hat{w}) = [\hat{w} - w^u_i] D(\hat{w}) = [p^m(w^\sigma_i) - w^\sigma_i] D(p^m(w^\sigma_i)) = \pi^\sigma_i(w^\sigma_i, \hat{w}), \tag{6} \]

with \( w^u_i = \hat{w} D(p^m(\hat{w}))/2D(\hat{w}) \) and \( w^\sigma_i \) implicitly defined as \( \hat{w} = w^\sigma_i D(p^m(w^\sigma_i))/2D(w^\sigma_i) \). The next lemma shows that the best response is indeed unique, and is therefore a best best-response function.\(^{14} \)

**Lemma 3.** The best responses \( w_i(w_j) \) given by (3)- (5) are unique and so is \( \hat{w} \).

We can now describe the best-response function in more detail. The overbidding strategy implicitly determined by (3) is an increasing function in \( w_j \). The larger \( w_j \), the larger must be \( R_i \)'s offer to make both suppliers better off from accepting only its offer. The underbidding strategy defined by (4) is relevant in case of \( w_j \) being smaller than \( p^m(w_i) \).\(^{15} \) For a similar argument as above, (4) must be an increasing function in \( w_j \). The underbidding strategy characterized by (5) is chosen if \( w_j \) exceeds \( p^m(w_i) \). In this region it is no longer the case that \( w_i(w_j) \) is an increasing function in \( w_j \). The reason is that if \( w_j \), being above \( p^m(w_i) \), is getting larger, the offer is becoming less attractive to manufacturers: as \( w_j \) is particularly high, the effect that demand is small dominates the high margin. As a consequence, \( R_i \) can reduce its wholesale price as \( w_j \) increases, and suppliers still prefer to accept its offer rather than only that of \( R_j \). It follows from this discussion that it can never be part of a best response for a retailer to offer a wholesale price above \( \hat{w} \).

The best-response function is plotted in Figure 1. It exhibits a jump at \( \hat{w} \). This is due to the fact that retailers optimally switch from overbidding to underbidding at \( \hat{w} \).

\(^{14} \)The proof of this lemma is relegated to the appendix.

\(^{15} \)Note that that \( w_i < w_j \) because \( D(p^m(w_j))/2D(w_j) < 1 \).
Figure 1: Set of best response functions of a representative retailer $R_i$.

Given the set of best response functions, we are now able to characterize the equilibrium of the first stage where retailers propose their offers.

**Proposition 1.** The first-stage game has no pure strategy equilibrium. In the mixed-strategy equilibrium, retailers choose wholesale prices on a compact support $[\hat{w}, \bar{w}]$ with strictly positive density on all points, where $\bar{w}$ is implicitly given by $\hat{w} = \bar{w}D(p^m(\bar{w}))/2D(\bar{w})$ and $\bar{w} = \hat{w}D(p^m(\hat{w}))/2D(\hat{w})$. There is a mass point at $\hat{w}$.

We first explain why there is no pure-strategy equilibrium in the first stage. Suppose that retailer $j$ sets $w_j > \hat{w}$. $R_i$’s best response is then to offer the $w_i$ according to (4), that is, it optimally underbids $w_j$, so that both suppliers just accept both contracts and $R_i$ sells in the downstream market. The best response of $R_j$ is then to increase $w_j$ slightly compared to the original wholesale price and set $w_j$ according to (3) so that both suppliers just accept its offer. This, in turn, triggers $R_i$’s best response to slightly raise $w_i$, and so on. Both retailers mutually and gradually increase their wholesale price offers until $w_i$ reaches $\hat{w}$. At this point, instead of increasing $w_j$ slightly above $w_j = \bar{w}$, $R_j$ now optimally chooses to
underbid according to (4) by dropping $w_j$ to $w$, whereupon the process repeats with reversed roles. Therefore, there is no intersection point between the two best-response functions and a pure-strategy equilibrium fails to exist. The bounds of the mixing support can be found starting from $\hat{w}$—i.e., the wholesale price, which when charged by the rival, makes a retailer indifferent between overbidding and underbidding. This determines $\overline{w}$ as upper bound and $\underline{w}$ as lower bound. Thus, in equilibrium, retailers randomize offers and neither of them chooses a wholesale price above $\overline{w}$ and below $\underline{w}$.

The equilibrium mixed strategies are in plotted in Figure 2.

**Figure 2: Equilibrium Mixing Ranges, $(w_i, w_j) \in [\underline{w}, \overline{w}]$.**

The mixed-strategy equilibrium is reminiscent to Bertrand-Edgeworth cycles (see, e.g., Maskin, 1986). In these cycles, as well as in our model, a firm undercuts is rival’s price up to a point where prices are so low, that the firm does better by setting a price discretely higher than the rival and serve only a small number of consumers. However, a crucial difference to our model is that there is no marginal undercutting as a firm needs to lower its price considerably to ‘convince’ both suppliers to accept its offer. This leads to mass point in the distribution at $w = \hat{w}$, as we will explain next.
In fact, the above described discrete jump in mutual best responses not only determines the support over which retailers randomize wholesale price offers but also causes the mass point at \( \hat{w} \).\textsuperscript{16} To see this note that, in order to be willing to mix between all wholesale prices on \([\underline{w}, \overline{w}]\), retailers have to be indifferent between them. To show that this requires retailers to shift probability mass to \( \hat{w} \) suppose that this was not the case and retailers randomize over \([\underline{w}, \overline{w}]\) with a smooth differentiable distribution \( H(w) \). By the definition of \( w_j \), we must have that \( R_i \)'s profit from setting \( w_i = w \) equals the profit from setting \( w_i = \overline{w} \) at \( w_j \), that is, \( \pi_i(w, \hat{w}) = \pi_i(\overline{w}, \hat{w}) \). This can be written as

\[
(p^M(\overline{w}) - \overline{w})D(p^M(\overline{w})) = [\hat{w} - \overline{w}]D(\hat{w}). \tag{7}
\]

If both retailers were randomizing with a smooth differentiable distribution \( H(w) \), \( R_i \)'s expected profit from setting \( w_i = \overline{w} \) equals

\[
(p^M(\overline{w}) - \overline{w})D(p^M(\overline{w}))H(\hat{w}) = \int_{\underline{w}}^{\hat{w}} (p^M(\overline{w}) - \overline{w})D(p^M(\overline{w}))dH(w) \tag{8}
\]

and its expected profit from setting \( w_i = w \) equals

\[
\int_{\underline{w}}^{\hat{w}} D(w)(w - \overline{w})dH(w). \tag{9}
\]

We know from (7) that the integrand in (8) and (9) is the same if and only if \( w = \hat{w} \). However, the integration is over values of the wholesale price that are strictly smaller than \( \hat{w} \). This does not affect (8), as the integrand there is independent of the values of the rival wholesale price but implies that all values of the integration in (8) are smaller than \([\hat{w} - \overline{w}]D(\hat{w})\). Consequently, it follows that

\[
\int_{\underline{w}}^{\hat{w}} (p^M(\overline{w}) - \overline{w})D(p^M(\overline{w}))dH(w) > \int_{\underline{w}}^{\hat{w}} D(w)(w - \overline{w})dH(w).
\]

To ensure indifference between \( \overline{w} \) and \( \overline{w} \), the distribution must entail a mass point at \( \hat{w} \). This will not affect the profit in (8) as suppliers do not accept the contract of \( R_i \) in case \( w_i = \overline{w} \) and \( w_j = \hat{w} \) but discretely increases the profit in (9) as at least one supplier accepts both contracts if \( w_i = \overline{w} \) and \( w_j = \hat{w} \). In addition, as mass point at \( \hat{w} \) is the only possibility to achieve equality between (8) and (9).

In Proposition 1, we characterized the support of the mixing range and establish the

\textsuperscript{16}This is another difference to models of sales (Varian, 1980) or Bertrand-Edgeworth cycles (see, e.g., Maskin, 1986).
existence of a mass point. However, we did not determine the distribution functions in the two intervals \([\underline{w}, \hat{w}]\) and \((\hat{w}, \overline{w}]\). To do so, we first write down \(R_i\)'s expected profit. Let us denote the distribution function in the interval \(w_j \in [\underline{w}, \hat{w}]\) by \(G(w)\) and the distribution function in the interval \(w_j \in (\hat{w}, \overline{w}]\) by \(F(w)\). The mass point at \(\hat{w}\) then has a probability mass \(\alpha(\hat{w}) = 1 - \int_{\underline{w}}^{\hat{w}} dG(w) - \int_{\hat{w}}^{\overline{w}} dF(w)\).

To write the expected profit in the most concise form denote let us denote \(\Omega(w_i) \equiv w_iD(p^m(w_i))/2D(w_i)\), with \(\Omega(w_i) < w_i\). That is, if \(R_i\) charges a wholesale price of \(w_i \in (\hat{w}, \overline{w}]\), it successfully overbids \(R_j\)'s wholesale price only if \(w_j < \Omega(w_i)\). The formula therefore corresponds to the (overbidding) response formulated by (3).

\[\text{We can write down } R_i\text{'s expected profit as follows:}\]

\[
E[\pi_i(w_i)] = \begin{cases} 
\int_{\underline{w}}^{\hat{w}} D(w)(w - w_i)dF(w) + \alpha(\hat{w})D(\hat{w})(\hat{w} - w_i) & \text{if } w_i \in [\underline{w}, \hat{w}] \\
\int_{\Omega(w_i)}^{\hat{w}} D(w)(w - w_i)dG(w) & \text{if } w_i \in [\underline{w}, \hat{w}]
\end{cases}
\]

\[\int_{\hat{w}}^{\overline{w}} [p^m(w_i) - w_i] dF(w) + \int_{w_i}^{\overline{w}} D(w)(w - w_i)dG(w) & \text{if } w_i \in (\hat{w}, \overline{w}].\]

\[\text{(10)}\]

In the expected profit, we need to distinguish between the two intervals of the mixing range. When setting a wholesale price in the interval \([\underline{w}, \hat{w}]\), \(R_i\) obtains a strictly positive profit only if its price falls below the one of the rival by not too large an amount. Specifically, using our definition above, this is the case if \(w_i \leq w_j \leq \Omega^{-1}(w_i)\). This implies that if both wholesale price offers are in the interval \([\underline{w}, \hat{w}]\), \(R_i\) receives a strictly positive demand only if \(w_j \geq w_i\), since then both suppliers (also) accept \(R_i\)'s offer. The respective profit of \(R_i\) equals the first term of the first expression of (10). The same applies to the cases of \(R_j\) setting \(w_j = \hat{w}\), where \(R_j\)'s profit is the second term, and of \(R_j\) choosing a wholesale price from \((\hat{w}, \overline{w}]\) so that \(w_j \leq \Omega^{-1}(w_i)\), where \(R_j\)'s profit is the third term of the first expression of (10). If instead \(w_i \in (\hat{w}, \overline{w}]\), the profit of \(R_i\) is strictly positive if \(R_j\) charges either a higher wholesale price, i.e., \(w_j > w_i\), so that both suppliers also accept \(R_i\)'s offer or if \(R_j\)'s wholesale price is particularly low, i.e., \(w_j < \Omega(w_i)\), so that both suppliers accept solely \(R_i\)'s offer. The profit of \(R_i\) in the latter case is represented by the first term and \(R_i\)'s profit in the first case by the second term of the second expression of (10).

Retailers are only willing to mix between all wholesale price offers \(w_i \in [\underline{w}, \overline{w}]\) if they involve the same expected profit. To solve for the equilibrium mixing probabilities, we can differentiate both expressions with respect to \(w_i\) and set the resulting terms equal to zero.\(^{18}\) Using Leibniz's rule and differentiating the first equation of (10)—the one relevant

\(^{17}\)Similarly, successful overbidding by \(R_j\) requires that \(w_j = \Omega^{-1}(\hat{w}_i)\).

\(^{18}\)Since expected profits must be identical for all \(w_i \in [\underline{w}, \overline{w}]\), the effect of a change in \(w_i\) on the expected profits must be zero.
for \( w_i \in [w, \hat{w}] \)—with respect to \( w_i \) yields

\[
0 = -\int_{\hat{w}}^{w_i} D(w)G'(w) \, dw - \left( \left( \Omega^{-1} \right)'(w_i) \right) \{D(\Omega^{-1}(w_i))(\Omega^{-1}(w_i) - w_i)G'(\Omega^{-1}(w_i))\} \\
- \alpha(\hat{w})D(\hat{w}) - \int_{w_i}^{\hat{w}} D(w)F'(w) \, dw.
\]

(11)

Similarly, differentiating the second term of (10)—the one relevant for \( w_i \in [\hat{w}, w] \)—with respect to \( w_i \) yields

\[
0 = F(\Omega(w_i)) \left[ (p^m)'(w_i) - 1 \right] + F'(\Omega(w_i)) \Omega'(w_i) [p^m(w_i) - w_i] - \int_{w_i}^{\hat{w}} D(w)G'(w) \, dw.
\]

(12)

Let us now consider some wholesale price \( w_i \in [w, \hat{w}] \), which we denote by \( \dot{w} \). Through the definition of the function \( \Omega(\cdot) \) there is a corresponding wholesale price \( w_i \in [\hat{w}, w] \), denoted by \( \ddot{w} \), such that \( \dot{w} = \ddot{w}D(p^m(\ddot{w}))/2D(\ddot{w}) = \Omega(\ddot{w}) \), and therefore \( \dot{w} = \Omega^{-1}(\ddot{w}) \). Using the symmetry of the equilibrium, we can then rewrite (11) and (12) as

\[
0 = -\int_{\hat{w}}^{\ddot{w}} D(w)G'(w) \, dw - \left( \left( \Omega^{-1} \right)'(\ddot{w}) \right) \{D(\ddot{w})(\ddot{w} - \dot{w})G'(\ddot{w})\} \\
- \alpha(\ddot{w})D(\ddot{w}) - \int_{\ddot{w}}^{\hat{w}} D(w)F'(w) \, dw
\]

(13)

and

\[
0 = F(\ddot{w}) \left[ (p^m)'(\ddot{w}) - 1 \right] + F'(\ddot{w}) \Omega'(\ddot{w}) [p^m(\ddot{w}) - \ddot{w}] - \int_{\ddot{w}}^{\hat{w}} D(w)G'(w) \, dw.
\]

(14)

In the appendix, we take the derivative of (13) with respect to \( \dot{w} \) and the derivative of (14) with respect to \( \ddot{w} \). This gives us two equations which are free of integrals. They constitute a system of two equations depending on the distribution functions \( G(\ddot{w}) \) and \( F(\dot{w}) \) and their higher-order derivatives. Solving this system of differential equations for \( G(\ddot{w}) \) and \( F(\dot{w}) \) can be done for any combination of points \( \dot{w} \in [w, \hat{w}] \) and \( \ddot{w} \in [\hat{w}, \overline{w}] \), which then yields the implicit solution for the distribution function. This is summarized in the following proposition.

**Proposition 2.** The distribution function \( F(\dot{w}) \) for all \( \dot{w} \in [w, \hat{w}] \) and the distribution function \( G(\ddot{w}) \) for all \( \ddot{w} = \in [\hat{w}, \overline{w}] \) is implicitly given by the solution to the equations (13) and (14).
The solution can only be obtained in implicit form as the equations involves both higher-order derivatives of the implicit functions $\Omega(\ddot{w}_i)$ and $\Omega^{-1}(\dot{w}_i)$ as coefficients of the differential equation. Hence, it has neither constant coefficients nor is it homogeneous. Consequently, no general method to solve this equation exists and, in addition, the non-constant coefficients are functions in implicit form. Below we will provide a numerical solution for the example with linear demand.

It is clear from the general analysis that with buyer power, the equilibrium outcome is fundamentally different from the one with seller power. The findings with respect to welfare and surplus allocation are formulated in the following Theorem.

**Theorem 1 (Implications of Buyer Power)** With perfect Bertrand competition in the upstream and the downstream market, consumer prices in the mixed-strategy equilibrium with buyer power are above those in the (standard zero-profit) Bertrand equilibrium with seller power. In the former, suppliers and retailers obtain positive profits, whereas consumers pay higher final-good prices. Welfare is lower with buyer power than with seller power.

With buyer power, the wholesale price offers among which retailers randomize in the mixed-strategy equilibrium are strictly above zero. A direct implication is that in stage three, consumer prices will be either equal to the respective monopoly price or equal to the higher wholesale price offer, given this offer constrains the pricing decision of the retailer with the lower offer and given that at least one supplier has accepted this lower wholesale price offer. The retailer that serves the market and the supplier(s) which accepted that retailer’s offer obtain a positive margin and, consequently, consumer surplus with buyer power is lower than with seller power. The reason is that the case where suppliers propose the wholesale price offers involves perfect competition at both layers of the vertical structure. This results in the socially desirable equilibrium in which margins are zero for both suppliers and retailers. However, in case that retailers have bargaining power, the only decision that is left to upstream firms is whether to accept or reject the respective offers proposed by the retailers. In this scenario, retailers compete in both retail prices and wholesale price offers. While the first still involves the socially beneficial property of a Bertrand equilibrium with homogenous goods that retailers opt to undercut the rival’s price to win its consumers away, the second operates in the opposite direction: in order to avoid that none of the suppliers will accept its offer in stage two, a representative retailer must offer a wholesale price strictly above the suppliers’ marginal cost.

Although the direction of the effects of buyer power are obvious in the general model, it does not allow to quantify the effects on surplus allocation. In that sense it is especially
relevant for antitrust policy to give an idea of the deadweight loss that is inflicted by buyer power. In order to address this issue, we set up the model with a linear demand specification in the subsequent section allowing us to quantify the implications of buyer power formulated in Theorem 1.

**Linear demand example**

Consider that retail demand is given by the simple specification \( D(p) = 1 - p \). All other assumptions imposed in the general demand case remain unchanged. In the third stage, the optimal retail pricing decisions are given as follows. In case that both suppliers solely accepted \( R_i \)'s offer, the solution to (2) is given by \( p^m_i(w_i) = (1 + w_i)/2 \). The realized demand is \( D(p^m_i) = (1 - w_i)/2 \) and \( R_i \) obtains a monopoly profit of \( \pi^m_i = (1 - w_i)^2/4 \). In case that wholesale price offers differ and the high offer constrains the pricing decision of the retailer with the low offer and at least one supplier accepted both offers, the retail price will be equal to the high wholesale price offer as outlined above.

Turning to the second stage, assume again that \( w_i \leq w_j \) for \( i \neq j \in \{1,2\} \) and for expositional simplicity, suppose that

\[
 w_i = w \leq w + \Delta = w_j. \tag{15}
\]

Again we first look at the case where \( w_j < p^m_i(w_i) \), that is, \( \Delta < (1 - w)/2 \). Hence, the maximum retail price \( R_i \) can set in case that at least one supplier accepted both offers is given by \( p_i = w + \Delta \). The set of payoffs resulting from all possible combinations of the suppliers’ strategies under (15) is provided in strategic form representation in the appendix, which is the same as Table 1 but with linear demand. In equilibrium, in which both suppliers accept the offer of \( R_i \) and at least one accepts both offers is payoff-dominant if \( \Delta \leq 3w \).

If instead \( \Delta > 3w \), both suppliers are better off from solely accepting the high wholesale price offer. However, given that \( \Delta < (1 - w)/2 \), offering \( \Delta = 3w \), so that both suppliers only accept the high offer requires that \( w \leq 1/7 \).\(^{19}\) Consequently, the maximum possible wholesale price offer to outbid the rival is given by \( 4/7 \).

For sake of completeness consider finally the case where \( \Delta \geq (1 - w)/2 \). This implies that the retailer with the low wholesale price offer can always set the monopoly retail price once a supplier has accepted its offer. If one supplier does so, she obtains a profit of \((w/2)(1 - w)\) given that her rival only accepts the offer of the retailer with the high wholesale price. Comparing this profit with the one in case both suppliers accept only the high offer yields that the latter is larger if \( \Delta \geq 1/2 - w + (1/2)\sqrt{1 - 8w(1 - w)} \). As above, \( w + \Delta \) sufficiently

\(^{19}\)Solving \((1 - w)/2 = 3w\), one obtains \( w = 1/7 \).
high to outbid the rival’s offer requires \( w \leq 1/7 \) implying again that the maximum possible wholesale price offer to outbid the rival equals 4/7.

In the first stage, retailers decide on their optimal wholesale price offers given the suppliers’ strategies in the second stage. The best-response function of \( R_i \) is

\[
w_i(w_j) = \begin{cases} 
   w_j + 3w_j & \text{if } w_j \leq \hat{w} \\
   \frac{w_j}{4} & \text{if } w_j \in \left[\hat{w}, \frac{4}{7}\right] \\
   1 - \frac{\sqrt{1 - 2w_j(1 - w_j)}}{2} & \text{if } w_j > \frac{4}{7},
\end{cases}
\]

(16)

where \( \hat{w} = \frac{1}{38} (11 - 3\sqrt{5}) \) is the solution to (6) under the linear demand function. A detailed derivation of \( \hat{w} \) is provided in the following.

As outlined in the analysis with the general demand function, \( w_j > 4/7 \) will not occur in equilibrium. A retailer \( R_i \)’s best response is either to overbid \( R_j \)’s offer by proposing \( w_i = 4w_j \) so that both suppliers accept only its offer or to underbid \( R_j \)’s offer by offering \( w_j/4 \), so that suppliers accept both contracts. The profit of \( R_i \) from the first strategy is \( \pi_i = (1/4)(1 - 4w_j)^2 \) and the one from the second strategy is \( \pi_i = (3/4)(1 - w_j)w_j \). Equating both profit functions and solving for \( w_j \) yields \( \hat{w} = (1/38) (11 - 3\sqrt{5}) \). From (16), it then follows that the upper bound of the support \( \hat{w} \) over which retailers randomize wholesale price offers equals \( 4\hat{w} = (4/38) (11 - 3\sqrt{5}) \), and, similarly, the lower bound \( \underline{w} \) is given by \( \hat{w}/4 = (1/152) (11 - 3\sqrt{5}) \).

It follows that under the linear demand specification, the equilibrium of the first stage is in mixed strategies where wholesale prices are randomized according to the same distribution function \( G(w) \) for \( w \in [\underline{w}, \hat{w}] \) and \( F(w) \) for \( w \in [\hat{w}, \overline{w}] \), with \( \underline{w} \approx 0.0282, \hat{w} \approx 0.1129 \) and \( \overline{w} \approx 0.4518 \). Both retailers have a mass point at \( \hat{w} \).

We can next write down \( R_i \)’s expected profits over the mixing support \([\underline{w}, \overline{w}]\). We stick to the notation introduced in the case of the general demand function, denoting a wholesale price offer \( \hat{w}_i \) if \( w_i \in (\underline{w}, \hat{w}) \) and \( \hat{w}_i \) if \( w_i \in [\hat{w}, \overline{w}] \). The function \( \Omega(w_i) = 4w_i \) in the linear demand case, which implies \( \hat{w}_i = 4\hat{w}_i \). \( R_i \)’s expected profit in terms of \( \hat{w} \) are given by

\[
E[\pi_i(w_i)] = \begin{cases} 
   \int_{\underline{w}}^{\hat{w}} (1 - w)(w - \hat{w})G'(w) \, dw + \alpha(\hat{w})(1 - \hat{w})(\hat{w} - \underline{w}) \\
   + \int_{\hat{w}}^{\overline{w}} (1 - w)(w - \hat{w})F'(w) \, dw \text{ if } w_i \in [\underline{w}, \hat{w}], \\
   F(\hat{w}) \left(\frac{1}{4}(1 - 4\hat{w})^2\right) + \int_{4\hat{w}}^{\overline{w}} (1 - w)(w - 4\hat{w}G'(w) \, dw \text{ if } w_i \in (\hat{w}, \overline{w}].
\end{cases}
\]

(17)

In the appendix, we show how to proceed in order to solve (17) for the mixing probabilities. There we show that although employing the linear demand specification avoids terms
in implicit form, we still need to solve a non-homogeneous fourth-order differential equation, which does not exhibit constant coefficients. The reason is that in the scenario of asymmetric Bertrand competition, the profit of the retailer with the lower wholesale price offer depends on the mixing strategy of the rival. This is in contrast to standard models involving mixed-strategy equilibria, in which the probability of quoting the lowest price depends on the mixing strategy of the rival, but the profit itself is independent of the rival’s price.

Even though we are not not able to analytically solve for the mixing probabilities, the model with the linear demand specification provides us with a basis for employing a numerical solution algorithm. In doing so, we obtain clear results on the surplus allocation and the welfare consequences. In the following we explain our approach in detail. Note that the solution is an exact numerical one and not only a simulation.

In order to solve for the mixed strategy equilibrium, we need to construct the joint probability density function $H'(w)$ that yields equal expected profits for each value of $w$ of both retailers on the domain $[w, \bar{w}]$. We fragment this interval into gridpoints and determine the probability mass on each gridpoint. Specifically, we approximate $H'(w)$ defined over the interval $[w, \bar{w}] = [w, \hat{w}) \cup \{\hat{w}\} \cup (\hat{w}, \bar{w}]$ with $2M + 1$ gridpoints. Here, the interval $[w, \hat{w})$ is represented by $M$ equally spaced points and, similarly, the interval $(\hat{w}, \bar{w}]$ is also partitioned into $M$ equally spaced points. Finally, the wholesale price offer $\{\hat{w}\}$ is represented by one single gridpoint. Note that because of the fragmentation into gridpoints we do not need to segment into two different probability density functions for $w \in [w, \hat{w})$ and $w \in [\hat{w}, \bar{w}]$.

We index gridpoints by $m$ when referring to $R_i$’s actions (i.e., wholesale price offers) within its mixing range and by $n$ when referring to $R_j$’s actions within its mixing range, where $m, n \in \{1, \ldots, 2M + 1\}$. Hence, we have a total number of action pairs $\{m, n\}$ that equals $(2M + 1)^2$.20

We proceed as follows. To calculate the expected profits of both retailers for each action pair $\{m, n\}$, we first pin down the respective best retail-price responses given the suppliers’ acceptance decision and the according consumer demands. Given this, we compute the best responses in the wholesale price offers. Then, we compute the symmetric randomizing strategy that assigns a probability density to each action involving identical expected profits along $[w, \bar{w}]$, that is, $H'_m$ at each gridpoint $m$ within $R_i$’s mixing support and analogously $H'_n$ at each gridpoint $n$ within $R_j$’s mixing support. This allows us finally to solve for the retailer’s expected profits. The equilibrium pricing decisions in the third stage evolve as described at beginning of our example with the linear demand function. Similarly, in the first stage, the retailers’ set of best responses in wholesale prices are according to (16). Following the discussion of the model section, $R_i$’s profits at each gridpoint $m$, given an

\[20\text{We set } M=50 \text{ in the numerical solution.}\]
action \( n \in \{1, \ldots, 2M + 1\} \), chosen by \( R_j \) are given by

\[
\pi_{mn} = \pi(w_m, w_n) = \begin{cases} 
\frac{(1-w_m)^2}{4} & w_m > 4w_n \\
(w_n - w_m)(1 - w_n) & w_m < w_n \cap 4w_m > w_n \\
0 & \text{otherwise}
\end{cases}
\]

Thus, in analogy to (17), the expected profits of \( R_i \) from choosing action \( m \) and considering \( H'_n \) as given are written as:

\[
E[\pi_{mn}] = \sum_n H'_n \pi_{mn} = E[\pi] \quad \forall m.
\]

The last equality of (18) follows from the condition that a mixed-strategy equilibrium requires identical expected profits for each \( m \) within the mixing support. By symmetry it must be that \( H'_m = H'_n \quad \forall \, m,n \). This gives us \( 2M + 1 \) equations. Finally, since \( H'_n \) is a probability distribution, we have

\[
\sum_n H'_n = 1.
\]

Therefore, we have \( 2M + 2 \) equations, and the same number of unknowns. These are the probability mass on each gridpoint \( m \) (denoted by \( H'_m \)) for \( 2M + 1 \) gridpoints and the expected profit \( E[\pi] \).

Figure 3 plots the probability density function and the respective cumulative distribution function of the optimal strategy mix of a representative retailer along the support \([w, \hat{w}]\). As shown in the discussion of Proposition 1, retailers shift probability mass to \( \hat{w} \) so that the joint density function \( H'(w) \) can be split into the two separate density functions \( G'(w) \) and \( F'(w) \).\(^{21}\) What can be seen is that retailers assign higher probabilities to wholesale prices in the lower interval \([w, \hat{w}]\) than to wholesale prices in the upper interval \((\hat{w}, w]\).

As explained above, the system described by (18) and (19) consists of \( 2M + 2 \) equations and \( 2M + 2 \) unknowns, that is, \( 2M + 1 \) for \( H'_n \) and one for \( E[\pi_{mn}] \). Using a conventional Newton algorithm, we obtain the exact numerical solution of this fixed-point problem. This solution of the system (18) and (19) allows us to compute the distribution of surpluses. The expected aggregated surplus of the retailers is given by

\[
E[\Pi^R] = 2E[\pi] = \sum_m \sum_n H'_m H'_n D(p_{mn})(p_{mn} - w_{mn}) = 0.0664. \tag{20}
\]

Similarly, the expected aggregated surplus obtained by the suppliers is

\(^{21}\)The same logically applies to the respective cumulative distribution function.
Equilibrium probability density function

Equilibrium cumulative distribution function

The expected consumer surplus is given by

$$E[\Pi^C] = \sum_m \sum_n H'_m H'_n D(p_{mn})(1 - p_{mn}) = 0.3085.$$  (22)

Finally, the expected deadweight loss that accrues under buyer power is computed as

$$E[DWL] = \sum_m \sum_n H'_m H'_n \frac{p_{mn}(1 - D(p_{mn}))}{2} = 0.0452.$$  (23)

Table 2 summarizes the results obtained under buyer power and puts them in relation with those obtained under seller power. There are two striking observations. First, consumers strongly suffer from buyer power. Their aggregated surplus is almost 40 percent lower compared to the case where the suppliers propose the wholesale price offers. Second, a large part of the lost consumer surplus is captured by suppliers. This implies that suppliers strongly benefit from passing bargaining power onto retailers.

5 Conclusion

This paper analyzed a simple model of a vertically related industry, in which both upstream and downstream firms sell a homogeneous product at their respective layer. We compared
Table 2: Welfare implications

<table>
<thead>
<tr>
<th></th>
<th>retailer profits</th>
<th>manufacturer profits</th>
<th>consumer surplus</th>
<th>deadweight loss</th>
</tr>
</thead>
<tbody>
<tr>
<td>buyer power</td>
<td>0.0664</td>
<td>0.0799</td>
<td>0.3085</td>
<td>0.0452</td>
</tr>
<tr>
<td>seller power</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
</tr>
</tbody>
</table>

the effects of buyer power in the wholesale market (i.e., downstream firms making take-it-or-leave-it offers) with supplier power. Whereas marginal cost pricing and zero profits emerge in the unique equilibrium with market power on the seller side, buyer power leads to prices above marginal cost and positive profits at both layers. There exists a unique mixed-strategy equilibrium in wholesale prices, which contains a mass point. Surprisingly, suppliers obtain positive profits in this equilibrium—that is, in case they do not have any market power—but no profit when they make offers to downstream firms. Employing a linear demand example shows that the loss in consumer surplus from buyer power is substantial. As a consequence, our paper provides a new rationale why buyer power is detrimental for consumers, which is independent of market concentration in the downstream stage and the size of firms. It therefore contradicts the long-standing wisdom that, in the wholesale market, the incentives of downstream firms and buyers are aligned.

Our analysis can be extended in many directions. First, we focused on the case of linear wholesale contracts. However, contracts between upstream and downstream firms are often more sophisticated and include fixed payments. Incorporating non-linear tariffs (such as two-part tariffs) in our analysis would not change our main conclusions as the argument why buyer power is anticompetitive would still hold. But it could give new insights on how the contractual form affects market outcomes, that is, are two-part tariffs are more or less competitive than linear wholesale prices. Second, we focused on a scenario with homogeneous products. This allows us to consider the benchmark of supplier power in the simplest way which leads to marginal-cost pricing in both the upstream and the downstream market. In this respect, buyer power can never strictly improve the market outcome, which is already efficient with supplier power. Allowing for product differentiation can provide us with new insights on the precise conditions for buyer power to be welfare-inferior compared to supplier...
power. For example, it may allow us to determine if our insights remain valid if products are sufficiently differentiated either at the upstream or the downstream level, or at both.
Appendix

Proof of Lemma 3.

This proof shows that the best-response functions are unique. We start with the case of
overbidding. The best-response is then characterized by (3). It is evident that for \( w_j = 0 \),
we have \( w_i = 0 \) as well. Taking the total derivative of (3) yields, denoting \( p^m(w_i) \) by \( p_i^m \),
\[
dw_j = \frac{1}{2D(w_i)^2} \left( D(p_i^m) + w_i \frac{\partial D(p_i^m)}{\partial p} \frac{\partial p_i^m}{\partial w_i} \right) D(w_i) - \frac{\partial D(w_i)}{\partial w_i} w_i D(p_i^m) \, dw_i. \tag{24}
\]
Rearranging yields
\[
\frac{dw_i}{dw_j} = \frac{2D(w_i)}{D(p^M(w_i)) \left( 1 + \epsilon_{D,w_i} - \epsilon_{D,p_i^m} \epsilon_{p_i^m,w_i} \right)}, \tag{25}
\]
where \( \epsilon_{D,w_i} = - \left[ \partial D(w_i)/\partial p \right] [w_i/D(w_i)] \), \( \epsilon_{D,p_i^m} = - \left[ \partial D(p_i^m)/\partial p \right] [p_i^m/D(p_i^m)] \), and \( \epsilon_{p_i^m,w_i} = \left[ \partial p_i^m/\partial w_i \right] [w_i/p_i^m] \). That is, \( \epsilon_{D,w_i} \) is the demand elasticity at \( p = w_i \), \( \epsilon_{D,p_i^m} \) is the demand
elasticity at \( p = p_i^m \), and \( \epsilon_{p_i^m,w_i} \) is the elasticity of the monopoly price \( p_i^m \) with respect to
cost. It is easy to check that (25) is strictly positive, implying that there is a one-to-one
mapping between \( w_j \) and \( w_i \).

The best response determined by (4) reflects the case in which \( R_i \) underbids \( R_j \), but \( w_j \)
is sufficiently small so that it puts a limit on the downstream price that \( R_i \) optimally sets.
Following a similar argument as above, we obtain
\[
\frac{dw_i}{dw_j} = \frac{2D(w_i) \left( 1 + \epsilon_{D,w_i} - \epsilon_{D,p_i^m} \epsilon_{p_i^m,w_i} \right)}{D(p^M(w_i)) \left( 2D(w_i) \right)},
\]
which again implies that the best response function is unique.

The best response (5) occurs for values of \( w_j \) that are larger than \( p^M(w_i) \). It is obvious
that (4) and (5) are the same at \( w_j = p^M(w_i) \). Rewriting (5) to collect all terms with \( w_i \) on
the left-hand side and all terms with \( w_j \) on the right-hand side yields
\[
2w_i D(p^M(w_i)) = w_j D(p^M(w_j)). \tag{26}
\]
Taking the derivative of left-hand side of (26) with respect to \( w_i \) gives
\[
2 \left( D(\cdot) + w_i (\partial D(\cdot)/\partial p^m(\cdot))(\partial p^m(\cdot)/\partial w_i) \right).
\]
Using the Envelope Theorem, we obtain
\[
2\partial D(\cdot)/\partial p^m(\cdot) \left( w_i (1 + \partial p^m(\cdot)/\partial w_i) - p^m(w_i) \right). \tag{27}
\]
Similarly, taking the derivative of the right-hand side of (26) with respect to \( w_j \) and employing the Envelope Theorem yields

\[
\frac{\partial D(\cdot)}{\partial p_m(\cdot)} (w_j (1 + \frac{\partial p_m(\cdot)}{\partial w_j}) - p_m(w_j)).
\] (28)

Suppose that \( w_j \) converges to the choke-off price \( \bar{p} \). Thus, the right-hand side of (26) converges to 0 as \( p_m(w_j) \to \bar{p} \). The best response of \( R_i \) is to set \( w_i = 0 \), which allows \( R_i \) to reap the monopoly profit and implies that the left-hand side of (26) is zero as well. In addition, as \( w_j \to \bar{w} \), the slope of the right-hand side of (5), given by (28), converges to \( 2(\frac{\partial D(\cdot)}{\partial p_m(\cdot)}) (-p_m(w_i)) \), which is positive. Thus, it follows that for large values of \( w_j \geq \tilde{w} \), we have \( \partial w_i/\partial w_j < 0 \). A similar argument holds for all \( w_j > p_m(w_i) \) because an increase in \( w_j \) now implies that the offer becomes less attractive for suppliers—the reduction in demand dominates the larger margin—which allows \( R_i \) to reduce \( w_i \). Therefore, the best response is again unique.

Finally, we show that uniqueness of \( \hat{w} \), that is, there exists a unique \( w_j \) at which (6) holds. Let us denote \( R_i \)'s overbidding wholesale price implicitly determined by (3) by \( w_o^i \) and the underbidding price defined by (4) by \( w_u^i \). The monopoly profit from overbidding is given by

\[
\pi_o^i = [p_m(w_o^i(w_j)) - w_o^i(w_j)] D(p_m(w_o^i(w_j))).
\] (29)

Taking the derivative of (29) with respect to \( w_j \) yields

\[
(\partial w_o^i/\partial w_j)\{[(\partial p_m(\cdot)/\partial w_i^o)(p_m^o(\cdot)) - p_m^o(\cdot)](\partial D(\cdot)/\partial p_m(\cdot)) + D(\cdot)] - D(\cdot)\}.
\]

Invoking the Envelope Theorem, it follows from the optimization problem of \( R_i \) in the retail market that at given \( w_j \), we have \( (p_m(w_o^i(w_j)) - w_o^i(w_j))(\partial D(\cdot)/\partial p_m(\cdot)) + D(\cdot) = 0 \). Since \( w_o^i(w_j) \) is monotonically increasing in \( w_j \), as shown above the above expression is strictly negative. As a consequence, \( \pi_o^i \) is a monotonically decreasing function in \( w_j \) Note that the maximum \( w_j \) at which \( R_i \) can submit an overbidding offer is below the choke-off price and given by \( \bar{p}D(p_m(\bar{p})/2D(\bar{p}) \).

Similarly, the profit from underbidding by (4) is given by

\[
[w_j - w_u^i(w_j)] D(w_j).
\] (30)

In contrast to the case of overbidding outlined above, if \( w_j \) equals the choke-off price, there
is a \( w_i \) at which \( R_i \) successfully underbids and sets the monopoly retail price. It follows that (30) is larger than (29) for high values of \( w_j \).

Considering the other extreme of \( w_j = 0 \), it is evident that overbidding is more profitable than underbidding as (30) will be zero, whereas (29) is strictly positive.

Thus, it remains to show that there is a unique point of intersection between (29) and (30). Comparing the two derivatives of (29) and (30) and using (6), we can show that at any intersection point, (30) must cross (29) from below. It follows that there is a unique intersection point.

**Differential equations**

The derivative of (13) with respect to \( \dot{w}_i \) can be formulated as follows:

\[
0 = G'(\ddot{w}) \left\{ \left( (\Omega^{-1})' (\ddot{w}) \right)^2 D'(\ddot{w})(\ddot{w} - \dot{w}) + D(\ddot{w}) (\Omega^{-1})' (\ddot{w}) \left( (\Omega^{-1})' (\ddot{w}) - 2 \right) + (\Omega^{-1})'' (\ddot{w})(\ddot{w} - \dot{w}) \right\} \\
+ (\Omega^{-1})' (\ddot{w}) G''(\ddot{w}) D(\ddot{w})(\ddot{w} - \dot{w}) + D(\ddot{w}) F'(\ddot{w}).
\]

Similarly, the derivative of (14) with respect to \( \ddot{w}_i \) can be written down as follows:

\[
0 = 2 F'(\ddot{w}) \Omega'(\ddot{w}) [p^m(\ddot{w}) - 1] + F''(\ddot{w}) (\Omega')^2 (\ddot{w}) [p^m(\ddot{w}) - \ddot{w}] \\
+ F(\ddot{w}) (p^m)''(\ddot{w}) + F'(\ddot{w}) \Omega''(\ddot{w}) [p^m(\ddot{w}) - \ddot{w}] + D(\ddot{w}) G'(\ddot{w}).
\]

**Strategic form representation of the 2nd stage supplier profits - linear demand function**

Given that \( w_i = w \leq w + \Delta = w_j \) and that \( w_j < p^m(w_i) \), all possible payoffs that can be obtained by the suppliers in the second stage are represented by the following strategic form representation.

\[
S_k
\]

<table>
<thead>
<tr>
<th>( S_i )</th>
<th>( \text{accept both} )</th>
<th>( \text{accept } R_j )</th>
<th>( \text{accept } R_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{accept both} )</td>
<td>( \frac{w[1-(w+\Delta)]}{2}, \frac{w[1-(w+\Delta)]}{2} )</td>
<td>( w[1-(w+\Delta)], 0 )</td>
<td>( \frac{w[1-(w+\Delta)]}{2}, \frac{w[1-(w+\Delta)]}{2} )</td>
</tr>
<tr>
<td>( \text{accept } R_j )</td>
<td>( 0, w[1-(w+\Delta)] )</td>
<td>( \frac{w+\Delta[1-(w+\Delta)]}{4}, \frac{w+\Delta[1-(w+\Delta)]}{4} )</td>
<td>( 0, w[1-(w+\Delta)] )</td>
</tr>
<tr>
<td>( \text{accept } R_i )</td>
<td>( \frac{w[1-(w+\Delta)]}{2}, \frac{w[1-(w+\Delta)]}{2} )</td>
<td>( w[1-(w+\Delta)], 0 )</td>
<td>( \frac{w[1-w]}{4}, \frac{w[1-w]}{4} )</td>
</tr>
</tbody>
</table>
Differential equations - linear demand function

With linear demand, the derivatives can be written as

\[ 0 = (1 - \dot{w}) F'(\dot{w}) + 48 \dot{w}(1 - 4 \dot{w}) G''(4 \dot{w}) - 48 w G'(4 \dot{w}) - 4(1 - 4 \dot{w}) G'(4 \dot{w}) + 12(1 - 4 \dot{w}) G'(4 \dot{w}) \]  

and

\[ 0 = (1 - 4 \dot{w})^2 \frac{64}{64} F''(\dot{w}) - \frac{(1 - 4 \dot{w})^4}{4} F'(\dot{w}) + \frac{1}{2} F'(\dot{w}) - (4 \dot{w} - 1) G'(4 \dot{w}). \]

In a next step, we solve (32) for \( G'(\cdot) \) and differentiate with respect to \( \ddot{w} \). Substituting both \( G'(\cdot) \) and \( G''(\cdot) \) into (31), we obtain the following third-order differential equation

\[ 0 = (3 - 27 \dot{w}) F'(\dot{w}) + \frac{(4 \dot{w} - 1)}{16} \left( 3 \dot{w}(1 - 4 \dot{w}) F'''(\dot{w}) + (2 - 80 \dot{w}) F''(\dot{w}) \right) - 4 F'(\dot{w}). \]

As outlined in the case with a general demand function, (33) does not have constant coefficients and it is non-homogeneous. The existence of non-constant coefficients in (33) stems from the fact that when underbidding the rival so that both contracts are accepted, not only the probability determining the expected profit, but also the profit itself is integrated over \( R_i \)'s offered wholesale price.

\[ ^{22} \text{When differentiating (32), the arguments of } G'(\cdot) \text{ and } G''(\cdot) \text{ are } \ddot{w}, \text{ while, when inserting back into (31), the arguments are again written as } 4 \dot{w}. \]
References


