Consumer Stockpiling as a Form of Behavioural-Based Price Discrimination

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Abstract

In this paper, we investigate the effect of behavioural-based price discrimination (BBPD) on a storable good market. We set up a two-period monopoly model with consumer stockpiling. In equilibrium, when the product differentiation is high relative to the transaction cost, consumer stockpiling behaviour can be used as a device for firm to perform such price discrimination. The results of welfare show that consumer stockpiling improves consumer surplus and profit despite the associated BBPD.

Key Words: Consumer Inventory; Dynamic Pricing; Price Discrimination

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1 Introduction

It is frequently observed that consumer’s demand involves a pattern of intertemporal demand shift through advance-purchase, such as fixed price energy tariff, fixed rate mortgage, and long term contract like gym membership, mobile phone contract and subscription market etc. In these examples, consumers pay a fixed price for the service they are about to receive in the future. One may argue that it is large uncertainty of future and risk preferences that triggers consumer to bring in their future demand at current fixed price. However, in some other market where risk preference is absent such as consumer product market, there is also many evidences suggest that consumers often buy more for future consumption for a wide range of product, including bottled soft drink, coffee, pasta, and laundry detergent, among many others.

For both consumer stockpiling and long term contract, consumers become inactive in the future. This allows seller to recognise early buyer from others and provides an opportunity for seller to price discriminatingly to different consumers. Despite the literature of behavioural-based price discrimination (BBPD) having received a wide attention in recent years, analysis of such stockpiling-based BBPD remains rare. Moreover, associated welfare analysis remains ambiguous in the literature. Addressing these omissions is important for the implications of consumer policy in relevant market.

This paper aims to help fill these gaps and makes two main contributions. Firstly, it provides a dynamic model that considers consumer stockpiling behaviour. In equilibrium we show how consumer stockpiling can be used by firm as a special device to perform BBPD against consumers. Secondly and more substantially, we provide a welfare analysis of such effects of BBPD, and demonstrate that such price discrimination through stockpiling always improves both consumer surplus and total welfare. Hence, policymakers should not be concerned by such a form of price discrimination.

More specifically, we borrow the random utility choice model (Perloff and Salop 1985) to propose a two-period differentiated monopoly that incorporates consumer stockpiling
behaviour. In each period, each consumer wishes to consume at most one unit, but is allowed to purchase the second unit to stockpile for future consumption. If consumers purchase in period 2, then a unit of transaction cost is incurred.

The model suggests that in any equilibrium, there is positive consumer stockpiling. Depending on the level of product differentiation, there are two cases in which all consumers stockpile and one case in which a proportion of consumers stockpile. In the corner solutions, all consumers stockpile in advance to save the expected expenditure on transaction cost in period 2. The firm endogenously sets a profit maximised price when the product differentiation level is sufficiently low relative to transaction cost. When the product differentiation level is moderate relative to transaction cost, this profit maximised price is constraint and thus the firm sets a lower price than the benchmark case. In the interior case where the level of product differentiation is higher relative to the transaction cost, consumers with higher match value stockpile in period 1, leaving those with low match value being active in period 2. Here, both period 1 and period 2 prices are lower than the benchmark case. This is because the firm needs to charge a lower price to low match value consumers in period 2. Then, in period 1 pricing, the firm needs to set its period 1 price higher than the period 2 price to sustain positive consumer stockpiling such that it can segment consumers to different groups according to their match value. This shows how the firm uses stockpiling as a device to identify consumers by their match values and charges discriminating prices against consumers.

We then examine the welfare effects. Particularly, we compare the welfare effects of BBPD with that of the normal pricing scheme. Despite some previous literature suggests that welfare effects are difficult to capture in a model of consumer stockpiling (Hendel and Nevo, 2013). Here, we provide a clear prediction relative to normal pricing benchmark that being able to stockpile always increases both firm’s profits and consumers surplus. This is because with BBPD, i) consumers can buy at lower prices, and ii) consumers can save the transaction cost. It also increases the firm’s profits. This is because even if performing BBPD
means to charge lower prices than normal pricing scheme, profit loss can be compensated by the increased sale.

Finally, we provide a brief extension of an alternative transaction cost assumption. It was originally assumed consumers to incur transaction cost in period 2. Here, it is modified to be incurred for the second trip only. In this extension, we show how this alternative makes no difference to the welfare effects of BBPD if we assume market coverage.

As a bi-product, the findings of this paper can also be used as an alternative explanations of price discrimination on some other industries that exist long-term contracts. These contracts allow consumers to pay a fixed-price in advance for the services and products that they receive in the future. Similar to consumer stockpiling, these contracts induce future demand to be shifted forward. It then follows that the monopolist can identify its previous consumers. Given this, the monopolist may choose to charge new consumers a different price. Common examples include telecommunication markets, gyms membership, magazine subscriptions and bank services.

In regards to the literature, this paper firstly relates to the literature on consumer stockpiling. There are some marketing literature that focuses on the underestimation of increased demand from stockpiling in a price promotion period. (See Gupta, 1988; Bell et al, 1999; Gedenk et al, 2010; etc.). However, this does not allow for endogenous pricing and therefore is insufficient for an analysis of price discrimination. For some other studies that allow for endogenous pricing (e.g. Anton and Das Varma, 2005; Hosken and Reiffen, 2007; Guo and Villas-Boas, 2007), they either consider a quantity competition or considers equilibria in which consumers don’t stockpile. This limits an analysis of welfare effects of stockpiling. Different from these above-mentioned paper, Hendel and Nevo (2003) study intertemporal price discrimination when consumers can store for future consumption. There are two types of consumers: price-sensitive consumer who stockpile for future, and less price-sensitive consumers who do not. Their result suggests that the welfare effect of BBPD is ambiguous, while we provide a clear and crisp result that how BBPD can strictly increase consumer
surplus and total welfare.

More broadly, this paper is related to the wider literature on BBPD. Rossi et al. (1996) has pointed out that firms in many industries can price discriminate on the basis of purchase history of consumers. Since then, BBPD becomes a hot topic in the field of industrial organisation and quantitative marketing. In the literature, apart from some comprehensive surveys made by Armstrong (2006) and Fudenberg and Villas-Boas (2007). Most studies on BBPD focus on competitive price discrimination. Some of them focuses on consumer poaching where firm charges one price to its loyal consumer and a lower price to its rival’s consumer. (see Fudenberg and Tirole, 2000; Hawswald and Marquez, 2006; Villas-Boas, 1999). Some other papers connect switching cost and BBPD (Chen, 1997; Shaffer and Zhang, 2000; and Taylor, 2003). They suggest that firm offers discounted price to compensate switching cost and thus gains less profits. Villas-Boas (2004) examines a monopolist selling to overlapping generations of heterogeneous consumers. The equilibrium involves cycles in price being charged to new consumers. For the welfare, he draws a result that the monopolist is worse off than if it could not perform price discrimination from recognising previous consumers. Jing (2011) considers a monopolist selling experienced product market where consumers’ valuation can only be fully understood after purchase. Welfare effects is subject to the condition of the market. Our study differs from two perspectives. First, by solely focusing on stockpiling behaviour, we show how it can be used as devices towards BBPD, and secondly, we provide a welfare analysis of such effects of BBPD. Second, we demonstrate that that such price discrimination through stockpiling always improves both consumer surplus and total welfare.

This paper proceeds as follows. Section 2 introduces the model. Section 3 and 4 present the main equilibrium analysis, before Section 5 provides the analysis of welfare effects and Section 6 shows an extension of alternative assumption of transaction cost. Finally, Section 7 concludes. All proofs are in the appendix.
2 Model

2.1 Assumptions

Consider a single product monopoly with zero production costs. The firm sells storable product over two periods, \( t = 1, 2 \). There is a unit mass of risk-neutral consumers with quasi-linear preferences, each of whom consumes at most one unit of the product per period. The market is not fully covered in a sense that consumers may choose not to buy at all in any given period. For a given price \( p_t \), consumer \( m \)'s net utility of consuming one unit is \( u_m = \varepsilon_m - p_t \), where \( \varepsilon \) is a consumer specific match value. Each match value, \( \varepsilon \), that remains fixed throughout the game and is independently distributed across consumers with \( G(\varepsilon) \). We assume \( G(\varepsilon) \) is continuous and twice differentiable on \([0, b]\) where \( b > 0 \). In particular, we focus on the uniform distribution with \( G(\varepsilon) = \frac{\varepsilon}{b} \) and \( g(\varepsilon) = \frac{1}{b} \). The parameter, \( b \), is used and interpreted as the degree of product differentiation.

In our model, we assume that transactions are potentially costly for consumers. Such a transaction cost may be required in order to make a purchase and is independent of the number of units bought. Common examples includes the costs of visiting a firm or ordering a delivery. To ease exposition in the main model, we assume that transaction costs are zero in period 1, but equal to \( \kappa \in (0, b) \) in period 2.\(^1\) This captures the fact that repeat transactions are particularly costly for consumers and as we later show, it is the level of transaction costs in period 2, rather than period 1, that are important for consumers' stockpiling decisions. However, in Section 6, we show how this assumption can be relaxed to allow for positive transactions costs in both periods. For simplicity, we also suppose that all agents have a discount factor close to one, as most appropriate for products that are purchased frequently (e.g. bottles of cola).

We consider a one-shot game with two periods. In period 1, the firm chooses its period 1

\(^1\)To ensure that the whole market is active, \( \kappa \) cannot be too large, \( \kappa < b \). One can also easily extend the model to allow consumers to have positive stockpiling costs, \( s \geq 0 \), as consistent with the costs of storing a product. The results then hinge on the level of net transaction costs, \((\kappa - s)\), rather than \( \kappa \).
price, $p_1$. The firm is unable to commit to its period 2 price. Consumers learn match values and observe the period 1 price before making period 1 purchase and stockpiling decisions - they can choose to not buy at all, to buy one unit, or to stockpile by buying two units. In period 2, the firm sets its period 2 price, $p_2$. Any remaining consumers that did not stockpile in period 1 then observe this price and choose whether to buy one or zero units. We focus then seek an equilibrium with equilibrium prices $p_1^*$ and $p_2^*$.

### 2.2 Benchmark Analysis

We first briefly examine a benchmark case where consumer stockpiling is not feasible. In this case, the two periods are almost identical, apart from the transaction cost in period 2. In any period, a consumer will purchase one unit if his match value exceeds the cost of purchasing. As such, consumer will buy one unit in period 1 with probability $\Pr(\varepsilon - p_i \geq 0)$ and one unit in period 2 if $\Pr(\varepsilon - p_2 - \kappa \geq 0)$. Accordingly, firm’s demand in period 1 and period 2 can be written as,

\[
Q_1(p_1^{NS}) = 1 - G(p_1) = \frac{b - p_1}{b}, \quad Q_2(p_2^{NS} + \kappa) = 1 - G(p_2 + \kappa) = \frac{b - (p_2 + \kappa)}{b}
\]  

(1)

After applying the usual first order condition, one then obtain the non-storage equilibrium prices and quantity. In period 1, the firm sets $p_1^{NS} = \frac{b}{2}$, and the equilibrium quantity is $Q_1^{NS} = \frac{1}{2}$. In period 2 the firm sets $p_2^{NS} = \frac{b-\kappa}{2}$ and sells $Q_2^{NS} = \frac{b-\kappa}{2b}$. It can be seen that in period 2, the firm sets lower price. This is because the firm needs to offer a discounted price in period 2 to induce the consumers to incur the transaction cost. In aggregate, the firm earns $\pi^{NS} = \frac{(b-\kappa)^2 + b^2}{4b}$, whereas the equilibrium prices are increasing in the product differentiation $b$, and (weakly) decreasing in the transaction cost $\kappa$.
3 Equilibrium Analysis

We now start the main equilibrium analysis, where consumer stockpiling is feasible. Section 2.2 covers some important preliminary features of stockpiling decisions of consumers in period 1. Section 3.2 then endogenises the firm’s behaviour.

3.1 Consumers’ Decisions

We first characterise some features of consumer’s stockpiling decisions and demand in period 1 for a given period 1 price, $p_1$, and expected period 2 price, $p_e^2$. Then we consider period 2 demand, for a given period 2 price, $p_2$.

3.1.1 Period 1

Consider any given consumer $m$’s options with match value, $\varepsilon_m$, period 1 price, $p_1$ and expected period 2 price, $p_e^2$.

She could: i) choose to stockpile by buying two units in period 1 to gain

$$u_m^S = 2(\varepsilon_m - p_1)$$

ii) not to buy in period 1, but possibly to buy one unit in period 2 to gain

$$u_m' = \max\{\varepsilon_m - p_e^2 - \kappa, 0\}$$

iii) buy one unit in period 1 and possibly to buy one unit in period 2 to gain

$$E(u_m^{NS}) = (\varepsilon_m - p_1) + \max\{\varepsilon_m - p_e^2 - \kappa, 0\}$$

Then note the following. First, if buying in period 1 gives consumer $m$ negative payoffs, i.e. $\varepsilon_m - p_1 < 0$, then a) stockpiling in period 1 gives consumer $m$ a negative payoff as well, and
b) option iii) becomes dominated by option ii). Under this circumstance, consumer $m$ never buys in period 1, but may buy in period 2 only, depending on the period 2 price. Secondly, if $\epsilon_m - p_1 > 0$, consumer $m$ will never choose option ii) to buy one unit only in period 2 because this is dominated by option i) or iii). Thus, the consumer must instead choose between i) and iii) and so will prefer option i) to stockpile if $S_m = u_m^S - E(u_m^{NS}) \geq 0$. It can then be shown that an increase in consumer $m$’s match value weakly increases $S_m$. Now, we can state the following.

Lemma 1. If consumer $m$ with $\epsilon_m$ finds it optimal to stockpile in period 1, then so will any other consumer $k$ with $\epsilon_k > \epsilon_m$. If consumer $m$ with $\epsilon_m$ finds it optimal to not stockpile then so will any other consumer $l$ with $\epsilon_l < \epsilon_m$.

Lemma 1 supports the intuition that given the price of period 1 and expected period 2 price, whether consumer chooses to stockpile or not can be identified by their match value. Particularly, our model predicts that the consumers that are most likely to stockpile are those with relatively higher match values.

To proceed, it is useful to define $\bar{\epsilon}$ as the match value of marginal consumer who is indifferent between stockpiling in period 1, and to define $X(\bar{\epsilon})$ as the resulting level of stockpiling demand. As derived previously in (1), we also define $Q_1(.)$ as the level of consumer demand in period 1 absent the effects of stockpiling, and note that the firm’s total level of demand (observed demand) in period 1 therefore equals $\hat{Q}_1(.) = Q_1(.) + X(.)$. We can then state the following:

Lemma 2. Firm’s observed demand in period 1, $\hat{Q}_1(.)$, is:

$$\hat{Q}_{11}(.) = \begin{cases} 
2Q_1(p_1) = \frac{2(b-p_1)}{b} & \text{if } \bar{\epsilon} \leq p_1 \\
Q_1(p_1) + X(\bar{\epsilon}) = \frac{b-p_1}{b} + X(\bar{\epsilon}) & \text{if } \bar{\epsilon} \in (p_1, b) \\
Q_1(p_1) = \frac{b-p_1}{b} & \text{if } \bar{\epsilon} \geq b 
\end{cases}$$ (2)
This describes three cases. First, if $\bar{\varepsilon} \leq p_1$, all consumers stockpile. Hence, firm’s stockpiling demand is equal to its true period 1 demand, $X(.) = Q_1(.)$, and so firm’s total period 1 demand equals $\hat{Q}_1(.) = 2Q_1(p_1)$, where $Q_1(p_1)$ coincides with the demand in the benchmark, (1). If, instead, $\bar{\varepsilon} \in (p_1, b)$, then only some consumers stockpile. Here, firm’s observed demand equals $\hat{Q}_1(.) = Q_1(p_1) + X(\bar{\varepsilon})$ since an aggregate of $Q_1(p_1)$ consumers buy of which $Q_1(p_1) - X(\bar{\varepsilon})$ buy one unit, and $X(\bar{\varepsilon})$ buys two units. Finally, if $\bar{\varepsilon} \geq b$, then no consumers stockpile, and so period 1 observed demand just corresponds to the benchmark case period 1 demand.

3.1.2 Period 2

We now move on to consider period 2 demand. Similar to period 1, we can define $Q_2(.)$ as the level of consumer demand in period 2 absent the effects of stockpiling, as derived previously in (1). We can then state the following.

**Lemma 3.** The firm’s observed demand in period 2, $\hat{Q}_2$, is,

$$\hat{Q}_2(.) = \begin{cases} 
0 & \text{if } \bar{\varepsilon} \leq p_1 \\
Q_2(p_2 + \kappa) - X(\bar{\varepsilon}) = \frac{b - (p_2 + \kappa)}{b} - X(\bar{\varepsilon}) & \text{if } \bar{\varepsilon} \in (p_1, b] \\
Q_2(p_2 + \kappa) = \frac{b - (p_2 + \kappa)}{b} & \text{if } \bar{\varepsilon} \geq b
\end{cases} \quad (3)$$

If $\bar{\varepsilon} \leq p_1$, all consumers have stockpiled and so period 2 is inactive. However, if $\bar{\varepsilon} \in (p_1, b)$, then consumers with $\varepsilon \in (p_1, \bar{\varepsilon}]$ did not stockpile and so remain active. As in the benchmark, any such consumer will then buy one unit in period 2. It then follows that firm’s observed period 2 demand equals, $\hat{Q}_2 = Q_2(p_2 + \kappa) - X(\bar{\varepsilon})$. If $\bar{\varepsilon} \geq b$, no consumers stockpiled and so period 2 observed demand just equals $\hat{Q}_2 = Q_2(p_2 + \kappa)$. 

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3.2 Firm’s Decisions

Using backwards induction, we now consider the firm’s equilibrium decisions. We start from period 2 for a given level of consumer storage demand from period 1. We then derive the equilibrium levels of stockpiling demand for a given period 1 price and expected period 2 price, $X(p_1, p_2^e)$, where consumers’ expectations of period 2 prices are consistent with the equilibrium, $p_1 = p_2^*(X)$. Finally, given the equilibrium levels of stockpiling demand, we then solve for period 1 equilibrium price.

3.2.1 Period 2

From (3), we know the period 2 market is active only if $\bar{\varepsilon} \in (p_1, b]$. From Section 3.1.1, given the period 2 price, period 2 observed demand comprises of those consumers that i) desire to buy in period 2 such that $\varepsilon -(p_2^e + \kappa) \geq 0$ and ii) did not stockpile in period 1. Suppose a proportion of consumers, indexed by $X(\bar{\varepsilon}) \in (0, 1]$, have stockpiled in period 1, such that $Q_1(.) - X(\bar{\varepsilon})$ consumers are potentially active within the market in period 2. Now we can state the following for the period 2 equilibrium,

**Lemma 4.** Suppose $\bar{\varepsilon} \in (p_1, b]$ such that period 2 market is active. Then, if $b[1-X(\bar{\varepsilon})] - \kappa \geq 0$, then the unique period 2 is achieved in equilibrium with,

$$p_2^*(X(\bar{\varepsilon})) = \frac{b}{2b}(b[1-X(\bar{\varepsilon})] - \kappa) \geq 0 \tag{4}$$

and $\hat{Q}_2^*(.) = Q_2(p_2^*) - X(\bar{\varepsilon}) = \frac{b}{2b}[b(1-X(\bar{\varepsilon})) - \kappa] \geq 0$

The optimal period 2 price, $p_2^*(X(\bar{\varepsilon}))$ is positive if $X(\bar{\varepsilon}) < \frac{b-\kappa}{b}$. Intuitively, those consumers that stockpiled in period 1 are the consumers with the highest match values. Hence the demand in period 2 consists of consumers with lower match values. It indicates that firm acquires consumer’s match values and therefore sets period 2 discriminatory price by observing whether consumers have stockpiled and how many consumer have stockpiled. One can find that period 2 price has the following property. First, it is lower than period 2
equilibrium price in benchmark case \( p_{NS2}^* = \frac{b-\kappa}{2} \). When storage demand reduces to 0, it collapses to benchmark case period 2 equilibrium price. Thus, it can be inferred that consumer stockpiling is the reason of price discrimination in period 2. When stockpiling demand exists, i.e, \( X > 0 \), different groups of consumers, identified by whether they stockpiled or not, have differences in the match value of the same product. In addition, period 2 price is also subject to the level of \( \kappa \). This is because firm need to lower period 2 prices to attract and compensate consumers for the existence of transaction cost, which increases consumer’s expenses to buy in period 2. Having observed differences of consumers’ match value and the level of visit cost, firm can price discriminatingly in period 2 market.

### 3.3 Period 1

From the last section, we learned that period 2 price is determined by the level of stockpiling demand. In this section, we return to period 1 to examine the formation of stockpiling demand. We then solve for period 1 equilibrium price.

#### 3.3.1 Equilibrium Stockpiling Demand

Denote \( X(p_1, p_2^e) \) as firm’s equilibrium level of stockpiling demand, where consumers expectations are correct if \( p_1^e = p_2^e(X) \). As explained below, we can now state the following lemma.

**Proposition 1.** The unique stockpiling demand in period 1 can be expressed as follows:

\[
X^*(.) = \begin{cases} 
0 & \text{if } p_1 \geq \frac{b+\kappa}{2} \\
\frac{b+\kappa-2p_1}{b} & \text{if } \kappa < p_1 < \frac{b+\kappa}{2} \\
\frac{b-p_1}{b} & \text{if } p_1 \leq \kappa 
\end{cases}
\]  

(5)

When making the decisions of whether or not to stockpile, consumers optimally compare
between the cost of stockpiling in period 1 $p_1$, and the cost returning to buy a second unit in period 2 $p_2 + \kappa$. This comparison is subject to the match value of each consumer, the level of period 1 price and expected period 2 price. Proposition 1 displays three different scenarios of stockpiling demand in period 1.

First, consider the first case where no stockpiling demand is facilitated. Here, all consumers who have purchased in period 1 find that stockpiling is less attractive than buying just one unit. From Section 3.1.1, it must be the case that $S^m = u^S_m - E(u^{NS}_m) < 0 \forall \varepsilon_m$. If $E(u^{NS}_m) = max\{\varepsilon - p^*_2 - \kappa, \ 0\} = \varepsilon - p^*_2 - \kappa^2$, rearranging yields the condition of operating zero storage demand, $p_1 > p_2 + \kappa$. Inserting equilibrium period 2 prices of equation (5.4) with $X = 0$, shows that this case requires $p_1 > \frac{b - \kappa}{2} + \kappa = \frac{b + \kappa}{2}$.

Second, consider the intermediate case where $X \in (0, Q(p_1))$. In this case, there exists a consumer who is indifferent between stockpiling and buying 1 unit in each period with $S^m = u^S_m - E(u^{NS}_m) = 0 \forall \varepsilon_m$, within those who have purchased in period 1. Rearranging yields $p_1 = p_2 + \kappa$. Substituting equation (4) and isolating the expression of $X$ yields $X^* = \frac{b + \kappa - 2p_1}{b}$ and then get the condition $\kappa < p_1 < \frac{b + \kappa}{2}$.

Third, consider the third case where $X = Q_1(p_1)$ such that all consumers who buy one unit in period 1 also stockpile. Here, $S^m < 0 \forall \varepsilon_m$. Rearranging yields, $p_1 < p_2(X = Q(p_1) + \kappa$. Inserting equation (4) with $X = Q(p_1) = \frac{b - p_1}{b}$ shows that this case requires $p_1 < \kappa$.

### 3.3.2 Equilibrium Period 1 Price

We now move to derive the equilibrium by solving for the period 1 equilibrium prices. Given changes of stockpiling demand in period 1 and period 2, firm’s profit maximisation problem can be written as,

$$
\pi^S = p_1 \cdot [Q_1(p_1) + X(p^*_1)] + p^*_2(X^*) \cdot [Q_2(p_2) - X(p^*_1)]
$$

If $E(u^{NS}_m) = 0$, then $S^m = 2(\varepsilon_m - p_1) < 0$. In this case consumer do not buy in period 1. The whole market is not active.
where firm receives period 1 demand \( \hat{Q}_1 = Q_1(p_1) + X(.) \) from (2) and (5), and (if active) sets a period 2 equilibrium price, \( p_2^* \), (4), and receives a period 2 equilibrium demand \( \hat{Q}_2 = Q_2(p_2) - X(.) \) from (3). To start with, we first cover the following important result.

**Proposition 2.** In any symmetric equilibrium with \( \kappa > 0 \), there is a positive level of stockpiling demand.

First, note that the benchmark case with no consumer stockpiling cannot qualify as an equilibrium when \( \kappa > 0 \). This is because zero stockpiling demand requires \( p_1 > p_2 + \kappa \). But using Lemma 5.1 and Lemma 5.4, one can find that the benchmark \( p_{1NS}^* = b < p_{2NS}^* + \kappa = \frac{b+\kappa}{2} \) implies the opposite. Hence, we now seek an equilibrium with \( X^* > 0 \). After defining \( Q_1^* \), \( Q_2^* \) and \( X^* \) as the relevant quantities evaluated at equilibrium prices, we can then state the following:

**Proposition 3.** There exists an unique equilibrium which is characterised as follows:

i) If product differentiation is low, \( b < 2\kappa \), then \( X^* = Q_1^* = \frac{1}{2} \), and \( \hat{Q}_2^* = Q_2^* - X^* = 0 \), where \( p_1^* = \frac{b}{2}, p_2^* = 0 \).

ii) If product differentiation is moderate, \( b \in [2\kappa, \frac{5\kappa}{2}] \), then \( X^* = Q_1^* = \frac{b-\kappa}{b} \), and \( \hat{Q}_2^* = Q_2^* - X^* = 0 \), where \( p_1^* = \kappa, p_2^* = 0 \).

iii) If product differentiation is high, \( b > \frac{5\kappa}{2} \), then \( X^* = Q_1^* = \frac{3\kappa}{2b} < Q_1^* = \frac{2b+\kappa}{4b} \) and \( \hat{Q}_2^* = Q_2^* - X^* = \frac{2b+\kappa}{4b} > 0 \), where \( p_1^* = \frac{b}{2} - \frac{\kappa}{4}, p_2^* = \frac{b}{2} - \frac{5\kappa}{4} \).

Proposition 3 establishes the unique equilibrium where consumers stockpile and where the firm sets prices endogenously. Depending on the level of product differentiation, there are three different cases. These can be understood as follows.

The first and the second cases suggest all consumers stockpile in equilibrium. i.e \( X^* = Q_1(p_1^*) = \frac{b-p_1^*}{b} \). In these two cases, consider the marginal consumer who is indifferent between stockpiling with \( u^S \equiv 2(\varepsilon - p_1^*) = (\varepsilon - p_1^*) + (\varepsilon - p_2^* - \kappa) \equiv u^S \). If this consumer were to deviate from equilibrium by not stockpiling, she should rationally expect an equilibrium period 2 price \( \lim_{X^* \to Q_1(p_1^*)} p_2^* = \frac{1}{2}(p_1^* - \kappa) \). Now, her benefits from not stockpiling can therefore be
expressed as $u^{NS} = (\varepsilon - p_1^*) + (\varepsilon - (\frac{p_1^*-\kappa}{2}) - \kappa) = 2\varepsilon - \frac{3p_1^*}{2} - \frac{\kappa}{2}$. Hence, this consumer will stockpile as required in equilibrium only if $u^S > u^{NS} \iff p_1^* \leq \kappa$. Moreover, using a similar logic to Proposition 5.1, this condition is sufficient for all consumers to stockpile in equilibrium. Therefore, any equilibrium with $X^* = Q_1(p_1^*)$ requires $p_1^* \leq \kappa$. To derive $p_1^*$, provided that the period 1 price is less than $\kappa$, the firm selects $p_1$ to maximise (6), where $X = Q_1(p_1)$, such that (6) becomes $\pi = 2p_1Q_1$ subject to $p_1 \leq \kappa$. After applying the normal first order condition, this leads to $p_1^* = \min\{\frac{b}{2}, \kappa\}$.

Hence, in the first case when the product differentiation level is sufficiently low relative to the level of transaction cost, $b < 2\kappa$, all consumers are inclined to stockpile in an effort of avoiding the relatively high transaction cost in period 2. This implies that the firm does not need to reduce its period 1 price to attract consumers to stockpile and firm’s profit is unconstrained at $p_1^* = \frac{b}{2}$. In this case, the equilibrium level of stockpiling demand is $X^* = Q_1^* = \frac{1}{2}$.

In contrast, in the second case when the product differentiation level is moderate relative to the level of transaction cost such that $b \in [2\kappa, \frac{5\kappa}{2}]$. In this case, the firm’s profit maximisation price $\frac{b}{2}$ is bound by the $p_1^* \leq \kappa$ and therefore sets $p_1^* = \kappa < \frac{b}{2}$ to ensure the marginal consumer who is indifferent between stockpiling and not is just willing to stockpile with $u^S = u^{NS}$. In this case, the equilibrium level of stockpiling demand is $X^* = Q_1^* = \frac{b-\kappa}{b}$.

Finally, consider the third case with a higher level of product differentiation level relative to the level of transaction cost, $b > \frac{5\kappa}{2}$. Here, only a strict positive proportion of consumers with high match values stockpile in advance. This leaves the remaining consumers with low match values being active in period 2. Hence, in response to the consumers with relatively lower match values, the firm optimally sets a lower price in period 2, to maintain its market demand. Consequently, as the period 2 price goes down, the period 1 price also falls. Intuitively, from the discussion of Proposition 5.1, we know that $X \in (0, Q_1(\cdot))$ requires $p_1^* = p_2^* + \kappa$. Thus, to ensure that a positive interior proportion of consumers are willing to stockpile, we require $p_1^* = p_2^* + \kappa$, and that this price relationship then uniquely pins
down the proportion of consumers who stockpile. In particular, the appendix then shows that \( p_1^* = \frac{b}{2} - \frac{\kappa}{4} = p_2^* + \kappa = \frac{b}{2} - \frac{5\kappa}{4} + \kappa \) and \( X^* = \frac{3\kappa}{26} \). Finally to ensure that equilibrium is well-defined with non-negative prices, it is necessary that \( b > \frac{5\kappa}{2} \).

The last case in which only an interior proportion of consumers stockpile can also be understood from the perspective of a special form of BBPD. This is because being able to stockpile actually gives firm an opportunity to use consumers’ stockpiling behaviour as a device to perform price discrimination for the unit of the product that will be consumed in period 2. Specifically, the firm sets period 1 price lower than the benchmark period 1 price to attract consumers with high match values to stockpile. By doing so, the firm is able to identify different groups of consumers with different match values from stockpiling behaviour. As a result, period 2 market only consists of those with lower match values. Having segmented the consumers, the firm then sets a even lower period 2 price to the remaining consumers.

4 Comparative Static Analysis

Before considering the welfare effects, we now analyse how the equilibrium level of stockpiling demand, \( X^* \), varies with product differentiation and the transaction cost.

Corollary 1. In equilibrium, the level of stockpiling demand \( X^* \) is weakly decreasing in the level of product differentiation, \( b \), and increasing in the size of the transaction cost, \( \kappa \).

Corollary 1 illustrates how stockpiling demand varies with respect to exogenous market parameters.

In the first equilibrium case from Proposition 3 where the product differentiation is sufficiently low relative to the level of transaction cost, \( b < 2\kappa \), equilibrium stockpiling demand is \( X^* = \frac{1}{2} \). This is independent with any exogenous factors.

In the second equilibrium case where the product differentiation is moderate relative to transaction cost such that \( b \in [2\kappa, \frac{5\kappa}{2}] \), the equilibrium stockpiling \( X^* = \frac{b - \kappa}{b} \) is increasing.
in the product differentiation level and decreasing in the transaction cost. Intuitively, the
demand function of period 1 demand and stockpiling demand are subject to the level of
product differentiation, \( Q_1(.) = X(.) = \frac{b-p_1}{b} \). As \( b \) increases, both stockpiling and period 1
demand increases. Similarly, the firm sets \( p_1^* = \kappa \) in this case, as the level of transaction cost
goes up, both period 1 demand and stockpiling demand go down.

In the last equilibrium case where the product differentiation is high relative to transac-
tion cost, \( b > \frac{5\kappa}{2} \), such that some consumers stockpile, \( X^* \in (0, Q_1(.) \), the equilibrium level
of stockpiling demand increases with product differentiation and decreases with transaction
cost. To get the intuition, first, consider a unit change in product differentiation, \( b \). Holding
\( X^* \) constant, both period prices increase, but the period 1 price increases by more such that
\( p_1^* > p_2^* + \kappa \). As a result, consumers are less inclined to stockpile and \( X^* \) reduces until the
point where \( p_1^* = p_2^* + \kappa \) is restored. Next, consider a unit change in the transaction cost, \( \kappa \).
Compared to the period 1 price, the price 2 price is more responsive to the transaction cost.
Holding \( X^* \) constant, \( p_1^* \) decreases, while \( p_2^* + \kappa \) increases such that \( p_1^* < p_2^* + \kappa \). As a result,
consumers are more inclined to stockpile and \( X^* \) increases until the point where \( p_1^* = p_2^* + \kappa \)
is restored.

5 Welfare

Having characterised the equilibrium, we now consider the welfare effects. Here, we define
define the total welfare as the sum of aggregate consumer surplus, \( CS \), and firm profits, \( \pi \).

\[
W(.) = CS(.) + \pi(.) \tag{7}
\]

5.1 Benchmark

Recall from the Section 2.2 that in the benchmark case where stockpiling is prohibited,
the firm sets a period 1 equilibrium price, \( p_1^{NS} = \frac{b}{2} \), and a period 2 price \( p_2^* = \frac{b-\kappa}{b} \), with
demand $Q_1(.) = \frac{b-p_1}{b}$, and $Q_2(.) = \frac{b-(p_2+\kappa)}{b}$. Equilibrium firm profits then equal $\pi^{*NS} = p_1^*Q_1^* + p_2^*Q_2^* = \frac{b^2+(b-\kappa)^2}{4b}$. Given the levels of transaction costs, one can then also define consumer surplus in period 1 as $CS_1 = \int_{p_1^*}^{b} Q_1(p_1) dp_1$ and consumer surplus in period 2 as $CS_2 = \int_{p_2^*+\kappa}^{b} Q_2(p_2 + \kappa) dp_2 \equiv \int_{p_2^*-\kappa}^{b-\kappa} Q_2(p_2) dp_2$, such that total consumer surplus equals $CS^{*NS} = \frac{b^2+(b-\kappa)^2}{8b}$. After expanding (7), one then obtains $W^{*NS} = \frac{3(b^2+(b-\kappa)^2)}{8b}$.

5.2 Main Model

By comparing these benchmark welfare values to the welfare values evaluated at equilibrium price $p_1^*$, we can now consider the welfare effects of stockpiling.

**Proposition 4.** The possibility of consumer stockpiling always increases the firm’s equilibrium profits, consumer surplus and total welfare.

Proposition 4 summarises the welfare effects of stockpiling. This can be understood as follows.

First, consider the first equilibrium case where product differentiation is extremely low relative to transaction cost such that $b < 2\kappa$. The prices are the same as the benchmark case, so no extra demand is stimulated. However, being able to stockpile brings demand forward from period 2 to period 1. This allows consumer to buy their period 2 unit in period 1 without incurring the transaction cost. It therefore increases consumer surplus. Meanwhile, the firm also benefits from it because as suggested by benchmark case, the firm needs to offer a discounted price in period 2 to induce the consumers to incur the transaction cost. Thus, if all consumers stockpile in period 1, the firm can sell the period 2 demand at a higher period 1 equilibrium price. In aggregate, social welfare, which is given as the sum of consumer surplus and firm’s profit as (7), increases in this case.

Second, consider the second equilibrium case where the product differentiation is moderate relative to transaction cost such that $b \in [2\kappa, \frac{5\kappa}{2}]$. Similar to the first case, all consumer stockpile. However, the firm now sets a lower period 1 price than in the benchmark. Thus,
consumers benefit not only from bringing forward their consumption to avoid the transaction costs, but also from a lower price compared to the benchmark. It can be shown in the appendix that selling at this price still increases the firm’s profit because this price attracts more consumers to buy and stockpile. From above, total welfare therefore increases in this case.

Finally, consider the third equilibrium case where the product differentiation is higher relative to transaction cost such that $b > \frac{5e}{7}$. The period 2 market is active and only a proportion of consumers stockpile, while the firm sets both period 1 and period 2 prices lower than the benchmark case. The reduced prices have two effects. First, they create more demand in both periods. These increase consumer surplus. Second, the reduced price in period 1 attracts consumers to stockpile. Stockpiling consumers are better from saving the transaction cost expenditure in period 2. Thus, consumer surplus are better off. It can then be shown in the appendix that the firm is also better off because the increased sales compensate for the reduced prices. Hence, as both the consumers and the firm benefit, total welfare also rises in this case.

Proposition 4 can also be understood in terms of BBPD. The previous literature suggests that the effects of price discrimination and stockpiling model is typically complex and difficult to derive (Hendel and Nevo, 2013). But in the final stockpiling case of our analysis, we provide a crisp and clear prediction of it. When stockpiling is feasible, the firm sets a lower period 1 price to identify consumers with relatively high match values by attracting them to stockpile, and increase market demand. In period 2, after acknowledging that now only consumers with lower match values are active, the firm then sets its period 2 price lower than its period 1 price for these consumers in a way that benefits both the firm and the consumers.

In other words, the firm would like to enable stockpiling rather than not while policy-makers would encourage consumers to stockpile in an effort of improving consumer surplus and total social welfare.
6 Alternative Transaction Cost Assumption

It was originally assumed that consumers incur a transaction cost if they make a purchase in period 2. We will now show how our results remain robust under a more realistic assumption where consumers incur the transaction cost only if they return to make a second transaction with the firm. However, in order to maintain tractability, this weaker transaction cost assumption requires us to make an additional assumption that the market is covered. In particular, this requires all consumers to consume (but not purchase) a unit in each period. Formally, this is consistent with consumers’ match values that are distributed on the interval \([a, b]\) where \(0 < a < b\) and where \(a\) is large enough to ensure that consumers always consume. Given this, option ii) in Section 3.1.1 becomes invalid. Any given consumer \(m\) only chooses between i) stockpiling in period 1 with utility: \(u^s_m = 2(\varepsilon_m - p_1)\) and, iii) buying 1 unit in each period with expected utility: \(E(u^{NS}_m) = \varepsilon_m - p_1 + (\varepsilon_m - p_2 - t)\).\(^3\) depending the one that maximises their utility. Now this extension with alternative transaction assumption coincides with the Lemma 1 of the main model. Using the same backward induction method as we did in the main model, it follows that the value of storage demand \(X(p_1)\) the following pricing equilibrium and welfare results under this alternative visit cost assumption are now subject to the lower bound of the distribution of consumer’s match value. Accordingly, our result of the welfare effects remains robust.

7 Conclusions

It is often observed that consumers stockpile for future consumption. What we focus in this paper is that how the stockpiling can be used as a device towards BBPD and its welfare implications. Based on a storable product, we set up a two-period monopoly. In the unique equilibrium, due to the existence of transaction costs, consumers stockpile. Depending on the level of product differentiation, there are two cases where all consumers stockpile and one case

\(^3\)It requires that \(a\) is sufficient large such that \(a - p_1 < t\).
where a proportion of consumers stockpile. In the latter case, where product differentiation level is high relative to transaction cost, higher match value consumers stockpile in advance while consumers with lower match value do not. Hence, the firm can segment consumers according to their match value and perform BBPD.

In regards to welfare effects, We show that being able to stockpile always increases aggregate consumer surplus and firm profits despite any potential BBPD. For the firm, this BBPD prompts it to optimally select lower prices in a way that increases its profits from the resulting increase in market demand. For the consumers, their surplus increases due to i) being able to stockpile and thereby reduce their expenditure on transaction costs, and ii) the reduced prices. Hence, policymakers should not be concerned by such a form of price discrimination.

We hope that future research can build on our work in at least three ways. First, further work should generalise, expand, and test out findings to develop the implications of competing market where more than one firms are in the market. Second, future work would be useful if more consumer factors, such as uncertainty and risk aversion, are taken into account. Finally, and more generally, we hope that future research can build on our framework to analyse further storable product related questions.
Appendix

Proof of Lemma 1. : Period 1 consumer’s choice can be summarised as follows: i), \( u'_m = 2(\varepsilon_m - p_1) \), ii), \( u'_m = \max\{\varepsilon_m - p_2^1 - t, 0\} \), and iii), \( E(u^NS_m) = (\varepsilon_m - p_1) + \max\{\varepsilon_m - p_2^1 - t, 0\} \). First consider if \((\varepsilon_m - p_1) < 0\), then i) will be a dominated strategy since \( u'_m < 0 \). Furthermore, ii) will be superior than iii). Thus, if purchasing in period 1 gives consumer a negative payoff such that \((\varepsilon_m - p_1) < 0\), option ii), which is irrelevant to storage demand, is the dominating strategy. On the other hand, if \((\varepsilon_m - p_1) > 0\), ii) will be the dominated strategy. Under such a case consumer will choose to stockpile if \( S_m = u'_m - E(u^NS_m) > 0 \). One can verify that \( S_m \) is weakly increasing in \( \varepsilon_m \) by taking first order derivatives of \( S_m \) to \( \varepsilon_m \), which equals to zero. This completes the proof.

Proof of Lemma 4. : Suppose the match value of marginal consumer \( \bar{\varepsilon} \in [0, b) \), such that firm have positive storage demand. Then one can use \( \pi_2 = p_2 Q_2(.) \) with (3) to derive firm’s period 2 price for a given level of stockpiling demand. Applying normal first order condition yields, \( p^*_2(.) = \frac{1}{2}[b(1 - X(\bar{\varepsilon})) - \kappa] \geq 0 \).

Proof of Proposition 1. : Once we have derived firm’s period 2 equilibrium price, we can use it to derive period 1 equilibrium level stockpiling demand. Firstly, consider consumers’ stockpiling decisions. From Section 3.1, we know consumer optimally compares the cost of stockpiling and the cost of waiting until period 2. If \( S_m = u'_m - E(u^NS_m) = 0 \), consumers are indifferent between stockpiling. By construction, this indigence requires \( p_1 = p_2(X) + \kappa \).

In the case of no stockpiling demand that is suggested by the top line of (5). In this case, \( X = 0 \), such that no consumer finds it optimal to stockpile. It must then follow that \( p_1 > p_2(0) + \kappa \). From (4), \( p^*_2 = \frac{b - \kappa}{2} \) when \( X = 0 \). Therefore, this case requires, \( p_1 > \frac{b - \kappa}{2} + \kappa \), rearranging yields \( p_1 > \frac{b + \kappa}{2} \).

Second, consider another ‘corner case’ in which all consumer stockpiles. In this case, all consumer who has bought in period 1 finds it optimal to stockpile, such that \( Q_1(.) = X(.) = \)
It must then follow that $p_1 < p_2(b - p_1) + \kappa$. By construction, it requires $p_1 < \kappa$.

Lastly, consider the intermediate case in which some consumer stockpiles. In this case, there exists an unique level of equilibrium stockpiling, $X \in (0, Q_1(.))$, such that $p_1 = p_2(X) + \kappa$ holds for firm. To obtain such $X$, one can insert $p^*_2$ from (4) and isolate the expression of $X$ to yield $X = \frac{b + \kappa - 2p_1}{b}$.

Finally, note that the levels of stockpiling and associated conditions in (5) are continuous when $p_1 > 0$.

**Proof of Proposition 3.** a) If $b < \frac{5\kappa}{2}$ such that all consumer stockpiles, firm’s profit maximisation function can be written as,

$$\pi = p_1[Q_1(.) + X(.)]$$

where $Q_1(.) = X(.) = \frac{b - p_1}{b}$. Applying the normal first order condition yields, $p_1 = \frac{b}{2}$. Note that, for all consumer stockpiling to be facilitated, it also requires $p_1 < \kappa$. Therefore, if $b < 2\kappa$, firm charges period 1 equilibrium price, $p^*_1 = \frac{b}{2}$. If $2\kappa < b < \frac{5\kappa}{2}$, firm charges $p^*_1 = \kappa$.

b). If $b > \frac{5\kappa}{2}$, firm’s profit maximisation problem can be written as,

$$\pi^S = p_1 \cdot [Q_1(p_1) + X(p^*_1)] + p^*_2(X^*) \cdot [Q_2(p_2) - X(p^*_1)] \quad (8)$$

where $Q_1(p_1) = \frac{b - p_1}{b}$ and $Q_2(p_2) = \frac{b - (p_2 + \kappa)}{b}$, and where $p^*_2(X^*)$ and $X(p^*_1)$ are given by (4) and (5).

After solving the first order condition of (6) with respect to $p_1^4$, one obtains

$$p^*_1 = \frac{b}{2} - \frac{\kappa}{4}$$

Together with $p^*_2 = \frac{b}{2} - \frac{5\kappa}{4}$, $X^* = \frac{3\kappa}{2b}$, $\pi^* = \frac{(2b - \kappa)^2}{8b} + \frac{\kappa^2}{b}$. This case requires $p^*_2 \geq 0$ or $b \geq (5\kappa/2)$.  

\footnote{The second order condition is given by, $\frac{\partial^2 \pi}{\partial p^2} = -\frac{4}{b} < 0$}
**Proof of Corollary 1.** The proof can be straightforwardly done by taking first order derivatives of \( X^* \) w.r.t \( z \in \{b, \kappa\} \) respectively.

**Proof of Proposition 4.** From Section 2.2, we know equilibrium price of benchmark case is \( p_1^{*NS} = \frac{b}{2} \), and \( p_2^{*NS} = \frac{b-\kappa}{2} \). In this case, firm’s profit function is \( \pi^{*NS} = p_1^{*NS}Q_1(.) + p_2^{*NS}Q_2(.) = (\frac{b-\kappa}{2} + b^2) \). From Section 5.1, we know the consumer surplus of benchmark case is \( CS^{*NS} = \frac{b^2}{8} + \frac{(b-\kappa)^2}{2b} \), and total welfare of the benchmark case is \( \frac{3[b^2 + (b-\kappa)^2]}{8b} \).

If stockpiling is feasible. From Section 3.3.2, there are three different cases,

a). If \( b < 2\kappa \), all consumer stockpiles while firm charges \( p_1^* = \frac{b}{2} \). In this case the profit function is \( \pi^* = 2p_1^*[Q_1(.)] = 2b \). It can then be inferred that being able to stockpile gives firm more profits from \( \pi^* - \pi^{*NS} = \frac{\kappa(2b-\kappa)}{4b} \geq 0 \). Consumer’s welfare is given by \( CS^* = 2\int_{p_1}^{b} Q_1(.) = \frac{b^2}{4} \). It can then be inferred that in this case being able to stockpile gives consumer more surplus from \( CS^* - CS^{*NS} = \frac{\kappa(2b-\kappa)}{8b} > 0 \). From above, it follows that social welfare also increases in this case.

b). If \( 2\kappa \leq b \leq \frac{5\kappa}{2} \), all consumer stockpiles while firm charges \( p_1^* = \kappa \). In this case, the profit function is \( \pi^* = 2p_1^*[Q_1(.)] = \frac{2\kappa(b-\kappa)}{b} \). It can then be inferred that being able to stockpile gives firm more profits from \( \pi^* - \pi^{*NS} = \frac{[(b-\frac{2\kappa}{2}) + \frac{\sqrt{2\kappa}}{2}][(b-\frac{2\kappa}{2}) - \frac{\sqrt{2\kappa}}{2}]}{4b} > 0 \) if \( 2\kappa \leq b \leq \frac{5\kappa}{2} \). Consumer’s welfare is given by \( CS^* = 2\int_{p_1}^{b} Q_1(.) = \frac{(b-\kappa)^2}{b} \). It can then be inferred that when \( 2\kappa \leq b \leq \frac{5\kappa}{2} \), being able to stockpile gives consumer more surplus from \( CS^* - CS^{*NS} = \frac{3[(b-\frac{2\kappa}{2}) + \frac{\sqrt{2\kappa}}{2}][(b-\frac{2\kappa}{2}) - \frac{\sqrt{2\kappa}}{2}]}{4b} > 0 \). From above, it follows that social welfare also increases in this case.

c). If \( b > \frac{5\kappa}{2} \), some consumer stockpiles while firm charges period 1 price \( p_1^* = \frac{b}{2} - \frac{\kappa}{2} \), and period 2 price \( p_2^* = \frac{b}{2} - \frac{5\kappa}{4} \). Firm’s profit maximisation function is now given by (6) and equals \( \pi^* = \frac{(2b-\kappa)}{8b} + \frac{\kappa^2}{b} \). It is straightforward to find that \( \pi^* - \pi^{*NS} = \frac{7\kappa^2}{8b} > 0 \).

For consumer surplus, If \( b > \frac{5\kappa}{2} \), in equilibrium where there is a strict positive proposition...
of consumer stockpile with $X^* = \frac{3\kappa}{2b} \in [0, Q_1(p_1)]$, we have $p_1^* = p_2^* + \kappa,$

$$CS = \int_{p_1^*}^{b} [Q_1(p_1) + X^*] dp_1 + \int_{p_2^* + \kappa}^{b} [Q_2(p_2 + \kappa) - X^*] dp_2$$

$$= \int_{p_1^*}^{b} Q_1(p_1) dp_1 + \int_{p_2^* + \kappa}^{b} Q_2(p_2) dp_2$$

$$= \frac{(\kappa + 2b)^2}{16b}$$

as $p_1^* = p_2^* + \kappa,$ and where $p_1^* = \frac{b}{2} - \frac{\kappa}{4}, p_2^* = \frac{b}{2} - \frac{5\kappa}{4}, X^* = \frac{3\kappa}{2b}, Q_1(p_1) = \frac{b-p_1}{b}, Q_2(p_2) = \frac{b-(p_2+\kappa)}{b}$.

Given the total consumer surplus of non-stockpiling benchmark case is $CS^{\ast NS} = \frac{b^2 + (b-\kappa)^2}{b}$

$$CS - CS^{\ast NS} = \frac{\kappa(8b - \kappa)}{16b}$$

This is strictly positive because $b > \frac{5\kappa}{2}$.

It then follows that both profits and consumer surplus increase, and therefore welfare, in this case.
References


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