Common Ownership, Institutional Investors, and Welfare*

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February 13, 2019

Abstract

This study evaluates the effects of institutional investors’ common ownership of firms competing in the same market. Overall, common ownership has two opposing effects: (a) it serves as a device for weakening market competition, and (b) it induces diversification, thereby reducing portfolio risk. We conduct a detailed welfare analysis within which the competition-softening effects of an increased degree of common ownership is weighted against the associated diversification benefits.

Keywords: Common ownership, institutional investors, market power, portfolio diversification.

JEL Classification Numbers: G11, G23, G28, L13, L41

(Version: output.tex)

*We thank Thomas Gehrig for insightful and valuable discussions. The views expressed here are the authors’ and not necessarily those of the Federal Reserve Bank of Atlanta or the Federal Reserve System.

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1. Introduction

Recent studies have attracted attention to the increasingly important phenomenon of common ownership of publicly traded firms (for example, Azar, Schmalz, and Tecu (2018), Seldeslachts, Newham, and Banal-Estanol (2017), Gilje, Gormley, and Levit (2017) and Schmalz (2018)). In particular, institutional investors often hold ownership stakes in competing firms belonging to the same industry. He and Huang (2017) present evidence that the proportion of US public firms with common institutional ownership has increased from below 10 percent in 1980 to about 60 percent in 2014. These public firms include institutional owners that simultaneously hold at least 5 percent of the common equity of rival firms in the same industry. Azar (2016) reports a similar trend by reference to a finding that the share of S&P 500 firms with overlapping owners holding at least 3 percent ownership stakes in firms belonging to the same industry has increased from 25 percent to 90 percent during the period from 2000 to 2010.

From a theoretical perspective, common ownership can be expected to soften competition because managers, who maximize the returns to their shareholders, internalize the effects their product market decisions have on rivals. This is the central mechanism developed in the model of overlapping intra-industry ownership by O’Brien and Salop (2000). Several empirical studies have recently estimated the effects on competition of common ownership in different industries. Azar, Schmalz, and Tecu (2018) present evidence to support the conclusion that common ownership raised airfares by relating these prices to measures of concentration which are adjusted to take common ownership into account. Further, Azar, Raina, and Schmalz (2016) show similar effects of common ownership on spreads and fees for banking products.\(^1\) Newham, Seldeslachts, and Banal-Estañol (2018) explore the effects of common ownership on entry. They employ data from the pharmaceutical industry to empirically establish that common ownership with the incumbent brand firm reduces the probability of generic entry.

In this analysis we focus on a configuration where consumers can allocate their savings into one of two competing institutional investors. The institutional investors channel their investments

\(^1\)Gramlich and Grundl (2017) apply different methods for estimating the effects of common ownership on competition in banking, finding mixed and very small effects.
to acquire ownership in product market firms operating in a duopolistic industry. We can think of these institutional investors as pension funds. We initially show that an increased degree of common ownership relaxes the intensity of product market competition. However, an increased degree of common ownership also reduces the risks in the intra-industry portfolios of the institutional owners. Therefore, an increased degree of common ownership defines an interesting tradeoff between relaxed competition in the product market and improved risk diversification in the asset market for risk-averse savers. A detailed welfare analysis associated with this tradeoff is the main contribution of our study.

Our welfare analysis reveals that the socially optimal degree of common ownership is importantly influenced by two factors: (i) the degree of risk aversion and (ii) the relative weight society assigns to consumption of the final product versus that assigned to returns on savings via institutional investors. A low relative weight assigned to consumption of the final product can be interpreted as a society that encourages savings for retirement and discourages excessive consumption. We characterize in detail how the socially optimal degree of common ownership with risk neutrality depends on the relative weight placed on consumption of the final good. In particular, we find that under risk neutrality complete ownership specialization, i.e. no common ownership at all, is socially optimal as long as the relative weight on consumption of the final good is sufficiently high. Further, we show analytically that with risk aversion, and for the class of utility functions with constant relative risk aversion (CRRA), an increase in the degree of risk aversion increases the socially optimal degree of common ownership. The intuition is that the institutional investors offer more diversified investment portfolios to their savers if there is a higher degree of common ownership, and the value savers attach to diversification is increasing as a function of the degree of risk aversion.

Our analysis is linked to a category of theoretical models which have characterized the effects of common ownership or overlapping ownership on market performance by applying industrial economics approaches. This category of models includes O’Brien and Salop (2000), López and Vives (Forthcoming), and Shy and Stenbacka (2018), and it is broadly surveyed in Section 2 of Schmalz (2018). Our present analysis extends this approach to a welfare analysis within which
the competition-softening effects of an increased degree of common ownership can be weighted against the associated diversification benefits.

The recent advances in the analysis of the effects of common ownership have initiated an intense debate among economists and legal scholars regarding policy implications. Elhauge (2016) and Posner, Scott Morton, and Weyl (2017) have proposed the introduction of rules to restrict the ability of institutional owners to hold ownership stakes in several firms operating in the same industry. Other researchers, for example, Lambert and Sykuta (2018) and Ginsburg and Kolvers (2018), have forcefully raised arguments against such restrictions. The debate has also entered the policy arena. For example, in its resolution, dated 19 April 2018, in response to the European Commission’s annual report on competition policy, the European Parliament calls on the Commission to “take all necessary measures to deal with the possible anti-competitive effects of common ownership” and to “investigate...the effects of common ownership on European markets, particularly on prices and innovation.” Our welfare analysis could be viewed as a central component in arguments required to derive effects-based policy implications because it highlights the tradeoff between competition-relaxing effects and diversification benefits associated with an increased degree of common ownership.

It should be pointed out that intra-industry common ownership is not the only way in which institutional investors in general, and pension funds in particular, can diversify their portfolios. Of course, diversification can also be accomplished by mixing equity from different industries as well as fixed income obligations without owning multiple competing firms within the same industry. However, as frequently observed, investors can further diversity their portfolios by acquiring stocks of competing firms within the same industry, and it is the focus of the present study to analyze this particular aspect. To achieve this goal we characterize the welfare tradeoff induced by an increased degree of common ownership as we balance the competition-relaxing effects against the associated diversification benefits.

This study is organized as follows. Section 2 designs a static duopoly model in order to measure how the share value of institutional investors varies with the degree of their common owner-
ship in firms competing in the same product market. Section 3 solves for the equilibrium profits of the firms and investors as functions of the degree of common ownership. Section 4 analyzes how common ownership affects investors’ portfolio risk. Section 5 conducts the welfare analysis of common ownership. Section 6 presents concluding comments. Appendices provide algebraic derivations.

2. Duopoly competition, institutional investors, and common ownership

Following Shy and Stenbacka (2018) we introduce institutional investors into a modified duopoly model with two firms competing based on production decisions in the product market. The two producing firms are owned by two institutional investors with ownership in both. This section investigates how the investment value of institutional investors is influenced by the degree of their common ownership of the producing firms.

The producing firm 1 and firm 2 are engaged in Cournot quantity competition in an industry facing an aggregate inverse demand function

\[ p = \alpha - \beta(q_1 + q_2), \quad \text{where } \alpha > 0, \ \beta > 0. \] (1)

The variable \( p \) denotes the price of a homogeneous product (or service) sold in this market and \( q_1 \) and \( q_2 \) are the quantities produced and sold by firms 1 and 2, respectively. Let \( \pi_1 \) and \( \pi_2 \) denote the profits earned by firms 1 and 2, receptively. Then, assuming zero marginal costs, the producing firms’ profits as functions of quantities produced are given by

\[ \pi_1(q_1, q_2) = pq_1 = [\alpha - \beta(q_1 + q_2)]q_1 \quad \text{and} \quad \pi_2(q_1, q_2) = pq_2 = [\alpha - \beta(q_1 + q_2)]q_2. \] (2)

Finally, (net) consumer surplus derived from the product market will be evaluated according to

\[ CS(Q) = \int_0^Q (\alpha - \beta x)dx - pQ = \frac{\beta Q^2}{2}, \] (3)

where \( Q = q_1 + q_2 \) is aggregate industry output and \( p \) is substituted from (1).
2.1 Common ownership

Firms 1 and 2 (the producers) are co-owned by two institutional investors labeled $A$ and $B$, as illustrated in Figure 1 and formalized in Assumption 1 below.\(^3\) Let $\mu$ denote investor $A$’s share of ownership in firm 1, and also investor $B$’s share of ownership in firm 2.

\[
\begin{align*}
\text{Institutional Investor } A & \quad \pi_A = \mu \pi_1 + (1 - \mu) \pi_2 \\
\text{Institutional Investor } B & \quad \pi_B = (1 - \mu) \pi_1 + \mu \pi_2
\end{align*}
\]

\begin{itemize}
\item[$\mu$] Firm 1
\item[1 - $\mu$] 1 - $\mu$
\item[$\mu$] Firm 2
\end{itemize}

\textbf{Figure 1:} The shares of common ownership in firms 1 and 2 by institutional investors $A$ and $B$.

\textbf{Assumption 1.} Institutional investor $A$ owns a (weak) majority share $\mu$ in firm 1, whereas institutional investor $B$ owns a (weak) majority share $\mu$ in firm 2. Formally, $\mu \in \left[\frac{1}{2}, 1\right]$.

In an industry with imperfect product market competition, common ownership induces owners to internalize the strategic externalities between the firms. Hansen and Lott (1996) and O’Brien and Salop (2000) have developed formal models to capture such effects, but they have not explored the welfare implications of common ownership in order to balance diversification benefits against competition-softening effects.

For reasons of transparency we assume that institutional investors $A$ and $B$ are the sole owners of firms 1 and 2. Therefore, Assumption 1 implies that institutional investor $A$ owns a minority share $(1 - \mu < 50\%)$ in firm 2, whereas institutional investor $B$ owns a minority share $(1 - \mu < 50\%)$ in firm 1. In view of Figure 1 and Assumption 1, the profits earned by the institutional investors,

\(^3\)According to Zingales (2012) (p.233), the share of individuals’ ownership of publicly traded equity has decreased dramatically since the 1920s. Azar, Schmalz, and Tecu (2018) estimate the ownership share of institutional investors (such as mutual funds and pension funds) of US publicly traded firms to presently be in the 70–80 percent range.
as functions of quantity produced by firms 1 and 2, are

\[
\pi_A(q_1, q_2) = \mu \pi_1(q_1, q_2) + (1 - \mu) \pi_2(q_1, q_2),
\]

(4a)

\[
\pi_B(q_1, q_2) = (1 - \mu) \pi_1(q_1, q_2) + \mu \pi_2(q_1, q_2),
\]

(4b)

where \(\pi_1\) and \(\pi_2\) are defined in (2).

2.2 Introducing risks

We introduce risks by modeling firms that can fail. A failure of a firm is an extreme manifestation of production cost uncertainty, but to focus exclusively on the central underlying economic mechanism, we only model an extreme case where a significant cost increase forces a firm to exit the industry.

Formally, we introduce three probabilities: Let \(\phi^{II}\) be the probability that both firms fail (the letter “phi” stands for “failure”). Also, let \(\phi^I\) be the probability that one firm fails while the other does not (two possible events). Finally, \(\phi^0\) denotes the probability that neither firm fails. Therefore,

\[
\phi^{II} + 2\phi^I + \phi^0 = 1.
\]

(5)

The probability structure assumed in equation (5) is general in the sense that it can capture both dependent and independent failure events. For example, the case of independence is captured by a binomial distribution with a failure probability \(f \in (0, 1)\). In the binomial case, (5) is simplified to \(\phi^{II} = f^2, \phi^I = f(1 - f)\) (two events), and \(\phi^0 = (1 - f)^2\). However, equation (5) assumes a more general probability structure to capture possible correlations between failures of the two firms. Such correlations may stem from possible aggregate industry declines or temporary industry downturns, and could become an important factor in determining investment portfolios.

2.3 Firms’ decision process and sequence of events

The literature does not provide a consistent method or a consensus regarding the modeling of how ownership structure actually translates into control of firms’ decisions. OECD (2017) presents an extensive discussion of this issue. Because institutional investor A is the majority shareholder in
firm 1, A can determine firm 1’s output level either through the exercise of direct influence or through the control of underlying managerial incentives. This means that investor A controls the production of firm 1, while taking into consideration the profit derived from its minority ownership share in firm 2. Similarly, institutional investor B determines the output level produced by firm 2 taking into account its minority ownership share in firm 1.4

The sequence of events is as follows:

First Stage: The fate of each producing firm is realized according to the probabilities defined in the discussion preceding equation (5).

Second State: Investor A (maintaining a majority share in firm 1) determines the output of firm 1 (if firm 1 does not fail), and investor B (maintaining a majority share in firm 2) determines the output of firm 2 (if firm 2 does not fail).

3. Equilibrium in the product market

There are four possible events that could be realized in the First Stage. The simplest case, with probability \( \phi^I \), both firms fail and exit the market. In this case, profits and consumer surplus are

\[
\pi^I_A = \pi^I_B = \pi^I_1 = \pi^I_2 = q^I_1 = q^I_2 = 0 \quad \text{and} \quad CS^I = 0. \tag{6}
\]

Next, with probability \( \phi^I \) firm 1 survives and firm 2 exits. Also, with probability \( \phi^I \) firm 1 exits and firm 2 remains. This is the monopoly case which is derived in Appendix A. In this case,

Either: \( q^I_1 = \frac{\alpha}{2\beta}, q^I_2 = 0, p^I = \frac{\alpha}{2}, \pi^I_1 = \gamma \frac{1}{4}, \pi^I_2 = 0, \pi^I_A = \frac{\mu\gamma}{4}, \pi^I_B = \frac{(1 - \mu)\gamma}{4}, \) and \( CS^I = \gamma \frac{4}{8} \)  
Or: \( q^I_1 = 0, q^I_2 = \frac{\alpha}{2\beta}, p = \frac{\alpha}{2}, \pi^I_1 = 0, \pi^I_2 = \gamma \frac{1}{4}, \pi^I_A = \frac{(1 - \mu)\gamma}{4}, \pi^I_B = \frac{\mu\gamma}{4}, \) and \( CS^I = \gamma \frac{4}{8} \),

where \( \gamma = \frac{\alpha^2}{\beta} \). Note that (7) displays the payoffs of two separate events, each occurs with probability \( \phi^I \). The subscripts “1f” indicates the event when firm 1 fails and “2f” the event when firm 2 fails.

4 An alternative modeling method would be to assume that a firm’s production decision is made in order to maximize the total portfolio value of its investors, weighted by the proportion of ownership held by these investors. Such an approach has been applied by Hansen and Lott (1996) as well as O’Brien and Salop (2000). The associated distinction between profit maximization and shareholder value maximization and its role for the analysis of strategic competition is discussed in Antón et al. (2018).
Finally, with probability $\phi^0$, neither firm fails, so that the product market is characterized by two competing firms. For given investors’ ownership shares $\mu$ and $1 - \mu$, investor $A$ determines the output of firm 1 $q_1$ to maximize (4a) and investor $B$ determines the output $q_2$ to maximize (4b).

As shown in Appendix A, the Cournot-Nash equilibrium production levels, the corresponding price, profits and consumer surplus are
\[
q_1^0 = q_2^0 = \frac{\alpha \mu}{\beta(2\mu + 1)}, \quad p^0 = \frac{\alpha}{2\mu + 1}, \quad \pi_A^0 = \pi_B^0 = \pi_1^0 = \pi_2^0 = \frac{\gamma \mu}{(2\mu + 1)^2}, \quad \text{and} \quad CS^0 = \frac{2\gamma \mu^2}{(2\mu + 1)^2}, \quad (8)
\]
where $\gamma = \alpha^2/\beta$.

Recall that the parameter $\mu$ measures the degree of common ownership. Specifically, $\mu = 0.5$ implies that investors $A$ and $B$ have equal ownership shares in both firms 1 and 2. In contrast, $\mu = 1$ implies that each producing firm is owned by a single investor (firm 1 is owned by investor $A$ and firm 2 is owned by investor $B$). Appendix A derives the following conclusions.

**Result 1.** Suppose neither firm fails (probability $\phi^0$), so the product market operates as a duopoly controlled by investors $A$ and $B$.

(a) Moving towards more equal co-ownership ($\mu$ decreases towards $\frac{1}{2}$) increases price, reduces aggregate industry profit, and increases all profits. Formally,
\[
\frac{\partial p^0}{\partial \mu} < 0, \quad \frac{\partial Q^0}{\partial \mu} > 0, \quad \frac{\partial \pi_1^0}{\partial \mu} = \frac{\partial \pi_2^0}{\partial \mu} < 0, \quad \frac{\partial \pi_A^0}{\partial \mu} = \frac{\partial \pi_B^0}{\partial \mu} < 0. \quad (9)
\]

(b) The maximum degree of common ownership ($\mu = \frac{1}{2}$) implements the monopoly solution where aggregate investors’ profit equals the monopoly profit level $\pi_A^0 + \pi_B^0 = \frac{\gamma}{3}$.

(c) The highest degree of market competition is achieved with specialization such that each investor fully owns only one firm ($\mu = 1$). In this case, the market performance is that of the standard Cournot duopoly competition.

Result 1 is illustrated in Figure 2. Sliding in the upward and leftward direction on the curve corresponds to a reduction in $\mu$ from $\mu = 1$ towards $\mu = \frac{1}{2}$ which lowers aggregate industry output and increases price and profits. In the limit, equal ownership $\mu = \frac{1}{2}$ generates the highest aggregate industry profit corresponding to a single-firm monopoly operation given in (7).\footnote{Similar results were obtained in Rubinstein and Yaari (1983) in a game-theoretic formulation with a perfectly-}

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8
\[ p = \alpha - \beta Q = \alpha - \beta (q_1 + q_2) \]

\[ \mu = \frac{1}{2} \quad \text{(equal ownership \& monopoly solution)} \]

\[ \frac{\alpha}{2\mu + 1} \]

\[ \frac{\alpha}{2\beta} \]

\[ \frac{2\alpha}{\beta(2\mu + 1)} \]

\[ \frac{2\alpha}{3\beta} \]

\[ \frac{\mu}{\beta} \]

\[ \mu = 1 \quad \text{(single ownership)} \]

\[ \mu = 1 \quad \text{(duopoly solution)} \]

Figure 2: Equilibrium price \( p^0 \) and aggregate industry output \( Q^0 \) under varying degrees of common ownership \( \mu \), for the case that neither firm fails (probability \( \phi^0 \)).

The effect of common ownership on consumer surplus \( CS \) is part of a comprehensive welfare analysis to be conducted in the next section.

4. The effect of common ownership on portfolio risks

Recall that the parameter \( \mu \) measures the share of investor A in firm 1 and the share of investor B in firm 2. We can view each investor as a manager of a portfolio containing two assets. This section investigates the effects of varying \( \mu \) on various statistics that characterize the portfolio of investor A. By symmetry, the conclusions of this analysis equally apply to the portfolio managed by investor B.

The portfolio of investor A consists of two assets which we denote by \( A_1 \) and \( A_2 \) (ownership shares in firms 1 and 2, respectively). In view of the equilibrium profit returns from these ownerships corresponding to the three possible uncertain events given in (6), (7), and (8), we write

\[ \pi^H_{A_1} = \pi^H_{A_2} = 0, \quad \text{either:} \quad \left[ \pi^I_{A_1} = \frac{\mu \gamma}{4} \right. \quad \text{and} \quad \pi^I_{A_2} = 0 \quad \text{or:} \quad \left[ \pi^I_{A_1} = 0 \quad \text{and} \quad \pi^I_{A_2} = \frac{(1 - \mu) \gamma}{4} \right. \]

\[ \pi^0_{A_1} = \frac{\mu \gamma \mu}{(2\mu + 1)^2}, \quad \text{and} \quad \pi^0_{A_2} = \frac{(1 - \mu) \gamma \mu}{(2\mu + 1)^2}. \quad (10) \]

competitive stock market, and in Rotemberg (1984) in a model where managers maximize a weighted sum of investors’ utilities under circumstances where these investors hold shares in multiple firms within the same industry.
Taking into consideration that investor $A$ owns a share $\mu$ in firm 1 and $1 - \mu$ in firm 2, the first expression in (10) is the profit return on assets 1 and 2 in the event that both firms (assets) fail, hence zero. $\pi_{A1}$ is the return on asset 1 when only asset 2 fails (probability $\phi^I$). Similarly, $\pi_{A2}$ is the return on asset 2 when only asset 1 fails (also probability $\phi^I$). Subscripts “$A1f$” and “$A2f$” denote the component in investor $A$’s portfolio that yields no return in the case that firm 1 or firm 2 fails, respectively. $\pi_A^0$ and $\pi_A^0$ are the profits made from assets 1 and 2, respectively, when neither asset fails.

The expected returns of the two assets in investor $A$’s portfolio (10) are

$$E[\pi_A] = \phi^H \pi_A^H + \phi^I \pi_A^I + \phi^I \pi_A^I + \phi^0 \pi_A^0$$ and $E[\pi_A] = \phi^H \pi_A^H + \phi^I \pi_A^I + \phi^I \pi_A^I + \phi^0 \pi_A^0$. (11)

Further, the variances of each asset in investor $A$’s portfolio are

$$Var[\pi_A] = \phi^H (\pi_A^H - E[\pi_A])^2 + \phi^I (\pi_A^I - E[\pi_A])^2 + \phi^I (\pi_A^I - E[\pi_A])^2 + \phi^0 (\pi_A^0 - E[\pi_A])^2,$$

$$Var[\pi_A] = \phi^H (\pi_A^H - E[\pi_A])^2 + \phi^I (\pi_A^I - E[\pi_A])^2 + \phi^I (\pi_A^I - E[\pi_A])^2 + \phi^0 (\pi_A^0 - E[\pi_A])^2.$$

The variances given in (12) are plotted in Figure 3. Note that $Var[\pi_A]$ increases with $\mu$, whereas $Var[\pi_A]$ declines with $\mu$ because higher values of $\mu$ correspond to a larger ownership share in firm 1 and a smaller share in firm 2.

Using the formula $Cov[X, Y] = E[XY] - E[X]E[Y]$, the covariance between the two assets in the portfolio of investor $A$ is

$$Cov[\pi_A, \pi_A] = \phi^H \pi_A^H \pi_A^H + \phi^I \pi_A^I \pi_A^I + \phi^I \pi_A^I \pi_A^I + \phi^0 \pi_A^0 \pi_A^0 - E[\pi_A]E[\pi_A],$$

where the returns are given in (10) and $E[\pi_A]$ and $E[\pi_A]$ are computed in (11).

In order to compute the variance of the entire portfolio managed by investor $A$, we define the (value-based) portfolio’s weights of the two assets as

$$s_A = \frac{E[\pi_A]}{E[\pi_A] + E[\pi_A]}$$ and $s_A = 1 - s_A = \frac{E[\pi_A]}{E[\pi_A] + E[\pi_A]}$. (14)

Intuitively, (14) defines the weight on asset $A1$ as the ratio of its expected return divided by the
expected return of the entire portfolio managed by investor $A$. The weight on asset $A_2$ is similarly defined.

We are now ready to specify the variance of the entire portfolio managed by investor $A$ as

$$\text{Var}[\pi] = s_{A_1}^2 \text{Var}[\pi_{A_1}] + (1 - s_{A_1})^2 \text{Var}[\pi_{A_2}] + 2s_{A_1}(1 - s_{A_1})\text{Cov} [\pi_{A_1}, \pi_{A_2}].$$

(15)

The solid curve in Figure 3 exhibits the portfolio variance as a function the majority ownership parameter $\mu$. According to Figure 3, as $\mu$ increases towards 1 so that investor $A$‘s portfolio consists mainly (only) of asset $A_1$, the variance of $A$‘s entire portfolio increases, and the return on asset $A_2$ does not play any role. In contrast, as $\mu$ declines towards $\frac{1}{2}$, investor $A$‘s portfolio become more diversified with equal expected returns on each asset and hence equal portfolio weights (14). We can therefore summarize this section with the following result.

**Result 2.** Increasing an investor’s majority share $\mu$ in one producing firm while reducing the minority share in the other firm increases the investor’s portfolio variance. Portfolio variance is minimized when each investor maintains an equal share in each of the product market rivals.

Overall, Result 1 means that an increased degree of common ownership (lower $\mu$) relaxes competition in the product market, whereas more ownership specialization intensifies competition.
Result 2, in its turn, means that an increased degree of common ownership reduces portfolio risks. Thus, based on the combination of Result 1 and Result 2 an increased degree of common ownership defines an interesting tradeoff between the competition-relaxing effects in the product market and risk diversification in the asset market for risk-averse savers. In the next Section we will conduct a welfare analysis to assess this tradeoff.

5. Welfare evaluations of common ownership

The economy analyzed in this paper consists of two separate groups of agents: A group of buyers whose aggregate welfare is summarized by a function of the aggregate consumer surplus $CS$, and a group of individuals who save (say, for retirement) in institutional investors $A$ or $B$. The welfare of the latter group is summarized by functions of the earnings of the investment funds, $\pi_A$ and $\pi_B$.

To connect the two groups of consumers we need to define a social welfare function with weights assigned to each group. Let $U$ be an increasing weakly concave and differentiable utility function, and let $\omega$ ($0 < \omega < 1$) be the weight in social welfare assigned to consumers in the product market. Then, the expected total welfare function $EW$ is defined by

$$EW = \omega \left[ EU(CS) \right] + (1 - \omega) \left[ EU(\pi_A) + EU(\pi_B) \right],$$

where $E$ is the expectation operator and $CS$, $\pi_A$ as well as $\pi_B$ are random payoffs distributed according to the three event probabilities (5) and the corresponding three event realizations derived in (6), (7), and (8).

The parameter $\omega$ determines the weight society assigns to consumer surplus relative to earnings from savings via institutional investors. The parameter $\omega$ also captures the outcome of influence activities directed to political decision makers by financial institutions (for lower $\omega$).

We next investigate the expected welfare $EW(\mu)$ as a function of the degree of common ownership $\mu$. In light of (5)–(8), the expected consumer welfare is $EU(CS) = 2\phi^I U(CS^I) + \phi^0 U(CS^0)$. Equation (7) implies that $\partial CS^I / \partial \mu = 0$. Hence, the effect of an increase in common ownership $\mu$
on expected consumer utility is given by

\[
\frac{\partial EU(CS)}{\partial \mu} = \phi^0 U'(CS^0) \frac{4\gamma \mu}{(2\mu + 1)^3} > 0. \tag{17}
\]

This means that increased ownership concentration (higher \(\mu\)) generates expected benefits to consumers because it reduces market power of the producing firms.

From (5)–(8) we can calculate that the expected utility associated with the earnings of institutional investors to be \(EU(\pi_A) + EU(\pi_B) = 2\phi^I[U(\pi_A^I) + U(\pi_B^I)] + \phi^0[U(\pi_A^0) + U(\pi_B^0)]\). Differentiation with respect to \(\mu\) shows that

\[
\frac{\partial [EU(\pi_A) + EU(\pi_B)]}{\partial \mu} = 2\phi^I \frac{\gamma}{4} \left[ U'(\frac{\gamma \mu}{4}) - U'(\frac{\gamma(1 - \mu)}{4}) \right] + 2\phi^0 U'(\frac{\gamma \mu}{(2\mu + 1)^2}) \frac{\gamma(1 - 2\mu)}{(2\mu + 1)^3} < 0. \tag{18}
\]

The first term in (18) is zero with risk neutrality, whereas it is strictly negative with risk aversion. The second term is strictly negative for \(\mu > 1/2\). Overall, (18) means that the expected return from ownership of institutional investors decreases with more specialized ownership for the institutional investors (higher \(\mu\)).

The welfare changes computed in (17) and (18) establish a tradeoff between expected consumer utility and expected utility associated with ownership of institutional investors in response to changes in the degree of common ownership \(\mu\). This yields the following general conclusion.

**Result 3.** An increased degree of common ownership by institutional investors of product market firms (lower \(\mu\)) decreases expected consumer utility in the product market and increases expected utility associated with earnings generated by institutional investors.

Overall, the tradeoff defined in Result 3 is influenced by two factors: the relative weights placed on the two groups of consumers (\(\omega\)) and the degree of risk aversion. For reasons of tractability we first analyze this tradeoff by focusing on risk neutrality. Subsequently, we will explore the role played by risk aversion by focusing on the class of utility functions with a constant relative risk aversion (CRRA).
5.1 Welfare evaluation with risk neutrality

In this subsection we focus on risk neutrality, which means a constant marginal utility associated with consumption and investor returns. In light of (16), (17) and (18) the effect of $\mu$ on expected welfare is formally captured by

$$\frac{\partial E\bar{W}}{\partial \mu} = \frac{2\gamma \phi^0 U'(CS^0)}{(2\mu + 1)^3} \left[ \omega 2\mu + (1 - \omega) (1 - 2\mu) \right],$$

(19)

where we have exploited that the marginal utility is constant under risk neutrality.

The derivative (19) is strictly increasing when $\omega > 1/2$, leading to the conclusion that $\mu = 1$ maximizes welfare when $\omega > 1/2$. For lower values of $\omega$, maximization of welfare generates an interior solution

$$\mu^*(\omega) = \frac{1 - \omega}{2(1 - 2\omega)}.$$  

(20)

However, the interior solution violates feasibility ($1/2 \leq \mu^*(\omega) \leq 1$) if $\omega > 1/3$. Therefore, $\mu = 1$ maximizes welfare also for $\omega \geq 1/3$. Consequently, we can formulate the following result.

**Result 4.** Suppose that consumers as well as savers are risk neutral. The institutional investors’ degree of common ownership that maximizes total welfare (16) is given by

$$\mu^*(\omega) = \begin{cases} \frac{1 - \omega}{2(1 - 2\omega)} & \text{if } 0 \leq \omega < \frac{1}{3} \\ 1 & \text{if } \frac{1}{3} \leq \omega \leq 1. \end{cases}$$

(21)

Result 4 is displayed in Figure 4.

Figure 4 shows that the welfare-maximizing degree of common ownership by institutional investors, $\mu^*$, declines towards $\frac{1}{2}$ as the weight assigned to consumers ($\omega$ in (16)) declines towards 0. This case formally captures the intuition that maximum common ownership ($\mu^* = \frac{1}{2}$) is optimal with a welfare function that disregards buyers’ welfare. The socially optimal $\mu^*$ is a strictly increasing function of the relative weight placed on consumers ($\omega$) until we reach the level $\omega = \frac{1}{3}$. Finally, for $\omega$ exceeding $\frac{1}{3}$ it is socially optimal that the institutional owners have no common ownership. This means specialized ownership on behalf of the institutional investors in the sense that the each of the two institutional owners concentrate their investment to one of the producing
firms, and that there is no overlap in the ownership. In Figure 4 this feature is captured by the fact that $\mu^* = 1$ for $\omega > 1/3$.

5.2 Welfare evaluation with risk aversion

We now shift the attention to a configuration with risk averse consumers and savers. All our computations in this subsection will focus on the class of utility functions with constant relative risk aversion (CRRA). More precisely, we focus on utility functions $U(y) = y^\theta$ for $0 < \theta \leq 1$. For this utility function, the Arrow-Pratt measure of relative risk aversion is $(1 - \theta)$. A higher value of $\theta$ means that the agents have a lower coefficient of relative risk aversion. And, broadly spoken, a lower value of $\theta$ captures more risk averse agents, whereas $\theta = 1$ corresponds to risk neutrality.

Using the three payoff realizations (6), (7), and (8), and substituting the utility function $U(y) = y^\theta$ into the general welfare function (16) yields

$$
E(W) = \omega \left[ 2\phi^I (\gamma) + \phi^0 \left( \frac{2\gamma \mu^2}{(2\mu + 1)^2} \right)^\theta \right] \quad \text{consumers' utility } EW_c
$$

$$
+ (1 - \omega) \left\{ 2\phi^I \left[ \left( \frac{\mu \gamma}{4} \right)^\theta + \left( \frac{(1 - \mu) \gamma}{4} \right)^\theta \right] + \phi^0 \left[ \left( \frac{\gamma \mu}{(2\mu + 1)^2} \right)^\theta + \left( \frac{\gamma \mu}{(2\mu + 1)^2} \right)^\theta \right] \right\} \quad \text{savers' utility } EW_s
$$

Differentiating (22) with respect to $\mu$ yields a first-order condition consisting of a high-order polynomial. Therefore, obtaining an algebraic closed-form solution as a basis for an explicit char-
acterization of the socially optimal level of common ownership ($\mu^*$) is not feasible. For that reason we initially apply numerical simulations to illustrate how increased risk aversion (decrease in $\theta$) affects the welfare-maximizing level of $\mu^*$. Figure 5 displays three graphs corresponding to different degrees of risk aversion: $\theta \in \{0.4, 0.6, 0.8\}$.

![Figure 5](image_url)

**Figure 5**: Expected total welfare $EW$ defined in (22) as functions of the magnitude of majority shares $\mu$ and the risk aversion parameter $\theta \in \{0.4, 0.6, 0.8\}$. Note: The figure is drawn according to $\gamma = 64$, $\omega = 0.5$, and probabilities $\phi^H = \phi^I = \phi^0 = 0.25$.

The top graph in Figure 5 corresponds to consumers and savers with low risk aversion where $\theta = 0.8$. Under equal weights ($\omega = \frac{1}{2}$), the welfare-maximizing majority ownership share is $\mu^* = 0.75$, meaning that some degree of investor portfolio diversification is socially optimal. The middle curve in Figure 5 depicts total welfare under higher risk aversion $\theta = 0.6$, where more portfolio diversification (more equal ownership with $\mu^* \approx 0.66$) is socially optimal. Finally, the bottom curve in Figure 5 corresponds to the highest degree of risk aversion ($\theta = 0.4$). For this case the simulation shows that the socially optimal ownership configuration is characterized by a higher degree of diversification as $\mu^* \approx 0.62$. Overall, these simulations suggest that a higher degree of risk aversion (lower $\theta$) tends to induce a higher degree of common ownership (lower $\mu^*$) in the social optimum. We will next theoretically verify this hypothesis.

The analysis of risk neutral consumers and savers in subsection 5.1, and in particular Figure 4, demonstrates the role played by the weight parameter $\omega$ in the social welfare function (16). In order to highlight the particular effects of risk aversion in a transparent way we will focus on the
case with equal weights $\omega = 1/2$. In Appendix B we prove the following result for $\omega = 1/2$ and a sufficiently large $\gamma (\gamma > 9)$.

**Result 5.** Suppose $\omega = \frac{1}{2}$ and $\gamma > 9$. Then, an increase in risk aversion (lower $\theta$) increases the socially optimal degree of common ownership (lower $\mu^*$).

The intuition behind Result 5 can be formulated as follows. With a higher degree of common ownership the institutional investors offer more diversified investment portfolios to their savers. The value savers derive from diversification is increasing as a function of the degree of risk aversion. This is the mechanism for why the socially optimal degree of common ownership increases with risk aversion. It should be emphasized that the socially optimal degree of common ownership balances the gains from diversification against the offsetting effects on consumer surplus, and that this tradeoff is importantly determined also by the parameter $\omega$. This tradeoff explains why our simulations illustrated in Figure 5 yields interior solutions for the socially optimal degree of common ownership. It should also be remembered that Result 5 is formulated for $\omega = 1/2$ precisely like the simulations illustrated in Figure 5.

6. **Conclusion**

We show that the socially optimal degree of common ownership is determined by two factors: (i) the degree of risk aversion and (ii) the relative weight society assigns to consumer surplus associated with the final good compared with the returns on savings via institutional investors. We demonstrate that under risk neutrality complete ownership specialization with no common ownership at all is socially optimal if the relative weight on consumption of the final good is sufficiently high. Further, we establish analytically that with risk aversion, and for the class of utility functions with constant relative risk aversion (CRRA), an increase in the degree of risk aversion increases the socially optimal degree of common ownership.

Our model characterizes the effects of common ownership through the production decisions under oligopoly competition within the framework of exogenously given default probabilities. It remains an interesting challenge for future research to investigate alternative channels regarding the effects of common ownership on industry performance and welfare. It could be a fruitful
approach to separately explore the effects of common ownership within the framework of an oligopoly model where the competing firms explicitly make decisions regarding risks or survival probabilities. Through such an approach, the risks would be an endogenous feature, making it possible to characterize the effects of common ownership on investments in innovation.

Appendix A Algebraic derivations for Section 3

Derivation of the equilibrium values (7). With no loss of generality we focus on the case in which firm 2 fails, so firm 1 becomes a monopoly seller. Setting \( q_2 = 0 \) into (4a) and maximizing with respect to \( q_1 \) yields the first-order condition

\[
0 = \frac{\partial \pi_A}{\partial q_1} = \alpha \mu - 2q_1 \beta \mu \quad \text{hence} \quad q_1 = \frac{\alpha}{2\beta}
\]

which is the monopoly output level. The second-order conditions is \( \frac{\partial^2 \pi_A}{\partial (q_1)^2} = -2\beta \mu < 0 \). Substituting into the demand function (1) yields the monopoly price \( p = \alpha/2 \). Substituting into (4a) and (4b) obtains the profit of investors \( A \) and \( B \), respectively. Substituting into (3) obtains the consumer surplus under monopoly, all are given in (7).

Derivation of the equilibrium values (8). Differentiating (4a) with respect to \( q_1 \) and (4b) with respect to \( q_2 \) yields

\[
0 = \frac{\partial \pi_A}{\partial q_1} = -2q_1 \beta \mu - q_2 \beta + \alpha \mu, \quad \text{and} \quad 0 = \frac{\partial \pi_B}{\partial q_2} = -2q_2 \beta \mu - q_1 \beta + \alpha \mu.
\]

The second-order conditions are \( \frac{\partial^2 \pi_A}{\partial (q_1)^2} = \frac{\partial^2 \pi_B}{\partial (q_2)^2} = -2\beta \mu < 0 \). The equilibrium output levels and price in (8) are obtained by solving the system of two first-order conditions (A.2) for \( q_1 \) and \( q_2 \), and then substituting into (1) to obtain \( p \). Substituting back into (4a) and (4b) yields the profits (8). Finally, substituting the equilibrium price and output levels into (3) obtains the equilibrium net consumer surplus \( CS \).

Derivation of Result 1. Using (8) and \( \frac{1}{2} < \mu < 1 \), for every \( i = 1, 2 \) and \( j = A, B \), \( \partial q_i / \partial \mu = \alpha / [\beta (2 \mu + 1)^2] > 0 \), \( \partial p / \partial \mu = -2 \alpha / (2 \mu + 1)^2 < 0 \), and \( \partial \pi_i / \partial \mu = \partial \pi_j / \partial \mu = \gamma (1 - 2 \mu) / (2 \mu + 1)^3 < 0 \). That proves part (a).
To prove part (b), substituting $\mu = \frac{1}{2}$ into (8) yields $\pi_A = \pi_B = \gamma/8$, and hence $\pi_A + \pi_B = \gamma/4$.

To prove part (c), substituting $\mu = 1$ into (8) yields $q_1 = q_2 = \alpha/(3\beta)$, $p = \alpha/3$, and $\pi_1 = \pi_2 = \pi_A = \pi_B = \alpha^2/(9\beta) = \gamma/9$, which are the standard Cournot duopoly competition equilibrium values.

**Appendix B  Proof of Result 5**

We consider the utility function $U(y) = y^\theta$ with $0 < \theta \leq 1$. Let $\omega = 1/2$. Using (16), (17), and (18), the necessary first-order condition characterizing the socially optimal degree of common ownership is given by

\[
\frac{\partial EW}{\partial \mu} = \phi \gamma \mu + \frac{2\gamma \mu^2}{(2\mu + 1)^2} U'(\frac{2\gamma \mu^2}{(2\mu + 1)^2}) + \phi \gamma \mu \left[ U'(\frac{\gamma \mu}{4}) - U'(\frac{\gamma (1 - \mu)}{4}) \right]
\]

\[
+ \phi \gamma \mu \left[ U'(\frac{\gamma (1 - \mu)}{4}) - U'(\frac{\gamma \mu}{4}) \right] = 0. \tag{B.1}
\]

We assume the second-order condition $\frac{\partial^2 EW}{\partial \mu^2} < 0$ to be satisfied, meaning that the first-order condition (B.1) is also a sufficient condition for the socially optimal degree of common ownership $\mu^*$. By differentiating (B.1) with respect to $\theta$ we find the effect $\theta$ on the socially optimal degree of common ownership from

\[
\frac{\partial^2 EW}{\partial \mu^2} + \frac{\partial^2 EW}{\partial \mu \partial \theta} \frac{\partial \mu^*}{\partial \theta} = 0. \tag{B.2}
\]

In order to apply (B.2) we need to characterize the second-order mixed derivative $\frac{\partial^2 EW}{\partial \mu \partial \theta}$. We can express the second-order mixed derivative as

\[
\frac{\partial^2 EW}{\partial \mu \partial \theta} = \phi' \gamma \mu A(\mu, \theta) + \phi^0 \frac{2\gamma \mu^2}{(2\mu + 1)^2} B(\mu, \theta) + \phi \gamma \mu \left[ U'(\frac{\gamma \mu}{4}) - U'(\frac{\gamma (1 - \mu)}{4}) \right], \tag{B.3}
\]

where we have introduced the notation that $\frac{\partial U'(\mu, \theta)}{\partial \theta} = U'_\theta(\mu, \theta)$ and where

\[
A(\mu, \theta) = U'_\theta(\frac{\gamma \mu}{4}) - U'_\theta(\frac{\gamma (1 - \mu)}{4}), \tag{B.4}
\]

\[
B(\mu, \theta) = U'_\theta(\frac{2\gamma \mu^2}{(2\mu + 1)^2}) - U'_\theta(\frac{\gamma \mu}{(2\mu + 1)^2}), \tag{B.5}
\]
and

\[ C(\mu, \theta) = U'(\frac{\gamma \mu}{(2\mu + 1)^2}). \]  

(B.6)

The utility function \( U(y) = y^\theta \) has the feature that \( U'(y) = y^{\theta-1}(1 + \theta \ln y) \). Therefore, with \( y = \frac{\gamma \mu}{(2\mu + 1)^2} \) it can be seen that \( \gamma > 9 \) is a sufficient condition for \( U'(y) > 0 \). This is also a sufficient condition to guarantee that \( U'(y) \) is strictly convex, because \( U''_\theta(y) = y^{\theta-1} \ln y(2 + \theta \ln y) > 0 \). These properties imply that \( A(\mu, \theta) > 0 \), \( B(\mu, \theta) > 0 \) and \( C(\mu, \theta) > 0 \). In light of (B.3) we can therefore draw the conclusion that \( \frac{\partial^2 EW}{\partial \mu \partial \theta} > 0 \). Consequently, from (B.2) we conclude that

\[ \frac{\partial \mu^*}{\partial \theta} = -\frac{\partial^2 EW}{\partial \mu \partial \theta} \frac{\partial^2 EW}{\partial \mu^2} > 0. \]  

(B.7)

This means that stronger risk aversion (lower \( \theta \)) induces a higher degree of socially optimal common ownership (lower \( \mu^* \)).

References


