Credit Market Frictions and Business Cycle Dynamics in an Oil-Rich Emerging Economy*

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Abstract

We develop a small open economy dynamic stochastic general equilibrium (DSGE) model for Nigeria, a major oil exporter, incorporating a financial accelerator mechanism as a credit market friction facing domestic firms seeking investment. The small open economy DSGE model also incorporates oil revenue, endogenous fiscal policy, incomplete exchange rate pass-through and credit-constrained consumers. For an oil exporting economy with significant exposure to external shocks, introducing this mechanism serves as an important amplification device for replicating business cycle dynamics. The Bayesian estimation results indicate the relative importance of incorporating financial frictions and other exogenous shocks in a non-industrial oil exporting economy model. The oil-exporter economy DSGE model is useful for monetary policy analysis and simulations.

JEL Classification: E37, E52, E58

Keywords: Oil-exporter economy, DSGE, small open economy, Bayesian estimation, monetary policy rules, credit market frictions.

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1 INTRODUCTION

Access to credit and other financial services remains a major challenge to households and firms in many emerging market and developing economies. Contingent upon the level of financial development, recent surveys indicate that a significant proportion of households and firms still have limited access to financial markets in some of these economies.\(^1\) To meet exigent obligations, credit constrained agents may resort to informal financial arrangements such as unregistered money lenders, pawnshops, friends and family for cash advances. Ultimately, when and where such services are unavailable or non-existent, some households and firms may depend solely on wages, revenue, savings and retained profits for financing. Thus, limited access to finance affects significantly, the consumption pattern and productivity of these agents over the business cycle.\(^2\)

Indeed, the crucial role financial development plays in economic growth is well documented in literature (see, for example Levine (1997)). Nevertheless, it is also common knowledge that financial access facilitates poverty alleviation, a fact that has motivated several research to extensively examine the impact of financial intermediation and access to credit on emerging market and developing economies.\(^3\) Specifically, these studies highlight how finance affects economic activities (see, Holmstrom and Tirole (1997), Levine et al. (2000) and Gertler and Kiyotaki (2010)). World Bank, 2012 for example shows how a dearth of financing disproportionately affects the performance of small and medium sized enterprises (SMEs) in some developing countries. Thus, the literature on the finance-economic growth nexus elucidates how finance affects output developments, poverty levels, income distribution, and macroeconomic and financial stability.

In line with the above, and within the purview of this study, there is a growing research interest in analyzing and modeling how credit market frictions affect business cycle fluctuations. This field of study focuses on how endogenous factors in credit markets work within formal modeling frameworks to amplify and propagate macroeconomic shocks. In this regard, some papers analyze the impact of deteriorating credit conditions on macroeconomic outcomes during economic downturns using dynamic general equilibrium models. Bolstered by recent experiences, including a series of financial crises and the reverberating effects of financial contagion that follow, these models characterize credit market imperfections indicating how asymmetric

\(^1\)see, Rabobank (2005) and EFInA (2016) for a survey on access to credit in developing countries.
\(^2\)For example, Findex survey on Nigeria indicates only approximately 65 per cent of the adult population.
\(^3\)For reading convenience we refer to emerging markets and developing economies in this paper simply as emerging economies.
information affects the lending behavior of financial institutions.

In a two-period model, Bernanke and Gertler (1989) in a seminal paper shows temporary shocks are more persistent when credit conditions are tight. The study indicates that a credit crunch leads to a fall in entrepreneurs’ net worth which consequently reduces investment over consecutive periods. Bernanke et al. (1999), from here forward referred to as BGG, introduce such credit market frictions into a calibrated dynamic stochastic general equilibrium (DSGE) model with sticky prices in a more rigorous manner by incorporating a financial accelerator mechanism. Beyond evaluating the role of credit markets during financial crisis and economic downturns, BGG shows that such mechanisms can explain cyclical fluctuations in some macroeconomic variables and amplify real and nominal shocks to the economy. Similarly, Christensen and Dib (2008) estimate sticky price DSGE models to assess the importance of financial frictions in the amplification and propagation of transitory shocks. Comparing two variations of a DSGE model, one with and one without a financial accelerator, the authors estimate via a maximum likelihood procedure using post-war US data. The results show that depending on the initial shock, that the financial accelerator mechanism is an important determinant of fluctuations in investment.

Attempts have also been made to extend the research to emerging economies. Elekdag et al. (2005) develop a small open economy model in which entrepreneurs finance their investment partially, using foreign currency. The authors estimate using Bayesian techniques to assess the significance of balance sheet-related frictions in credit markets in emerging market economies through the financial accelerator mechanism. The estimation results in Elekdag et al. (2005) supports how including the financial accelerator in standard DSGE models act via balance sheets to magnify macroeconomic shocks, and thus increase the volatility in real and financial variables. Merola (2010) introduced financial frictions into a two-sector small open economy model to analyze the impulse response functions to various structural shocks. Starting from a basic model, frictions are incorporated step-by-step to analyze how outcomes are affected by each additional friction. Adding features into the DSGE model one after the other gives a clearer understanding of the model’s behaviour. The study considers frictions that includes, foreign currency denominated debt, sticky wages, habit in consumption, and a financial accelerator mechanism.

Saad et al. (2011) estimates a macroeconomic model with financial frictions evaluating the relationship between the banking sector and macroeconomic outcomes in Morocco. The paper compares models with and without financial frictions and shows that the integrating these mechanisms affects the impulse responses. The impulse responses generated describe agents behaviour and is useful for evaluating financial
stability.

The literature on estimated DSGE models with financial frictions is still emerging. Perhaps the closest to our study is Gabriel et al. (2016) who consider the role of the BGG financial accelerator in the monetary transmission mechanism. The paper develops an open-economy DSGE model of the Indian economy, building up in stages several standard features for emerging economies including; credit constrained consumers, a financial accelerator mechanism and an exchange rate pass-through. The authors find that financial frictions are important features in small-open economy DSGE models.

Nigeria is a small open emerging economy which depends largely on oil and gas for exports and fiscal revenue. Accounting for over 90.0 per cent of total exports, the economy is highly vulnerable to fluctuations in oil prices. Recent statistics on financial inclusion in Nigeria indicate that 41.6 per cent of the 96.4 million adult population remains financially excluded.4 Of these only 33 per cent have access to credit of which 76.3 per cent still borrow from family and friends to meet urgent needs. The report indicates that only 2.7 per cent of the banked public have access to credit facilities. The proportion of the financially included adult population that are banked is 38.3 per cent.

In this paper, we extend a small open economy dynamic stochastic general equilibrium (DSGE) model for Nigeria, a major oil exporter, incorporating a mechanism for the ‘financial accelerator as a credit market friction. For an oil exporting economy with significant exposure to external shocks, introducing a financial accelerator serves as an important amplification device for replicating the impact of business cycle dynamics. Thus, we develop an open economy model with oil revenue, endogenous fiscal policy, incomplete exchange rate pass-through, credit-constrained consumers and a financial accelerator facing domestic firms seeking to finance their investment.

Following Olekah and Oyaromade (2007) which presented a small-scale DSGE model of the Nigerian economy given a pseudo-estimation of the model by VAR methodology, development of DSGE models is now routine and estimation are successfully undertaken via Bayesian estimation methods for Nigeria (see, Adebiyi and Mordi (2010), Adebiyi and Mordi (2012) and CBN (2013)). However, literature on estimating DSGE models with a financial accelerator for an oil exporter is relatively sparse. Babilla et al. (2016) in a recent study examines the effect of credit market imperfections on macroeconomic outcomes in Nigeria, by developing a small open economy DSGE model with financial frictions. The study shows that exter-

\footnote{EFInA (2016) reports "adults who do not have or use deposit money banks, other formal or any informal financial services. If they borrow or remit they do this through family and friends; if they save, they save at home."}
Figure 1: Adult Population Access to Financial Markets in Nigeria 2008 - 2016

Figure 2: Adult Population Access to Financial Services in Percentages 2008 - 2016
nal finance premium in the new Keynesian framework improves the ability of DSGE models to fit macroeconomic data and economic intuition. The modeling outcome implies that, DSGE models that incorporate financial frictions perform well in explaining real macroeconomic variables in Nigeria. Our study contributes to existing literature by examining the effects incorporating more rigourously a framework to account for credit market frictions such as a ‘financial accelerator’ has on a non-industrial oil producer economy. Extending these works, our paper also incorporates credit-constraint consumers in a fully micro-founded DSGE model with an oil sector and endogenous fiscal policy for Nigeria.

The main objective of this study is to highlight the relative importance of incorporating financial frictions, particularly the financial accelerator in a small open economy model for a non-industrial oil exporter. Following the introduction, section 2 presents the main features incorporated into DSGE model for Nigeria analyzed in this paper. We estimate via Bayesian maximum likelihood methods, analyse the results and stress some policy implications in sections 3, 4 and 5, respectively. Section 6 concludes.

2 Small Open Economy DSGE Oil-producer Model

We present in this section, the distinctive modelling features incorporated into the two-bloc small open economy DSGE model analyzed in this paper for the non-industrial oil producer economy. The Smets-Wouters type small open-economy DSGE model features nominal price rigidities a la Calvo (1983) (staggered prices and imperfect competition). The model also incorporates other features peculiar to emerging economy oil-producers including credit-constrained consumers, international financial frictions, incomplete exchange rate pass-through to import and export prices and oil revenue. A descriptive presentation of the core structure of the model can be found in Gabriel et al. (2010) which presents the model development step-by-step. A summary of the core of the DSGE model is in the Appendix A.

2.1 Dynamic Model

An Open Economy NK Model with Financial Frictions

Following the core structure of the model summarized in the Appendix, we introduce some important features for the Nigerian economy. First, we incorporate the two financial frictions that capture household heterogeneity in the form of Ricardian and non-Ricardian (credit-constrained) consumers, then a financial accelerator mecha-
nism to typify borrowing constraints facing domestic firms. Including these modules recognises crucial transmission channels through which shocks are propagated and provides conceptual basis for understanding the impact of adverse economic conditions such as periods of financial stress and economic recessions. The financial accelerator mechanism links borrowing cost to the net worth of firms in the model in order to amplify real and nominal shocks (see, Carlstrom and Fuerst (1997), Bernanke et al. (1999) and Gertler et al. (2003)).

First, $\lambda$ is the proportion of credit-constrained consumers who have no income from monopolistic retail firms. These non-Ricardian households consume wage income given by

$$C_{1,t} = \frac{W_t h_t}{P_t}$$  \hspace{1cm} (1)

The Ricardian consumers thus consume $C_{2,t}$. Composite consumption $C_t$ is equivalent to

$$C_t = \lambda C_{1,t} + (1 - \lambda)C_{2,t}$$  \hspace{1cm} (2)

given $0 < \lambda < 1$.

Introducing financial frictions in the model, the first step is to add liability dollarization as wholesale firms access funds from domestic and foreign financial intermediaries in two currencies in proportions that are exogenously determined. Thus risk premium and financial stress is given as $\varphi \in [0, 1]$, hence the expected cost, given $P_t$ the consumption price index and $R_{n,t}$ nominal interest rate will be

$$\Theta_t \varphi E_t \left[ (1 + R_{n,t}) \frac{P_{C,t}}{P_{C,t+1}} \right] + \Theta_t (1 - \varphi) E_t \left[ (1 + R^*_{n,t}) \frac{P^*_{C,t}}{P^*_{C,t+1}} \frac{RER_{C,t+1}}{RER_{C,t}} \right]$$

$$= \Theta_t \left[ \varphi E_t [(1 + R_{t+1})] + (1 - \varphi) E_t \left[ (1 + R^*_{t+1}) \frac{RER_{C,t+1}}{RER_{C,t}} \right] \right]$$  \hspace{1cm} (3)

If $\varphi = 1$ or if UIP holds this becomes $(1 + \Theta_t) E_t [1 + R_{t+1}]$. In (3), $RER_{C,t} = \frac{P^*_{C,t}}{P_{C,t}}$ is the real exchange rate, $R_t \equiv \left[ (1 + R_{n,t-1}) \frac{P_t}{P_{t-1}} \right] - 1$ equals the ex post real interest rate over $[t - 1, t]$ and $\Theta_t \geq 0$ is the external finance premium.

Secondly, external finance premium $\Theta_t$ will have firms equate expected returns to expected borrowing costs as follows

$$E_t [1 + R_{k,t+1}] = E_t \left[ \Theta_{t+1} \left( \varphi E_t [(1 + R_{t+1})] + (1 - \varphi) E_t \left( (1 + R^*_{t+1}) \frac{RER_{C,t+1}}{RER_{C,t}} \right) \right) \right]$$  \hspace{1cm} (4)

where

$$\Theta_t = s \left( \frac{N_t}{Q_{t-1} K_t} \right) ; \quad s'(\cdot) < 0$$  \hspace{1cm} (5)
In (5), \( N_t \) is net worth and \( Q_{t-1}K_t - N_t \) is now the requirement firms must meet to access funding externally. Therefore \( \frac{Q_{t-1}K_t - N_t}{N_t} \) represents the leverage ratio, and hence (4) and (5) implies capital cost is an increasing function of the leverage.

Assuming entrepreneurs exit under the probability \( 1 - \xi_e \), then firms net worth accumulates in line with

\[
N_{t+1} = \xi_e V_t + (1 - \xi_e)D_t^e
\]

(6)

where \( D_t^e \) are transfers made exogenously to new entrants from existing firms, while \( V_t \) is the net value carried to the next period. \( D_t^e \) is consistent with a balance growth path and is given by

\[
V_t = (1 + R_{k,t})Q_{t-1}K_t - \Theta_t \left[ \varphi (1 + R_t) + (1 - \varphi)(1 + R_t) \frac{RER_{e,t}}{RER_{e,t-1}} \right] (Q_{t-1}K_t - N_t)
\]

(7)

where \( R_{k,t} \) is the \textit{ex post} return

\[
1 + R_{k,t} = \frac{(1 - \alpha) \frac{Y^W}{Y_t} \frac{Y^W_t}{Y^W_t} + (1 - \delta)Q_t}{Q_{t-1}}
\]

(8)

Demand for capital is then given by

\[
E_t[1 + R_{k,t+1}] = \frac{E_t \left[ (1 - \alpha) \frac{Y^W}{Y_t} \frac{Y^W_{t+1}}{Y^W_{t+1}} + (1 - \delta)Q_{t+1} \right]}{Q_t}
\]

(9)

Finally, exiting entrepreneurs consume the residual equity so that their consumption

\[
C_t^e = \frac{1 - \xi_e}{\xi_e} N_t
\]

(10)

which becomes a part of total consumption.

**Fiscal Policy**

**Oil Sector**

We introduce the oil sector into the model by first specifying oil output \( Y^O \) as exogenously determined by the ratio of long-run oil contribution to real output. Since this is an ‘enclave sector’ we do not model the production linkages as developed in other papers. In the same vein, oil price follows Hamilton (1983) as an AR(1) process with
a random walk without a drift as elucidated in Bernanke et al. (1997), hence

\[ \log p_t^O = \log p_{t-1}^O + \varepsilon_{t}^{pO}, \]

where, \((\varepsilon_{t}^{pO} \sim i.i.d. N(0, \sigma_{pO}^2))\) is the shock to oil price.

Government revenue is simply aggregated as \(r_t^O\) which proxies royalties as well as profit taxes paid each period from sales of crude oil.\(^5\)

\[ T_t^O = \tau_t^O p_t^O y_t^O. \]

Given that gross nominal interest and the ex post real interest rates relationship is the Fischer equation

\[ 1 + \Pi_t = \frac{1 + R_{n,t-1}}{1 + \Pi_t} \quad (11) \]

Government debt will accumulate following

\[ D_t = 1 + R_t D_{t-1} + G_t - T_t^O \quad (12) \]

Then in terms of ratios to GDP (12) we have

\[ d_t = 1 + R_t d_{t-1} \frac{Y_{t-1}}{Y_t} + g_t - t_t^O = \frac{1 + R_t}{1 + \Delta Y_t} d_{t-1} + g_t - t_t^O \]

\[ (13) \]

\[ \Delta Y_t \equiv \frac{Y_t - Y_{t-1}}{Y_{t-1}} \text{ being the real growth rate of GDP}^6. \] Thus, government spending to GDP is

\[ g_t = t_t^O + d_t - \frac{1 + R_t}{1 + \Delta Y_t} d_{t-1} \quad (14) \]

A debt-GDP ratio rule closes the government side which returns the target steady-state debt-GDP ratio \(d\) at a rate that depends on \(\rho_d\) and an exogenous random shock that captures political and implementation uncertainty regarding the rule.

In a balanced growth steady state we have

\[ g = t^O + d + \frac{1 + R}{1 + \Delta Y} d \quad (15) \]

\(^5\)Resource production is subject to royalties at a rate of \(\tau_t^Roy\) and profit taxes at a rate of \(\tau_t^{div}\) determined by existing oil fiscal regime.

\(^6\)Debt, government spending and tax to GDP ratios are \(d_t \equiv \frac{D_t}{Y_t}, g_t \equiv \frac{G_t}{Y_t}\) and \(t_t^O \equiv \frac{T_t^O}{Y_t}\), respectively.
Thus for $R > \Delta Y$, $g < t^O$. In the model calibrate $t^O$ to be consistent with observed long-run values for $d$ and $g$ and 15.

**THE CENTRAL BANK**

The conventional Taylor-type policy rule we have $R_{n,t}$ the policy variable specified as:

$$\log \left( \frac{R_{n,t}}{R_n} \right) = \rho \log \left( \frac{R_{n,t-1}}{R_n} \right) + \theta_s \log \left( \frac{\Pi_t}{\Pi} \right) + \theta_s \log \left( \frac{S_t}{S} \right) + \epsilon_{MPS,t} \quad (16)$$

$$\log \frac{R_{n,t}}{R_n} = \rho_r \log R_{n,t-1}/R_n + (1 - \rho_r)(\theta_s E_t[\log \Pi_{t+j}]/\Pi$$

$$+ \theta_s \log S_t/S) + \epsilon_{MPS,t+1} \quad (17)$$

(17) is an ‘implementable’ rule that does not require knowledge of the output gap. Market clearing equation is therefore

$$Y_t = C_{H,t} + I_{H,t} + C^*_{H,t} + I^*_{H,t} + G_t \quad (18)$$

**Shocks** Five domestic shocks for technology, mark-up, UIP, consumption and investment conclude the home module:

$$\log \frac{A_{t+1}}{A} = \rho_t \log \frac{A_t}{A} + \epsilon_{a,t+1} \quad (19)$$

$$\log \frac{MS_{t+1}}{MS} = \rho_{ms} \log \frac{MS_t}{MS} + \epsilon_{ms,t+1} \quad (20)$$

$$\log \frac{UIP_{t+1}}{UIP} = \rho_{UIP} \log \frac{UIP_t}{UIP} + \epsilon_{uiP,t+1} \quad (21)$$

$$\epsilon_{r,t+1} = \rho_r \epsilon_{r,t} + \nu_{r,t+1} \quad (22)$$

$$\log \frac{\Psi_{C,t+1}}{\Psi_C} = \rho_{\psi_c} \log \frac{\Psi_{C,t}}{\Psi_C} + \epsilon_{\psi_c,t+1} \quad (23)$$

$$\log \frac{\Psi_{I,t+1}}{\Psi_I} = \rho_{\psi_i} \log \frac{\Psi_{I,t}}{\Psi_I} + \epsilon_{\psi_i,t+1} \quad (24)$$
The rest of the world processes are exogenously determined by AR1 shocks

\[
\log \frac{R_{n,t}^*}{R_{n}^*} = \rho_r^* \log \frac{R_{n,t-1}^*}{R_{n}^*} + \epsilon_{r,t+1}^* \tag{25}
\]

\[
\log \frac{\Pi_{t+1}^*}{\Pi^*} = \rho_\pi^* \log \frac{\Pi_t^*}{\Pi^*} + \epsilon_{\pi,t+1}^* \tag{26}
\]

\[
\log \frac{C_{t+1}^*}{C^*} = \rho_c^* \log \frac{C_t^*}{C^*} + \epsilon_{c,t+1}^* \tag{27}
\]

\[
\log \frac{I_{t+1}^*}{I^*} = \rho_i^* \log \frac{I_t^*}{I^*} + \epsilon_{i,t+1}^* \tag{28}
\]

and

\[
U_{C,t}^* = U \tag{29}
\]

\[
h_t^* = h \tag{30}
\]

\[
Y^* = C^* + I^* \tag{31}
\]

3 Calibration and Estimation

3.1 Bayesian Estimation

We estimate the non-linear DSGE model presented earlier using Bayesian methods in this section

3.2 Data

Six quarterly observable macroeconomic variables including output, investment, consumption, inflation rate, real effective exchange rate (REER) and nominal interest rate form the data set used in the estimation of the system. The secondary data are obtained from sources including National Bureau of Statistics Nigeria, the Statistical Bulletin of Central Bank of Nigeria Statistical and the International Financial Statistics (IFS) of the International Monetary Fund. The estimated period is from 2002:3-2015:4 after due consideration for issues such as the existence of structural breaks in the data. We use TRAMO-SEATS throughout for seasonal adjustments and apply the Hodrick-Prescott filter with \(\lambda = 1600\) in order to obtain stationary data for the real output (GDP), investment(INV) and consumption(CONS).

The corresponding measurement equations of the model are specified as:
\[
\begin{bmatrix}
\log(GDP_t)/GDP_t \\
\log(INV_t)/INV_t \\
\log(CONS_t)/CONS_t \\
\log(CPI_t - CPI_{t-1}) - \log(\Pi) \\
3MDR_t/4 - 3MDR_t/4 \\
\log(REER_t/REER_t)
\end{bmatrix}
= \begin{bmatrix}
\log(Y_t) + mey_t \\
\log(INV_t) + mec_t \\
\log(CONS_t/CONS_t) + mei_t \\
\log(\Pi) \\
\log(REER_t/REER_t)
\end{bmatrix}
\]

### 3.2.1 Calibrated Parameters

We fix a number of parameters including structural parameters set to correspond to data sample means. This ensures that all calibrated parameters are reflective of the observed steady state values. The settings are listed in Table 1.

Finally, trade data is applied to we calibrate \( \omega_C \) and \( \omega_I \). From (A.88) we have

\[
\begin{align*}
\text{cs}_{\text{imp}} & \equiv \frac{\text{C-imports}}{\text{GDP}} = \frac{P_F C_F}{P_C Y} = c_y(1 - w_C) \left( \frac{P_F}{P_C} \right)^{-\mu_C} \\
\text{is}_{\text{imp}} & \equiv \frac{\text{I-imports}}{\text{GDP}} = \frac{P_F I_F}{P_C Y} = i_y(1 - w_I) \left( \frac{P_F}{P_I} \right)^{-\mu_I} \\
\text{cs}_{\text{exp}} & \equiv \frac{\text{C-exports}}{\text{GDP}} = (1 - \omega_C^*) \left( \frac{P_H}{P_C RER_C} \right)^{-\mu_C^*} \frac{Y^*}{c_y^* Y} = \frac{C^*_H}{Y} \\
\text{is}_{\text{exp}} & \equiv \frac{\text{I-exports}}{\text{GDP}} = (1 - \omega_I^*) \left( \frac{P_H}{P_I RER_I} \right)^{-\mu_I^*} \frac{Y^*}{i_y^* Y} = \frac{I^*_H}{Y} \\
\text{tb} & \equiv \frac{TB}{P_C Y} = \text{oil} + \text{cs}_{\text{exp}} + \text{is}_{\text{exp}} - \text{cs}_{\text{imp}} - \text{is}_{\text{imp}}
\end{align*}
\]

where \( \text{oil} \equiv \frac{TB}{P_C Y} \). Hence using data for shares \( \text{cs}_{\text{imp}}, \text{is}_{\text{imp}}, \text{cs}_{\text{exp}}, \text{is}_{\text{exp}} \) and \( \text{oil} \), we can calibrate \( \omega_C \) and \( \omega_I \) making these variables to be solved in the steady state.

Using data for Nigeria foreign trade statistics for 2014 (see, NBS (2015)) we obtain \( \text{cs}_{\text{imp}} = 0.10, \text{is}_{\text{imp}} = 0.15, \text{cs}_{\text{exp}} = 0.23 \) and \( \text{is}_{\text{exp}} = 0.02 \) for \( \text{TB} = 0 \).

We can choose units of home and foreign output so that \( P_H = P_F = 1 \). Then \( P_C = P_I = 1 \). The remaining calibration is \( k = \frac{Y^*}{Y}, c_y^* \) and \( i_y^* \).

### 3.2.2 Prior Distribution of Estimated Parameters

The theoretical underpinnings of the model and findings from previous related studies are used to estimate parameters for the Nigerian economy. Thus, prior distributions are set as provided by foreknowledge in the Table 2.
Calibrated parameter | Symbol | Value for Nigeria
--- | --- | ---
Discount factor | $\beta$ | 0.99
Depreciation rate | $\delta$ | 0.025
Risk premium - scaling | $k_B$ | 1.00
FA risk premium | $\theta$ | 1.00
Risk premium elasticity | $\chi_B$ | 0.05

**Implied steady state relationship**

<p>| | | | | |</p>
<table>
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| hours worked/time available | $h$ | 0.35 | calibrated to hit proportion of hours worked
| Preference parameter | $\varrho$ | | |
| Imported investment share | $i_{s_{\text{import}}}$ | 0.15 | |
| Imported consumption share | $c_{s_{\text{import}}}$ | 0.10 | |
| Exported investment share | $i_{s_{\text{export}}}$ | 0.02 | |
| Exported consumption share | $c_{s_{\text{export}}}$ | 0.23 | |
| oil revenues/GDP | $oil$ | 0.08 | |

**Table 1:** Calibrated Parameters

### 3.3 Posterior Estimates

Two optimization routines are used to obtain the joint distribution of estimated parameters. The first optimization routine produces the Hessian matrix and posterior model, while the Hessian matrix is applied to generate samples from the posterior distribution using the algorithm Metropolis-Hastings. The procedures are undertaken to obtain reasonable estimates by running 150,000 draws of the MCMC-MH algorithm and adjusting the variance-covariance matrix so acceptance rates range between 20 and 30 per cent.\(^7\) Table 3, reports estimates of posterior means alongside the 95% confidence intervals for all estimated parameters.

Estimations are reported in Table 3 for both the standard DSGE model without financial frictions for the oil exporter (No FF) and for a model with financial frictions which include the financial accelerator mechanism (FF).

We find the posterior estimates as well as confidence intervals in the columns on the right of the Table. These are reasonable results similar to earlier studies (see, for example Gabriel *et al.* (2016)). The estimates for the calvo parameter are quite plausible between 2-4 Quarters being less stickier for the model with financial frictions. Results obtained for consumption habits, $\alpha$, $\sigma$ and $\zeta$ are also as expected for a nonindustrial small open economy.

The policy parameters capture the response to inflation suggesting the CBN is aggressive towards curtailing inflation with $\theta_{p_{\text{in}}}$ above 1. Degree of policy smoothing high at $\rho_r$ estimated at 0.83. The standard deviations are large as expected for an

\(^7\)See Schorfheide (2000) provides more details on the estimation procedures.
<table>
<thead>
<tr>
<th>S/N</th>
<th>Parameter</th>
<th>Notations</th>
<th>Prior distribution</th>
<th>Author(year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Risk aversion</td>
<td>$\sigma$</td>
<td>Inv. gamma</td>
<td>2.00 4.00</td>
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<tr>
<td>2</td>
<td>Labour share</td>
<td>$\alpha$</td>
<td>Beta</td>
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<td>3</td>
<td>Consumption habit</td>
<td>$h_c$</td>
<td>Beta</td>
<td>0.70 0.10</td>
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<tr>
<td>4</td>
<td>Investment adjustment</td>
<td>$\phi_i$</td>
<td>Inv. gamma</td>
<td>4.00 3.00</td>
</tr>
<tr>
<td>5</td>
<td>Calvo-prices</td>
<td>$\zeta$</td>
<td>Beta</td>
<td>0.75 0.15</td>
</tr>
<tr>
<td>6</td>
<td>Substitution elasticity</td>
<td>$\chi$</td>
<td>Normal</td>
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</tr>
<tr>
<td>7</td>
<td>Liquidity constrained consumers</td>
<td>$\lambda$</td>
<td>Beta</td>
<td>0.40 0.10</td>
</tr>
<tr>
<td>8</td>
<td>Feedback from expected inflation</td>
<td>$\theta_p$</td>
<td>Normal</td>
<td>2.00 1.00</td>
</tr>
<tr>
<td>9</td>
<td>Feedback from exchange rate</td>
<td>$\theta_r$</td>
<td>Inv. gamma</td>
<td>0.001 0.01</td>
</tr>
<tr>
<td>10</td>
<td>Interest rate smoothing</td>
<td>$\rho_r$</td>
<td>Beta</td>
<td>0.700</td>
</tr>
<tr>
<td>11</td>
<td>Technology</td>
<td>sd ($\epsilon_A$)</td>
<td>Inv. gamma</td>
<td>2.00 4.00</td>
</tr>
<tr>
<td>12</td>
<td>Government spending</td>
<td>sd ($\epsilon_G$)</td>
<td>Inv. gamma</td>
<td>3.00 4.00</td>
</tr>
<tr>
<td>13</td>
<td>Price mark-up</td>
<td>sd ($\epsilon_{MS}$)</td>
<td>Inv. gamma</td>
<td>2.00 3.00</td>
</tr>
<tr>
<td>14</td>
<td>Monetary policy</td>
<td>sd ($\epsilon_{mps}$)</td>
<td>Inv. gamma</td>
<td>3.00 4.00</td>
</tr>
<tr>
<td>15</td>
<td>Foreign Interest rate</td>
<td>sd ($\epsilon_r$)</td>
<td>Inv. gamma</td>
<td>3.00 4.00</td>
</tr>
<tr>
<td>16</td>
<td>Foreign inflation rate</td>
<td>sd ($\epsilon_{mf}$)</td>
<td>Inv. gamma</td>
<td>3.00 4.00</td>
</tr>
<tr>
<td>17</td>
<td>Foreign consumption</td>
<td>sd ($\epsilon_c$)</td>
<td>Inv. gamma</td>
<td>2.00 3.00</td>
</tr>
<tr>
<td>18</td>
<td>Oil Price</td>
<td>sd ($\epsilon_{oil}$)</td>
<td>Inv. gamma</td>
<td>3.00 4.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S/N</th>
<th>Parameter</th>
<th>Notations</th>
<th>Prior distribution</th>
<th>Author(year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Risk aversion</td>
<td>$\sigma$</td>
<td>Inv. gamma</td>
<td>Berg et al. (2013), Agenor and Montiel (1999)</td>
</tr>
<tr>
<td>2</td>
<td>Labour share</td>
<td>$\alpha$</td>
<td>Beta</td>
<td>Gabriel et al. (2010)</td>
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<td>3</td>
<td>Consumption habit</td>
<td>$h_c$</td>
<td>Beta</td>
<td>Adebiyi and Mordi (2010), CBN (2013)</td>
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<td>4</td>
<td>Investment adjustment</td>
<td>$\phi_i$</td>
<td>Inv. gamma</td>
<td>CBN (2013)</td>
</tr>
<tr>
<td>5</td>
<td>Calvo-prices</td>
<td>$\zeta$</td>
<td>Beta</td>
<td>CBN (2013)</td>
</tr>
<tr>
<td>6</td>
<td>Substitution elasticity</td>
<td>$\chi$</td>
<td>Normal</td>
<td>CBN (2013)</td>
</tr>
<tr>
<td>7</td>
<td>Liquidity constrained consumers</td>
<td>$\lambda$</td>
<td>Beta</td>
<td>EFInA (2010)</td>
</tr>
<tr>
<td>8</td>
<td>Feedback from expected inflation</td>
<td>$\theta_p$</td>
<td>Normal</td>
<td>CBN (2013)</td>
</tr>
<tr>
<td>9</td>
<td>Feedback from exchange rate</td>
<td>$\theta_r$</td>
<td>Inv. gamma</td>
<td>CBN (2013)</td>
</tr>
<tr>
<td>10</td>
<td>Interest rate smoothing</td>
<td>$\rho_r$</td>
<td>Beta</td>
<td>Garcia (2010), Adebiyi and Mordi (2010)</td>
</tr>
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<td>11</td>
<td>Technology</td>
<td>sd ($\epsilon_A$)</td>
<td>Inv. gamma</td>
<td>Adolfsson et al. (2008)</td>
</tr>
<tr>
<td>12</td>
<td>Government spending</td>
<td>sd ($\epsilon_G$)</td>
<td>Inv. gamma</td>
<td>Authors’ estimates</td>
</tr>
<tr>
<td>13</td>
<td>Price mark-up</td>
<td>sd ($\epsilon_{MS}$)</td>
<td>Inv. gamma</td>
<td>Authors’ estimates</td>
</tr>
<tr>
<td>14</td>
<td>Monetary policy</td>
<td>sd ($\epsilon_{mps}$)</td>
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<td>Foreign consumption</td>
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</tr>
<tr>
<td>18</td>
<td>Oil Price</td>
<td>sd ($\epsilon_{oil}$)</td>
<td>Inv. gamma</td>
<td>Authors’ estimates</td>
</tr>
</tbody>
</table>

**Table 2: Priors**

emerging economy being close to the priors.

4 **EMPIRICAL APPLICATION**

After having shown the model estimates and the assessment of relative model fit to its other rivals with different restrictions, we use them to investigate a number of key macroeconomic issues in Nigeria.

4.1 **IMPULSE RESPONSE ANALYSIS**

In this section we study (the estimated posterior) impulse responses for four selected shocks: a domestic technology shock, government spending shock, monetary policy shock and an oil price shock. Figures below plot the mean responses corresponding to a positive one standard deviation of the shocks’ innovation and are derived from the mean posteriors.

Overall, the impulse responses derived from the posteriors estimates show that incorporating a financial accelerator mechanism magnifies shocks to investment and output (see, Gertler et al. (2003)). The impulse responses show that with the inclusion of a financial accelerator mechanism, investment shock are amplified. Investment is important to production in the oil sector and may be largely impacted when oil prices fall. In addition, due to the simple specification of fiscal policy in the model, the impulse responses show that for fiscal policy to serve as an effective macroeconomic
Figure 3: Estimated Impulse Responses to a Positive Productivity Shock

Figure 4: Estimated Impulse Responses to a Government Spending Shock
Figure 5: Estimated Impulse Responses to a Monetary Policy Shock

Figure 6: Estimated Impulse Responses to a Oil Price Shock
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Prior distribution</th>
<th>Posterior distribution&lt;sup&gt;◊&lt;/sup&gt;</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Density</td>
<td>Mean</td>
<td>S.D/df No FF FF</td>
</tr>
<tr>
<td>Investment adjustment</td>
<td>$\phi_i$</td>
<td>Inv. gamma 4.00 3.00</td>
<td>1.43 [1.03:1.77] 1.40 [1.03:1.77]</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$\sigma$</td>
<td>Inv. gamma 2.00 4.00</td>
<td>1.57 [0.51:2.79] 1.07 [0.55:1.57]</td>
</tr>
<tr>
<td>Consumption habit</td>
<td>$b_C$</td>
<td>Beta 0.70 0.10</td>
<td>0.68 [0.49:0.86] 0.70 [0.53:0.88]</td>
</tr>
<tr>
<td>Calvo prices</td>
<td>$\xi$</td>
<td>Beta 0.75 0.15</td>
<td>0.55 [0.40:0.73] 0.63 [0.50:0.76]</td>
</tr>
<tr>
<td>Labour share</td>
<td>$\alpha$</td>
<td>Beta 0.60 0.10</td>
<td>0.78 [0.76:0.80] 0.79 [0.77:0.80]</td>
</tr>
<tr>
<td>Import Pass-through parameter</td>
<td>$\theta$</td>
<td>Beta 0.50 0.20</td>
<td>0.36 [0.21:0.51] 0.36 [0.21:0.51]</td>
</tr>
<tr>
<td>Substitution elasticity (H/F goods)</td>
<td>$\mu$</td>
<td>Normal 1.50 0.20</td>
<td>1.41 [1.07:1.73] 1.37 [1.06:1.70]</td>
</tr>
<tr>
<td>Substitution elasticity (varieties)</td>
<td>$\zeta$</td>
<td>Normal 7.00 0.50</td>
<td>6.98 [6.17:7.78] 6.98 [6.20:7.83]</td>
</tr>
<tr>
<td>Financial frictions</td>
<td>External finance premium elasticity (F)</td>
<td>$\chi$</td>
<td>Inv. gamma 0.03 4.00</td>
</tr>
<tr>
<td>Inverse of Leverage (F)</td>
<td>$n_k$</td>
<td>Beta 0.50 0.15</td>
<td>- 0.49 [0.35:0.65]</td>
</tr>
<tr>
<td>Proportion of RT consumers</td>
<td>$\lambda$</td>
<td>Beta 0.40 0.10</td>
<td>- 0.38 [0.22:0.53]</td>
</tr>
<tr>
<td>Interest rate smoothing</td>
<td>$\rho_e$</td>
<td>Beta 0.50 0.10</td>
<td>0.82 [0.78:0.87] 0.83 [0.79:0.88]</td>
</tr>
<tr>
<td>Feedback from expected inflation</td>
<td>$\theta_{bh}$</td>
<td>Normal 1.50 0.01</td>
<td>1.498 [1.482:1.515] 1.499 [1.483:1.516]</td>
</tr>
<tr>
<td>Feedback from X-rate</td>
<td>$\theta_{\phi}$</td>
<td>Inv. gamma 0.005 0.001</td>
<td>0.005 [0.004:0.007] 0.005 [0.004:0.007]</td>
</tr>
<tr>
<td>AR(1) coefficient</td>
<td>Technology</td>
<td>$\rho_A$</td>
<td>Beta 0.85 0.10</td>
</tr>
<tr>
<td>Government spending</td>
<td>$\rho_G$</td>
<td>Beta 0.85 0.10</td>
<td>0.85 [0.71:0.99] 0.85 [0.72:0.99]</td>
</tr>
<tr>
<td>Price mark-up</td>
<td>$\rho_{MS}$</td>
<td>Beta 0.50 0.10</td>
<td>0.50 [0.34:0.66] 0.50 [0.33:0.66]</td>
</tr>
<tr>
<td>Foreign Interest rate</td>
<td>$\rho_{fe}$</td>
<td>Beta 0.50 0.10</td>
<td>0.49 [0.34:0.65] 0.49 [0.35:0.64]</td>
</tr>
<tr>
<td>Foreign inflation rate</td>
<td>$\rho_{f}$</td>
<td>Beta 0.50 0.10</td>
<td>0.47 [0.33:0.62] 0.48 [0.33:0.62]</td>
</tr>
<tr>
<td>Foreign consumption</td>
<td>$\rho_{fc}$</td>
<td>Beta 0.50 0.10</td>
<td>0.51 [0.36:0.69] 0.51 [0.35:0.68]</td>
</tr>
<tr>
<td>Standard deviation of AR(1) innovations/LLD. shocks</td>
<td>Technology</td>
<td>$sd(\epsilon_A)$</td>
<td>Inv. gamma 2.00 4.00</td>
</tr>
<tr>
<td>Government spending</td>
<td>$sd(\epsilon_G)$</td>
<td>Inv. gamma 3.00 4.00</td>
<td>3.14 [2.85:4.57] 2.74 [2.04:3.46]</td>
</tr>
<tr>
<td>Price mark-up</td>
<td>$sd(\epsilon_{MS})$</td>
<td>Inv. gamma 2.00 4.00</td>
<td>2.06 [0.51:3.42] 1.76 [0.53:1.98]</td>
</tr>
<tr>
<td>Monetary policy</td>
<td>$sd(\epsilon_{MP})$</td>
<td>Inv. gamma 3.00 4.00</td>
<td>0.79 [0.62:0.96] 0.77 [0.60:0.94]</td>
</tr>
<tr>
<td>Foreign Interest rate</td>
<td>$sd(\epsilon_f)$</td>
<td>Inv. gamma 3.00 4.00</td>
<td>2.19 [1.14:3.16] 2.40 [1.29:3.37]</td>
</tr>
<tr>
<td>Foreign inflation rate</td>
<td>$sd(\epsilon_{fr})$</td>
<td>Inv. gamma 3.00 4.00</td>
<td>1.37 [0.89:1.83] 1.14 [0.76:1.54]</td>
</tr>
<tr>
<td>Foreign consumption</td>
<td>$sd(\epsilon_c)$</td>
<td>Inv. gamma 2.00 3.00</td>
<td>11.69 [5.12:27.67] 3.24 [0.47:8.09]</td>
</tr>
<tr>
<td>Oil Price</td>
<td>$sd(\epsilon_{oil})$</td>
<td>Inv. gamma 3.00 4.00</td>
<td>2.67 [0.80:4.09] 2.39 [0.76:4.03]</td>
</tr>
</tbody>
</table>

**Table 3: Priors and Posterior Estimates**

<sup>◊</sup> Notes: we report posterior means and 90% probability intervals (in parentheses) based on the output of the Metropolis-Hastings Algorithm. Sample period: 2003:II to 2015:IV.

management tool during a crisis, government spending needs to be productive to impact positively on consumption.
5 Policy Discussions and Recommendations

There are strong implications for the conduct of monetary policy with the introduction of financial frictions in the DSGE model for Nigeria. Firstly, incorporating credit constrained consumers implies that the consumptions pattern of these households are only dependent on changes to real wages. Hence, the estimates show a significant proportion of composite consumption is driven by the behaviour of non-optimizing households which are quite insensitive to changes and movements in interest rates. Consequently, the heterogenous households in the DSGE model impacts negatively on the effectiveness of monetary policy as a stabilisation instrument since the transmission of monetary shocks via the interest rate channel is limited. The issue of limited access to financial markets pervades many emerging economies as revealed in the model estimates which further stresses the need for the central bank to implement policies that promote innovations for financial inclusion of a greater proportion of households. Implicitly, it may be inferred that increasing households access to financial markets should positively impact on productivity in an economy dominated by individual-owned small business holdings.

Secondly, incorporating a financial accelerator mechanism highlights crucial elements of the structure of an oil exporter economy exposed to the vagaries of oil prices. Particularly, for a nonindustrial oil exporter economy that faces large external shocks and depends on financing from abroad, liability dollarization alongside oil price shocks affects firms capacity to smooth production over the business cycle. The intuition that the net worth of firms deteriorates during adverse economic conditions helps amplify shocks to investment for the emerging economy oil producer. The implication is that during crisis periods firms would find it difficult borrowing to boost productivity. Similarly, there are lessons for the fiscal authorities seeking to borrow from international capital markets during an economic downturn. For nonindustrial oil producer economies, the external risk premium rises and the cost of borrowing increases substantially when oil price falls and foreign reserves are depleted. Thus, policy makers should consider building fiscal buffers as the most viable option by saving for the ‘rainy days’ and spend out of savings to boost the economy when required.
6 CONCLUSIONS AND FUTURE RESEARCH

Overall, the models estimated in this study are quite insightful. For an oil exporting economy with significant exposure to external shocks, introducing a financial accelerator mechanism serves as an important amplification device for replicating business cycle dynamics. The Bayesian estimation results indicate the relative importance of incorporating financial frictions and other exogenous shocks in a nonindustrial oil exporting economy model. The oil-exporter economy DSGE model is useful for monetary policy analysis and simulations.

In view of the foregoing, future research may consider characterizing the dual structure of the Nigerian Economy with a two-sector formal/informal division of the nonindustrial oil producer DSGE model. The introduction of a formal/informal sector that captures the unofficial economy may significantly determine the sign and magnitude of impulse responses given the SOE features of the Nigerian economy.

REFERENCES


A Summary of Small Open Economy Model

A.1 Dynamic Model

For the small open economy as \( \nu \to 0 \) and \( w^*_C \to 1 \), from

\[
 w_C = 1 - (1 - \nu)(1 - \omega_C) ; \quad w^*_C = 1 - \nu(1 - \omega^*_C) \quad (A.1)
\]

In (A.1), \( \omega_C, \omega^*_C \in [0, 1] \) are a parameters that captures the degree of ‘bias’ in the two blocs. If \( \omega_C = \omega^*_C = 1 \) we have \( w_C = w^*_C = 1 \), i.e., autarky, while \( \omega_C = \omega^*_C = 0 \) gives us the case of perfect integration with \( w_C = \nu \) and \( w^*_C = 1 - \nu \), i.e., weights are in proportion to the proportions of goods produced in the two countries.

we have that \( \frac{1 - \nu}{1 - \omega_C} (1 - w^*_C) \to 1 - \omega_C^* \). Similarly, \( \frac{1 - \nu}{1 - \omega^*_C} (1 - w_C) \to 1 - \omega_C^* \).

\[
 \Lambda_{C,t} : \quad \frac{1}{1 + R_{m,t}} = \beta E_t \left[ \frac{\Lambda_{C,t+1}}{\Lambda_{C,t} \Pi_{t+1}} \right] \quad (A.2)
\]

\[
 W_t, \quad \frac{P_{C,t}}{P_{C,t}} = \frac{\Lambda_{L,t}}{\Lambda_{C,t}} = -\frac{\Lambda_{h,t}}{\Lambda_{C,t}} \quad (A.3)
\]

\[
 C_{2,t} : \quad \Lambda_{C,t} = (1 - \theta) C_{2,t}^{1 - \theta}(1 - \phi) - (1 - h_t) \theta(1 - \phi) \quad (A.4)
\]

\[
 \lambda_{h,t} = -C_{2,t}^{1 - \theta}(1 - \phi) - (1 - h_t) \theta(1 - \phi) - 1 \quad (A.5)
\]

\[
 C_{1,t} = \frac{W_t h_t}{P_{C,t}} \quad (A.6)
\]

\[
 C_t = \lambda C_{1,t} + (1 - \lambda) C_{2,t} \quad (A.7)
\]

\[
 \left( \frac{P_{F,t}}{P_{C,t}} \right) : \quad 1 = \left[ w_C \left( \frac{P_{H,t}}{P_{C,t}} \right)^{1 - \mu_C} + (1 - w_C) \left( \frac{P_{F,t}}{P_{C,t}} \right)^{1 - \mu_C} \right]^{\frac{1}{1 - \mu_C}} \quad (A.8)
\]

\[
 \frac{P_{H,t}}{P_{C,t}} = \frac{1}{[w_C + (1 - w_C) T_t^{1 - \mu_C}]^{\frac{1}{1 - \mu_C}}} \quad (A.9)
\]

where \( T_t \equiv \frac{P_{F,t}}{P_{H,t}} \)

\[
 C_{F,t} = w_C \left( \frac{P_{H,t}}{P_{C,t}} \right)^{-\mu_C} C_t \quad (A.10)
\]

\[
 C_{F,t} = (1 - w_C) \left( \frac{P_{F,t}}{P_{C,t}} \right)^{-\mu_C} C_t \quad (A.11)
\]

\[
 C_{H,t}^* = (1 - \omega_C^*) \left( \frac{P_{H,t}}{P_{C,t} \text{REER}_{C,t}} \right)^{-\mu_C} C_t^* \quad (A.12)
\]
\[ H_t : \quad H_t - \xi_H \beta E_t[\Pi_{H,t+1}^{\xi} H_{t+1}] = Y_t \Delta_{C,t} \]  
(A.13)

\[ J_t : \quad J_t - \xi_H \beta E_t[\Pi_{H,t+1}^{\xi} J_{t+1}] = \frac{1}{1 - \xi} MS Y_t \Delta_{C,t} MC_t \]  
(A.14)

\[ \Pi_{H,t} : \quad 1 = \xi_H \Pi_{H,t}^{\xi-1} + (1 - \xi_H) \left( \frac{J_t}{H_t} \right)^{1-\xi} \]  
(A.15)

\[ MC_t = \frac{P_{H,t}^W}{P_{C,t}} = \frac{P_{H,t}^W / P_{C,t}}{P_{H,t} / P_{C,t}} = \frac{W_t h_t}{P_{C,t}^2} \]  
(A.16)

\[ h_t : \quad Y_t^W = (A_t h_t)^\alpha K_t^{1-\alpha} \]  
(A.17)

\[ Y_t = (1 - c) Y_t^W \]  
(A.18)

\[ \frac{P_{H,t}^W}{P_{C,t}} = \frac{P_{H,t}^W}{P_{C,t}} = MC_t \frac{P_{H,t}}{P_{C,t}} \]  
(A.19)

\[ Q_t : E_t \left[ 1 + R_{t+1} \right] = \frac{E_t \left[ \frac{P_{H,t}^W}{P_{C,t}} \left( 1 - \alpha \right) \frac{Y_t}{R_{t+1}} + (1 - \delta) Q_{t+1} \right]}{Q_t} \]  
(A.20)

\[ R_t : \quad 1 + R_t = \frac{1 + R_{n,t-1}}{1 + \Pi_t} \]  
(A.21)

\[ K_{t+1} = (1 - \delta) K_t + (1 - S(X_t)) I_t \]  
(A.22)

\[ X_t = \frac{I_t}{I_{t-1}} \]  
(A.23)

\[ S(X_t) = \frac{\phi_t}{2} (X_t - (1 + g))^2 \]  
(A.24)

\[ I_t : \quad \frac{P_{I,t}}{P_{C,t}} = Q_t (1 - S(X_t) - X_t S'(X_t)) + E_t \left[ \frac{Q_{t+1} S'(X_{t+1})}{(1 + R_{t+1}) \frac{P_{R,t+1}}{P_{I,t}}} \right] \]  
(A.25)

\[ I_{H,t} = w_I \left( \frac{P_{H,t}/P_{C,t}}{P_{I,t}/P_{C,t}} \right)^{-\mu_I} I_t \]  
(A.26)

\[ I_{F,t} = (1 - w_I) \left( \frac{P_{F,t}/P_{C,t}}{P_{I,t}/P_{C,t}} \right)^{-\mu_I} I_t \]  
(A.27)

\[ I_{H,t}^* = (1 - \omega_I) \left( \frac{P_{H,t}/P_{C,t}}{P_{I,t}/P_{C,t}} \right)^{-\rho_I} I_t^* \]  
(A.28)

\[ \frac{P_{I,t}}{P_{C,t}} = \left[ w_I \left( \frac{P_{H,t}}{P_{C,t}} \right)^{1-\mu_I} + (1 - w_I) \left( \frac{P_{F,t}}{P_{C,t}} \right)^{1-\mu_I} \right]^{\frac{1}{1-\rho_I}} \]  
(A.29)

\[ Y_t : \quad Y_t = C_{H,t} + I_{H,t} + C_{F,t} + I_{F,t} + G_t \]  
(A.30)

\[ S_t = \frac{RER_{C,t} \Pi_t}{RER_{C,t-1} \Pi_t} \]  
(A.31)
\[ \Pi_{F,t} : \quad \frac{\tau_t}{\tau_{t-1}} = \frac{\Pi_{F,t}}{\Pi_{H,t}} \]  
(A.32)

\[ \tau_t : \quad \text{RER}_{C,t} = \frac{1}{[1 - w_C + w_C \tau_t^{\mu_C - 1}]^{1/\mu_C}} \]  
(A.33)

\[ \text{RER}_{t,t} = \frac{1}{[1 - w_I + w_I \tau_t^{\mu_I - 1}]^{1/\mu_I}} \]  
(A.34)

\[ \Pi_t = [w_C(\Pi_{H,t})^{1-\mu_C} + (1 - w_C)(\Pi_{F,t})^{1-\mu_C}]^{1/\mu_C} \]  
(A.35)

\[ \log(1 + R_{n,t})/(1 + R_n) = \rho_t \log(1 + R_{n,t-1})/(1 + R_n) + (1 - \rho_t)(\theta_s E_t[\log \Pi_{t+1}] / \Pi + \theta_s \log S_t/S) + \epsilon_{t,t+1} \]  
(A.36)

\[ \text{RER}_{t}^* = \frac{\Lambda_{C,t}^*}{\Lambda_{C,t}} \]  
(A.37)

\[ 1 + R_{t}^* = \frac{1 + R_{n,t-1}^*}{1 + \Pi_t^*} \]  
(A.38)

\[ \frac{1}{(1 + R_{n,t}^*) \phi(S_tB_{F,t}^*/P_{H,t}Y_t)} S_tB_{F,t}^* = S_tB_{F,t-1}^* + TB_t \]  
(A.39)

\[ \phi(S_tB_{F,t}^*/P_{H,t}Y_t) = \exp \left( \frac{\phi_B S_tB_{F,t}^*/P_{H,t}Y_t}{P_{H,t}Y_t} \right) ; \phi_B < 0 \]  
(A.40)

\[ TB_t = P_{H,t}Y_t - P_{C,t}C_t - P_{I,t}I_t - P_{H,t}G_t \]  
(A.41)

Then the real exchange rate is given by

\[ \text{RER}_{C,t} = \text{RER}_{t}^d \text{RER}_{t}^r \]  
(A.42)

\[ \text{RER}_{t}^d : 0 = E_t \left[ \frac{\Lambda_{C,t+1}}{\Lambda_{C,t}} \frac{\text{RER}_{t+1}}{\Pi_{t+1}^*} \frac{1}{\phi(S_tB_{F,t}^*/P_{H,t}Y_t) \exp(\epsilon_{UIP,t+1})} \left( \frac{1}{\phi(S_tB_{F,t}^*/P_{H,t}Y_t) \exp(\epsilon_{UIP,t+1})} - \frac{\text{RER}_{t+1}^d}{\text{RER}_{t}^d} \right) \right] \]  
(A.43)
Shocks:

\[
\log \frac{A_{t+1}}{A_t} = \rho_a \log \frac{A_t}{A} + \epsilon_{a,t+1} \tag{A.44}
\]
\[
\log \frac{G_{t+1}}{G_t} = \rho_g \log \frac{G_t}{G} + \epsilon_{g,t+1} \tag{A.45}
\]
\[
\log \frac{M_{S,t+1}}{M_S} = \rho_{mS} \log \frac{M_{S,t}}{M_S} + \epsilon_{mS,t+1} \tag{A.46}
\]
\[
\log \frac{UIP_{t+1}}{UIP_t} = \rho_{UIP} \log \frac{UIP_t}{UIP} + \epsilon_{UIP,t+1} \tag{A.47}
\]
\[
\epsilon_{r,t+1} = \rho_r \epsilon_{r,t} + \nu_{r,t+1} \tag{A.48}
\]

If the ROW is not modelled explicitly we close the model with exogenous AR1 shocks

\[
\log(1 + R_{n,t}^*)/(1 + R_{n}^*) = \rho_s^* \log(1 + R_{n,t-1}^*)/(1 + R_{n}^*) + \epsilon_{r,t+1} \tag{A.49}
\]
\[
\log \frac{\Pi_{t+1}^*}{\Pi_t^*} = \rho_{\pi}^* \log \frac{\Pi_t^*}{\Pi_t^*} + \epsilon_{\pi,t+1} \tag{A.50}
\]
\[
\log \frac{C_{t+1}^*}{C_t^*} = \rho^* \log \frac{C_t^*}{C_t^*} + \epsilon_{ct,t+1} \tag{A.51}
\]
\[
\log \frac{I_{t+1}^*}{I_t^*} = \rho^* \log \frac{I_t^*}{I_t^*} + \epsilon_{i,t+1} \tag{A.52}
\]
\[
\log \frac{\Lambda_{t+1}^*}{\Lambda_t^*} = \rho^* \log \frac{\Lambda_t^*}{\Lambda_t^*} + \epsilon_{\lambda,t+1} \tag{A.53}
\]

Otherwise \(R_{n,t}^*, \Pi_{t}^*, C_{t}^* \) and \(I_{t}^* \) are modelled as in Appendix A.

A.2 Steady State

First assume zero growth in the steady state: \( g = g^* = 0 \) and non-negative inflation. Then we have
\[ R_n : \quad 1 + R_n = (1 + R_n^*)\phi \left( \frac{SB}{P} \right) \] (A.54)

\[ \frac{W}{P} = -\frac{U_L}{U_C} \] (A.55)

\[ U_C = (1 - \varrho)C_2^{(1-\varrho)(1-\sigma)-1}(1 - L)^\varrho(1-\sigma) \] (A.56)

\[ U_L = -C_2^{(1-\varrho)(1-\sigma)} \varrho(1 - L)^\varrho(1-\sigma)-1 \] (A.57)

\[ C_1 = \frac{WL}{P_C} \] (A.58)

\[ C = \lambda C_1 + (1 - \lambda)C_2 \] (A.59)

\[ \frac{P_F}{P_C} : \quad 1 = \left[ w_C \left( \frac{P_H}{P_C} \right)^{1-\mu_C} + (1 - w_C) \left( \frac{P_F}{P_C} \right)^{1-\mu_C} \right]^\frac{1}{1-\mu_C} \] (A.60)

\[ \frac{P_H}{P_C} = \frac{1}{[w_C + (1 - w_C)T^{1-\mu_C}]^{\frac{1}{1-\mu_C}}} \] (A.61)

\[ C_H = w_C \left( \frac{P_H}{P_C} \right)^{-\mu_C} C \] (A.62)

\[ C_F = (1 - w_C) \left( \frac{P_F}{P_C} \right)^{-\mu_C} C \] (A.63)

\[ C_H^* = (1 - \omega_C^*) \left( \frac{P_H}{P_C RER_C} \right)^{-\mu_C} C^* \] (A.64)

\[ H(1 - \xi_H \beta) = YU_C \] (A.65)

\[ J(1 - \xi_H \beta) = \frac{1}{1 - \frac{1}{\zeta}} YU_C MC \] (A.66)

\[ MC : \quad H = J \] (A.67)

\[ MC = 1 - \frac{1}{\zeta} = \frac{C_2}{\alpha Y \frac{P_H}{P_C}} \] (A.68)

\[ Y = (1 - c)(AL)^{\alpha} K^{1-\alpha} \] (A.69)
\[
\frac{P^W_H}{P_C} = MC \frac{P^H_H}{P_C} \\
K = \frac{(1 - \alpha)MC \frac{P^H_H}{P_C}Y}{(R + \delta)Q} \\
1 + R = \frac{1 + R_n}{\Pi} \\
I = (g + \delta)K \\
X = 1 \\
S(X) = S'(X) = 0 \\
Q = \frac{P_I}{P_C}
\]

\[
I_H = w_I \left( \frac{P_H}{P_C} \right)^{-\mu_I} I \\
I_F = (1 - w_I) \left( \frac{P_F}{P_C} \right)^{-\mu_I} I \\
I^*_H = (1 - \omega^*_I) \left( \frac{P_H}{PRER} \right)^{-\mu^*_I} I^* \\
\frac{P_I}{P_C} = \left[ w_I \left( \frac{P_H}{P_C} \right)^{1-\mu_I} + (1 - w_I) \left( \frac{P_F}{P_C} \right)^{1-\mu_I} \right]^\frac{1}{1-\mu_I} \\
Y = C_H + I_H + EX_C + EX_I + G_t \\
EX_C = C_{H,t}^* = (1 - \omega^*_C) \left( \frac{P_H}{P_CPRER} \right)^{-\mu^*_C} C^* \\
EX_I = I_{H,t}^* = (1 - \omega^*_I) \left( \frac{P_H}{P_I PRER_I} \right)^{-\mu^*_I} I^* \\
RER_C = \frac{1}{[1 - w_C + w_C T^{\mu_C - 1}]^{1-\mu_C}} \\
RER_I = \frac{1}{[1 - w_I + w_I T^{\mu_I - 1}]^{1-\mu_I}} \\
R_n^* = \beta(1 + R_n^*) \\
1 + R^* = \frac{1 + R_n^*}{\Pi^*}
\]

The problem now there are 31 variables but only 30 SS equations! The model is only complete if we pin down the steady state of the foreign assets or equivalently the trade balance. In other words there is a unique model associated with any choice of the long-run assets of our SOE.\(^8\).

\(^8\)The same point applies to government debt when we introduce fiscal policy.
Our missing equation is therefore the trade balance

\[ TB = P_H Y - P_C C - P_I I - P_H G = P_H EX_C - (P_C C - P_H C_H) + P_H EX_I - (P_I I - P_H I_H) \]

\[ \text{Net Exports of C-goods} \quad \text{Net Exports of I-goods} \]

(A.88)

using (A.81), for some choice of \( TB \), say zero.

The problem now is that we need to force the non-linear model to this steady state even when the latter may not be completely accurate. A way of doing this is to add a term \( \theta_H \log(TB_t/TB) \) to the Taylor rule with a very small \( \theta_H > 0 \) so that when there is a trade surplus the rule makes the nominal exchange rate appreciate slightly.

Finally we calibrate \( \omega_C \) and \( \omega_I \) using trade data. From (A.88) we have

\[ c_{\text{imp}} \equiv \frac{\text{C-imports}}{\text{GDP}} = C_F \frac{P_F}{P_C} Y = c_y(1 - w_C) \left( \frac{P_F}{P_C} \right)^{-\mu_C} \]  
\[ (A.89) \]

\[ i_{\text{imp}} \equiv \frac{\text{I-imports}}{\text{GDP}} = I_F \frac{P_I}{P_I} Y = i_y(1 - w_I) \left( \frac{P_I}{P_I} \right)^{-\mu_I} \]  
\[ (A.90) \]

\[ c_{\text{exp}} \equiv \frac{\text{C-exports}}{\text{GDP}} = (1 - \omega_C^*) \left( \frac{P_H}{P_C RER_C} \right)^{-\mu_C} \frac{C^*_H}{Y} \]  
\[ (A.91) \]

\[ i_{\text{exp}} \equiv \frac{\text{I-exports}}{\text{GDP}} = (1 - \omega_I^*) \left( \frac{P_H}{P_I RER_I} \right)^{-\mu_I} \frac{I^*_I}{Y} \]  
\[ (A.92) \]

Hence using data for shares \( c_{\text{imp}}, i_{\text{imp}}, c_{\text{exp}} \) and \( i_{\text{exp}} \), we can calibrate \( \omega_C \) and \( \omega_I \) making these variables to be solved in the steady state.

Use data for Nigeria: \( c_{\text{imp}} = 0.10, i_{\text{imp}} = 0.15, c_{\text{exp}} = 0.23 \) and \( i_{\text{exp}} = 0.02 \) for \( TB = 0 \).

With balanced steady-state growth, the balanced growth steady state path (bgp) of the model economy with or without investment costs is given by \( Q = 1 \) and

\[ \frac{\bar{A}_{C,t+1}}{\bar{A}_{C,t}} = 1 + g_{\Delta C} = \left[ \frac{\bar{C}_{t+1}}{\bar{C}_{t}} \right]^{(1-\sigma)(1-\epsilon)-1} = (1 + g)^{(1-\epsilon)(1-\sigma)-1} \]  
\[ (A.93) \]

Thus from (A.2)

\[ 1 + R = \frac{(1 + g)^{1+\sigma(1-\epsilon)}}{\beta} \]  
\[ (A.94) \]
Similarly for the foreign bloc

\[ 1 + R^* = \frac{(1 + g^*)^{1+\sigma^* - 1}(1 - \sigma^*)}{\beta^*} \]  
(A.95)

It is then possible to have different preferences, inflation and growth rates provided

\[ \frac{1 + R_n}{1 + R^*_n} = \frac{\phi \left( \frac{SB}{P} \right)}{\Pi^* (1 + R^*)} = \frac{\Pi^* \beta^* (1 + g^*)^{1+\sigma^* - 1}(1 - \sigma^*)}{\Pi^* \beta (1 + g^*)^{1+\sigma^* - 1}(1 - \sigma^*)} \]  
(A.96)

which pins down the assets in the steady state.