Optimal Income Taxation in Developing Economies

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Abstract

This paper considers optimal linear income tax policy in a dynamic general equilibrium overlapping generations model of retirement savings and offspring's educational investment in an economy populated by altruistic agents. The model captures salient features of developing economies with two production sectors, formal and informal. The formal sector employs both capital and labor with taxation of factor incomes, while only labor is employed in the informal sector where wages are not taxed. There are two groups of workers differentiated by skill-type : skilled and unskilled. Skill-type of an agent is influenced by human capital investment undertaken by a parent.

Both non-redistributive and redistributive optimal policies are quantitatively characterized in a model calibrated to the Ghanaian economy where 88% of workers are in the informal sector. Optimal income taxation requires high capital income tax relative to labor income tax ranging from 29% in the case of non-redistributive policy to 40% for redistributive policy in contrast to optimal labor tax of 3% and 10% respectively. This substantial positive capital income tax mitigates the distortionary impact of labor income tax on human capital accumulation and indirectly taxes the unobserved earnings in the informal sector. It is found that neglecting informal sector and human capital in designing optimal tax policy leads to overestimation of labor income tax and underestimation of optimal capital income tax in developing countries with a large and significant informal sector and a low level of human capital.

Keywords: Optimal income taxation; education; informal sector; developing economies.
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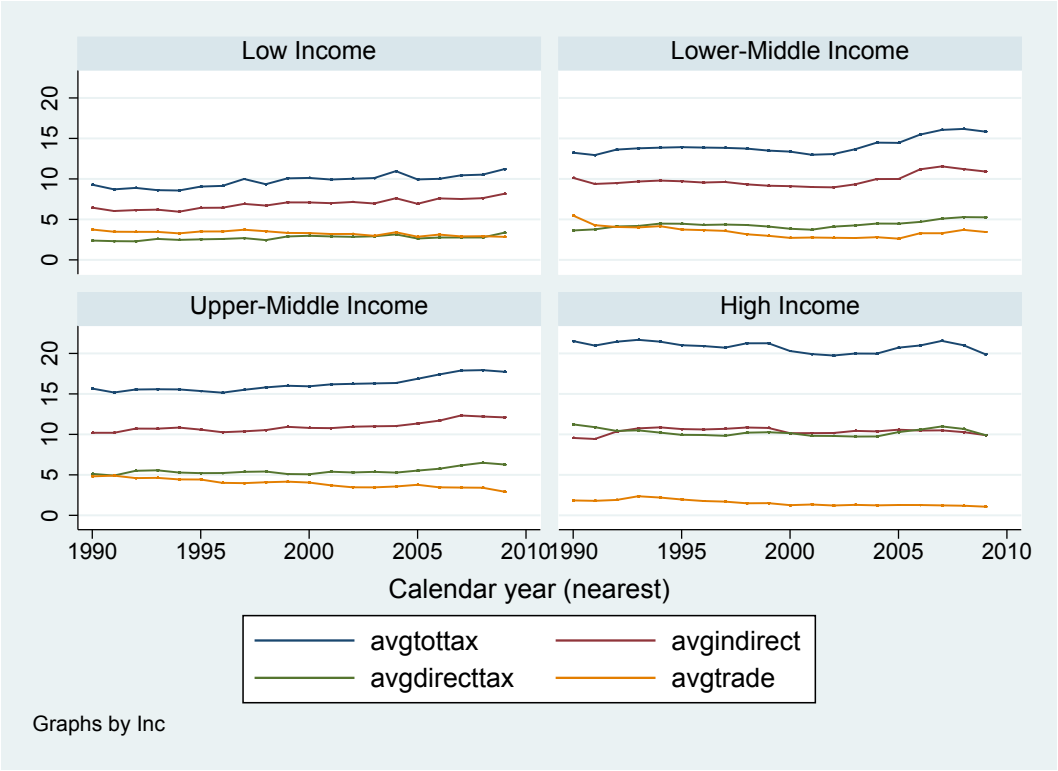
1 Introduction

Raising sufficient revenues for developmental spending on items such as infrastructure and human capital and for redistribution is a major concern in developing countries. Tax revenues in developing countries are low (averaging 13% in lower middle income countries compared to 24% percent in high income countries) with a structure of tax systems that favor indirect taxation (Penalosa and Turnovsky, 2005). Indirect taxes (VAT and consumption tax) constitute about 76% of tax revenue or 10% of GDP in developing countries (see Figure 1).

Figure 1: Tax Revenue components as percentage of GDP



Source: authors' calculations based on UNU-WIDER data.

In addition to heavy reliance on Value Added Tax (VAT) in developing countries (IMF, 2011)[1], there are concerns over regressivity of indirect taxation in general and VAT in particular. The limited capacity to raise more revenues through consumption taxes suggests

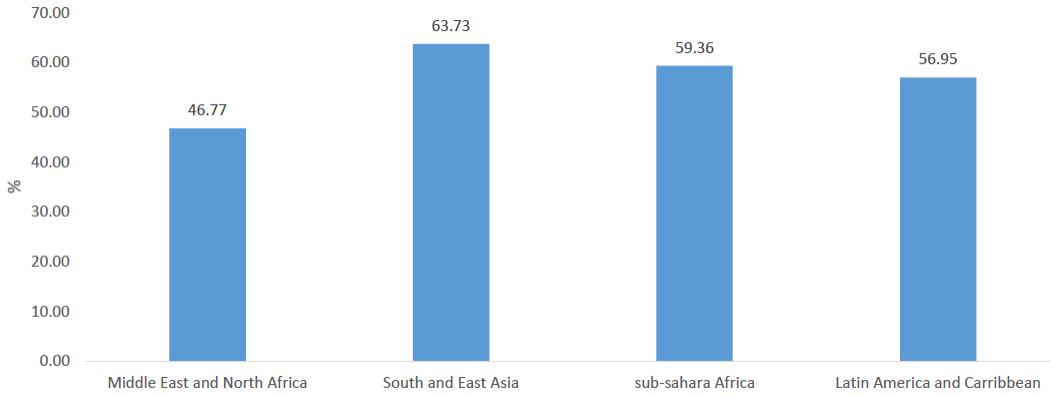
[1] Average VAT rates are in low income countries (16%), lower-middle income countries (13%), upper-middle income countries (15%) and high income countries (20%).

the need for developing economies to raise any additional resources from income tax rather than consumption tax (Turnovsky and Basher, 2009).

Moreover, the high and rising level of inequality in many developing countries[2] provides additional incentive for the use of redistributive tax and transfer systems through personal income tax (Kanbul et al, 2015). Indeed, recommendations for tax reforms in developing countries have always emphasized the need for increased use of direct taxation (see Ahmad and Stern,1991).

However, in developing countries, raising tax revenues through personal income tax is impeded by the large informal sector.[3] Estimates of the contribution of the informal economy to total (non-agricultural) economic activity range from 8.6% to 61.8% of gross value added (GVA) in developing and transitional economies (ILO, 2013). In terms of employment, ILO (2013) finds that informal employment accounts for 6.1 per cent to 83.6 per cent of the non-agricultural labor force in developing countries, with regional variations (see Figure 2). It is estimated that in Africa 90% of workers have informal jobs (ILO, 2009). More worrying is the finding that the size of the informal sector has been growing (Schneider, Buehn, and Montenegro, 2010; and Tafenau, Herwartz, and Schneider 2010).

Figure 2: Informal Employment Share (non-agriculture)



Source: authors' calculations based data from ILO (2013).

The presence of large informal sector does not only pose serious constraint on revenues generation but affect the optimal income tax policy (see Fortin et al, 1997; Cuff, 2011).

[2]Niño-Zarazúa, *et al* (2016) indicates that, sub-Sahara Africa, for examples, has the highest inequality in the world measured by Gini with an average Gini coefficient of 61.9% over the last 3 decades

[3]Typically, the informal sector firms do not pay tax and are less productive compared to formal sector firms. Informal sector exists in every economy particularly developing countries where it is large and significant part of economic activities.

The informal sector affects the efficiency cost of taxes, especially income tax that can be avoided (Kanbul et al, 2015).[4]

A higher labor income tax in the formal sector may also affect human capital investment since most of the formal sector workers are skilled/educated(see Amaral and Quintin, 2005)[5].

However, the fiscal implications of large informal and human capital investment with heterogeneous agents in developing countries are yet to be studied. More importantly, the optimal design of tax policy in developing countries has received very little attention.[6]

The objective of this paper is therefore to characterized optimal capital and income tax in an economy saddled with large informal sector in a simple overlapping generations model. We find the best way to generate revenues for financing of government expenditure and/or transfers with less damaging effect on the size of formal sector and human capital investment. To our knowledge this is the first paper that quantitatively characterized optimal income tax using a calibrated dynamic model in a developing country.

Ghana is used as a representative country to set up a benchmark economy and perform quantitative exercises. Ghana was selected for the following reasons: First, Ghana has large informal sector that accounts for about 88% of total employment. Second, through household surveys, we are able to observe labor income of individuals who are either employed in formal sector or informal sector. Finally, interest income is exempted from tax in Ghana (The Income Tax (Amendment) Act, 2016 (Act 907)).[7]. Therefore it will be important to know whether the policy is optimal or not.

We find that the optimal capital income tax is higher than optimal labor income tax. If the government do not pursue redistributive policy, optimal capital income tax is 29%. The corresponding labor income tax rate is 3%. A high capital income mitigates the human capital investment distortions created by labor income tax and also indirectly tax the

[4]Governments in developing countries with weak fiscal capacity faces both economic and institutional constraints in revenue generation. While the economic incentive constraints limit the use of high income tax rates to raise more fiscal resources as labor will move from formal to informal sector, the institutional constraints limit broadening of the income tax base due to low fiscal capacity in monitoring and compliance given rise to colossal informality (Besley and Persson, 2013)

[5]For instance, while about half of youth with tertiary education have informal sector job in developing countries, around 87% of youth who do not have primary education earn their living from informal employment (see Shehu and Nilsson, 2014)

[6]Exceptions are analytical and numerical optimal taxation works of Penalosa and Turnovsky (2005) and Kanbul et al (2015) in a framework with untaxed informal workers

[7]Indeed, a recently promulgated law in September 2015 that imposes personal interest income tax rate of 1% was abolished in February 2016 as it is perceived to create disincentive to savings

unobserved labor income in the informal sector. Without explicitly accounting for the role of informal sector leads to imposition of high labor income tax that forces labor to move to the non-taxable informal sector. Government generates low tax revenues as a result dwindling tax base leading 'recursive fiscal dillema'(Turnovsky and basher, 2009). Higher labor income tax also reduces benefits from human capital investment.

The rest of the paper is structured as follows: Section 2 reviews related literature with section 3 setting the model. Section 4 presents equilibrium features of the model and define competitive equilibrium while section 5 states the welfare measure. In section 6 discusses calibration strategy and parameters. We discuss optimal taxation results in section 7 and conclude in section 8.

2 Literature Review

This study contributes to the extensive literature that analyses the circumstances under which long run non-zero capital income tax is optimal.[8] A non-zero capital income tax is found to be optimal for efficient savings and capital utilization in studies that use overlapping generations model (see for example Atkinson and Sadmo, 1980; Chamley, 1985). In general, the desirability of non-zero capital income taxation emanates from government and/or the individuals constrained choices (Judd, 1999)[9].

From government constrained choice perspective[10], Correia (1996) has shown that capital income should be taxed when it is impossible to optimally tax other factors of production. In an overlapping generations model with altruistic agents set up, Cremer et al (2003) showed that if inherited wealth are not perfectly observable and cannot be taxed, then non-zero capital income tax is optimal in order to indirectly tax the inherited wealth.

Our study is closely related to the literature that analyses circumstances where government is unable to separately tax human capital and labor income (Jacobs and Bovenberg, 2009; Stantcheva, 2014, Davies et al, 2009; Peterman, 2016; Chen et al, 2011)[11] and factors employed in a particular sector (Penalosa and Turnovsky, 2005; Cerda and Saravia, 2013;

[8]The long run zero capital income tax seminal contribution proposition due to Judd (1985) and Chamley (1985) is one of the celebrated results in optimal taxation literature.

[9]For studies on household constraint choices and market imperfection on optimal capita income taxation, see also Aiyagari (1995), Imrohoruglu (1998), Conesa et al (2009) Hsu and Yang (2013).

[10]see also: Erosa (2002) and Garriga (2003), using life-cycle model of overlapping generations, have theoretically shown that if age-dependent taxes are not allowed, a non-zero capital income tax is optimal.

[11]In an infinitely lived agent model with human capital investment, Lucas (1990), Judd (1999), and Jones et al (1993,1997) still found both physical capital income and human capital income tax to be zero in finite time.

Kanbur et al, 2015; Doligalski, 2016).[12]

This paper focus on workers heterogeneity with respect to skill-type which is largely determined by parents human capital investment. The endogeneity of human capital investment has been shown to account for the desirability of non-zero capital income taxation (Jacobs and Bovenberg, 2009; Stantcheva, 2014; Davies et al, 2009; and Peterman,2016).[13]

In partial equilibrium framework, Jacobs and Bovenberg (2009) uses life cycle model with heterogeneous agents in terms of their abilities to acquire human capital and showed that capital income tax is positive. Since labor income tax distort human capital accumulation, positive capital income is needed to alleviate labor income distortions. In dynamic intergenerational model of human capital investment and bequests, Stantcheva (2014) finds that bequest taxes are generally non-zero in oder to influence education investment and work decisions indirectly.

With dynastic human family model with human and physical capital, Davies et al (2009) in a comprehensive analytical optimal policy framework that include time consistent income and consumption taxation and investment subsidies, showed that capital income should be taxed with higher rate than labor income tax in order to encourage working. The subsidy on investment is required to mitigate the distortionary impact of capital income taxation so that income tax will still be optimal at steady state. Peterman (2016) also quantitatively characterized the optimal linear capital income taxation and non-linear labor taxation in comprehensive general equilibrium model of overlapping generations with endogenous human capital accumulation and found that high capital income tax and flat but lower labor income tax as human capital amplifies the distortions associated with non-linear labor income tax.[14]

All these studies[15] except Peterman (2016), derived optimal labor income and capital income tax result analytically. Instead, we quantitatively characterized the optimal income taxation in family dynastic model with overlapping generations where parents undertake

[12]In infinitely agent model with labor market search frictions, a non-zero capital income tax is needed to correct distortions in the labor market in order to encourage hiring and labor supply (Domeij, 2005).

[13]Browning and Burbidge (1990) showed that positive capital income with zero labor income tax (or a subsidy on labor supply if the desired government revenue is small) is still optimal even if there is no redistribution so far as liquidity constraint is presence in the early stage of life cycle in a single agent model of human capital investments.

[14]In an infinitely lived agent model with labor market search frictions and endogenous human capital accumulation, Chen et al (2011) find that a higher capital income tax and reduced labor income tax are needed to cause a rise in wage discount and encourage more hiring and employment.

[15]Other related literature considers the effects of fiscal policy on education or human capital investment (see for example Heckman, 1976; Benabou, 2002; Erosa and Koreshkova, 2007)

human capital investment of their children by calibrating our model to data. Besides, we focus on old age savings that generate capital accumulation rather than bequest.[16]

Informality and/or informal sector have not been explicitly considered in optimal tax design literature especially in developing countries where it is large. Exceptions are Kanbur et al (2015), Cerda and Saravia (2013), Penalosa and Turnovsky (2005) and Doligalski (2016).[17] Penalosa and Turnovsky (2005) showed that the impossibility of taxing a sector leads to optimality of non-zero capital income tax. They relied on a two-sector model that include formal and non-taxable informal sectors. Both sectors employ homogeneous labor and capital as factors of production with high capital-labor ratio in the formal sector and considered capital and labor income taxation. The models have special relevant for developing countries as oppose to the canonical optimal taxation analysis, where government is assumed to possess the fiscal capacity to tax income from every sector. Penalosa and Turnovsky (2005) find that optimal capital income tax rate should be at least as large as labor tax rate in an economy with nontaxable informal sector. Since in the model both capital income taxation and labor income taxation are distortionary by shifting resources to informal sector, higher taxation of capital income raises more revenues since the taxable formal sector is capital intensive.

Cerda and Saravia (2013) however modeled formal and informal sector in monopolistic competition market with three tax instruments (labor tax, capital income tax and profit tax). Formal sector used both capital and labor as factors of production while labor is the only factor in the formal sector. The study found negative capital and labor income tax and positive profit taxes to avoid factor income taxation distortion of pushing both capital and labor into less productive informal sector.

This study, however, considers labor skill heterogeneity which is important for the sizes of formal and informal sectors and therefore fiscal policy in a perfect competitive market with labor income tax and capital income tax. In addition, we consider human capital investment with heterogeneous agents in three period life cycle model.

Amaral and Quintin (2006) consider two sector economy of formal and informal sectors framework with endogenous education with heterogeneous abilities in dynamic model to explain the key features of informal and formal sectors. He showed formal sector, that becomes capital intensive due to its manages ability and opportunity to expand through

[16]We conjecture that bequest would be more elastic to bequest taxes compared to life cycle savings we consider here. Thus,optimal labor income and bequest taxation may be different from what we consider in this study

[17]Our paper is also related to the literature that analyze fiscal implications of informality in developing counties (see for example Turnovsky and Basher, 2009; Busato, et al 2012; Fortin et al 1997;Prado, 2011)

borrowing, employs more skilled workers relative to unskilled workers due to the complementarity between physical and human capital. Though the present paper and Amaral and Quintin (2006) framework are similar, we rather focus on the design of optimal capital and labor income taxation

Kanbur et al (2015) study the effects of informality on optimal linear labor income tax and transfer system with exogenous earning capacities of agents by deriving the optimal tax formulas in a static model of welfare and poverty (max)minimization that features efficiency cost of taxation and society's value for redistribution. They found that the presence of informal sector alleviate the burden of higher taxes on the poor but also increases the elasticity of the tax base implying high efficiency cost of higher taxes. Doligalski (2016) also analyze optimal non-linear labor income tax in formal-informal framework where workers are differentiated based on differences in formal and informal productivities in static environment. In this paper, workers heterogeneity are treated as endogenous in a dynamic general equilibrium model to consider both linear labor income tax and capital income tax.

Moreover, these studies provide analytical and numerical findings of the effects of informality on optimal income taxation.[18] We use a calibrated data to quantitatively characterized optimal income taxation. This study considers optimal taxation policy in developing economies with large informal sector. We specifically, analyze the implications of large but low-productivity non-taxable informal sector and human capital investment on the design of optimal capital income and labor income taxes in dynastic human capital model.

3 The model

The model economy is populated by a continuum of three-period lived heterogeneous agents in overlapping generation setup. We combine key features of life cycle and dynastic models: life cycle periods of working and retirement necessitate savings and altruistic motives in dynastic models implies human capital investment in children. The agents are ex-ante heterogeneous in terms of age and skill-type. Every decision is made by young adult. All young adults have one unit of fixed time which is allocated between working in formal and informal sector of employment that produce identical good in a perfect competitive environment. There is government that collects capital income taxes and formal sector labor tax to finance general government consumption and lump-sum transfer payments.Households and firms take fiscal policies as given when making optimal decisions.

[18]Except Doligalski (2016) who calibrated the model to Columbian economy where about 58% of labor work in the informal sector but as noted earlier his model is static and do not deal with capital income tax.

3.1 Demography

The economy is populated by a continuum of three-period lived heterogeneous agents. Individuals are described as child, young and old adult over the life cycle. In the first period, individuals are children who do not work, make no economic decision but receive an education investment in school as chosen by their parents. Children then become young adults and entered with their own children in the beginning of their second period.Young adults choose the education or human capital investment of their children, e_i, how much to consume and save. Young adults retire and entered into the third period as old adults, consumed all their savings and die deterministically at the end of third period. Young adults, who are in their second period, are categorized into skilled and unskilled agents based on the education decision of the parents and luck in accumulating human capital in the first period as children.

3.2 Production Structure

There is a representative firm in each sector that produce an identical good that can either be consumed or saved for future consumption. All firms in both sectors are perfectly competitive in both goods and factors markets, albeit constraint in the formal sector, using a constant-return-to-scale technology. While capital is immobile and restricted to the formal sector, labor services are perfectly mobile across the two productive sectors. The total output in the economy is given by $Y = Y^f + Y^h$.

3.2.1 Formal Sector

There is a representative firm in the formal sector that employs physical capital and skilled labor as the only factors of production. The goods and capital markets in the formal sector are perfectly competitive, whereas labor market is not due to existence of constraints on formal-informal wage gap. Factors employed in the formal sector firms voluntarily pay taxes imposed by government. The formal sector production is given as

$$Y^f = A^f K^{\alpha_k} (L_s^f)^{\alpha_s} (L_u^f)^{\alpha_u}, \quad \alpha_k + \alpha_s + \alpha_u = 1$$

where physical capital share of income, skilled labor share of income, and unskilled labor share of income are given by α_k, α_s and α_u respectively. In terms of per worker, the formal sector production function can be written as

$$y^f = A^f k^{\alpha_k} (l_s^f)^{\alpha_s} (l_u^f)^{\alpha_u} \tag{1}$$

We assume that prices of input factors are determined in the perfect competitive markets subject to former sector constraint on formal-informal wage premium.[19] Profit maximization therefore implies that firms pay each production factor its marginal product at equilibrium.

Formal sector firm profit maximization problem is formally specified as follows:

$$\Pi = \underset{k,l_s^f,l_u^f}{Max}\{A^f k^{\alpha_k}(l_s^f)^{\alpha_s}(l_u^f)^{\alpha_u} - rk - w_s^f l_s^f - w_u^f l_u^f\} \tag{2}$$

subject to

$$(1-\tau_l)w_s^f/w_s^h = \kappa_s \tag{3}$$

$$(1-\tau_l)w_u^f/w_u^h = \kappa_u \tag{4}$$

where κ is the measure of after tax formal-informal wage gap. The first order conditions from firm profit maximization lead to the factor prices and formal sector constraint.[20]

$$r = (\alpha_k)A^f(k)^{\alpha_k-1}(l_s^f)^{\alpha_s}(l_u^f)^{\alpha_u} \tag{5}$$

$$w_s^f = (\alpha_s)A^f(k)^{\alpha_k}(l_s^f)^{\alpha_s-1}(l_u^f)^{\alpha_u} \tag{6}$$

$$w_u^f = (\alpha_u)A^f(k)^{\alpha_k}(l_s^f)^{\alpha_s}(l_u^f)^{\alpha_u-1} \tag{7}$$

$$(1-\tau_l)w_i^f/w_i^h = \kappa_i \tag{8}$$

3.2.2 Informal Sector

The representative firm that operates in the informal sector does not pay tax on the factors employed for its production leading to incompleteness in the tax system. Following the literature on informal sector that models informal sector firms as labor intensive relative to formal sector firms, we model informal sector firm to use only skilled and unskilled

[19] This is akin to Rauch (1991) general equilibrium model where firms avoid payments of minimum wage by operating in a small scale informal sector. Instead of minimum wage, this paper uses minimum formal-informal wage premium/gap to generate the informal sector employment and allow prices to be set competitively

[20] Note that though formal-informal wage premiums or constraints are exogenously imposed and fixed over time, skill premium is endogenously determined.

labor(See, for example, Turnovsky and Basher, 2009; Ihrig and Moe, 2004; and Cerda and Saravia, 2013)[21]. Thus, the production function and prices of inputs(skilled and unskilled labor) are given by

$$Y^h = A^h(L_s^h)^\gamma(L_u^h)^{1-\gamma}, \quad \gamma < 1$$

with per worker output of the form

$$y^h = A^h(l_s^h)^\gamma(l_u^h)^{1-\gamma} \tag{9}$$

$$w_s^h = (\gamma)A^h(l_s^h)^{\gamma-1}(l_u^h)^{1-\gamma} \tag{10}$$

$$w_u = (1-\gamma)A^h(l_s^h)^\gamma(l_u^h)^{-\gamma} \tag{11}$$

3.3 Government

Government taxes formal sector labor and capital incomes at respective rates, τ_L and τ_K, to finance exogenous lump sum transfer and general government consumption. Government's budget constraint in every period is given by

$$n_s\tau_L(l_s w_s^f) + (1-n_s)\tau_L(l_u w_u^f) + \tau_K rk = b + g \tag{12}$$

where $k = K/N^y$ is the capital per worker. On the right, the first term is transfer per worker to households and g is exogenous general government expenditure.

3.4 Education and Skill Acquisition

We assume that skilled-type of a child is determined in the first period of life by the education decision of the parent[22] and random element similar to the formulation of Caucutt, Imrohoroglu and Kumar (2003; 2006) and Cramer et al (2011).[23] The randomness implies

[21] In related literature, this formulation is similar to frameworks where capital is fixed in the informal sector and others where capital is abstracted. Doekpe (2004), for instance, used this type of production function for agricultural sector

[22] Household expenditure on education in sub-Sahara Africa covers about 25.5% of total expenditure on education. Primary(29%), Lower secondary (49%), Upper secondary (44%), Tertiary level (22%) lowest but the high unit cost makes it burdensome for parents. Even in developed countries, education expenses is a major burden on families at almost all stages of education as they pay fully the entire costs associated with primary and secondary schooling (see Stantcheva, 2014). At college level, parents, relatives and friends covered about 48% of total college expenses in 2016 (Sallie Mae, 2016).

[23] Cramer et al (2011), however added third component family background and parental genes.

that parental investment in education does not necessarily transform a child to a skilled young adult but increases the probability of becoming skilled (Doepke and Zilibotti, 2005). Every young adult has one child and decides the number of hours the child spend in school (with the teacher) for education purpose and pays a formal skilled wage per unit of time the child spend in school.[24]

We normalize school or teacher's time to 1 and therefore a parent has to choose $e_i \leq 1$ for his/her child's education. With positive time in school, the probability that a child of an $i-$type young adult becoming a skilled young adult in the second period is given by $\pi_i(e_i)$, with $1-\pi_i(e_i)$ probability that child becomes unskilled. We assume that $\pi_i(e_i)$ is an increasing and strictly concave function with $\pi \in (0,1)$ and $\pi(0) = 0$, implying that the probability of skill acquisition exhibit a diminishing return to educational time investment. Specifically, we assume human capital production function of the form:

$$\pi_i = b_i e_i^{\eta} \tag{13}$$

where $b_s \geq b_u$ to account for advantages that children of rich parents have in completing school or becoming skilled workers which are not being modeled. These advantages may be related to early childhood education or pre-school care and schooling advantages that rich children have (see Caucutt and Kumar, 2007).

Conditional on the parent choice of quantity of education,[25] the intergenerational mobility probability matrix between the skilled and unskilled states is given by

$$\begin{pmatrix} & \text{skilled} & \text{unskilled} \\ \text{skilled} & \pi(e_s^*) & 1-\pi(e_s^*) \\ \text{unskilled} & \pi(e_s^*) & 1-\pi(e_u^*) \end{pmatrix} \tag{14}$$

We calibrate our model of these transition probabilities to the empirical estimates from data.

3.5 Household's Problem

In this model economy, only young adults make economic decisions. A young adult cares about present and future consumption as well as future welfare of her child. The Bellman equation for an young adult agent with skill-type i who decides on present and future consumption, formal labor supply and child's human capital investment e_i,[26] to maximize her

[24] Since we focus on inequality in education investment and hence skill level, we abstract from public education.

[25] In fact, the education time could also be interpreted as education quality that would be bought by the parent

[26] We focus on education as the only way through which parents transfer resources to their children in this model and thus abstract from bequest. Indeed, in developing countries proportion of parents that plan to leave

lifetime utility is: [27]

$$V_i = \underset{c_i^y, c^{o\prime}, e_i, l_i \leq \bar{l}_i]}{Max} \left\{ u(C^y) + \beta u(C^{o\prime}) + \Psi\left(\pi_i(e_i)V_s(x) + (1-\pi_i(e_i))V_u(x)\right)\right\}, \qquad i = s, u \tag{15}$$

subject to

$$C_i^y + a_i' + e_i w_s^f = (1-\tau_L) l_i w_i^f + (1-l_i) w_i^h + b \tag{16}$$

$$C_i^{o\prime} = \left[1 + (1-\tau_K) r' - \delta\right] a_i' \tag{17}$$

$$0 \leq l_i \leq \bar{l}_i(\kappa_i) \leq 1 \tag{18}$$

where β is the subjective discount factor with respect to own future utility from consumption and Ψ is the general level of altruism factor that discounts child's future welfare. $\kappa_i \geq 1$ is a measure of employment constraint/distortion in the formal sector labor market that prevent agent from choosing the optimal formal sector labor supply.[28] We model out-of-pocket household expenses on education and abstract from any government intervention or subsidy.[29] Equation (17) indicates that young adult consumption, savings, and child's education expenditure should be equal to working life labor income reflecting absence of credit market to finance education investment.[30]

The utility function, $u(C^y)$ is twice differentiable with $u_{c^y}(C^y) \geq 0$ and $u_{c^y c^y}(C^y) < 0$. We use the CRRA utility function for empirical analysis of the form $u = \frac{C^{1-\sigma}}{1-\sigma}$. τ_L and τ_K are labor income tax rate and capital income tax rate respectively.[31]

bequest as oppose to *intervivos* will be very marginal. Bequest recipients in US is about only 3.7 % (Gale and Scholz, 1994)

[27] We abstract from labor-leisure choice by assuming that total labor supply is fixed. Indeed, leisure may be less valued and consumed in developing countries given the low level of consumption (see Turnosky and Basher, 2011). Moreover, the tax imposition may compel agents to evade and therefore operate in informal sector rather than reducing the hours work. The informal sector may serve as buffer to where hours not spend on formal production are employed(see Todaro,1989, for example)

[28] Isabel Gnther and Launov, (2012) find that informal employment is a mix of voluntary for attractive employment opportunity and involuntary where it serves as employment of last resort after rationed out from formal job. we model this phenomenon by allowing agents to choose but cannot exceed the constraint posed in the formal sector

[29] Thus $e_i w_s^f$ is the household expenditure on education

[30] This in particular is relevant for developing countries where higher education are usually sponsored by parents but children cannot be held liable to their parents borrowing to finance their education. see for example Benabou 2002 and Schoonbroodt and Tertilt (2014) for similar exposition where inability of parents to have leverage of children serves as a source of liquidity constraint. See also Picketty (2000).

[31] We concentrate on linear taxes as Peterman (2016) suggests that in the presence of human capital invest-

4 Equilibrium Features of the Model

4.1 Optimal Decisions

4.1.1 Formal-Informal Labor Decision

Young adults maximize labor income by allocating fixed time unit between working in formal and informal sectors. Thus, young adult chooses formal labor supply l_i to maximizes his life time utility. This is equivalent to maximizing the expected net wage/labor income by choosing formal labor supply, l_i. Thus, at optimum l_i where labor market clears

$$(1-\tau_i)w_i^f = w_i^h \quad \text{if} \quad \kappa = 1 \tag{19}$$

$$\text{else ;}$$

$$(1-\tau_i)w_i^f = \kappa_i w_i^h \quad \text{if} \quad \kappa > 1 \quad \text{and} \tag{20}$$

$$l_i = \bar{l}_i(\kappa_i) \tag{21}$$

where $\kappa_i \geq 1$ as noted above is a measure distortion in the labor market that prevents after tax formal wage to equalized informal wage at equilibrium. We assume that this distortion is constant over time.

4.1.2 Saving Decisions

$$u_{c^y} = \beta\left(1+(1-\tau_k)r' - \delta\right)u_{c^{o\prime}} \tag{22}$$

where u_{c^y} and $u_{c^{o\prime}}$ are marginal utilities of present and future consumption respectively.

4.1.3 Parent Education Decision

Young adults choose children's school time e_i to maximize his/her life time utility. Thus, at optimum e_i, marginal benefits from child's education must be equal to 'internalized' marginal cost of child education. This is given as

$$\Psi\pi_{e_i}(e_i)\Omega(x') = u_{c_i^y}w_s^f \tag{23}$$

ment, flat labor income and capital income taxes are optimal. Moreover, Erosa (2002) and Garriga (2003), suggest that either a non-linear labor income tax or a non-zero capital income tax could be used to mimic life cycle models of overlapping generations if age-dependent taxes are disallowed. It is also important to note that the flat income tax scheme with appropriate designed lump-sum transfer in itself leads to non-linear tax transfer scheme can achieve similar redistribution objective like non-linear tax system (Keen et al, 2008; Peichl, 2014).

where following Caucutt, Imrohoroglu and Kumar (2003; 2006), $\Omega(x') = (V_s(x') - V_u(x')$ and therefore $\Psi\pi_{e_i}(e_i)\Omega(x')$ is the marginal benefit of offspring's human capital investment given by the value to being skilled, $\Omega(x')$ weighted by the altruism factor Ψ and the marginal productivity of the human capital investment $\pi_{e_i}(e_i)$. $u_{c_i^y} w_s^f$ is the marginal cost of child's human capital investment composed of per unit cost of education (formal sector skilled wage) w_s^f weighted by marginal utility of consumption $u_{c_i^y}$ that will be lost.

Thus, the optimal decisions of skilled and unskilled young adult leads to the following[32]:

For skilled young adult,

$$\Psi\pi_{e_s}(e_s)\left(\Omega(x')\right) = u_{c_s^y} w_s^f$$

For unskilled young adult

$$\Psi\pi_{e_u}(e_u)\left(\Omega(x')\right) = u_{c_u^y} w_s^f$$

We derive the maximum lifetime utilities, $V_s(x)$ and $V_u(x)$, by evaluating the Bellman equation with optimal policy functions, including saving function $a_i'^*(n_s)$, formal labor supply ι_i^* and therefore maximized wage income and children education function $e_i^*(n_s)$ to get $V_s(x)$ and $V_u(x)$.

We subtract the value for unskilled from skilled worker to get the law of motion of the value to being skilled worker $\Omega(x)$,

$$\Omega(x) = CU_s - CU_u + \Psi\left(\pi_s(e_s^*) - \pi_u(e_u^*)\right)\left(\Omega(x')\right)$$

where CU_sconsumption utility of the skilled/rich and CU_uconsumption utility of the unskilled/poor.

As in Caucutt, Imrohoroglu and Kumar (2003; 2006), the value to being skilled has two component comprising $CU_s - CU_u$, the present increase in consumption utility from being skilled worker and a higher chance of realizing the future of being skilled $(\Omega(x'))$. Moreover, the endogenous altruism factor increases with the difference in investment in education between skilled and unskilled workers.

The law of motion for a fraction of skilled workers is given by

$$n_s' = n_s\pi\left(e_s^*(n_s)\right) + (1 - n_s)\pi\left(e_u^*(n_s)\right)$$

[32]Notice that the concavity of utility function and π as well as the fact that $w_s > w_u$implies that $e_s > e_u$ since for a given human capital investment $e_s = e_u$, the marginal cost of human capital investment is higher for the unskilled young adult relative to the skilled young adult. This results from the higher utility cost of human capital investment for unskilled relative to that of skilled workers.

and the law of motion for physical capital per worker is given by

$$k' = n_s a_s'^* + (1-n_s)a_u'^*$$

4.2 Definition of equilibrium

Definition 1. A competitive equilibrium of this economy is a collection of allocations $\{c, a_i', l_i, e_i\}$, a sequence of prices $\{w_s^f, w_s^h, w_u^h, r\}$, a government policy $\{\tau_l, \tau_k, b\}$ such that Given prices w_s^f, w_s^h, w_u^h, r; $\Omega(x)$; and government policies τ_l, τ_k, b

1. Young adults choose c_i, a_i', l_i, and e_i to solve their decision problem for each skill-type $i = s, u$

2. The representative firm in both formal and informal sectors chooses $k^\star$, $l_s^{f\star}$ and $l_u^{f\star}$ respectively to maximize profits so that prices of capital, and both types of workers are paid their marginal products.

3. Capital market clears, i.e $k' = n_s a' + (1-n_s)a_u'$

4. The formal sector labor market supply is $l_s^f = l_s n_s$ and $l_u^f = l_u(1-n_s)$ and informal sector with $l_s^h = (1-l_s)n_s$ and $l_u^h = (1-l_u)(1-n_s)$

5. The labor market clears. That is; $l_s = n_s$; and $l_u = 1-n_s$ where $l_s = l_s^f + l_s^h$ and $l_u = l_u^f + l_u^h$ [33]

6. Government balances its budget every period i.e
   $(1-n_s)\tau_l l_u^f w_u^f + n_s \tau_l l_s^f w_s^f = b + g$

7. The goods market clears, i.e.
   $c + \bar{e} + k' + g = y + (1-\delta)k$
   where $c = n_s(c_s^y + c_s^o) + (1-n_s)(c_u^y + c_u^o)$
   and $k = K/N^y$ and $y = y^f + y^h$ and $\bar{e} = (n_s e_s + (1-n_s)e_u)\, w_s^f$ and $g = (1-n_s)\left(\tau_l l_u^f w_u^f\right) + n_s\left(\tau_l l_s^f w_s^f\right) - b$

8. $\Omega(n_s)$ is consistent with young parents education investment decisions of their children.

9. The n_s' function that individuals posit is consistent with decisions rules

[33] Notice that we did not account for teachers who form part of the skilled labor force for the sake simplicity as the percentage of teachers in the working population is expected to be small. According to GSS (2013), education sector workers are about 2% of the workers in 2012 and therefore those teaching staff that would be involved in secondary and tertiary level will be fewer.

4.3 Steady State Equilibrium

A steady state is a competitive equilibrium with $n_s = n_s^*$ and $k = k^*$ with $n_s'^* = n_s^*$ and $k'^* = k^*$

On a steady state, all variables including wages, output per worker, capital/savings per worker, fraction of skilled young adults, and the value of being skilled are time-invariant. thus, $\Omega(x) = \Omega(x')$, hence

$$\Omega(x) = \frac{CU_s - CU_u}{1 - \Psi\left(\pi_s(e_s^*(n_s)) - \pi_u(e_u^*(n_s))\right)}$$

and $n_s' = n_s$

$$n_s = \frac{\pi(e_u^*(n_s))}{1 - (\pi(e_s^*(n_s)) - \pi(e_u^*(n_s)))}$$

and $k' = k$

$$k = n_s a_s^* + (1 - n_s) a_u^*$$

5 Welfare Measure

We measure social welfare by the average lifetime values of all agents who have entered the economy as young adults in steady state equilibrium. Following Caucutt, Imrohoroglu and Kumar (2003)[34] we use utilitarian welfare criterion given by

$$SW = \left(\frac{1}{(1-\sigma)}\right)\left(\mu n_s\left((c_s^y)^{1-\sigma} + \beta(c_s^{o\prime})^{1-\sigma}\right) + (1-\mu)(1-n_s)\left((c_u^y)^{1-\sigma} + \beta(c_u^{o\prime})^{1-\sigma}\right)\right) \tag{24}$$

where μ is the weight government attaches to welfare of the rich and therefore $1-\mu$ is the weight on the welfare of the poor.

Government maximizes the aggregate social welfare by choosing capital tax rate τ_k, and labor tax rate τ_l.

To compare optimal tax welfare with benchmark model, we use the consumption-equivalent variation (CEV), measured as ω and defined as

$\omega = \left(\frac{SW_{opt}}{SW}\right)^{\frac{1}{1-\sigma}} - 1$

where SW_{opt} represents social welfare measure at the optimal taxation economy and SW being the baseline economy where the taxes imposed are not optimal.

[34] and as suggested by Hammond (1988; cited in Cremer et al, 2003), we do not include altruistic component in the social welfare function.

6 Calibration

In our quantitative analysis, Ghana is used as a representative economy. Our strategy is to make the model economy consistent with the Ghanaian economy on important indicators such as capital output ratio, fraction of workers that have at least secondary education[35], probability of intergeneration transmission of education status, and government revenue as a percentage of GDP. In achieving this, we also ensure that the number of parameters jointly used to match the data are reduced. Our aim is to get a set of parameters in the calibrated model economy that will serve as our benchmark economy for further experiments or policy analysis.

The parameters used to calibrate our steady state benchmark economy are summarized in Table 1. There are two sets of the parameters: the first set of parameters discussed are either estimated from the data without the model or taking from the existing literature and second set consists of parameters that were jointly used to match key economic variables in the data.

Table 1: Parameters

Parameters	Interpretation	Value	Source/Target
Production			
γ	skilled labor share of informal output	0.204	w_s^h/w_u^h
α_k	capital share of formal output	0.4	IMF
α_s	skilled labor share of formal output	0.5255	w_s^f/w_u^f
A^f	formal sector productivity level	10	normalization
κ_s	Measure of formal skilled wage distortion	2.27	w_u^f/w_u^h
κ_u	Measure of formal unskilled wage distortion	1.34	w_s^f/w_s^h
Government Policy			
τ_L	Labor tax	0.1	Effective rate for median tax payer
τ_K	Capital tax	0.21	Nguyen-Thanh & Strupat (2012)
Calibrated Parameters			
A^h	Informal sector productivity level	4	Joint: capital-output ratio
β	Subjective discount factor	0.875	Joint: fraction of skilled workers
Ψ	altruism factor	0.7	Joint: mobility matrices
η	elasticity of human capital	0.2	Joint: govt revenue to GDP ratio
b_u	productivity measure of education (poor)	0.5	

[35] secondary school education is considered as the minimum level of education for a worker to be skilled. As argued by Caucut and Kumar (2003), return to secondary school education is substantially higher than the primary school which could be as low as 2% for some African countries (see Bigsten et. al., 2000)

The first set of the parameters are those which can be independently calibrated. For production parameters, we normalize formal sector productivity, $A_f = 10$. We set capital income share of output $\alpha = 0.4$ (IMF, 2007). For shares of skilled and unskilled labor income share, we use the employment share to match the skilled wage premium. From the data, we calculate the share of formal skilled workers and unskilled workers in the total employment which amounted to 9.3 per cent and 3 per cent respectively. We then match them to the skilled wage premium in the formal sector to derive skilled and unskilled labor share. Similarly, skilled and unskilled labor share in the informal sector is calculated by using the information in the informal sector. We choose the depreciation rate, $\delta = 1$ as we assume one generation last for 30 years hence the capital stock would have been completely depreciated.

For parameters related to fiscal policy, as stated above we assume that only the formal sector pay tax. Thus, we concentrate on labor income tax and capital (interest) income tax rates in the formal sector. Total tax revenue from these two sources in Ghana average about 5.4% of GDP. Since government also faces borrowing constraint in the model, we use revenue of 5.4% of GDP to represent exogenous government expenditure g. Thus, we choose government spending g so that it account for 5.4 per cent of GDP in the initial steady state equilibrium. Lump sum transfer is taking as residue of tax revenue minus government expenditure.

Capital income tax rate in Ghana is estimated as 21% which is equivalent to the sum of effective average tax rate on both equity and debt as measured by Nguyen-Thanh & Strupat (2012). Labor income tax is measured by the individual income tax rates levied directly on a persons income. The personal income tax system in Ghana is staggered depending on the taxable income and ranges from 0% to the maximum 25%. We chose 10% to represent labor income tax in our calibration as the rate that correspond to average tax rate of mean income earner in the formal sector.

6.0.1 Joint Calibration

Since parameters in the human capital productions functions are specific to our model, following Calcutt et al (2003), we normalize the productivity parameter of the skilled human capital technology, $b_s = 1$. Our model include both discount factor, (β) and altruism factor, (Ψ) which are usually proxied by only discount factor in infinitely models. Since there is no widely accepted empirical estimate of altruism factor(Leung and Chen, 2006), we jointly calibrate the remaining parameters in the human capital production functions in addition to the intergenerational altruism factor (Ψ) as well as the discount factor (β) so that the

model outcomes match the data including capital-output ratio, fraction of skilled workers, transition matrices and government expenditure to GDP ratio.

6.0.2 Calibration properties

From Table 2, it can be confirmed that the targeted moments of the model replicate well moments from the data. In fact, even for number of moments that were not targeted explicitly (skill premiums, share of sector employment by skilled type, share of capital and labor tax in GDP), calibration yields parameters that matches well with the data. The size of informal sector of about 62% is not significantly different from the sub-Saharan Africa which ranges from 36.2% to 60% as estimated by ILO (2014).[36] The model outcome of total household expenditure on education as a percentage on GDP of 2.32% is well within the sub-Saharan Africa range of 0.1% to 4.8% with an average of 1.5%.[37]

Table 2: Comparison of Model Outcome

Model Quantity	Interpretation	Data	Model(Benchmark)
n_s	fraction of skilled workers	24.39	25
K/Y	capital-output ratio	2.0	2.2
$b_s(e_s)^{\eta_s}$	prob(skilled|skilled)	54.39	50.01
$b_u(e_u)^{\eta_u}$	prob(skilled|unskilled)	20.43	16.64
g/y	Personal income tax as a % of GDP	5.4	5.4
$p_f = w_s^f/w_u^f$	Formal skill wage premium	2.0	2.1
$p_h = w_s^h/w_u^h$	Informal skill wage premium	1.18	1.24
p	skill income premium	1.64	1.80
$\tau_l * L.W/GDP$	share of Labor tax in GDP	2.11	2.25
$\tau_k * r.K/GDP$	share of capital tax in GDP	3.48	3.15
l_s^f	Formal skilled workers(%)	9.7	9.28
l_u^f	Formal unskilled workers (%)	3.14	2.76
l_s^h	Informal skilled Workers (%)	14.61	15.72
l_u^h	Informal Unskilled workers (%)	72.41	72.22
Y^h/Y	size of Informal sector (%)	36.2 - 61.8	62
$(n_s * e_s + (1-n_s)e_u)w_s^f/Y$	Household expenditure on education as a % of GDP	0.1-4.8	2.32

[36] As also reported in Table 2, estimates for some sub-Sahara African economies.

[37] UNESCO Institute for Statistics, 2011

7 Optimal Taxation

In this section, we quantitatively show results on optimal capital and income taxation at steady state. First, we find optimal tax rates when there is no redistribution. That is government solves for optimal combination of capital and labor income tax rates to raise revenue for fixed government expenditure. Second, we introduce a lump sum transfer to households for redistributive reasons. In both sections, we also look at experiments to diagnose the effect of each distortion on optimal taxation.

7.1 Optimal taxation without redistribution

We find optimal labor income tax and capital income tax to be 3% and 29.64% respectively as reported in Table 3. The optimal capital income tax is higher than the tax rate in the benchmark Ghanaian economy, while the labor income tax rate is lower. Moreover, the optimal capital income tax is relatively larger than that of labor income tax. This suggests that the distortion resulting from labor income tax (including displacing workers to less productive informal sector and fall in educational investment) is relatively large compared to the inter-temporal allocation of consumption distortion created by capital income tax. Hence, the government optimizes by reducing distortion created by labor income tax of discouraging human capital investment by setting low labor income tax and a higher capital income tax that discourage financial savings and increase human capital investment through substitution effect. The optimal capital income tax sacrifice efficiency in the level of financial and human capital investment for efficiency in composition of these investment (see Jacobs and Bovenberg, 2009). Hackman(1976) also finds positive substitution effect of capital income tax on human capital investment.

Aggregate effects The third column of Table 3 reports differences in some relevant aggregate variables in the optimal tax system and benchmark economy. Capital-output ratio falls due to higher capital income taxes under the optimal tax system which lower savings and therefore aggregate capital accumulation. The higher capital income tax also leads to marginal increase in educational investment and skill share of the population through substitution effect. The output per worker also rises from 3.35% to 3.38% equivalent to 0.94% due to an increase in labor supply in the high productive formal sector when labor income tax rate is low. The welfare improvement from benchmark model to the optimal tax economy is equivalent to 0.38 per cent increase in lifetime consumption as indicated by the social welfare measured in terms of consumption-equivalent variation (CEV).

Table 3: Optimal Tax vs Benchmark

Indicator	Benchmark	Optimal Tax	Change
Column	**1**	**2**	**3**
Capital income tax rate (%)	21	29.64	8.64
Labor income tax rate(%)	10	3.00	-7
Transition (Rich) (%)	50.1	50.17	0.007
Transition (Poor) (%)	16.64	17.1	0.46
Skilled Share (%)	25.0	25.55	0.55
K/Y	2.2152	2.113	-0.1022
Output per worker	3.3453	3.3768	0.0315
Skill premium	1.7961	1.8865	0.0904
Welfare Gain (CEV) (%)	-	0.37771	0.37771

7.1.1 Formal-informal labor supply, education and savings tax Distortions

In the model, labor income tax distorts formal-informal labor supply and human capital investment through its effect on future benefit of human capital. Capital income tax on the other hand, distort consumption-savings decisions directly and human capital investment indirectly. Capital income tax increases the relative present value of human capital investment to that of financial savings. Thus, due to the decline in after tax returns on savings as a result of capital income tax, individuals substitute human capital investment for financial savings.

Fixed sector labor supply We fix formal and informal labor supply for skilled and unskilled agents at the equilibrium level of the optimal-tax baseline economy. This is intended to shut down the tax distortion from informal-formal labor supply decision. Thus, in this counterfactual experiment, there would be no formal-informal labor supply effects of labor income tax. The column 2 of Table 4 indicates that by fixing sectorial labor supply hence eliminating such distortion, optimal labor income tax rises from 3% to 12% while optimal capital income tax reduces from 29.64% to 16.25%. Neglecting formal-informal labor decisions therefore leads to overestimation of labor income tax and underestimation of capital

income tax by 9 percentage points and 13.2 percentage points respectively. In other words, the presence of non-taxable informal sector limits the use of labor income tax to generate revenue for government expenditure. Thus, economies with large informal sector would be better off by imposing lower labor income tax.

It can also be argued that capital income tax is used to indirectly tax the unobserved labor income earn from the informal sector. Since financial savings will be proportional to after tax labor income that include formal sector wage income and informal sector wage income, capital income tax is a way of indirectly taxing income from the informal sector without generating the formal-informal distortion that would be created by labor income taxation of the formal income.

Fixed Education Investment To shut down the tax distortion imposed on human capital investment decision, we fix education investment for skilled and unskilled agents at the their equilibrium values of the optimal-tax baseline economy. In this way, labor income tax will not affect education decision and therefore no effect on the skill proportion of the population. Column 3 of Table 4 indicates that the optimal capital income tax falls from 29.64% in the baseline economy to 23.43% while the labor income tax rises from 3% to 6%. Thus, since there is no inter-temporal distortion of labor income tax, the government optimizes by reducing capital income tax and increase labor income tax to raise enough revenue for the exogenous expenditure. This implies that neglecting human capital decisions leads to an overestimated labor income tax.

No Labor income Tax Distortion In column 4 of Table 4, we fix both formal labor supply and education hours to the equilibrium level of the baseline economy. In this way, income tax will not have any effect on labor supply and human capital investment. The results depict optimal capital income tax of 21.05% and optimal labor income tax of 8% compared to the baseline rates of 29.64% and 3% for capital income tax and labor income tax respectively. Thus, the government finds it efficient to reduce capital income tax and increase labor tax which has no distortion in the economy. The implication is that without explicitly considering the taxation impacts of both informal sector and human capital investment, tax design, especially in developing countries, would be inefficient by overtaxing labor income and undertaxing capital income.

The high rate of capital income tax experiment with no labor distortion suggests the need for long run efficient capital income tax to increase capital accumulation as savings are for life cycle reasons (see Chameley, 1986; Atkinson and Sadmo, 1980). Taxation of

interest on savings is needed to ensure efficient allocation of consumption and accumulation of high capital formation. Thus, in spite of the fact that distortion of labor income tax has been effectively eliminated, it is still optimal to tax capital income.

Fixed asset level We fix the financial savings level to eliminate the inter-temporal distortions arising from capital income tax. Column 5 of Table 4 reports result of this counterfactual case. It indicates that optimal labor income tax should be zero with capital income tax rising from baseline optimal tax economy of 29.64% to 30.64% as all government expenditure is financed by capital income taxation. The implication is that making savings for old age an exogenous will lead to an overestimation of capital income taxes and understating the need for labor income tax that maximizes social welfare.

Table 4: Experiments

		Labor-Distortion			
Indicator	Baseline	Fixed labor	Fixed education	Both	Fixed Savings
Column	**1**	**2**	**3**	**4**	**5**
Capital income tax rate (%)	29.64	16.25	23.43	21.05	30.64
Labor income tax rate(%)	3.00	12.00	6.00	8.00	0.00
Transition (Rich) (%)	50.17	51.33	50.17	50.17	55.07
Transition (Poor) (%)	17.1	16.83	17.12	17.12	17.38
Skilled Share (%)	25.55	25.70	25.57	25.57	27.89
K/Y	2.113	2.0827	2.3053	2.2592	2.0681
Output per worker	3.3768	3.3715	3.4408	3.4199	3.4719
Skill premium	1.8865	1.7726	1.9087	1.8635	1.8393
Welfare Gain (CEV) (%)	0.37771	0.9242	0.8573	0.8563	3.0415

7.2 Optimal taxation with redistribution

The presence of incomplete market (or non-borrowing constraints) and agents heterogeneity make a case for introduction of redistributive policies. We therefore find optimal income tax of a redistributive government. The results is shown in Table 5.

With the incentive of reducing inequality, a redistributive government taxes labor income and capital income at high rates to generate more revenue. We find that the optimal redistributive optimal capital and labor income tax rates are 40% and 10% respectively. The welfare improvement from benchmark model where there is no redistribution to the optimal

redistributive tax economy is equivalent to 0.81 per cent increase in lifetime consumption as measured by consumption-equivalent variation (CEV).

The increase in labor income tax to finance lump sum transfer creates intra-temporal distortion by pushing workers into informal sector. It also generates inter-temporal efficiency concern of reducing the benefit from education. Without capital income tax, agents increase their investment in financial savings and reduce investment in human capital. To alleviate this labor tax distortion, government increase capital income tax to mitigate the negative effect of labor tax on human capital investment. Jacobs and Bovenberg (2010) also finds the need for capital income tax for mitigating purposes when labor income tax affect human capital investment.

Aggregate effects With redistribution, government generates higher revenues through labor income tax. This creates inter-temporal distortion on investment in human capital by discouraging parents from educational investment as net benefit from being skilled reduces. Parents rather save more for future consumption instead of investing in child education. The increase in savings leads to a higher capital output ratio and output per worker which rises by 0.3222 and 0.0389 respectively when compared to the baseline without redistribution. To alleviate this distortion on human capital investment, government raises tax rate on capital income. Despite the ameliorating effect of capital income tax, skilled population reduce by 1.01 percentage point from the baseline optimal equilibrium without redistribution.

Table 5: Optimal Redistribution Policy

Indicator	Benchmark	Baseline(No R)	Optimal R Tax	Change (Col3-Col2)
Column	**1**	**2**	**3**	**4**
Capital income tax rate (%)	21	29.64	40	10.36
Labor income tax rate(%)	10	3.00	10.00	7
Transition (Rich) (%)	50.1	50.17	48.46	-1.31
Transition (Poor) (%)	16.64	17.1	16.76	-0.34
Skilled Share (%)	25.0	25.55	24.54	-1.01
K/Y	2.2152	2.113	2.4352	0.3222
Output per worker	3.3453	3.3768	3.4157	0.0389
Skill premium	1.7961	1.884165	1.9141	0.0276
Transfer per worker	-	-	0.1141	0.1141
Welfare Gain (CEV) (%)	-	0.3777	0.8152	0.4375

The result indicates that in economies with high inequalities and liquidity constrained

agents, redistributing income from high to low and from old adults to young adults increases capital output ratio. However, redistribution exerts negative impact on human capital investment on the rich households thereby reducing the skilled labor force in the economy.

7.2.1 Formal-informal labor supply, education and savings tax Distortions

By introducing lump sum transfer that redistributes income from the rich to the poor, we conduct our experiment to check the effect of distortions on government optimal fiscal policy. From Table 6, we fix formal labor supply, education investment and assets to the steady state equilibrium levels as found in the baseline policy.

Table 6: Experiments-Redistribution

		Labor-Distortion			
Indicator	Baseline	Fixed labor	Fixed education	Both	Fixed Assets
Column	**1**	**2**	**3**	**4**	**5**
Capital income tax rate (%)	40	46	75.5	60	52
Labor income tax rate(%)	10	60	21	83	0.0
Transition (Rich) (%)	48.46	44.97	48.45	48.45	56.61
Transition (Poor) (%)	16.76	14.89	16.73	16.73	17.73
Skilled Share (%)	24.54	21.18	24.51	24.51	29.01
K/Y	2.4352	3.7286	5.115	3.9764	2.3052
Output per worker	3.4157	3.5915	4.3152	3.7709	3.64
Skill premium	1.9141	1.6643	2.5861	1.0999	1.9249
Transfer per worker	0.1141	0.6403	0.8376	1.0702	0.1723

Fixed sector labor supply As seen in the column 2 of Table 6, when labor supply is fixed, a redistributive government optimizes by taxing labor income at a high rate of 60% against 46% of capital income tax. Without formal-informal labor supply distortion, the government uses high labor income tax to raise adequate revenues for redistribution. The capital income tax increased by 6 percentage points to mitigate the effect of labor income tax on educational investment (see Jacobs and Bovenberg, 2010; Peterman, 2016).

Fixed Education Investment Column 3 of Table 6 reports counterfactual case when the inter-temporal distortion of labor income is completely shut down leaving only intra-

temporal labor movement between formal and informal sectors and inter-temporal savings effect. It is shown that both capital income tax and labor income tax increased from 40% and 10% to 75.5% and 21% respectively. Compared to results in column 2, it is clear that intra-temporal distortion resulting from labor income tax in the presence of informal sector is more profound, as it limits the use of labor income taxation for redistribution.

Fixed asset level Fixing asset level to baseline optimal savings level for both skilled and unskilled agents eliminates savings distortions of capital income tax. The result indicates a higher optimal capital income tax of 52% from 40% in the baseline optimal equilibrium. The optimal labor tax is 0% as all government expenditure and redistribution are financed by capital income tax as indicated in column 5 of Table 6.

7.3 Role of the Informal Sector

The optimal tax mix is significantly affected by the presence of non-taxable informal sector. As explained by Correira (1996), the inability to optimally tax a given factor has effect on the long run capital income tax (see also Judd, 1999). We therefore assume that the government has the capacity to tax income on factors that are employed in the informal sector in addition to the formal sector in order to disentangle the effect of non-taxable informal sector on optimal tax design.

Non-redistributive As reported in column 2 of Table 7, we find that when the informal sector can be taxed and government expenditure is low, optimal income tax features zero labor tax and positive capital income tax. Specifically, if government eliminate tax inefficiency by developing capacity to tax the informal sector, zero labor income tax is optimal. Government therefore raises revenues from capital income tax which amounted to 33.42%. This suggest that the inter-temporal distortion of the labor tax is quite stronger than that of capital income tax, hence zero labor income tax.[38] .

Moreover, since financial savings are for life cycle purposes hence less elastic to tax rates compared to human capital, it is more efficient to tax capital than labor. In pure life cycle model, Browning and Burbidge (1990), finds that with low level of desired government revenue, capital income should be taxed, possibly with consumption(labor) income subsidy if household faces liquidity constraint in the beginning cycle when human capital investment are undertaken. A model with altruistic parents undertaken human capital

[38]tax rates are restricted to be non-negative. In principle negative labor tax(labor subsidy) with high capital tax could be optimal (see Browning and Burbidge,1990)

Table 7: Taxation of Informal sector

	Non-Redistribitive		Redistributive	
Indicator	Baseline	Taxing Informal	Baseline	Taxing Informal
Column	**1**	**2**	**3**	**4**
Capital income tax rate (%)	29.64	33.42	40	68
Labor income tax rate(%)	3.00	0.00	10	71
Transition (Rich) (%)	50.17	50.12	48.46	43.71
Transition (Poor) (%)	17.1	17.27	16.76	15.26
Skilled Share (%)	25.55	25.72	24.54	21.33
K/Y	2.113	2.0810	2.4352	4.4529
Output per worker	3.3768	3.3896	3.4157	4.0378
Skill premium	1.8865	1.9326	1.9141	3.4173

investment as discuss in this paper also features the positive capital income tax with zero labor income tax.

Redistributive Column 4 of Table 7 reveals that taxing the informal sector leads to optimal capital and labor income tax of 68 per cent and 71 per cent respectively for redistributive government. Since there is no constraint on tax collection from the informal sector, the redistributive government increases both tax rate to finance higher transfer and reduce consumption inequality.

8 Sensitivity Analysis

8.1 Role of the formal-informal wage premium

In this section, we investigate the effect of exogenous formal-informal wage premium on formal sector labor supply. First, we analyze a situation where at equilibrium, formal after tax wage is exactly equal with informal wage. Thus, workers allocate their labor resources between formal and informal sector until after tax wages are equal in the two sectors.

Non-redistributive Column 2 of Table 8 reports optimal tax mix for government that raises tax revenues for exogenous government expenditure. Optimal capital income tax and labor income tax are 8.06% and 12% respectively compared to 29.64% and 3% in the baseline economy. Thus, by eliminating after tax wage gap between formal and informal

Table 8: Formal-Informal wage gap

	Non-Redistributive			Redistributive	
Indicator	Baseline($k_i{}^b$)	No Formal constraint ($k_i = 1$)	$k_i = 1.5 * k_i{}^b$	Baseline	No Formal constraint
Column	**1**	**2**	**3**	**4**	**5**
Capital income tax rate (%)	29.64	8.06	43.44	40	60
Labor income tax rate(%)	3.00	12.0	0.00	10	41
Transition (Rich) (%)	50.17	58.20	45.33	48.46	47.14
Transition (Poor) (%)	17.1	18.56	15.63	16.76	17.08
Skilled Share (%)	25.55	30.75	22.23	24.54	24.42
K/Y	2.113	1.4431	2.982	2.4352	3.0029
Output per worker	3.3768	3.4409	3.2414	3.4157	3.8269
Skill premium	1.8865	1.7958	1.9555	1.9141	1.8381

sectors, labor resources will be more efficiently allocated. This indeed is similar to elimination of distortion created by employing less than optimal labor services in the formal sector and more than optimal labor in formal sector. Hence, the government can tax labor income higher and reduce capital income tax in order to maximize social welfare.

Column 1 to column 3 suggest that when the formal-informal wage gap is high, capital income tax is preferable to labor income tax. The implied employment in the formal sector resulting from high formal-informal wage gap is relatively low and therefore labor income tax distortion created by pushing more workers into the informal sector is substantial. Hence optimal taxation requires high capital income tax which has no intra-temporal distortion and a lower or no labor income tax.

Redistributive The optimal redistributive tax is 41 per cent for labor income and 60 per cent for capital income. The elimination of constraint allows more labor resources to be employed in the more productive formal sector. This allows the government to increase labor income tax for the purpose of redistribution and a higher capital income tax to alleviate the burden of labor income tax on human capital investment.

8.2 Changes skilled Population

Since the efficiency of human capital investment is likely to affect educational investment and optimal income tax mix, we provide a sensitivity check to see the effect of changes in elasticity of human capital production. This helps to optimal tax implications of high proportion of skilled workers in the economy. We perform this task by using different values of the elasticity of human capital investment efficiency measured by η. A lower η implies that education investment technology is efficient in producing skilled labor.

Table 9: skilled Population

Indicator	$\eta = 0.15$	$\eta = 0.2$	$\eta = 0.25$
Capital income tax rate (%)	29.18	29.64	33.38
Labor income tax rate(%)	0.00	3.00	5.1
Transition (Rich) (%)	54.11	50.17	47.65
Transition (Poor) (%)	22.24	17.1	12.51
Skilled Share (%)	32.64	25.55	19.29
K/Y	1.7859	2.113	2.5234
Output per worker	3.4839	3.3768	3.2169
Skill premium	1.5390	1.8865	2.3907

The results are reported in Table 9. It is revealed that optimal capital and labor income tax should be 29.18% and 0.0% respectively. Thus, when education technology is very efficient in transforming children to skilled adults, only capital income should be used in order not to discourage investment in human capital. However, when η is increased from 0.2 to 0.25, both optimal labor income tax and capital income tax rises. η determines the welfare gains of capital taxes serving as subsidy on human capital investment through the production of more skilled workers. Thus, when human capital technology is very efficient then the gain through lower η outweighs the welfare cost of distorting the financial savings, hence less labor tax see Jacobs and Bovenberg, 2010).

9 Conclusion

We developed a three-period life cycle model of overlapping generations with human capital investment, financial savings, and formal-informal labor supply which are relevant for developing economies with large informal sector. Parents heterogeneity in terms of skill type and income are transmitted to their children through education investment and luck leading to income inequality that emerges endogenously.

The model is calibrated to Ghanaian economy to quantitatively characterized optimal capital and labor income tax. We find that optimal capital income tax is higher than what is

currently applied in Ghana with lower optimal labor income tax. We showed that neglecting informal sector in tax design would lead to understating optimal capital income tax and overstating labor income tax and the difference is quantitatively significant. The parental decision on educational or human capital investment of their offspring is also important for sharing the burden of income tax optimally. Non-consideration of education decision of parents leads to an underestimation of capital income tax by about 6.21 percentage points and overestimation of labor income tax by about 3 percentage points.

Since many developing countries have not been able to overcome the restraints on revenue generation efforts imposed by the presence of large informal sector, redirecting taxation to capital income could generate higher revenues with less distortion compared to the labor income tax. Moreover, since part of income generated from the informal sector is ultimately put into savings, capital income tax could be used to indirectly tax the informal sector labor income.

Our result suggest an important trade-off between promotion of skill acquisition and physical capital accumulation in developing countries. Thus, the effort of reducing inequality and informality while stimulating human capital and physical capital accumulation is a major challenge for developing countries. Policies to reduce inequality through higher taxation of labor income may lead to higher informality and fewer skilled population.

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